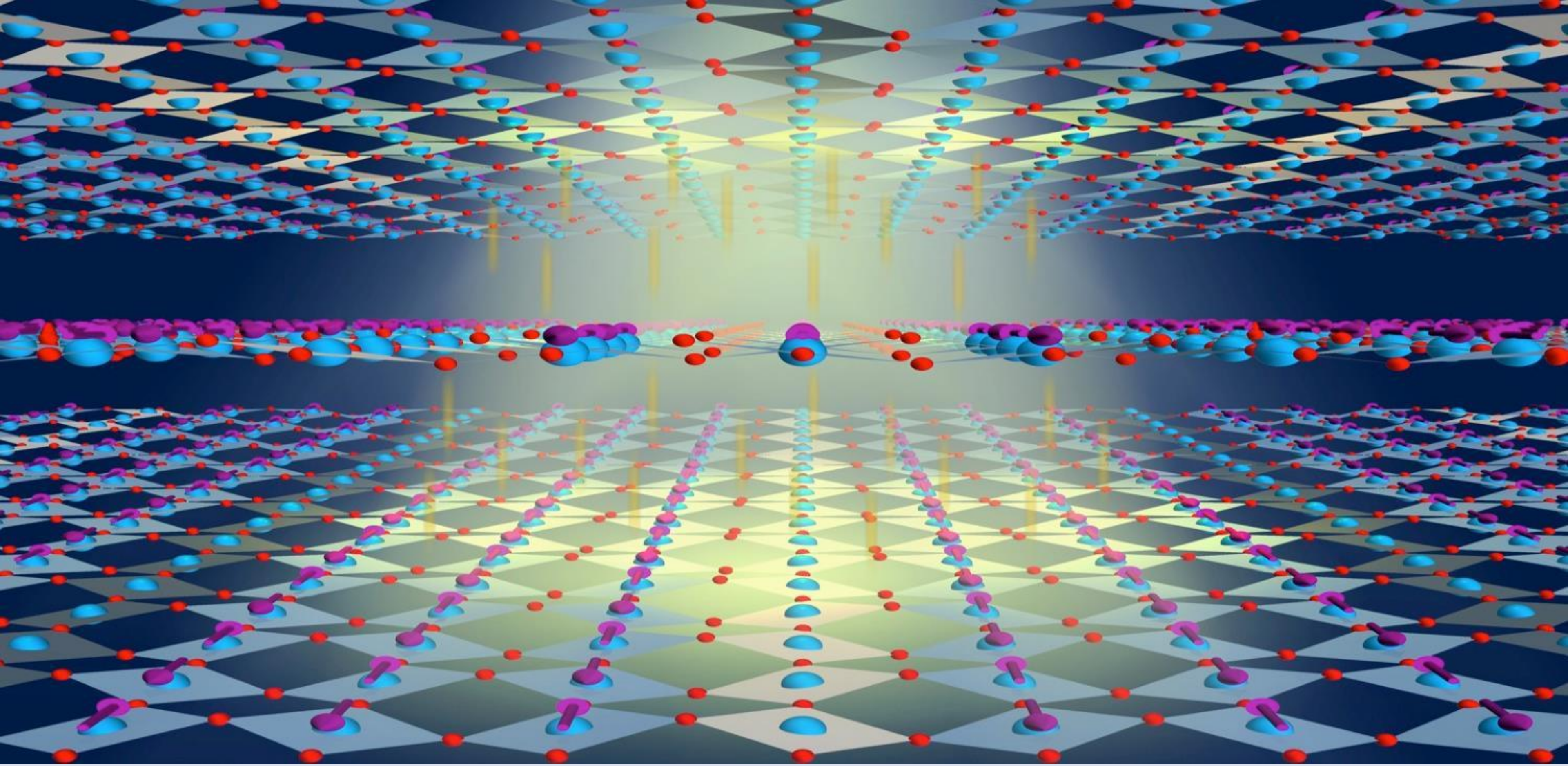


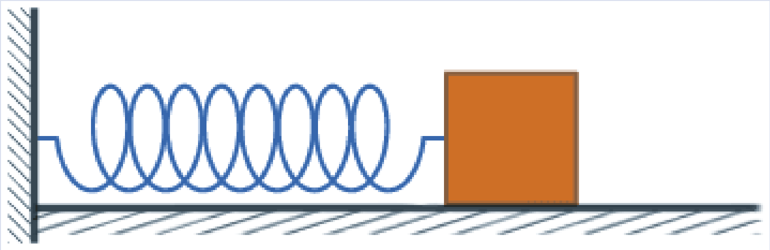
Non-stationary quantum many-body dynamics

Berislav Buca | Clarendon Laboratory, University of Oxford | 5 May 2021

Outline

- **Introduction**
- Motivation
- Definition of non-stationarity (complex long-time dynamics)
- **Contrast: Equilibration to stationarity (statistical mechanics)**
- **Dynamical symmetries**
- Closed quantum many-body systems
- Open quantum many-body systems
- **Examples of non-stationary closed quantum many-body systems**
- Heisenberg XXZ spin chain
- Spin lace (~magnetic azurite) (works both as an open example)
- **Examples of non-stationary open quantum many-body systems**
- Dephased spin-1/2 fermions in an optical lattice
- Spinor BECs in an optical cavity
- **Conclusion**

What is non-stationarity?



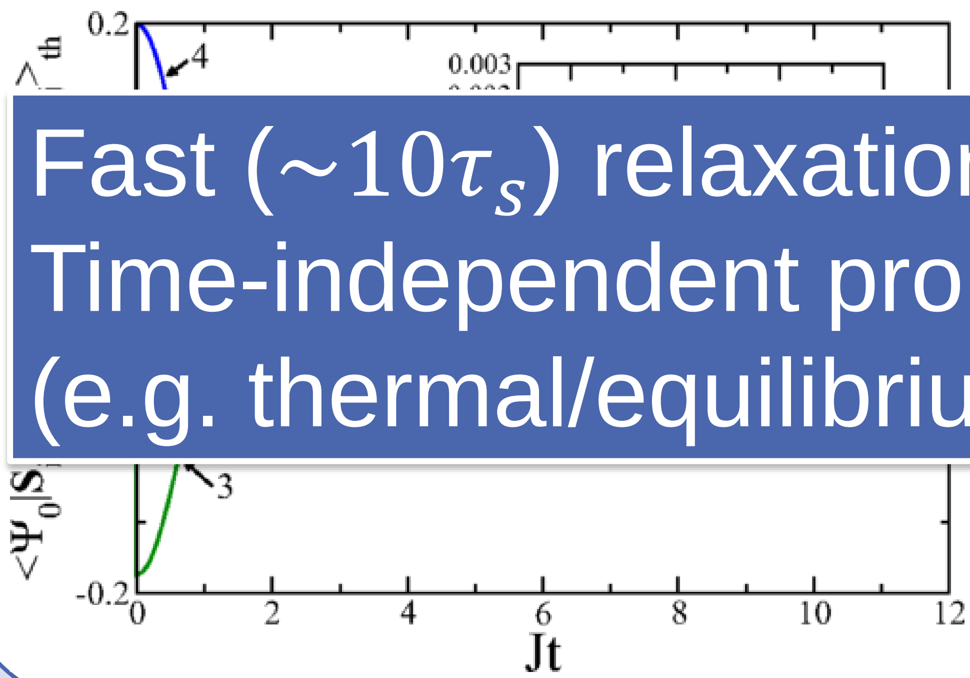
A simple harmonic oscillator

- We will focus on (physically measurable) **local** quantities (e.g. particle number, heat, charge, position of a particle, magnetization, …)
- Explicit long-time time-dependence of measurable quantities $\langle x(t) \rangle$
- What happens when we have a huge number of particles that interact with each other?
- Statistical mechanics: Equilibration on some time scale $\langle x(t) \rangle \rightarrow x_\infty$
- What is the time scale?

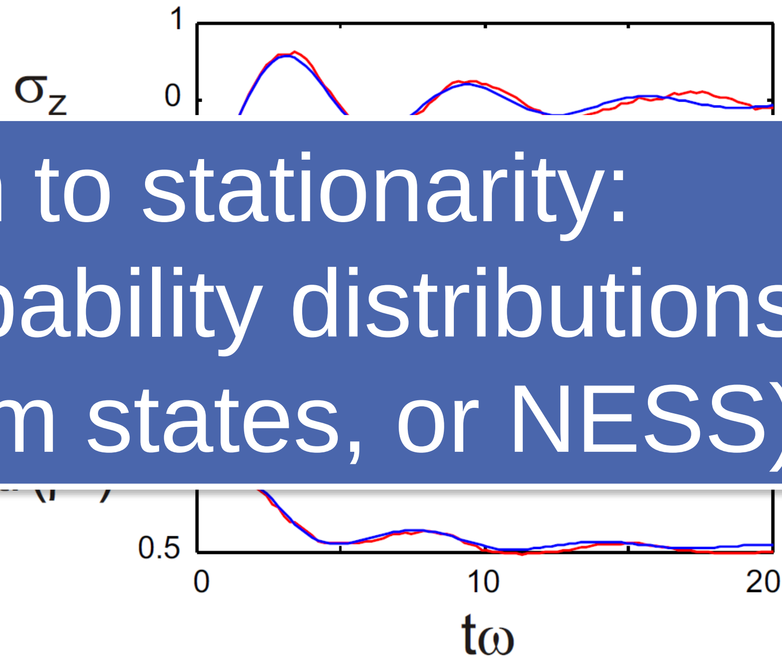
Ivan Airola, Wikimedia

Quantum physics

Relaxation in closed systems



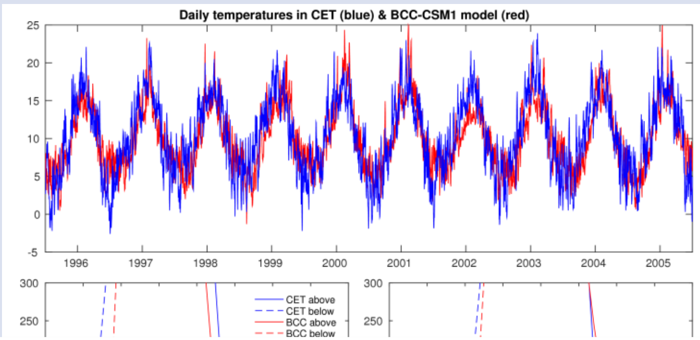
Open systems

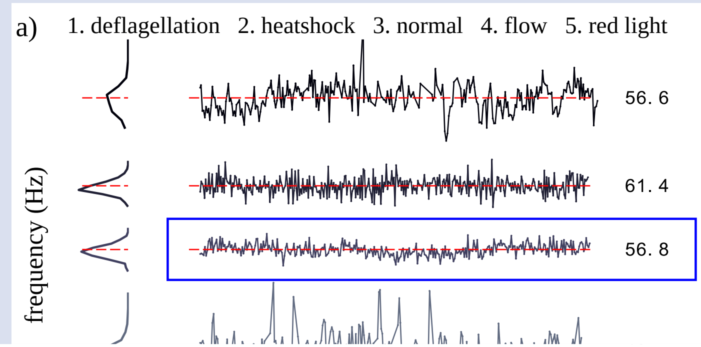


Fast ($\sim 10\tau_s$) relaxation to stationarity:
Time-independent probability distributions
(e.g. thermal/equilibrium states, or NESS)

J. Sirker, N. P. Konstantinidis, F. Andraschko, and N. Sedlmayr, Phys. Rev. A **89**, 042104 (2014)

Real world systems

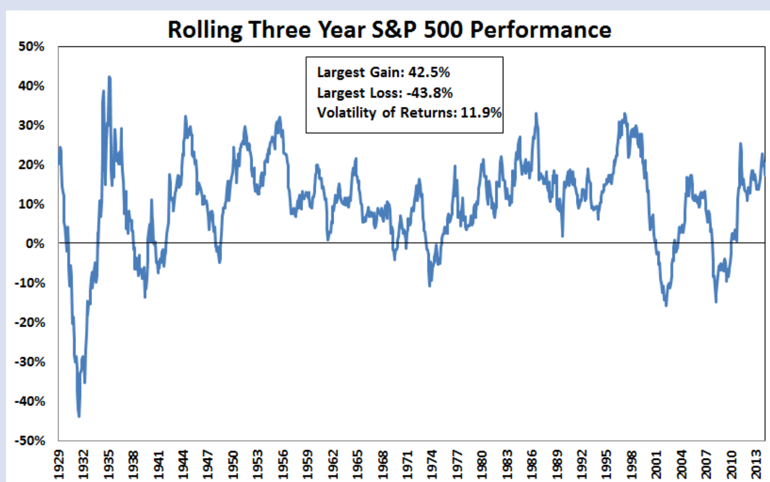




Non-stationary for extremely long time compared to microscopic scales!

Temperatures (daily)

Eukaryotic flagellum oscillations
(Wan, Goldstein. PRL 2014)



Complex dynamics

Goal:
Understand emergence of periodic time dynamics on a fundamental quantum level!

-P. W. Anderson, More is different

Naïve approach

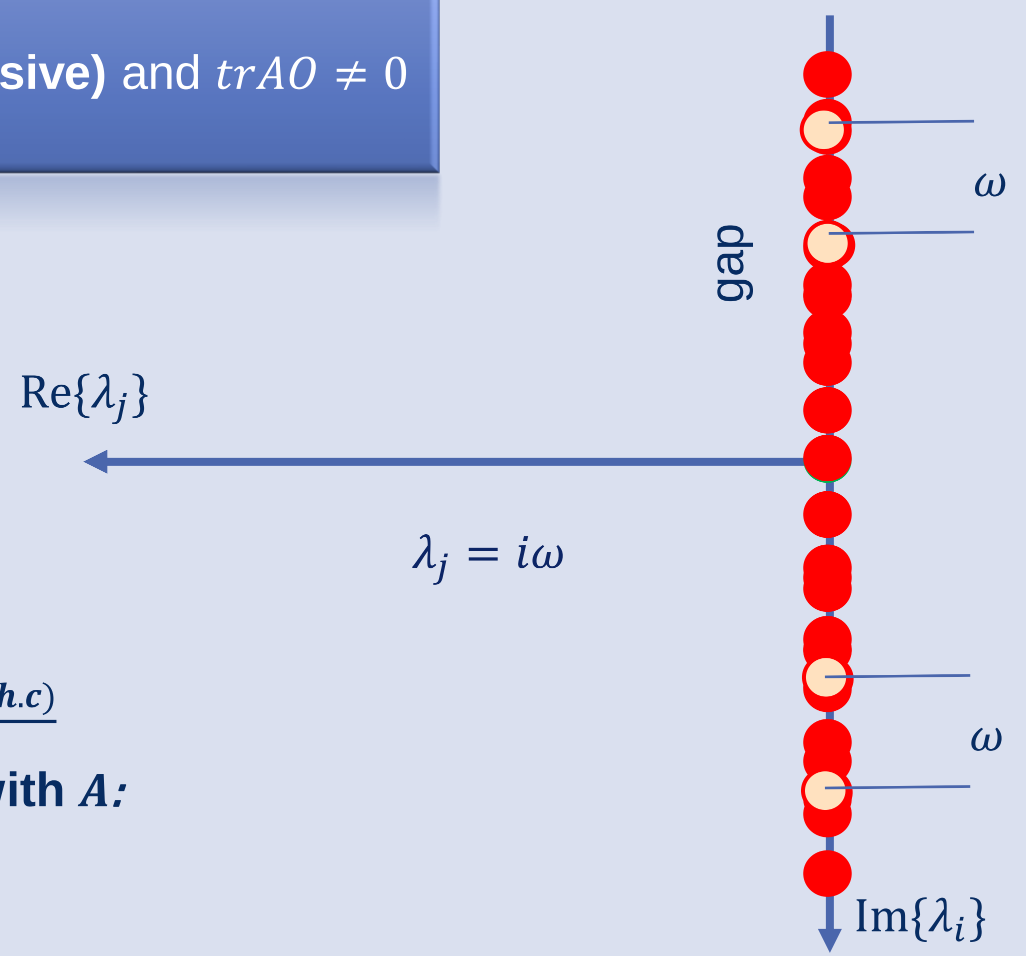
- We have a system with a macroscopic number of **interacting** particles
- Diagonalise: $H|\psi_k\rangle = E_k|\psi_k\rangle$
- Frequencies: $\omega_{k,j} = E_k - E_j$
- Solution for **generic** initial state $|\psi_0\rangle$ for **generic** observable O :
- $\langle O(t)\rangle = \sum_{j,k} e^{-i\omega_{j,k}t} \langle\psi_j|O|\psi_k\rangle\langle\psi_0|\psi_k\rangle\,\langle\psi_j|\psi_0\rangle$
- $\langle O(t\to\infty)\rangle$ goes to constant value $\langle O\rangle_\infty$ (statistical mechanics) (continuous spectrum – happens in finite time)

Randon

A sum of an infinite number of waves with random frequency and phase
Destructive interference - dephasing

Eigenfrequencies λ_i of the dynamics

Dynamical symmetries:
$[H, A] = -\omega A, \quad ||A|| \propto V$ **(extensive)** and $trAO \neq 0$



- Maximum entropy
- **Systems goes to t-GGE:**
- $\boldsymbol{\rho(t \to \infty) = \frac{exp(-\beta H + \mu Q + \mu_A\, e^{i\omega t} A + h.c)}{Z}}$
- **Operators that have overlap with** $\boldsymbol{A}$**:** $\boldsymbol{\langle AO \rangle \neq 0}$
- $\boldsymbol{\langle O(t \to \infty) \rangle = B\, exp(i\omega t) + h.c}$

Eigenvalues λ_i of the dynamics

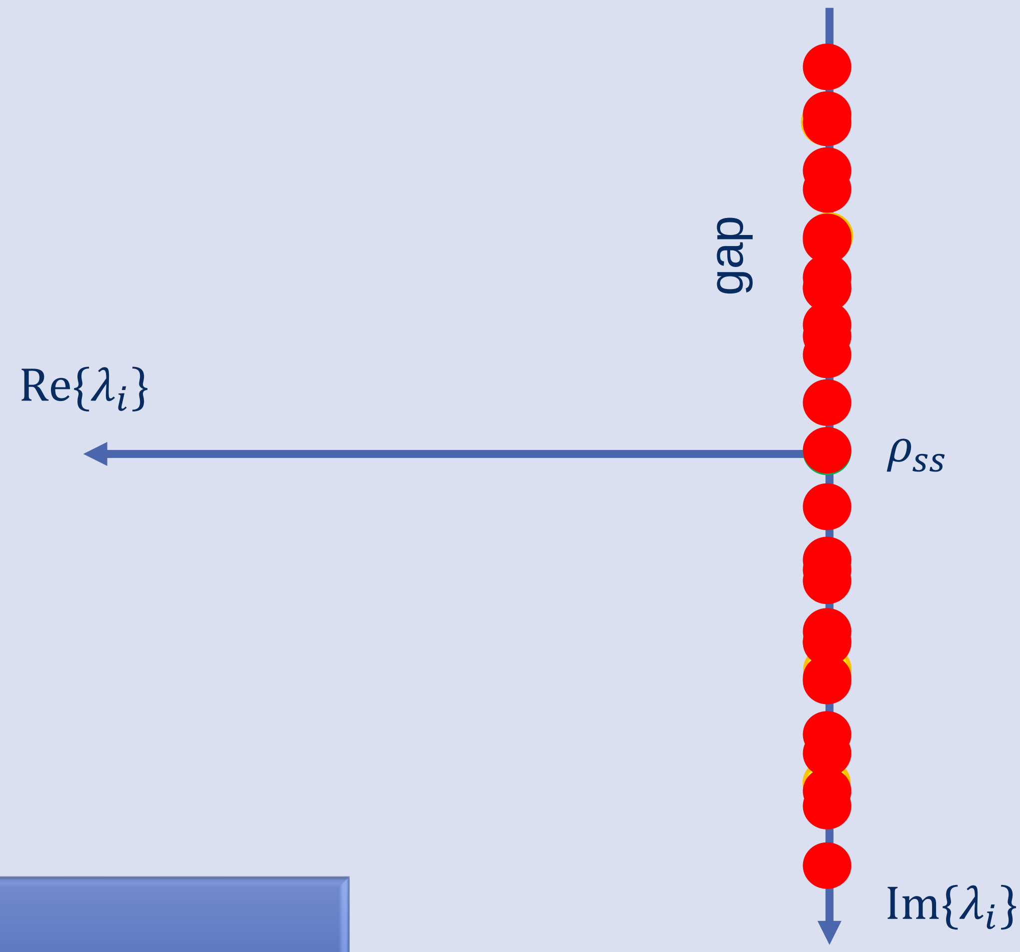
$$\dot{\rho} = \mathcal{L}\rho = -\mathrm{i}[H,\rho] + \sum_{\mu} \left(2\, L_\mu\, \rho\, L_\mu^\dagger - L_\mu^\dagger L_\mu\, \rho - \rho L_\mu^\dagger L_\mu\right)$$

$$\mathcal{L}\rho_i = \lambda_i \rho_i$$

$$\rho(t) = \sum_i e^{\lambda_i t}\, c_i \rho_i$$

Open system

- negative real parts
- dissipation
- not necessarily DFS!
- (often more general)



Necessary* and sufficient for ●
$[H,A] = -\omega A$ and $[A, L_\mu^\dagger] = [A, L_\mu] = 0 \quad \forall\mu$

Non-stationarity in **closed** many-body quantum systems: Examples

Example #1: Heisenberg XXZ spin-1/2 chain

Closed system example: XXZ

- Standard model of quantum magnetism (integrable)
- Add magnetic field to XXZ spin chain
- $H = \sum_j \sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \Delta \sigma_j^z \sigma_{j+1}^z + 2\mathrm{h}\, \sigma_{\mathrm{j}}^z$
- Dynamical symmetries:

$[H, Y(\phi)\,] = h\, mY(\phi)$(with $\Delta = \cos\eta, \eta = \frac{2l}{m}\pi$)

- Quasilocal

Example: For $\Delta = \cos 2\pi/3 = -1/2$

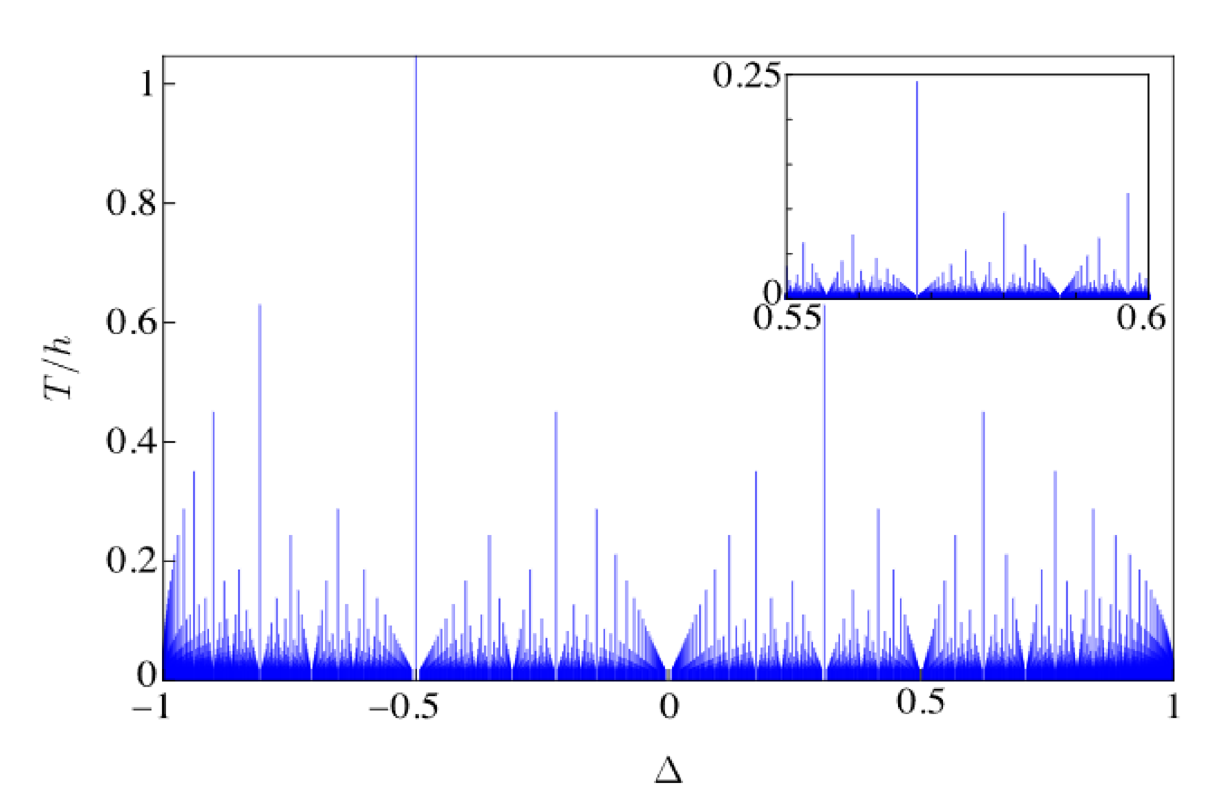
$Y(\phi) = \sum_j \csc\phi \left(\sigma_j^+ \sigma_{j+1}^+ \sigma_{j+2}^+\right)$ +higher operator terms

4,5 … n site terms

M Medenjak, B Buca, D Jaksch. Phys. Rev. B **102**, 041117 (2020). Construction of Y's: obtainable via Bethe Ansatz - Zadnik, Medenjak, Prosen, Nuc. Phys. B

Fractals!

m-point correlator (and higher) that have non-zero overlap with Y's will oscillate forever with period $T = 2\pi/(h\,m)$



Autocorrelation function at infinite temperature

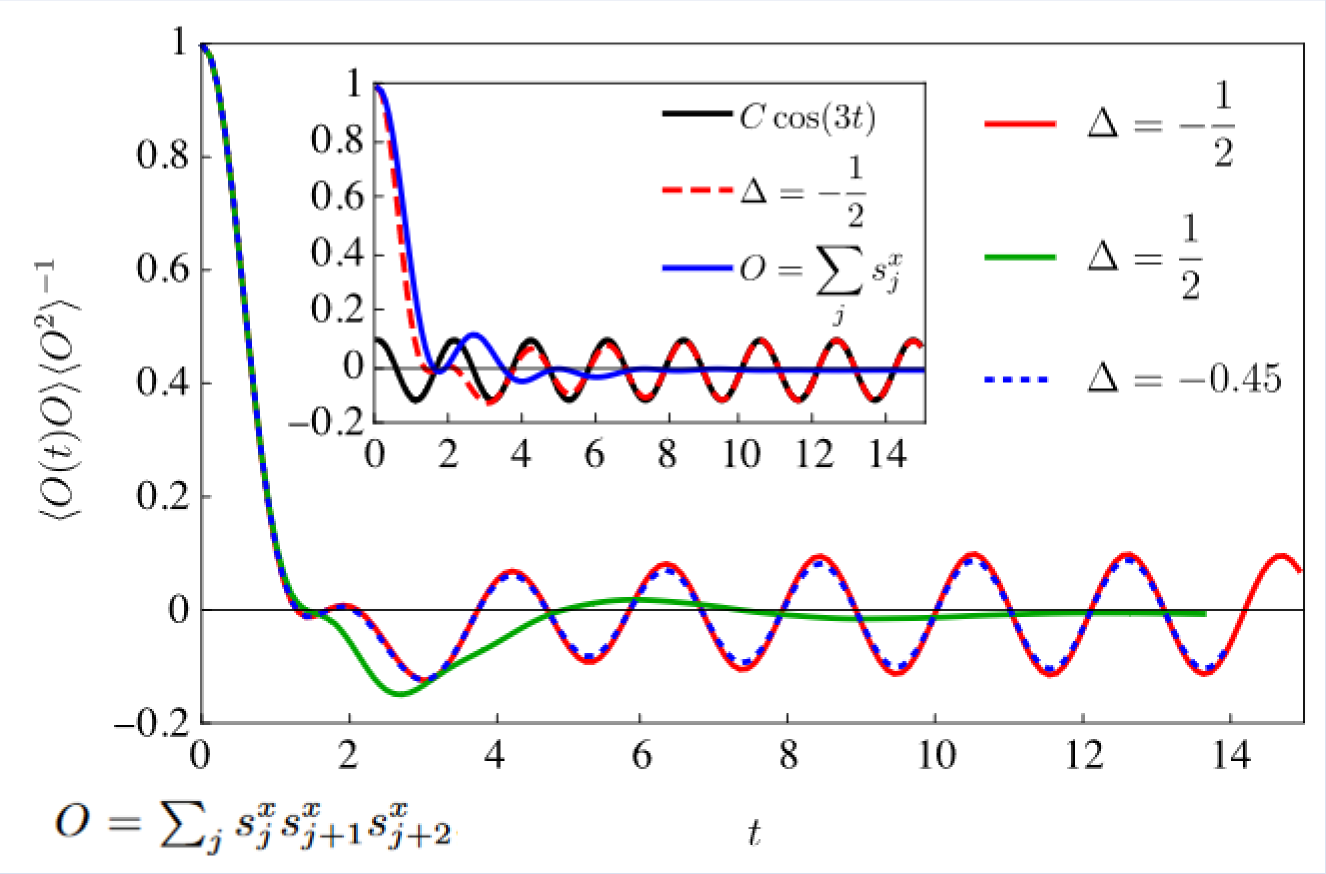
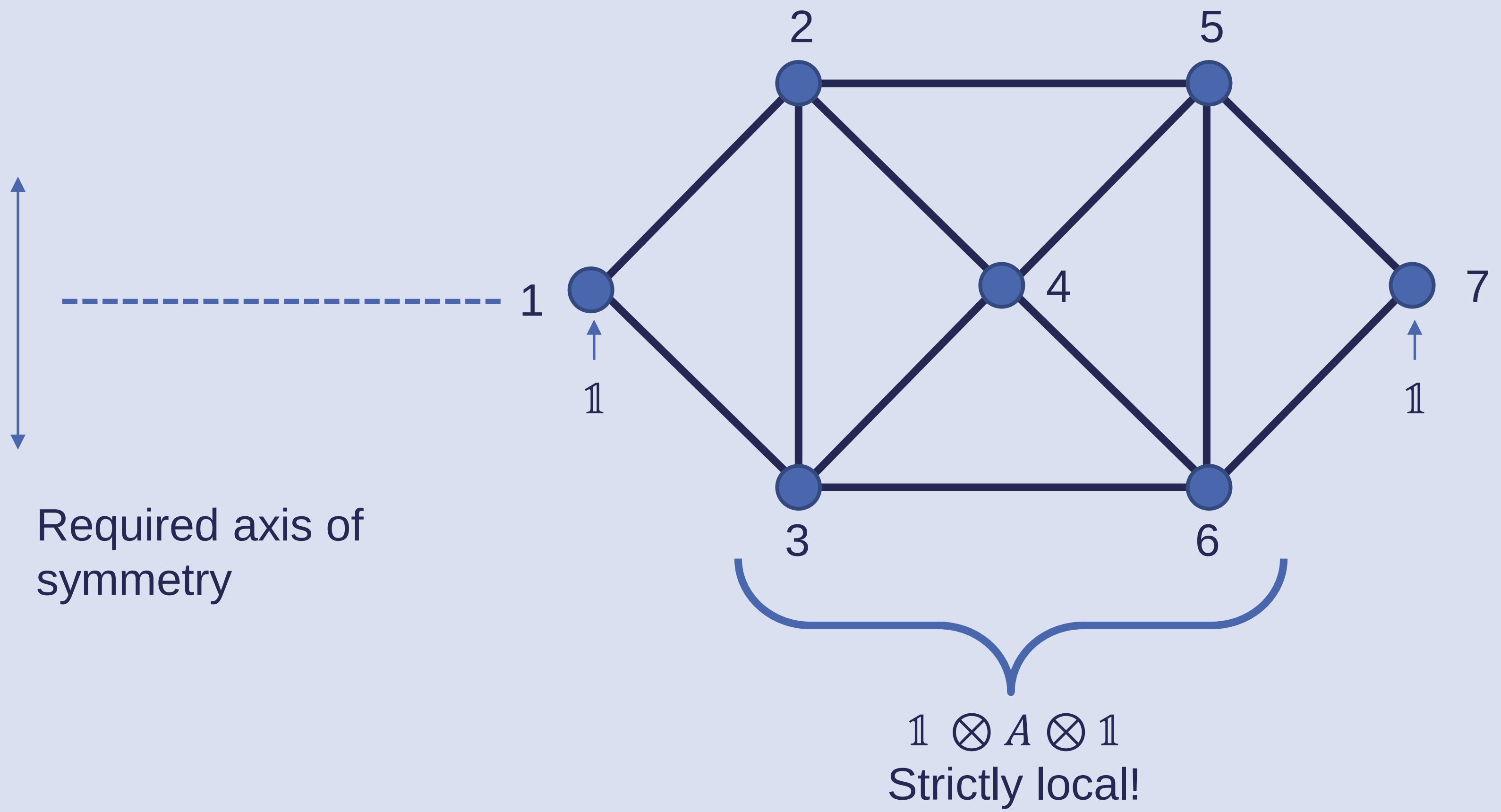


Figure 2: DMRG (N=100, h=1) and analytical result $C = \frac{1}{64}\left(\frac{27\sqrt{3}}{\pi} - 8\right)$,
With $\Delta = \cos({}^{2l}/_{m}\,\pi)$

Example #2: “Spin lace”

Closed system example: Spin lace

- (Almost) any reflection symmetric interaction on the below quasi-1D geometry



BB, A. Purkayastha, G. Guarnieri, M. Mitchison, D. Jaksch, J. Goold. arXiv:2008.11166 (2020).

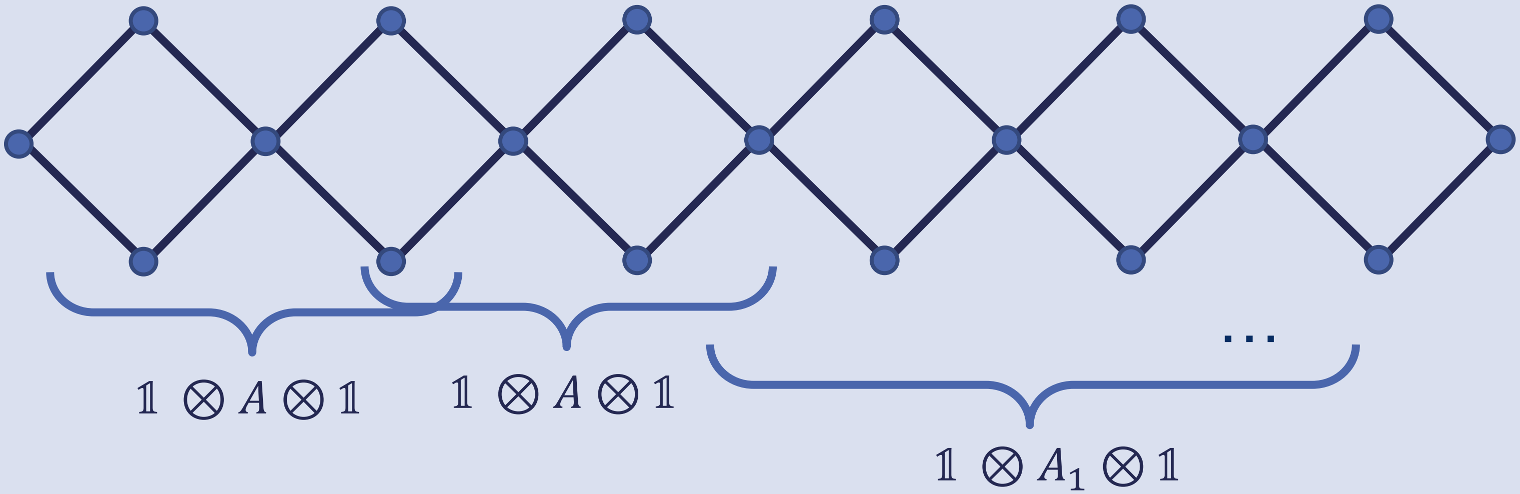
Stability

- Stable to any coupling (dissipation)
- $A = P_{|\uparrow\downarrow - \downarrow\uparrow\rangle} \otimes \sigma^{+} \otimes P_{|\uparrow\downarrow - \downarrow\uparrow\rangle}$
- NB Apart from A's there are also conservation laws $\mathrm{Q} = [A, A^{\dagger}]$ and conservation due to reflection

BB, A. Purkayastha, G. Guarnieri, M. Mitchison, D. Jaksch, J. Goold. arXiv:2008.11166 (2020).

Spin lace

- We could couple the spin-1/2 plaquette to anything
- Let us choose the following system ($k = 0,1, \dots \frac{N-4}{3}$)

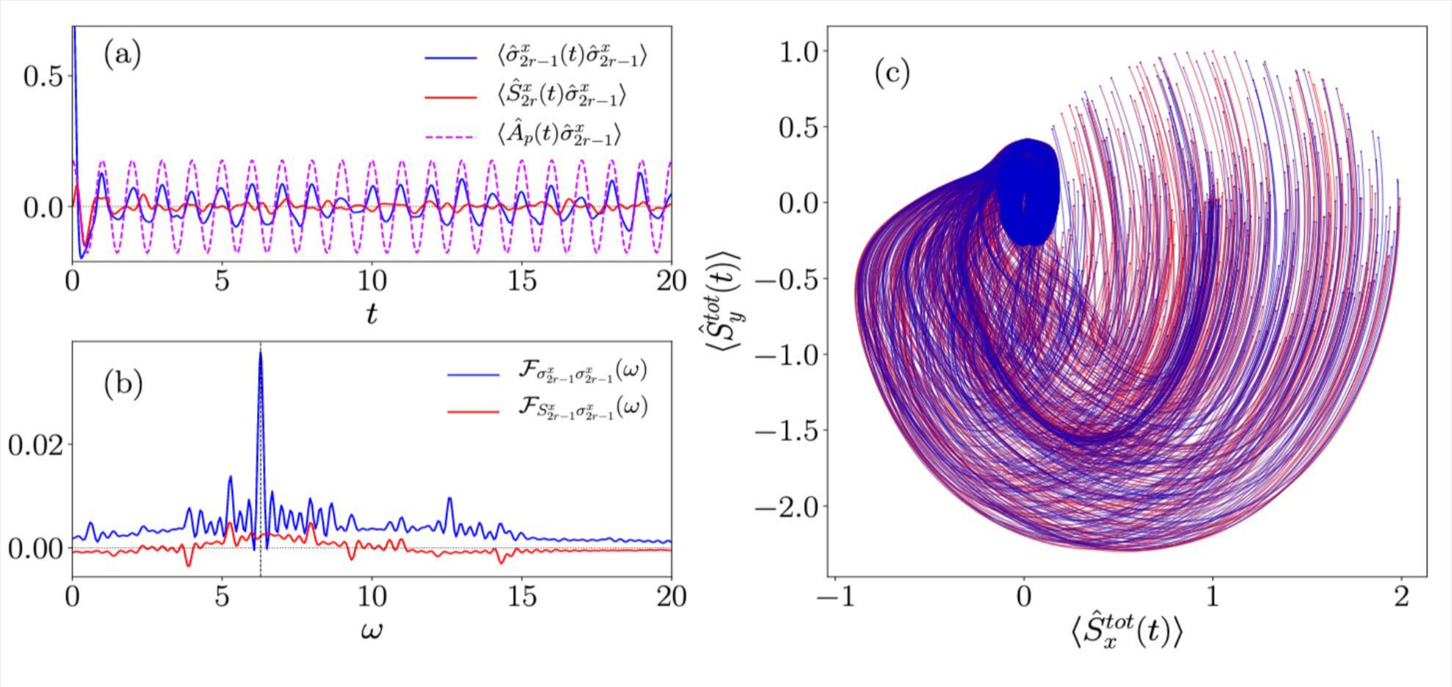


- Superextensive number of dynamical symmetries and conservation laws – superintegrable!
- **Stable to local perturbation of arbitrary strength including dissipative**
- Only way to break the above properties is to break the Z_2 reflection symmetry on each and every site

BB, A. Purkayastha, G. Guarnieri, M. Mitchison, D. Jaksch, J. Goold. arXiv:2008.11166 (2020).

Quantum many-body attractor

- We study linear response at infinite temperature
- Autocorrelation functions
- Observables that have overlap with $A(Q)$ will oscillate (relax to finite values)
- Other observables ergodic (relax to 0)

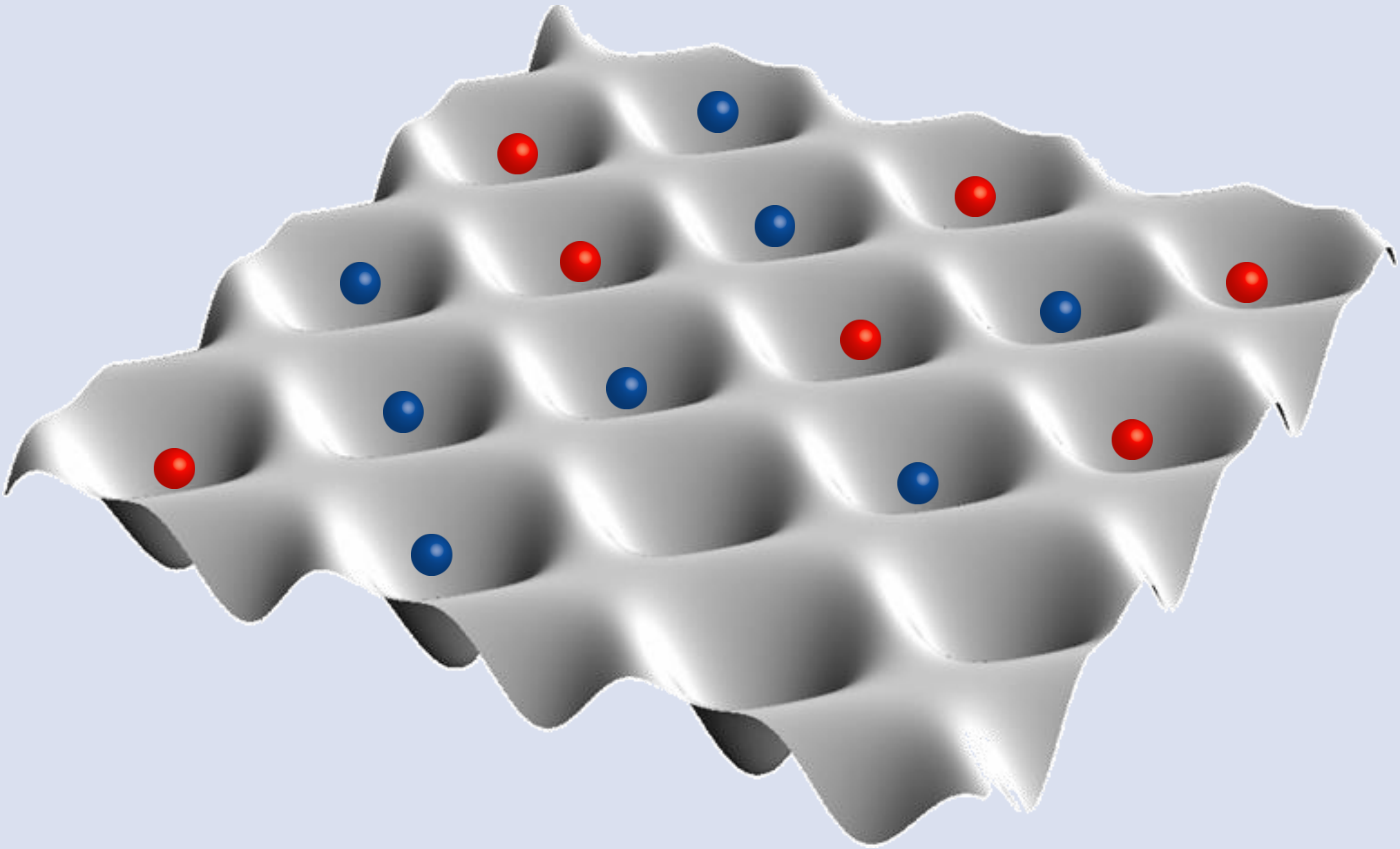


BB, A. Purkayastha, G. Guarnieri, M. Mitchison, D. Jaksch, J. Goold. arXiv:2008.11166 (2020).

Non-stationarity in **open** many-body quantum systems; Examples

Open system example #1: Interacting fermions with two spin states in an optical lattice

Open system example



BB, J. Tindall, and D. Jaksch, Nat. comms. 10, 1730 (2019)

Hubbard model and symmetries

- Hubbard Hamiltonian on a D-dimensional bipartite lattice with M sites

$$H_{\text{Hub}} = -t \sum_{\langle i,j \rangle,\sigma} \left(c_{i\sigma}^{\dagger} c_{j\sigma} + h.c. \right) + U \sum_{i} n_{i\uparrow} n_{i\downarrow} + \sum_{i} (\epsilon_i - \mu) n_i + \frac{B}{2} (n_{i\uparrow} - n_{i\uparrow})$$

- Spin symmetry

$$S^z = \sum_j S_j^z, \qquad S_j^z = \frac{1}{2}(n_{j,\uparrow} - n_{j,\downarrow}),$$

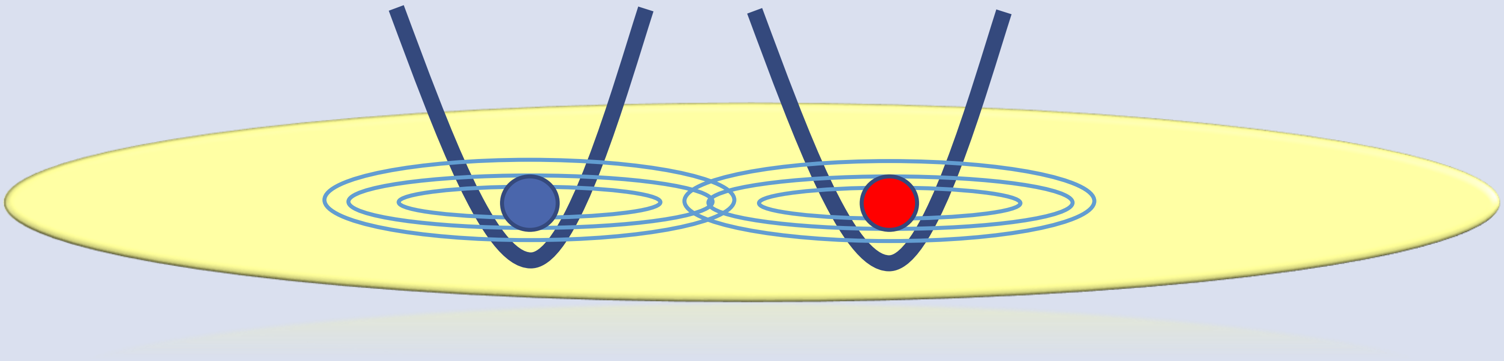
$$S^+ = \sum_j S_j^+, \qquad S_j^+ = c_{j,\uparrow}^{\dagger} c_{j,\downarrow}$$

$$S^- = \sum_j S_j^-, \qquad S_j^- = c_{j,\downarrow}^{\dagger} c_{j,\uparrow},$$

- with

$$[H, S^z] = 0, \qquad [H, S^{\pm}] = \pm B\, S^{\pm}$$

External symmetry selective dissipation



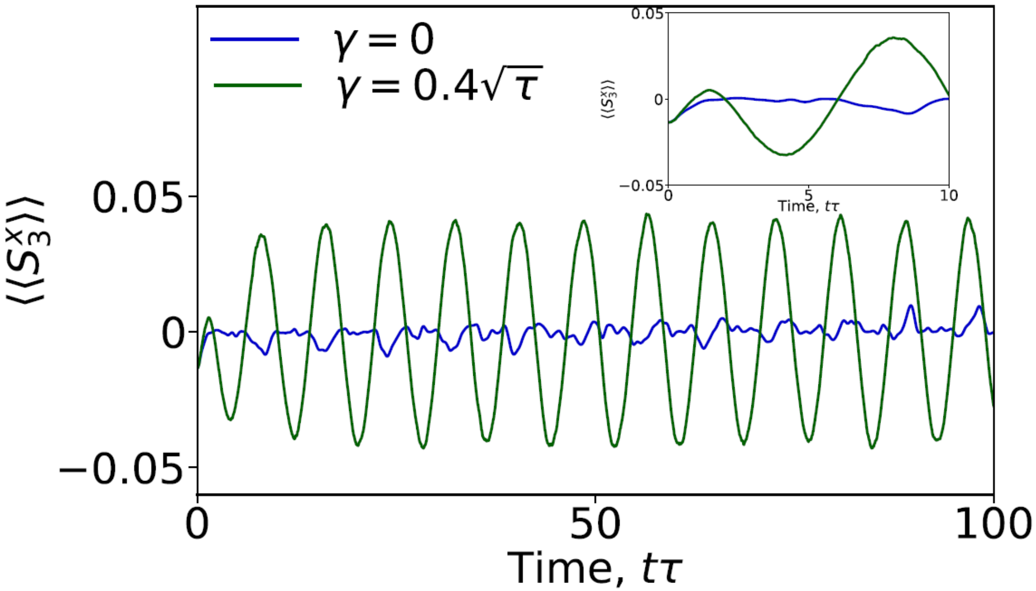
- $H_{\text{Hub}} = -t \sum_{\langle i,j \rangle,\sigma} \left(c_{i\sigma}^{\dagger} c_{j\sigma} + h.c. \right) + U \sum_i n_{i\uparrow} n_{i\downarrow} + \sum_i (\epsilon_i - \mu) n_i + \frac{B}{2} \, (n_{i\uparrow} - n_{i\uparrow})$

- Spin agnostic external dissipation will dephase the lattice wave function locally

$$L_\mu = \gamma_\mu \, n_\mu$$

M. Bruderer and D. Jaksch, New J. Phys. **8**, 87 (2006)

Disorder and temperature

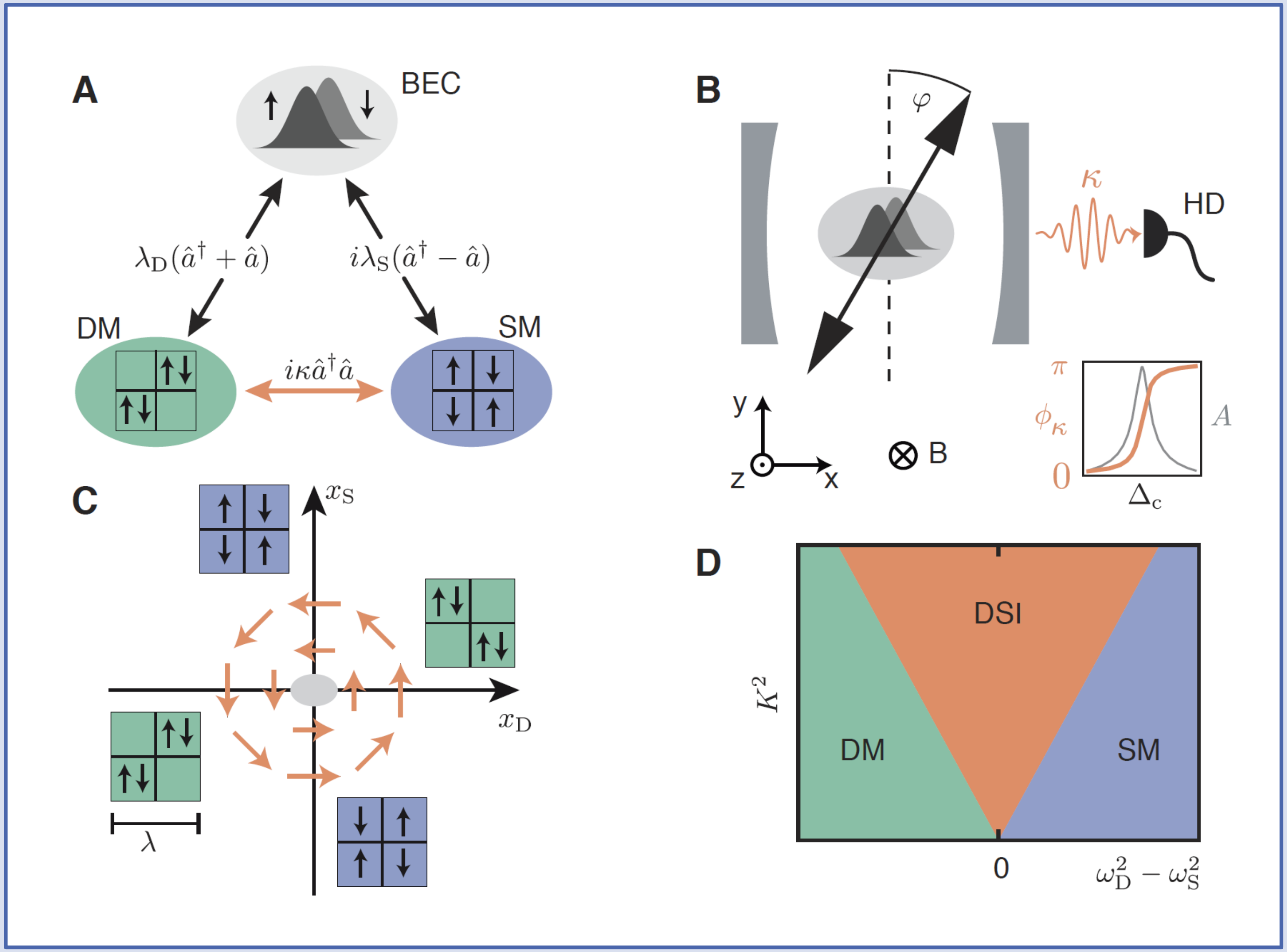


Initial ground state of the Hubbard model without disorder and $U = \sqrt{2}\tau$, $B = 0$
At $t = 0$ quench to $U = \tau$, $B = 0.8\tau$ and disorder switched

Essentially setup from: Phys. Rev. X 9, 041014 (2019), but for fermions; and Phys. Rev. X 7, 041047 (2017)

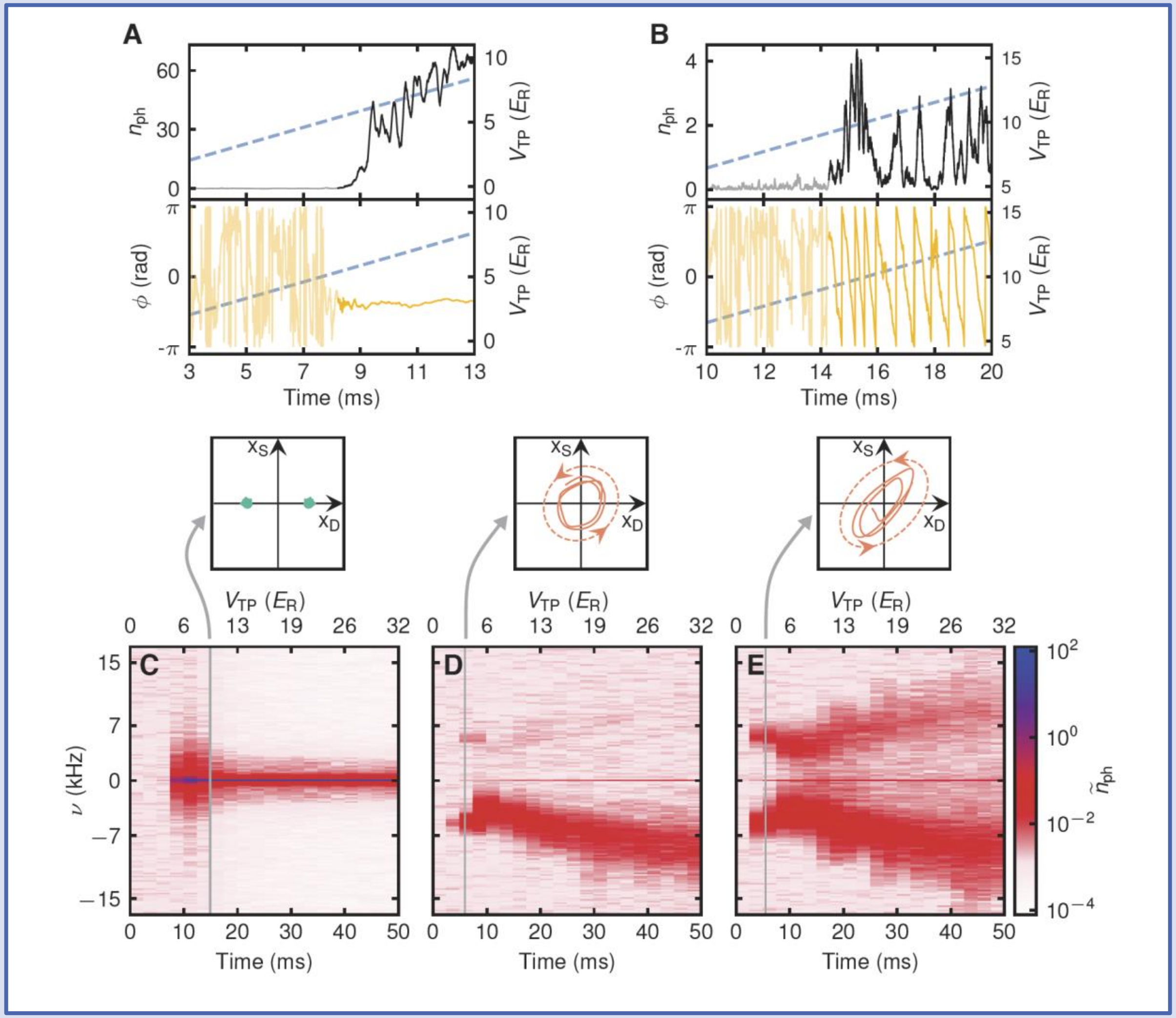
Open system example #2: Two component BEC in a lossy optical cavity

Approximate example: spinor BEC



N. Dogra, M. Landini, K. Kroeger, L. Hruby, T. Donner, T. Esslinger, Dissipation Induced Structural Instability and Chiral Dynamics in a Quantum Gas, *Science* 366.6472 (2019): 1496-1499.

Spinor BEC



N. Dogra, M. Landini, K. Kroeger, L. Hruby, T. Donner, T. Esslinger, Dissipation Induced Structural Instability and Chiral Dynamics in a Quantum Gas, *Science* 366.6472 (2019): 1496-1499.

Model

- We can model the experiment by a master equation

- $H = \hbar\omega a^\dagger a + \hbar\omega_0(S_{z+} + S_{z-})$
$$+\frac{\hbar}{\sqrt{N}}\left[\lambda_D\left(a^\dagger + a\right)(S_{x+} + S_{x-}) + i\lambda_S\left(a^\dagger - a\right)(S_{x+} - S_{x-})\right]$$

- $S_{\alpha,\pm}$ - collective spin operators + and – Zeeman states
- $\lambda_{D,S}$ - coupling (depend on the angle of the field)
- ω-detuning
- ω_0-bare energy
- and include cavity loss

$$\dot{\rho} = -\frac{i}{\hbar}[H_c, \rho] + \frac{\kappa}{2}\left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right)$$

Closely related to Dicke model with $\boldsymbol{\lambda_S = 0}$

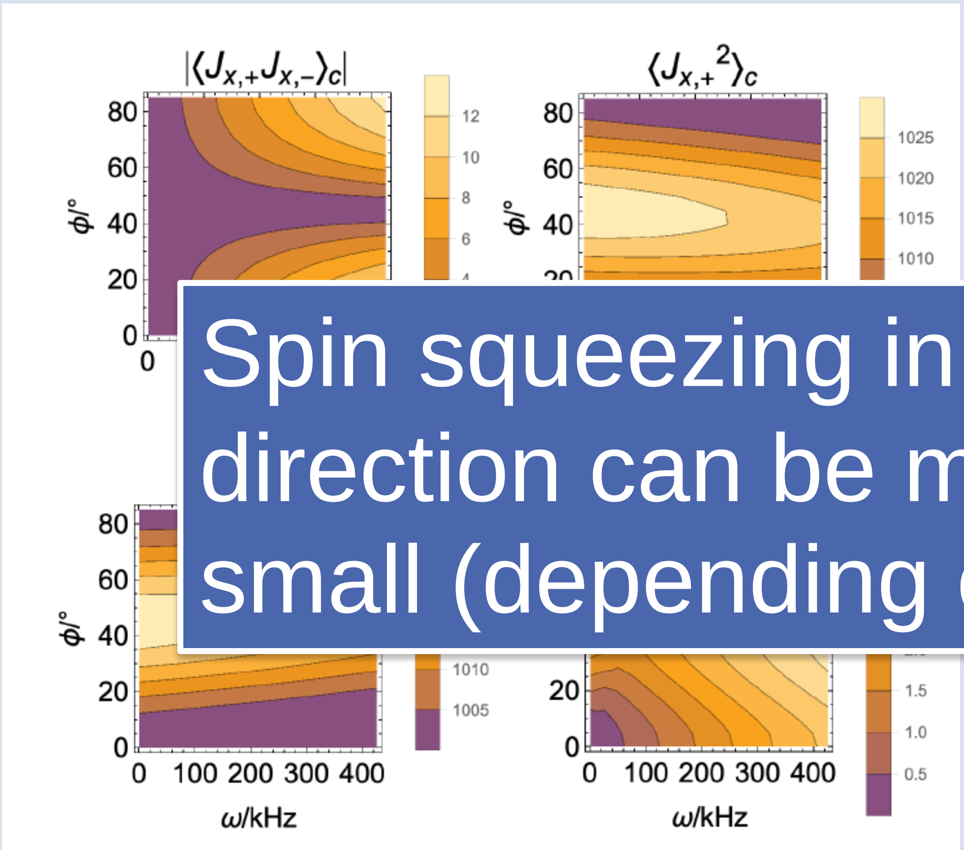
Beyond mean field

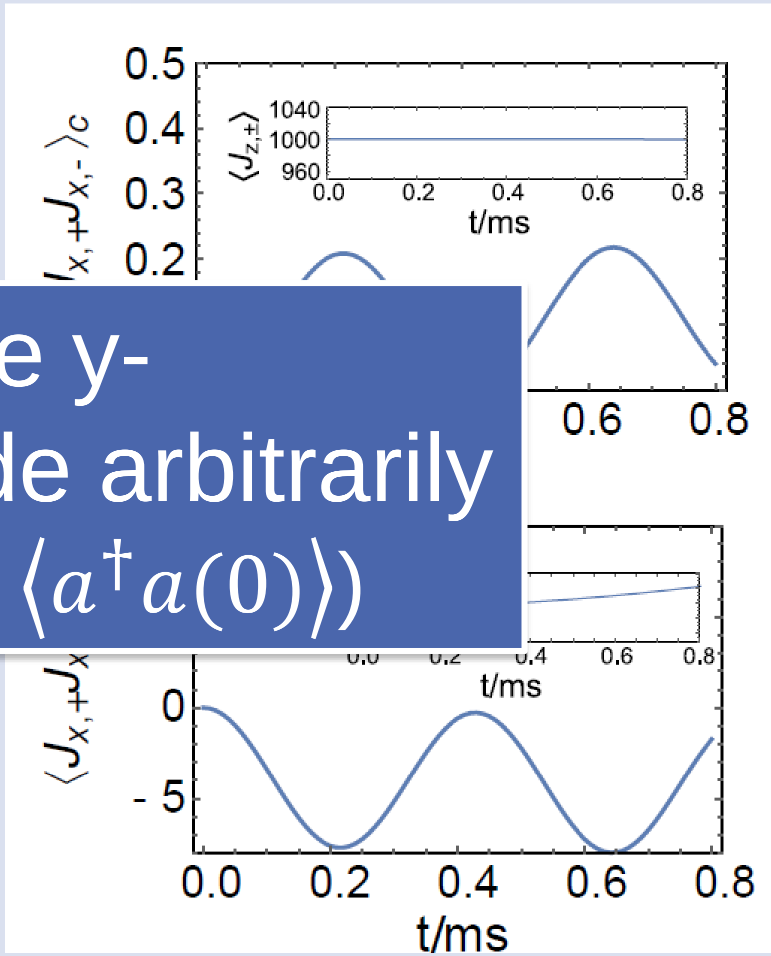
- Approximate dynamical symmetry in the strong loss limit $\kappa \to \infty$
- Perform perturbation theory in large $\kappa = \kappa' \gamma, \gamma \gg 1$
- In first (beyond 0) order the stationary state eigenvalue 0 is split into $\lambda = i\,(n-m)\omega_0 + O\left(\frac{1}{\gamma^2}\right), n, m = \pm 1, \pm 2, \dots$
- Quantum Zeno dynamics!

Higher order correlations

- We can access higher order correlations

$\lambda_D = 6.3kHz, \lambda_S = 7.25kHz, \omega = 46kHz$



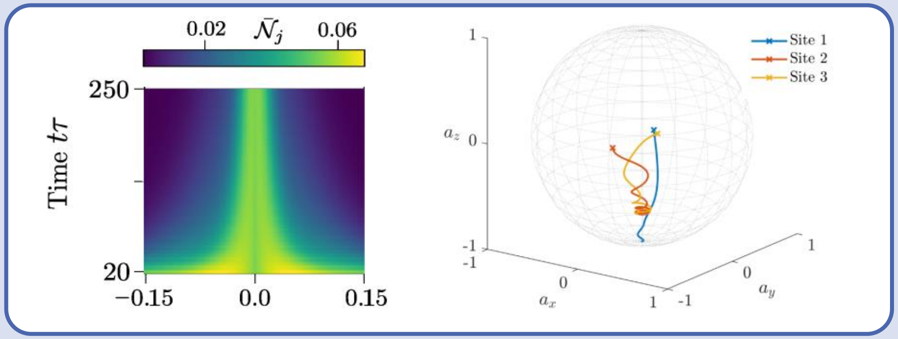


Spin squeezing in the y-direction can be made arbitrarily small (depending on $\langle a^{\dagger}a(0)\rangle$)

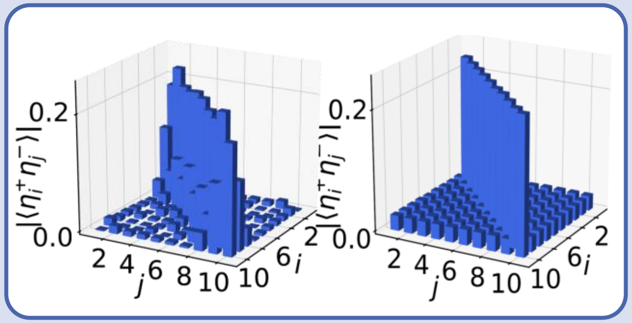
$\lambda_D = 9.6kHz, \lambda_S = 0.17kHz, \omega = 246kHz$

Summary and outlook

- **Goal: Understanding the emergence of complex dynamics from quantum laws and reconciliation with statistical physics.** Notion of **dynamical symmetries** crucial for non-stationary dynamics!
- Physical examples (Heisenberg XXZ spin chain, fermions in BEC, etc)
- Open questions: Hydro, transport in the presence of many-body non-stationarity, long-range interactions (dipolar) etc. Applications for metrology, signal filtering, quantum sensing, etc.
- DS for quantum scars (PRB102, 085140 (2020)), time crystals(PRL125, 060601 (2020))



Synchronization: BB, Booker, Jaksch. arXiv:2103.01808 ;
J. Tindall, C. Sanchez Munoz, BB and D. Jaksch, 2020 New J. Phys. **22** 013026



Dissipation induced η-pairs: J. Tindall, BB, J.R. Coulthard, D. Jaksch, Phys. Rev. Lett. **123**, 030603 (2019)

B. Buca, J. Tindall, and D. Jaksch, Nat. comms. 10, 1730 (2019)