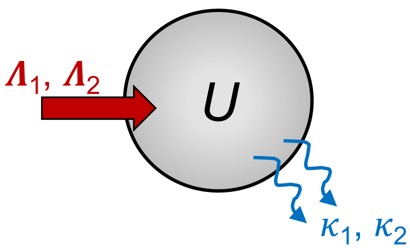
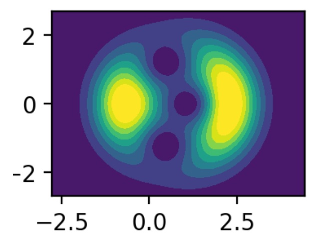
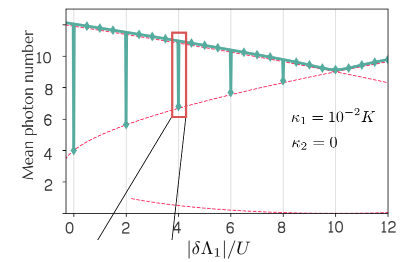
Driven-dissipative quantum systems & hidden time-reversal symmetries

Aashish Clerk, University of Chicago







- Exact descriptions via a **useful** quantum formulation of “detailed balance”
  (work with D. Roberts, A. Lingenfelter, arXiv:2011.02148)

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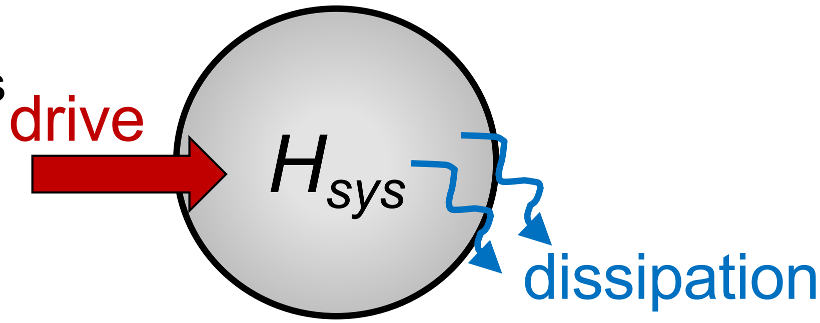
Driven dissipative quantum phenomena

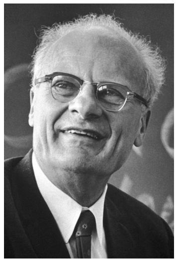
- Generic ingredients:
  - System with strong nonlinearity / interactions
  - Energy pumped in via external drives
  - Balanced by dissipation ⇒ non-thermal steady state



- Are there non-trivial cases where we can analytically describe steady-state properties?
  (non-perturbative, not semiclassics, not high-T limit, not MFT, etc.)

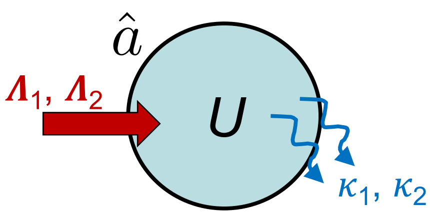
$$\hat{\rho}(t \to \infty) \equiv \hat{\rho}_{\mathrm{ss}}$$

- Closed systems: exact solutions for strongly interacting systems
  - Integrable quantum systems (Bethe ansatz methods)
    - e.g. Heisenberg models, Anderson model, Rabi model, ...
  - Analogous approaches for driven-dissipative problems?



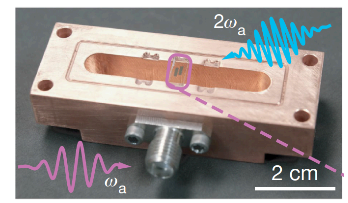
Exact solutions of nonlinear bosonic systems

- Single nonlinear quantum cavity, coherent driving and loss (linear & nonlinear):
  - Realized in many experiments (e.g. superconducting circuits)



$$\frac{d}{dt}\hat{\rho} = -i[\hat{H}_a, \hat{\rho}] + \kappa_1 \mathcal{D}\left[\hat{a}\right]\hat{\rho} + \kappa_2 \mathcal{D}\left[\hat{a}^2\right]\hat{\rho}$$

$$\hat{H}_a = \frac{U}{2}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a} - \Delta\hat{a}^\dagger\hat{a} + \left(\Lambda_1\hat{a}^\dagger + \frac{\Lambda_2}{2}\hat{a}^\dagger\hat{a}^\dagger + h.c.\right)$$



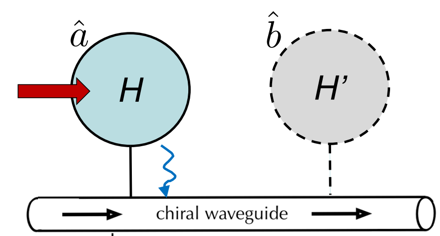
(Grimm et al, Nature. 2020)

- Exact, non-perturbative descriptions of the quantum steady state:

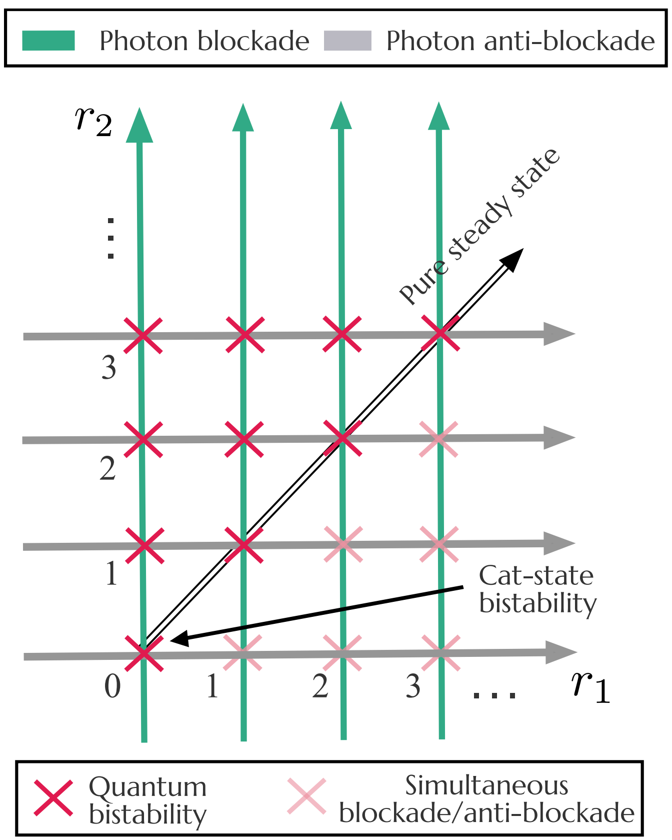
- Mapping to classical Fokker-Planck equation (complex-*P* method)
  - Drummond and Walls 1980, Drummond et al. 1981
    (see also Elliott and Ginossar 2016, Bartolo et al. 2016)

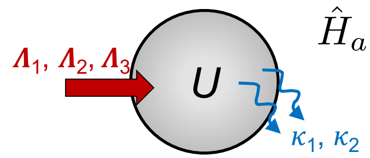
$$\langle p|\hat{\rho}_{ss}|q\rangle = \frac{1}{\mathcal{N}\sqrt{p!q!}}\sum_{m=0}^{\infty}\frac{1}{m!}\mathcal{F}_{m+p}(f,g,c)\,\mathcal{F}^*_{m+q}(f,g,c).$$

- Coherent quantum absorber method (CQA):
  - Linear driving & loss: Stannigel, Rabl and Zoller NJP 2012
  - Nonlinear driving/loss: Roberts & AC, PRX 2020



CQA solutions yield physical insights!





$$\hat{H}_a = \frac{U}{2}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a} - \Delta\hat{a}^\dagger\hat{a} + \left(\Lambda_1\hat{a}^\dagger + \frac{\Lambda_2}{2}\hat{a}^\dagger\hat{a}^\dagger + h.c.\right)$$

$$\hat{H}_{\text{new}} = \Lambda_3\hat{a}^\dagger\hat{a}^\dagger\hat{a} + \text{ h.c.}$$

$$\Phi_{+,\text{SB}}[z] \propto {}_1F_1\left[-r_1; -r_2; \sqrt{-8\lambda_2 z}\right] \quad \Rightarrow \quad \langle\alpha|\hat{\rho}|\alpha\rangle \propto |\Phi_{\text{SB}}[\alpha^*]|^2$$

Segal-Bargmann
"wavefunction in phase space"

- Steady state controlled by two parameters:

$$r_1 \simeq \frac{\Delta}{U} - i\frac{\Lambda_1}{\sqrt{\Lambda_2 U}} \qquad r_2 \simeq \frac{2\Delta}{U}$$

drives *detuning*

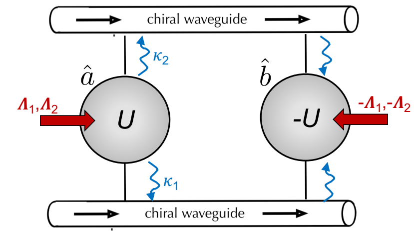
- Surprising behavior when r_1, r_2 are positive integers
  - Photon blockade, quantum bistability, …..

Roberts & AC, PRX 2020

Why are these systems solvable?

$$\hat{H}_a = \frac{U}{2}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a} - \Delta\hat{a}^\dagger\hat{a} + \left(\Lambda_1\hat{a}^\dagger + \frac{\Lambda_2}{2}\hat{a}^\dagger\hat{a}^\dagger + h.c.\right)$$

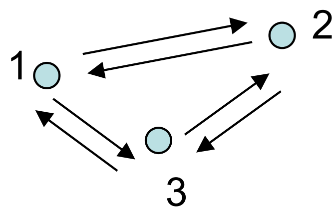
$$\frac{d}{dt}\hat{\rho} = -i[\hat{H}_a, \hat{\rho}] + \kappa_1\mathcal{D}\left[\hat{a}\right]\hat{\rho} + \kappa_2\mathcal{D}\left[\hat{a}^2\right]\hat{\rho}$$



- No obvious symmetry or small parameter
- Adding other innocuous looking terms destroy solvability (e.g. $\hat{H}' = \hat{a}^\dagger\hat{a}^\dagger\hat{a}^\dagger + h.c.$)

- IDEA: make a connection to **time-reversal symmetry**
  - For closed (non-dissipative, non-driven) systems:
    - CM: $(x(t), p(t))$ ✓ → $(x(-t), -p(-t))$ ✓
    - QM: exists an anti-unitary ***T*** such that $\hat{T}\hat{H}\hat{T}^{-1} = \hat{H}$
  - For dissipative systems, need something more general…..

Time reversal and detailed balance



$$\frac{d}{dt}p_j(t) = \sum_k \left[\Gamma_{k\to j}\, p_k(t) - \Gamma_{j\to k}\, p_j(t)\right]$$

- CM: Time-reversal symmetry ➔ <u>**detailed balance**</u>
- Implies "Onsager" time-symmetry of fluctuations

$$\boxed{\Gamma_{i\to j}\cdot p_i(\infty) = \Gamma_{\tilde{j}\to\tilde{i}}\cdot p_{\tilde{j}}(\infty)}$$

$$\overline{A(t)B(0)} = \overline{\tilde{B}(t)\tilde{A}(0)}$$

- "Quantum detailed balance" = ????
  - Define as a correlation function symmetry?
    (Agarwal 1973, Carmichael 1976, ….)

$$\frac{d}{dt}\hat{\rho} = -i[\hat{H},\hat{\rho}] + \sum_j \kappa_j \mathcal{D}\left[\hat{z}_j\right]\hat{\rho}$$

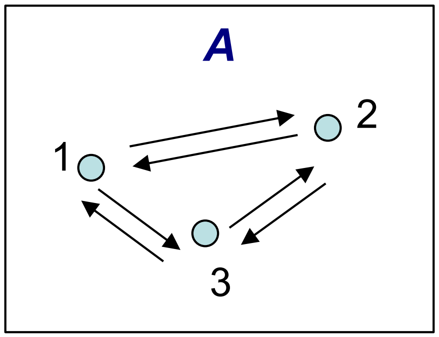
$$\boxed{\langle \hat{A}(t)\hat{B}(0)\rangle \stackrel{!}{=} \langle \hat{\tilde{B}}(t)\hat{\tilde{A}}(0)\rangle}$$

  - Issue: only holds for <u>**very limited**</u> class of system with "trivial" steady states *(Alicki 1978, ….)*

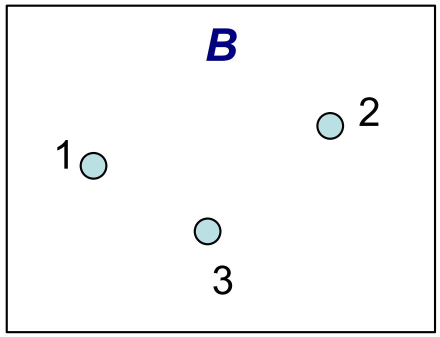
$$\hat{H} = \sum_n E_n|\phi_n\rangle\langle\phi_n| \quad\Rightarrow\quad \hat{\rho}_{ss} = \sum_n p_n|\phi_n\rangle\langle\phi_n|$$

Doubled-system formulation

- Equivalent **classical** formulation of detailed balance:



Original system, transition rates $\Gamma_{i \to j}$



Copy of original system, but **no dynamics**

- Start doubled classical system in a correlated state:

$$p_{AB}[n, m; t = 0] = p_{\rm ss}[n] \times \delta_{m,\tilde{n}}$$

- Detailed balance condition:

$$\overline{X_A(t) Y_B(0)} = \overline{Y_A(t) X_B(0)}$$

- Use this to motivate a definition of quantum detailed balance and "hidden" time reversal symmetry ***T***….

$$\frac{d}{dt}\hat{\rho} = -i[\hat{H}, \hat{\rho}] + \sum_j \kappa_j \mathcal{D}\left[\hat{z}_j\right]\hat{\rho}$$

  - Steady state of original system:

$$\rho_{\rm ss} = \sum_k p_k |k\rangle\langle k|$$

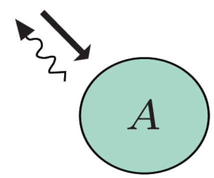
  - Correlated state of "doubled" system

$$|\psi_{\rm TFD}\rangle = \sum_k \sqrt{p_k}\, |k\rangle_A \left(\hat{T}|k\rangle_B\right)$$

Hidden time-reversal symmetry

- A fully quantum notion of "detailed balance" that has **operational utility**, and that can be present even for systems with non-trivial steady states

- Original system Lindblad master equation:
  (steady state could be non-trivial, no "obvious" detailed balance)



steady state:

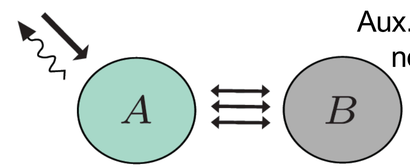
$$\hat{\rho}_{\rm ss} = \sum_n p_n |n\rangle\langle n|$$

($|n\rangle$ are not nec. energy eigenstates)

- Pick an anti-unitary operator ***T***
  - Use to construct a "thermofield double" entangled state

$$|\psi_{\rm TFD}\rangle = \sum_n \sqrt{p_n} |n\rangle_A \otimes \hat{T} |n\rangle_B$$

- Consider a "doubled" system, prepare in thermofield entangled state….



Aux. system B has no dynamics!

- ***T* represents a hidden time-reversal symmetry if for all operators we have:**

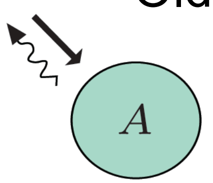
$$\langle \hat{X}_A(t) \hat{Y}_B(0) \rangle \stackrel{!}{=} \langle \hat{Y}_A(t) \hat{X}_B(0) \rangle$$

(see also Duvenhage and Snyman, J. Math Phys. A 2015)

Dueling detailed balance definitions

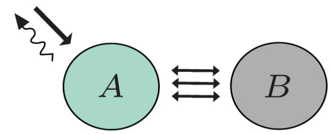
- ***Q: Is some anti-unitary T a meaningful symmetry of my open quantum system?***

Old: "Conventional" QDB



$$\langle \hat{X}(t)\hat{Y}(0)\rangle \stackrel{!}{=} \langle \hat{\tilde{Y}}(t)\hat{\tilde{X}}(0)\rangle$$

New: Hidden TRS



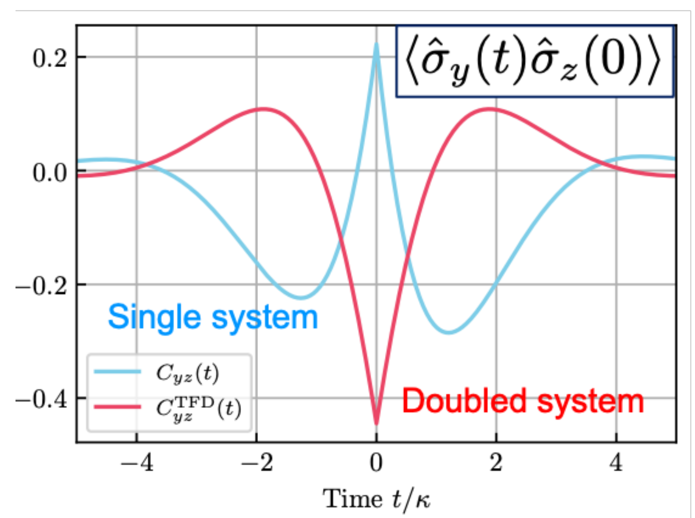
$$\langle \hat{X}_A(t)\hat{Y}_B(0)\rangle \stackrel{!}{=} \langle \hat{Y}_A(t)\hat{X}_B(0)\rangle$$

$$|\psi_{\mathrm{TFD}}\rangle = \sum_n \sqrt{p_n}|n\rangle_A \otimes \hat{T}|n\rangle_B$$

- Key: many systems violate conventional QDB, but nonetheless have "hidden TRS"
- Simple example: driven qubit with loss…

$$\frac{d}{dt}\hat{\rho} = -i\left[\Omega\hat{\sigma}_x, \hat{\rho}\right] + \kappa\mathcal{D}[\hat{\sigma}_-]\hat{\rho}$$

- Hidden TRS: $\hat{T}_h \propto \left(1 - \frac{2i\Omega}{\kappa}\hat{\sigma}_x\right)\hat{K}_z$



Dueling detailed balance definitions

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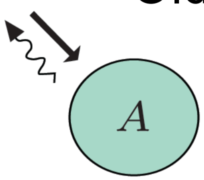

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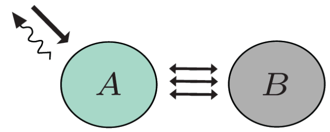
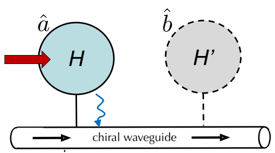
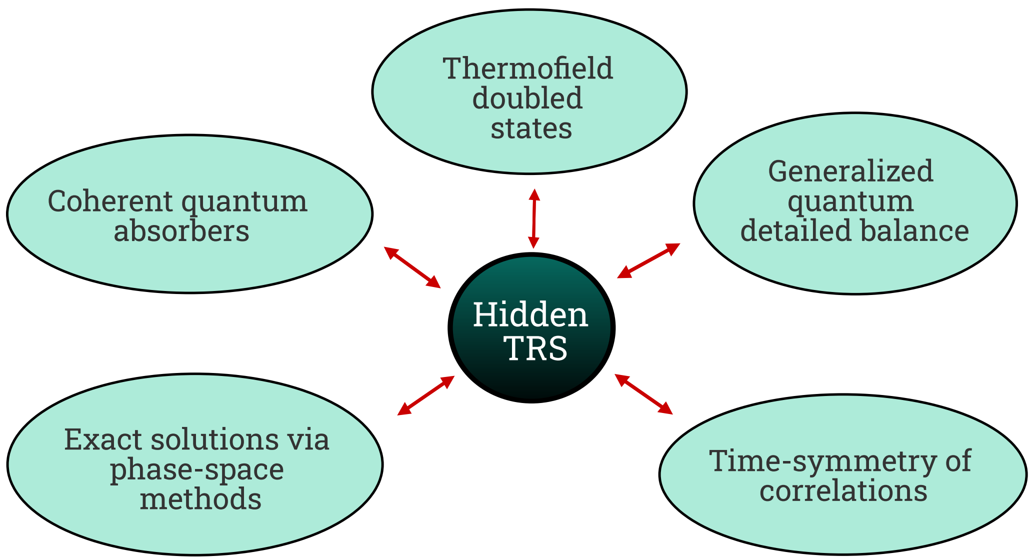
New: Hidden TRS



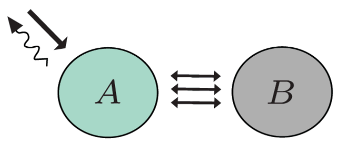
$$\langle \hat{X}_A(t)\hat{Y}_B(0)\rangle \overset{!}{=} \langle \hat{Y}_A(t)\hat{X}_B(0)\rangle$$





See D. Robert’s talk tomorrow

Hidden TRS enables exact solutions

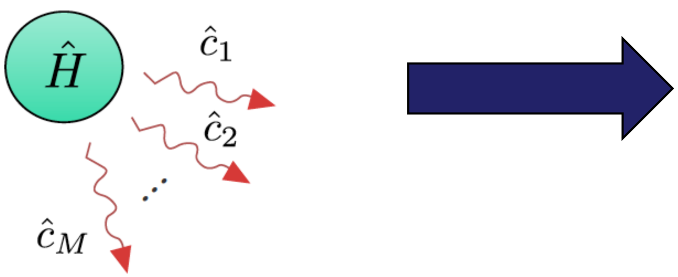


$$|\psi_{\text{TFD}}\rangle = \sum_n \sqrt{p_n}|n\rangle_A \otimes \hat{T}|n\rangle_B$$

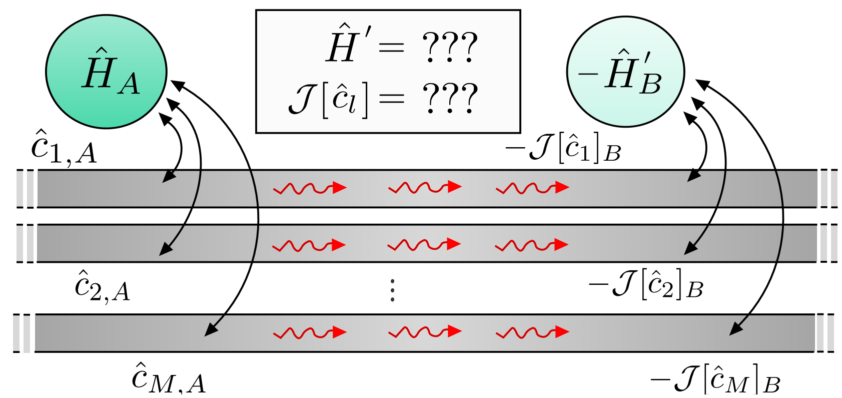
$$\langle \hat{X}_A(t)\hat{Y}_B(0)\rangle \stackrel{!}{=} \langle \hat{Y}_A(t)\hat{X}_B(0)\rangle$$

- The existence of hidden TRS places non-trivial constraints on dynamics
- Theorem: hidden TRS implies the existence of a “simple” absorber network

$$\hat{\rho}(t \to \infty) = ???$$

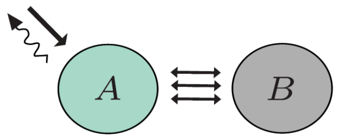


$$\frac{d}{dt}\hat{\rho}(t) = -i[\hat{H}, \hat{\rho}(t)] + \sum_{j=1}^{M} \mathcal{D}[\hat{c}_j]\,\hat{\rho}(t)$$



$$\hat{\rho}_{AB}(t \to \infty) = |\psi_{\text{TFD}}\rangle\langle\psi_{\text{TFD}}|$$

Hidden TRS enables exact solutions



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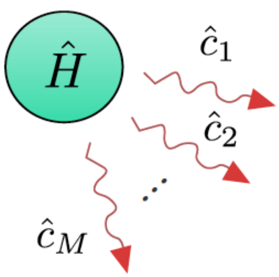
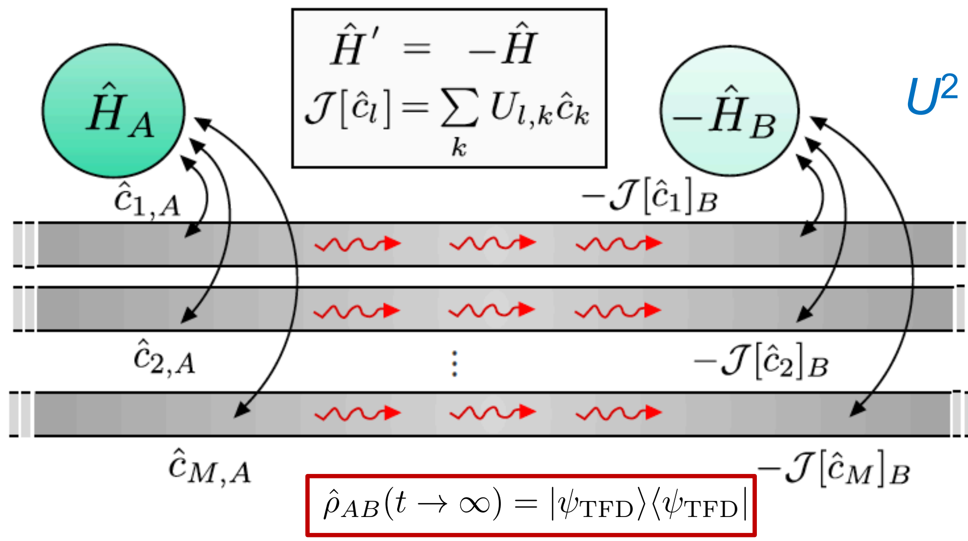
$$\hat{\rho}(t \to \infty) = ???$$

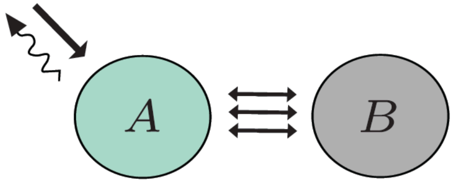




$U^2 = 1$

- Finding steady state reduces to finding a **highly constrained** pure state
- Not limited to bosons (works for spins!)

Hidden TRS: observable consequences



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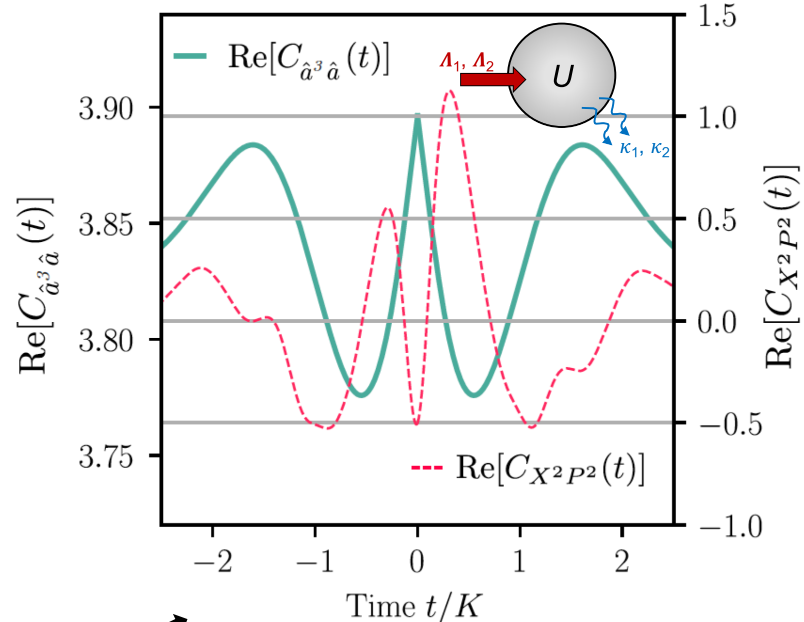
$$\langle \hat{X}_A(t)\hat{Y}_B(0)\rangle \overset{!}{=} \langle \hat{Y}_A(t)\hat{X}_B(0)\rangle$$

- Creating a doubled system and measuring TFD correlators difficult
  (but not impossible: recent Monroe group experiment, PNAS 2020)
- Good news: H-TRS ensures that **certain** single-system correlators exhibit time-symmetry
- e.g. driven nonlinear cavity…..

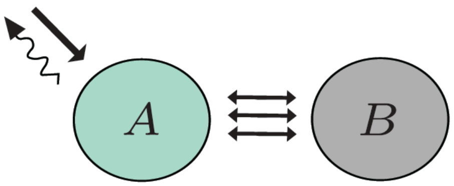
$$\hat{H}_a = \frac{U}{2}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a} - \Delta\hat{a}^\dagger\hat{a} + \left(\Lambda_1\hat{a}^\dagger + \frac{\Lambda_2}{2}\hat{a}^\dagger\hat{a}^\dagger + \Lambda_3\hat{a}^\dagger\hat{a}^\dagger\hat{a} + h.c.\right)$$

$$\langle \hat{a}^3(t)\hat{a}(0)\rangle \quad \text{vs.} \quad \left\langle \left[\hat{a}(t) + \hat{a}^\dagger(t)\right]^2 \left[\hat{a}(0) - \hat{a}^\dagger(0)\right]^2 \right\rangle$$



(D. Roberts, A. Lingenfelter and AC, arXiv:2011.02148)

Hidden TRS & thermal fluctuations



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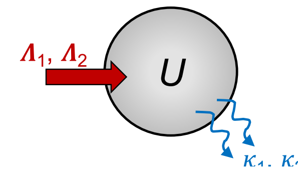
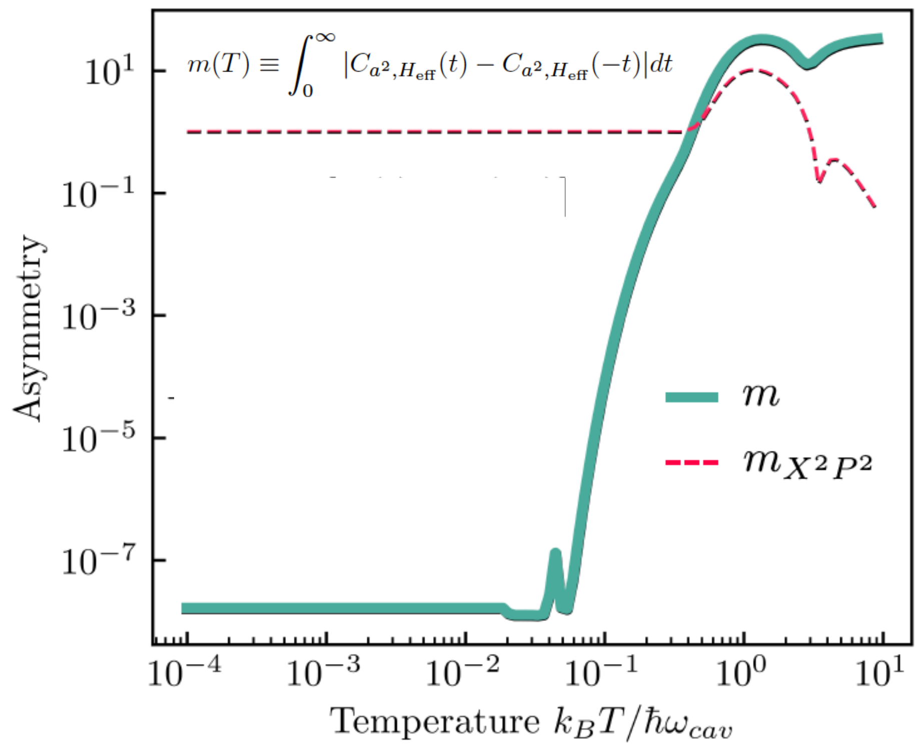
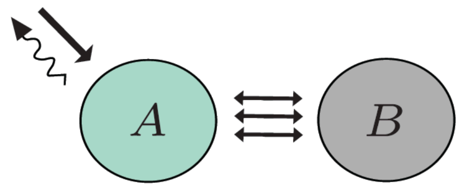


- Driven nonlinear cavity:
  - At high temperature, system has “classical” detailed balance
  - *Is “hidden TRS” just an extension of this to T=0?*

- Short answer: no!
  - Thermal fluctuations can in some cases destroy hidden TRS…
  - Phase transition?



Hidden TRS: future directions



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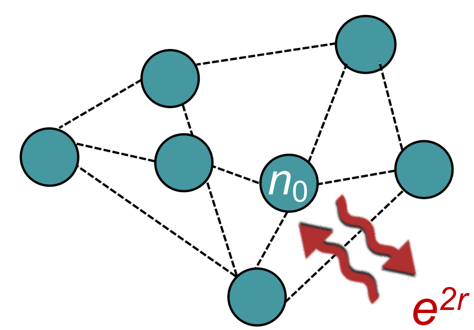
$$\langle \hat{X}_A(t)\hat{Y}_B(0)\rangle \stackrel{!}{=} \langle \hat{Y}_A(t)\hat{X}_B(0)\rangle$$

- A truly quantum notion of detailed balance that has operational utility
- Many interesting extensions & applications to pursue
  - Construct non-trivial, exactly solvable **lattice models**
  - Extension to spins, fermions……
  - Relation of hidden TRS and complexity / "quantumness" of steady state
  - Novel perturbation theory in symmetry-breaking fields
  - Extension to non-Markovian systems
  - ……



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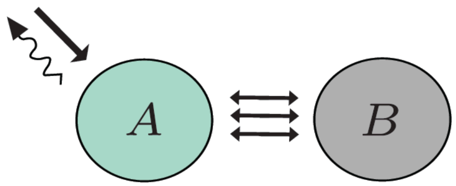
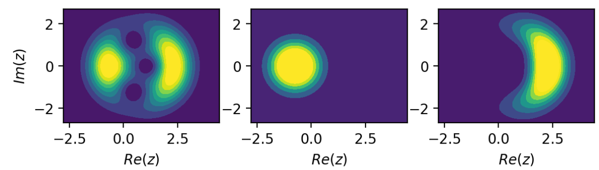
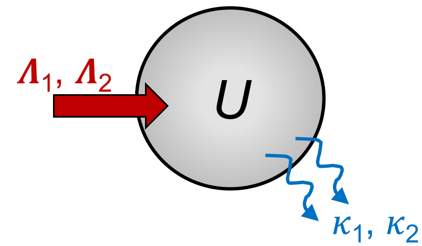
Conclusions



$$|\psi_{\mathrm{TFD}}\rangle = \sum_n \sqrt{p_n}|n\rangle_A \otimes \hat{T}|n\rangle_B$$





- New approach for exact solutions of driven-dissipative resonators
  (David Roberts and AC, PRX 10, 021022 2020)

- Hidden TRS in driven-dissipative quantum systems
  (David Roberts, A. Lingenfelter and AC, arXiv:2011.02148)
  - Non-trivial solutions related to a surprising symmetry
  - ***Underlies phase space solution methods (see D. Robert's talk tomorrow!)***

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