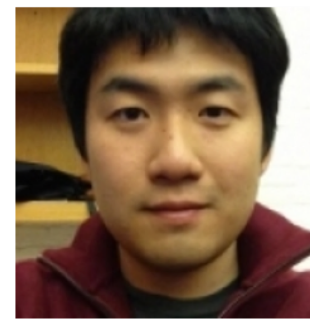
Symmetry enriched phases of quantum circuits

Y. Bao, S. Choi, EA, arXiv:2102.09164

Yimu Bao

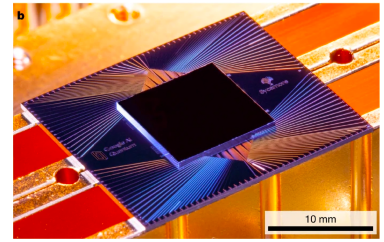
Soonwon Choi

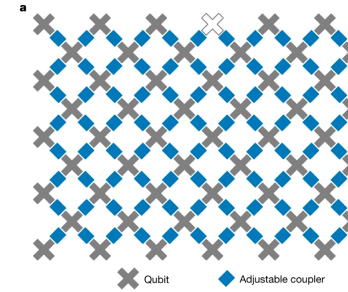


GORDON AND BETTY MOORE FOUNDATION

Huge progress in building controllable quantum circuits

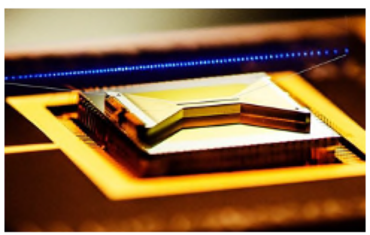
Google quantum supremacy demonstration with SC quantum circuits
Arute et. al. (Martinis, Roushan group) Nature 2019





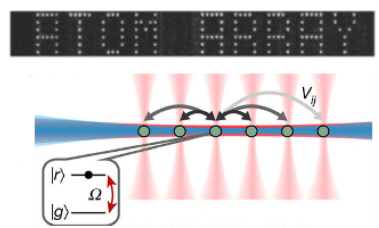
Other platforms:

Trapped ions



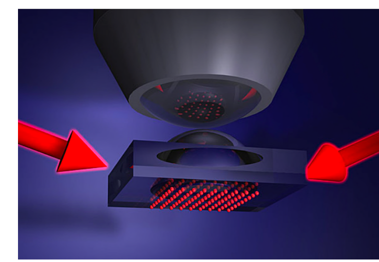
Monroe group @ UMD

Rydberg atoms on Tweezers



Lukin group @ Harvard

Ultra cold atoms



Griener group @ Harvard

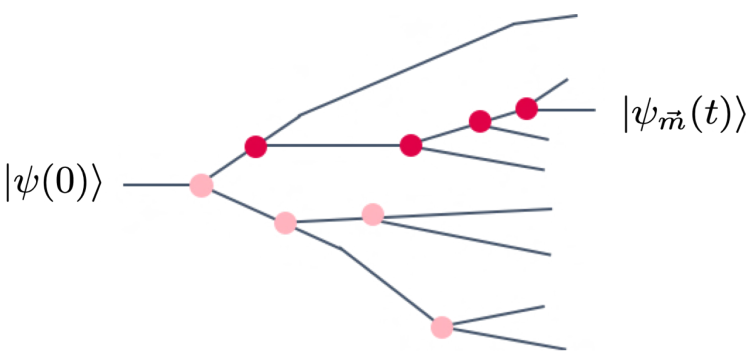
Present new paradigms for quantum many body physics.

Measurement induced phase transition in hybrid quantum circuits

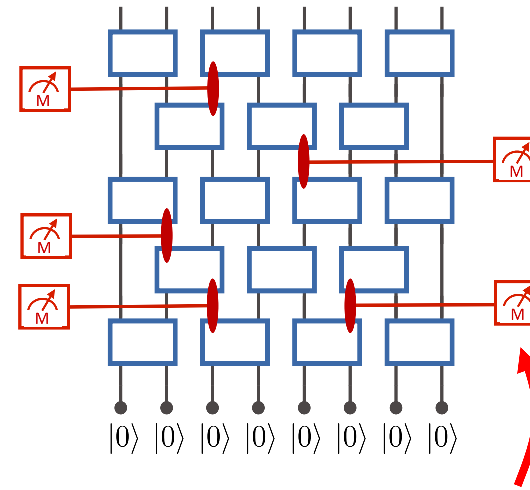
Skinner, Ruhman, Nahum PRX 2019;   Li, Chen, Fisher PRB 2018,
Chan et. al. PRB 2019, …

Simplest setting: a pure state undergoing stochastic evolution due to the measurements. Evaluate entanglement conditioned on measurement outcomes

$$\vec{m} = \{m_1, m_2, m_3, \ldots\}$$



$$\langle S_A \rangle_{\mathcal{U}} = \langle \sum_{\vec{m}} p_{\vec{m}} S_{A,\vec{m}} \rangle_{\mathcal{U}}$$



Measure with probability p

Project on measurement result:

$$|\psi\rangle \mapsto \frac{\hat{P}_\mu |\psi\rangle}{\sqrt{\langle\psi|\hat{P}_\mu|\psi\rangle}} \text{ with prob.} \langle \hat{P}_\mu \rangle$$

Measurement induced phase transition in hybrid quantum circuits

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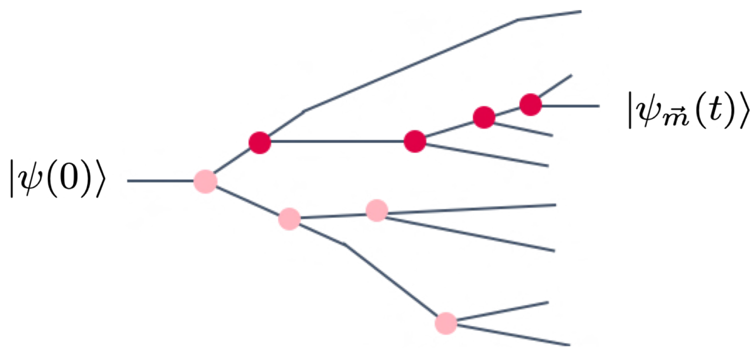
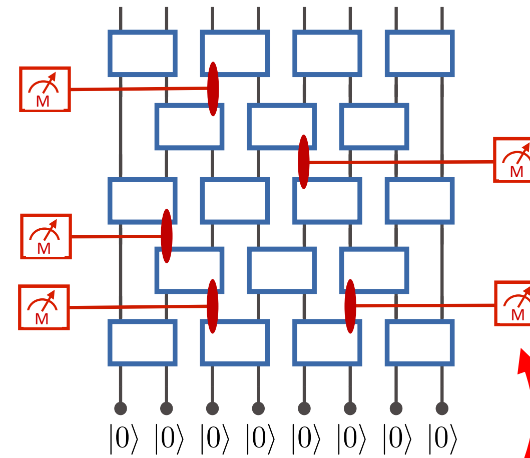
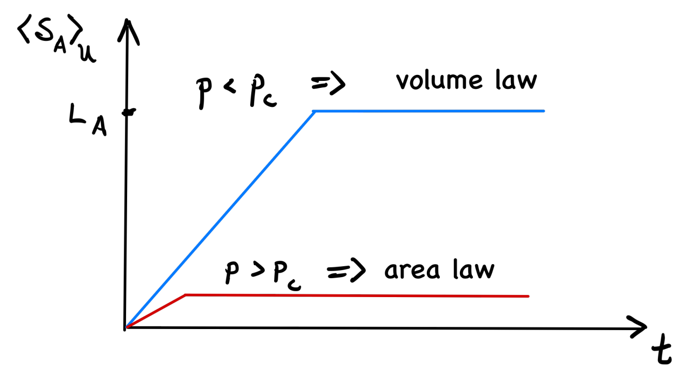


$$\langle S_A \rangle_{\mathcal{U}} = \langle \sum_{\vec{m}} p_{\vec{m}} S_{A,\vec{m}} \rangle_{\mathcal{U}}$$

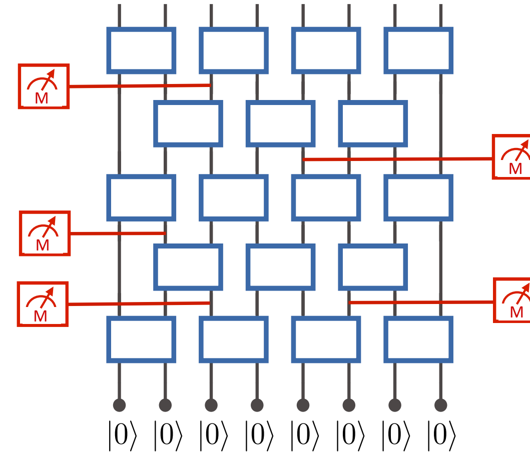


Measure with probability p



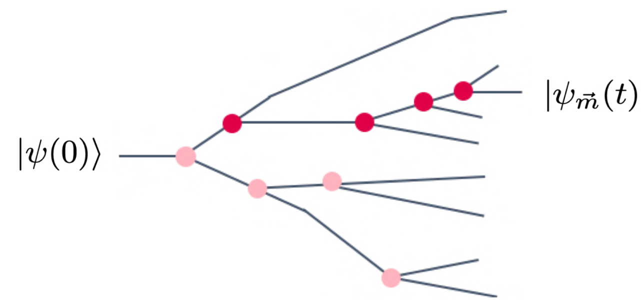
This talk

A hybrid quantum circuit generates a new kind of ensemble of quantum trajectories distinct from either the thermal ensemble or a quantum ground state



- How to characterize this ensemble theoretically and experimentally?

- What are the possible phases of the circuit ensemble?
  How do they depend on the symmetries of the circuit ?



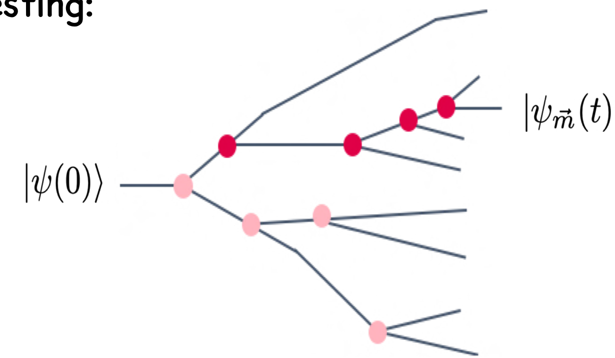
I will show that pertinent properties of the ensemble can be mapped to the ground state of an effective Hamiltonian with an <u>enlarged symmetry</u>.

→ New phases and phase transitions.

How to characterize the ensemble of trajectories

Expectation values of observables over the ensemble are not interesting:

$$\langle \hat{O} \rangle = \overline{\sum_m \mathrm{tr}\left(|\psi_m\rangle\langle\psi_m|\hat{O}\right)} = \mathrm{tr}\left(\rho_{\mathrm{av}}\hat{O}\right) \qquad \rho_{\mathrm{av}} \xrightarrow[t\to\infty]{} \mathbb{1}$$



We need to consider fluctuations over the trajectories:

$$\mathcal{O}_k = \overline{\sum_m p_m \left(\frac{\langle\psi_m|\hat{O}|\psi_m\rangle}{\langle\psi_m|\psi_m\rangle}\right)^k}$$

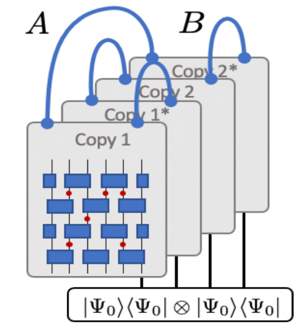
Can be captured by the dynamics of n-copies of the density matrix*:

$$\rho^{\otimes n} = \sum_m |\psi_m\rangle\langle\psi_m| \otimes |\psi_m\rangle\langle\psi_m| \otimes \ldots \otimes |\psi_m\rangle\langle\psi_m| \equiv |\rho^{(n)}\rangle\rangle$$

This also captures purities and Renyi entropies

* need auxiliary replicas
for correct averaging

Mapping the dynamics of n-copies to an effective ground state problem



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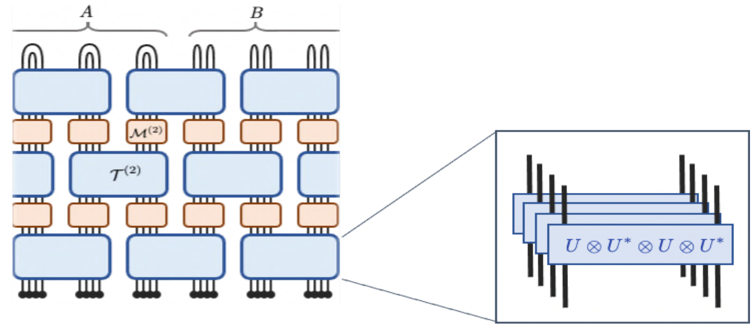
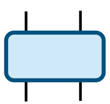
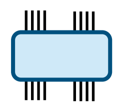
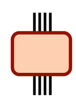




$$U_{ij} = e^{-i\theta_{ij}^{\alpha\beta}\tau_i^{\alpha}\tau_j^{\beta}}$$



$$= \overline{U_{ij} \otimes U_{ij}^* \otimes U_{ij} \otimes U_{ij}^*} = e^{-\delta t J \hat{h}_{\mathrm{u}}}$$



$$= (1 - \Gamma_\nu \delta t)\mathbb{1}^{\otimes 4} + \Gamma_\nu \delta t \sum_{m_\nu = \pm} P_{m_\nu}^{\otimes 4} = e^{-\delta t \Gamma \hat{h}_{\mathrm{M}}}$$

$$|\rho^{(2)}(t)\rangle\rangle = e^{-H_{\mathrm{eff}} t}|\rho^{(2)}(0)\rangle\rangle$$

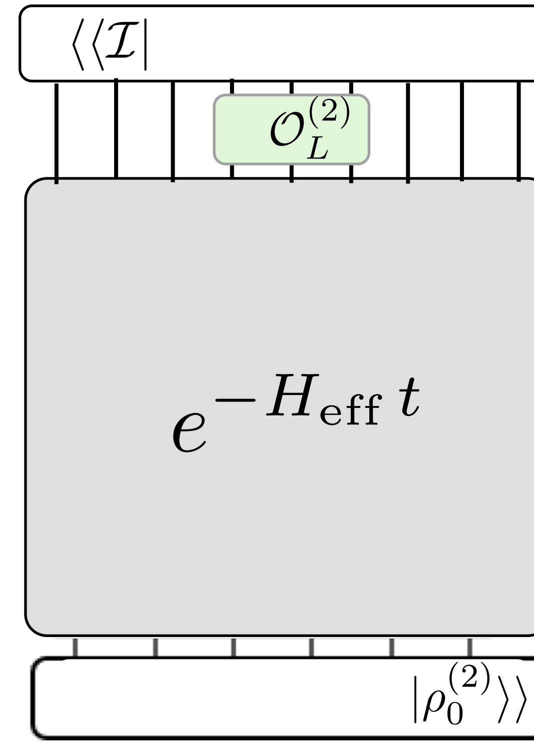
Imaginary time evolution of the unnormalized doubled density matrix !

Circuit quantities (moments) → boundary operators

$$\overline{\sum_m p_m \left(\frac{\langle \psi_m | \hat{O} | \psi_m \rangle}{\langle \psi_m | \psi_m \rangle} \right)^k} = \lim_{n \to 1} \frac{\langle\langle \mathcal{I} | \mathcal{O}_L^{(k)} | \rho_{\rm gs}^{(n)} \rangle\rangle}{\langle\langle \mathcal{I} | \rho_{\rm gs}^{(n)} \rangle\rangle}$$

$$\langle\langle \mathcal{I} | = \sum_{\sigma, \sigma'} \langle\langle \sigma, \sigma, \sigma', \sigma' |$$

Purity: $$e^{-S_A^{(2)}} \approx \frac{\langle\langle \mathcal{I} | (\mathrm{SWAP})_{\mathrm{A}} | \rho_{\rm gs}^{(2)} \rangle\rangle}{\langle\langle \mathcal{I} | \rho_{\rm gs}^{(2)} \rangle\rangle}$$



Intrinsic dynamical symmetry

The unitary gates and measurements preserve purity

➡ Symmetry to permutations between kets and separately between bras

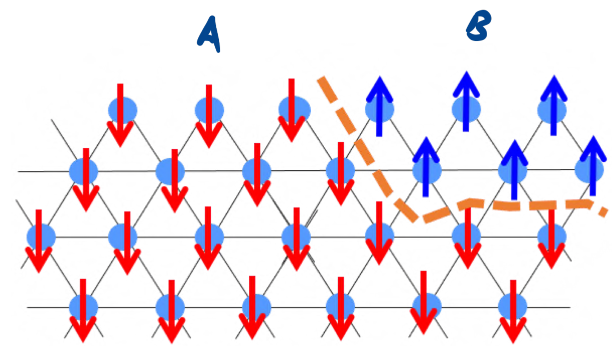
$$\rho^{\otimes n} = \sum_m |\psi_m\rangle\langle\psi_m| \otimes |\psi_m\rangle\langle\psi_m| \otimes \ldots \otimes |\psi_m\rangle\langle\psi_m| \equiv |\rho^{(n)}\rangle\rangle$$

$$(S_n \times S_n) \rtimes \mathbb{Z}_2^H$$

$\mathbb{Z}_2^H$ Due to conservation of hermiticity

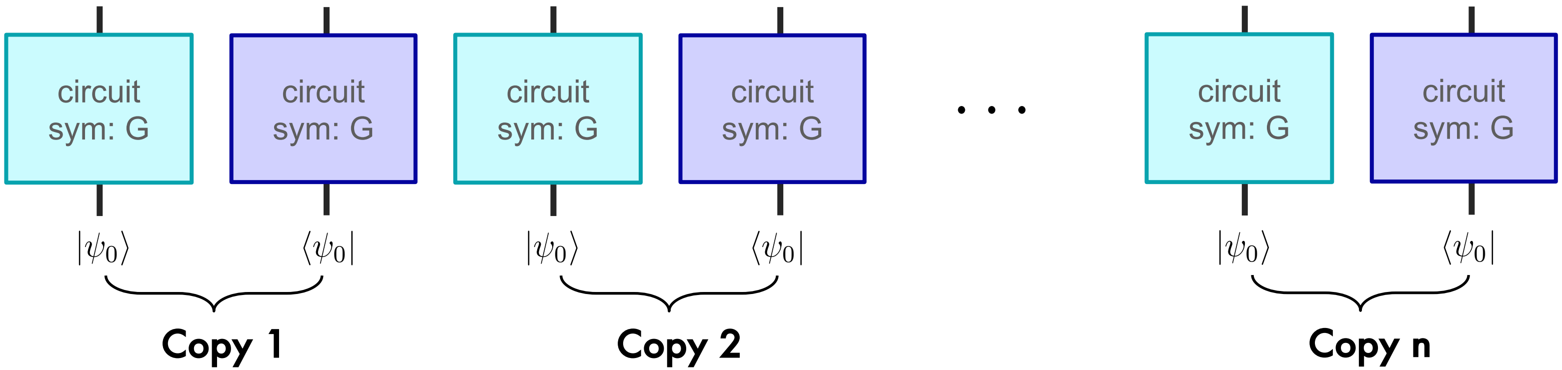
Breaking of the S_n (left) permutation symmetry ➡ volume law entropy

$$S_A^{(2)} \approx -\log\left[\langle\langle\mathcal{I}|(\mathrm{SWAP})_{\mathrm{A}}|\rho^{(2)}_{\mathrm{gs}}\rangle\rangle\right] + \log\left[\langle\langle\mathcal{I}|\rho^{(2)}_{\mathrm{gs}}\rangle\rangle\right] = \mathcal{F}_{dw}$$



Bao Choi and EA PRB 2020; Jian, You, Vasseur and Ludwig PRB 2020

Now add physical symmetry of the circuit elements G



Physical circuit symmetry

$G^{\otimes n} \times G^{\otimes n}$

$\rtimes$

Dynamical symmetry

$(\mathcal{S}_n \times \mathcal{S}_n) \rtimes \mathbb{Z}_2^{\mathbb{H}}$

$=$

Enlarged symmetry

$\left((G^{\otimes n} \rtimes \mathcal{S}_n) \times (G^{\otimes n} \rtimes \mathcal{S}_n)\right) \rtimes \mathbb{Z}_2^{\mathbb{H}}$

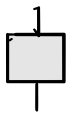
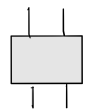
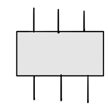
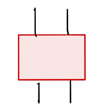
The phases of the circuit ensemble correspond to ground states of an effective Hamiltonian with an enlarged symmetry

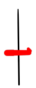
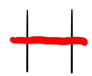
A richer phase structure than would be possible with the circuit symmetry alone

Example 1: qubit circuit with Z_2 symmetry

Symmetry generator: $\hat{\pi} = \prod_{j=1}^{L} Z_j$

Possible circuit elements:

 $= e^{-h_i \hat{Z}_i}$  $= e^{-iK_{ij} \hat{Z}_i Z_j}$,  , ...    $= e^{-iJ_{ij} \hat{X}_i \hat{X}_j}$

 = measure $\hat{Z}_i$  = measure $X_i X_j$

Enlarged symmetry for two copies:

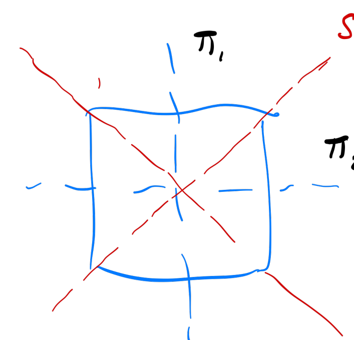
$$\mathcal{G}^{(2)} = (D_4 \times D_4) \rtimes \mathbb{Z}_2^{\mathbb{H}}$$

In this case it is sufficient to consider

$$\mathcal{G}^{(2)}_{\text{eff}} = D_4 (\rtimes \mathbb{Z}_2^{\mathbb{H}}),$$

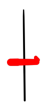
- Four area-law phases
- Six volume-law phases
- Topological phase protected by $Z_2 \times S_2$ symmetry

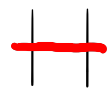
$$D_4 = (\mathbb{Z}_2 \times \mathbb{Z}_2) \rtimes \mathcal{S}_2$$



Warmup – measurement only model with Z_2 symmetry

[Sang, Hsieh, *arXiv:*2004.09509]

 = measure $\hat{Z}_i$

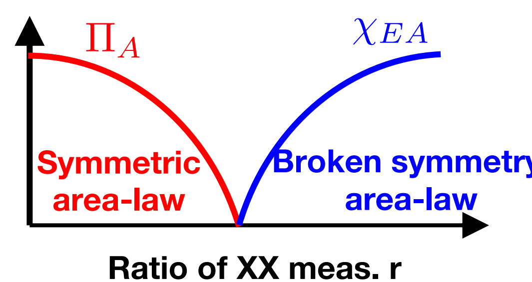
 = measure $X_i X_j$

Z measurements dominate:

- Random definite parity state in each trajectory $|\uparrow\uparrow\downarrow\uparrow\downarrow\cdots\uparrow\rangle$
- Nonvanishing subsystem parity variance $\Pi_A = \sum_m p_m \langle \prod_{i \in A} Z_i \rangle_m^2$

XX measurements dominate:

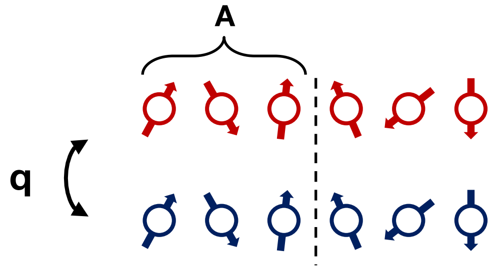
- Random broken symmetry state $|\rightarrow\rightarrow\leftarrow\rightarrow\leftarrow\ldots\rightarrow\rangle + |\leftarrow\leftarrow\rightarrow\leftarrow\rightarrow\ldots\leftarrow\rangle$
- Nonvanishing Edwards-Anderson (spin glass) correlation function $\chi_{EA} = \sum_m p_m \langle X_i X_j \rangle_m^2$



This is what we expect from just the physical circuit Z_2 symmetry.

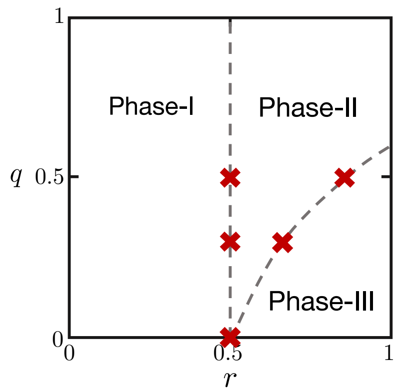
No signature of an enlarged symmetry in this case !

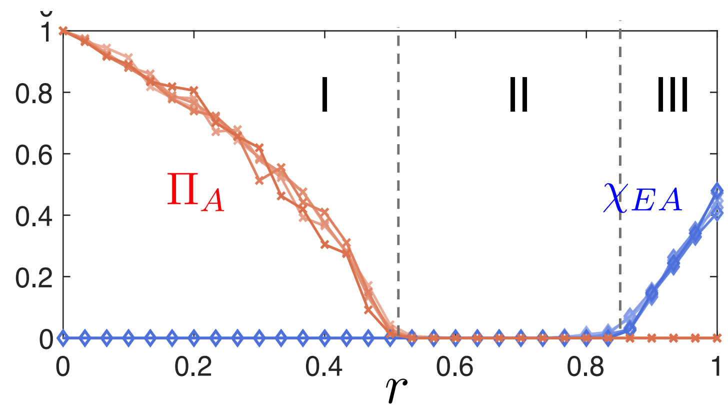
Two chain model



Chain-1: Unitary evolution unrestricted by symmetry.
Alone realizes a volume law phase (broken S_2 symmetry)

Chain-2: Measurements-only and restricted by Z2 symmetry
Alone realizes the symmetric and ordered area law phases



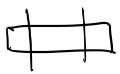


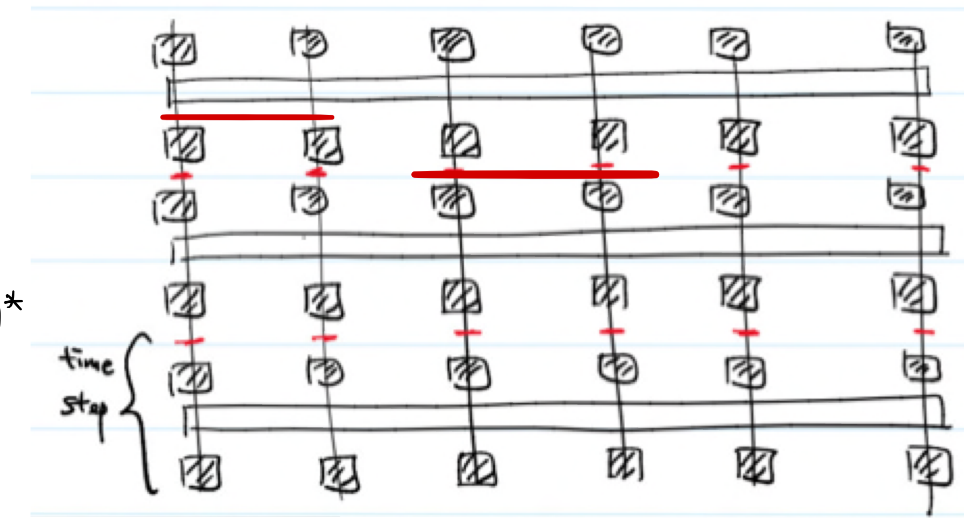
- q = 0 trivially realizes a broken symmetry volume law phase.
- q>0 critical point opens to a new phase.
- Three distinct volume law phases: direct evidence for enlarged symmetry.
- <u>Measurement protected quantum order</u> in a volume law state

Example 2: Gaussian fermion circuit

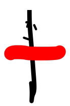
Symmetry: Z_2 fermion parity

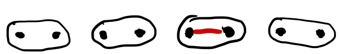
On site:  $= e^{-\vartheta_j \bar{\gamma}_j \gamma_j} \otimes V^* \otimes V \otimes V^*$

Hopping:  $= e^{\delta t J_i \gamma_{i-1} \bar{\gamma}_i} \otimes U^* \otimes U \otimes U^*$

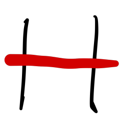


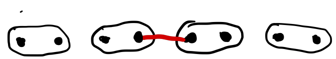
On site parity measurement:

 = measure $i\bar{\gamma}_j \gamma_j$



Bond parity measurement:

 = measure $i\gamma_{j-1}\bar{\gamma}_j$



Previous results:

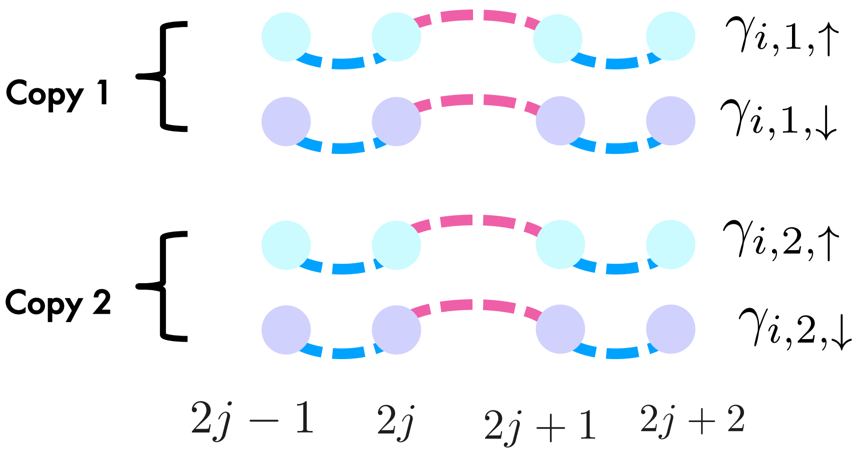
1. Absence of volume law phase for any non vanishing measurement strength

   X. Cao, A. Tilloy and A. De Luca, SciPost 2019

2. Numerical simulations indicated transition from a phase with log(L) entanglement to area law.

   O. Alberton, M. Buchhold, S. Diehl, arXiv:2005.09722

Enlarged symmetry



Purely unitary dynamics:

Four identical free Majorana chains

➡ SU(2)xSU(2) ("spin" x "eta" symmetry)

Add measurements: $(U(1) \rtimes Z_2) \times (U(1) \rtimes Z_2)$

The effective conserved fermions are not local to a single copy

$$c_{2j-1,\uparrow} = \frac{\gamma_{2j-1,1\uparrow} + i\gamma_{2j-1,2\uparrow}}{2} \qquad c_{2j-1,\downarrow} = \frac{\gamma_{2j-1,1\downarrow} - i\gamma_{2j-1,2\downarrow}}{2}$$

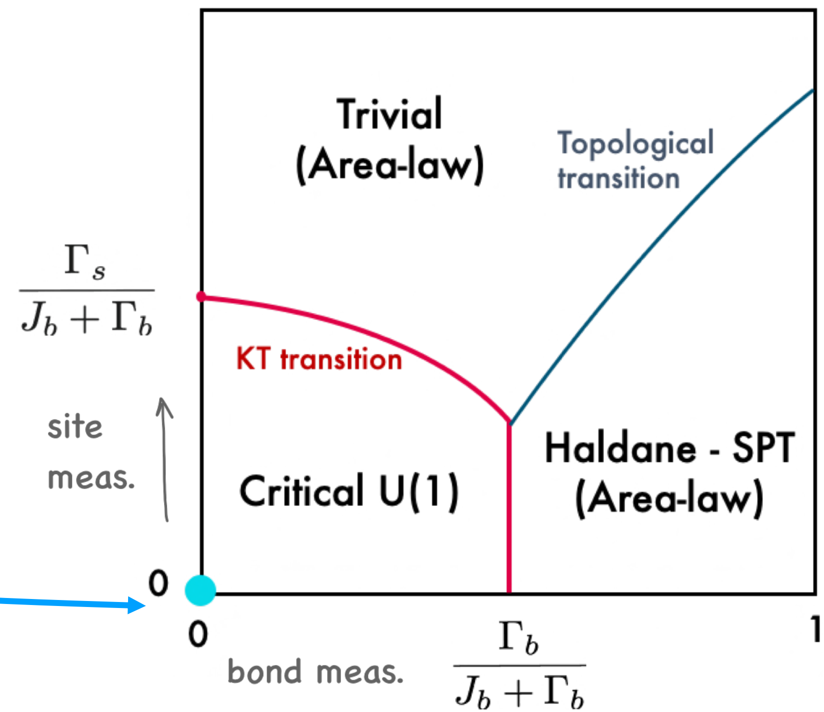
Effective Hamiltonian: spin-1 model

$$\mathcal{H}_Q = \sum_{j=1}^{L} (-J_b - \Gamma_b)\left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y\right) + (-J_b + \Gamma_b) S_j^z S_{j+1}^z + \Gamma_s \left(S_j^z\right)^2$$

Hopping   Bond meas.   Site meas.

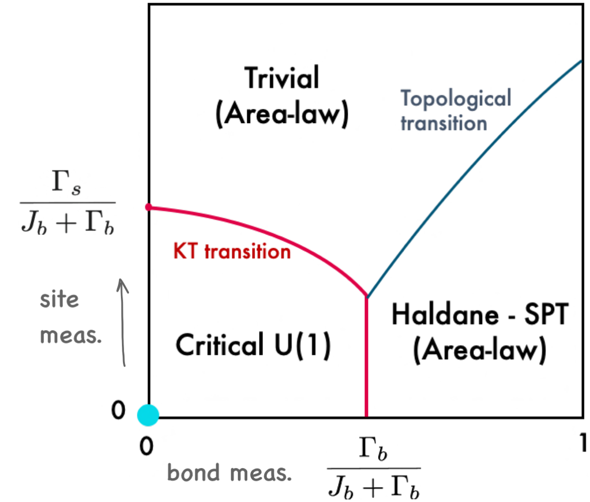
SU(2) symmetry only
for purely unitary dynamics

SU(2) ferromagnet allows
Broken symmetry -> volume law



Low energy theory

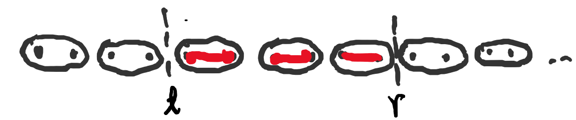
$$H = \frac{1}{2}\int dx \left[K(\nabla\hat{\theta})^2 + \frac{1}{K}(\nabla\hat{\phi})^2\right] - g\int dx \cos(2\hat{\phi})$$



Diagnostics:

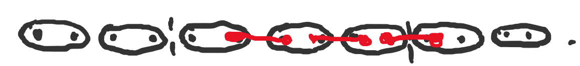
Site parity variance:

$$[\Pi_s^{(2)}] \sim [\sin\phi_\ell \sin\phi_r]_{boundary} \sim |x|^{-\frac{K}{4}}$$

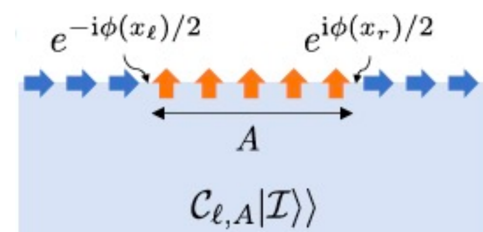


Bond parity variance:

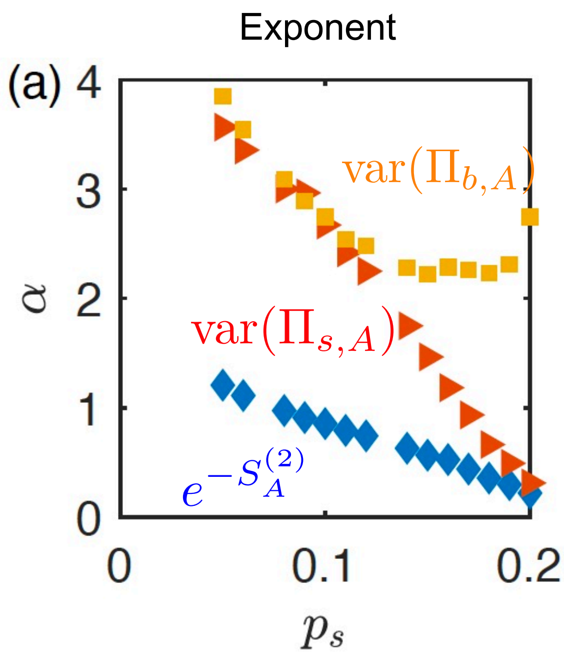
$$\Pi_b^{(2)} \sim [\cos\phi_\ell \cos\phi_r]_{bnd} \sim |x|^{-\frac{K}{4}}$$

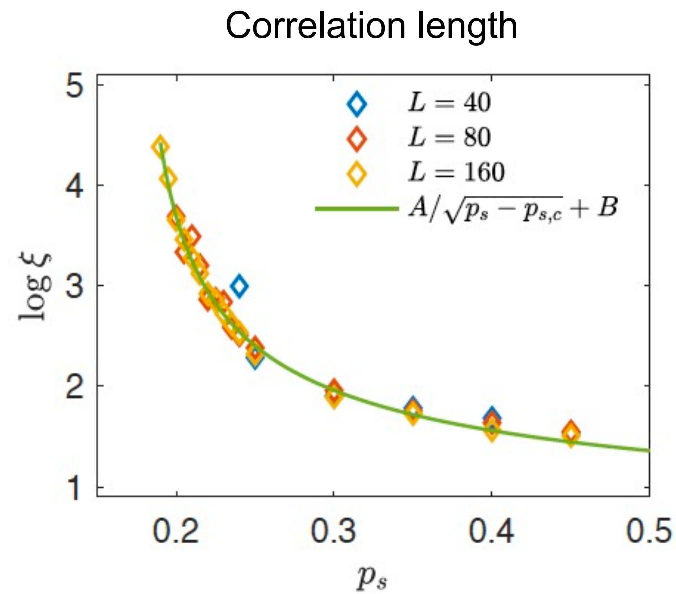


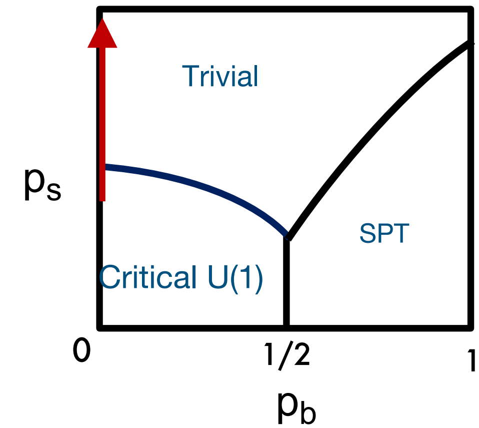
Purity: $e^{-S_A^{(2)}} \sim [e^{-i\phi_\ell/2}\, e^{i\phi_r/2}] \sim |x|^{-\frac{K}{16}}$



Numerical simulation of gaussian Majorana model

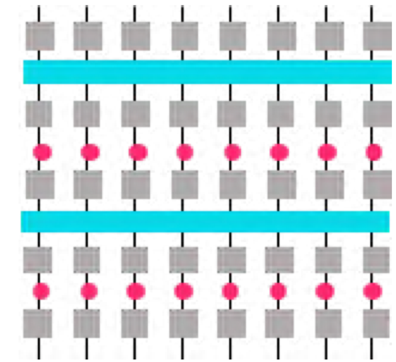






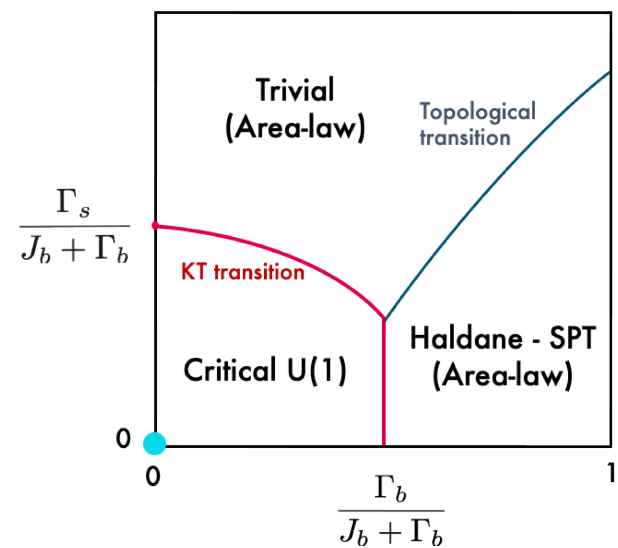
Consistent with a KT transition

Summary and conclusions



- Phases of hybrid quantum circuits can be classified as ground states of an effective Hamiltonian with enlarged effective symmetry

- Example 1: Qubit circuit with Z2 symmetry
  Phases are classified by a larger D4 symmetry
  Broken symmetry volume law states
  SPT phase protected by Z2 x S2 (?)



- Example 2: Gaussian fermion circuit
  Z2 fermion parity enlarged to U(1)
  Evidence for KT transition (critical to area law)

Questions and outlook

- Replica limit? Non replica treatment?
- Can the time-reversal-like (transpose) symmetry be spontaneously broken?
- More natural and scalable probes - Fisher information
- Continuous symmetry – interplay with hydrodynamics?
- Applications to protecting quantum information?
  Measurement preparation of topological states / anyons ?