

# Electric-field effects on localized spins

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Republic  
of Poland



Foundation for  
Polish Science



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# Handbook of Magnetism and Magnetic Materials

Editors: **Coey**, Michael, **Parkin**, Stuart (Eds.)



36 chapters

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## Magnetic Nanoparticles

Majetich, S.

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## Dilute Magnetic Materials

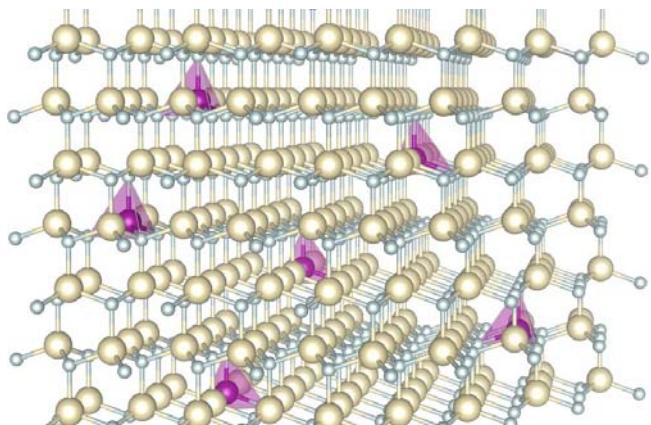
Bonanni, A., Dietl, T., Ohno, H.

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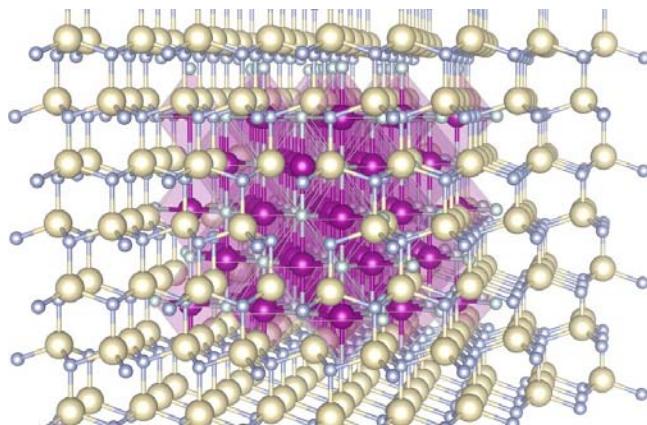
## Artificially Engineered Magnetic Materials

Marrows, C.

## Spatial distribution of magnetic component

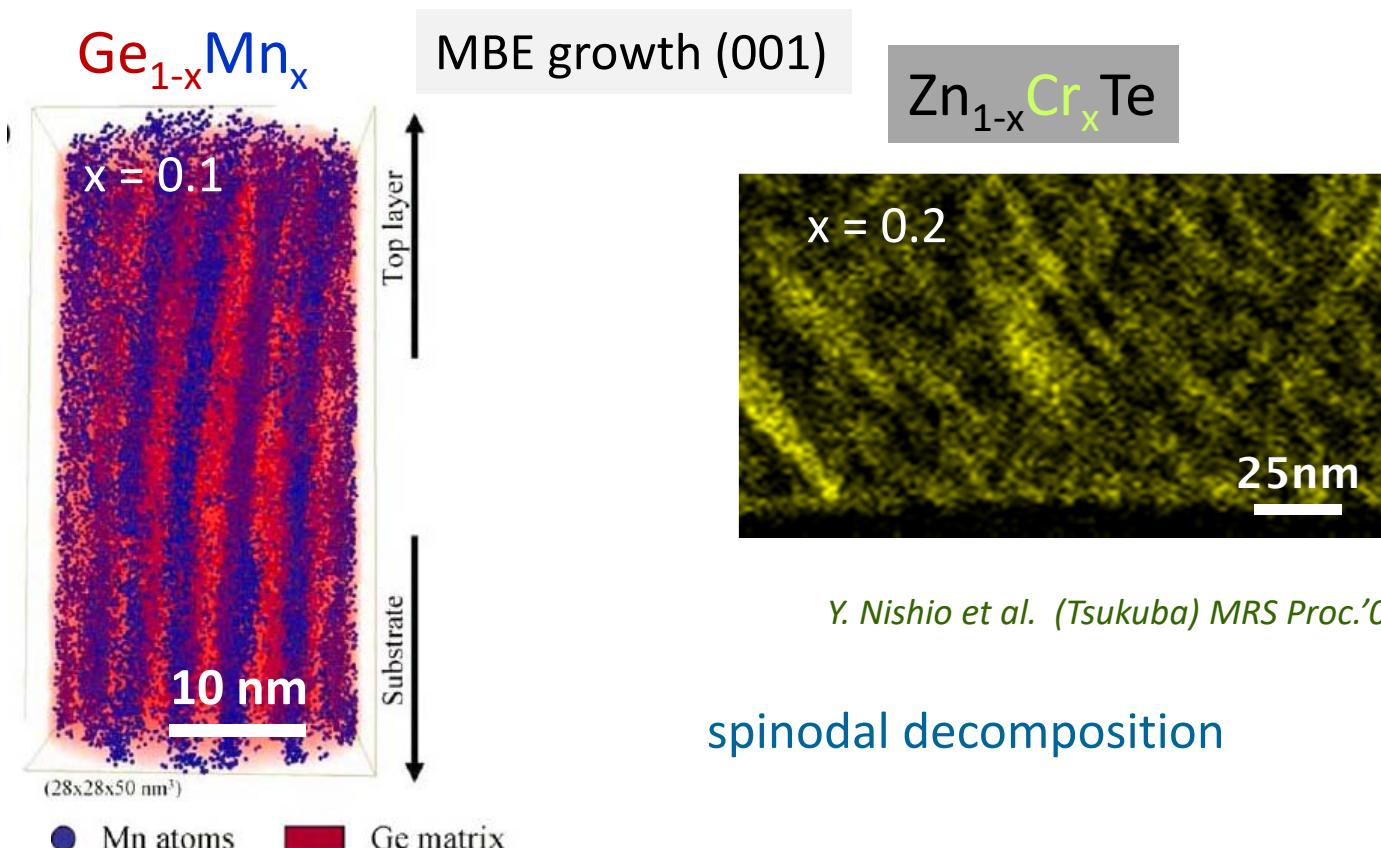


random



non-random

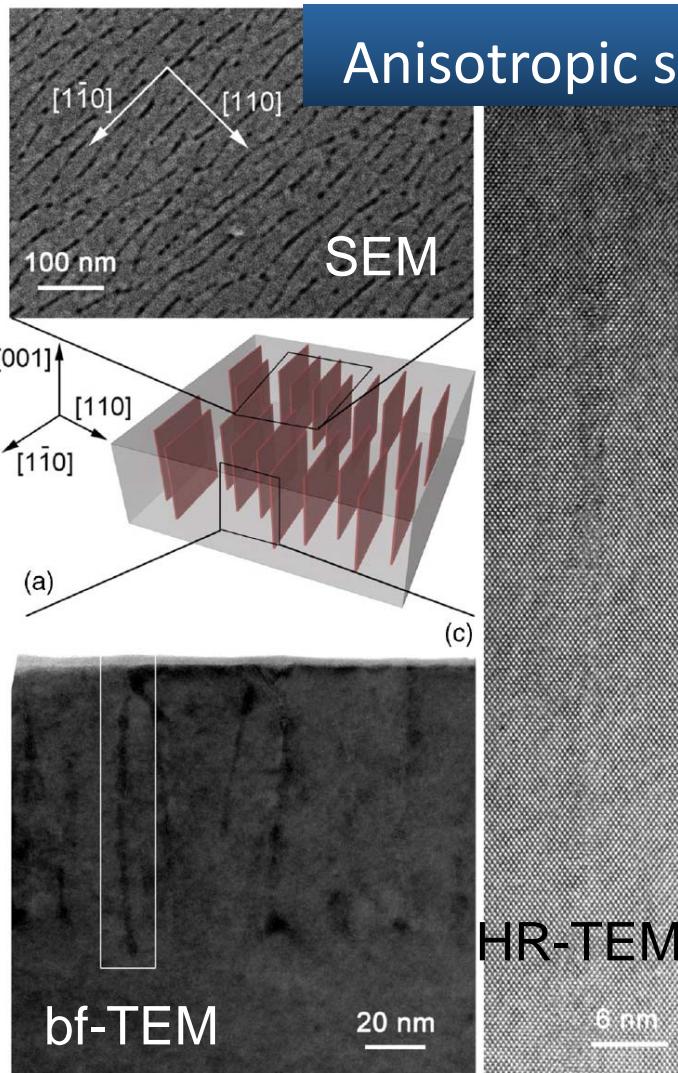
## Aggregation of magnetic cations - nanocolumns



*I. Mouton et al. [Rouen, Grenoble] JAP'2012*

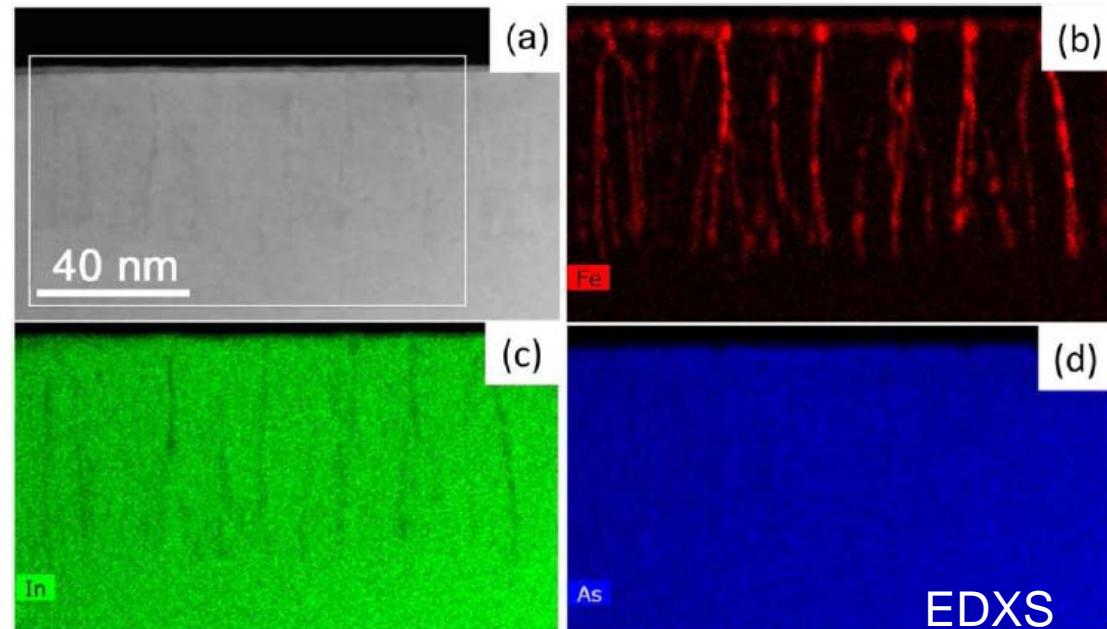
when self-organized growth of magnetic nanostructures will be competitive with top-down approaches dominating today?

T. Dietl et al., Spinodal nanodecomposition in semiconductors doped with transition metals, *Rev. Mod. Phys.* **87**, 1311 (2015)



## Anisotropic spinodal decomposition

$(001)\text{In}_{1-x}\text{Fe}_x\text{As}, x = 0.04$



Fe-rich nanoplates oriented along [-110]  
 $[110] \neq [-110]$  nematicity  $D_{2d} \rightarrow C_{2v}$

Ye Yuan et al. [Dresden, Warsaw] PRMater'2018

## Examples of nematicity

GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As QW AMR in QHE/FQHE  $D_{2d} \rightarrow C_{2v}$

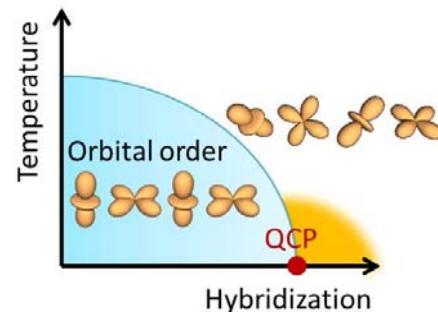
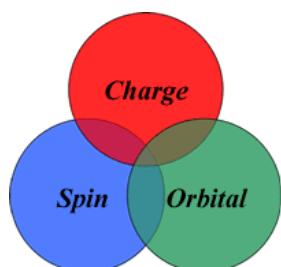
Ga<sub>1-x</sub>Mn<sub>x</sub>As magnetic anisotropy, AMR  $D_{2d} \rightarrow C_{2v}$

Cu<sub>x</sub>Bi<sub>2</sub>Se<sub>3</sub> anisotropy of sc gap  $D_{3d} \rightarrow C_{2v}$

....

Literature model:

spontaneous symmetry breaking by ordering of charge  
or spin or orbital ... degrees of freedom at low temperatures

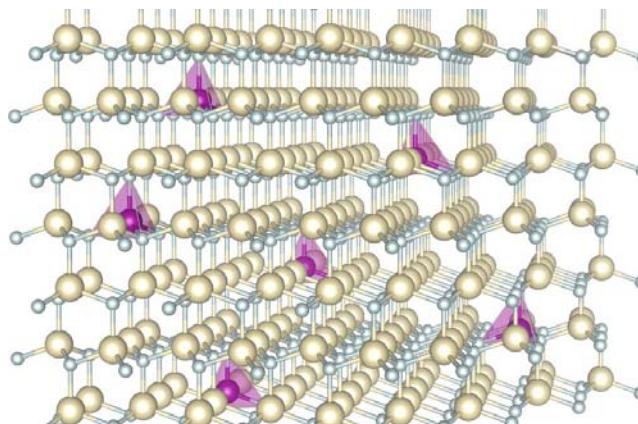


In most cases symmetry breaking (nematicity) originates from quenched anisotropic spinodal decomposition of alloy components/impurities/defects/... appearing during the growth

theory: *M. Birowska et al. [Warsaw] PRL'2012*

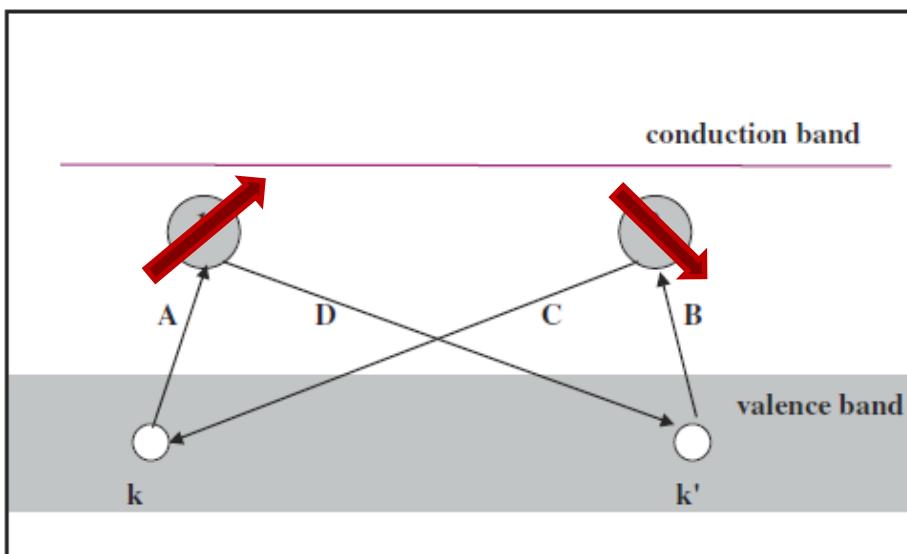
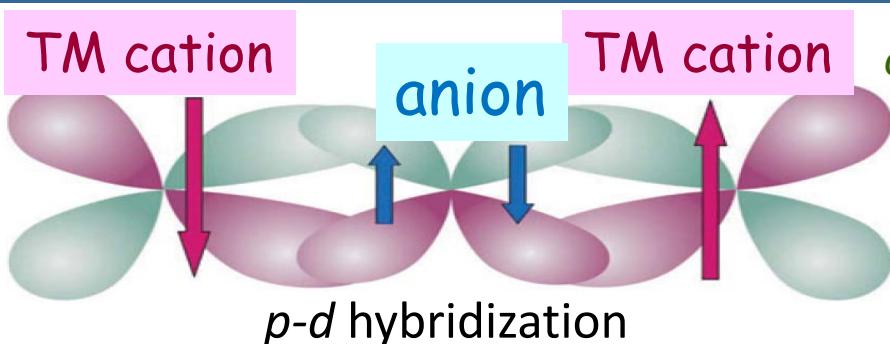
*Ye Yuan et al. [Dresden, Warsaw] PRMater'2018*

Random distribution, no carriers



spin-spin interactions?

## Superexchange in II-VI/III-V DMS



cf. P.W. Anderson, J.B. Goodenough, J. Kanamori

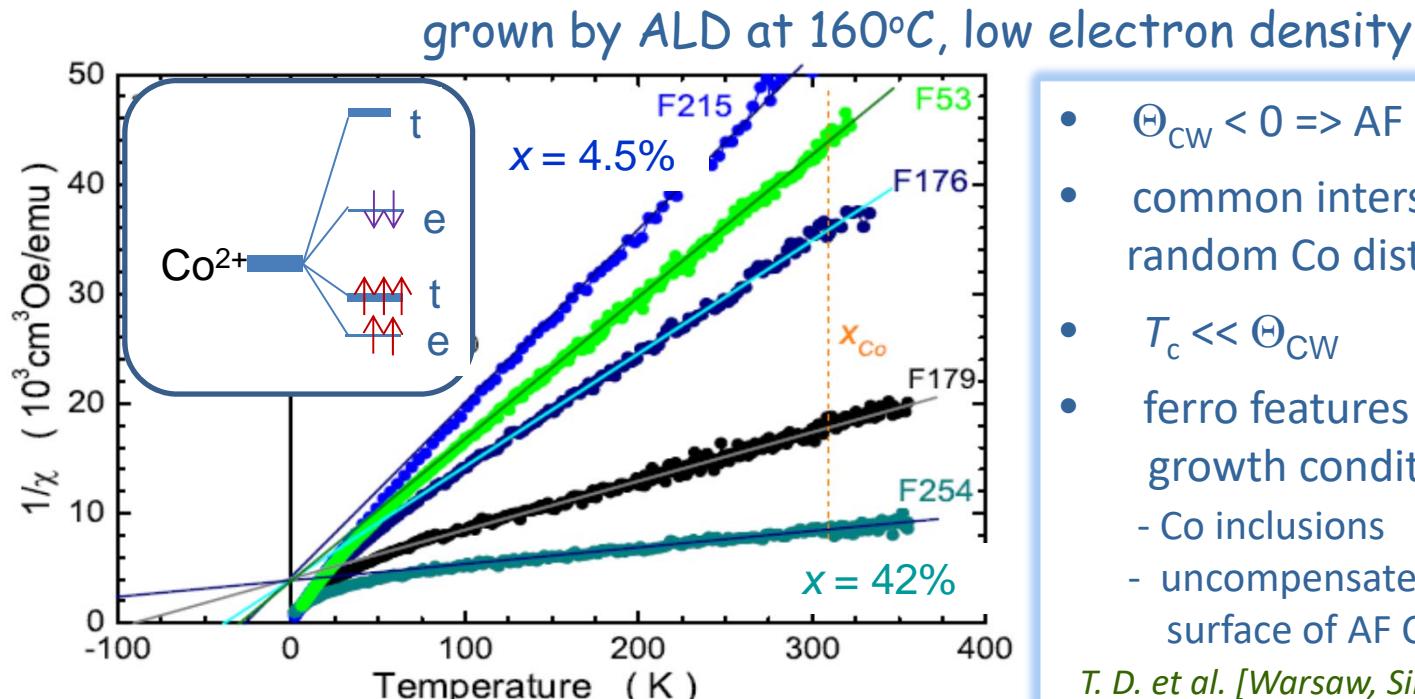
- always present in magnetic compounds
- 4th order in  $V_{pd}$
- short range  $J_{ij} = J_0 \exp(-R_{ij}/b)$
- usually antiferromagnetic, e.g., for  $\text{Mn}^{2+}, \text{Fe}^{3+}, \text{Co}^{2+}$  in tetrahedral DMS

J. Spalek et al. [Purdue, Krakow, Warsaw] PRB'1986  
B.E. Larson et al. [Harvard] PRB'1988

- can be ferromagnetic, e.g., for  $\text{Mn}^{3+}, \text{Cr}^{2+}, \text{V}^{2+}$  in tetrahedral DMS

J. Blinowski et al. [Warsaw, Schottky] PRB'1996

## wz-Zn<sub>1-x</sub>Co<sub>x</sub>O – 1/χ vs. T AF superexchange

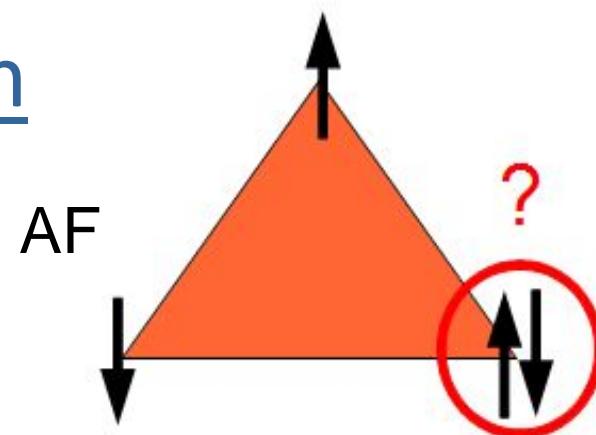


M. Sawicki et al. [Warsaw] PRB'2013

cf. A. Ney et al. [Duisburg-Essen, Pacific, Grenoble] PRL'2008

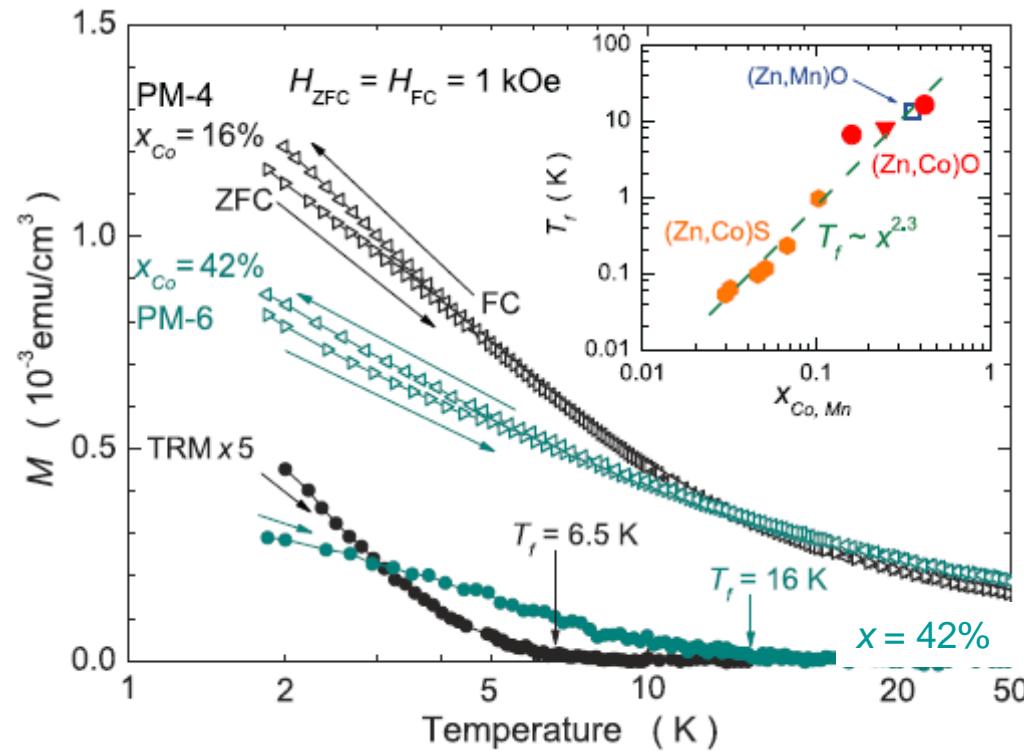
## Random antiferromagnets

### Frustration



e.g., C. Lacroix, P. Mendels, F. Mila "Introduction to Frustrated Magnetism", Springer 2011

# Spin-glass freezing in $\text{Zn}_{1-x}\text{Co}_x\text{O}$



$$T_f \ll \Theta_{\text{CW}}$$

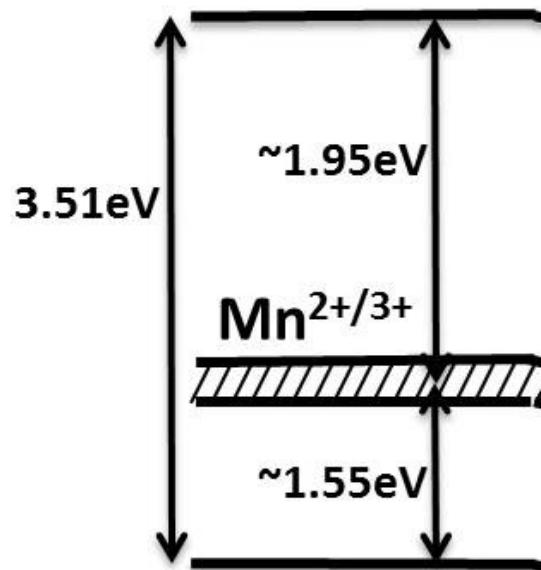
*M. Sawicki et al. [Warsaw] PRB'2013*

## Semiinsulating GaN:Mn by AMMONO/Unipress

M. Zajac et al. [Warsaw] APL'2001

A. Wolos et al. [Warsaw] PRB'2004

Mn – deep acceptor in GaN :-:



Mn – ideal for making S.I. GaN (-:

M. Zajac et al. [AMMONO/Unipress] Prog. Crys. Growth'2018



# Magnetism and its control in wz-Ga<sub>1-x</sub>Mn<sub>x</sub>N epilayers

## Collaborators:

D. Sztenkiel, M. Foltyn, G. P. Mazur,  
K. Gas, C. Śliwa, M. Sawicki



R. Adhikari, A. Bonanni



MOVPE/Adv. charac.



D. Hommel  
**PORT**  
Polish Center for  
Technology Development

MBE



N. Gonzalez Szwacki, J. A. Majewski



## Ga-subsititutional Mn acceptor in wurtzite GaN

- forms mid-gap  $\text{Mn}^{2+}/\text{Mn}^{3+}$  ( $A^-/A^0$ ) level (impurity band)

*T. Graf et al. [Schottky] pss'2003; Wolos et al. [Warsaw] PRB'2004*

→ GaN:Mn excellent semi-insulating material  
(up to at least 10% of Mn)

- hole binding energy: mostly  $p-d$  hybridization

→ Zhang-Rice polaron;  $\text{Mn}^{3+} = \text{Mn}^{2+} + h$

*T. D. et al. [Warsaw, Tohoku] PRB'2002*

- $\text{Mn}^{3+}$  high spin configuration  ${}^5T_2$  ( $S = 2, L = 2$ )

confirmed by high-field EPR, EXAFS, XANES, XES

*A. Bonanni et al. [Linz, Warsaw, Grenoble] PRB'2010'2011, Sci. Rep.'2012*

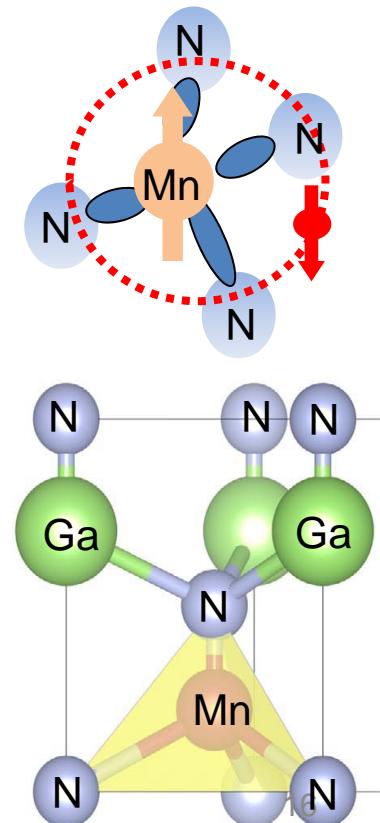
→ Jahn-Teller distortion along one of  $<100>$  cubic axes

→ wz trigonal distortion along  $[111]$  cubic axis

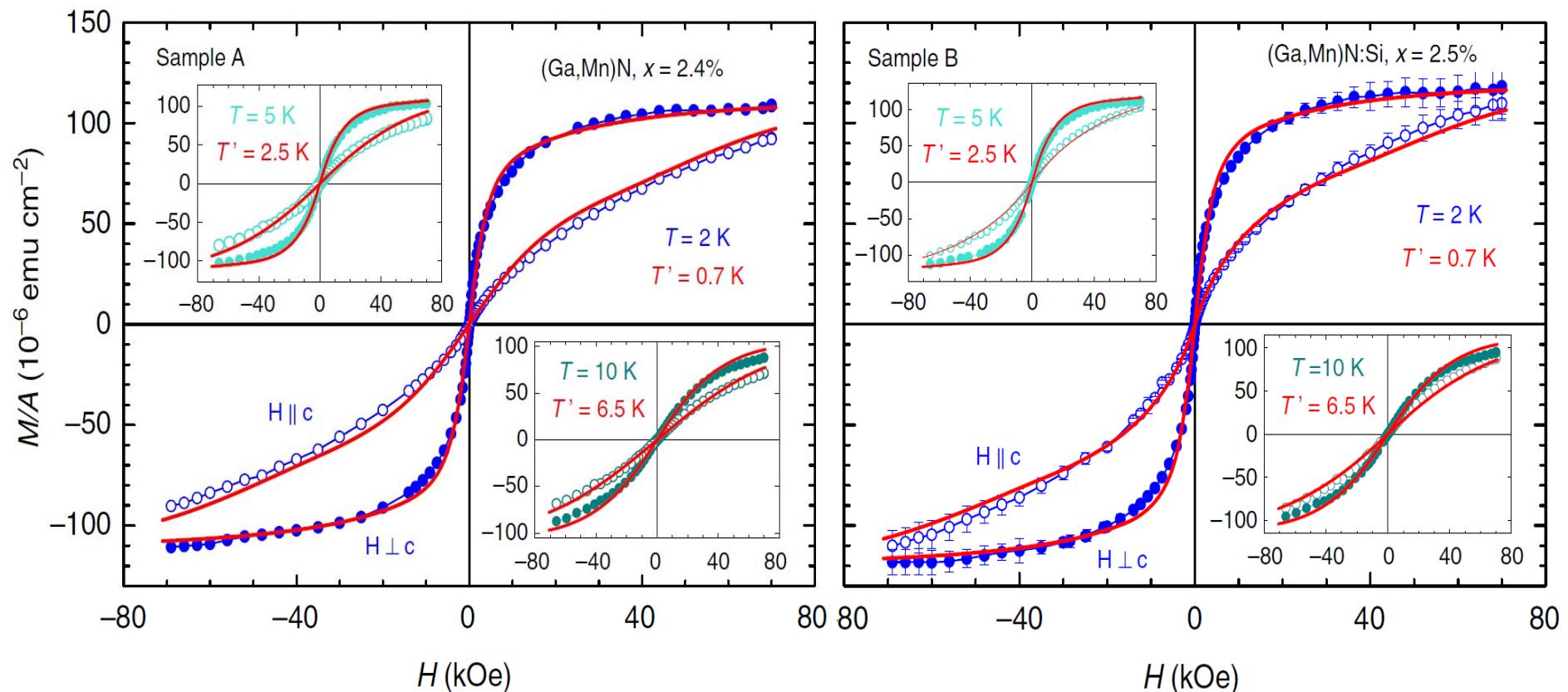
*J. Gosk et al. [Warsaw] PRB'2005; W. Stefanowicz et al. [Warsaw, Linz] PRB'2010*

*cf. F. Virot et al. [Marseille, Mostaganem] JPC'2010*

- crystal-field parameters known [x-ray, optics,  $M(T,H)$ ]



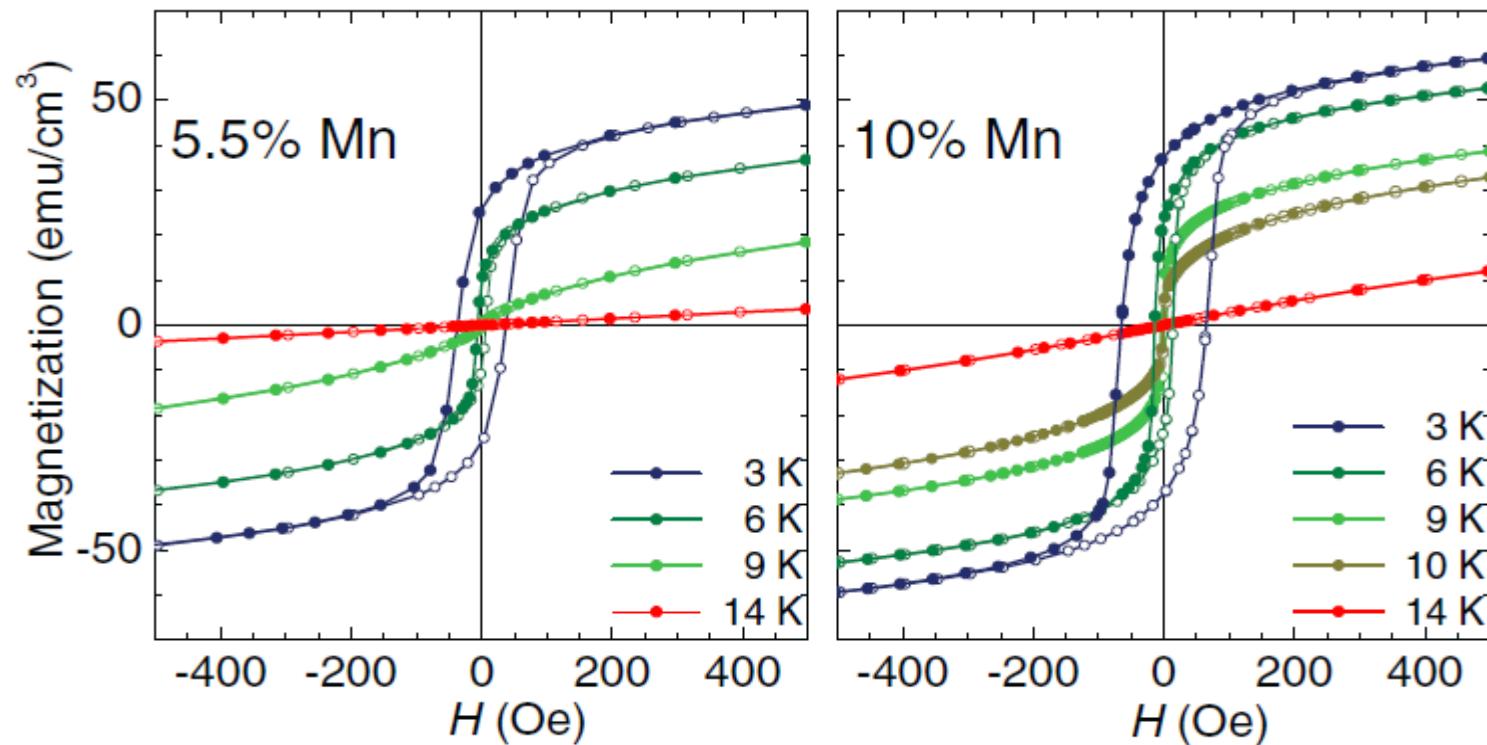
## $M(T,H)$ in paramagnetic in wz-Ga<sub>1-x</sub>Mn<sub>x</sub>N: exp. vs. theory



$$\text{Mn}^{3+} \quad \mathcal{H} = \mathcal{H}_{\text{TE}} + \mathcal{H}_{\text{JT}} + \mathcal{H}_{\text{TR}} + \mathcal{H}_{\text{SO}} + \mathcal{H}_Z$$

$\text{Mn}^{2+} \sim 10\%, \text{ Mn}^{3+} \sim 90\%$

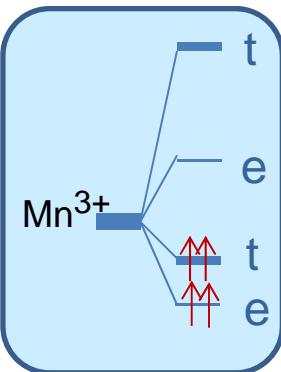
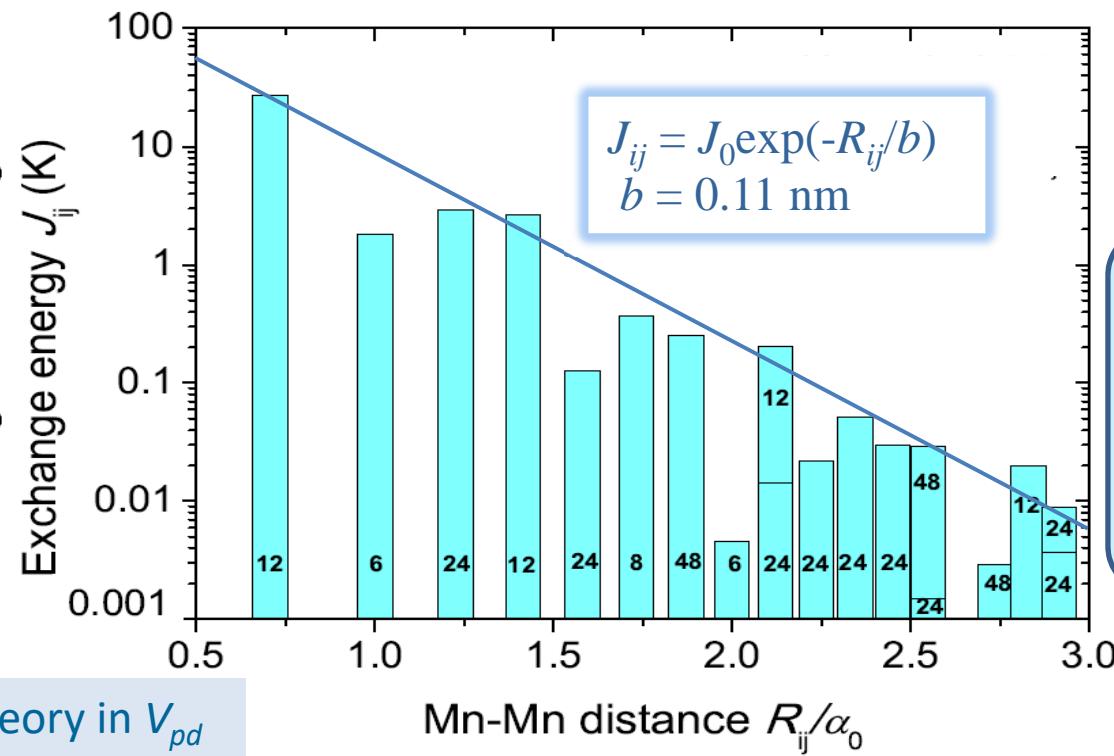
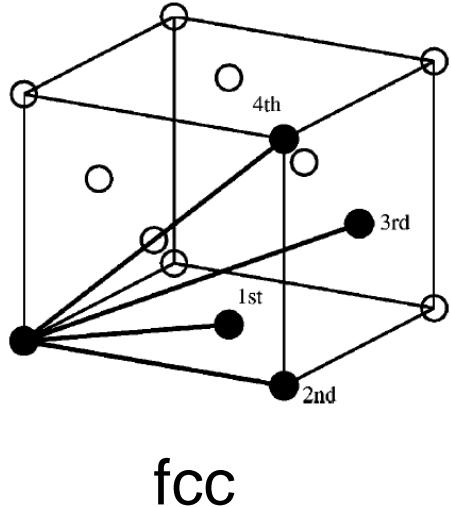
## Exp.: Low temperature ferromagnetism in wz-Ga<sub>1-x</sub>Mn<sub>x</sub>N



A. Bonanni *et al.*, M. Sawicki *et al.* [Linz, Warsaw, Bremen, Athens] PRB'2011, 2012, 2013  
cf. E. Sarigiannidou *et al.* [Grenoble] PRB'2006

## Theory: Ferromagnetic superexchange in $\text{Ga}_{1-x}\text{Mn}_x\text{N}$ with $\text{Mn}^{3+}$ TBA

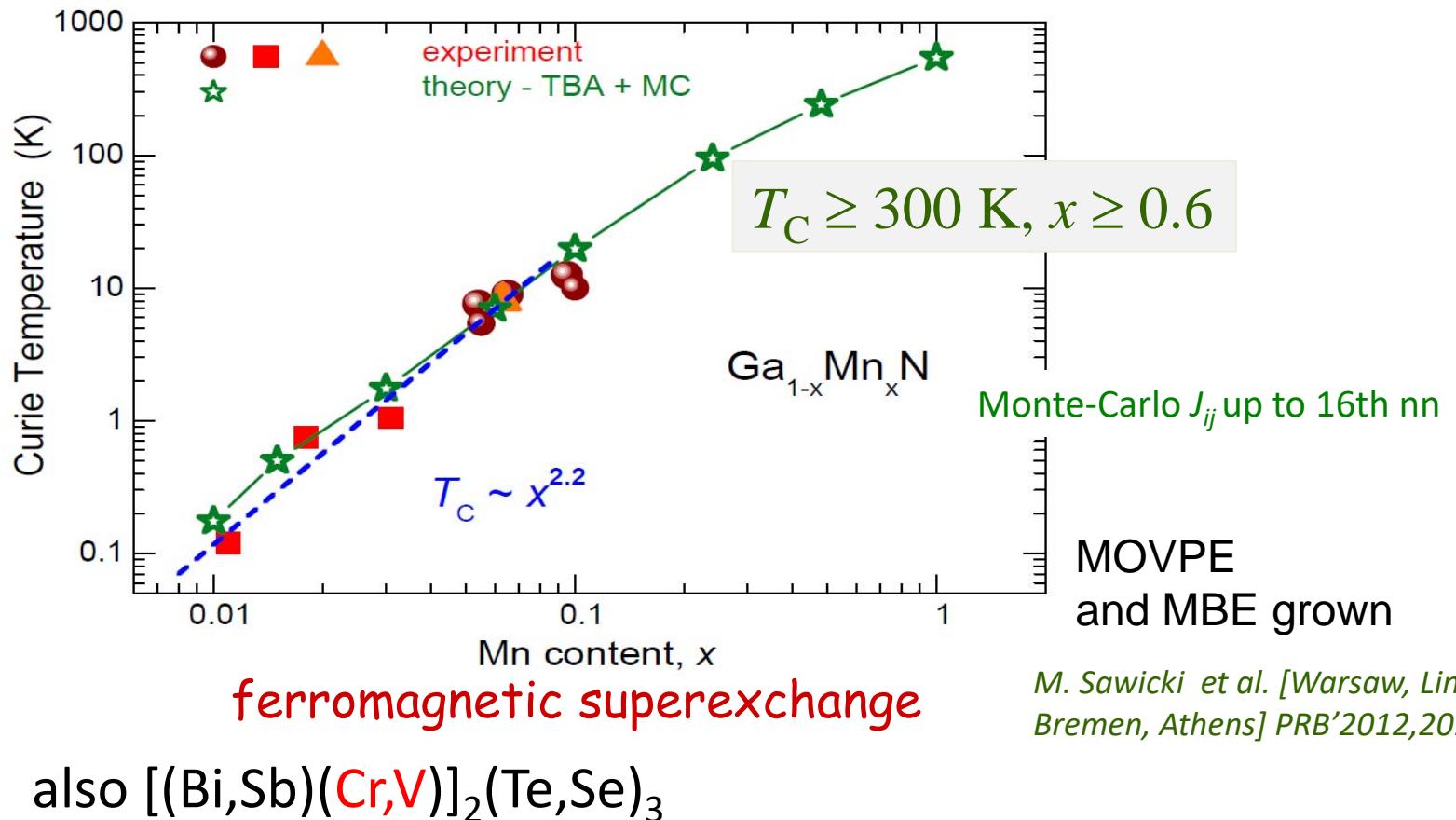
$$H_{ij} = -J_{ij}\mathbf{S}_i\mathbf{S}_j$$



4th order perturb. theory in  $V_{pd}$   
parameters from XAS, PE

S. Stefanowicz et al. [Warsaw, Bremen, Athens] PRB'2013

# Ferromagnetism of semi-insulating wz-Ga<sub>1-x</sub>Mn<sub>x</sub>N with Mn<sup>3+</sup> expl/theory



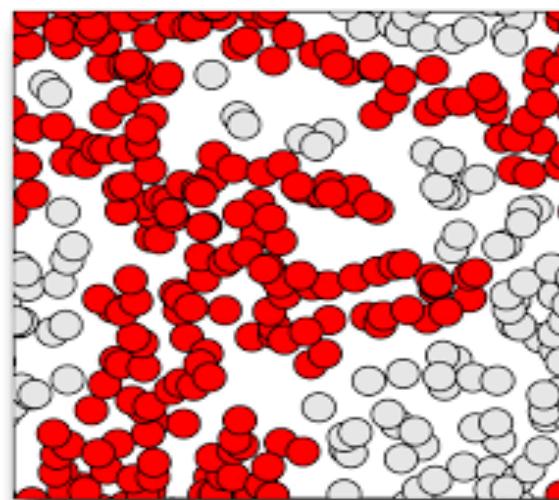
Room temperature ferromagnetic features in DMS (or in nominally non-magnetic semiconductors or oxides) result from spinodal decomposition or contamination

*G. Karczewski et al. [Warsaw, Wuerzburg] JSNM'2003*

*T. Dietl et al. [Warsaw,Osaka,Linz,Grenoble, Tsukuba,Tokyo] Rev. Mod. Phys.'2015*

## Random ferromagnets

### Percolation



e.g., A. Aharony, D. Stauffer "Introduction To Percolation Theory" Taylor & Francis, 2003

# Percolation theory of dilute magnets $A_{1-x}TM_x$

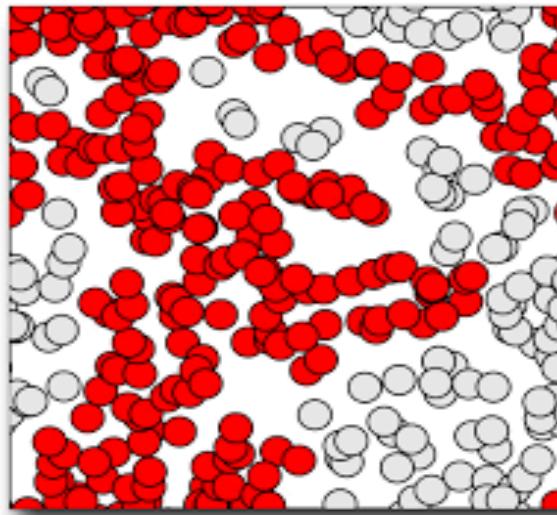
percolation theory  
(overlapping spheres)

$$r_{ij}^{(cr)} = 0.87(N_0x)^{-1/3}$$

$$J_{ij} = J_0 \exp(-r_{ij}/b)$$

spins  $ij$  talk if  $J_{ij} \geq T$

$$T_c = T_0 \exp[-0.87(N_0x)^{-1/3}/b]$$



I.Ya. Korenbilt et al. [Leningrad] Phys. Lett.' 1973

# Testing percolation theory in wz-Ga<sub>1-x</sub>Mn<sub>x</sub>N

percolation theory  
(overlapping spheres)

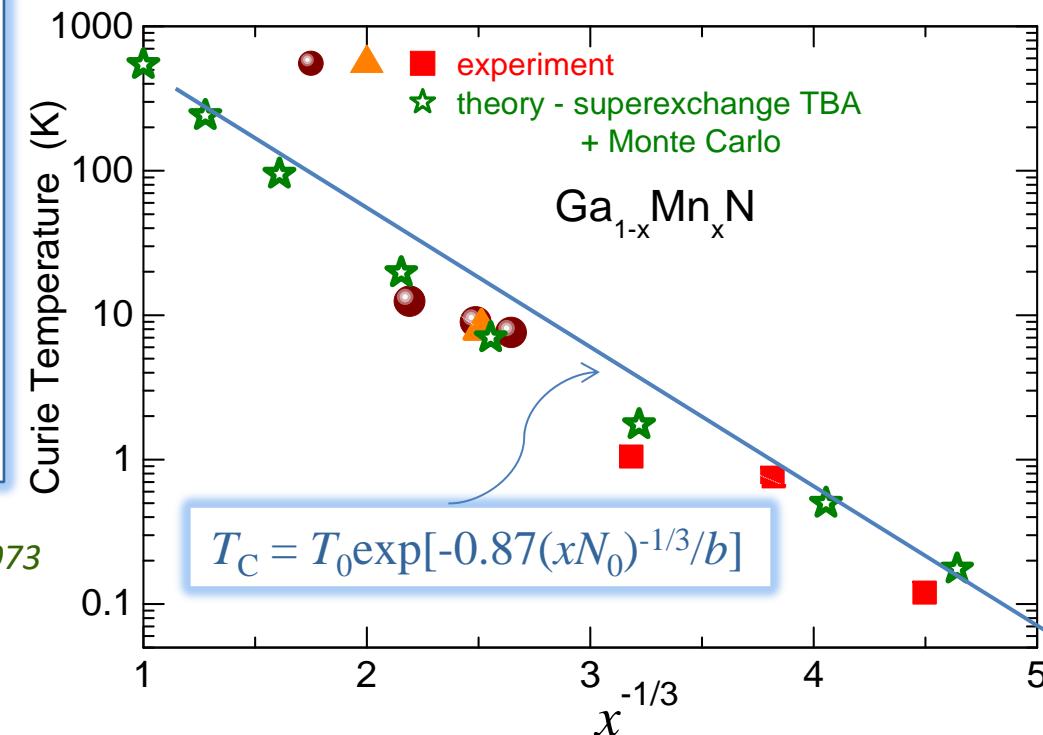
$$r_{ij}^{(cr)} = 0.87(N_0x)^{-1/3}$$

$$J_{ij} = J_0 \exp(-r_{ij}/b)$$

spins  $ij$  talk if  $J_{ij} \geq T$

$$T_c = T_0 \exp[-0.87(N_0x)^{-1/3}/b]$$

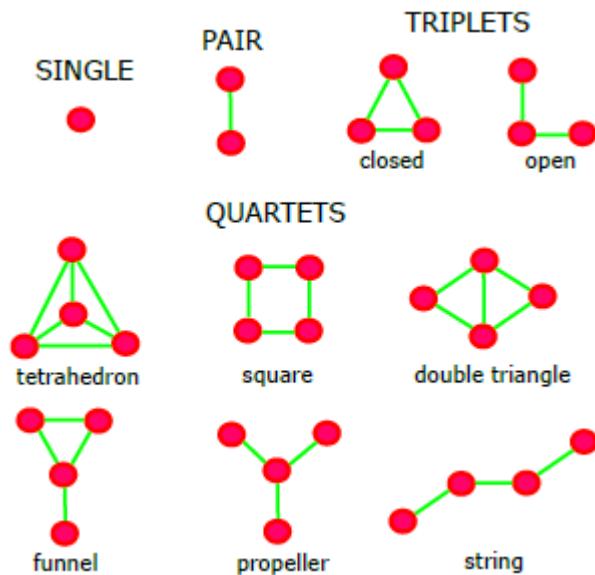
I.Ya. Korenbilt et al. [Leningrad] Phys. Lett.' 1973



random Mn distribution proven

## Two computational developments for $M(H)$

**$M(H)$  from  
quantum levels of ferro coupled**



D. Sztenkiel arXiv:2006.12945  
also NJP -112309.R1 (2020)

**$M(H)$  from atomistic LLG eqn.  
for coupled classical spins**

$$\frac{\partial \mathbf{S}_i}{\partial t} = -\frac{\gamma}{1 + \alpha_G^2} [\mathbf{S}_i \times \mathbf{H}_{eff}^i + \alpha_G \mathbf{S}_i \times (\mathbf{S}_i \times \mathbf{H}_{eff}^i)]$$

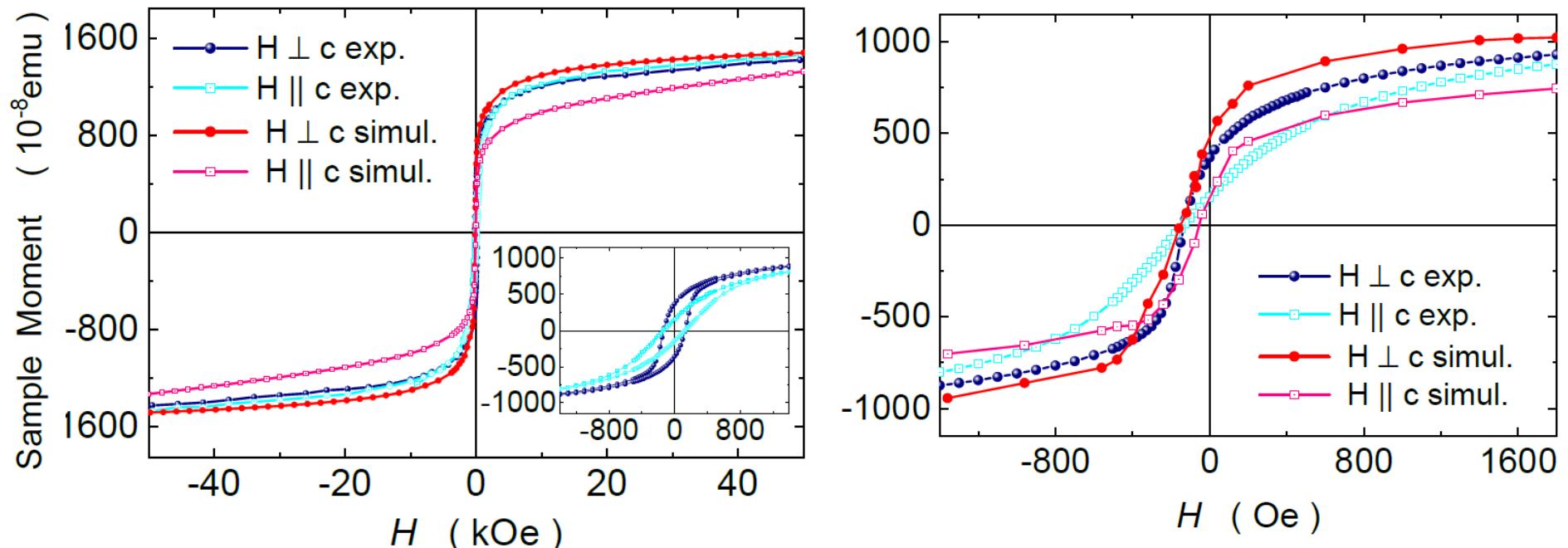
$$\mathbf{H}_{eff}^i = -\frac{1}{\mu_S} \frac{\partial \mathcal{H}}{\partial \mathbf{S}_i} + \mathbf{H}_{th}^i(t)$$

$$\mathcal{H} = \mathcal{H}_{Exch} + \mathcal{H}_{Trig} + \mathcal{H}_{JT} + \mathcal{H}_Z$$

$$\mathcal{H}_{Exch} = - \sum_{i \neq j} J_{i,j} \mathbf{S}_i \mathbf{S}_j$$

D. Sztenkiel et al., JEMS 2020  
cf. R. F. L. Evans et al. [York] JPC'2014

## Experimental (blue) vs. atomistic LLG (red) $M(H)$ at 2 K, $x = 6\%$



8640 Mn spins,  $t = 280$  ns

→ hystereses due to in-plane barriers caused by JT effect

[macrospin model would give square hysteresis]

## Electric field effects in wz-Ga<sub>1-x</sub>Mn<sub>x</sub>N

D. Sztenkiel et al., Nat. Commun. 7, 13232 (2016); JEMS 2020

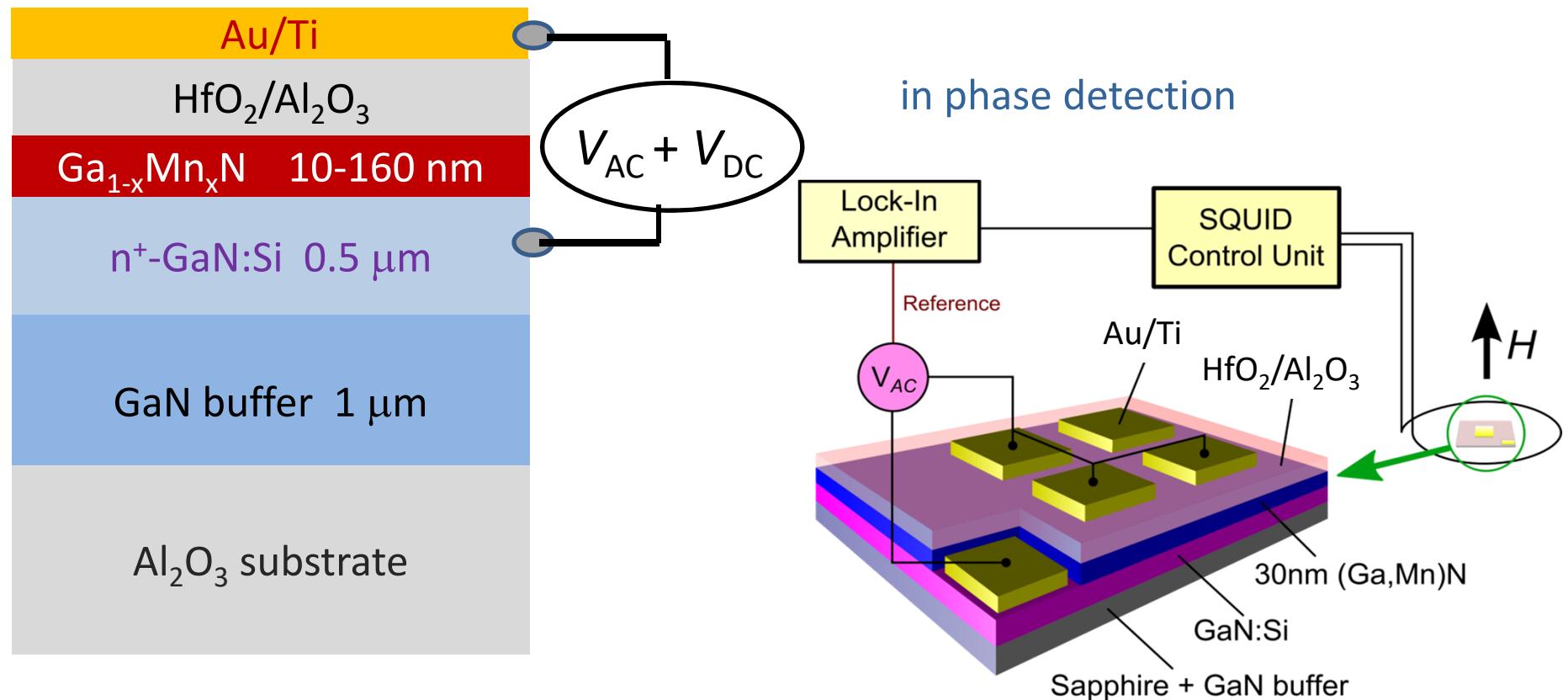
$$M_E^{(i)} = \alpha_{ji} \mathcal{E}_j + \beta_{jki} \mathcal{E}_j B_k + \dots$$

$\alpha_{ji} \neq 0$  if  $\mathcal{T}$  and  $\mathcal{I}$  broken

$\beta_{jki} \neq 0$  if  $\mathcal{I}$  broken

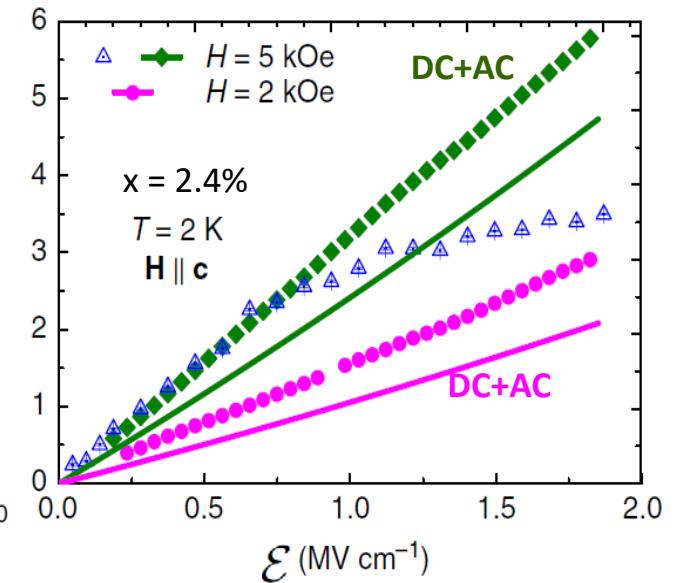
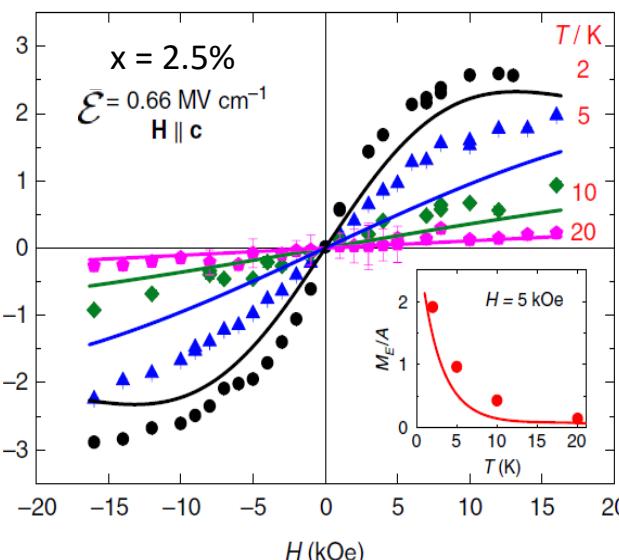
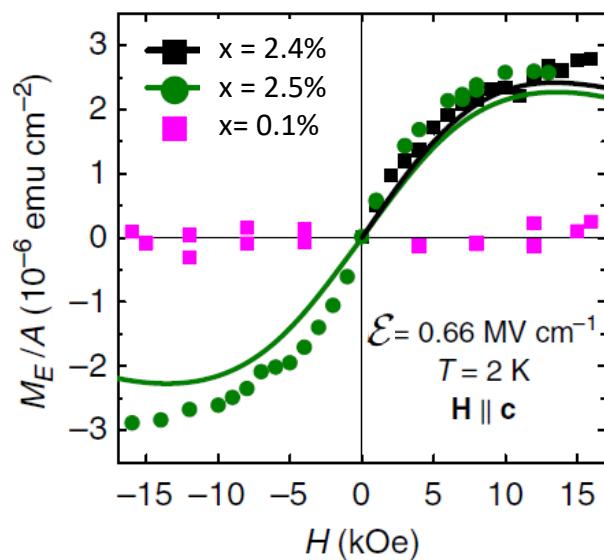
$\mathcal{I}$  broken in wz crystals

## Capacitor structure and expl set-up



## $M_E$ in paramagnetic $\text{Ga}_{0.98}\text{Mn}_{0.02}\text{N}$ – exp. vs. theory

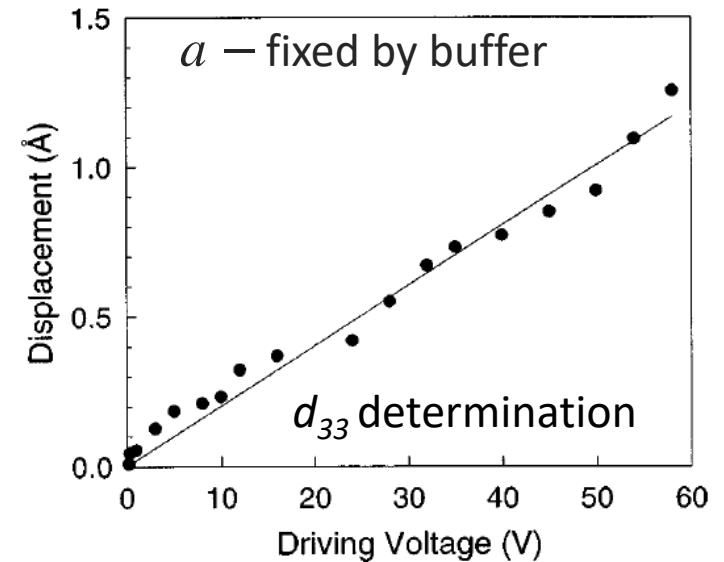
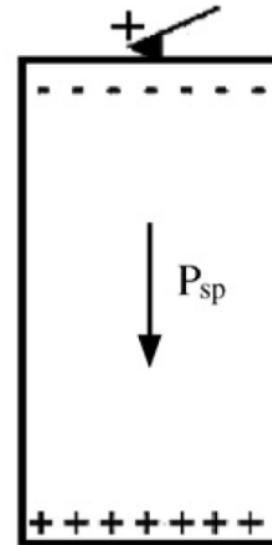
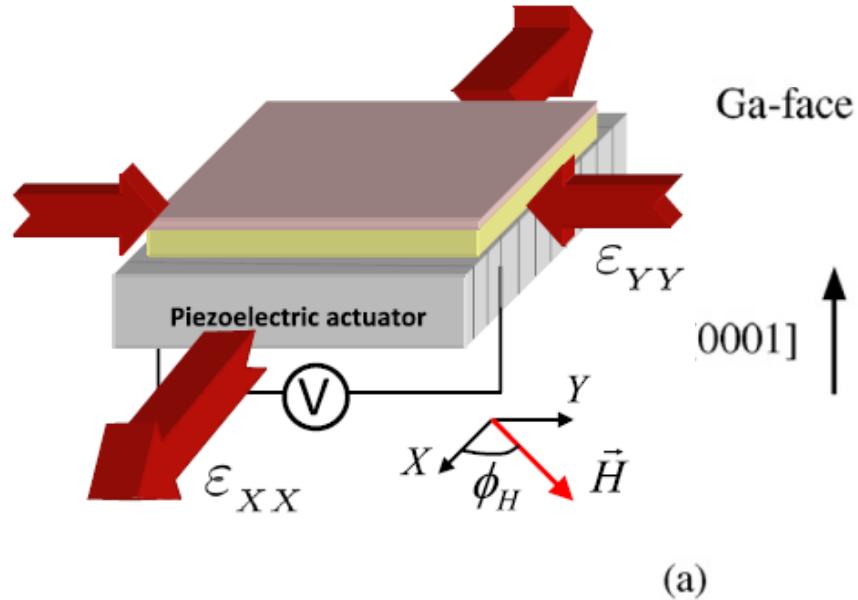
$$M_E^{(i)} = \alpha_{ji} \cancel{\mathcal{E}_j} + \beta_{jki} \mathcal{E}_j B_k + \dots$$



$$M_E^{(\max)} \approx 0.04 M_{\text{Sat}}$$

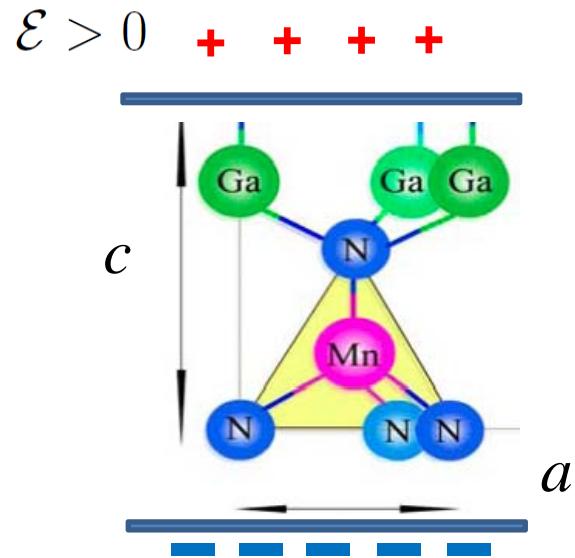
no adjustable parameters

# Piezoelectricity of wz-GaN



I.L. Guy et al. [Macquarie U.] APL'1999

## Piezo-electromagnetic effect (PEME) in wz-Ga<sub>1-x</sub>Mn<sub>x</sub>N



The dominant effect  
of the electric field:

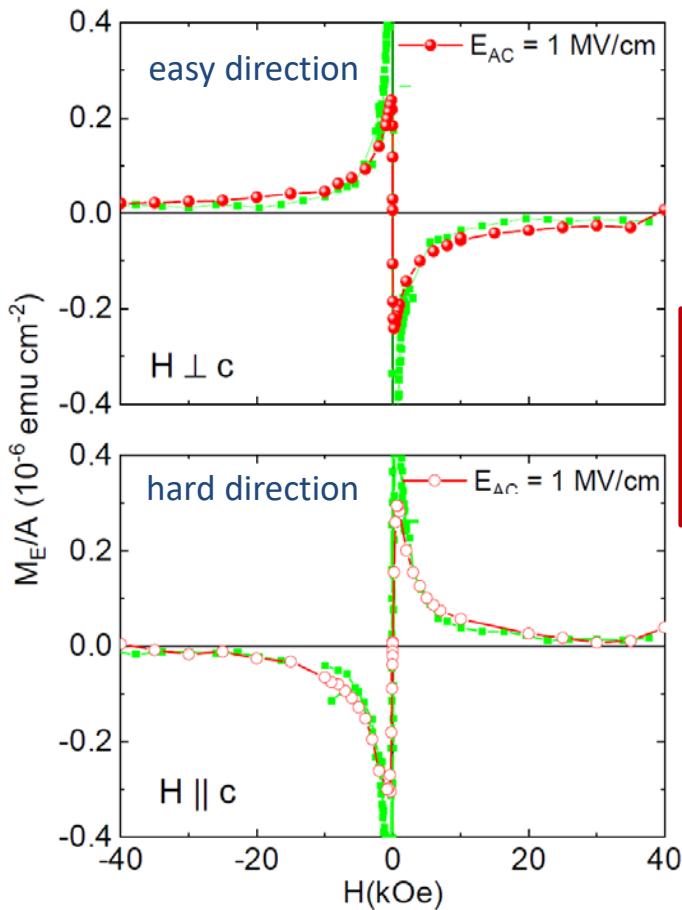
$$\mathcal{H}_{\text{Trig}} \propto c(\mathcal{E})/a - (8/3)^{1/2}$$

$$\Delta c(\mathcal{E})/c = d_{33}\mathcal{E}$$

$\varepsilon_{33}$  up to 0.1%

# Electric field effects in ferromagnetic wz-Ga<sub>1-x</sub>Mn<sub>x</sub>N

$x = 6\%$   
 $T = 2 \text{ K}$

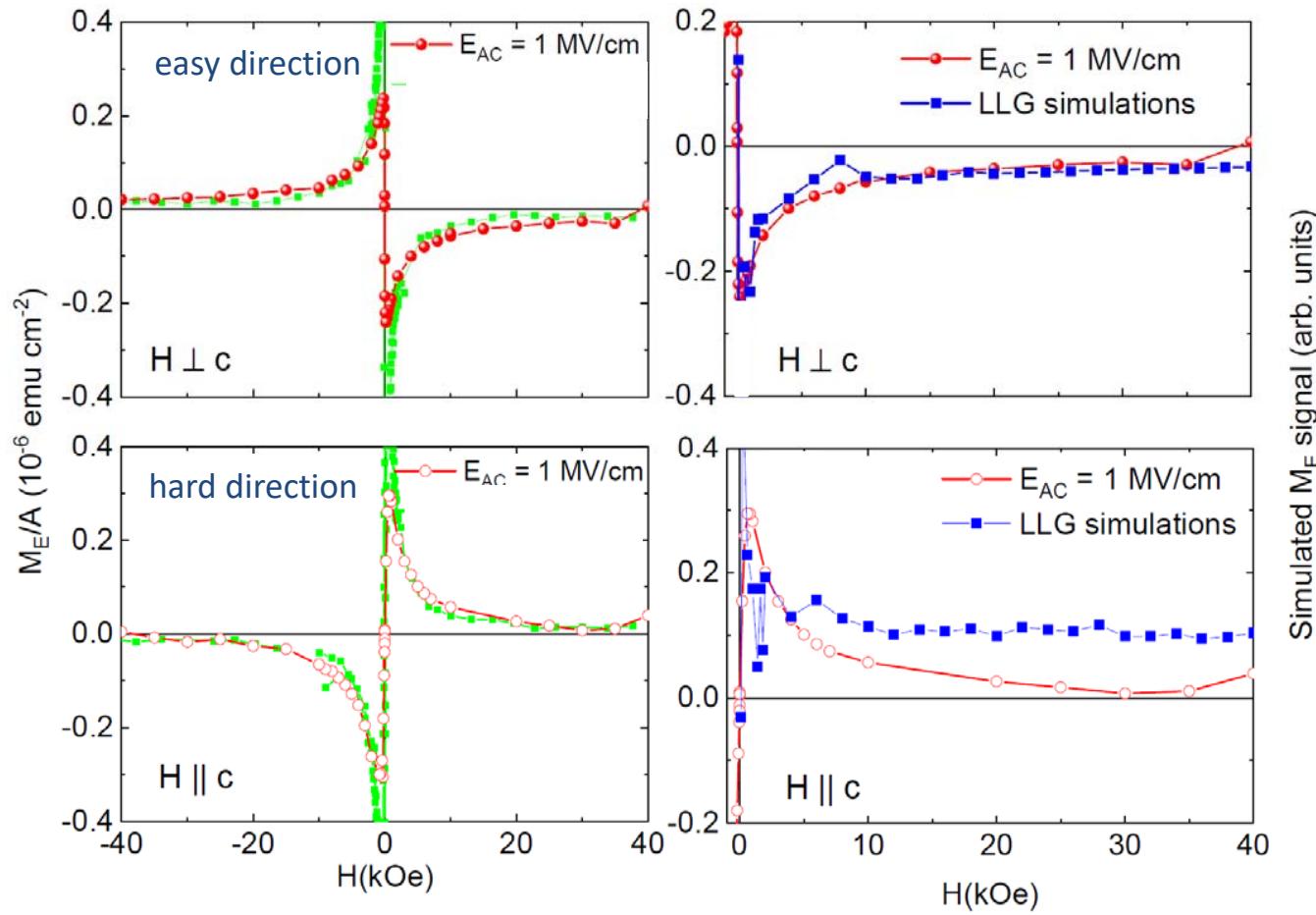


- opposite sign of  $\Delta M_{\perp}(\mathcal{E})$  and  $\Delta M_{\parallel}(\mathcal{E})$
- vanish in high fields

$$M_E^{(\max)} \approx 0.003 M_{\text{Sat}}$$

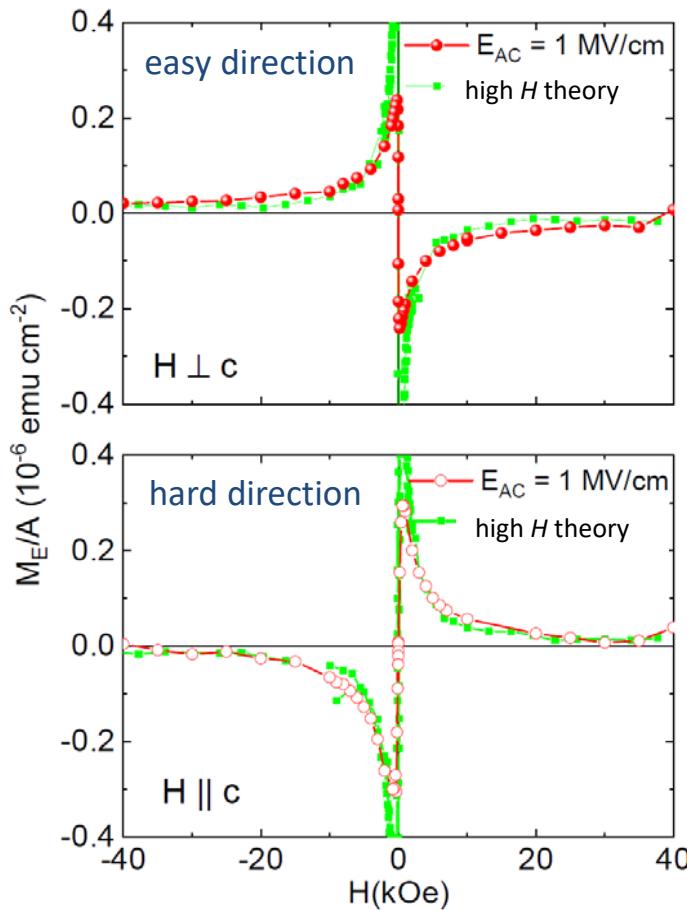
## Exp. results vs. atomistic LLG

$x = 6\%$   
 $T = 2 \text{ K}$



## Exp. results vs. qualitative model

$x = 6\%$   
 $T = 2 \text{ K}$



$$\mathbf{M} = \mathbf{M}(\mathbf{H}_{\text{eff}}) = \mathbf{M}(\mathbf{H} + \mathbf{H}_{\text{an}})$$

$$M_E^{(i)} = \frac{\partial M_i}{\partial \mathbf{H}_{\text{eff}}} \frac{\partial \mathbf{H}_{\text{eff}}}{\partial \mathcal{E}} \mathcal{E}$$

$$\mathbf{H}_{\text{TR}} = b[c(\mathcal{E})/a - (8/3)^{1/2}] [-M_x, M_z]$$

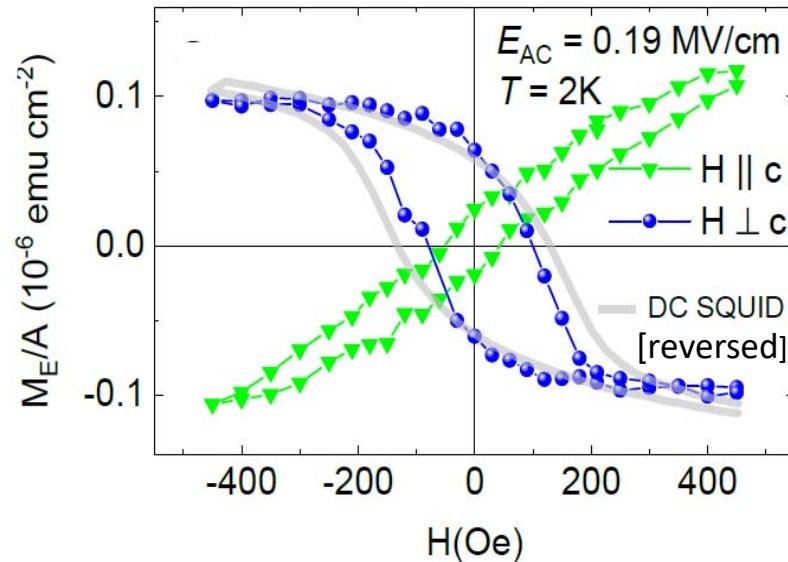
$$M_E^{(x)} = -b \frac{\partial M_x}{\partial H_{\text{eff}}^{(x)}} M_x d_{33}(c/a) \mathcal{E}$$

$$M_E^{(z)} = b \frac{\partial M_z}{\partial H_{\text{eff}}^{(z)}} M_z d_{33} \mathcal{E}$$

in high fields  $\frac{\partial M_{x,z}}{\partial \mathbf{H}_{\text{eff}}} \rightarrow \frac{\partial M_{x,z}}{\partial \mathbf{H}}$

## PEME in weak $H$ fields

$x = 6\%$



in weak magnetic fields time-reversal  
symmetry broken only by  $\mathcal{M}$

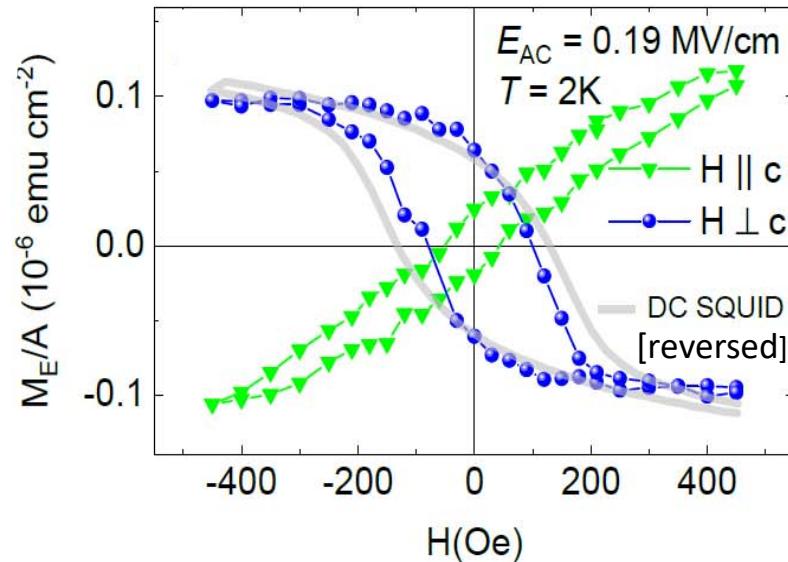
$$M_E^{(x)} = -b \frac{\partial M_x}{\partial H_{\text{eff}}^{(x)}} M_x d_{33}(c/a) \mathcal{E}$$

$$M_E^{(z)} = b \frac{\partial M_z}{\partial H_{\text{eff}}^{(z)}} M_z d_{33} \mathcal{E}$$

$$M_E^{(i)} = \alpha_{ji} \mathcal{E}_j + \beta_{jkl} \mathcal{E}_j B_k + \dots$$

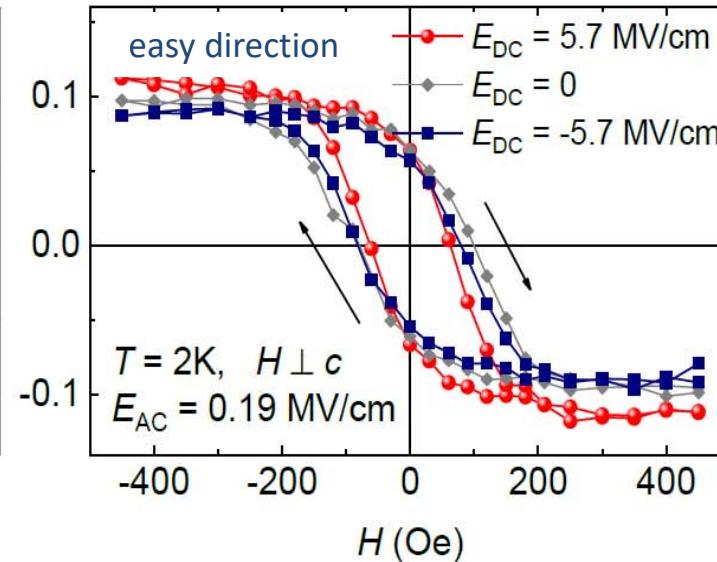
## PEME in weak $H$ fields

$x = 6\%$



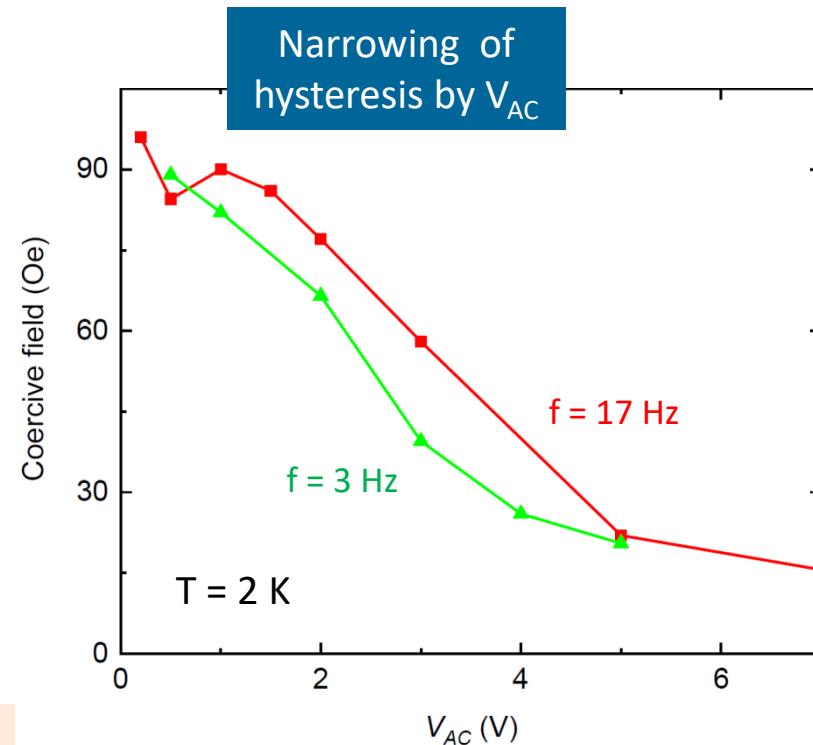
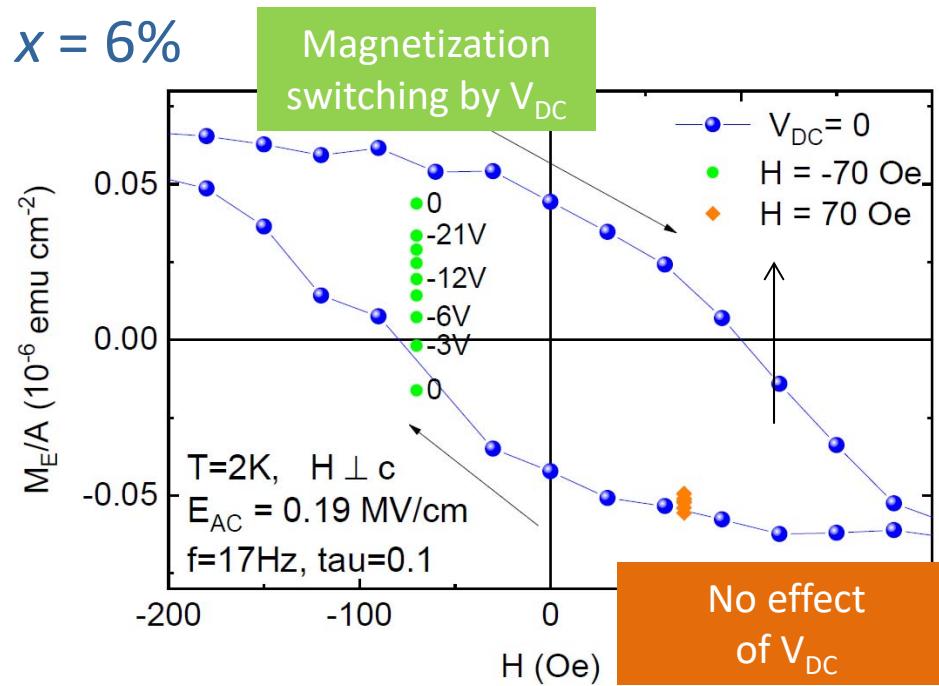
in weak magnetic fields time-reversal symmetry broken only by  $M$

$$M_E^{(i)} = \alpha_{ji} \mathcal{E}_j + \beta_{jki} \cancel{\mathcal{E}_j B_k} + \dots$$



hysteresis width determined by JT  
 $\rightarrow$  independent of  $\mathcal{E}_{DC}$

## Magnetization switching



### Models:

- heating by capacitor charging
- precessional switching *D. Sztenkiel et al. JEMS 2020*

# Hole-mediated ferromagnetism in Mn-doped semiconductors

- holes due to Mn itself

$(\text{In,Mn})\text{As}$ ;  $(\text{Ga,Mn})\text{As}$  *H. Ohno et al. [IBM, Tohoku] PRL'92, APL'96*  $T_C$  up to 200 K

$(\text{Sb,Mn})_2\text{Te}_3$ ;  $(\text{Bi,Mn})_2\text{Te}_3$  *Choi et al. [Ulsan] pps(b)'04*  $T_C$  up to 20 K

- holes due to acceptor impurities

$(\text{Cd,Mn})\text{Te:N}$ ,  $(\text{Zn,Mn})\text{Te:P}$   $T_C$  up to 5 K

*TD, Cibert et al. [Grenoble, Warsaw] PRB'97, PRL'97, PRL'03*

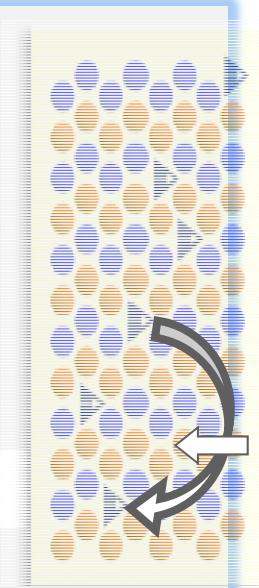
$(\text{K,Ba})(\text{Zn,Mn})_2\text{As}_2$  *K. Zhao al. [Beijing, Columbia U.] Nat. Commun.'13*  $T_C$  up to 230 K

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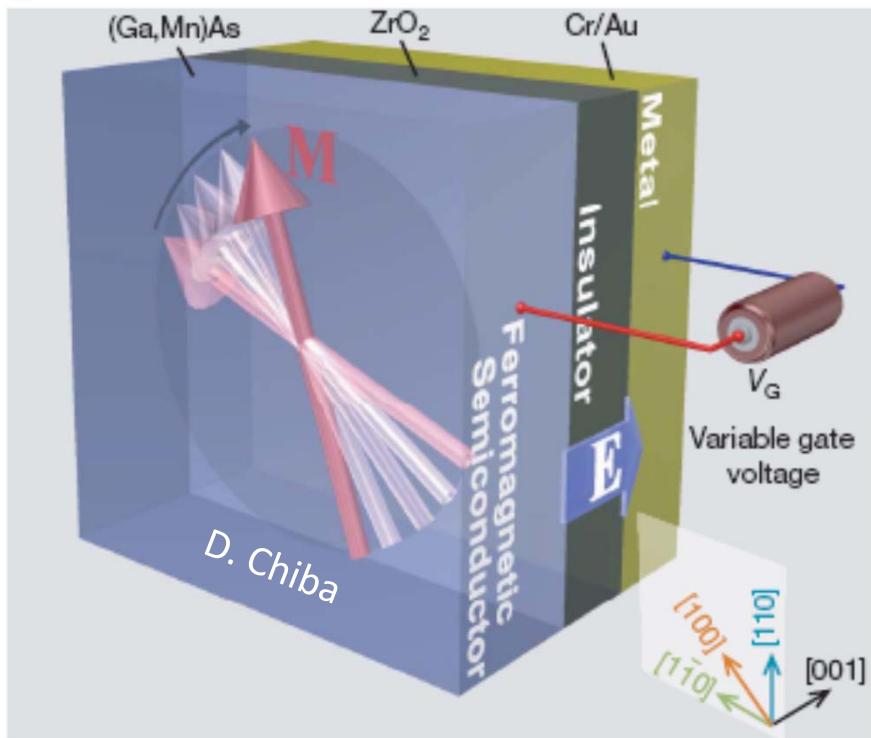
$(\text{Pb,Sn,Mn})\text{Te}$  *T. Story et al. [Warsaw] PRL'86*  $T_C$  up to 10 K

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# Magnetization manipulation by gating



$$\operatorname{div} \mathcal{E}(z) = \rho(z)/\epsilon \Rightarrow T_C(z)$$

also multilayers, interfaces, junctions, ...

cf. T. Dietl, H. Ohno, RMP'2014

F. Matsukura, Y. Tokura, H. Ohno, Nat. Nano.'2015

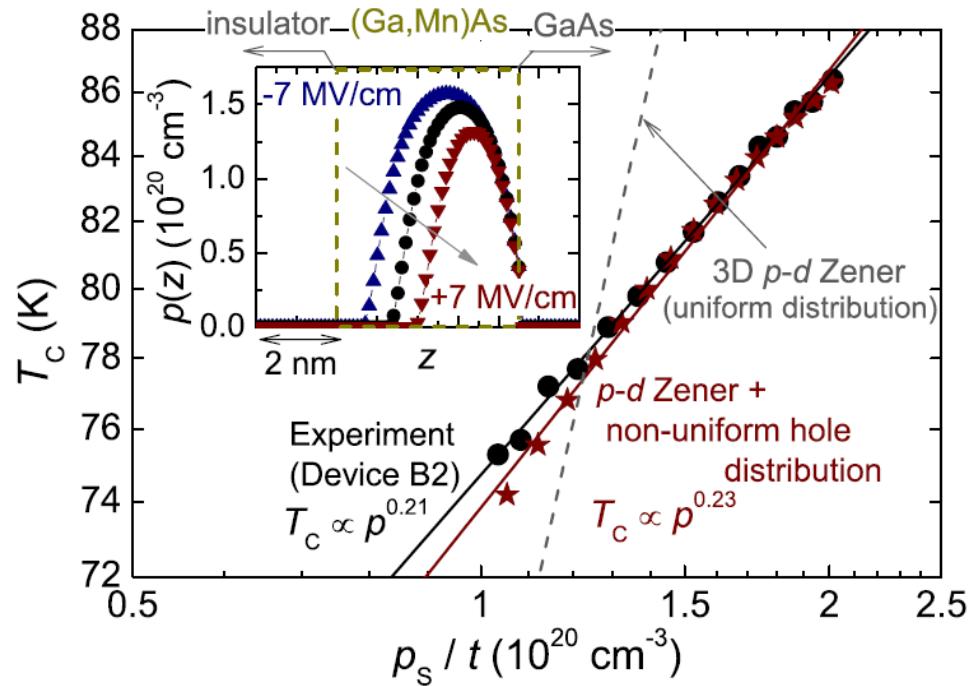
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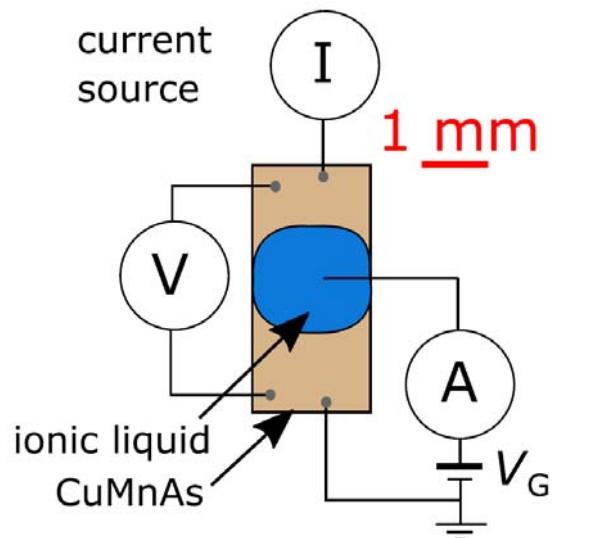
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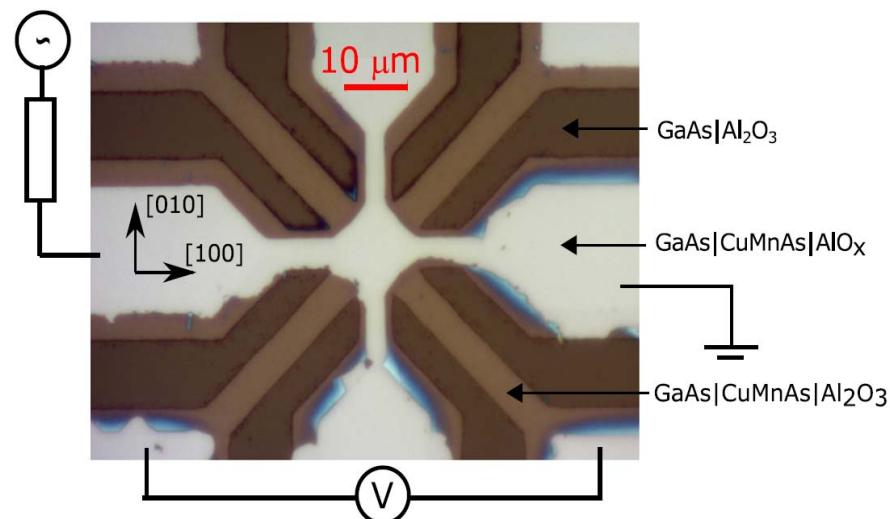
*Y. Nishitani et al. [Tohoku, Warsaw] PRB.'2010*

## Gating semimetallic CuMnAs at 300 K

MBE: 10 nm tetragonal CuMnAs on (001) GaAs  
protected by 2.5 nm oxidized Al



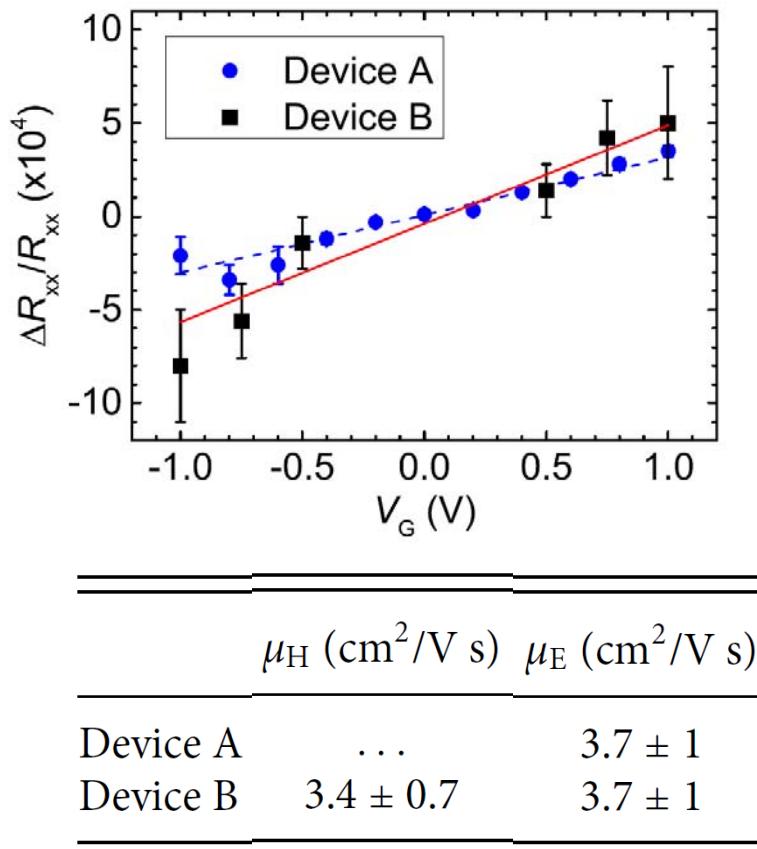
ionic liquid DEME-TFSI  
 $C/S = (4.4 \pm 0.8) \times 10^{-7} \text{ F/cm}^2$



$$p_{\text{Hall}} = (5 \pm 1) \cdot 10^{21} \text{ cm}^{-3}$$

M. Grzybowski et al. [Warsaw, Nottingham] AIP Adv.'2019

## Field effect and mobility at room temperature



field mobility

$$\mu_E = \frac{L}{fWC/S} \frac{1}{R_{xx}^2} \frac{\partial R_{xx}}{\partial V_G}$$

as  $\mu_H = \mu_E$  no evidence for:

- multicarrier transport
- anomalous Hall effect
- electrochemical interfacial reactions

## Summary: concepts in dilute magnetic materials

spinodal decomposition

self-organization

high  $T_c$  ferromagnetism

anisotropic spinodal decomposition

nematicity

antiferromagnetic superexchange

ferromagnetic superexchange

frustration

percolation

dilute ferromagnetic insulators

dilute ferromagnetic p-type semiconductors

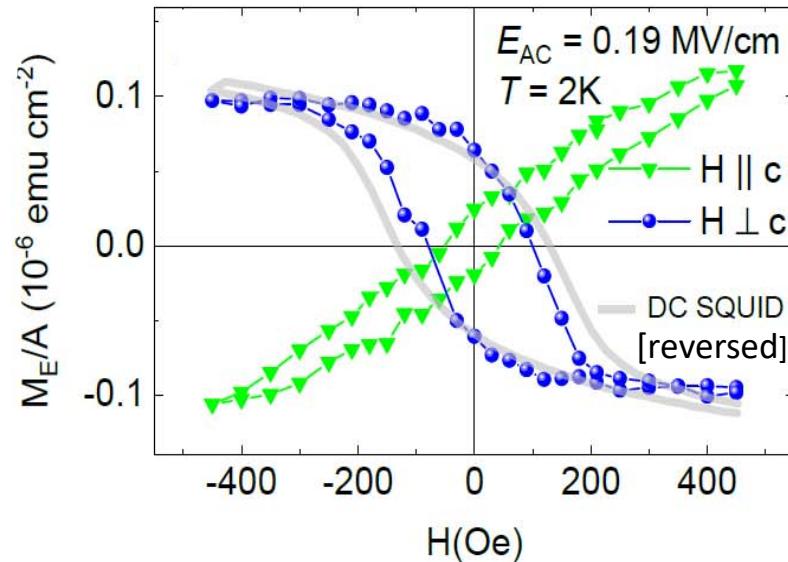
piezo-electromagnetic effects

gating effects

$T_c$  in electrically non-uniform systems

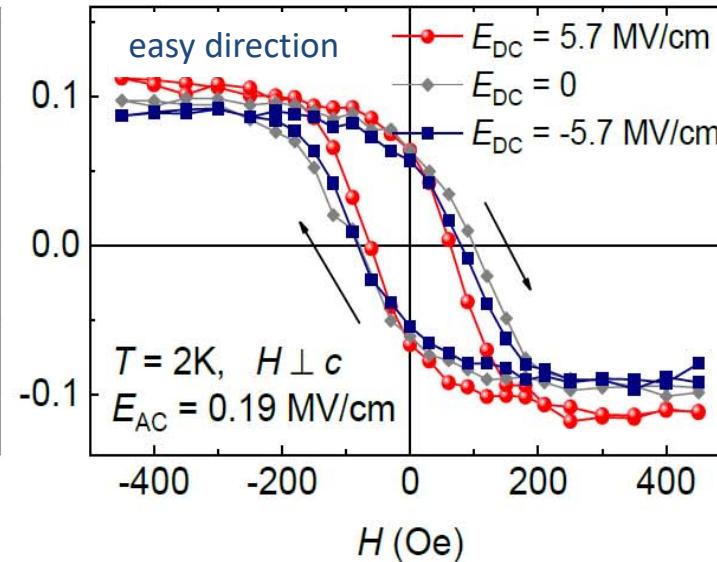
## PEME in weak $H$ fields

$x = 6\%$



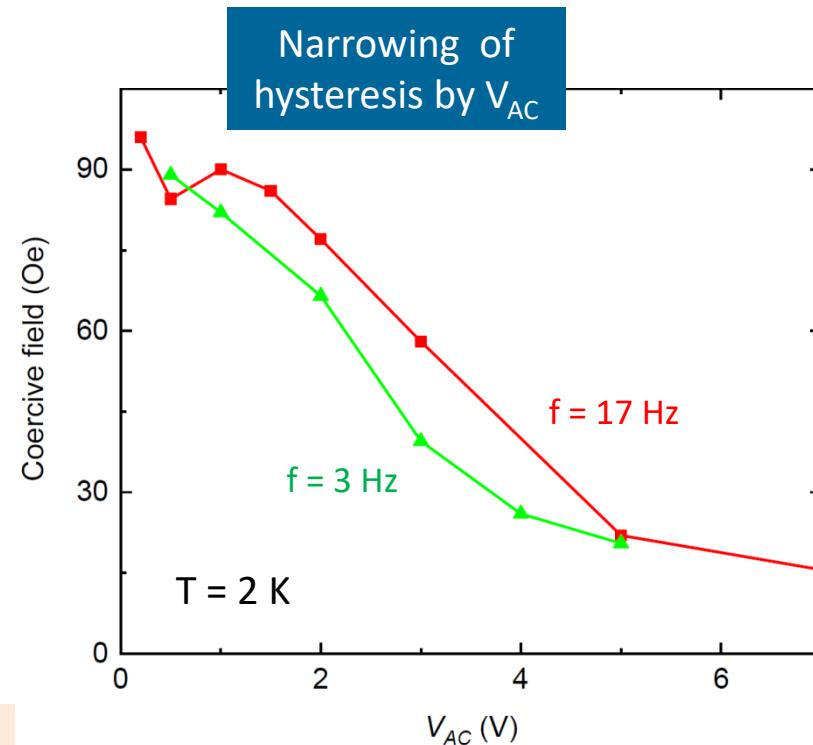
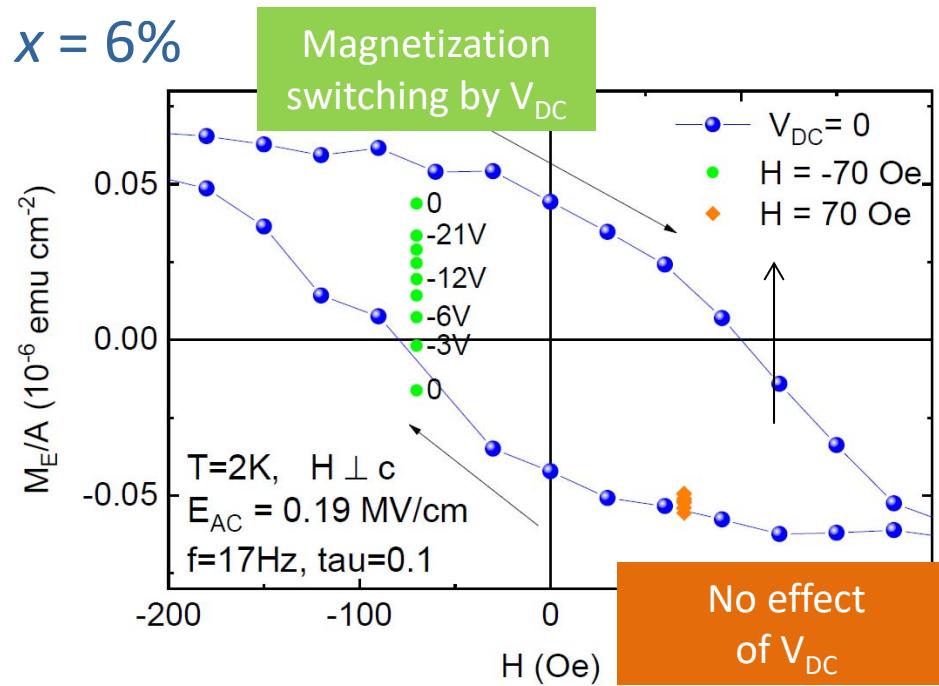
in weak magnetic fields time-reversal symmetry broken only by  $M$

$$M_E^{(i)} = \alpha_{ji} \mathcal{E}_j + \beta_{jki} \cancel{\mathcal{E}_j B_k} + \dots$$



hysteresis width determined by JT  
 $\rightarrow$  independent of  $\mathcal{E}_{DC}$

## Magnetization switching



### Models:

- heating by capacitor charging
- precessional switching *D. Sztenkiel et al. JEMS 2020*

# Hole-mediated ferromagnetism in Mn-doped semiconductors

- holes due to Mn itself

$(\text{In,Mn})\text{As}$ ;  $(\text{Ga,Mn})\text{As}$  *H. Ohno et al. [IBM, Tohoku] PRL'92, APL'96*  $T_C$  up to 200 K

$(\text{Sb,Mn})_2\text{Te}_3$ ;  $(\text{Bi,Mn})_2\text{Te}_3$  *Choi et al. [Ulsan] pps(b)'04*  $T_C$  up to 20 K

- holes due to acceptor impurities

$(\text{Cd,Mn})\text{Te:N}$ ,  $(\text{Zn,Mn})\text{Te:P}$   $T_C$  up to 5 K

*TD, Cibert et al. [Grenoble, Warsaw] PRB'97, PRL'97, PRL'03*

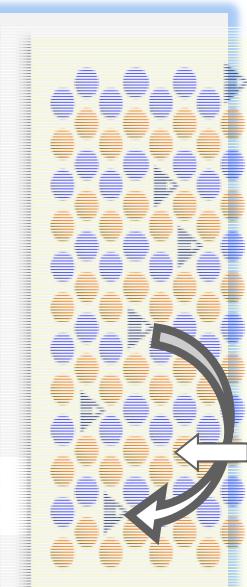
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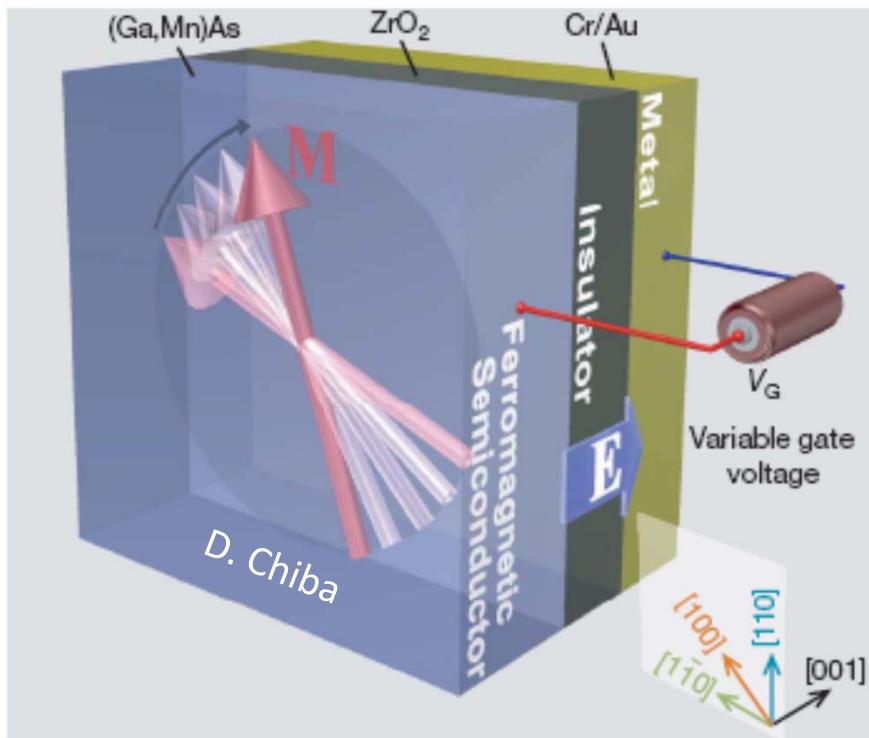
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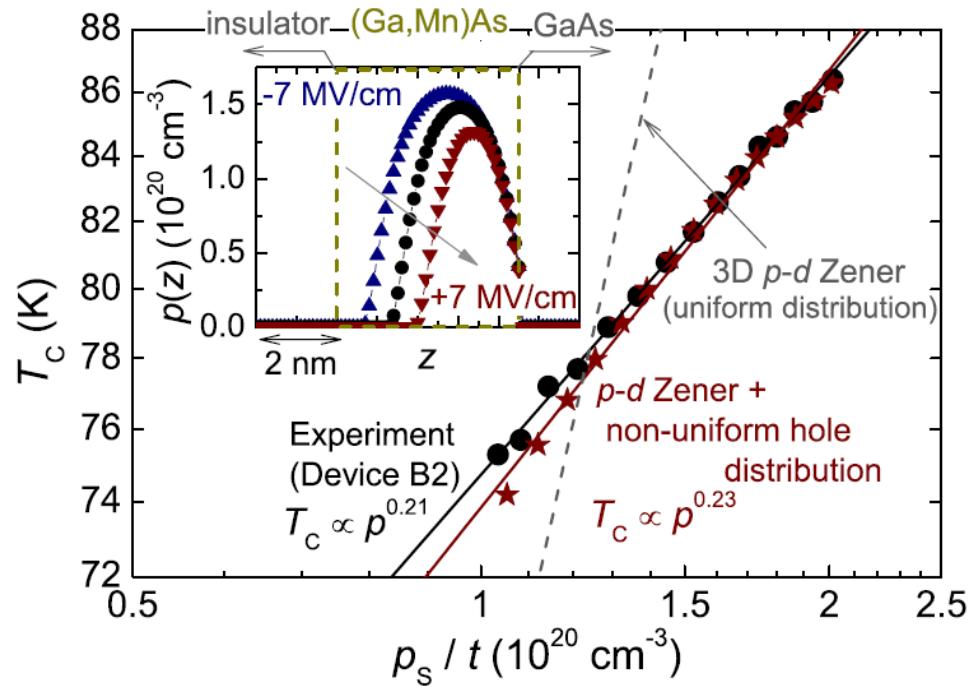
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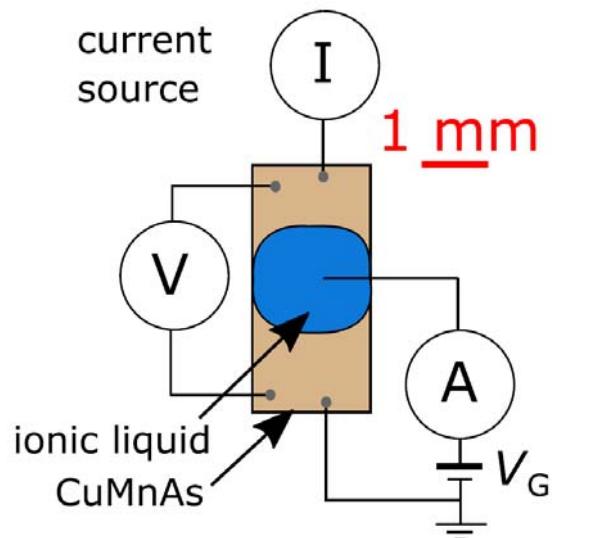
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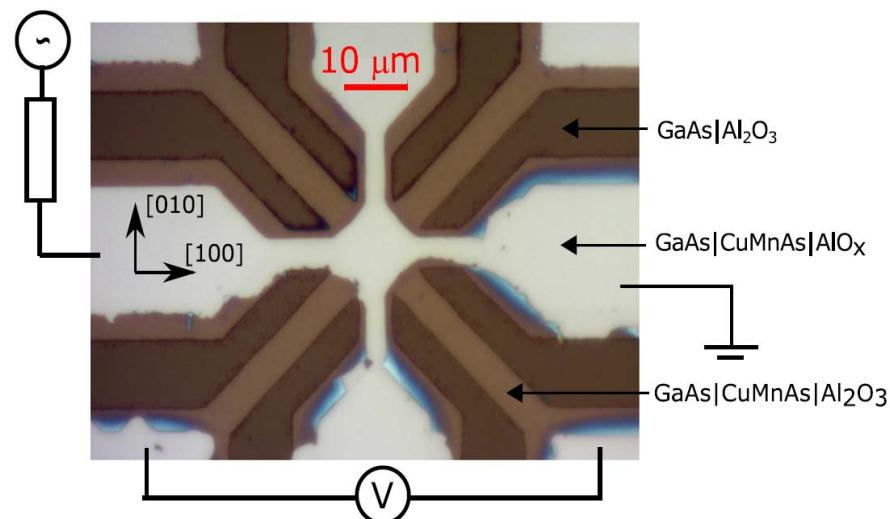
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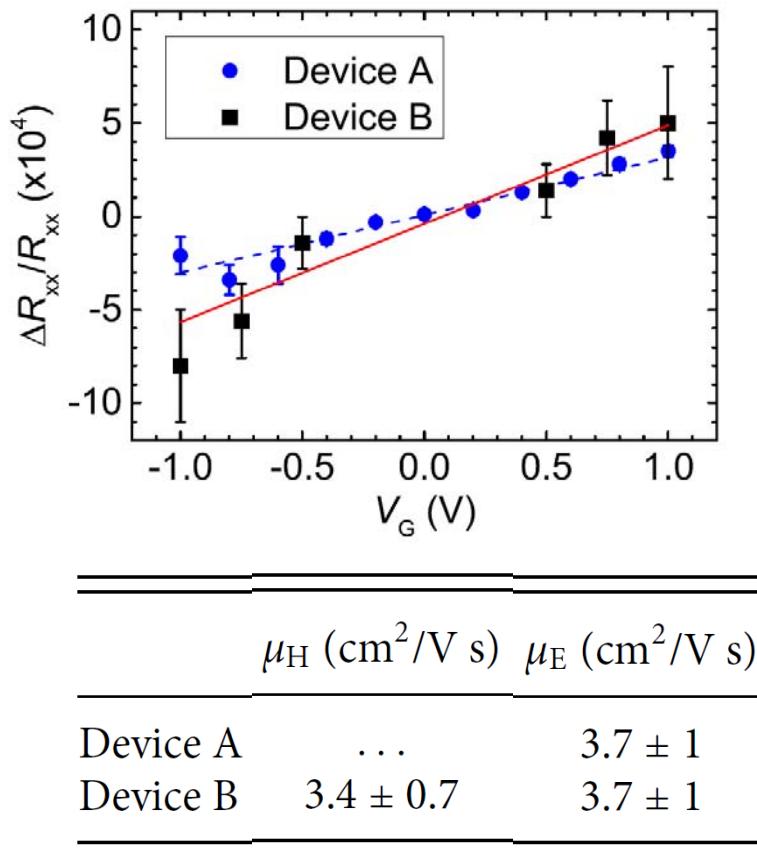
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