

# Half-integer charge injection by a Josephson junction without excess noise

- *Majorana edge modes*
- *edge vortex injection, braiding, fusion*
- *fractional charge transfer*
- *the noise paradox*

arXiv:2005.08655, with

Fabian Hassler, Aurélien Grabsch, Michał Pacholski,  
Dima Oriekhov, Omrie Ovdad, Inanc Adagideli

# Summer of 1999: superconducting Majoranas debut

PHYSICAL REVIEW B

VOLUME 61, NUMBER 14

1 APRIL 2000-II

## **Quasiparticle localization in superconductors with spin-orbit scattering**

T. Senthil and Matthew P. A. Fisher

*Institute for Theoretical Physics, University of California, Santa Barbara, California 93106-4030*

(Received 21 June 1999)

[arXiv:cond-mat/9906290](https://arxiv.org/abs/cond-mat/9906290)

## **A. Majorana metal**

[arXiv:cond-mat/9906453](https://arxiv.org/abs/cond-mat/9906453)

PHYSICAL REVIEW B

VOLUME 61, NUMBER 15

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## **Paired states of fermions in two dimensions with breaking of parity and time-reversal symmetries and the fractional quantum Hall effect**

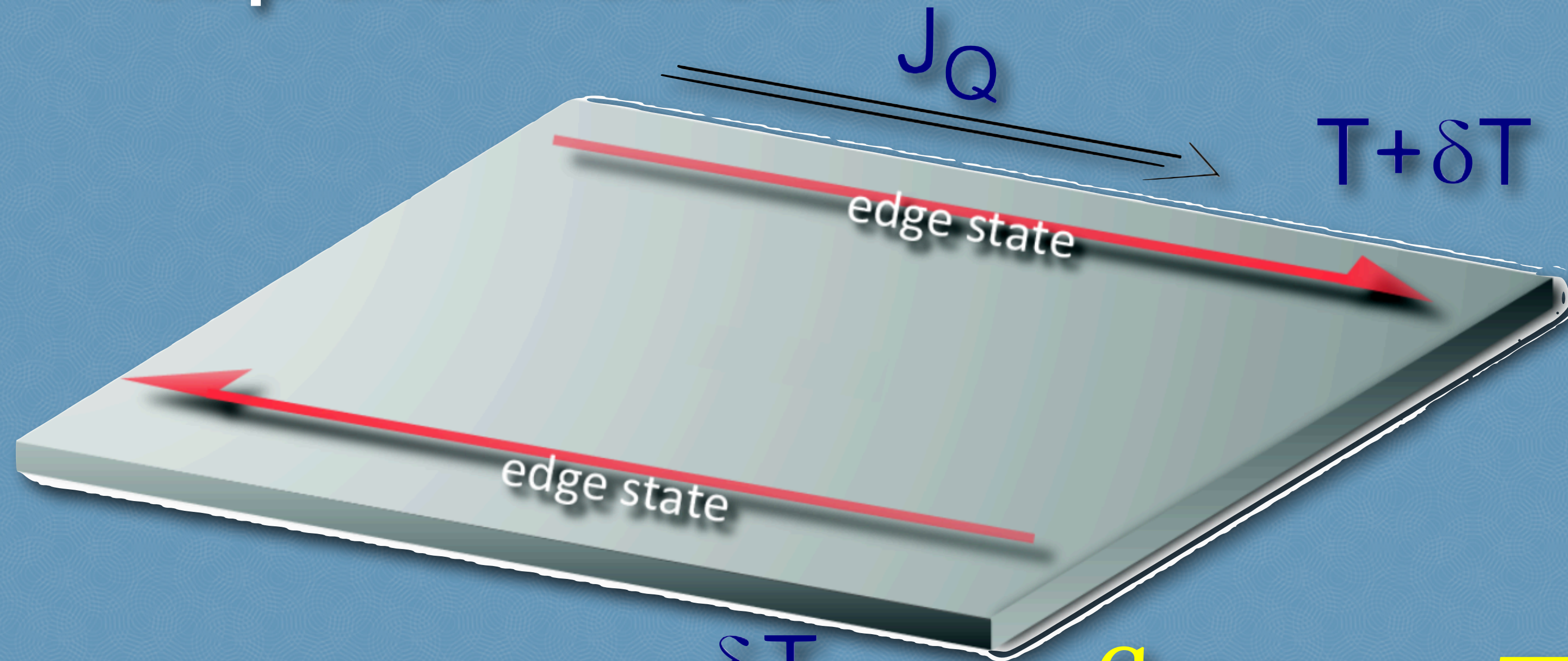
N. Read and Dmitry Green

*Departments of Physics and Applied Physics, Yale University, P.O. Box 208120, New Haven, Connecticut 06520-8120*

(Received 30 June 1999)

## **B. Majorana fermions, edges and vortices**

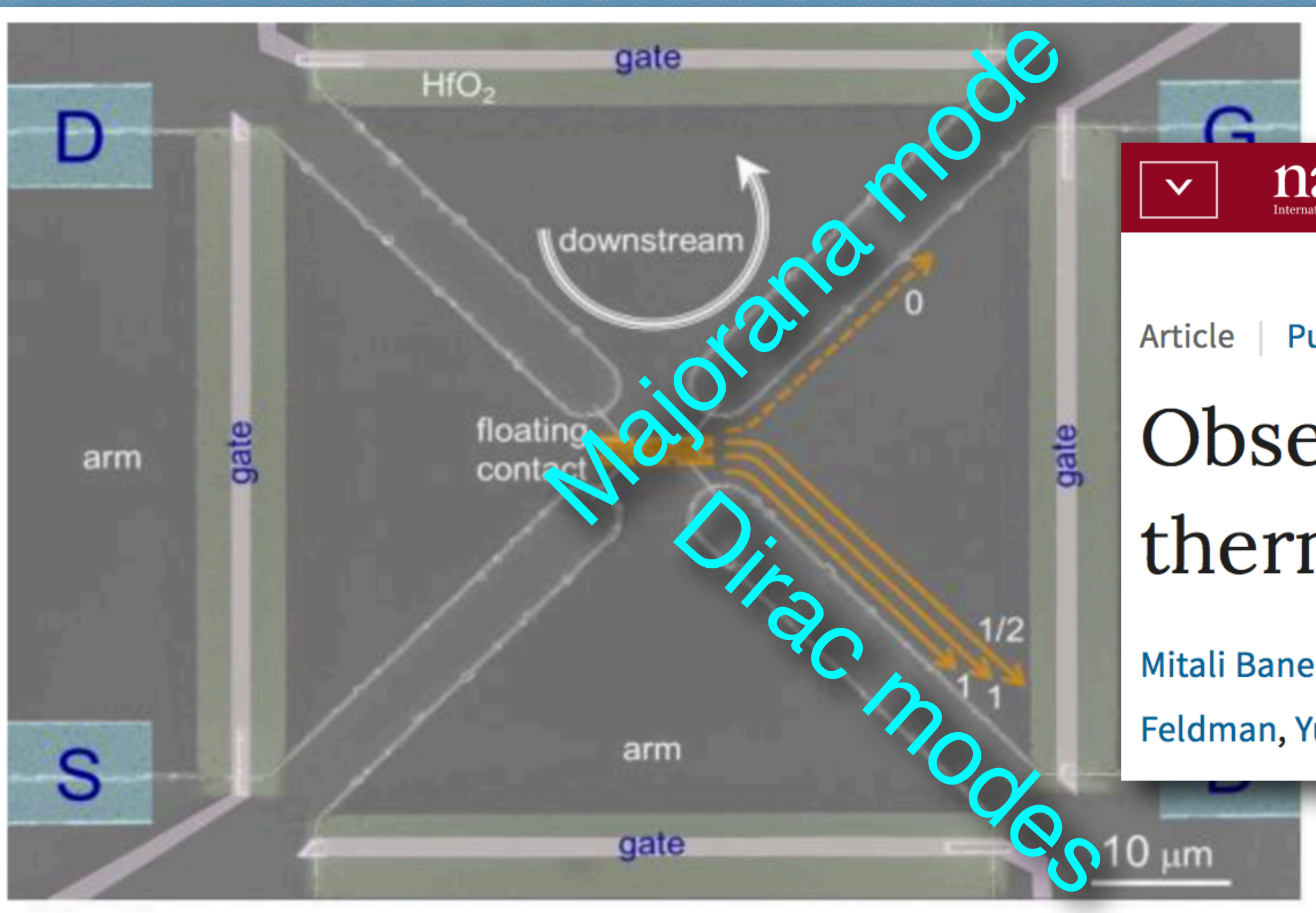
# Majorana edge modes of a topological superconductor



“superconducting QHE”  $\delta T$   $g_{\text{Majorana}} = \pi^2 k_B^2 T / 6h$

half-integer quantised thermal conductance

2 Majoranas = 1 ordinary (Dirac) fermion,  
so this is half the “text book” value



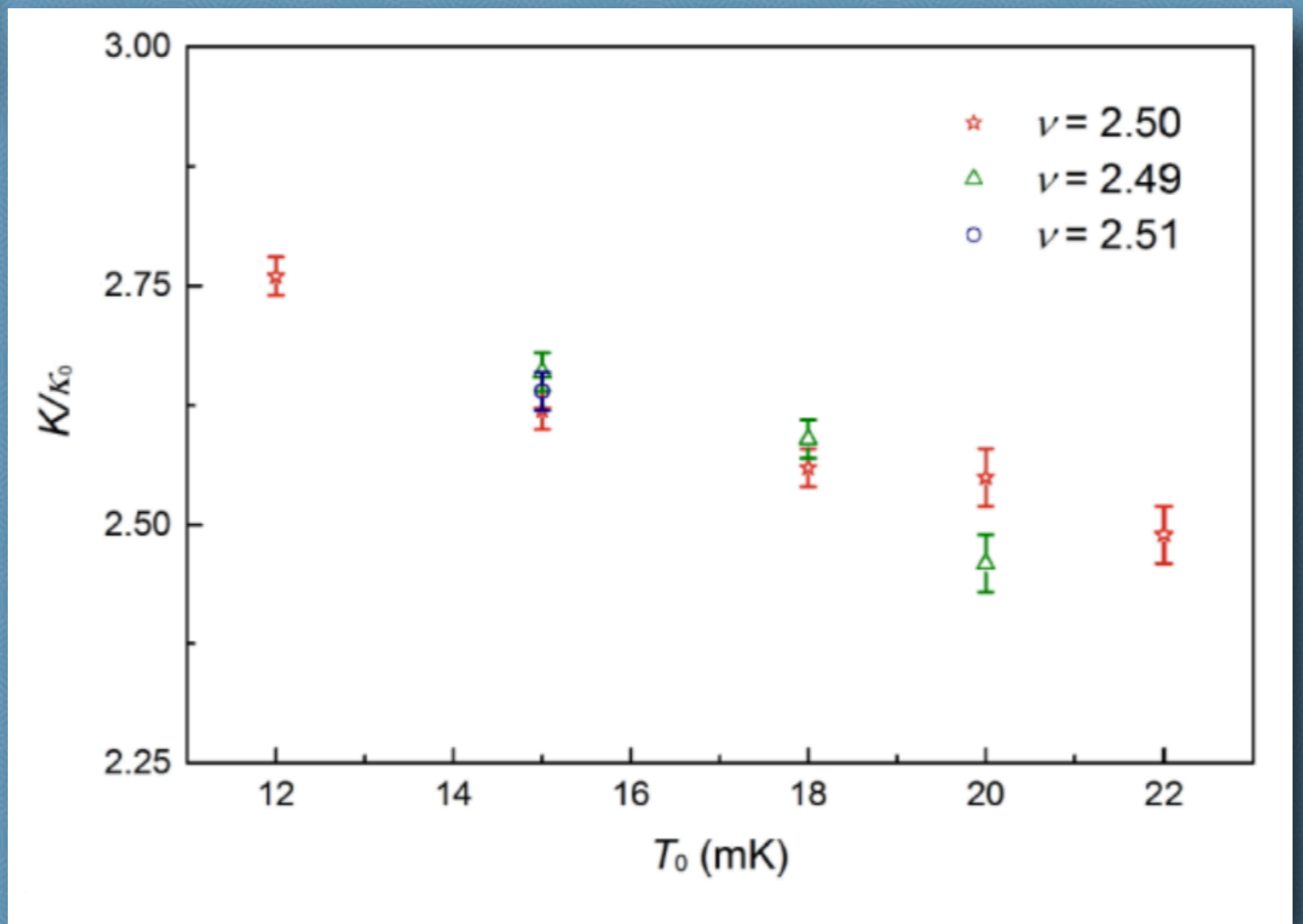
**nature**  
International journal of science

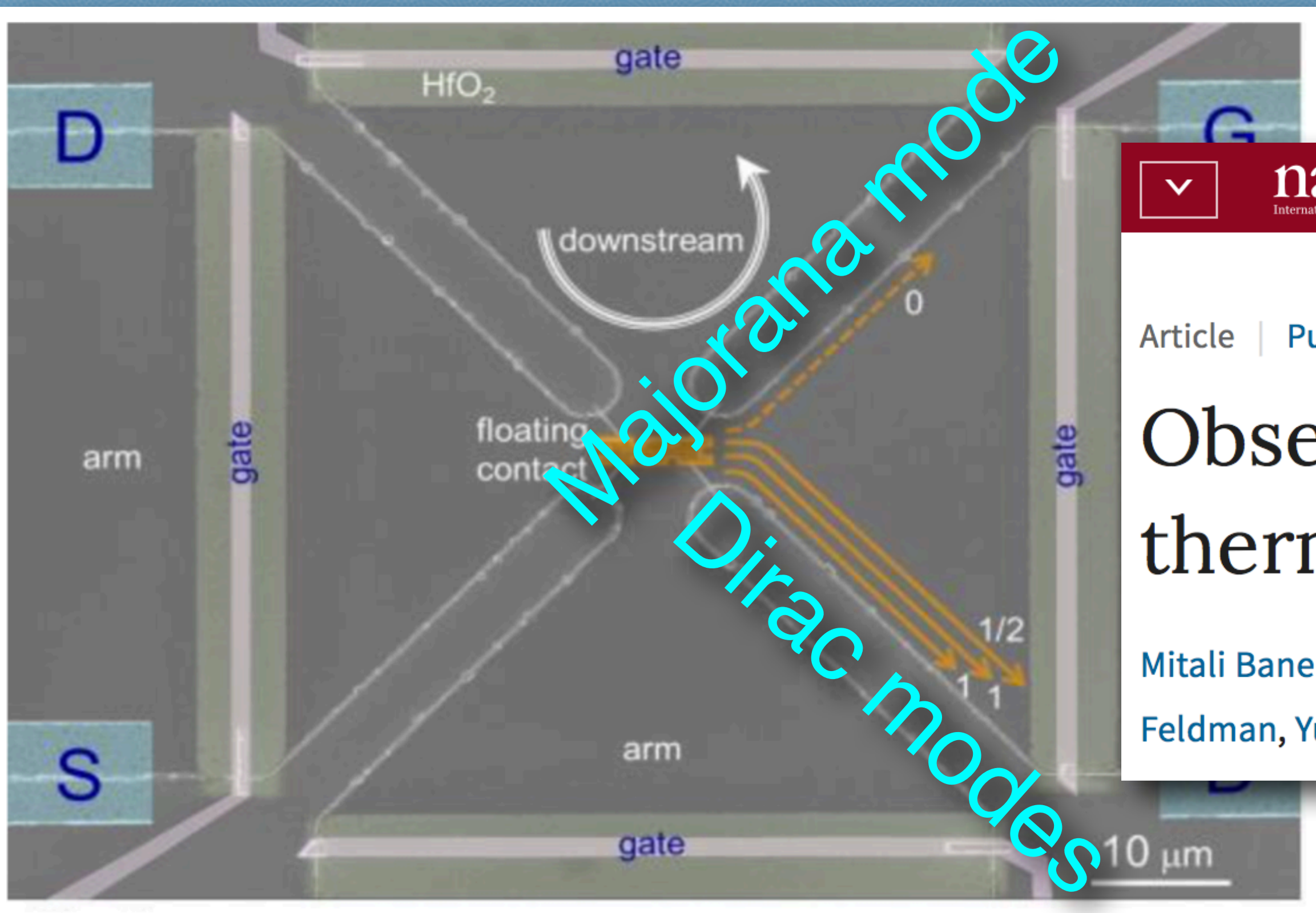
Article | Published: 04 June 2018

## Observation of half-integer thermal Hall conductance

Mitali Banerjee, Moty Heiblum , Vladimir Umansky, Dima E. Feldman, Yuval Oreg & Ady Stern

$\nu=5/2$  quantum Hall effect,  
Majorana mode predicted  
by Read & Green (2000)





**nature**  
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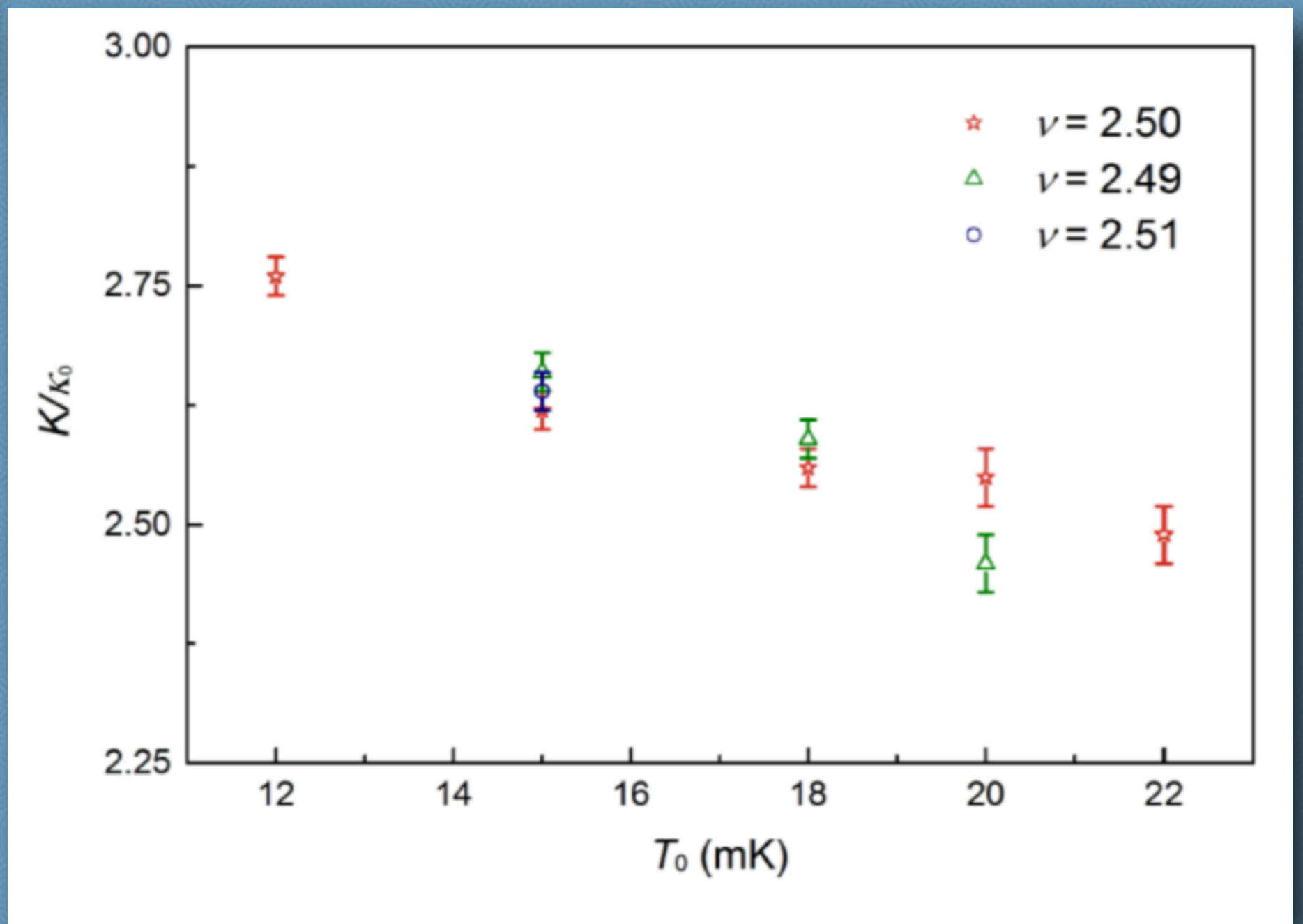
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*superconducting  
realisation?*





**'Angel particle'** which is both matter and anti-matter discovered in ...

[The Independent](#) - Jul 20, 2017

Physicists believe they have discovered a **particle** that is both matter and anti-matter, an idea that was first theorised 80 years ago. When the ...

Scientists discover "**angel particle**" that is its own antiparticle

[New Atlas](#) - Jul 20, 2017

Scientists find "**Angel Particle**" — a particle that's its own antiparticle

[ZME Science](#) - Jul 21, 2017

Quantum Leap: '**Angel Particle**' That Is Both Matter and Anti-Matter ...

[Sputnik International](#) - Jul 21, 2017

An experiment proposed by Stanford theorists finds evidence for the ...

Highly Cited - [Stanford University News](#) - Jul 20, 2017

**'Angel particle'** which is both matter and anti-matter is discovered ...

In-Depth - [Daily Mail](#) - Jul 21, 2017



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ZME Science -

Quantum Leap - What Is Both Matter and Anti-Matter ...

Space.com - Particle Physicists find evidence for the ...

ArXiv - Particle Physicists find evidence for the ...

Phys.org - Particle Physicists find evidence for the ...

Angel particle: Matter and anti-matter is discovered ...

In-Depth - Daily Mail - Jul 21, 2017



## The 'Angel Particle' is still missing

New research suggests that the discovery of a new particle — the Majorana fermion — was a false alarm, but the search for the 'angel particle' goes on.

## Majorana: The So-Called Angel Particle Was Not Discovered

UNIVERSITY PARK, Pa., Jan. 16, 2020 — A research team at Penn State, partnering with the University of Würzburg in Germany, has flagged a 2017 discovery of Majorana fermion as a false alarm.



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ZME Science -

Quantum Leap - What Is Both Matter and Anti-Matter ...

Space.com - Particle physicist finds evidence for the ...

ArXiv -

Most Cited - News - Jul 20, 2017

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*new evidence:*

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New research suggests that the discovery of a new particle — the Majorana fermion — was a false alarm, but the search for the 'angel particle' goes on.

## Spectroscopic fingerprint of chiral Majorana modes at the edge of a quantum anomalous Hall insulator/superconductor heterostructure

Junying Shen, Jian Lyu, Jason Z. Gao, Ying-Ming Xie, Chui-Zhen Chen, Chang-woo Cho, Omargeldi Atanov, Zhijie Chen, Kai Liu, Yajian J. Hu, King Yau Yip, Swee K. Goh, Qing Lin He, Lei Pan, Kang L. Wang, Kam Tuen Law, and Rolf Lortz

PNAS January 7, 2020 117 (1) 238-242; first published December 18, 2019 <https://doi.org/10.1073/pnas.1910967117>

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# NONABELIONS IN THE FRACTIONAL QUANTUM HALL EFFECT

Gregory MOORE

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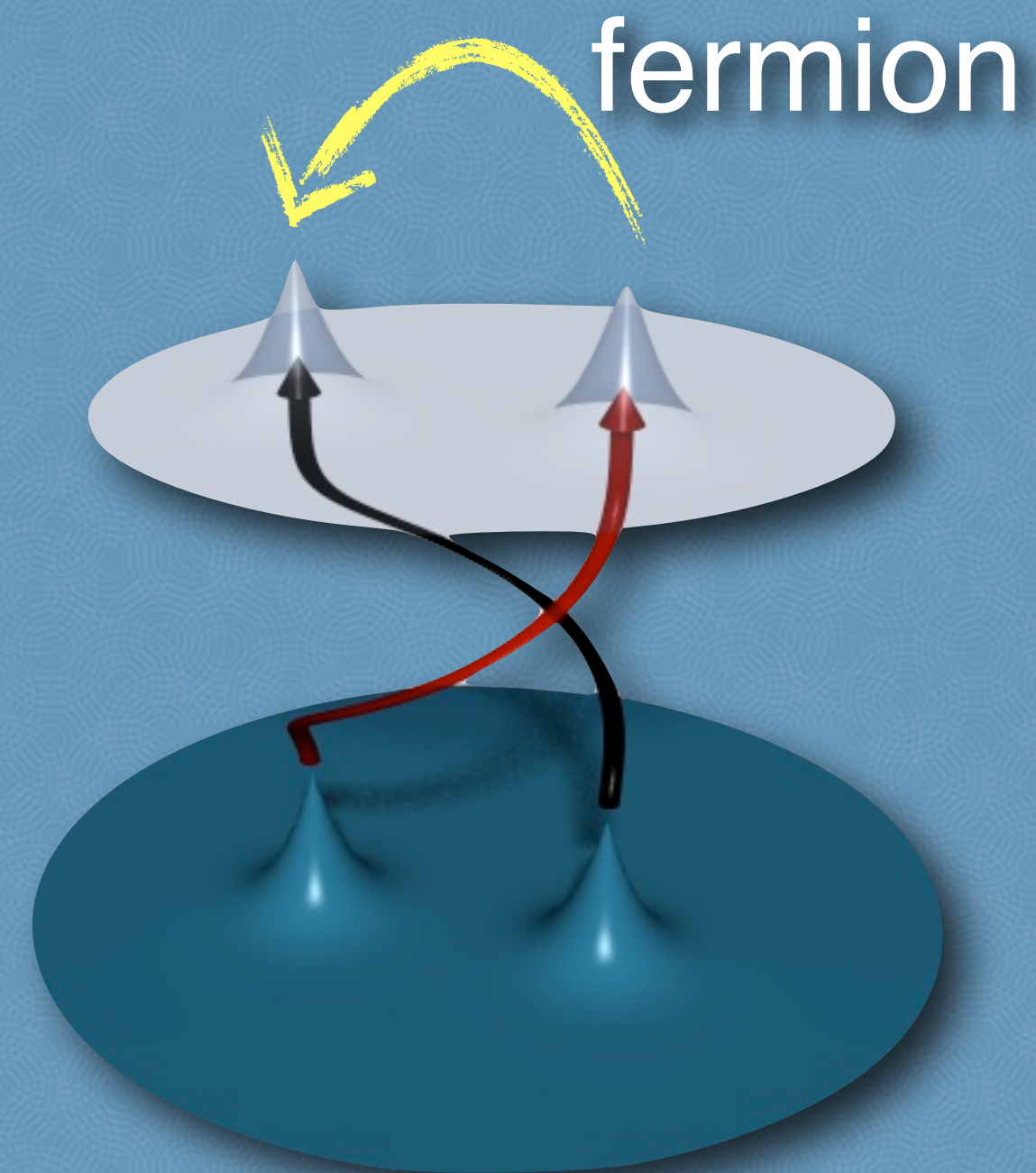
Received 31 May 1990

(Revised 5 December 1990)

Non-Abelian Statistics of Half-Quantum Vortices in  $p$ -Wave Superconductors

D. A. Ivanov

Phys. Rev. Lett. **86**, 268 – Published 8 January 2001



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*two vortices which move around each other  
exchange a fermion:*

*non-Abelian braiding → topological quantum computation*

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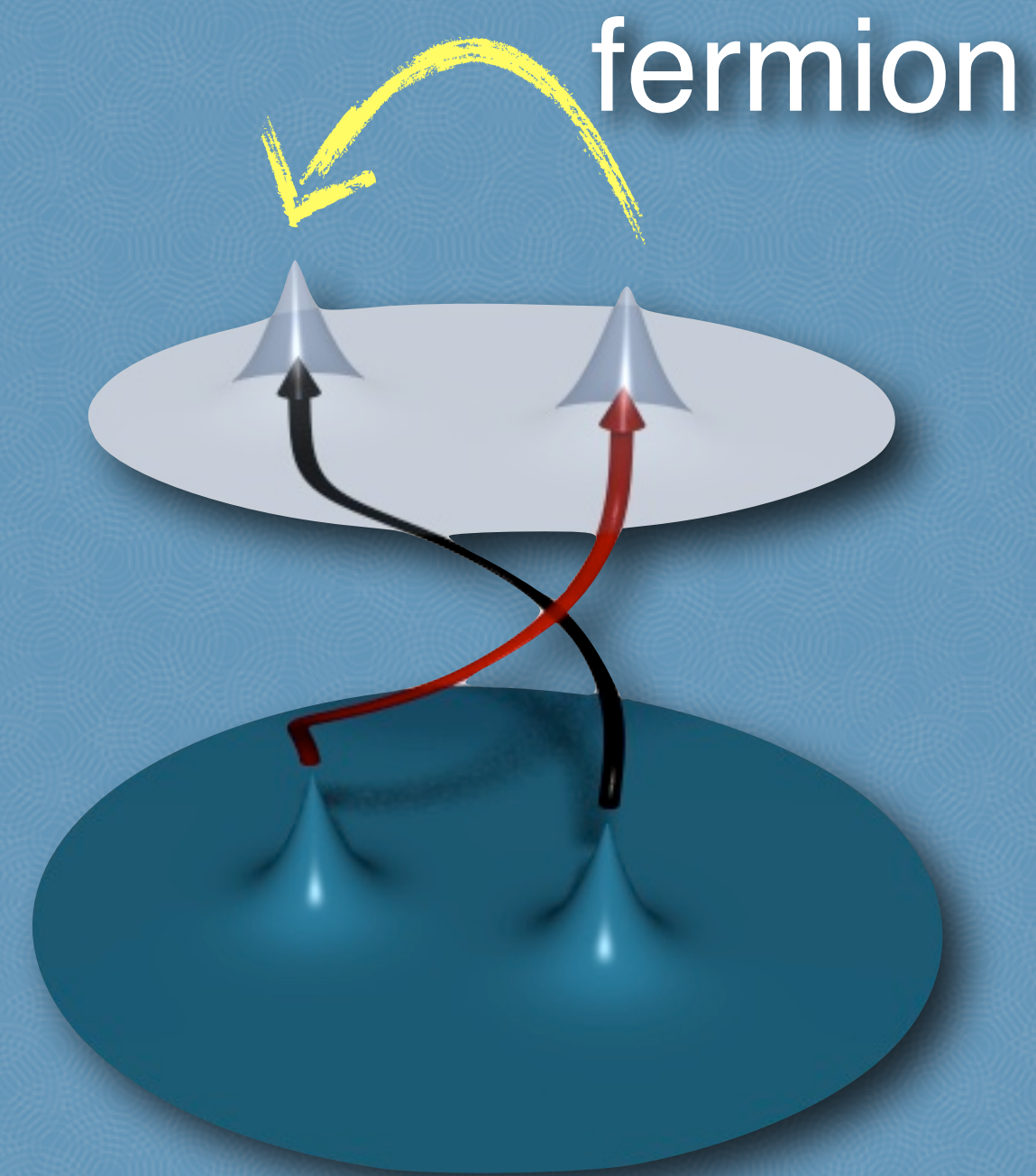
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*non-Abelian braiding → topological quantum computation*

*Zhang's question [PNAS 115, 10938 (2018)]:  
how to braid Majorana edge modes?*

# Proposal for edge vortex braiding

PRL **122**, 146803 (2019); SciPost **6**, 022 (2019); **8**, 013 (2020)

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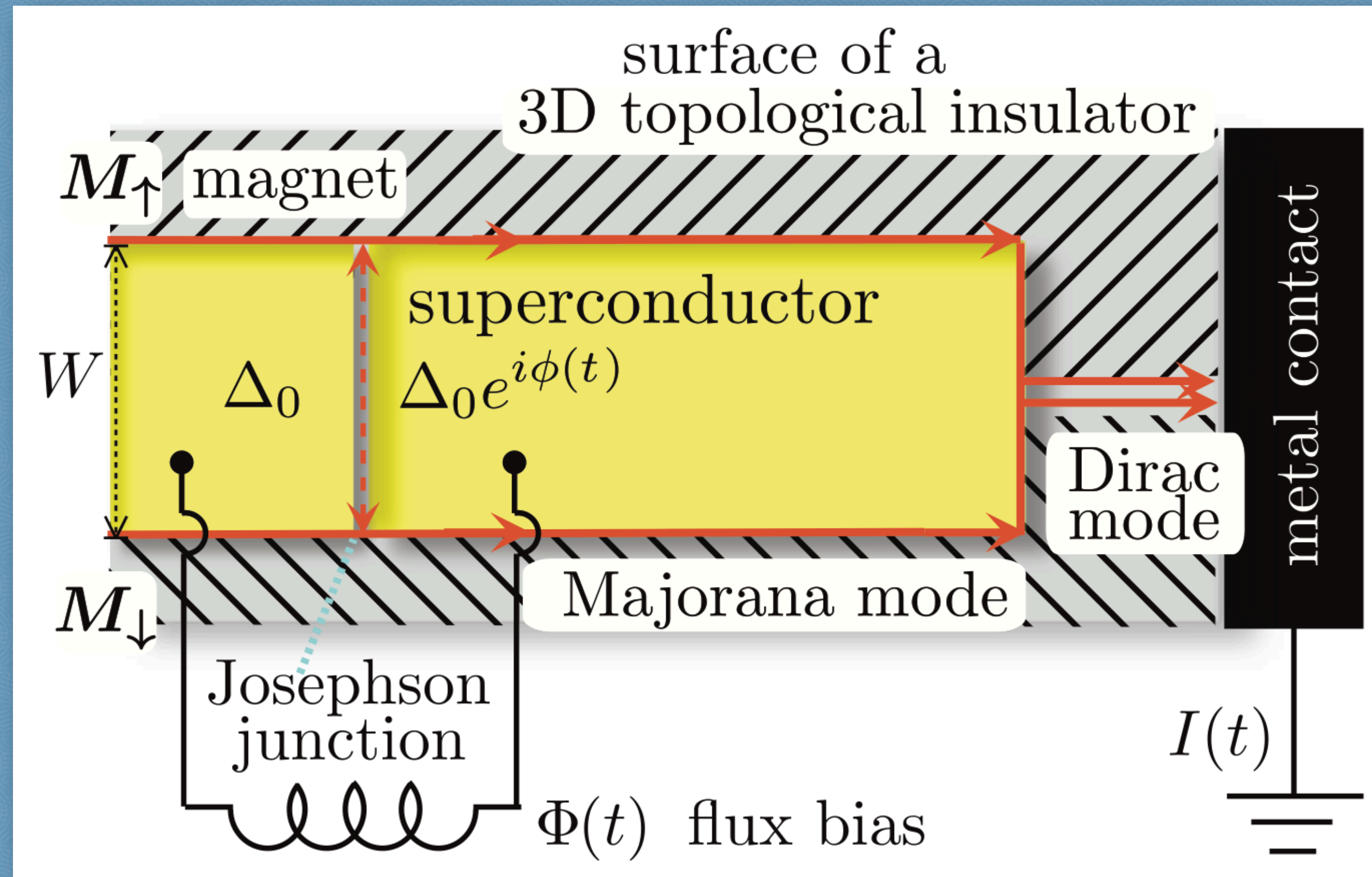
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- a  $2\pi$  phase slip in a Josephson junction injects a pair of vortices in the edge mode
- a pair of edge vortices can carry charge  $\pm e/2$
- an edge vortex can be braided with a bulk vortex, exchanging charge  $e$

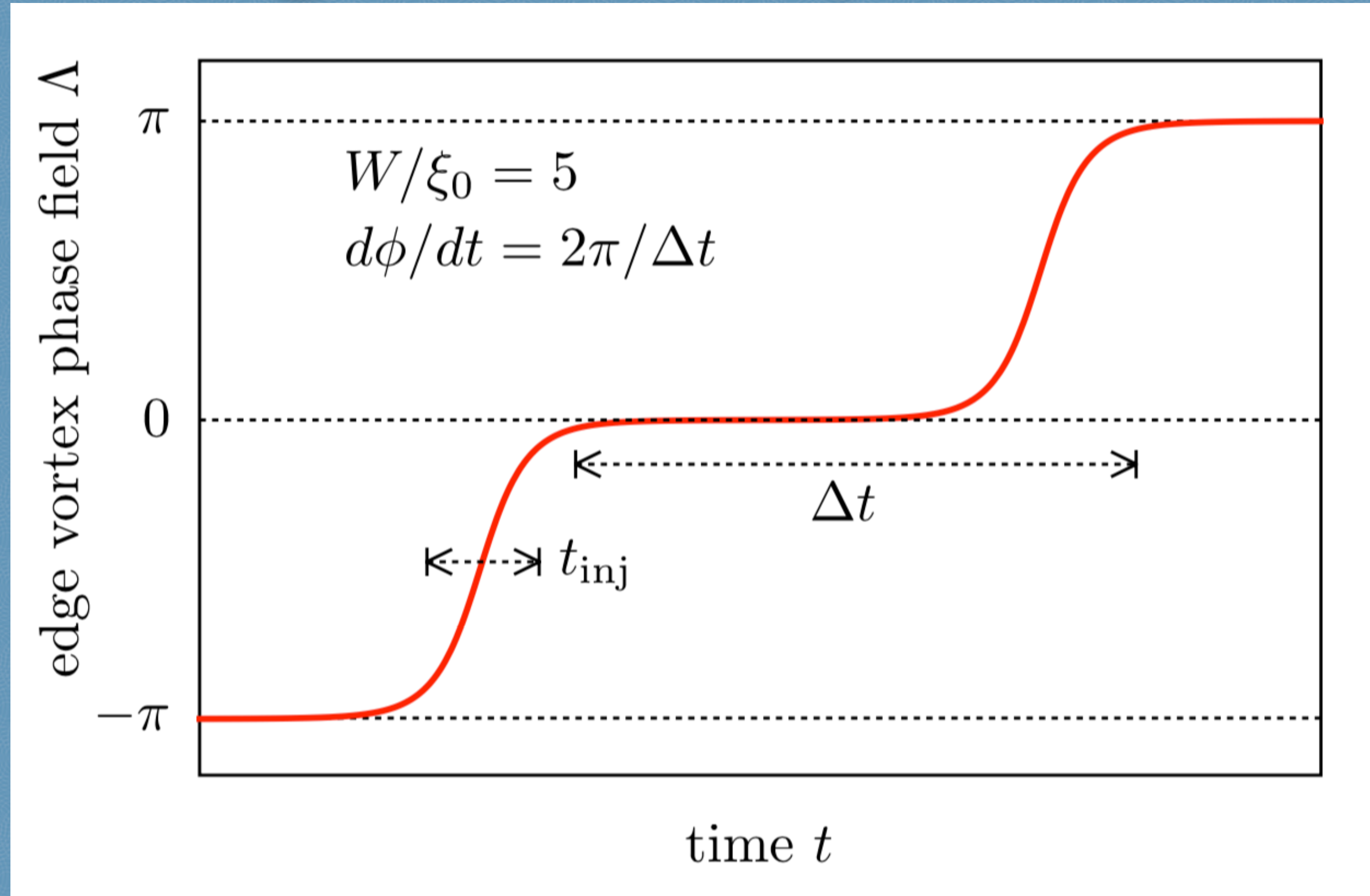
# Edge vortex primer (1)



$\hbar/2e$  flux increment advances the superconducting phase  $\varphi$  by  $2\pi$   
 $\Rightarrow \pi$ -phase domain wall for unpaired fermions ("edge vortex"),  
propagating chirally along the edge

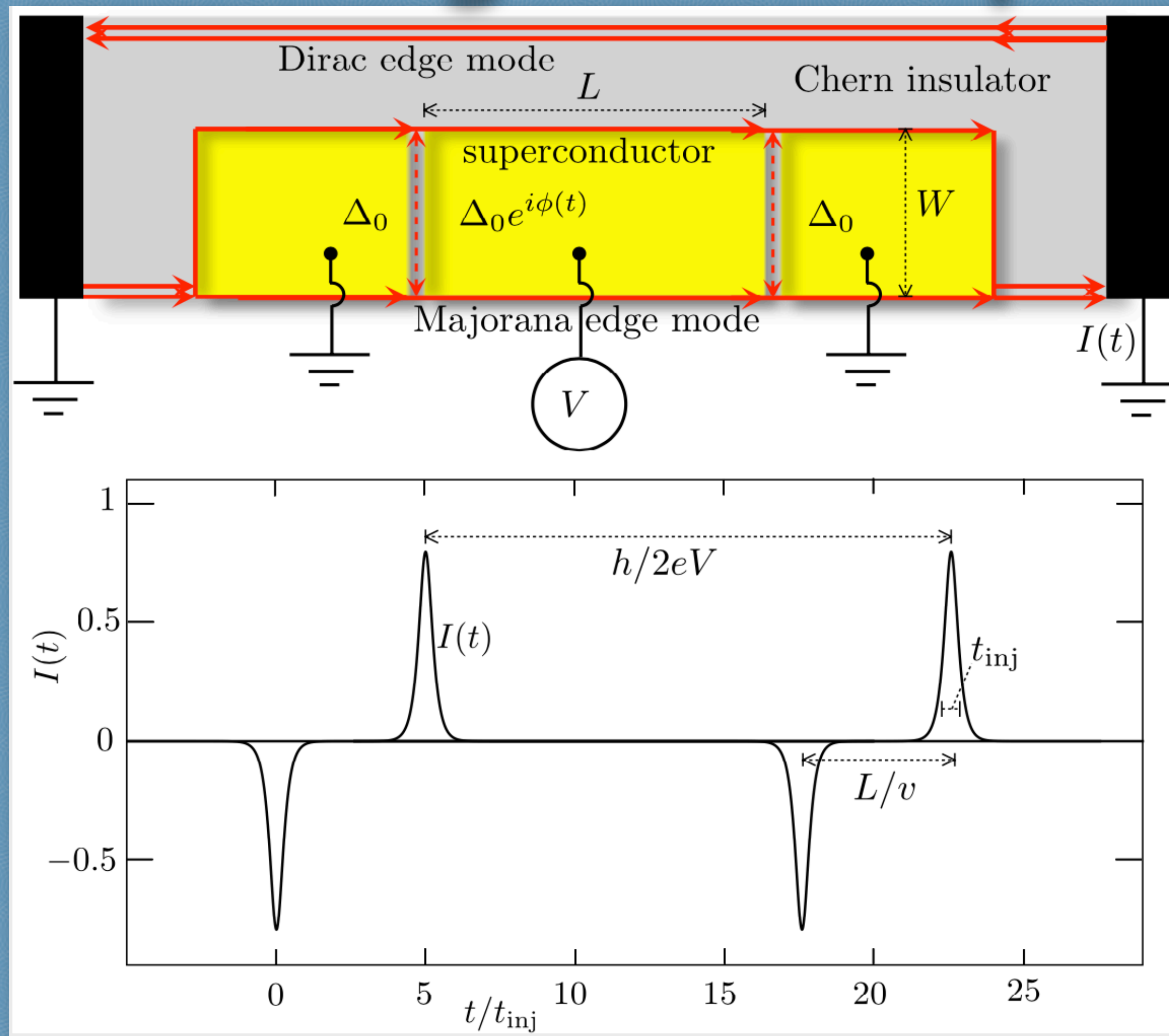


# Edge vortex primer (2)



*separation of time scales:* width of the  $\varphi$ -phase domain walls is smaller than their spacing by a factor  $\xi/W$  (coherence length/junction width)

# Edge vortex primer (3)

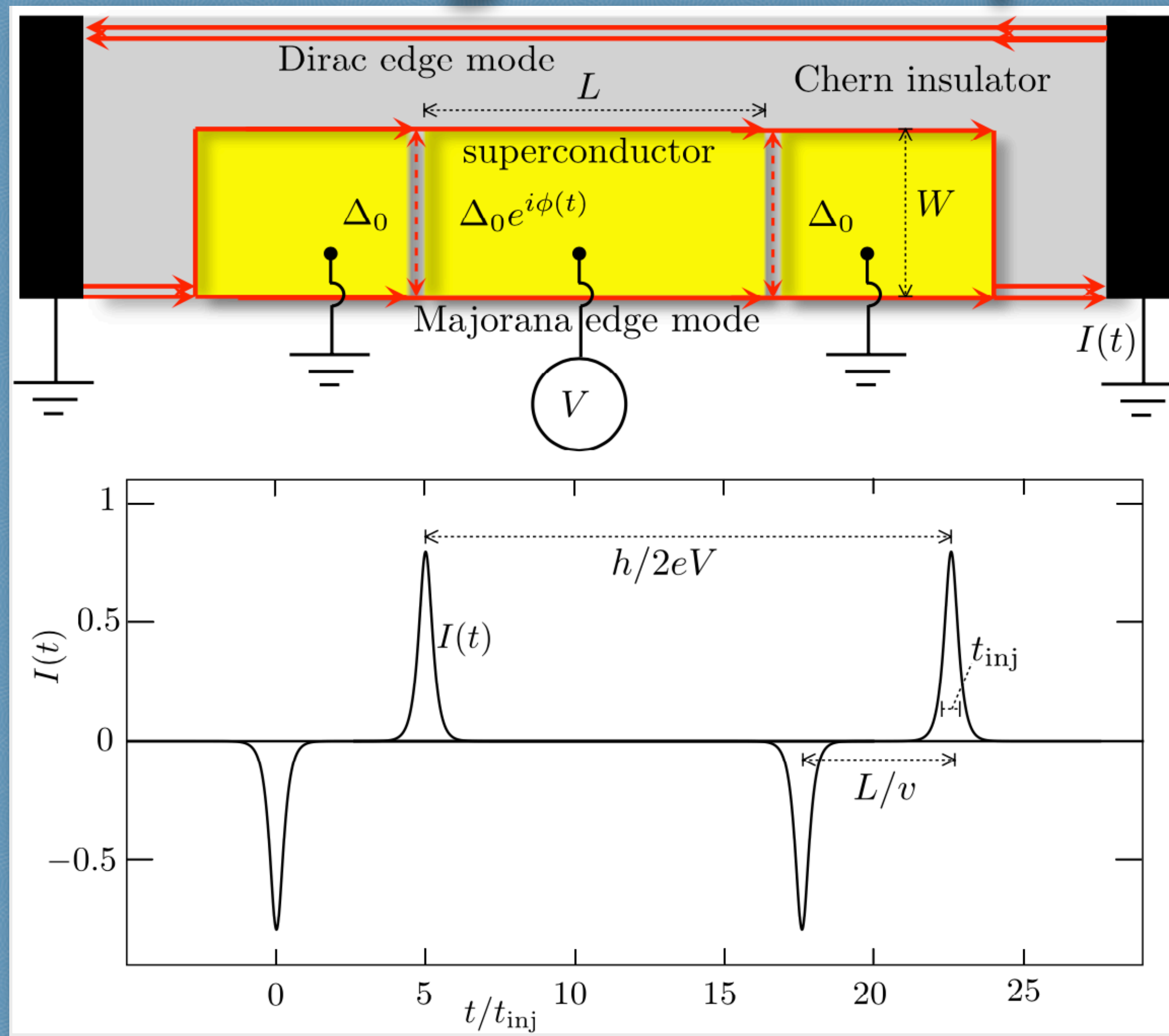


$$I(t) = \frac{e}{2\pi} \frac{d\Lambda}{dt}$$

(Brouwer formula)

Fusion of edge modes injects a current pulse. Injected charge  $Q = \pm e/2$  per  $\pi$ -phase for each vortex pair.

# Edge vortex primer (3)



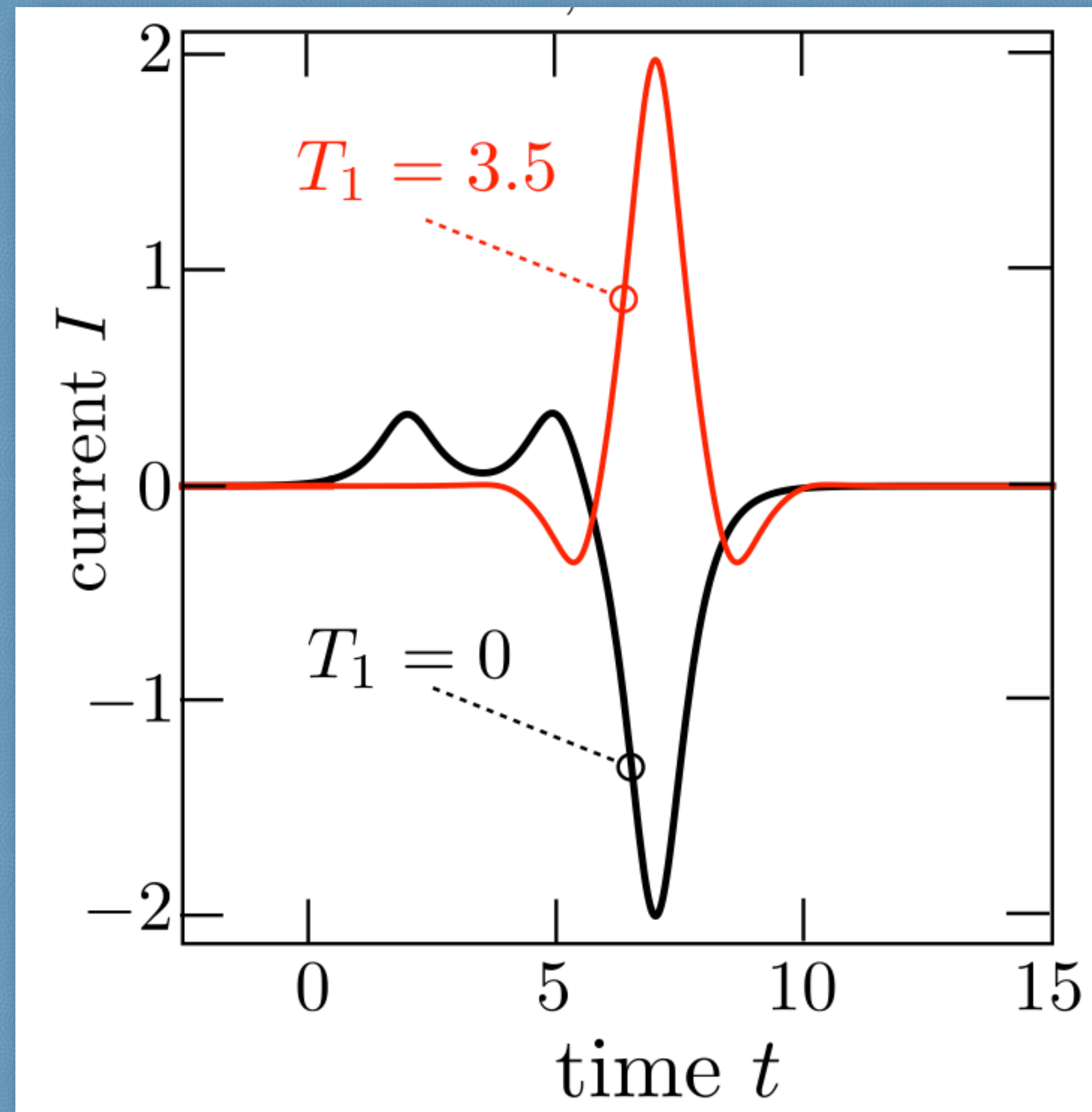
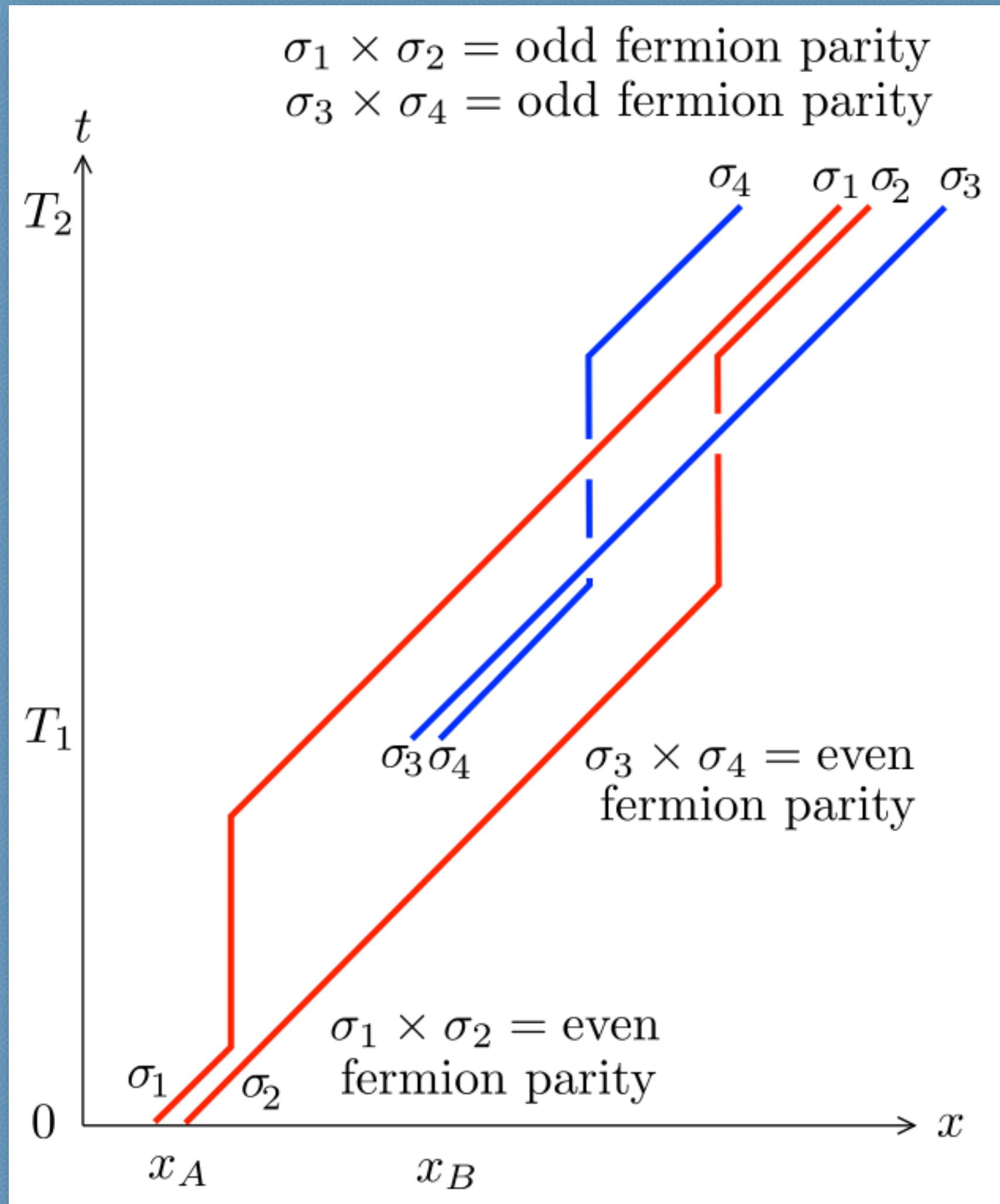
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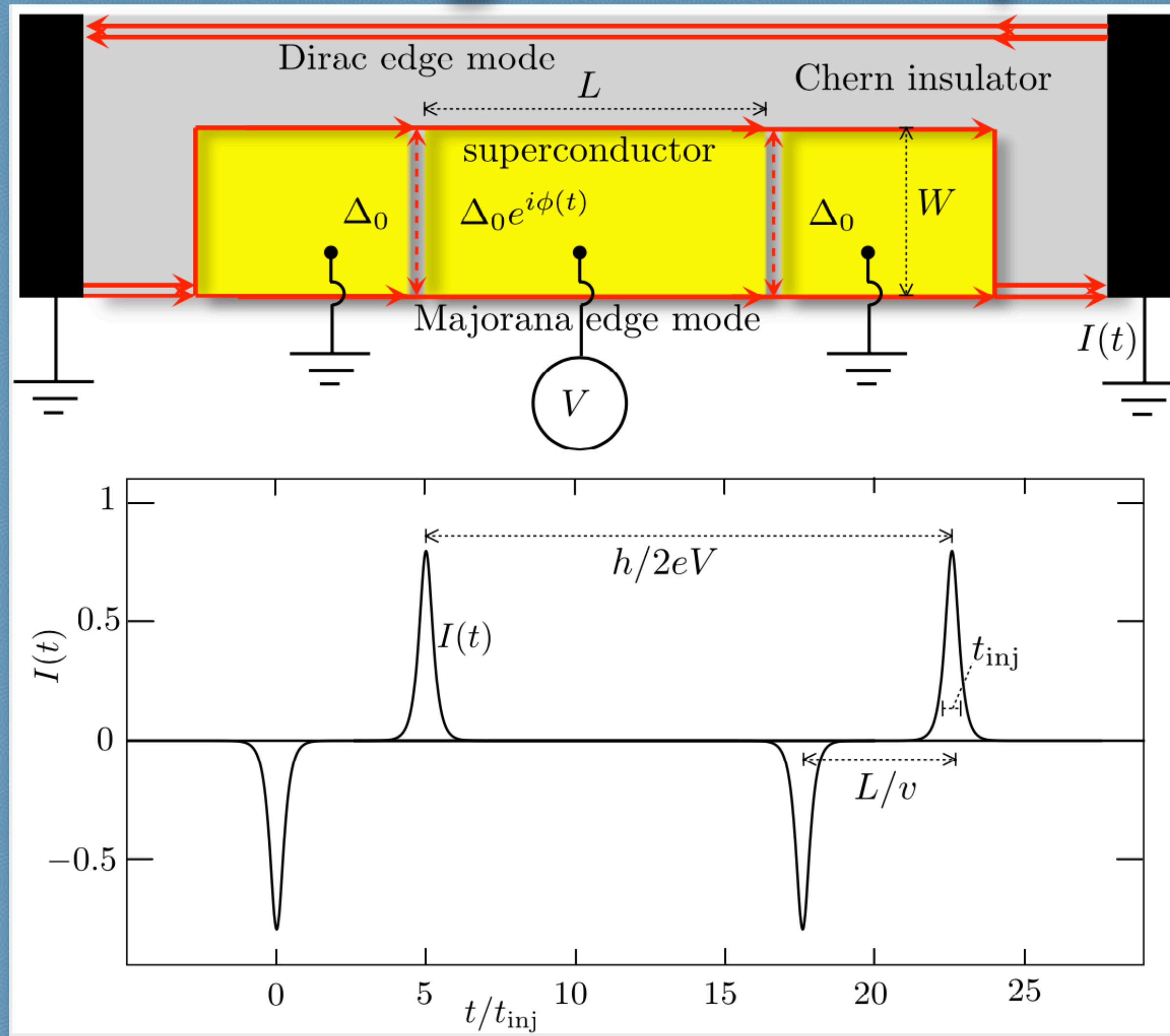
*braiding flips the sign of the charge*

# Edge vortex primer (4)



*braiding of two pairs of edge vortices*

# Edge vortex primer (3)

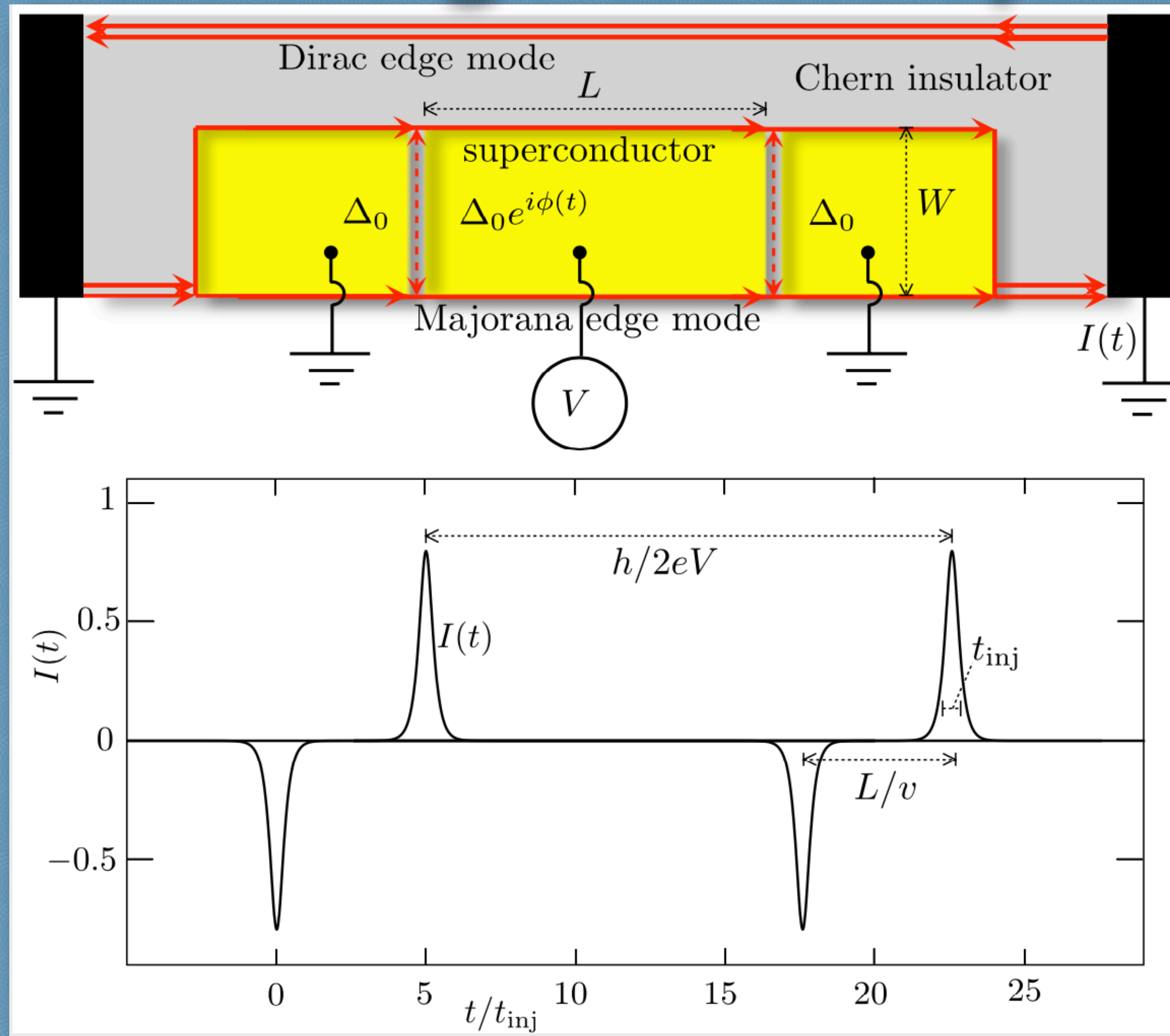


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# Edge vortex primer (3)



$$I(t) = \frac{e}{2\pi} \frac{d\Lambda}{dt}$$

(Brouwer formula)

Fusion of edge modes injects a current pulse. Injected charge  $Q = \pm e/2$  per  $\pi$ -phase for each vortex pair – *on average*.

*is the fractional charge a sharp observable, without quantum fluctuations?*

**Fractional charge, a sharp quantum observable**

S. Kivelson and J. R. Schrieffer

*Institute for Theoretical Physics and Department of Physics, University of California,  
Santa Barbara, California 93106*

(Received 28 December 1981)

calculation for the  $n=2$  and  $n=3$  cases, questions have been raised<sup>6</sup> as to whether the charge of a soliton is in fact a sharp quantum observable. That is, are the quantum fluctuations of the soliton charge about its fractional average value vanishingly small or is the fractional value  $Q_s$  simply a quantum average of several integer values? In the latter case, each individual measurement of the charge would yield an integer value, and only the mean of these observed values would be fractional.

Zero-modes in a *gapped* system have a fractional charge without quantum fluctuations. Does this carry over to *gapless* Majorana modes?

# The noise paradox

Only integer charge can enter a normal metal. For half-integer charge transfer this would require binomial statistics: either charge  $e$  or charge  $0$  is transferred with equal probability  $\Rightarrow$  variance =  $e^2/4$

And yet: a straightforward calculation using the bosonisation technique gives zero charge noise.  
How is this possible?



# Bosonisation approach to electron counting statistics (Levitov, Lee, Lesovik 1996)

bosonic density operator  $\rho$ , in terms of the Majorana fields  $\psi_1, \psi_2$

$$\hat{\rho}(x) = -\frac{1}{2}i\hat{\psi}_1(x)\hat{\psi}_2(x),$$

commutator

$$[\hat{\rho}(x), \hat{\rho}(x')] = \frac{i}{2\pi} \frac{\partial}{\partial x} \delta(x - x'),$$

vortex field in terms of the phase profile  $\Lambda$

$$\hat{\sigma}(t) = \exp\left(-i \int dx \hat{\rho}(x) \Lambda(x, t)\right),$$

$$|t\rangle = \hat{\sigma}(t)|0\rangle.$$

$$\hat{\sigma}^\dagger(t)\hat{\rho}(x)\hat{\sigma}(t) = \hat{\rho}(x) + \frac{1}{2\pi} \frac{\partial}{\partial x} \Lambda(x, t),$$

$$I(x, t) = ev \langle t | \hat{\rho}(x) | t \rangle = ev \langle 0 | \hat{\sigma}^\dagger(t) \hat{\rho}(s) \hat{\sigma}(t) | 0 \rangle = \frac{ev}{2\pi} \frac{\partial}{\partial x} \Lambda(x, t)$$

we have recovered the Brouwer formula

because the conjugation of the vortex field  $\sigma$  with the charge density  $\rho$  is a c-number, there are no fluctuations around the average.

$$C(\xi) = \ln \langle t | e^{i\xi \hat{Q}} | t \rangle \quad \text{cumulant generating function}$$

$$= \ln \left\langle 0 \left| \exp \left( i\xi \int dx \hat{\sigma}^\dagger(t) \hat{\rho}(x) \hat{\sigma}(t) \right) \right| 0 \right\rangle$$

$$= \frac{i\xi}{2\pi} \int dx \frac{\partial}{\partial x} \Lambda(x, t)$$

Linear in  $\xi$ , so no charge fluctuations

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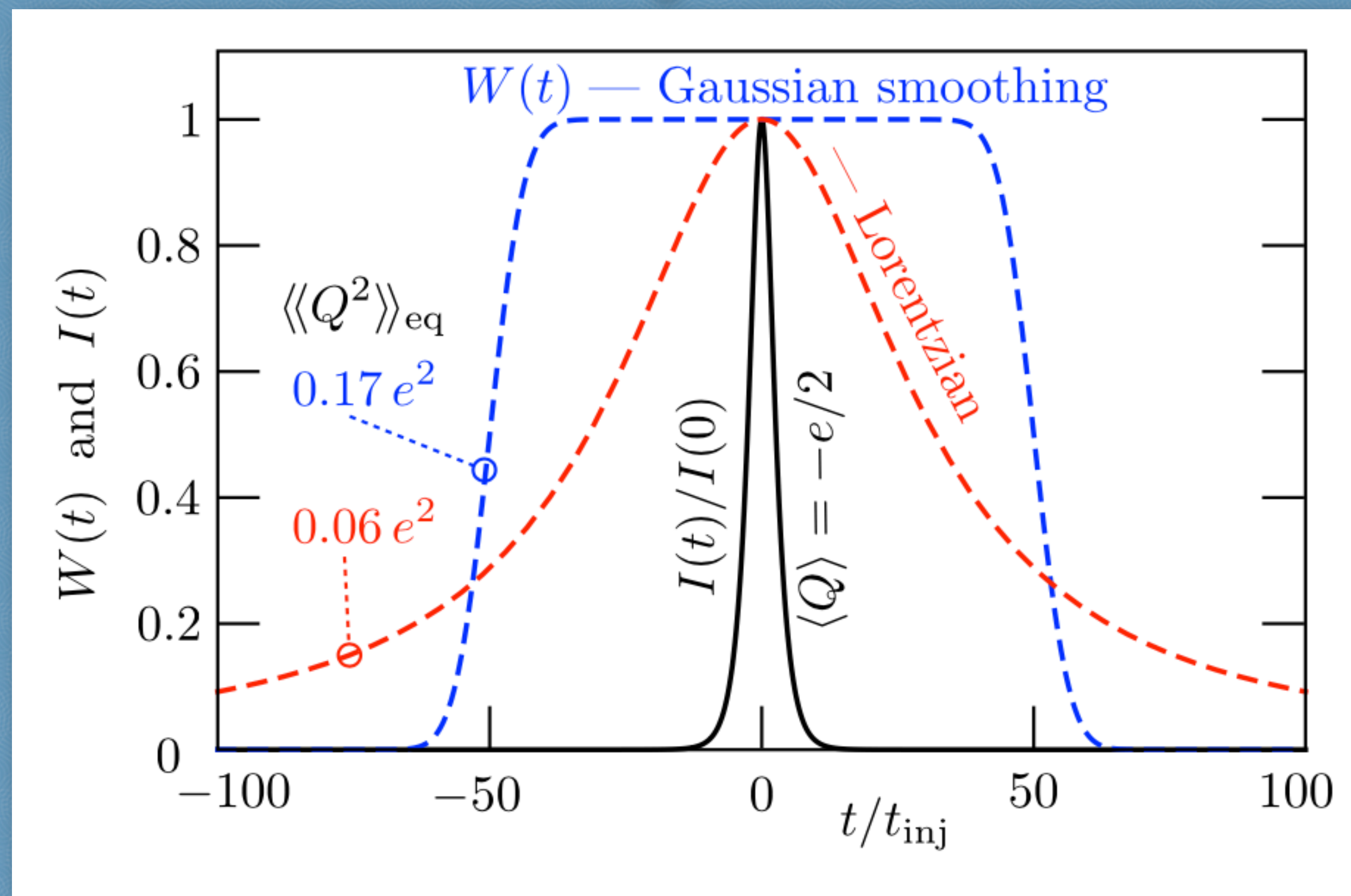
$$= \ln \left\langle 0 \left| \exp \left( i\xi \int dx \hat{\sigma}^\dagger(t) \hat{\rho}(x) \hat{\sigma}(t) \right) \right| 0 \right\rangle$$

$$= \frac{i\xi}{2\pi} \int dx \frac{\partial}{\partial x} \Lambda(x, t) + \ln \langle 0 | e^{i\xi \hat{Q}} | 0 \rangle \quad \text{except}$$

linear in  $\xi$ , so no charge fluctuations

equilibrium fluctuations

# The noise paradox – *resolved*

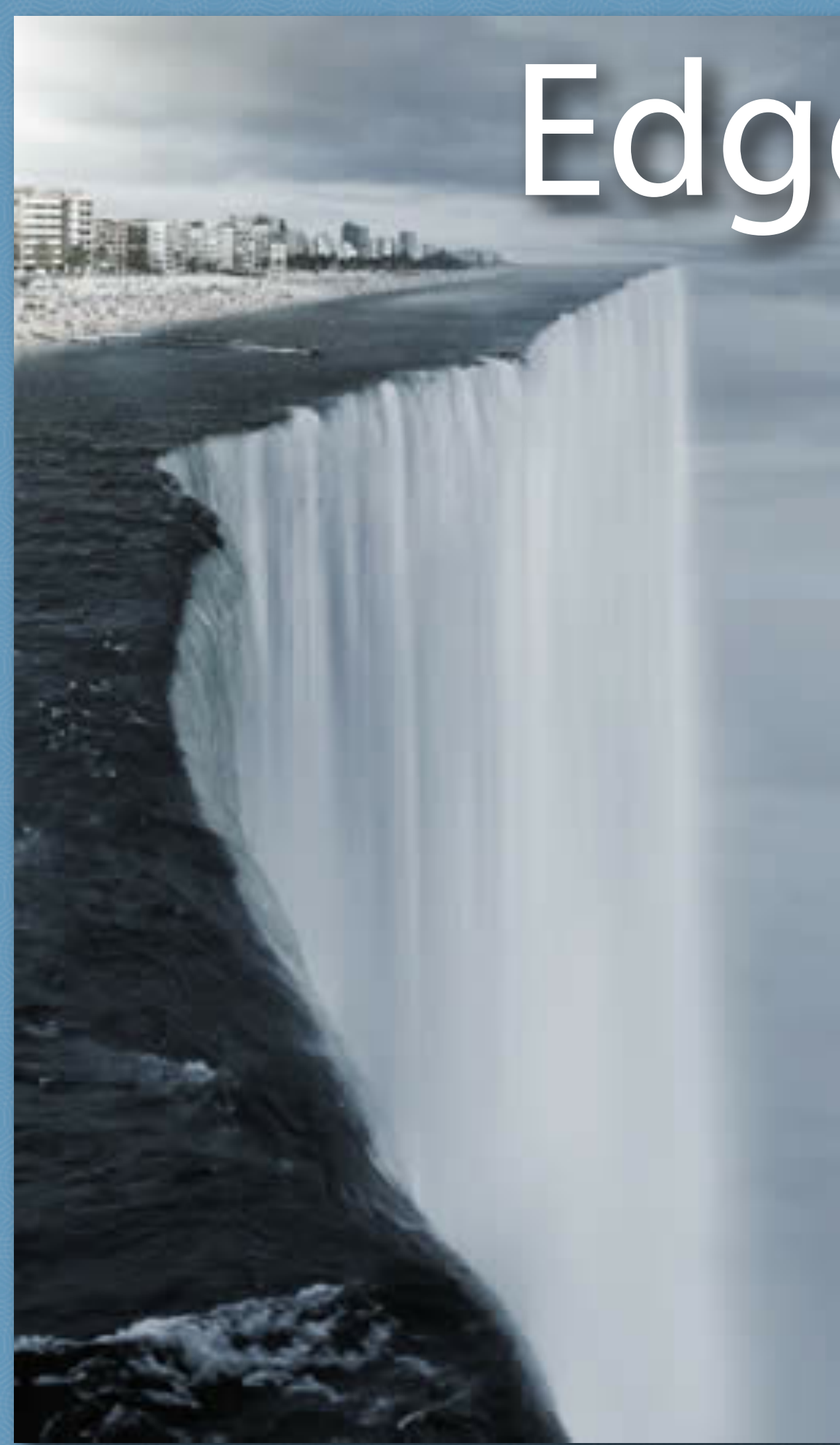


The integer value constraint of a charge counting measurement is avoided by the equilibrium noise, which persists at zero temperature in a gapless system.

$$\langle Q \rangle = \frac{e}{2\pi} \int_{-\infty}^{\infty} dt W(t) \Lambda'(t)$$

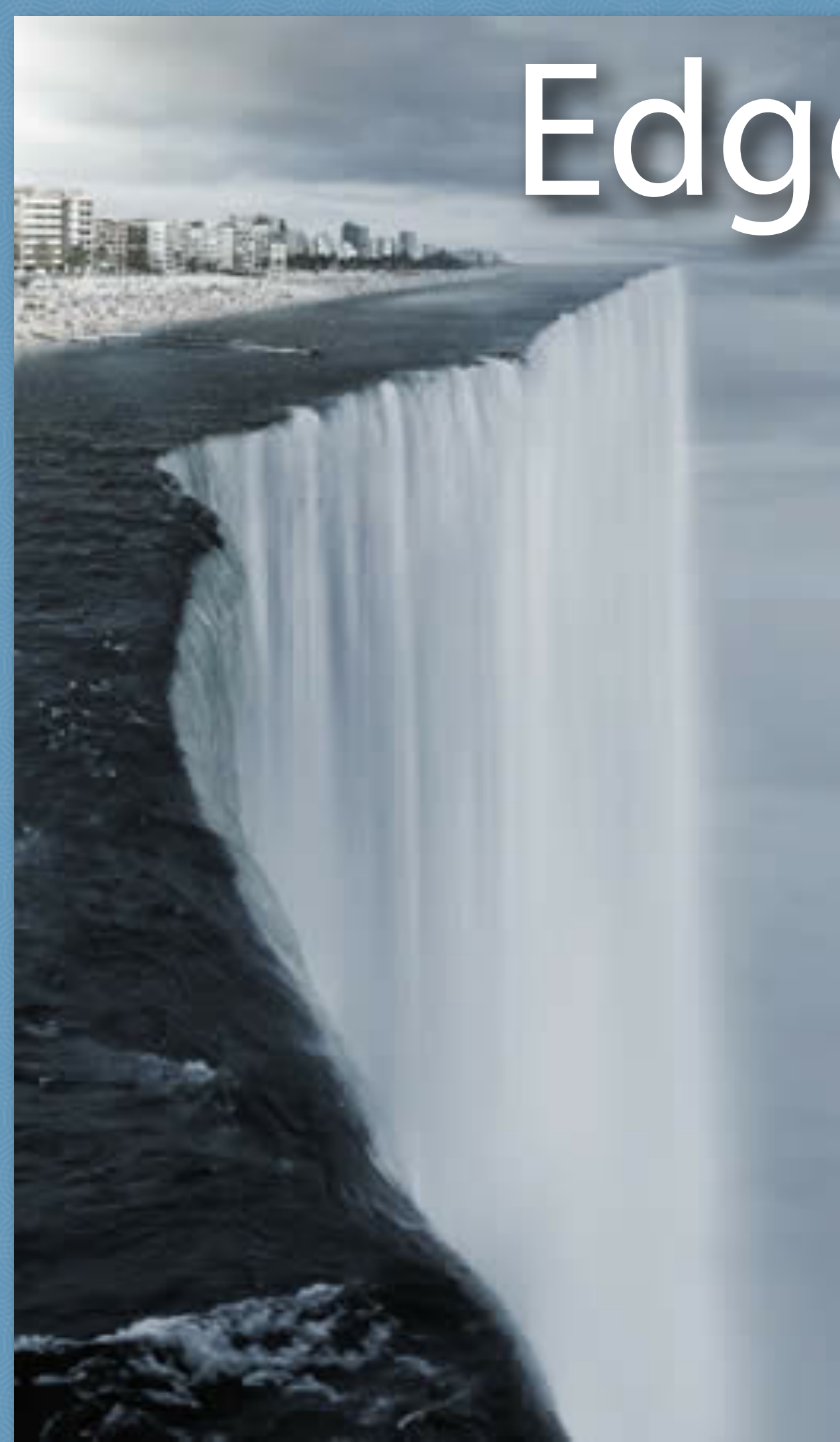
$$\lim_{T \rightarrow 0} \langle\langle Q^2 \rangle\rangle_{eq} = \frac{e^2}{4\pi^2} \int_0^{\infty} dE E |W(E)|^2$$

# Edge vortices rule!

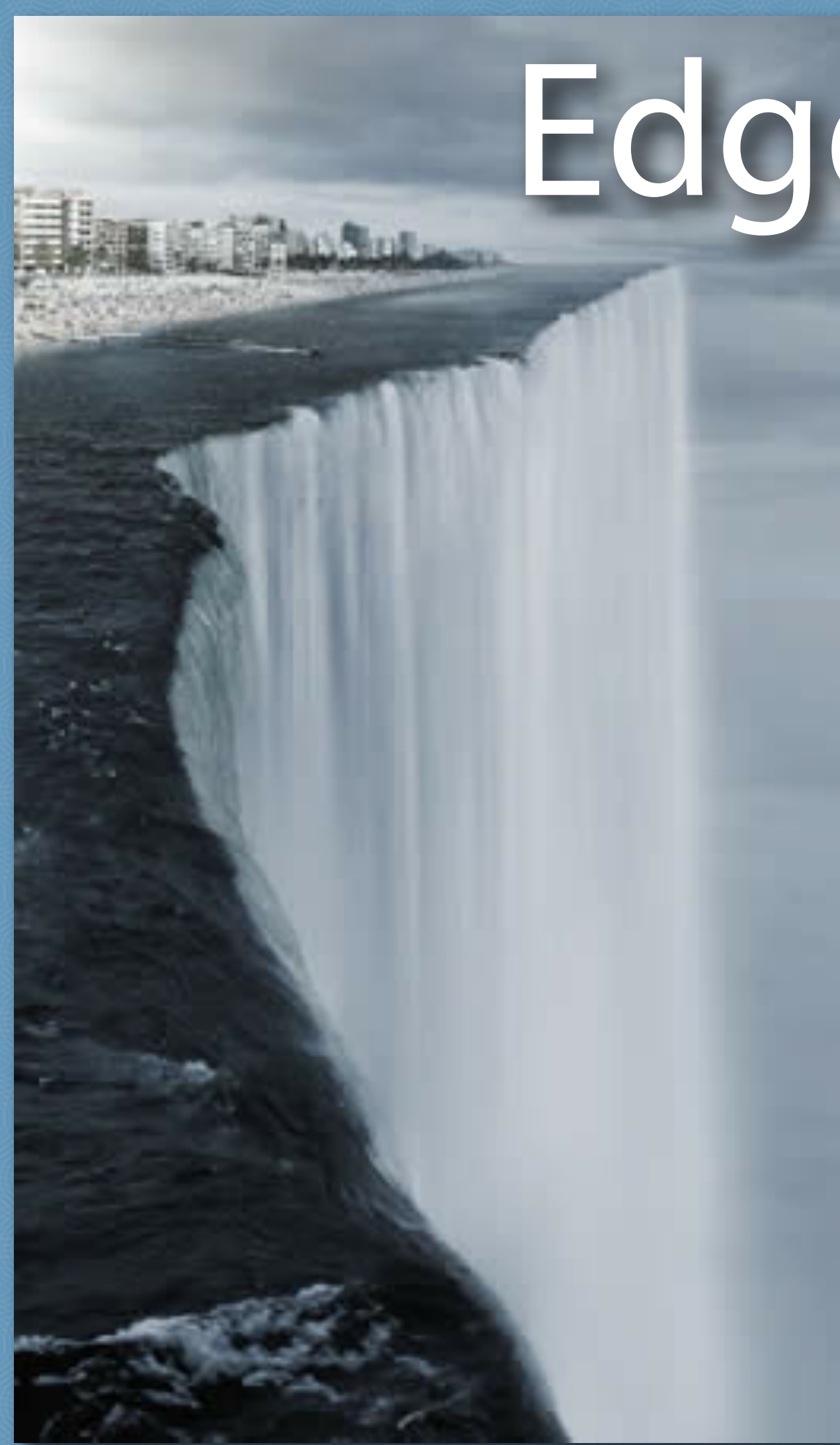


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- mobile counterpart to Majorana zero-modes

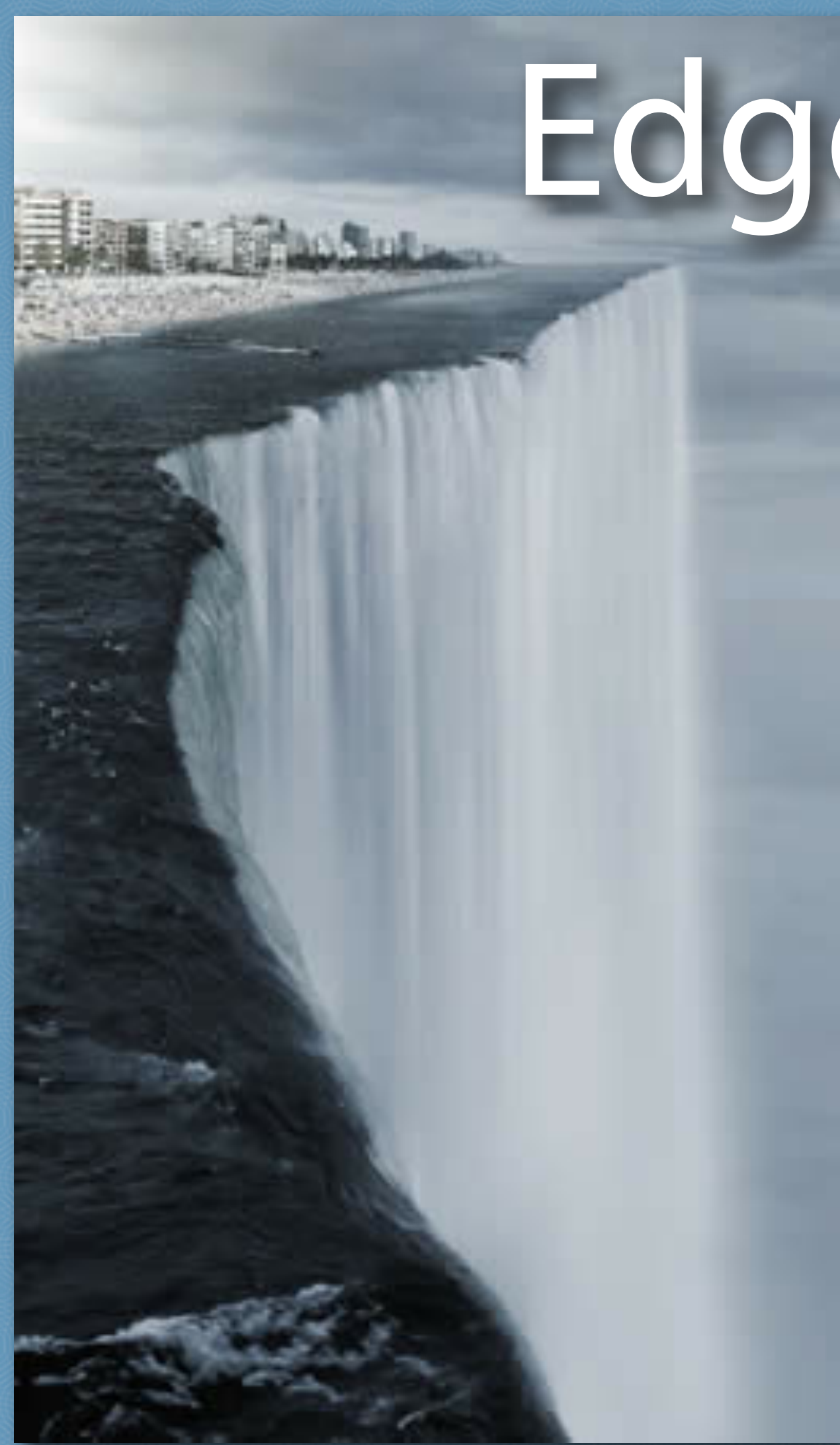


# Edge vortices rule!



- mobile counterpart to Majorana zero-modes
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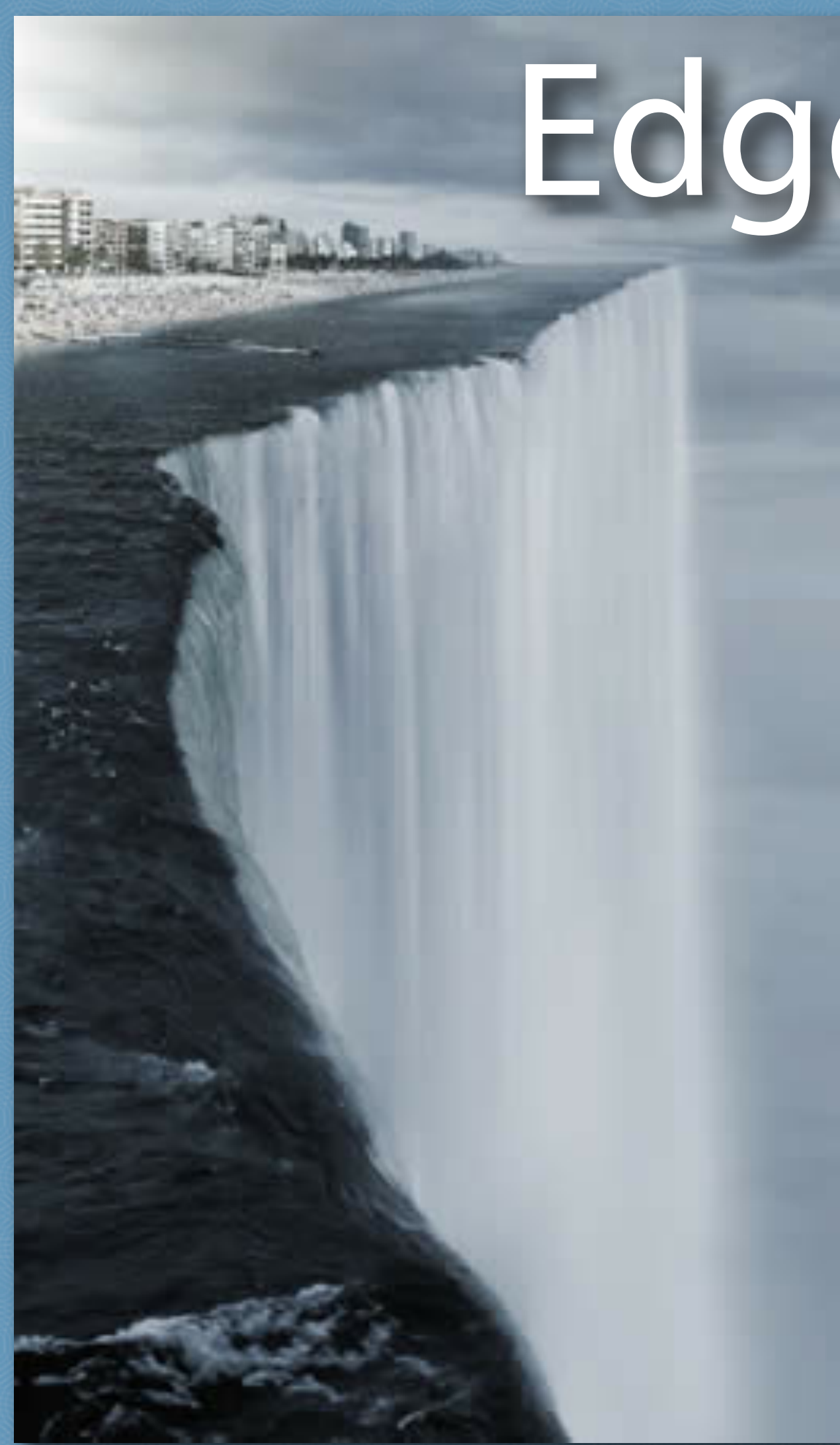
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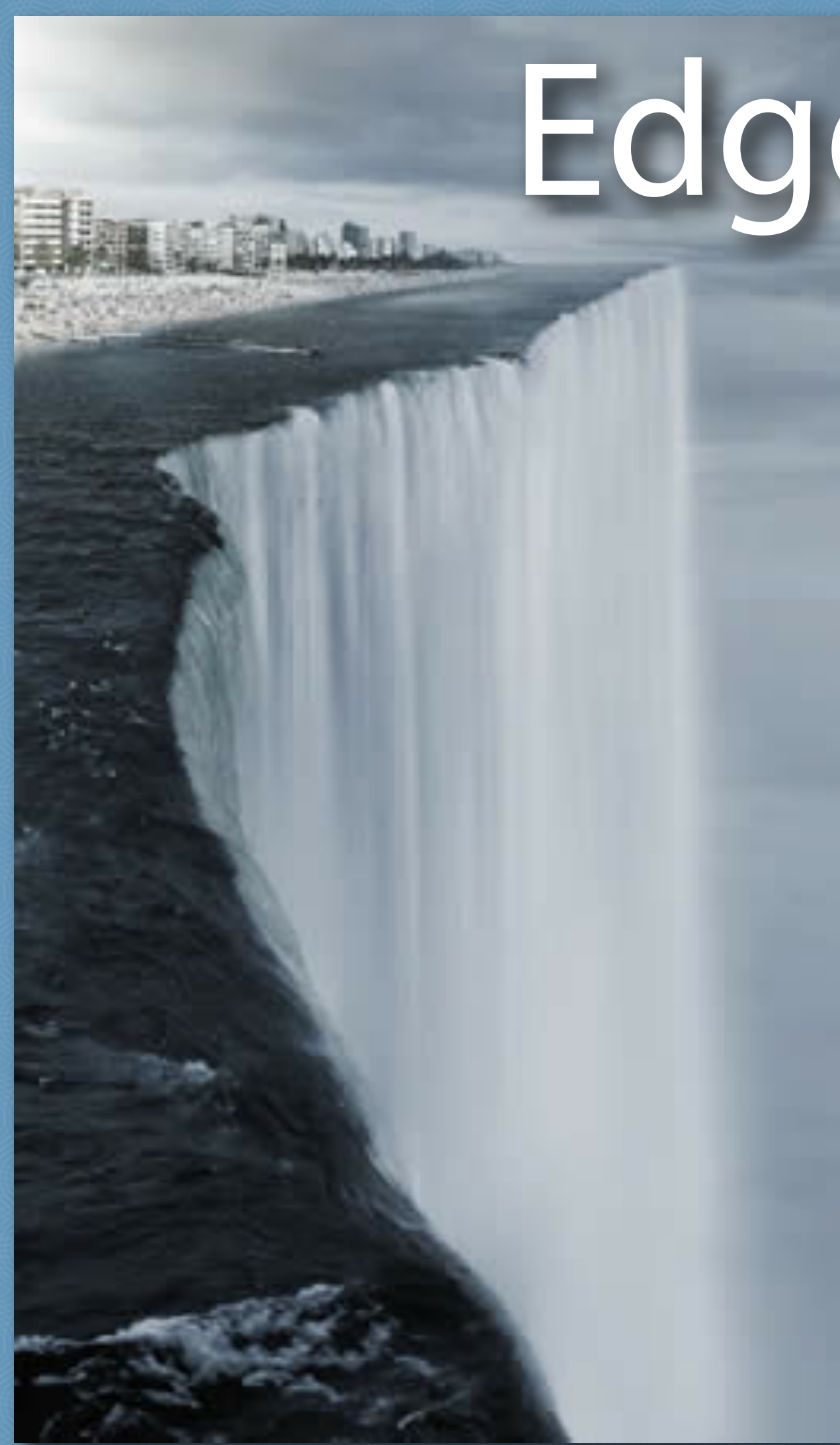


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- mobile counterpart to Majorana zero-modes
- can be injected on-demand by a Josephson junction
- chiral motion implements braiding
- electrical detection as a  $\pm e/2$  charge pulse
- sharp observable (no excess noise)