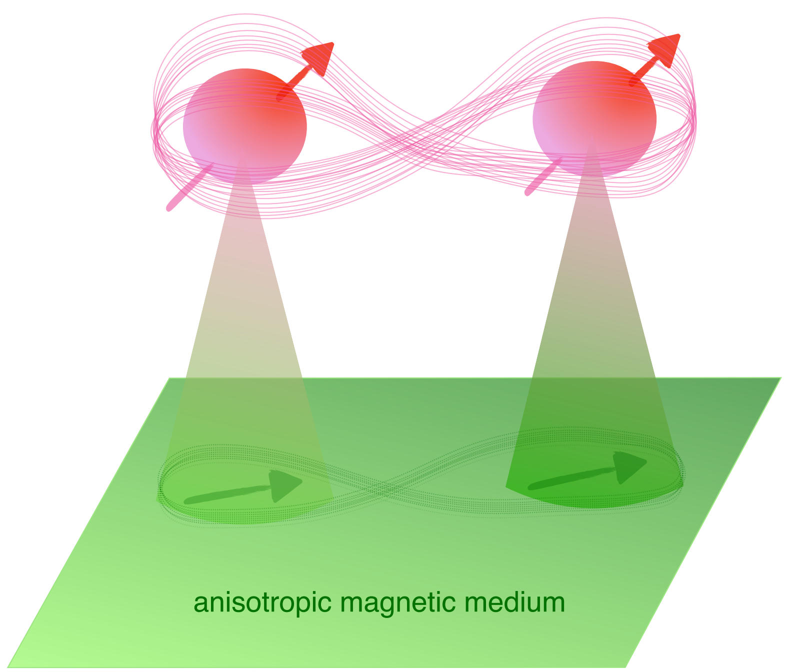
Squeezing spin entanglement out of magnons

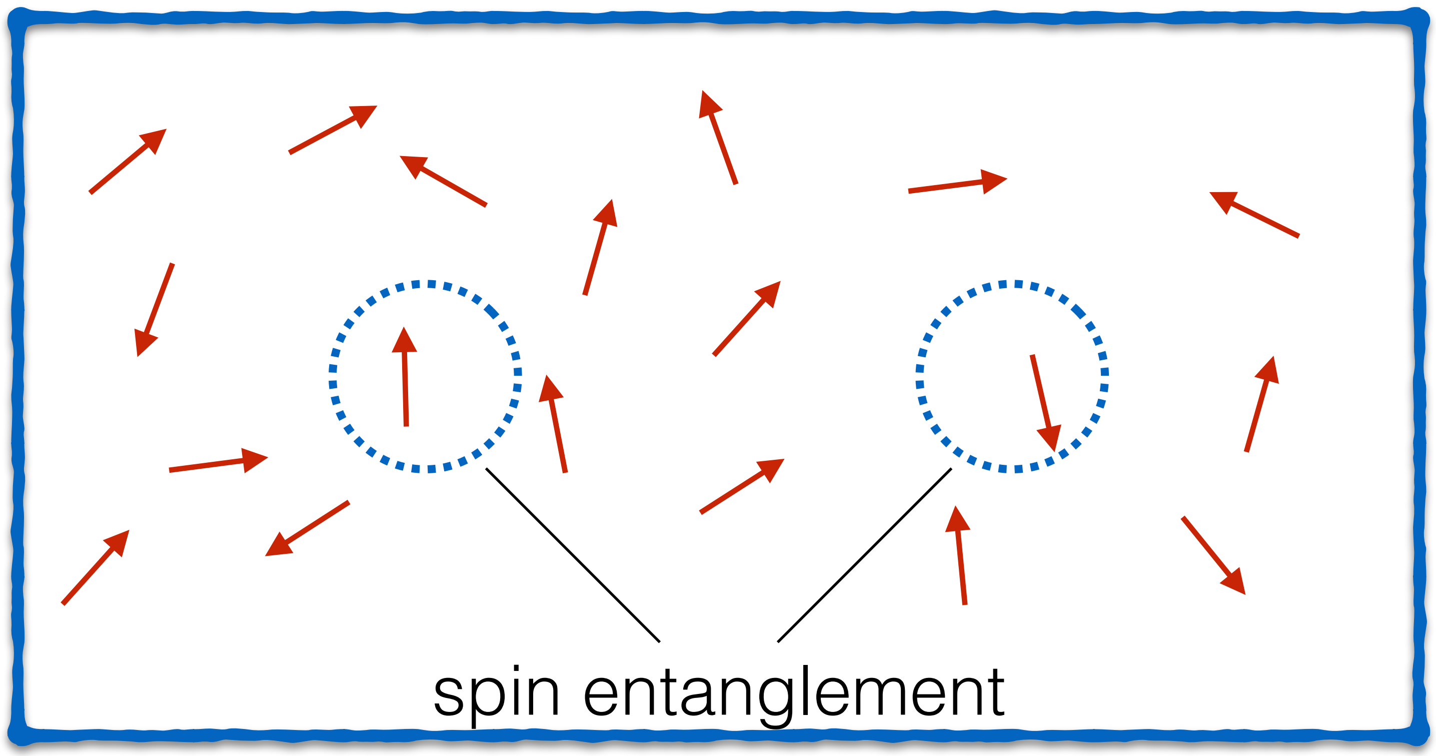
ordered magnetic media as a quantum-information resource



Yaroslav Tserkovnyak

w/ Se Kwon Kim and Ji Zou

Motivation



- How quantum are magnetic materials, in and out of equilibrium, from the quantum-information perspective?
- New experimental probes of materials (optical, electrical)?
- *Can dynamic instabilities (microwave or thermoelectrically induced), including magnon BEC, be useful in tuning quantum entanglement?*
- *The role of dissipation (non-Hermitian dynamics: exceptional points etc.)*

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Editors' Suggestion

Tuning entanglement by squeezing magnons in anisotropic magnets

Ji Zou,[1] Se Kwon Kim,[1,2] and Yaroslav Tserkovnyak[1]

[1] *Department of Physics and Astronomy, University of California, Los Angeles, California 90095, USA*

[2] *Department of Physics and Astronomy, University of Missouri, Columbia, Missouri 65211, USA*

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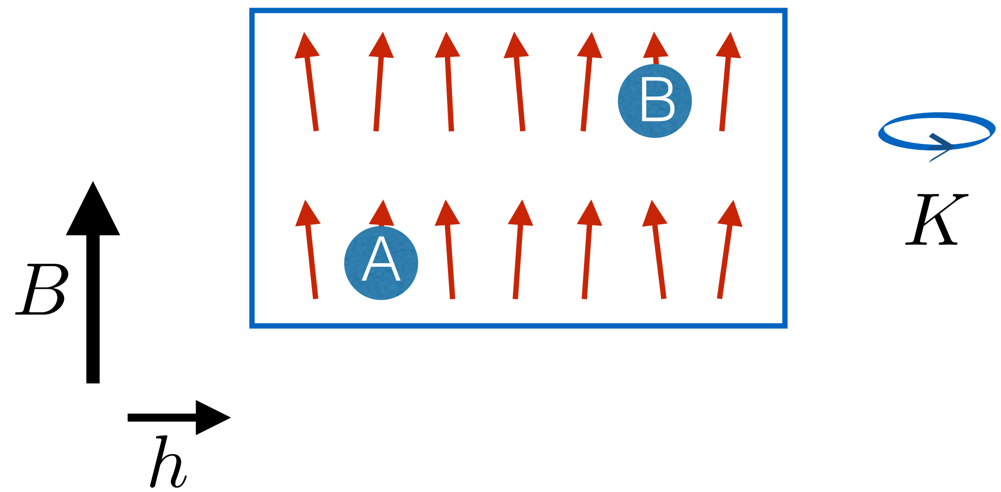
We theoretically study the entanglement between two arbitrary spins in a magnetic material where magnons naturally form a general squeezed coherent state in the presence of an applied magnetic field and axial anisotropies. Employing concurrence as a measure of entanglement, we demonstrate that spins are generally entangled in thermodynamic equilibrium, with the amount of entanglement controlled by the external fields and anisotropies. As a result, the magnetic medium can serve as a resource to store and process quantum information. We furthermore show that the entanglement can jump discontinuously when decreasing the transverse magnetic field. This tunable entanglement can be potentially used as an efficient switch in quantum-information processing tasks.

Model



In the magnon expansion:

$$H = \sum_{\mathbf{k}} (2J\mathbf{k}^2 + B) a^\dagger_{\mathbf{k}} a_{\mathbf{k}} + \frac{K}{2} \sum_{\mathbf{k}} (a^\dagger_{-\mathbf{k}} a^\dagger_{\mathbf{k}} + a_{\mathbf{k}} a_{-\mathbf{k}}) - \frac{h\sqrt{N_0}}{2} (a^\dagger + a) + \cdots$$

exchange+Zeeman　　anisotropy　　transverse field

Let us start at zero temperature and a small macrospin nanomagnet

Coherent vs squeezed magnons

$$H = B a^\dagger a + \frac{K}{2}\underbrace{(a^\dagger a^\dagger + aa)}_{\text{squeezing}} - \frac{h\sqrt{N_0}}{2}\underbrace{(a^\dagger + a)}_{\text{displacement}} + \cdots$$

resonance frequency: $\omega = \sqrt{B^2 - K^2}$ magnon number: $N = \langle a^\dagger a \rangle$

N_0 is the total number of spin-1/2 sites

- Transverse field h alone would produce a *coherent* state with magnon #:

$$N_c = N_0 (h/2B)^2$$

- A sole anisotropy K would result in a *squeezed* state with magnon #:

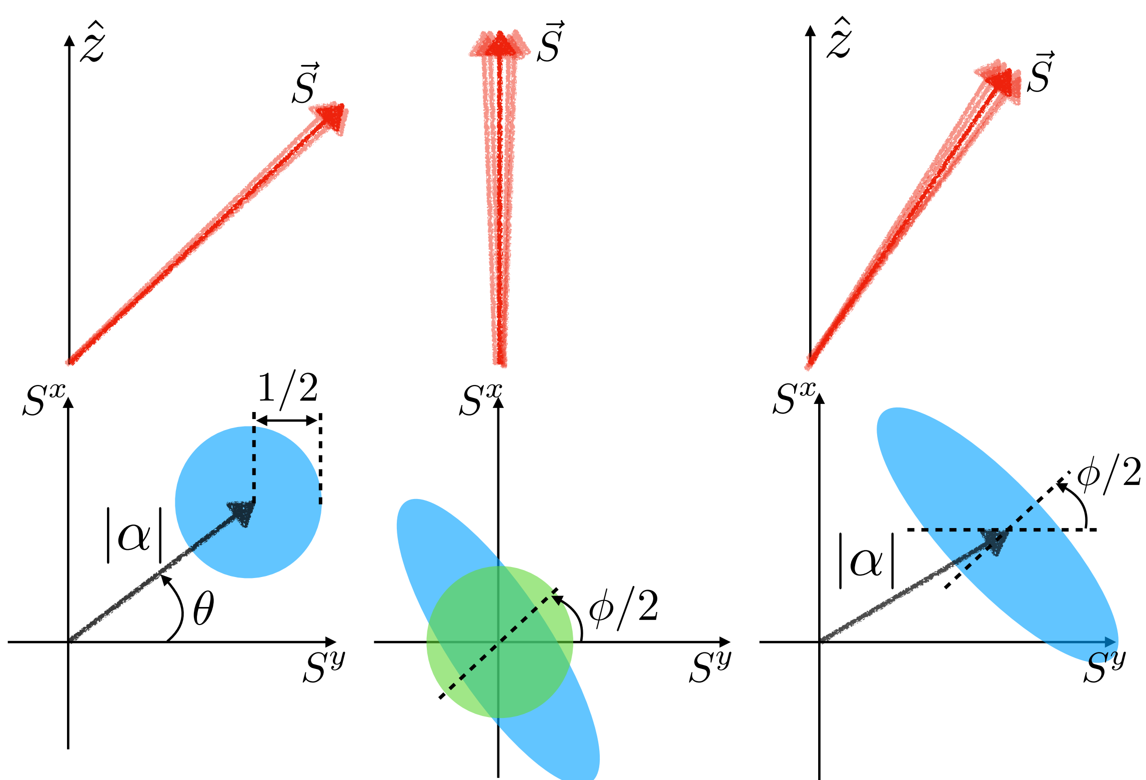
$$N_s = (B - \omega)/2\omega$$

Squeezed coherent state

$$|\psi\rangle = D(\alpha)S(\zeta)|0\rangle$$

displacement operator: $D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$

squeezing operator: $S(\zeta) = e^{[\zeta^* a^2 - \zeta (a^\dagger)^2]/2}$



(a) coherent state (b) squeezed vacuum state (c) squeezed coherent state

$$\alpha = |\alpha| e^{i\theta} \propto h$$

$$\zeta = |\zeta| e^{i\phi} \propto K$$

the ellipse aspect ratio is $e^{2|\zeta|}$

The coherent state is merely a tilted magnet

Entanglement in the squeezed coherent state

- While displacing (tilting) a magnetic state is clearly inconsequential to entanglement, squeezing does produce a finite *entanglement of formation*

- The associated two-spin *concurrence* is:

$$\mathcal{C} = \frac{2}{N_0} \frac{\sqrt{N_s}}{\sqrt{N_s+1}+\sqrt{N_s}} \to \frac{1}{N_0} \quad \text{@ large } N_s$$

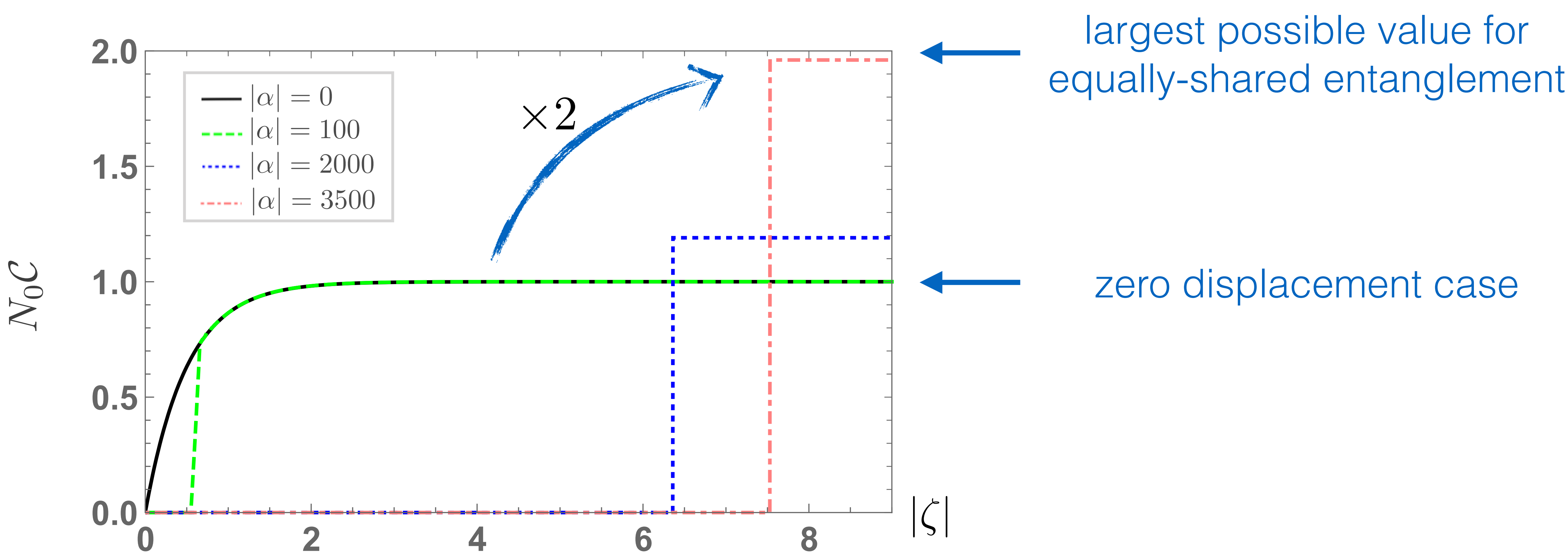
- Most interestingly, squeezing a displaced state exhibits a jump-like behavior in concurrence (with doubling of the above limiting concurrence) at the threshold determined by:

$$N_s = N_c^2/2N_0$$

The total magnon #: $N = \underbrace{N_c}_{|\alpha|^2} + \underbrace{N_s}_{\sinh^2|\zeta|}$

Entanglement jump by squeezing

$$|\psi\rangle = D(\alpha)S(\zeta)|0\rangle$$



The doubling of the concurrence by displacing the squeezed state is due to accessing the odd-parity magnon states, which are absent in the squeezed-vacuum state

Fock state

- For an N-magnon Fock state, we find the concurrence:

$$\mathcal{C} = 2\frac{N}{N_0}\left(1 - \sqrt{1 - 1/N}\,\right)$$

- It reduces from the maximal allowed value by 1/2, when N increases

- Agrees with the well known result for the *Dicke state*, which is a generalization of a maximally-entangled *Bell state*, obtained by flipping N out of N_0 spin-1/2 states in a symmetric fashion

$N = 1$ is the maximally-entangled state, which saturates the upper bound of $2/N_0$ for concurrence (when the entanglement is symmetrically shared):

$$|{\downarrow\uparrow}\cdots{\uparrow}\rangle + |{\uparrow\downarrow}\cdots{\uparrow}\rangle + |{\uparrow\uparrow}\cdots{\downarrow}\rangle$$

Formalism

two-qubit concurrence: $\mathcal{C}(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$

square roots of eigenvalues,
in decreasing order, of

$$\rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$$

density matrix: $$\rho_{ij} = \begin{bmatrix} \langle k_i^+ k_j^+ \rangle & \langle \sigma_i^- k_j^+ \rangle & \langle k_i^+ \sigma_j^- \rangle & \langle \sigma_i^- \sigma_j^- \rangle \\ \langle \sigma_i^+ k_j^+ \rangle & \langle k_i^- k_j^+ \rangle & \langle \sigma_i^+ \sigma_j^- \rangle & \langle k_i^- \sigma_j^- \rangle \\ \langle k_i^+ \sigma_j^+ \rangle & \langle \sigma_i^- \sigma_j^+ \rangle & \langle k_i^+ k_j^- \rangle & \langle \sigma_i^- k_j^- \rangle \\ \langle \sigma_i^+ \sigma_j^+ \rangle & \langle k_i^- \sigma_j^+ \rangle & \langle \sigma_i^+ k_j^- \rangle & \langle k_i^- k_j^- \rangle \end{bmatrix}$$

in the basis $\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$

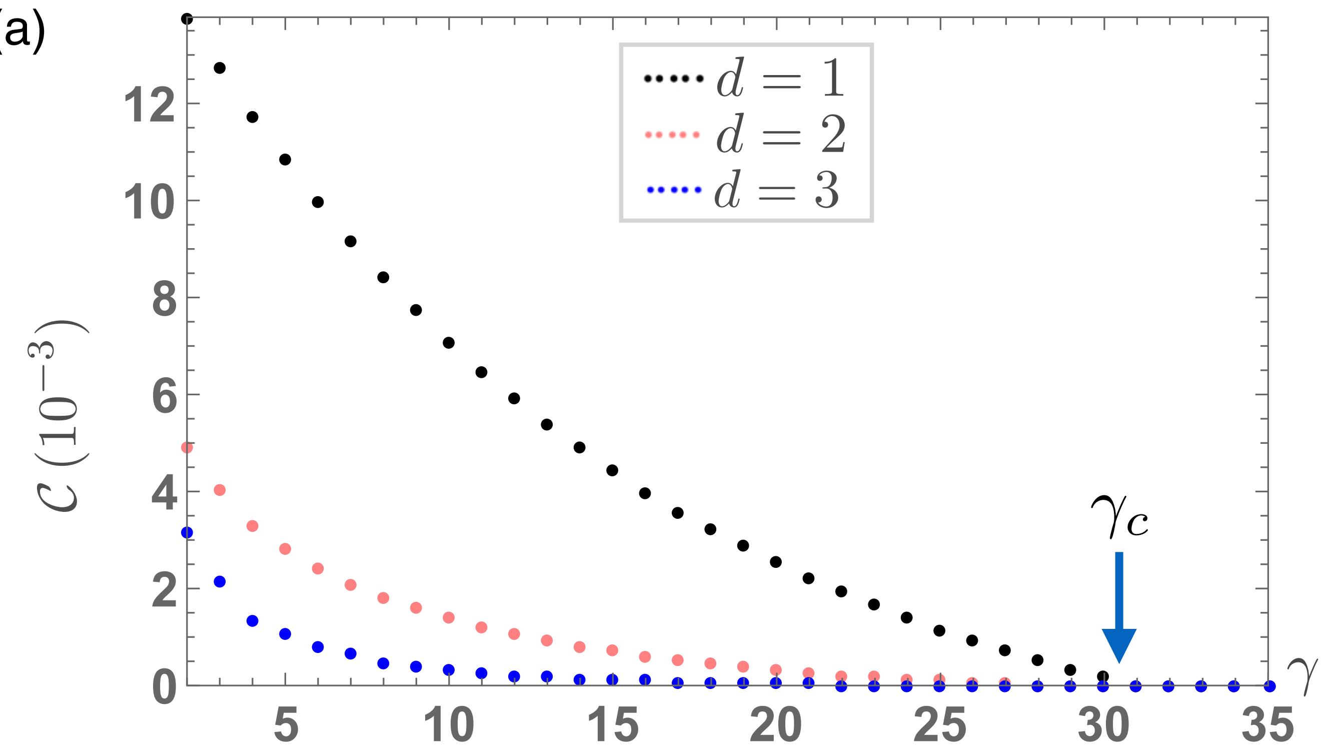
is evaluated by expressing the spin operators

$$k^\pm = (1 \pm \sigma_z)/2\,, \quad \sigma^\pm = (\sigma^x \pm i\sigma^y)/2$$

in terms of the magnon operators and calculating the resultant correlators

Thermodynamic limit

(a)



concurrence as a function of distance (in units of lattice spacing)

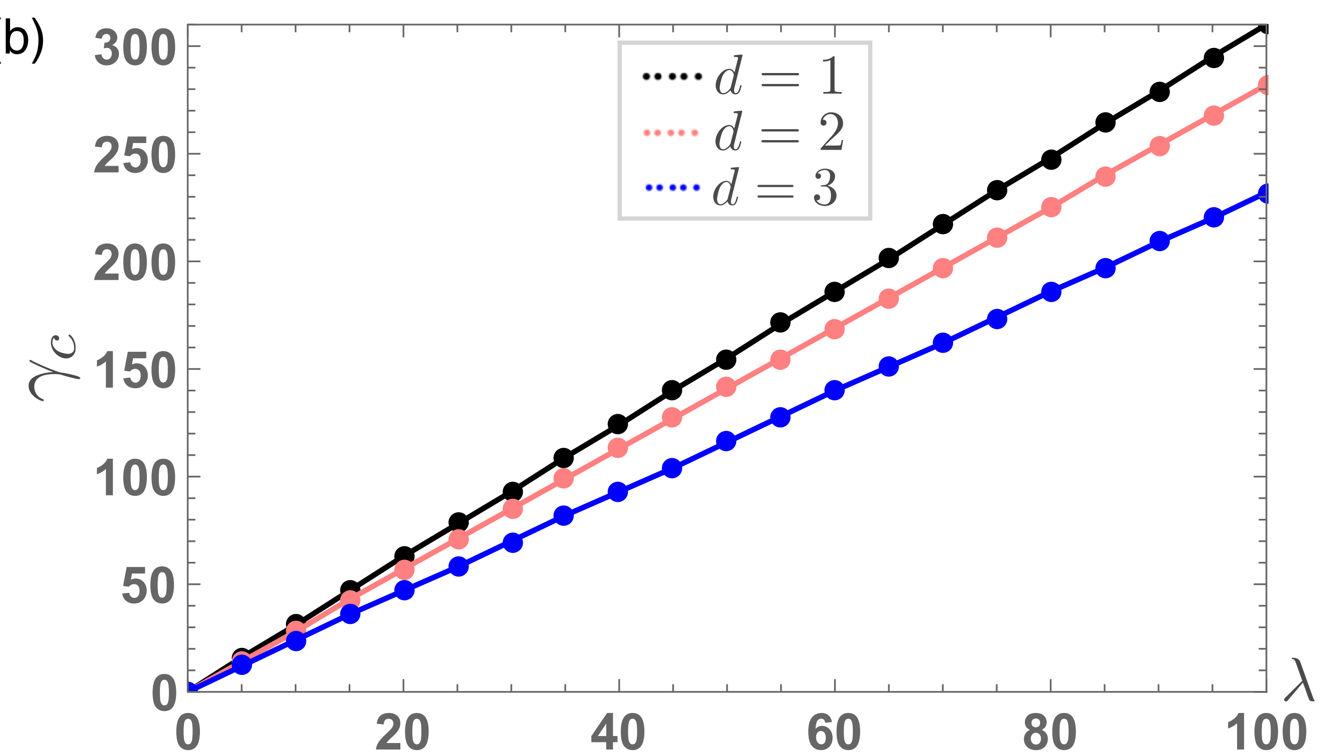
$$K/B = 1/2$$

anisotropy

$$\lambda = \sqrt{J/B} = 10$$

exchange length

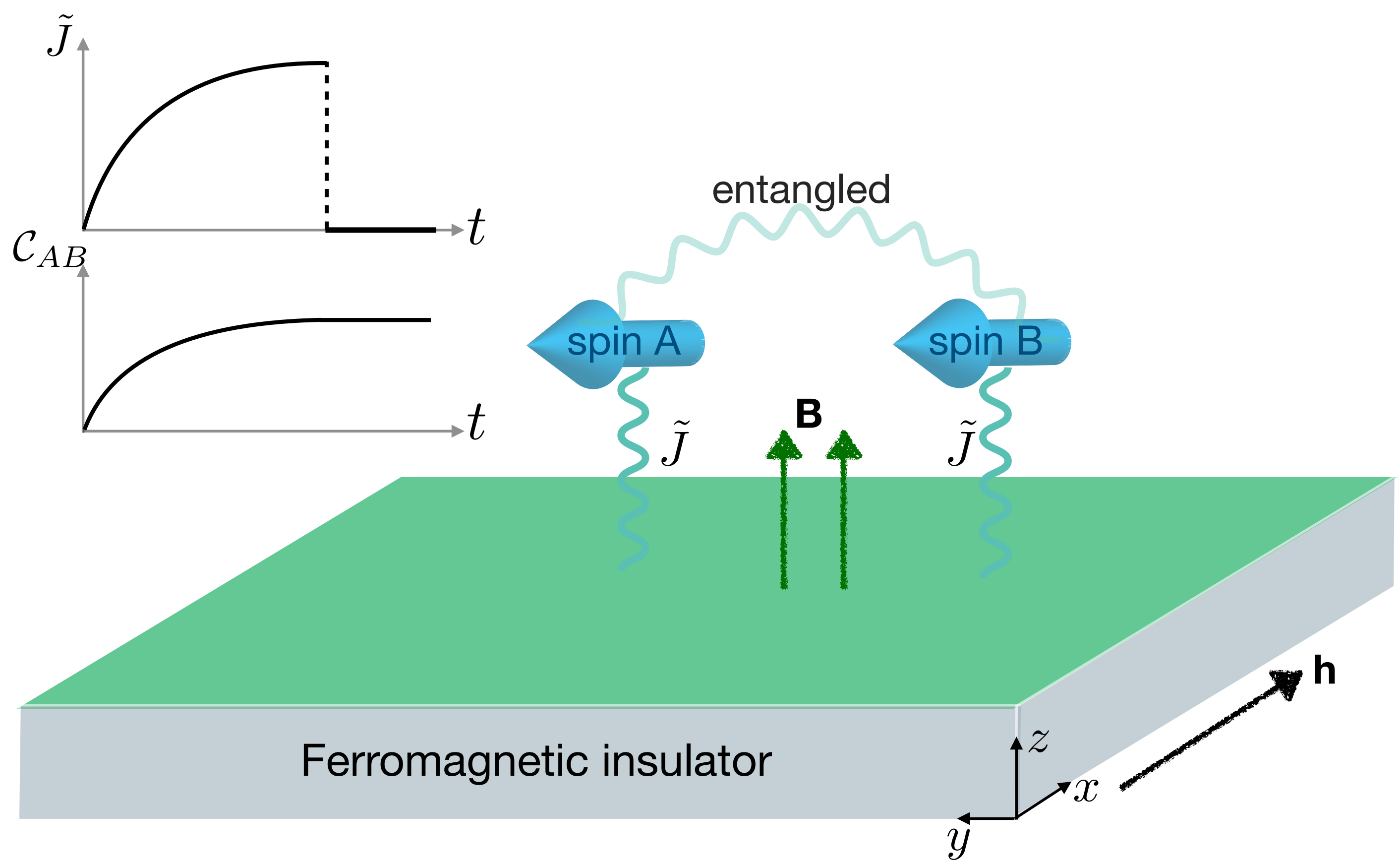
(b)



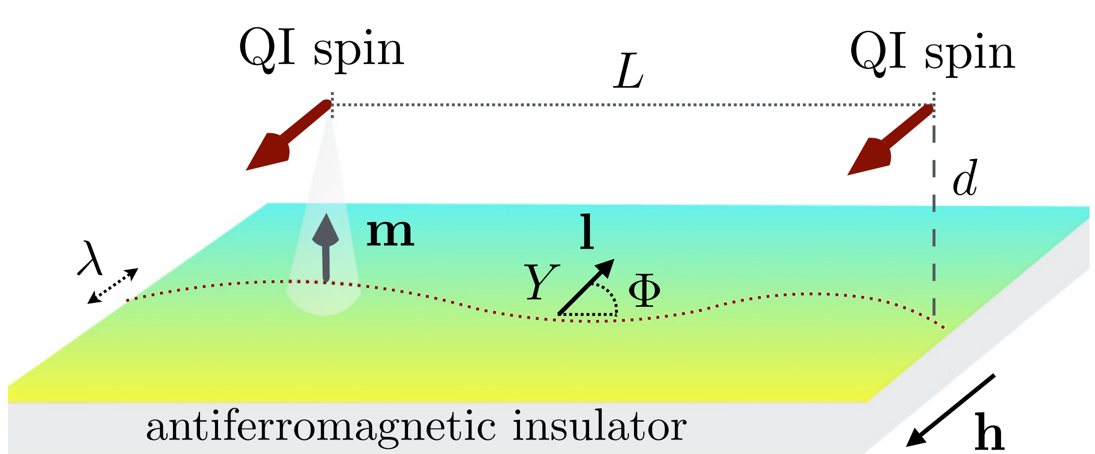
critical distance γ_c as a function of the exchange length

quantum correlations decrease with dimensionality d

Imprinting the entanglement by proximity



Spins A, B could be proximal NV centers or other imbedded quantum impurities or quantum dots, whose spectral properties and/or effective interaction $\tilde{J}$ with the magnetic substrate could be dynamically tuned



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Entangling distant spin qubits via a magnetic domain wall

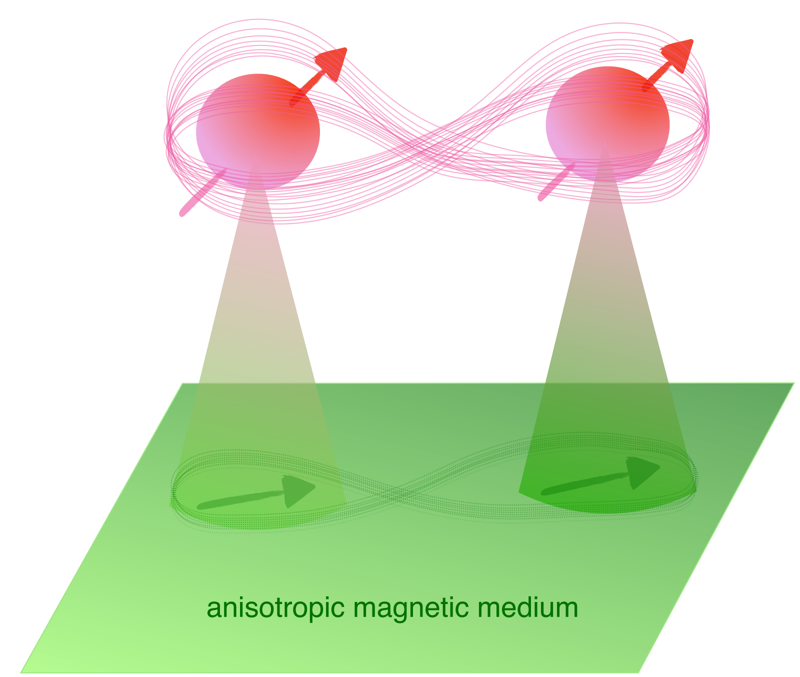
B. Flebus and Y. Tserkovnyak

Some take-home messages

- Intrinsic entanglement of magnetic materials, even simple ferromagnets, has rich structure, which opens new ways to think of the fundamental (quantum) material properties

- From the quantum-information perspective, it is intriguing to think of magnets as natural entanglement reservoirs, which could be distilled for practical tasks like quantum teleportation

- By imprinting the bulk entanglement onto integrated quantum impurities/dots, we can tap into established optical and electrical probes to measure spin dynamics and spin-spin correlations



Thank you!