# Topological properties and Hall effects

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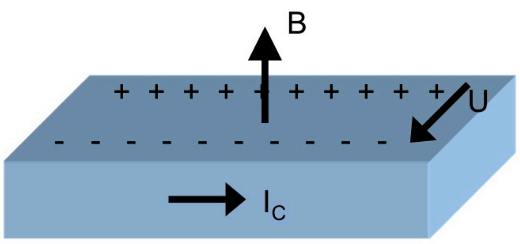
# Outline

- Introduction
- Topological Hall effect
- Anomalous Hall effect for coplanar antiferromagnets
- Magnetic spin Hall effect and spin current vorticity
- Summary

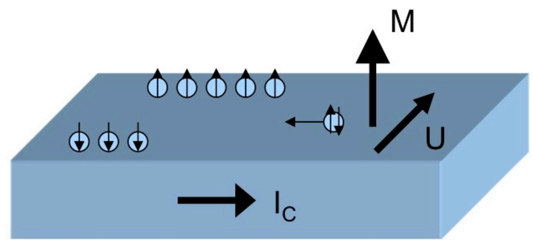
# Transversal transport coefficients

# The Hall trio

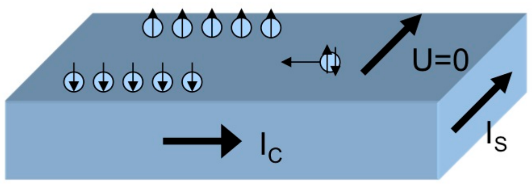
Hall effect
1879




Anomalous Hall effect
1881




Spin Hall effect
2004




Lorentz force

Berry curvature

$$\mathbf{\Omega}_n(\mathbf{k})$$

spin-orbit interaction:

$$\mathbf{s}\cdot\mathbf{L}$$


Nagaosa, Sinova et al., Rev. Mod. Phys. **82**, 1539 (2010)
Sinova, Valenzuela, Wunderlich, Back, and Jungwirth. Rev. Mod. Phys. **87**,1213 (2015)

# Semiclassical equation of motion

Change of momentum:

$$\hbar\dot{\mathbf{k}} = -e\mathbf{E} \qquad - \; e\dot{\mathbf{r}} \times \mathbf{B}(\mathbf{r})$$

Lorentz force

Change of position:

$$\dot{\mathbf{r}} = \frac{\partial E_n(\mathbf{k})}{\hbar\partial\mathbf{k}} - \dot{\mathbf{k}} \times \mathbf{\Omega_n}(\mathbf{k})$$

Anomalous velocity

M.-C. Chang and Q. Niu, Phys. Rev. B **53**, 7010 (1996)

# Ohm's law and conductivity tensor

$$\mathbf{j} = \hat{\sigma} \cdot \mathbf{E} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix} \mathbf{E}$$

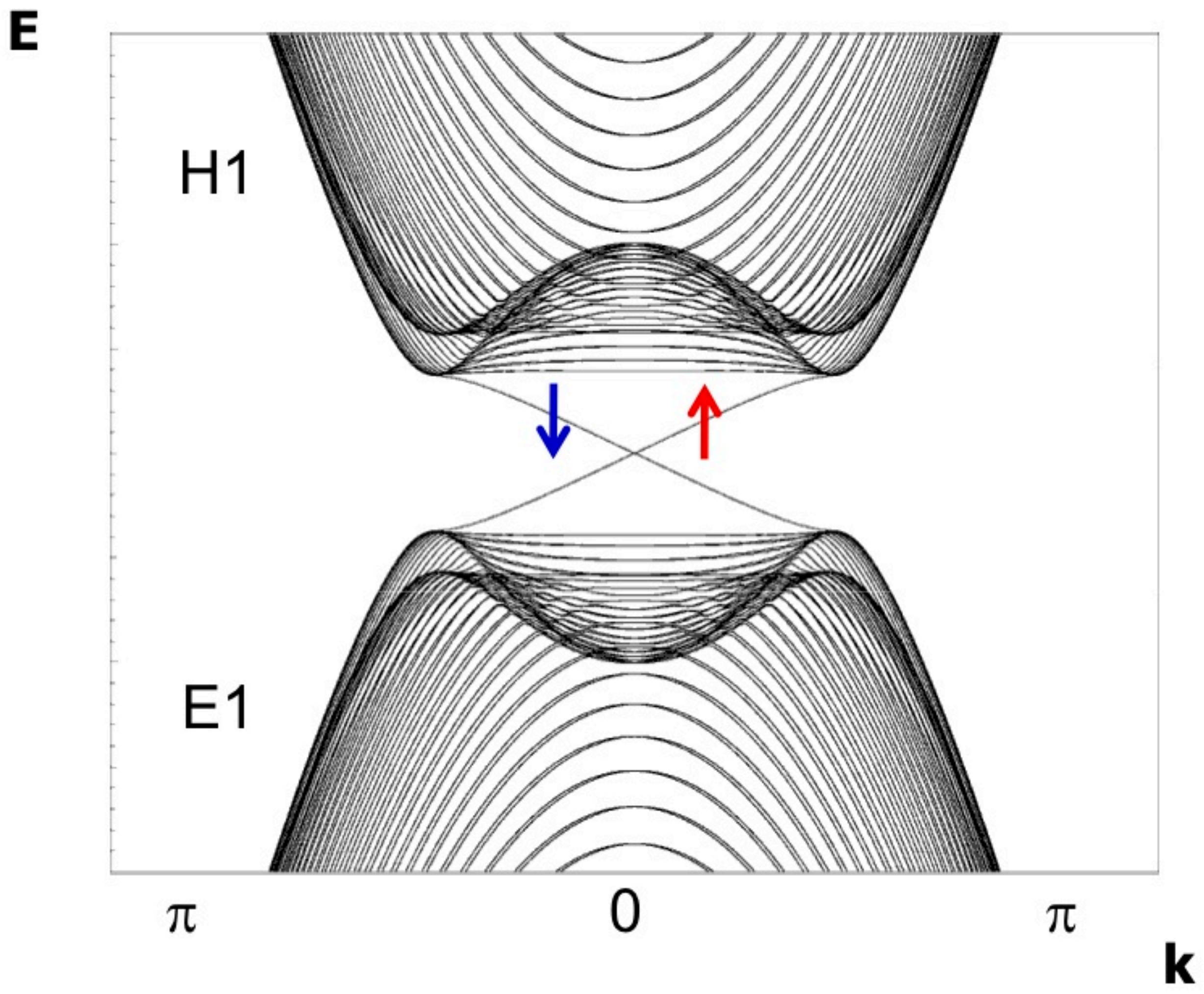
## Anomalous Hall conductivity:

$$\sigma_{\alpha\beta} = -\frac{e^2}{\hbar}\frac{1}{(2\pi)^3}\sum_n \int_{BZ} \mathrm{d}^3k\; f_n(\vec{k})\Omega_n(\vec{k})$$

## Berry curvature:

$$\Omega_n(\vec{k}) = 2i\hbar^2 \sum_{m\neq n} \frac{\langle n|\underline{v}_{\vec{k},\alpha}|m\rangle\langle m|\underline{v}_{\vec{k},\beta}|n\rangle}{\left(\varepsilon_{n\vec{k}} - \varepsilon_{m\vec{k}}\right)^2}$$

# Topological surface state of a $Z_2$ TI




B. A. Bernevig, T. L. Hughes, S. C. Zhang, Science **314**, 1757 (2006)

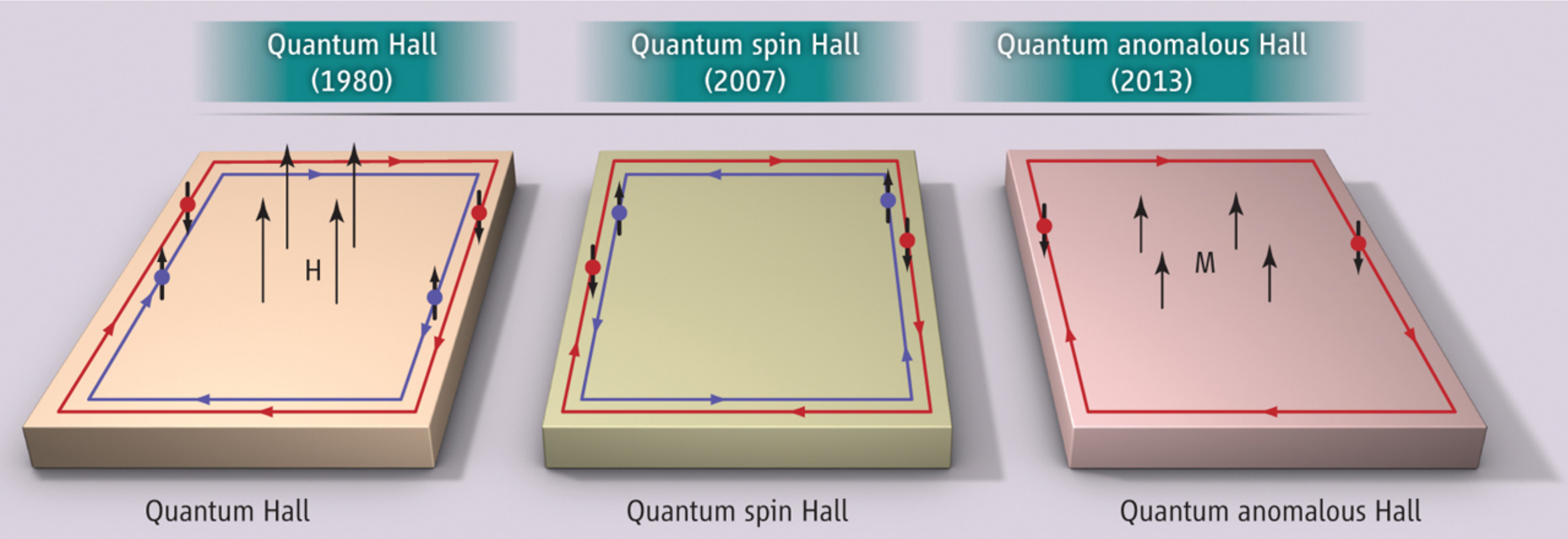
Quantum anomalous Hall conductivity and Chern number:

$$\sigma_{\alpha\beta} = -\frac{e^2}{\hbar}\frac{1}{(2\pi)^3}\sum_n \int_{BZ} \mathrm{d}^3k \; f_n(\vec{k})\Omega_n(\vec{k}) = -\frac{e^2}{\hbar}\,C$$

Berry curvature:

$$\Omega_n(\vec{k}) = 2i\hbar^2 \sum_{m\neq n} \frac{\langle n|\underline{v}_{\vec{k},\alpha}|m\rangle\langle m|\underline{v}_{\vec{k},\beta}|n\rangle}{\left(\varepsilon_{n\vec{k}} - \varepsilon_{m\vec{k}}\right)^2}$$

# The quantum Hall trio




S. Oh, Science **340**,153-154 (2013)

Börge Göbel

Alexander Mook

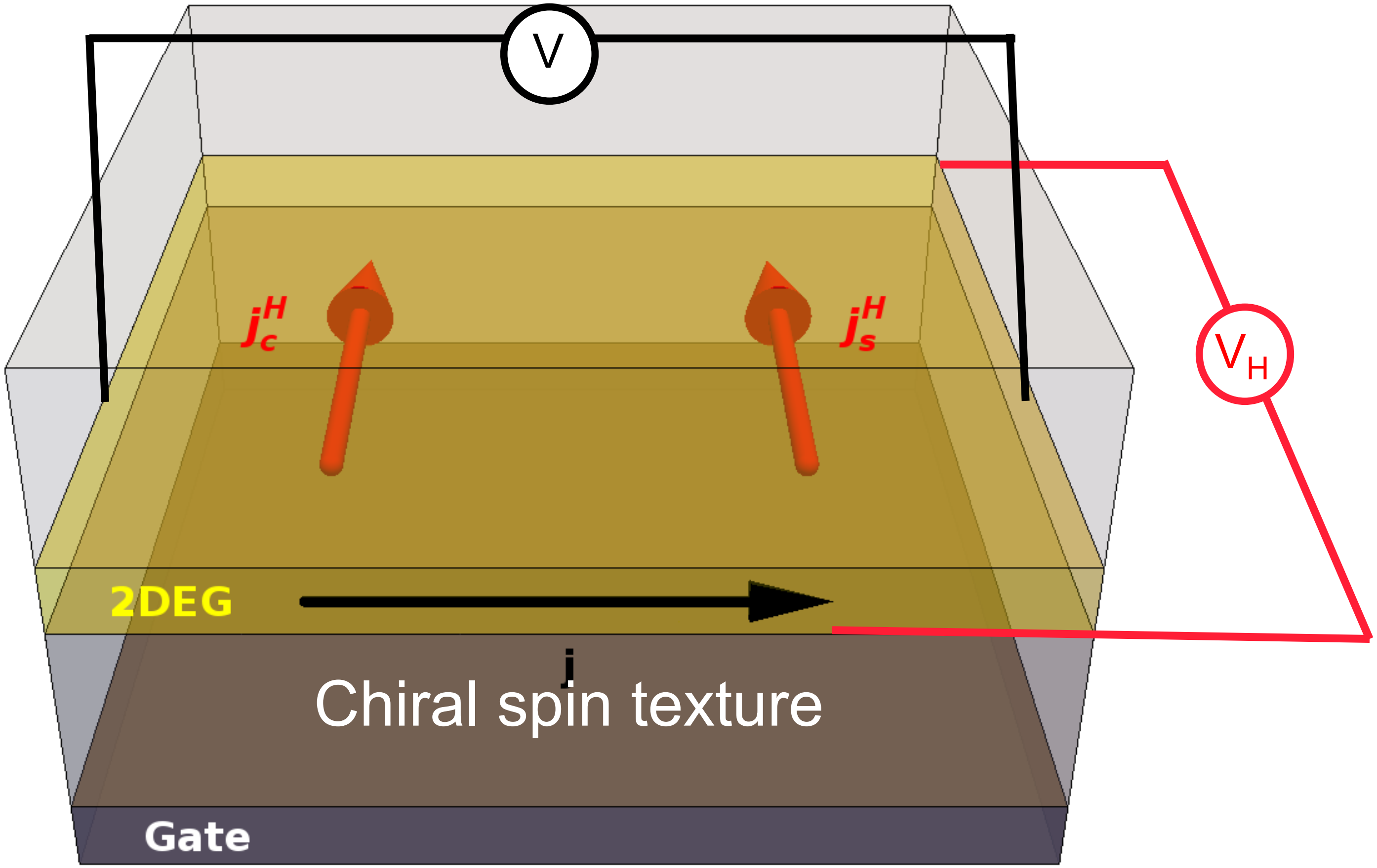
Jürgen Henk

# Topological Hall effect

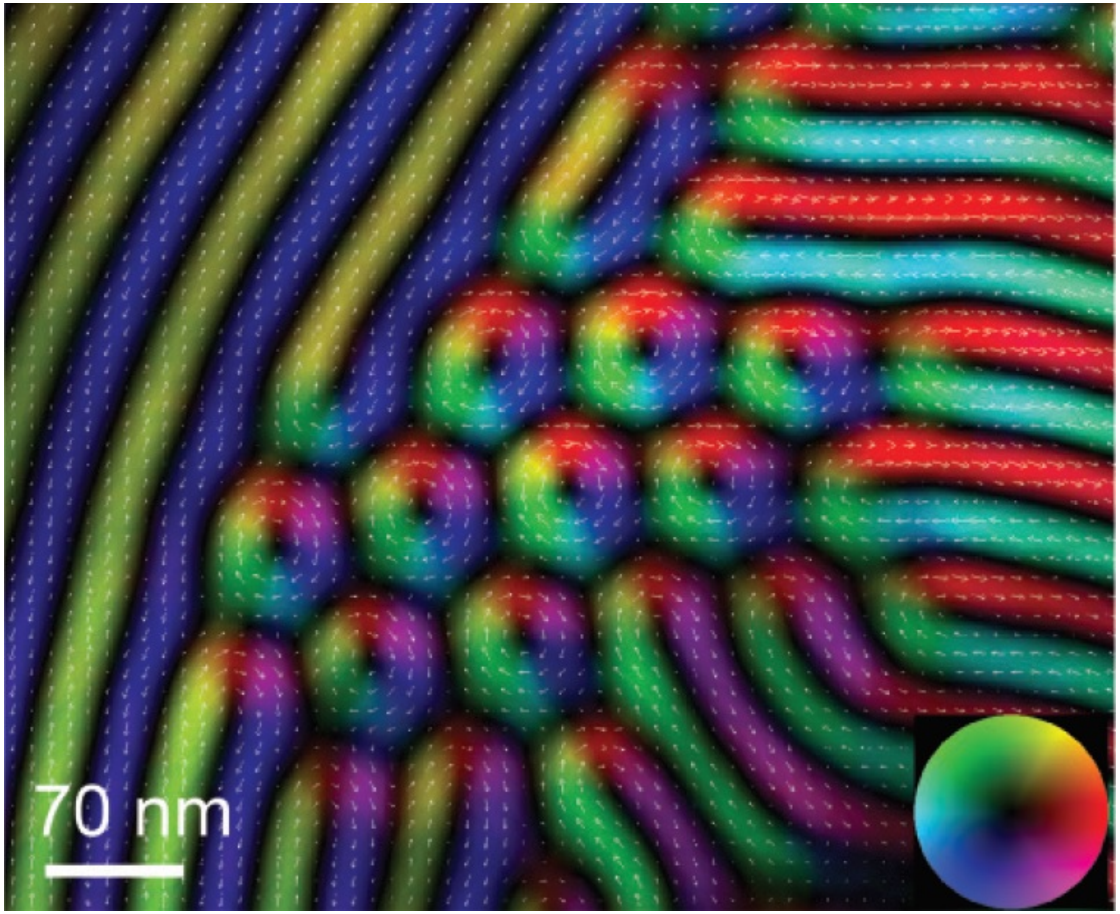
# Topological Hall effect

B. Göbel, A. Mook, J. Henk, and I. M., Phys. Rev. B **95**, 094413 (2017)
B. Göbel, A. Mook, J. Henk, and I. M., New J. Phys. **19**, 063042 (2017)
B. Göbel, A. Mook, J. Henk, and I. M., Eur. Phys. J. B **91**, 179 (2018)
B. Göbel, A. Mook, J. Henk, and I. M., phys. stat. sol. (b) **257**, 1900518 (2020)

# Experiment to measure the THE

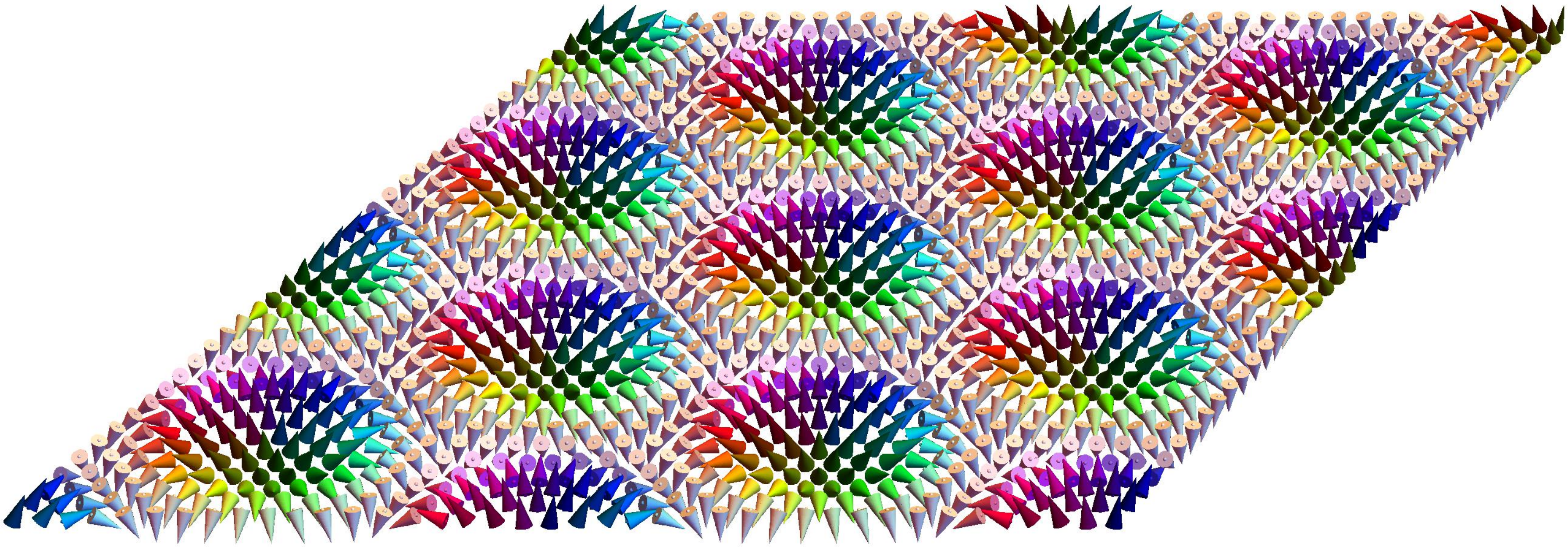


# Skyrmions




M. Nagao et al., Experimental observation of multiple-q states for the magnetic skyrmion lattice and skyrmion excitations under a zero magnetic field. Phys. Rev. B **92**, 140415 (2015)

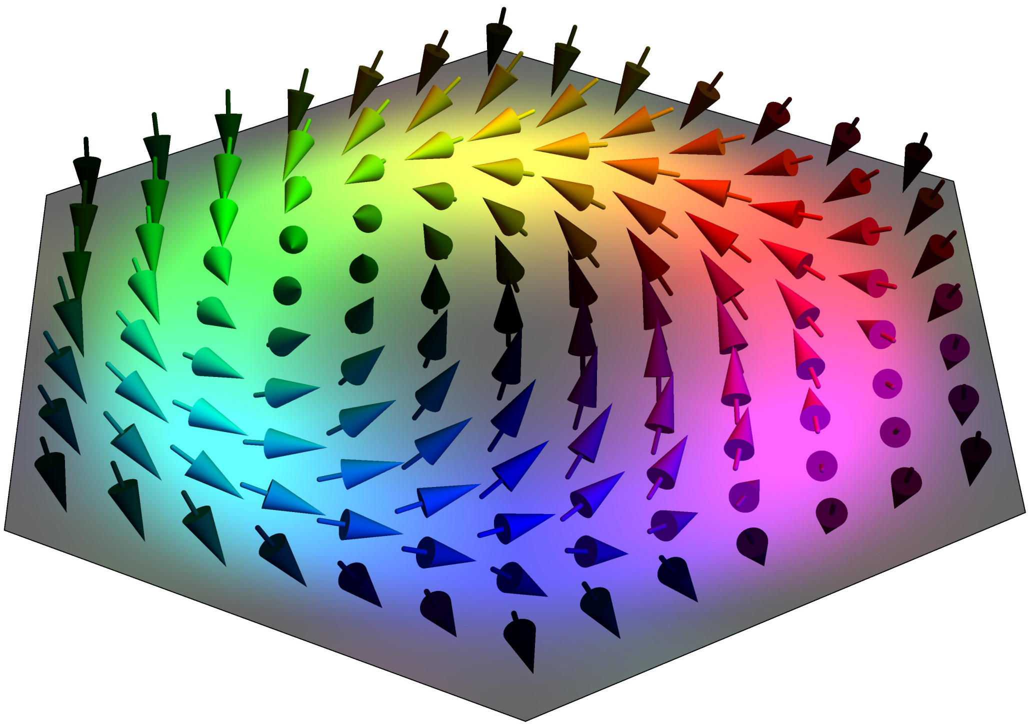
# Skyrmion lattice



B. Göbel, A. Mook, J. Henk, and I.M., Phys. Rev. B **95**, 094413 (2017)

# Skyrmion – background spin texture

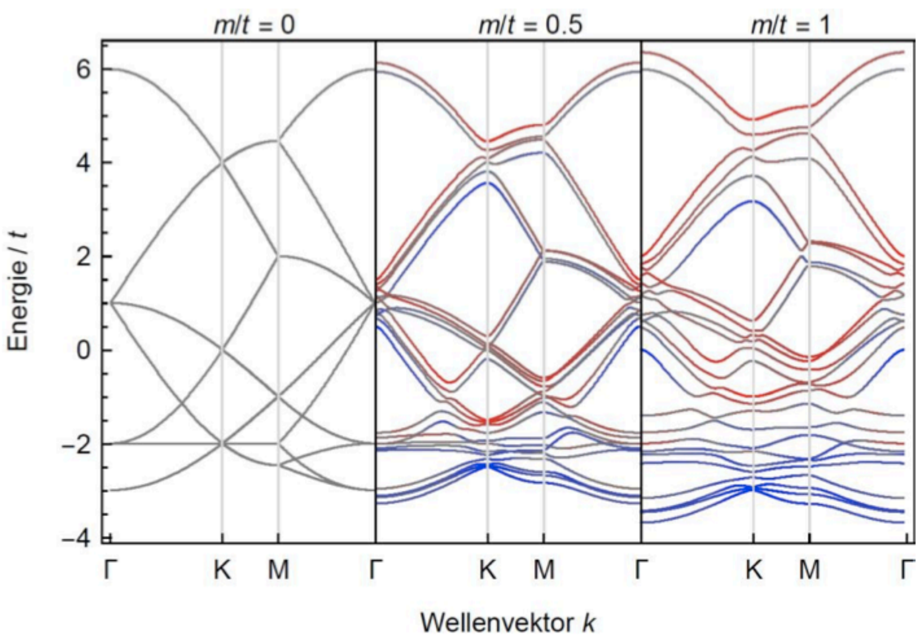
$$\mathbf{m}_i$$

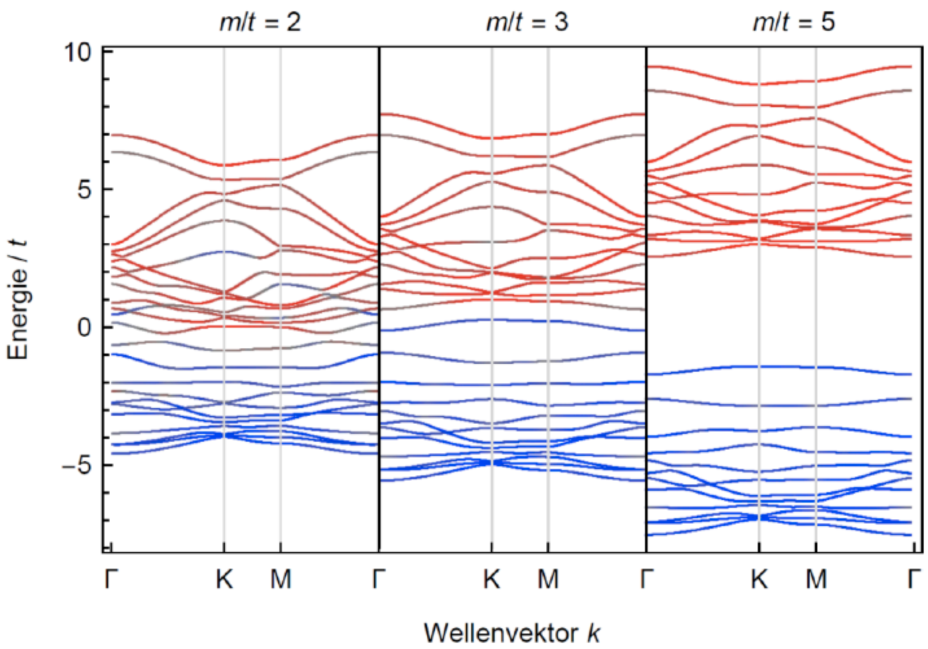


$$\hat{H} = \sum_{ij} t_{ij} c_i^{+} c_j - J \sum_i c_i^{+} \hat{\sigma} c_i \cdot \mathbf{m}_i$$

# Electron bandstructure in background spin texture

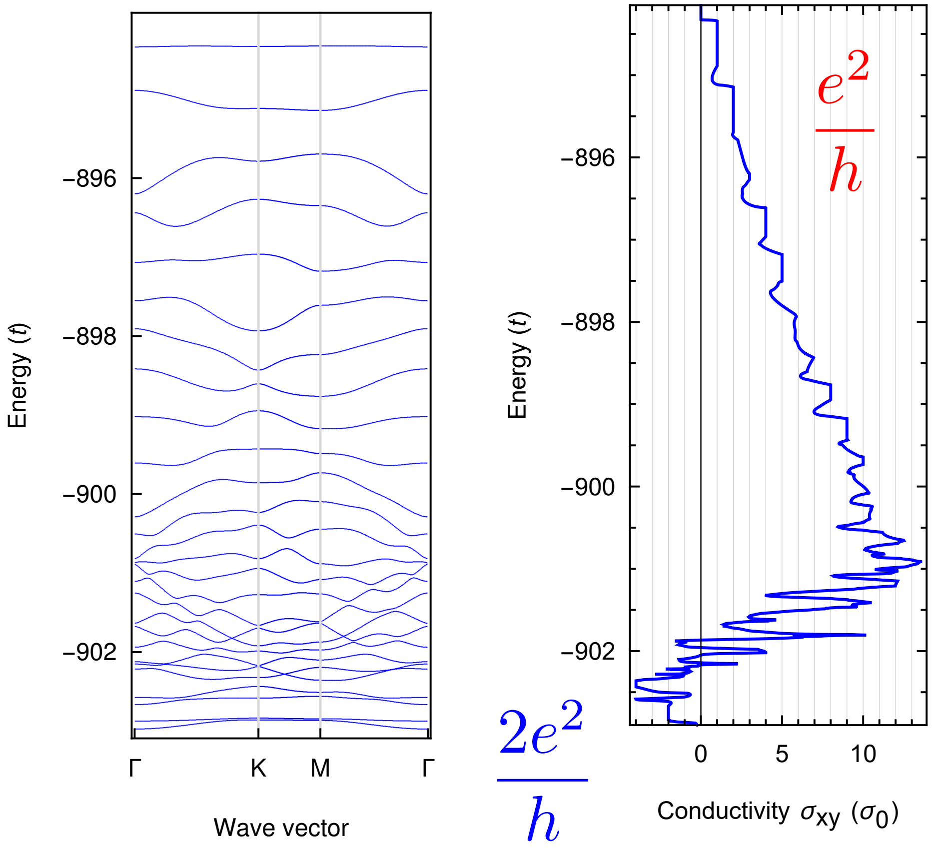
$$\hat{H} = \sum_{ij} t_{ij} c_i^{+} c_j - J \sum_i c_i^{+} \hat{\sigma} c_i \cdot \mathbf{m}_i$$



# Electrons in the skyrmion field and THE

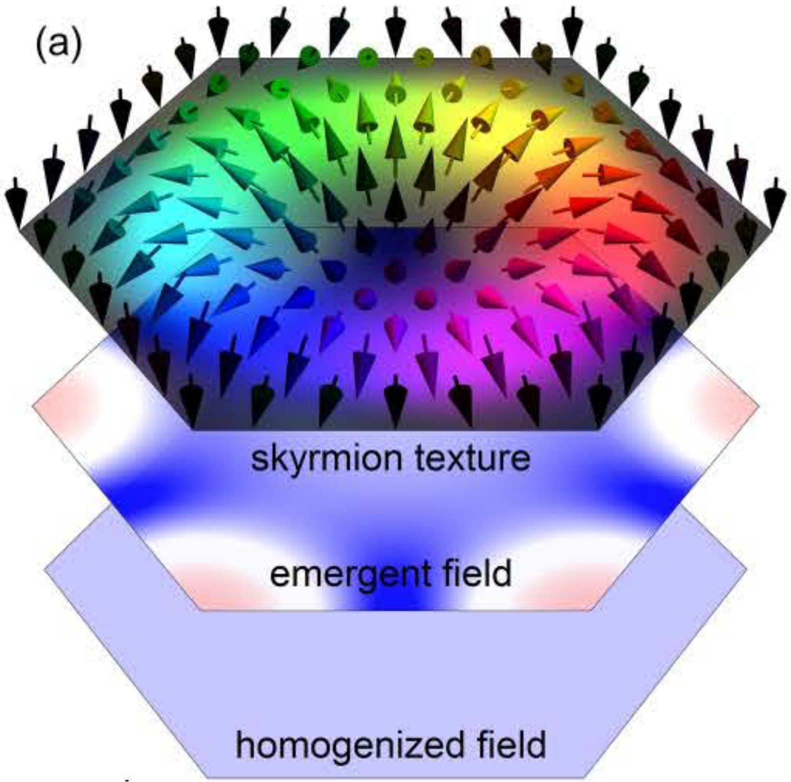
$$\sigma_{\alpha\beta} = \frac{e^2}{h}\frac{1}{2\pi}\sum_n \int_{BZ} \mathrm{d}^2k \; f_n(\vec{k})\Omega_n(\vec{k})$$



# From THE to QHE

# Spin texture, skyrmion number and emergent field

$$N_{sk} = \frac{1}{4\pi}\int_{xy} d^2r\ \mathbf{m}(\mathbf{r}) \cdot [\frac{\partial \mathbf{m}(\mathbf{r})}{\partial x} \times \frac{\partial \mathbf{m}(\mathbf{r})}{\partial y}]$$

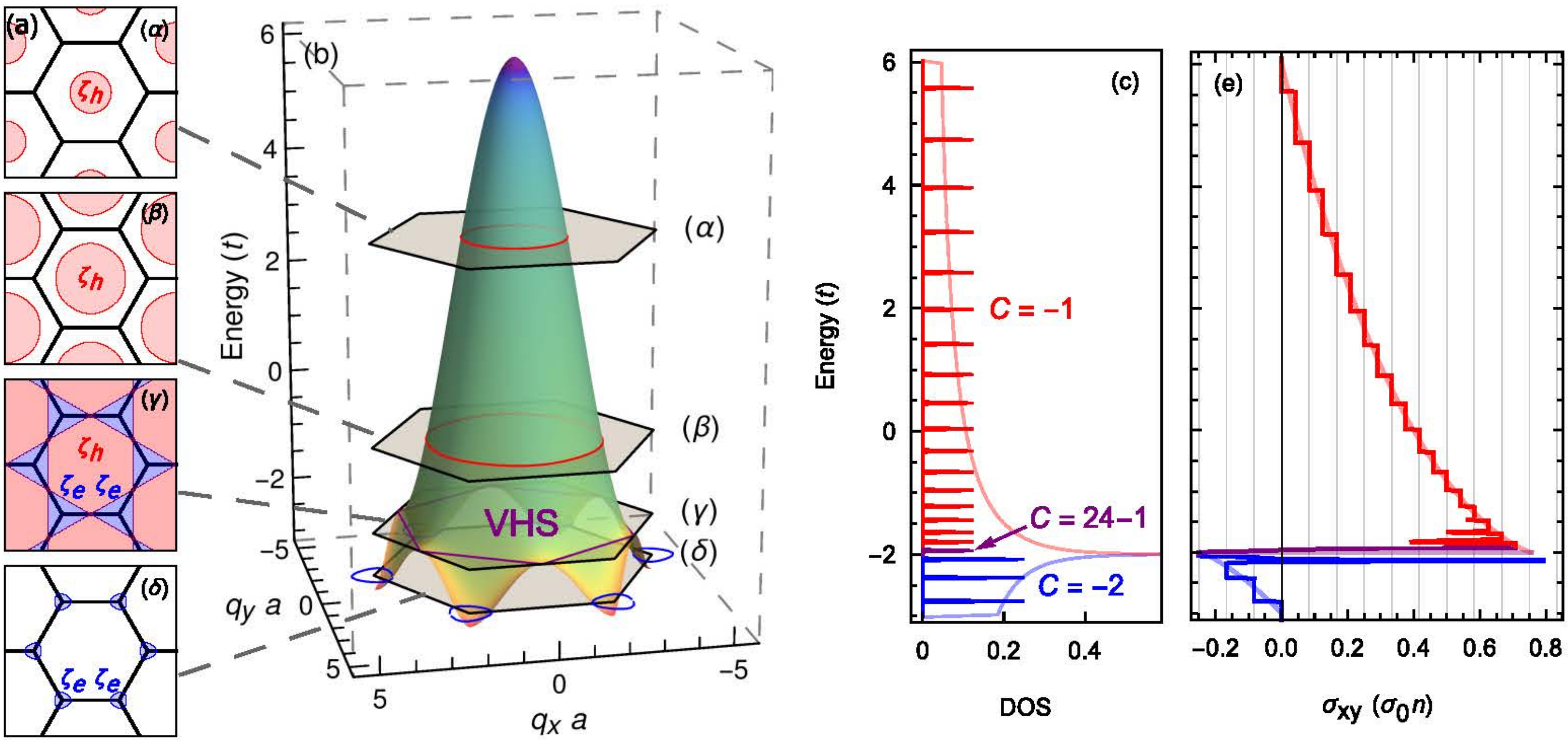



Tomek Schulz, R Ritz, Andreas Bauer, Madhumita Halder, Martin Wagner, Chris Franz, Christian Pfleiderer, Karin Everschor, Markus Garst, Achim Rosch, Nat. Phys. 8, 301 (2012)

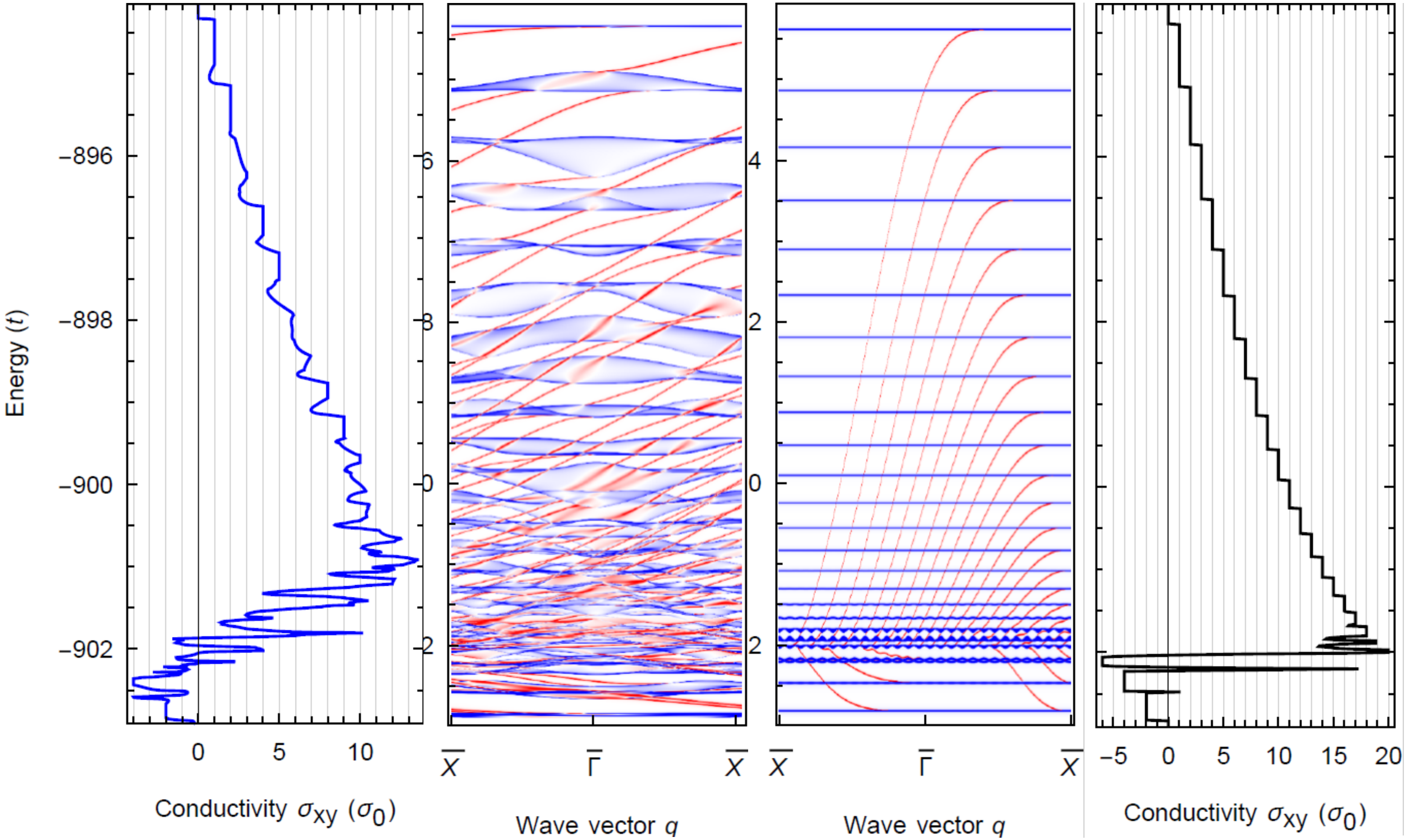
Keita Hamamoto, Motohiko Ezawa, and Naoto Nagaosa, Phys. Rev. B **92,** 115417 (2015)

# Free electrons in a triangular lattice

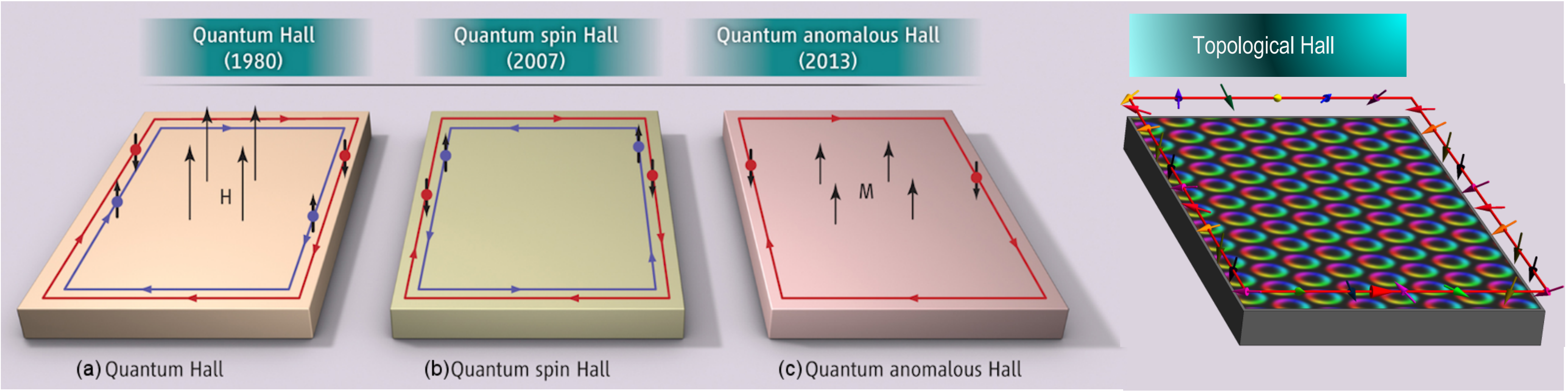
$$H = \frac{(\mathbf{p} - e\,\mathbf{A})^2}{2m}$$



# Comparison of THE, QHE, and surface bandstructure



# The quantum Hall trio and THE




S. Oh, Science **340**,153-154 (2013)

# Summary

- Anomalous velocity is the origin of the transversal transport coefficients: spin and anomalous Hall effect and quantum spin and anomalous Hall effect, as well as the topological Hall and quantum topological Hall effect!
- THE can be mapped onto the QHE

Oliver Busch

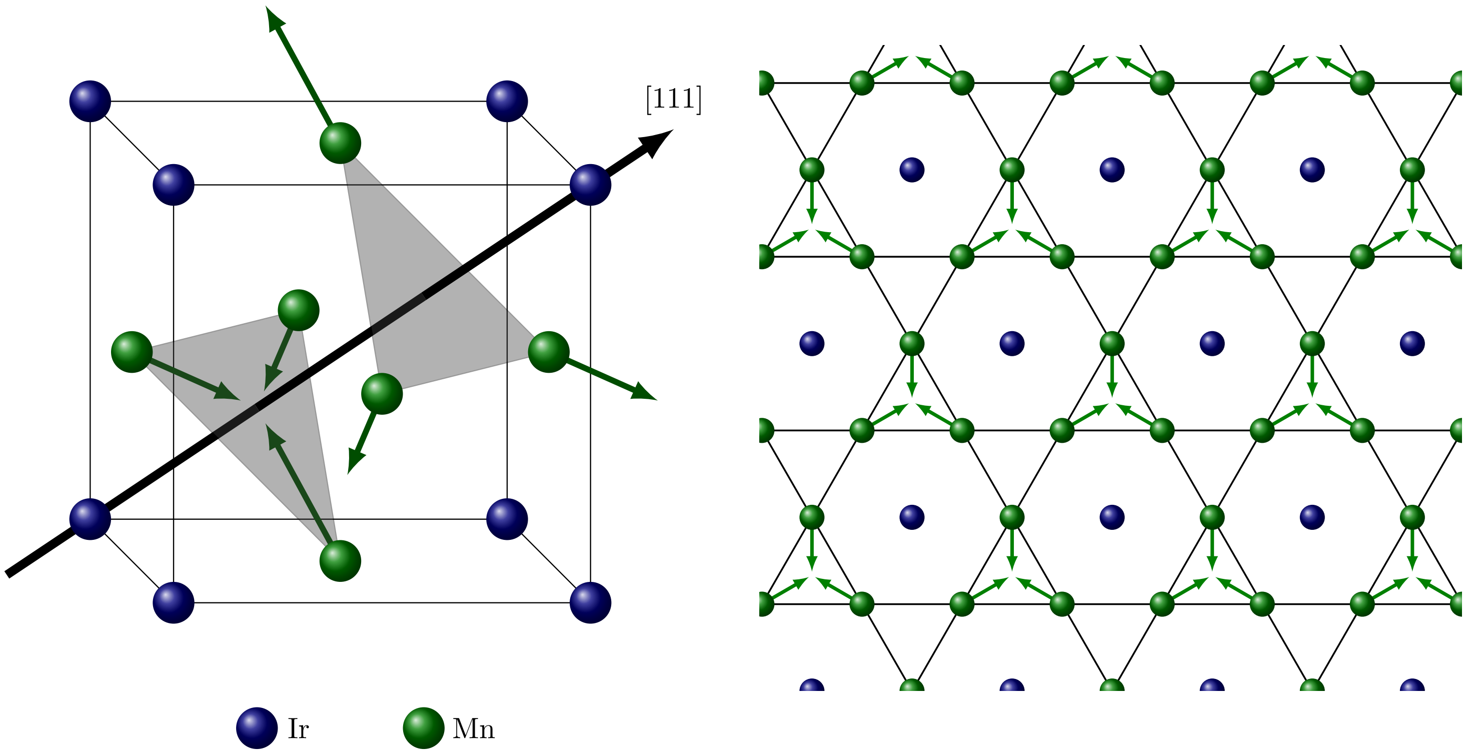
Börge Göbel

# Anomalous Hall effect in coplanar antiferromagnets

# Anomalous Hall effect in coplanar antiferromagnets

Oliver Busch, Börge Göbel, and I. M., Phys. Rev. Research **2**, 033112 (2020)

# Coplanar antiferromagnets




Unit cell with triangular magnetic order (left).

An individual (111) plane that forms a kagome lattice (right).

# Large AHE in coplanar antiferromagnets

$Mn_3X$ (X=Ir, Rh, Pt) $Mn_3X$ (X=Ga, Ge, Sn)

Hua Chen, Qian Niu,and Allan H MacDonald, “Anomalous Hall effect arising from noncollinear antiferromagnetism,”Phys. Rev. Lett.**112**, 017205 (2014)

Jürgen Kübler and Claudia Felser, “Non-collinear antiferromagnets and the anomalous Hall effect,” EPL **108**, 67001 (2014)

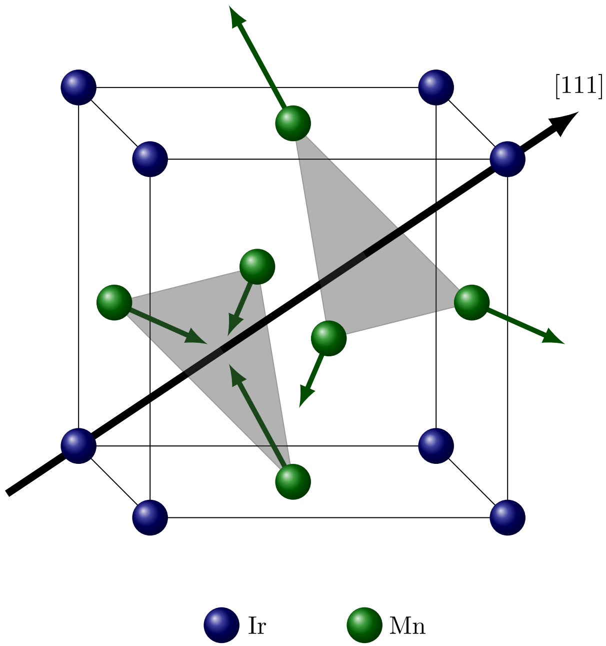
Satoru Nakatsuji, Naoki Kiyohara, and Tomoya Higo, “Large anomalous Hall effect in a non-collinear antiferromagnet atroom temperature,” Nature **527**, 212 (2015)

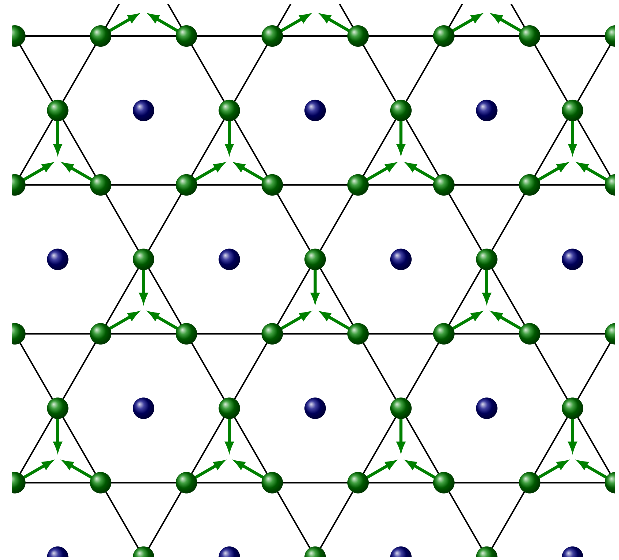
Yang Zhang, Yan Sun, Hao Yang, Jakub Železný, Stuart Parkin, Claudia Felser, and Binghai Yan, “Strong anisotropic anomalous Hall effect and spin Hall effect in the chiral antiferromagnetic compounds $Mn_3X$ (X = Ge, Sn, Ga, Ir, Rh, and Pt),” Phys. Rev. B **95**, 075128 (2017)

M.-T. Suzuki, T. Koretsune, M. Ochi, and R. Arita, „Cluster multipole theory for anomalous Hall effect in antiferromagnets“, Phys. Rev. B 95, 094406 (2017)

Xiaodong Zhou, Jan-Philipp Hanke, Wanxiang Feng, FeiLi, Guang-Yu Guo, Yugui Yao, Stefan Bl Blügel, and Yuriy Mokrousov, “Spin-order dependent anomalous Hall effect and magneto-optical effect in the noncollinear antiferromagnets $Mn_3XN$ with X = Ga, Zn, Ag, or Ni,” Physical Review B **99**,104428 (2019)

# Large AHE in coplanar antiferromagnets



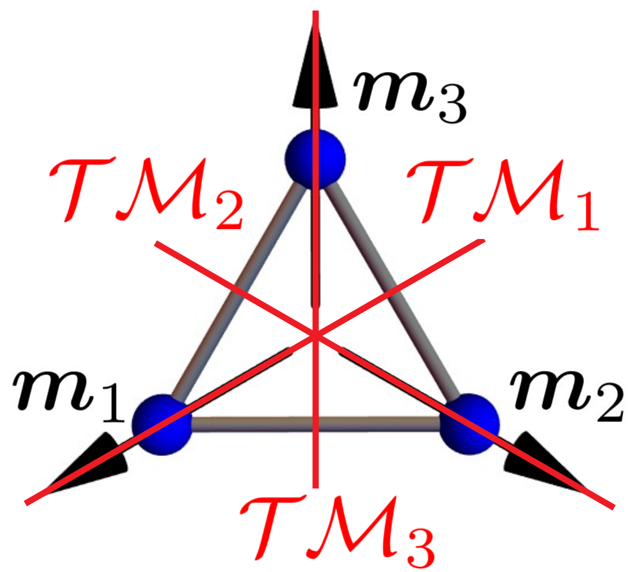
Usually: anomalous Hall resistance ~ M
Now: M = 0

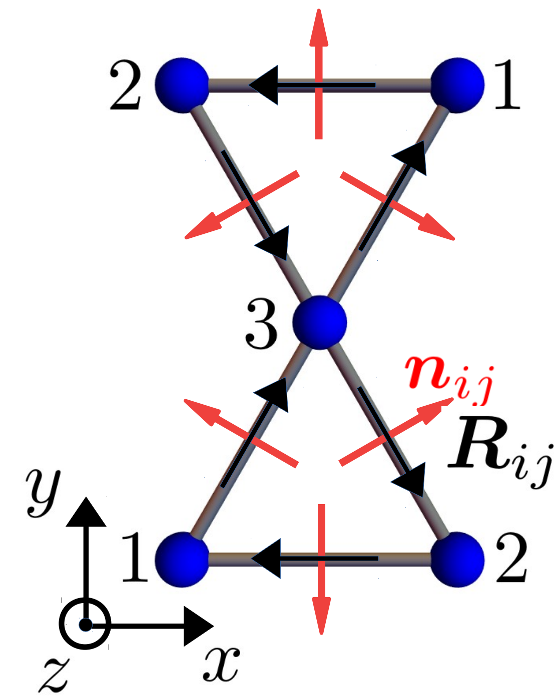
# Model Hamiltonian

$$H = t\sum_{\langle ij\rangle} c_i^\dagger c_j + m\sum_i c_i^\dagger(\vec{m}_i\cdot\vec{\sigma})c_i + i\lambda\sum_{\langle ij\rangle} c_i^\dagger(\vec{n}_{ij}\cdot\vec{\sigma})c_j$$

Hund's coupling

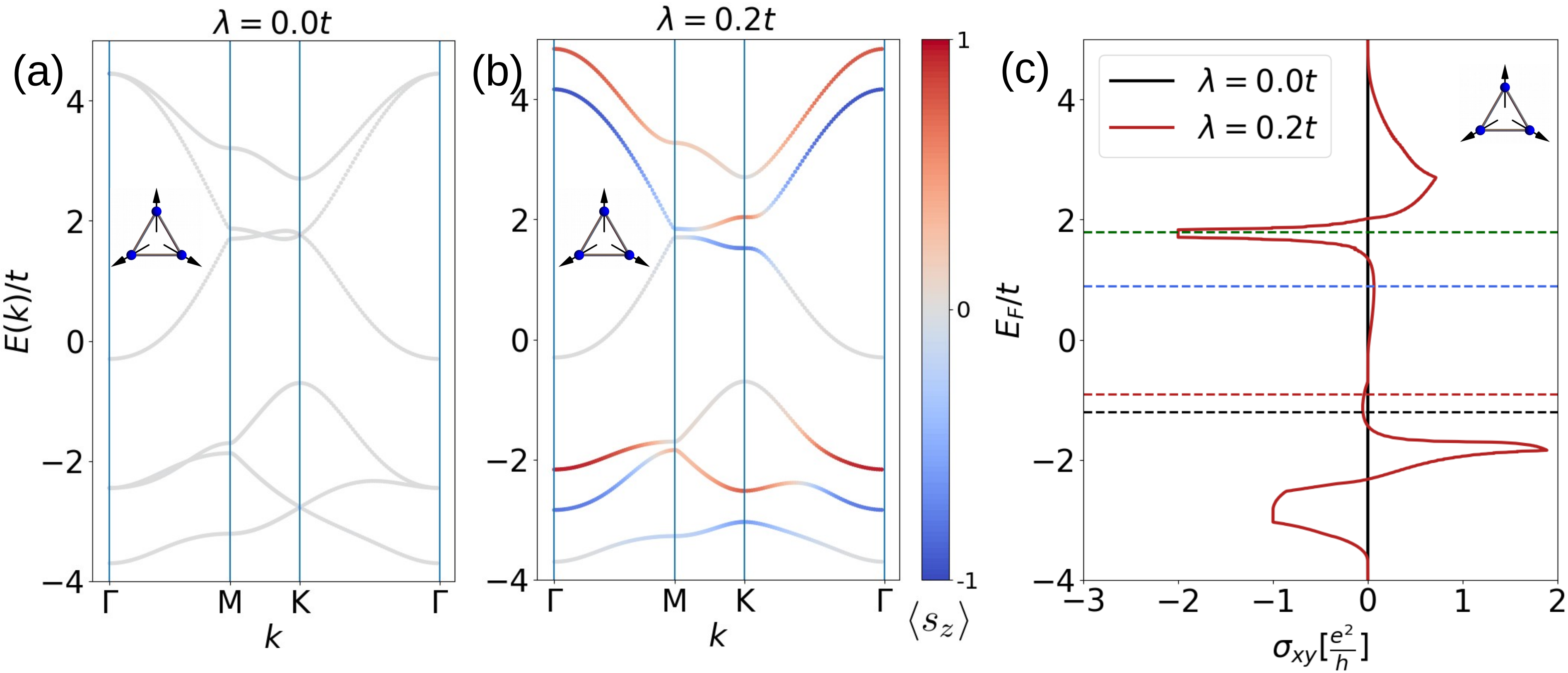
spin orbit coupling




Hua Chen, Qian Niu, and Allan H MacDonald, Phys. Rev. Lett. **112**, 017205 (2014)

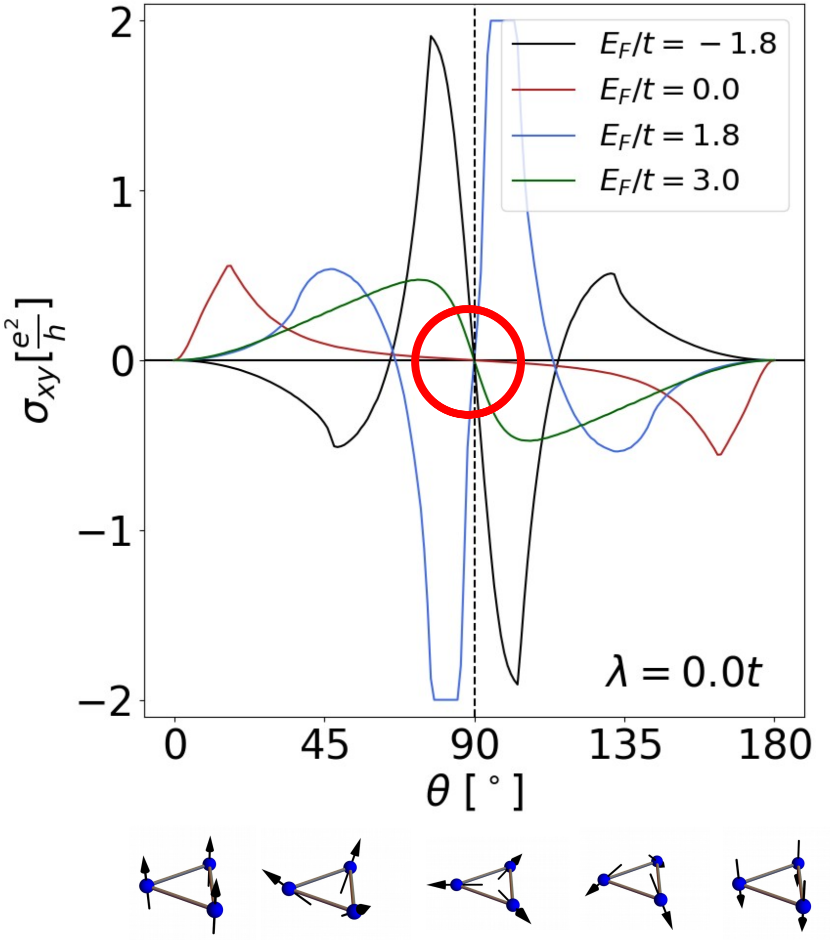
# Bandstructure and AHE

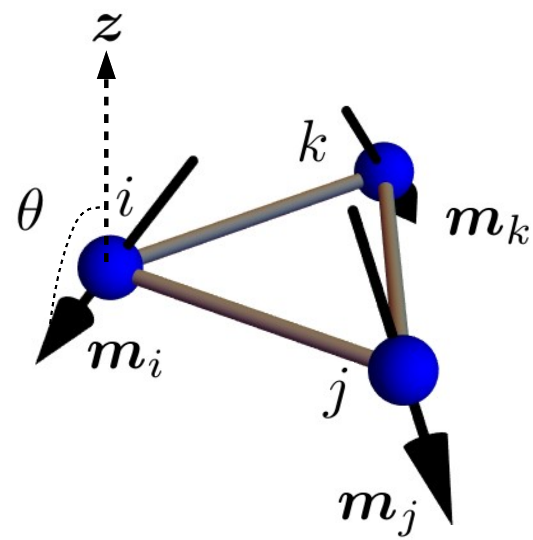



without and with spin orbit coupling

Oliver Busch, Börge Göbel, and I. M., Phys. Rev. Research **2**, 033112 (2020)

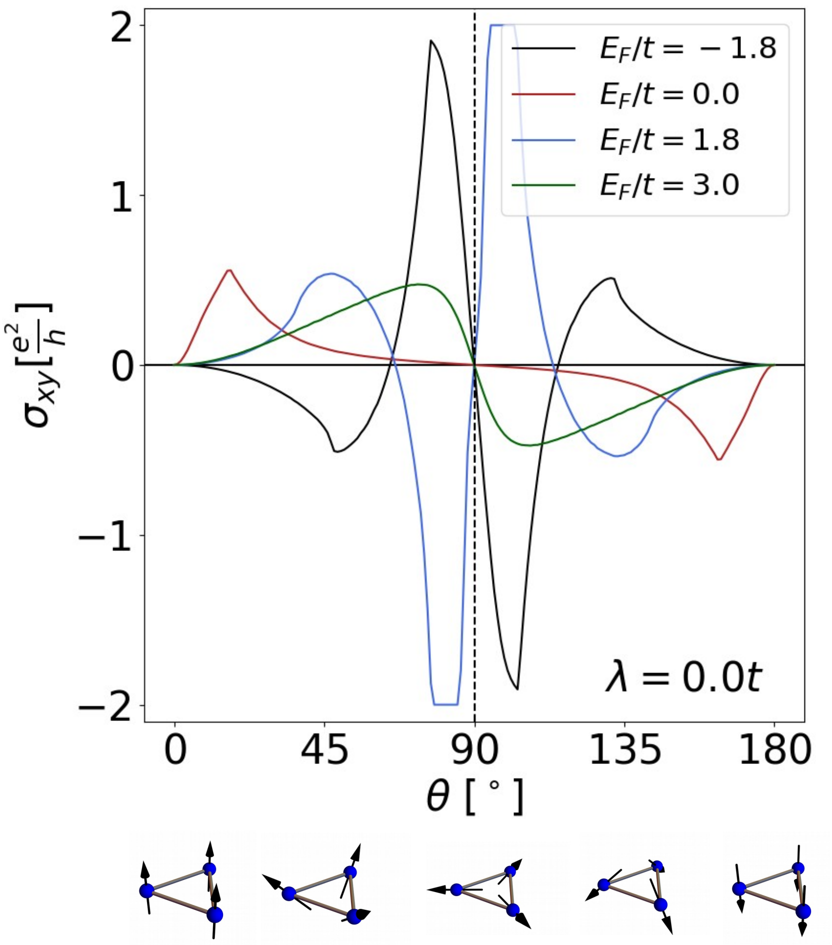
# AHE under out-of-plane tilting of the spin texture

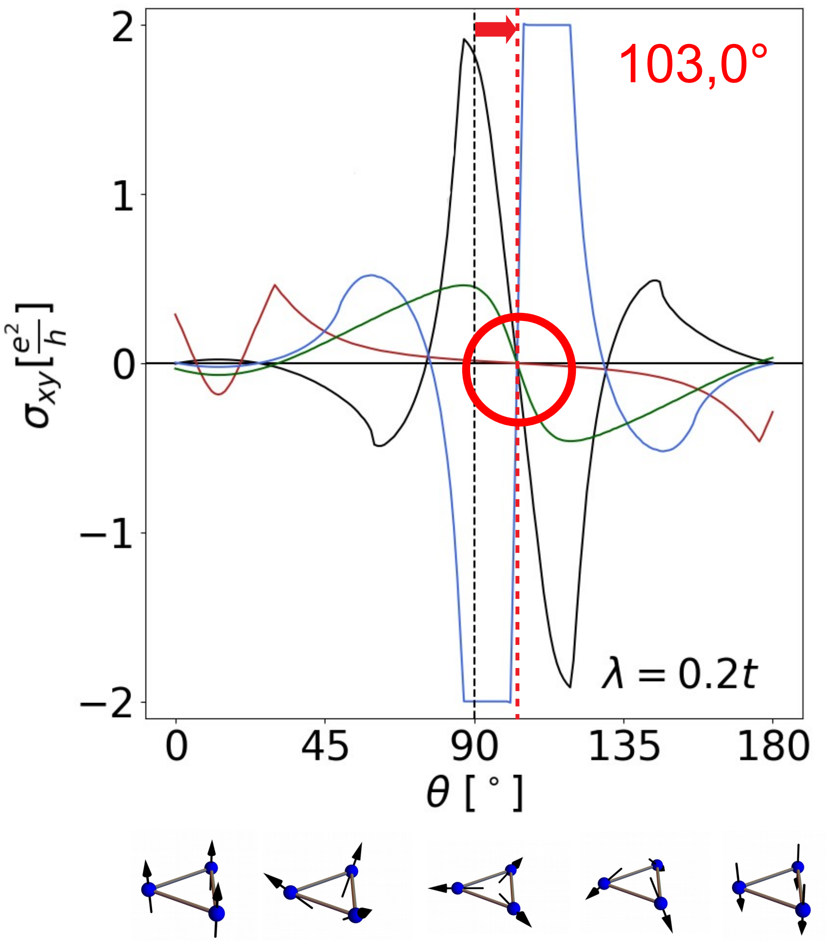



without spin orbit coupling

# AHE under out-of-plane tilting of the spin texture

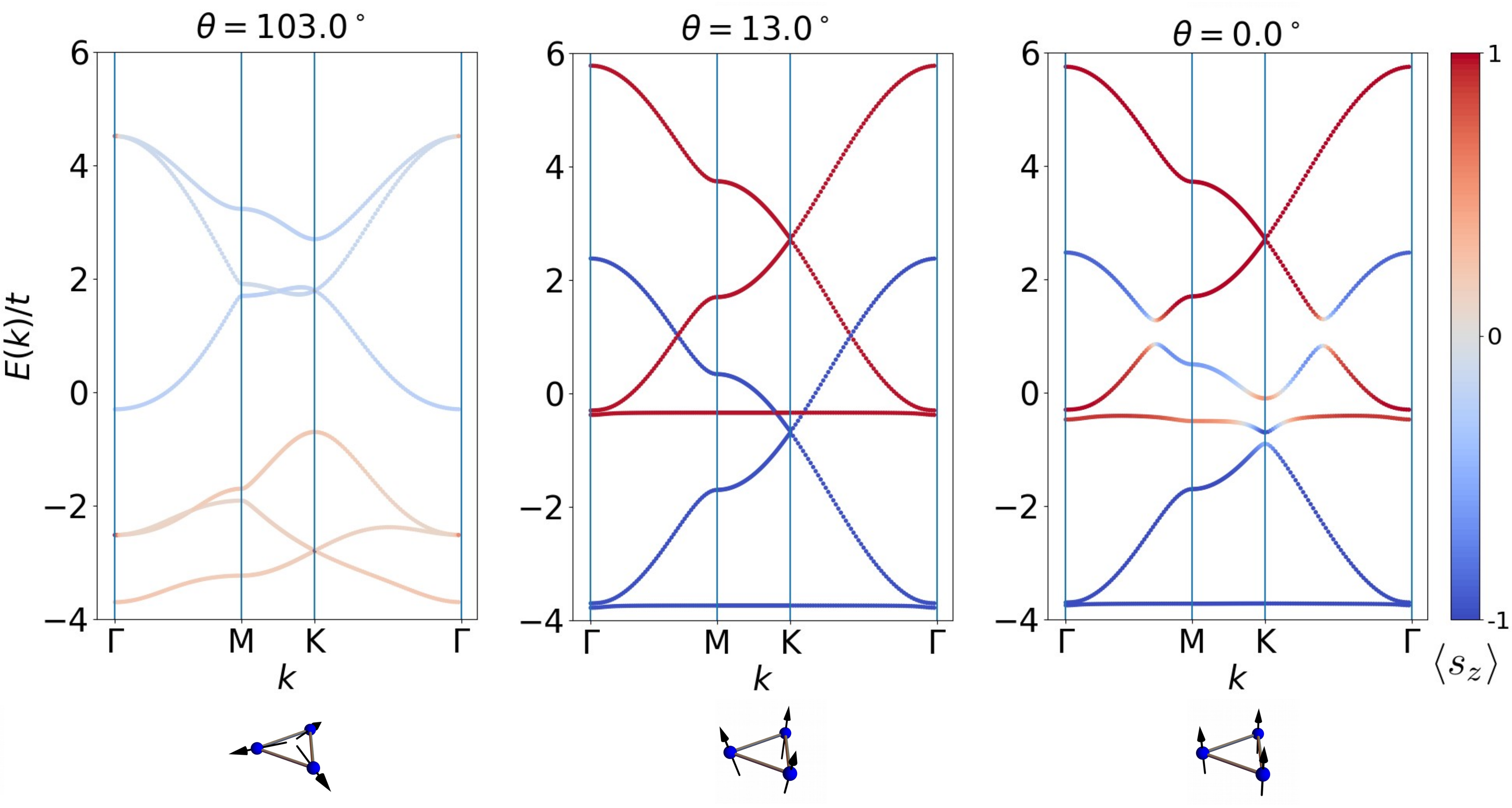



without spin orbit coupling

with spin orbit coupling

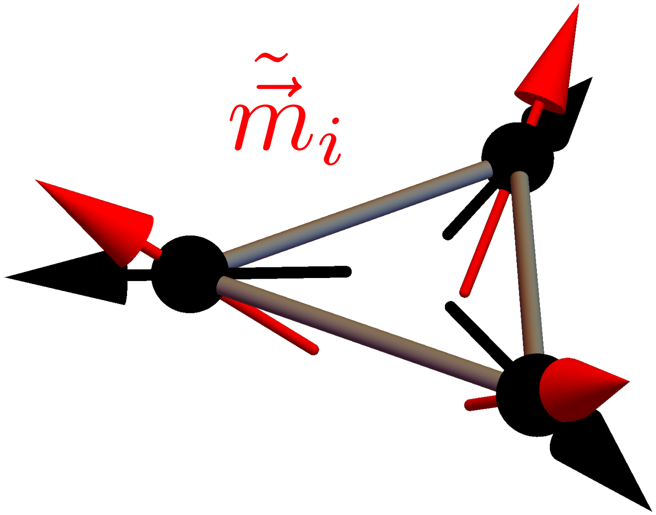
# Bandstructure with SOC under tilting

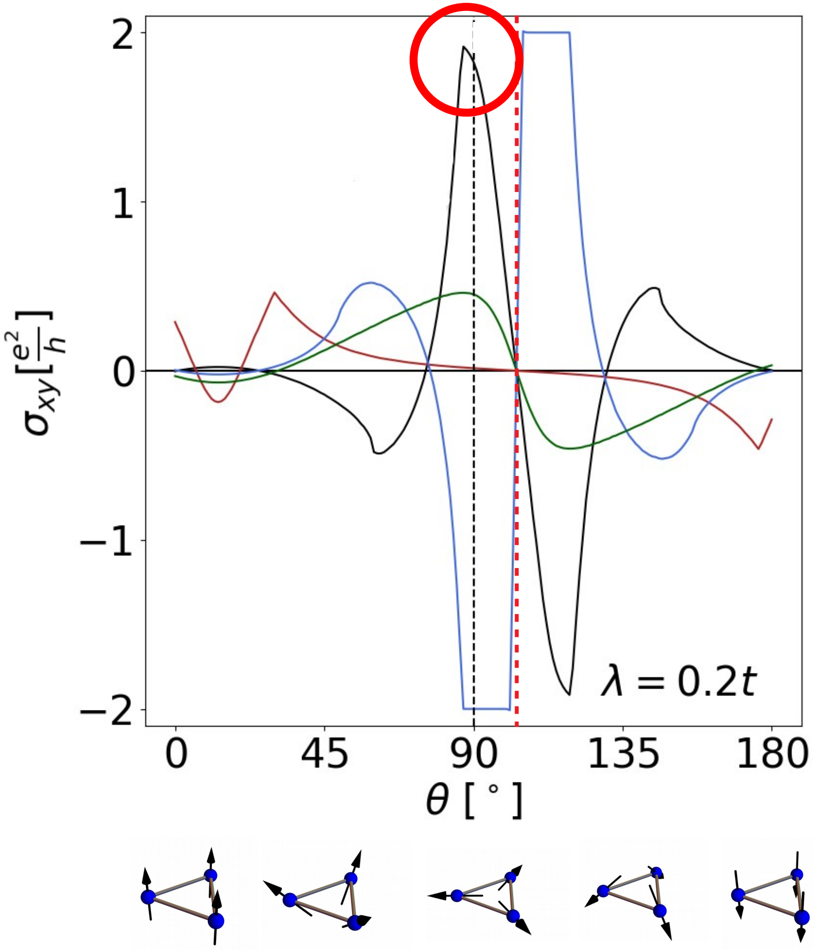


# Virtual spin texture after unitary transformation

$$H = \sum_{\langle i,j \rangle} t_{ij}^{\text{eff}} d_i^\dagger d_j + m \sum_i d_i^\dagger (\tilde{\vec{m}}_i \cdot \vec{\sigma}) d_i + i\tilde{\lambda} \sum_{\langle i,j \rangle} d_i^\dagger (\vec{n}_{ij} \cdot \vec{\sigma}) d_j$$

Virtual spin texture:

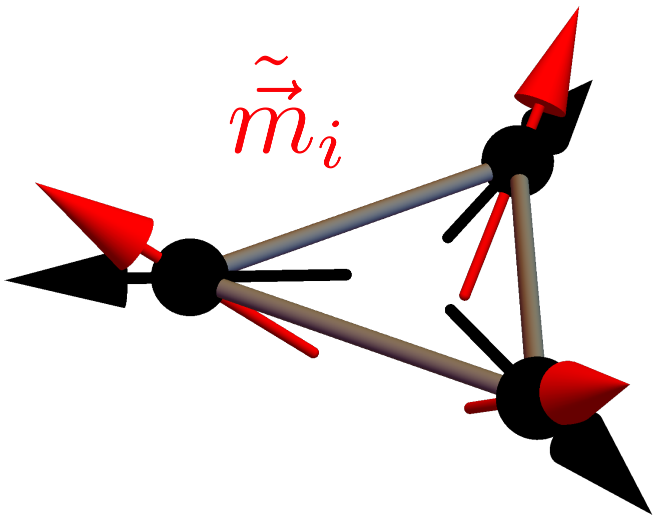


# Virtual spin texture after unitary transformation

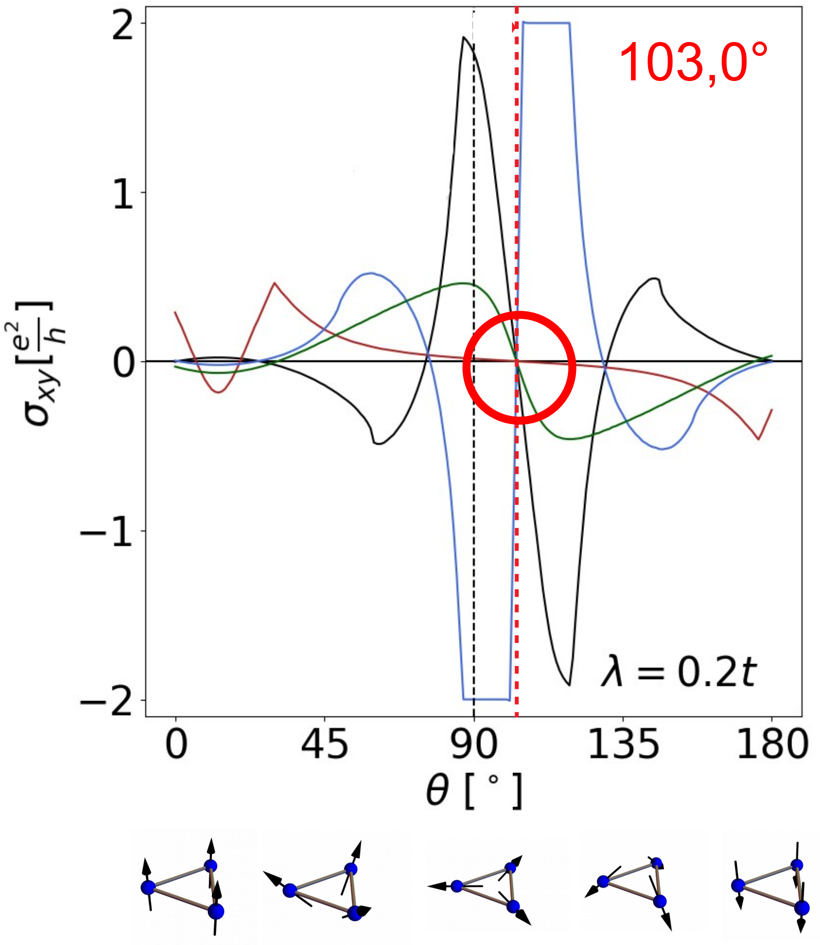
$$H = \sum_{\langle i,j \rangle} t^{\text{eff}}_{ij} d_i^\dagger d_j + m \sum_i d_i^\dagger (\tilde{\vec{m}}_i \cdot \vec{\sigma}) d_i + i\tilde{\lambda} \sum_{\langle i,j \rangle} d_i^\dagger (\vec{n}_{ij} \cdot \vec{\sigma}) d_j$$

Virtual spin texture:



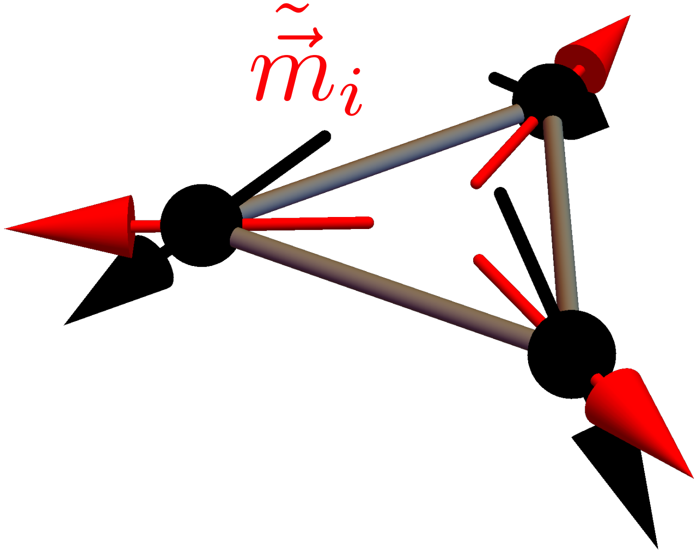

Effective spin orbit coupling:

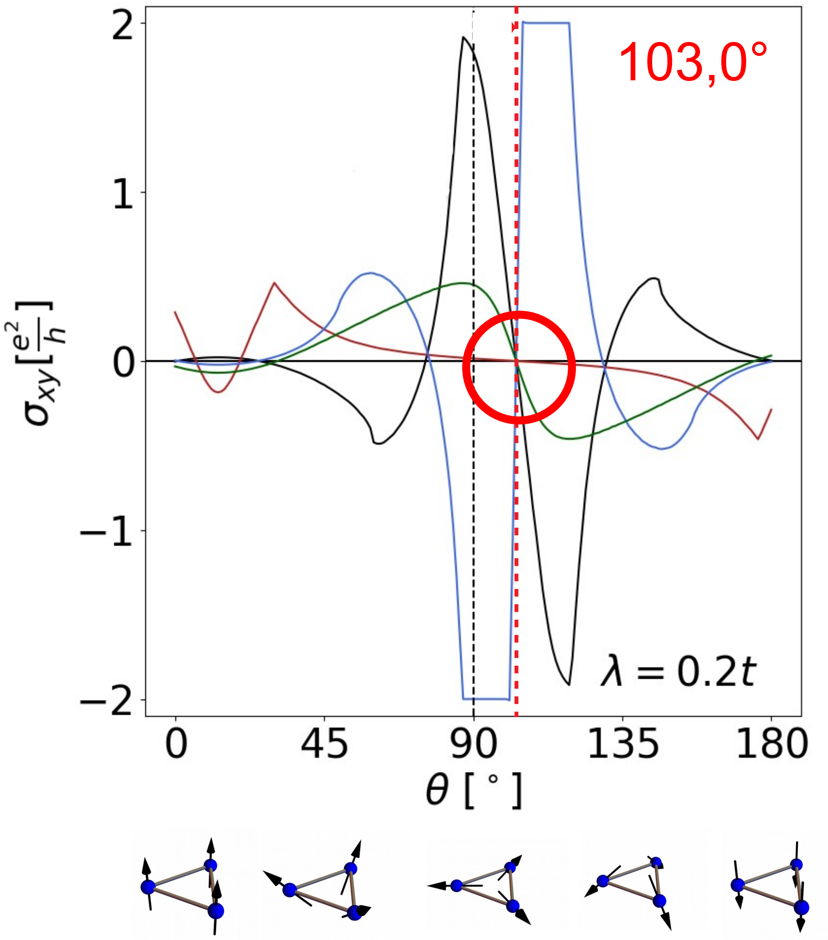
$$\tilde{\lambda} = \frac{\sqrt{3}}{2} \sin(\alpha)\, t + \cos(\alpha) \lambda$$



# Virtual spin texture at 103°

$$H = \sum_{\langle i,j \rangle} t_{ij}^{\text{eff}} d_i^\dagger d_j + m \sum_i d_i^\dagger (\tilde{\vec{m}}_i \cdot \vec{\sigma}) d_i \qquad \tilde{\lambda} = 0$$




Virtual spin texture is coplanar!

# Summary of AHE in coplanar antiferromagnets

- Microscopic origin of the anomalous Hall effect for Kagome antiferromagnets
- Spin-orbit coupling causes a virtual spin texture responsible for the Hall conductivity instead of the measureable spin texture
- Virtual spin texture gives rise to a THE proportional to the opening angle of the magnetic moments and an AHE proportional to the resulting out-of-plane moment

Robin Neumann

Annika Johansson

Alexander Mook

Jürgen Henk

# Magnetic spin Hall effect and spin current vorticity

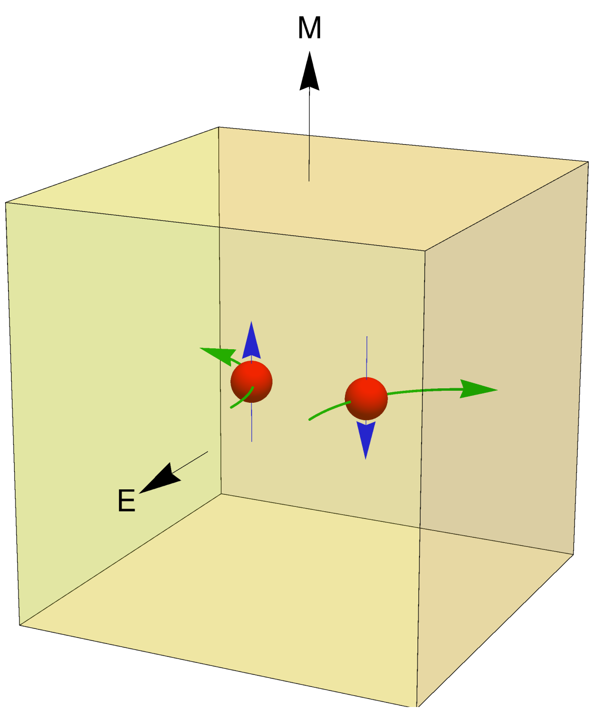
# Magnetic spin Hall effect and spin current vorticity

Robin R. Neumann, Alexander Mook, Jürgen Henk, and I. M.,
Phys. Rev. Research **2**, 023065 (2020)

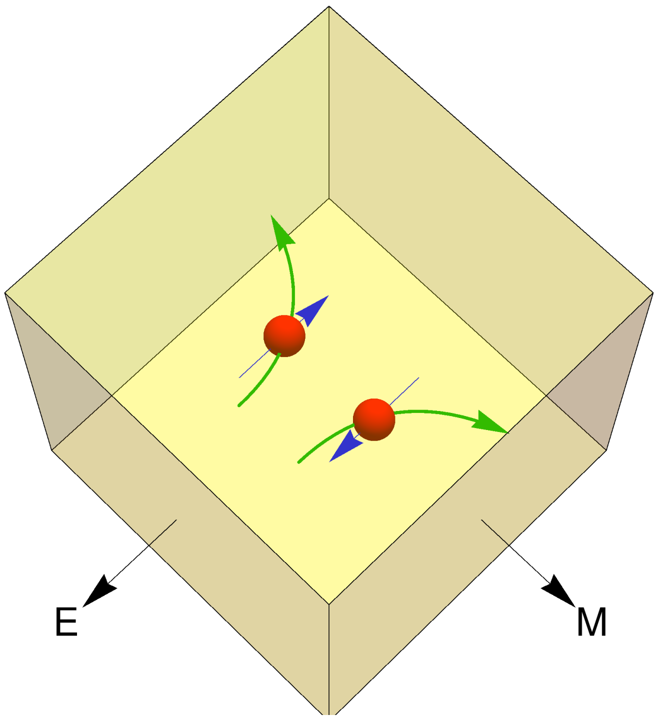
Jakub Železný, Yang Zhang, Claudia Felser, and Binghai Yan, Spin-polarized current in noncollinear antiferromagnets, Phys. Rev. Lett. 119, 187204 (2017)
Hua Chen, Qian Niu, and Allan H. MacDonald, “Spin Halleffect without spin currents in magnetic insulators,” (2018),arXiv:1803.01294.
Motoi Kimata, Hua Chen, Kouta Kondou, Satoshi Sugimoto,Prasanta K. Muduli, Muhammad Ikhlas, Yasutomo Omori,Takahiro Tomita, Allan H. MacDonald, Satoru Nakatsuji, and Yoshichika Otani, “Magnetic and magnetic inverse spin Hall effects in a non-collinear antiferromagnet,” Nature (2019),10.1038/s41586-018-0853-0

# Anomalous versus magnetic spin Hall effect

AHE




MSHE




Spin-polarized current perpendicular to **M**

Spin current parallel to **M**

Jakub Železný, Yang Zhang, Claudia Felser, and Binghai Yan,
Phys. Rev. Lett. 119, 187204 (2017)

# Spin conductivity tensor

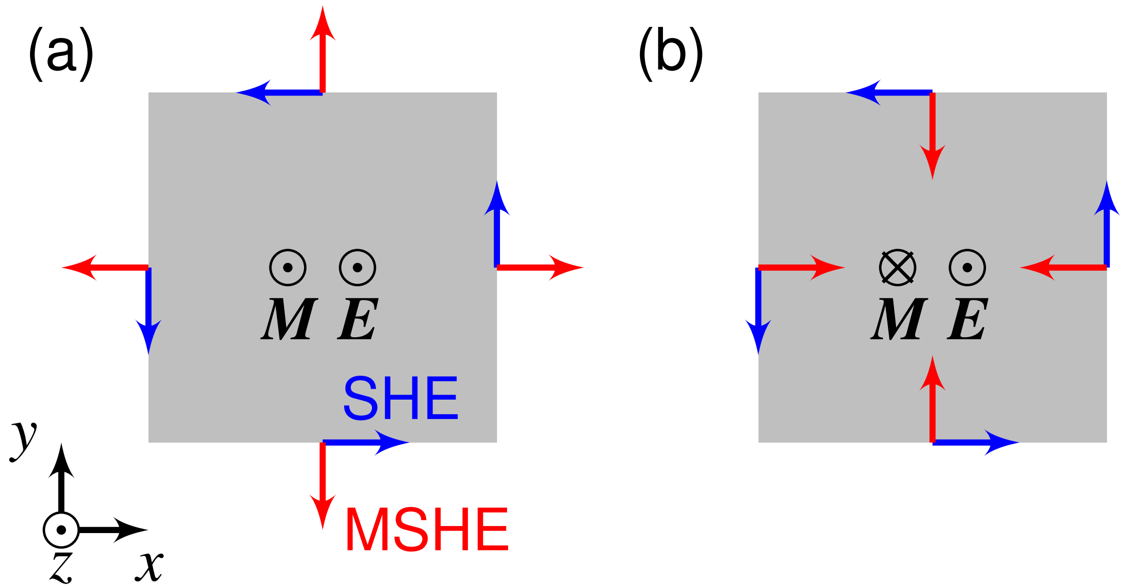
Time-reversal symmetry: $\sigma^{\gamma}_{\mu\nu} = \sigma^{\gamma,\text{odd}}_{\mu\nu} + \sigma^{\gamma,\text{even}}_{\mu\nu}$

$$\sigma^{\gamma,\text{odd}}_{\mu\nu} = \frac{\hbar^2 \varGamma}{V} \sum_{n,m,\vec{k}} \frac{f_{m\vec{k}} - f_{n\vec{k}}}{\varepsilon_{n\vec{k}} - \varepsilon_{m\vec{k}}} \frac{\mathrm{Re}\left(\langle n|\underline{J}^{\gamma}_{\vec{k},\mu}|m\rangle\langle m|\underline{J}_{\vec{k},\nu}|n\rangle\right)}{\left(\varepsilon_{n\vec{k}} - \varepsilon_{m\vec{k}}\right)^2 + (\hbar\varGamma)^2}$$

$$\sigma^{\gamma,\text{even}}_{\mu\nu} = -\frac{\hbar}{V} \sum_{n,m,\vec{k}} \left(f_{m\vec{k}} - f_{n\vec{k}}\right) \frac{\mathrm{Im}\left(\langle n|\underline{J}^{\gamma}_{\vec{k},\mu}|m\rangle\langle m|\underline{J}_{\vec{k},\nu}|n\rangle\right)}{\left(\varepsilon_{n\vec{k}} - \varepsilon_{m\vec{k}}\right)^2 + (\hbar\varGamma)^2}$$

Charge current: $\underline{J}_{\vec{k},\nu}$ Spin current: $\underline{J}^{\gamma}_{\vec{k},\mu}$

# Spin accumulation under time-reversal symmetry




MSHE: $$\sigma_{\mu\nu}^{\gamma,\mathrm{odd}}[\{\vec{m}_i\}] = -\sigma_{\mu\nu}^{\gamma,\mathrm{odd}}[-\{\vec{m}_i\}]$$

SHE: $$\sigma_{\mu\nu}^{\gamma,\mathrm{even}}[\{\vec{m}_i\}] = \sigma_{\mu\nu}^{\gamma,\mathrm{even}}[-\{\vec{m}_i\}]$$

# Contributions to the MSHE

Antisymmetric part defined as MSHE vector:

$$\vec{\sigma}^{\gamma}_{\mathrm{MSHE}} \equiv \begin{pmatrix} \sigma^{\gamma,(\mathrm{a})}_{yz} \\ \sigma^{\gamma,(\mathrm{a})}_{zx} \\ \sigma^{\gamma,(\mathrm{a})}_{xy} \end{pmatrix} = \frac{1}{2\Gamma V} \sum_{n,\vec{k}} \vec{J}^{\gamma}_{n\vec{k}} \times \vec{J}_{n\vec{k}} \left( -\frac{\partial f_{n\vec{k}}}{\partial \varepsilon} \right)$$

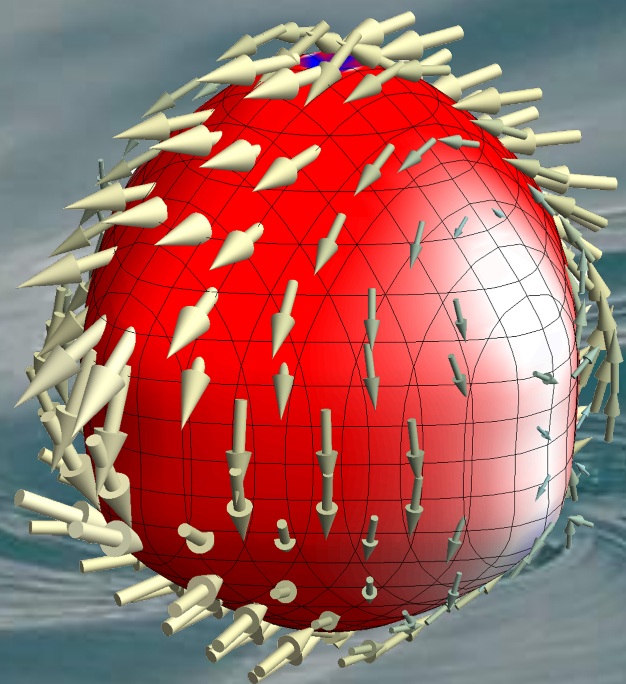
T=0:

$$\vec{\sigma}^{\gamma}_{\mathrm{MSHE}} = \frac{e}{2\hbar\Gamma(2\pi)^3} \sum_{n} \oint_{\varepsilon_n=\varepsilon_{\mathrm{F}}} \hat{\vec{v}}_{n\vec{k}} \times \vec{J}^{\gamma}_{n\vec{k}} \,\mathrm{d}S$$

Spin current vorticity:

$$\vec{\sigma}^{\gamma}_{\mathrm{MSHE}} = \frac{e}{2\hbar\Gamma(2\pi)^3} \vec{\omega}^{\gamma}(\varepsilon_{\mathrm{F}})$$

# Spin current vorticity in the Fermi sea



**MSHE is nonzero if there is an integrated sense of rotation of the spin current in the Fermi sea!**

# Summary

- MSHE is nonzero if we have nonzero spin current vorticity in the Fermi sea
- MSHE can occur in any ferromagnet
- MSHE can occur in antiferromagnets with MLG that permits a magnetic toroidal moment
- MSHE can occur in Kagome magnets
- MSHE can occur in Rashba materials with warping
- Magnetic Spin Nernst Effect for magnons is expected

# Collaborations and funding

S. Blügel, Y. Mokrousov, FZ Jülich
H. Ebert, D. Ködderitzsch, LMU München
B. Gyorffy, M. Gradhand,
University of Bristol University
F. Munoz, Santiago

A. Fert, A. Barthelemy,
M. Bibes, Paris
W. Wulfhekel,
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