SPICE workshop 'Dissipative Phases of Entangled Quantum Matter'
May 03 2021
Mainz, Germany (zoom)

Institute for Theoretical Physics University of Cologne

Measurement Induced Phases and Phase Transitions in Fermion Chains

Sebastian Diehl

Institute for Theoretical Physics, University of Cologne

work with

Ori Alberton, Michael Buchhold, SD — PRL 126, 170602 (2021)

Michael Buchhold, Yuri Minoguchi, Alex Altland, SD — arXiv:2102.08381

€ € € :

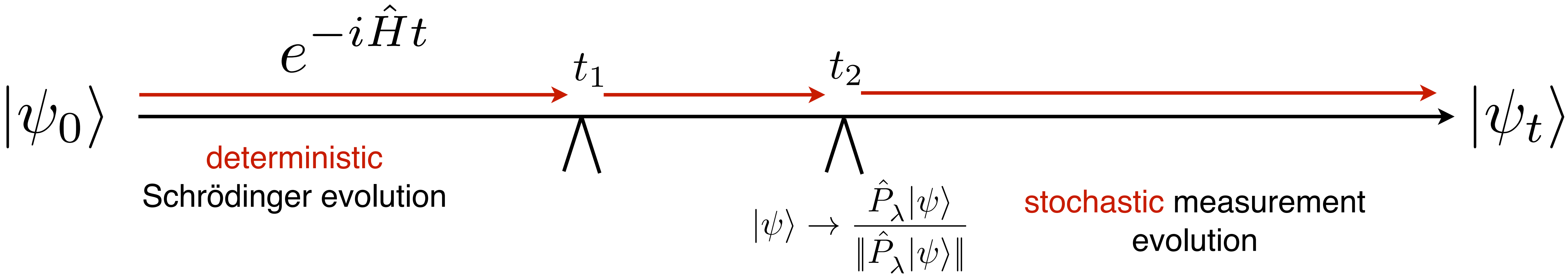
DFG Deutsche Forschungsgemeinschaft

erc European Research Council
Established by the European Commission

Introduction

Small quantum systems: Measurements

- two types of quantum dynamics



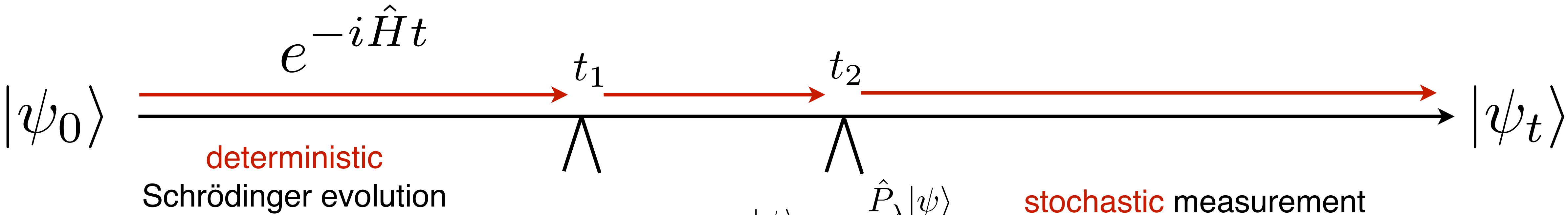
for measurement observable $\hat{M} = \sum_\lambda m_\lambda |\lambda\rangle\langle\lambda| \equiv \sum_\lambda m_\lambda \hat{P}_\lambda$

- dynamics non-trivial (eigenstates not shared) once $[\hat{H}, \hat{M}] \neq 0$

Introduction

Small quantum systems: Measurements

- two types of quantum dynamics



$$|\psi\rangle \to \frac{\hat{P}_\lambda|\psi\rangle}{\|\hat{P}_\lambda|\psi\rangle\|}$$

stochastic measurement evolution

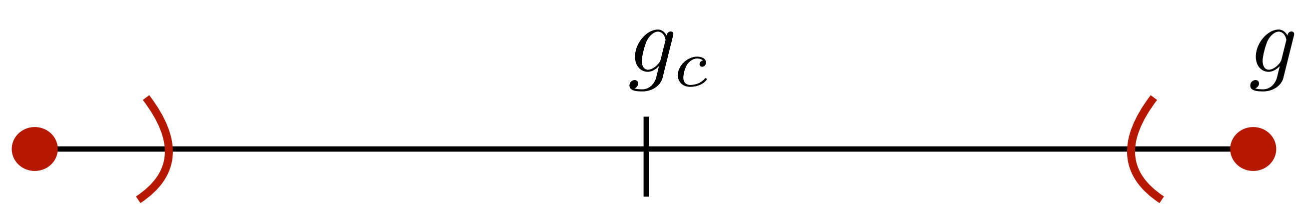
for measurement observable $\hat{M} = \sum_\lambda m_\lambda |\lambda\rangle\langle\lambda| \equiv \sum_\lambda m_\lambda \hat{P}_\lambda$

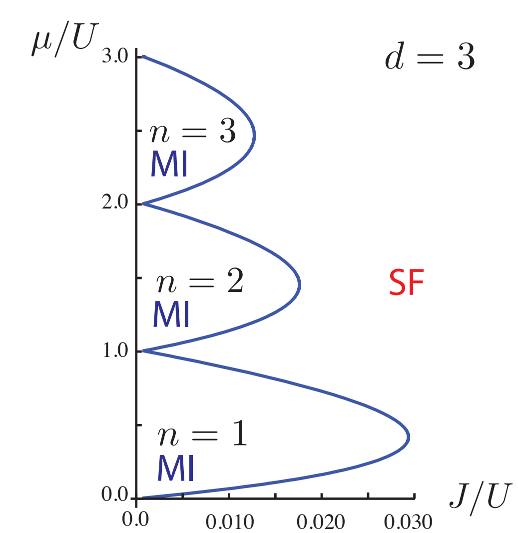
- dynamics non-trivial (eigenstates not shared) once $[\hat{H}, \hat{M}] \neq 0$

Many-body systems: Phase transitions

- non-commuting operators lead to (quantum) phase transitions

$$\hat{H} = \hat{H}_1 + g\hat{H}_2 \qquad [\hat{H}_1, \hat{H}_2] \neq 0$$





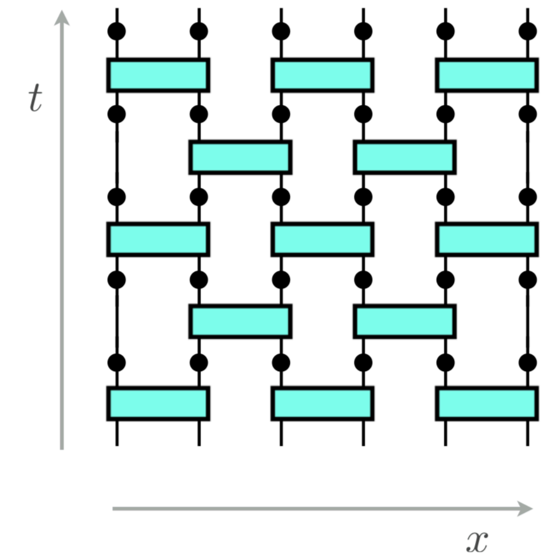
e.g. Mott-insulator to superfluid transition in cold atoms

➡ combine measurement and many particles: similar scenario?

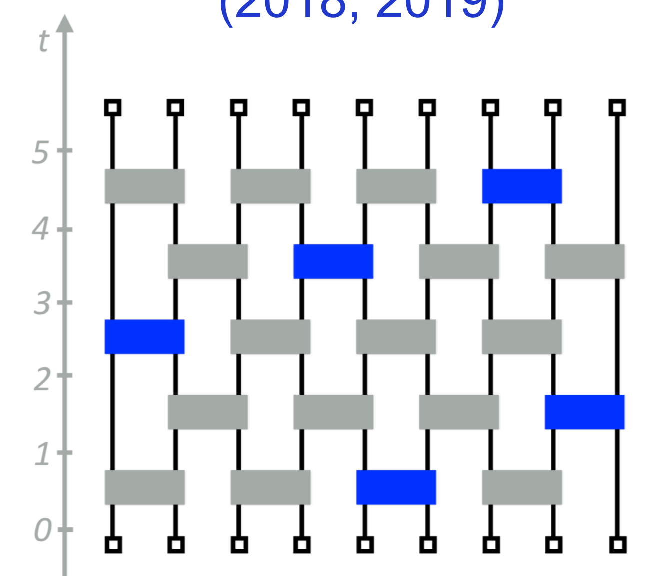
Entanglement Phase Transitions in Random Circuits

- model and key ingredients:
  - randomly chosen local entangling unitary gates
  - projective local measurement of non-commuting observables

Skinner, Ruhman, Nahum PRX (2019)

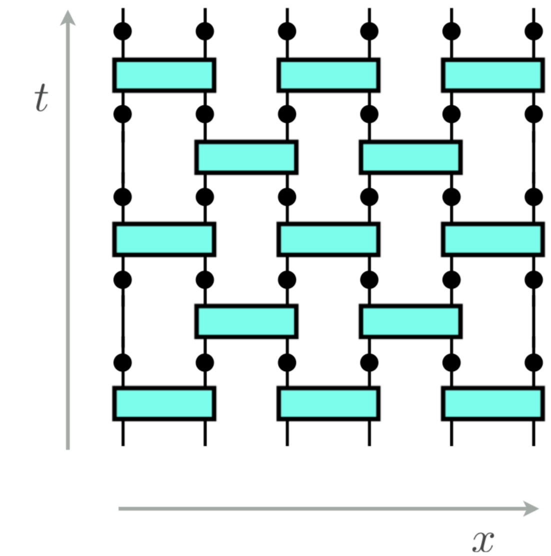


Li, Chen, Fisher, PRB (2018, 2019)

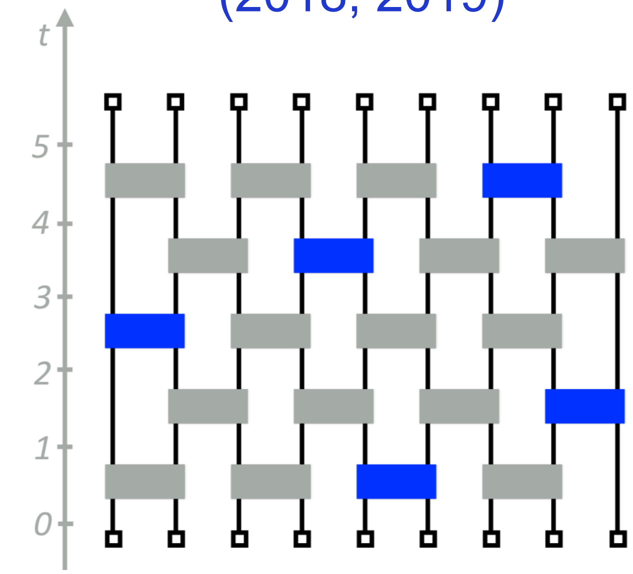


Entanglement Phase Transitions in Random Circuits

Skinner, Ruhman, Nahum
PRX (2019)



Li, Chen, Fisher, PRB
(2018, 2019)



- model and key ingredients:
  - randomly chosen local entangling unitary gates
  - projective local measurement of non-commuting observables

- basic picture: competition in many-body context (measure σ_i^z)

$g = 0$ — $g^{-1} = 0$

$$g = \frac{\text{\# measurements/time}}{\text{\# unitaries/time}}$$

chaotic dynamics

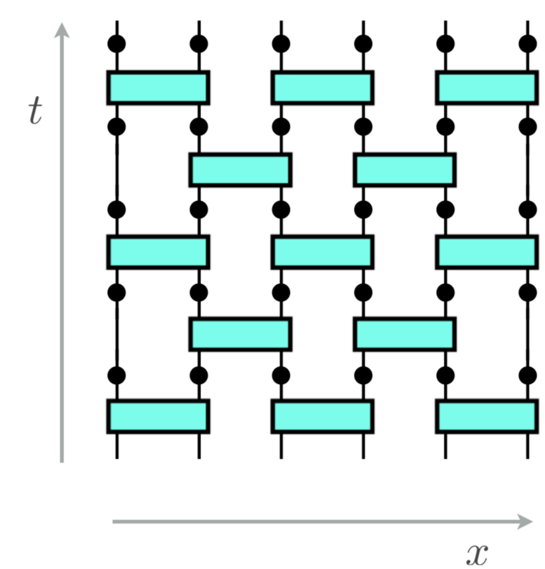
➡ volume law entanglement growth

product state $\prod_i |\sigma_i\rangle \quad \sigma_i = \uparrow, \downarrow$

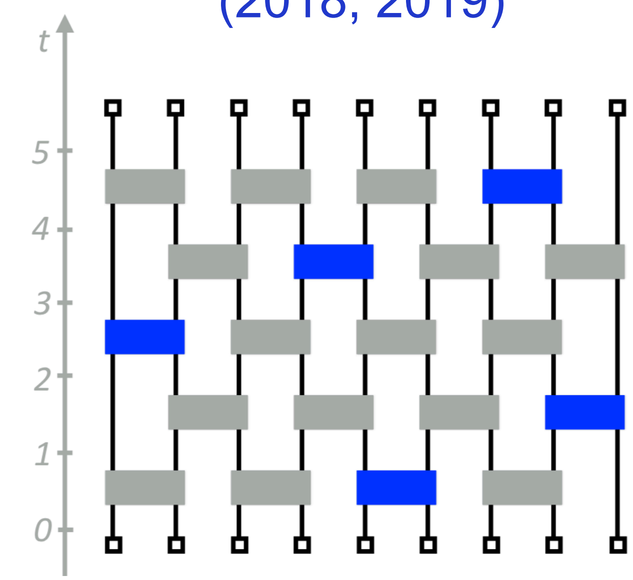
➡ area law entanglement growth

Entanglement Phase Transitions in Random Circuits

Skinner, Ruhman, Nahum PRX (2019)

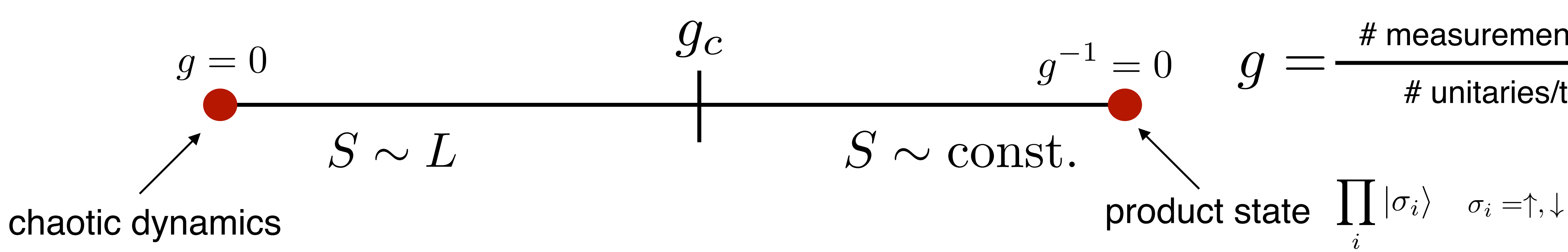


Li, Chen, Fisher, PRB (2018, 2019)



- model and key ingredients:
  - randomly chosen local entangling unitary gates
  - projective local measurement of non-commuting observables

- basic picture: competition in many-body context (measure σ_i^z)



$$g = \frac{\#\text{ measurements/time}}{\#\text{ unitaries/time}}$$

➡ volume law entanglement growth

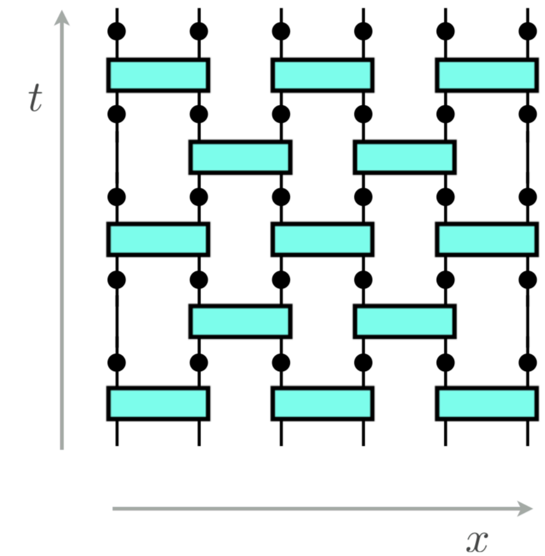
➡ area law entanglement growth

➡ Phase transition in entanglement growth at finite competition ratio g

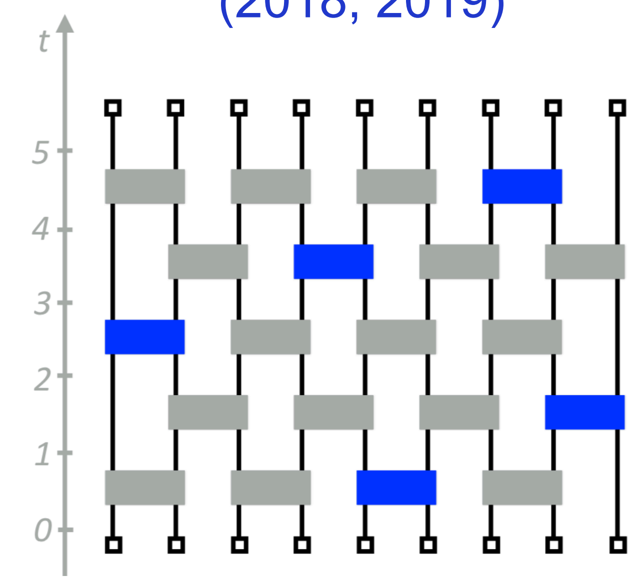
Entanglement Phase Transitions in Random Circuits

- model and key ingredients:
  - randomly chosen local entangling unitary gates
  - projective local measurement of non-commuting observables

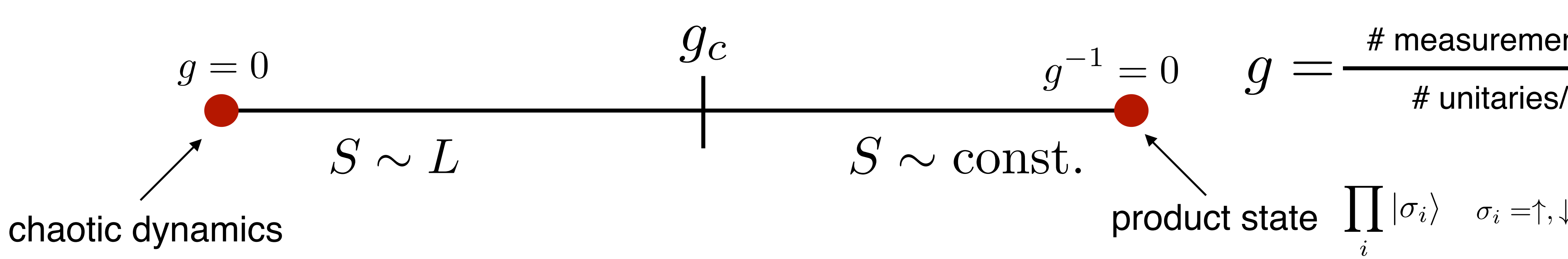
Skinner, Ruhman, Nahum PRX (2019)



Li, Chen, Fisher, PRB (2018, 2019)



- basic picture: competition in many-body context (measure σ_i^z)



$$g = \frac{\text{\# measurements/time}}{\text{\# unitaries/time}}$$

➡ volume law entanglement growth

➡ area law entanglement growth

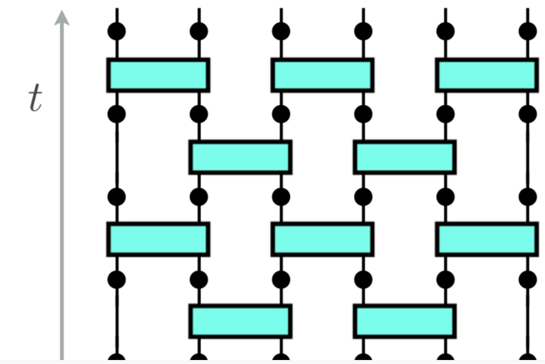
- Physical pictures
  - quantum error correction — Choi, Bao, Qi, Altman, PRL (2020); Fan et al. arxiv (2020); Li Fisher PRB (2021)
  - purification transition — Gullans, Huse, PRX (2020); Gopalakrishnan, Gullans arxiv (2020)
  - mapping to stat mech models — Jian, You, Vasseur Ludwig, PRB (2020), Nahum et al., PRX Quantum (2021)

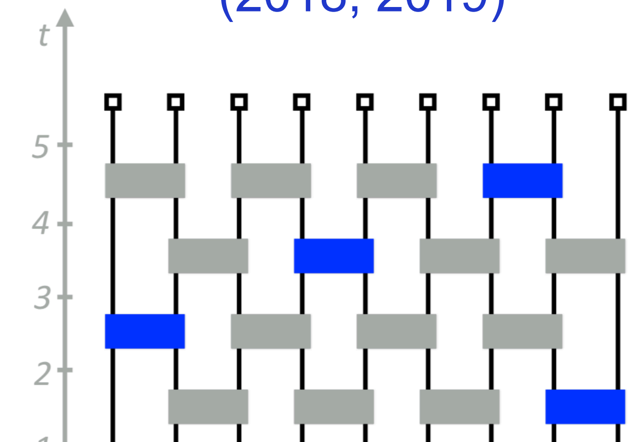
➡ Phase transition in entanglement growth at finite competition ratio g

Entanglement Phase Transitions in Random Circuits

Skinner, Ruhman, Nahum PRX (2019)

Li, Chen, Fisher, PRB (2018, 2019)





- model and key ingredients:
    - randomly chosen local entangling unitary gates
    - projective local measurement of non-

Outline

- Model: monitored fermion chain
- New entanglement transition from log scaling to area law
- Effective field theory for measured Dirac fermions

➡ volume law entanglement growth

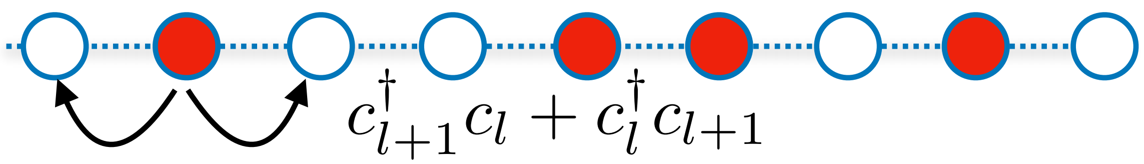
➡ area law entanglement growth

- Physical pictures
    - quantum error correction — Choi, Bao, Qi, Altman, PRL (2020); Fan et al. arxiv (2020); Li Fisher PRB (2021)
    - purification transition — Gullans, Huse, PRX (2020); Gopalakrishnan, Gullans arxiv (2020)
    - mapping to stat mech models — Jian, You, Vasseur Ludwig, PRB (2020), Nahum et al., PRX Quantum (2021)

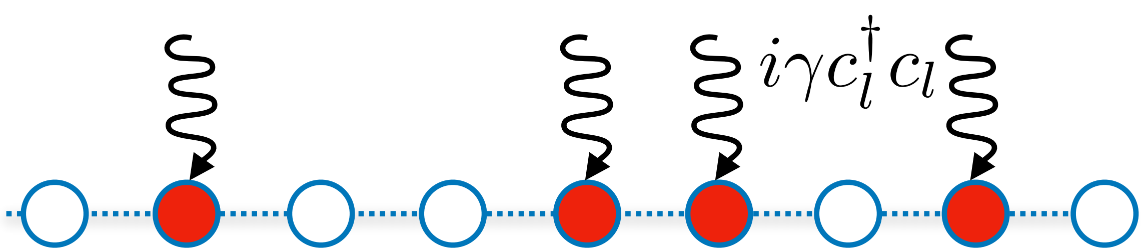
➡ Phase transition in entanglement growth at finite competition ratio g

Entanglement Phase Transition in a Monitored Fermion Chain

Hamiltonian:

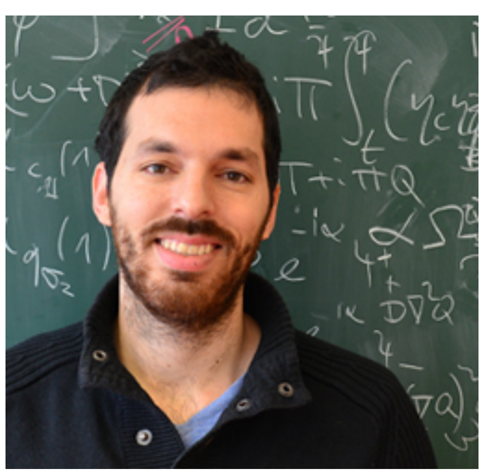


entanglement growth

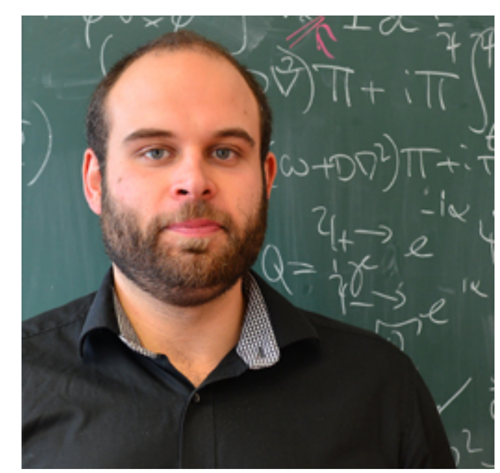


Monitoring:

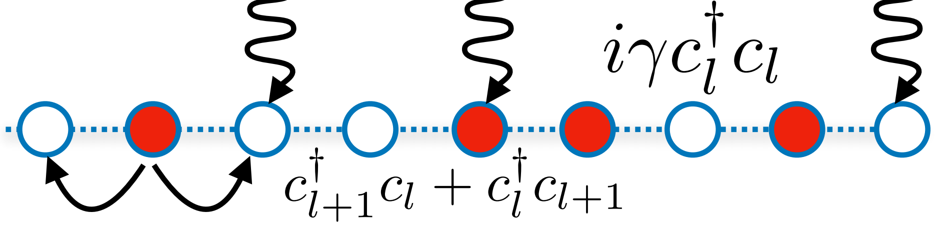
entanglement saturation



O. Alberton, M. Buchhold, SD,
PRL 126, 170602 (2021)



Monitored Fermion Dynamics



- Weak continuous measurements: Quantum state diffusion

Belavkin (1987); Gisin, Percival (1993)

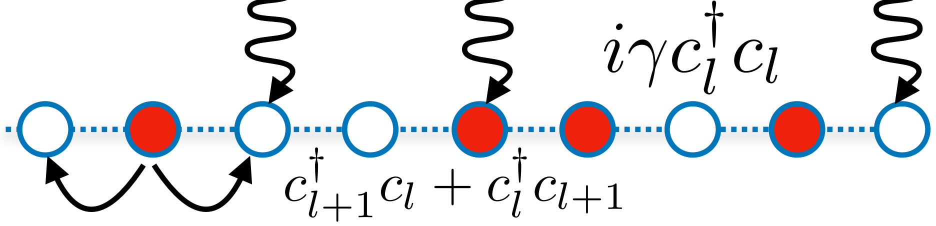
Review: Jacobs, Steck, Contemp. Phys. (2006)

$$d|\psi_t\rangle = dt(-i\hat{H} - \tfrac{\gamma}{2}\sum_l \hat{M}_l^2|\psi_t\rangle + \sum_l dW_l \hat{M}_l|\psi_t\rangle$$

Gaussian white noise

Szyniszewski, Romito, Schomerus, PRB (2019)

Monitored Fermion Dynamics



- Weak continuous measurements: Quantum state diffusion

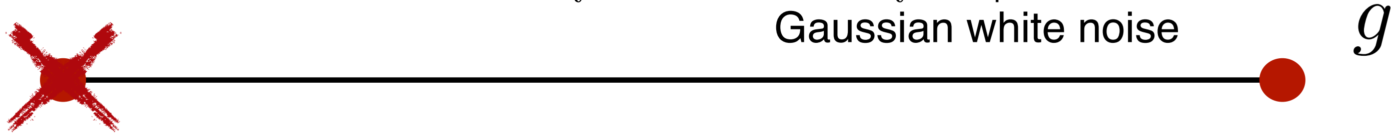
Belavkin (1987); Gisin, Percival (1993)

Review: Jacobs, Steck, Contemp. Phys. (2006)

$$d|\psi_t\rangle = dt(-i\hat{H} - \tfrac{\gamma}{2}\sum_l \hat{M}_l^2|\psi_t\rangle + \sum_l dW_l \hat{M}_l|\psi_t\rangle$$

Gaussian white noise

- competition: $g = \frac{\gamma}{J}$



Szyniszewski, Romito, Schomerus, PRB (2019)

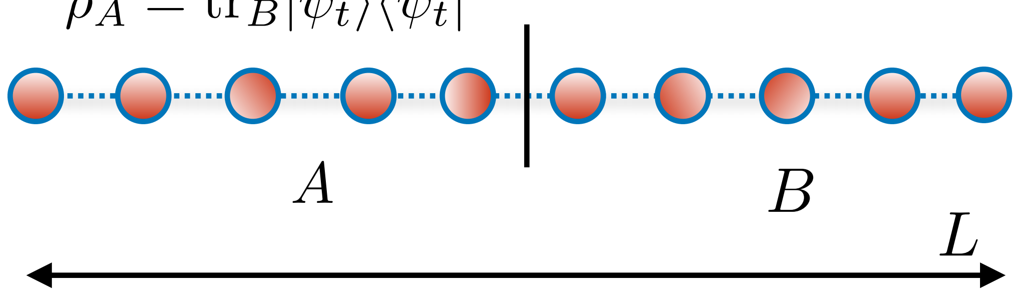
- unitary dynamics: hopping

$$H = -J\sum_l \left(c_l^\dagger c_{l+1} + c_{l+1}^\dagger c_l\right)$$

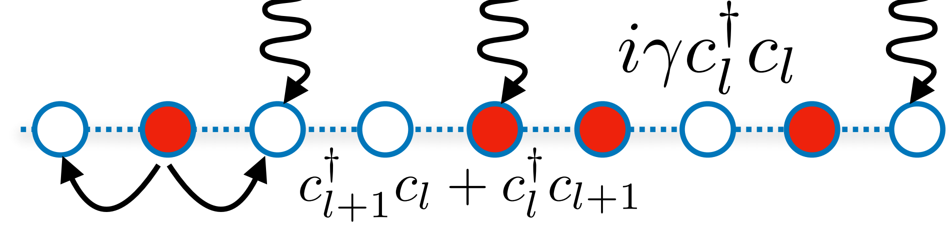
➡ volume law entanglement entropy

$$S_{vN}(L/2, L) = \mathrm{tr}\rho_A \log(\rho_A) \overset{t\to\infty}{\sim} L$$

$\rho_A = \mathrm{tr}_B|\psi_t\rangle\langle\psi_t|$



Monitored Fermion Dynamics



- Weak continuous measurements: Quantum state diffusion

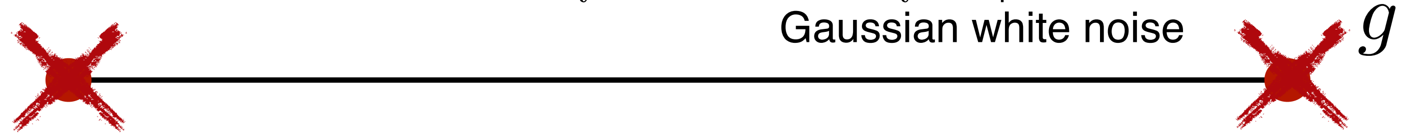
Belavkin (1987); Gisin, Percival (1993)

$$d|\psi_t\rangle = dt(-i\hat{H} - \tfrac{\gamma}{2}\sum_l \hat{M}_l^2|\psi_t\rangle + \sum_l dW_l \hat{M}_l|\psi_t\rangle$$

Gaussian white noise

Review: Jacobs, Steck, Contemp. Phys. (2006)

- competition: $g = \dfrac{\gamma}{J}$



Szyniszewski, Romito, Schomerus, PRB (2019)

- unitary dynamics: hopping

$$H = -J\sum_l \left(c_l^\dagger c_{l+1} + c_{l+1}^\dagger c_l\right)$$

- measurement operators $\hat{M}_l = \hat{n}_l - \langle \hat{n}_l\rangle_t$
- H = 0: continuous collapse into dark state

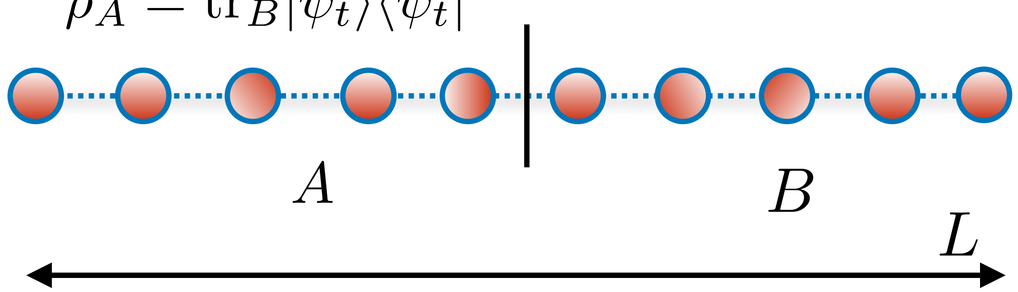
$$\hat{M}_l|\psi_t\rangle = 0 \quad \text{for} \quad \hat{n}_l|\psi_t\rangle = n_l|\psi_t\rangle$$

eigenstate of measurement operator

➡ volume law entanglement entropy

$$S_{vN}(L/2, L) = \mathrm{tr}\rho_A \log(\rho_A) \overset{t\to\infty}{\sim} L$$

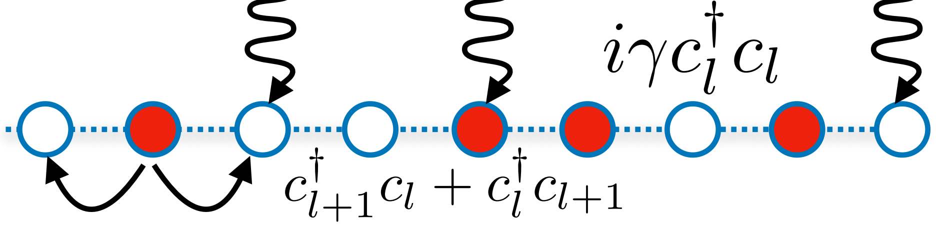
$\rho_A = \mathrm{tr}_B|\psi_t\rangle\langle\psi_t|$



➡ area law entanglement entropy

$$S_{vN}(L/2, L) = s_0$$

Monitored Fermion Dynamics



- Weak continuous measurements: Quantum state diffusion

Belavkin (1987); Gisin, Percival (1993)

Review: Jacobs, Steck, Contemp. Phys. (2006)

$$d|\psi_t\rangle = dt(-i\hat{H} - \tfrac{\gamma}{2}\sum_l \hat{M}_l^2|\psi_t\rangle + \sum_l dW_l \hat{M}_l|\psi_t\rangle$$

Gaussian white noise

- competition: $g = \dfrac{\gamma}{J}$

Szyniszewski, Romito, Schomerus, PRB (2019)

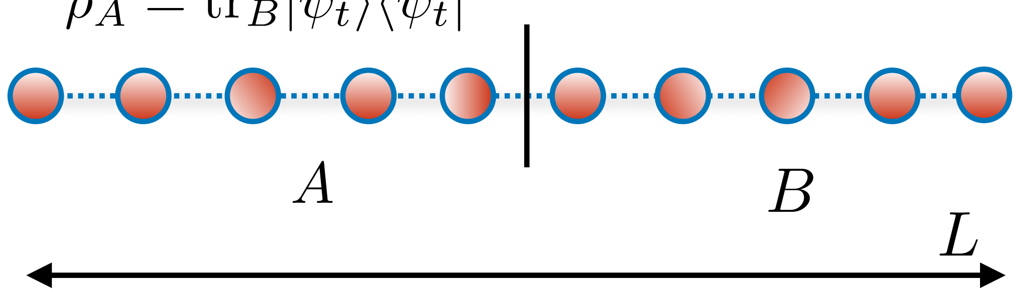
- unitary dynamics: hopping

$$H = -J\sum_l \left(c_l^\dagger c_{l+1} + c_{l+1}^\dagger c_l\right)$$

➡ volume law entanglement entropy

$$S_{vN}(L/2, L) = \mathrm{tr}\rho_A \log(\rho_A) \overset{t\to\infty}{\sim} L$$

$\rho_A = \mathrm{tr}_B|\psi_t\rangle\langle\psi_t|$



- measurement operators $\hat{M}_l = \hat{n}_l - \langle\hat{n}_l\rangle_t$
- H = 0: continuous collapse into dark state

$$\hat{M}_l|\psi_t\rangle = 0 \quad \text{for} \quad \hat{n}_l|\psi_t\rangle = n_l|\psi_t\rangle$$

eigenstate of measurement operator

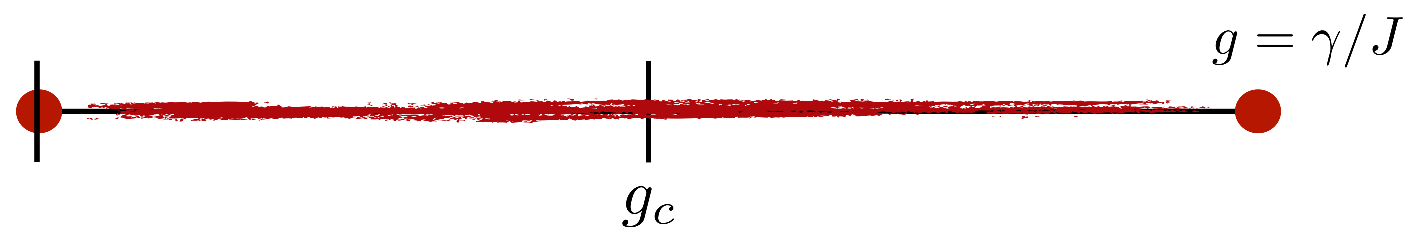
➡ area law entanglement entropy

$$S_{vN}(L/2, L) = s_0$$

- caveat: $|\psi_{t\to\infty}\rangle$ is a random variable

➡ consider trajectory ensemble to extract information

Monitored Fermion Dynamics: Extracting Information



- usual observables:

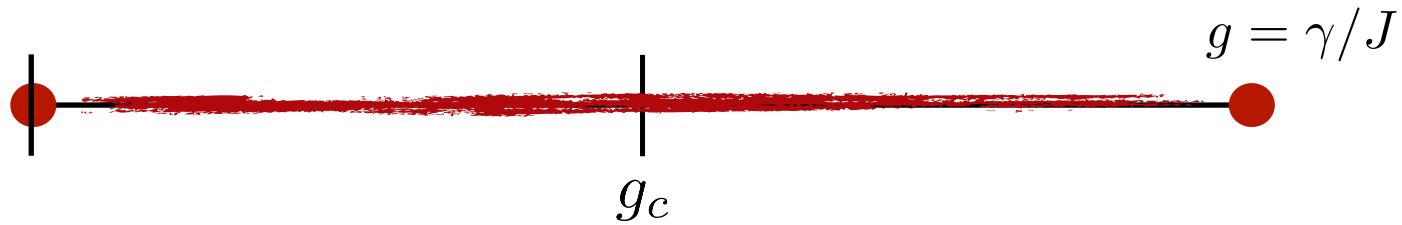
$$\langle \hat{A} \rangle = \langle \psi_t | \hat{A} | \psi_t \rangle \xrightarrow[\text{e.g. trajectories}]{\text{statistical average}} \overline{\langle \hat{A} \rangle} = \overline{\langle \psi_t | \hat{A} | \psi_t \rangle} = \mathrm{tr}[\hat{A}\overline{\hat{\rho}_t}] \qquad \hat{\rho}_t = |\psi_t\rangle\langle\psi_t|$$

quantum average

state projector

- Problem: Hermitian measurement operators => $\overline{\hat{\rho}_t} \sim \mathbf{1}$

Monitored Fermion Dynamics: Extracting Information



- usual observables:

$$\langle \hat{A} \rangle = \langle \psi_t | \hat{A} | \psi_t \rangle \xrightarrow[\text{e.g. trajectories}]{\text{statistical average}} \overline{\langle \hat{A} \rangle} = \overline{\langle \psi_t | \hat{A} | \psi_t \rangle} = \mathrm{tr}[\hat{A}\overline{\hat{\rho}_t}] \qquad \hat{\rho}_t = |\psi_t\rangle\langle\psi_t|$$

quantum average

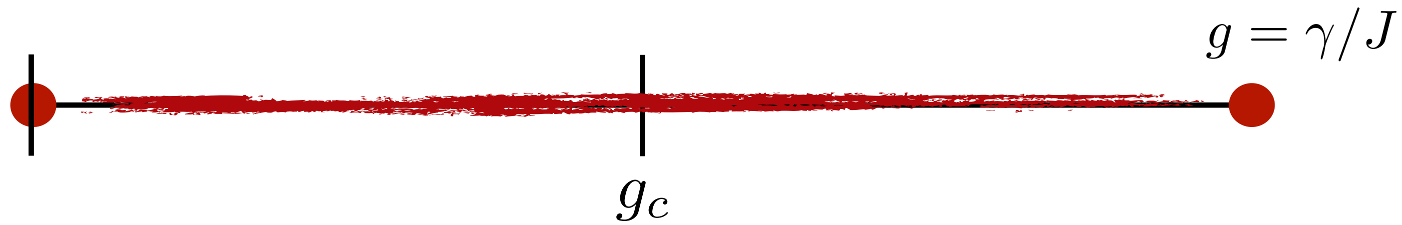
state projector

- Problem: Hermitian measurement operators => $\overline{\hat{\rho}_t} \sim \mathbf{1}$

➡ use state-dependent observables $\overline{\langle \hat{A}(|\psi\rangle) \rangle} = \mathrm{tr}\overline{\hat{A}(\hat{\rho})\hat{\rho}}$

more promising, because in general $F(\overline{\hat{\rho}}) \neq \overline{F[\hat{\rho}]}$

Monitored Fermion Dynamics: Extracting Information



- usual observables:

$$\langle \hat{A} \rangle = \langle \psi_t | \hat{A} | \psi_t \rangle \xrightarrow[\text{e.g. trajectories}]{\text{statistical average}} \overline{\langle \hat{A} \rangle} = \overline{\langle \psi_t | \hat{A} | \psi_t \rangle} = \mathrm{tr}[\hat{A}\overline{\hat{\rho}_t}] \qquad \hat{\rho}_t = |\psi_t\rangle\langle\psi_t|$$

quantum average

state projector

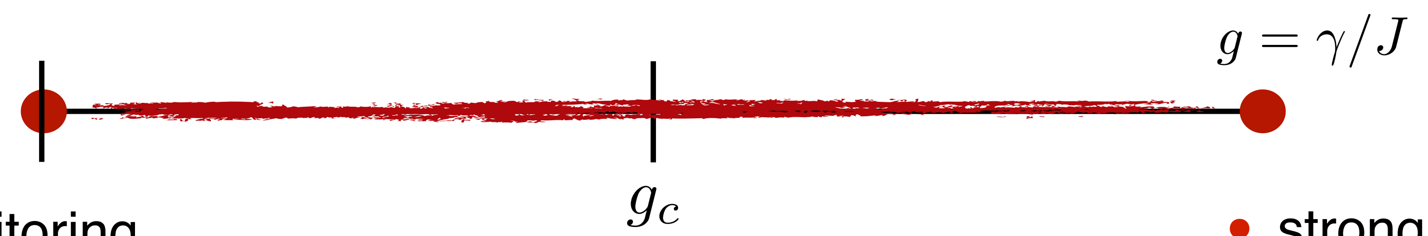
- Problem: Hermitian measurement operators => $\overline{\hat{\rho}_t} \sim \mathbf{1}$

➡ use state-dependent observables $\overline{\langle \hat{A}(|\psi\rangle) \rangle} = \mathrm{tr}\overline{\hat{A}(\hat{\rho})\hat{\rho}}$

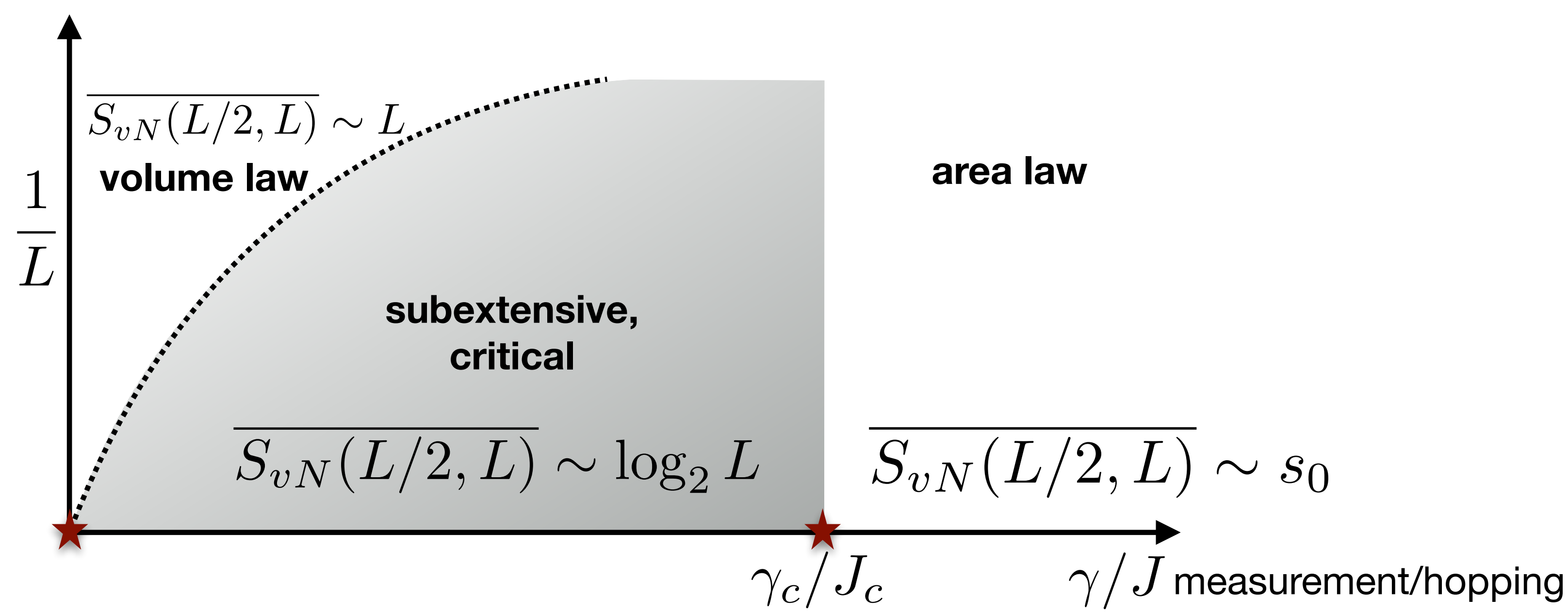
more promising, because in general $F(\overline{\hat{\rho}}) \neq \overline{F[\hat{\rho}]}$

- examples:
  - von Neumann entropy $\overline{S_{vN}(l, L)} = \overline{\langle \log(\rho_A) \rangle}$ arbitrarily high power of state projector
  - correlation function $\overline{\langle \hat{n}_i \rangle \langle \hat{n}_j \rangle}$ quadratic in state projector

Trajectory Ensemble Phase Diagram



- weak monitoring
- strong monitoring

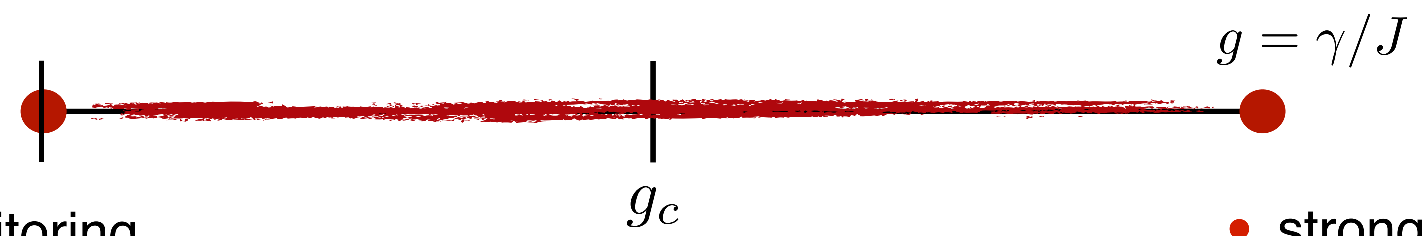


➡ extended subextensive, critical phase at intermediate monitoring

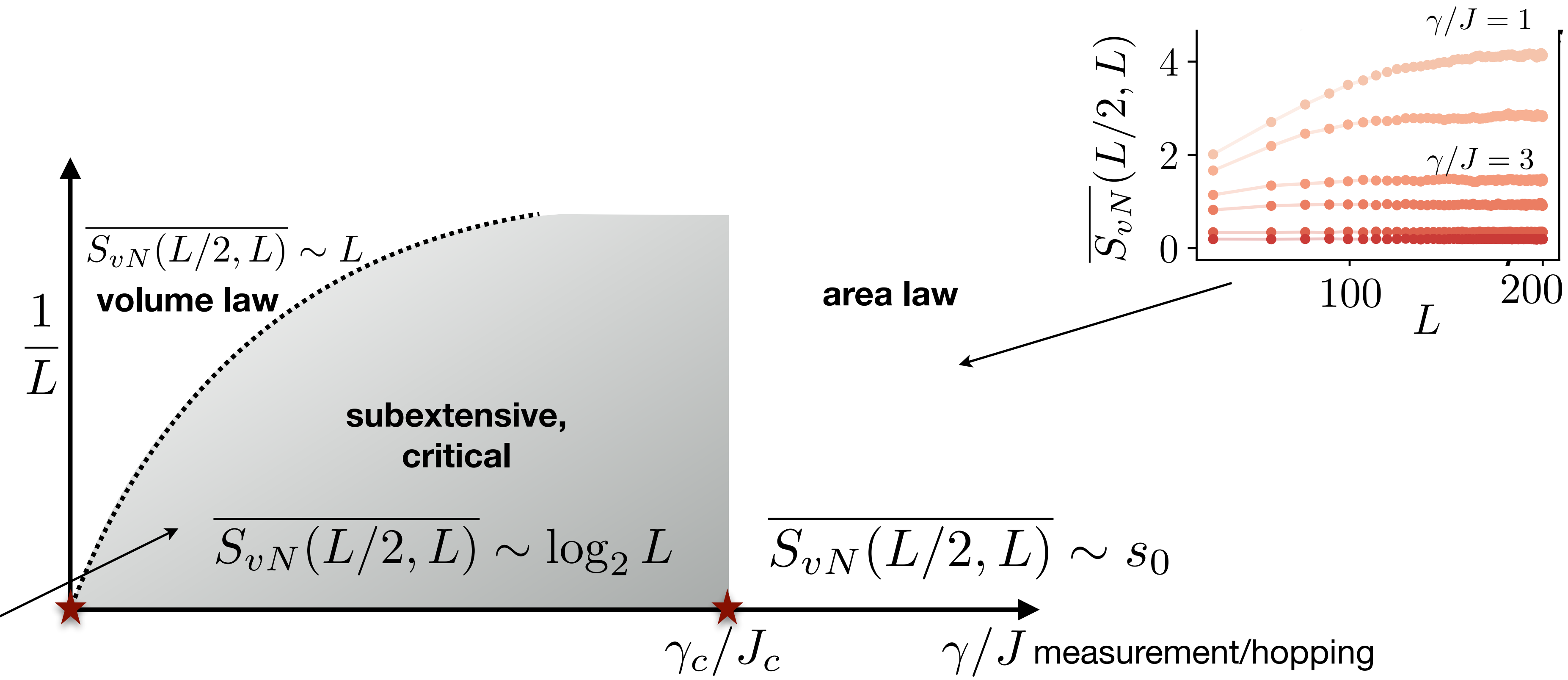
➡ new phase transition for physical measurement protocol

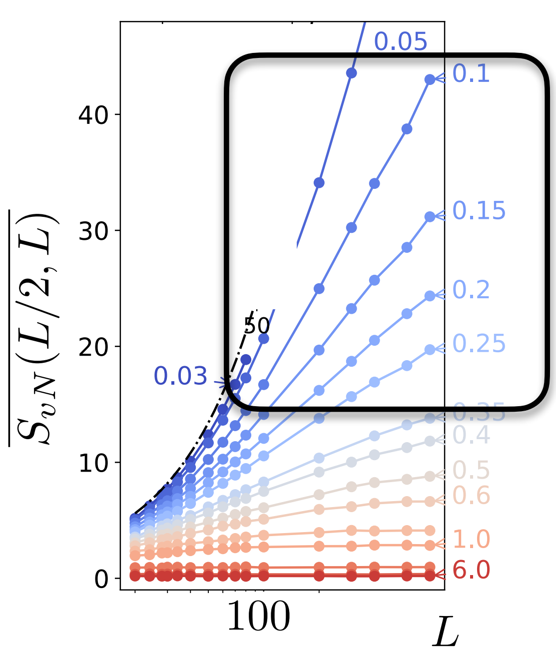
critical phase but no transition for non-unitary circuits: Chen, Li, Fisher, Lucas PRR (2020)

Trajectory Ensemble Phase Diagram



- weak monitoring
- strong monitoring



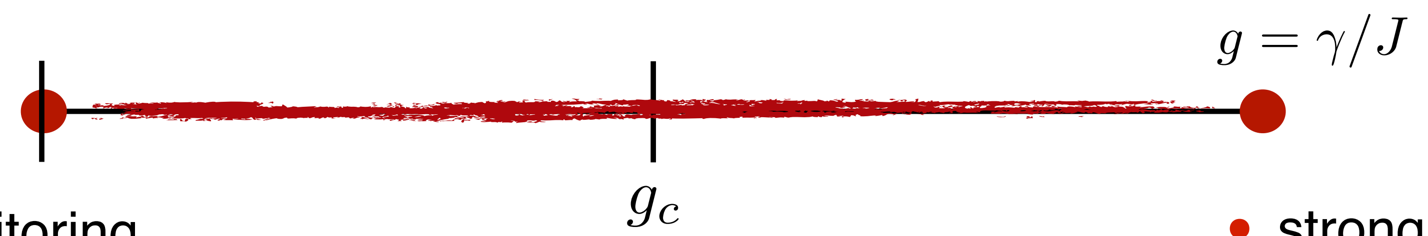


➡ extended subextensive, critical phase at intermediate monitoring

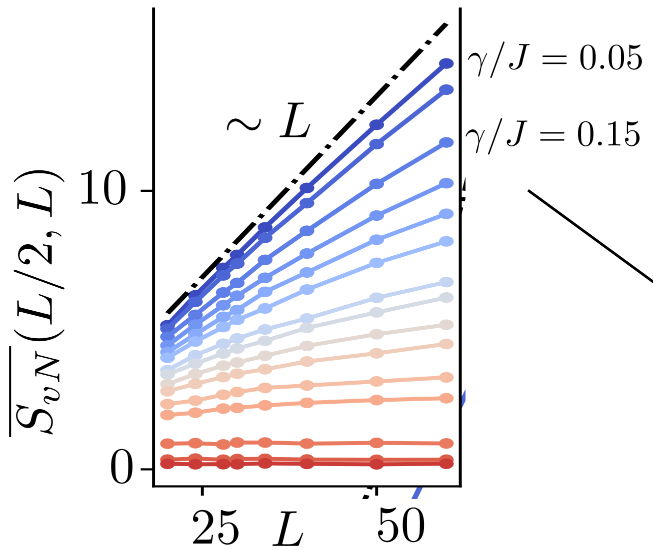
➡ new phase transition for physical measurement protocol

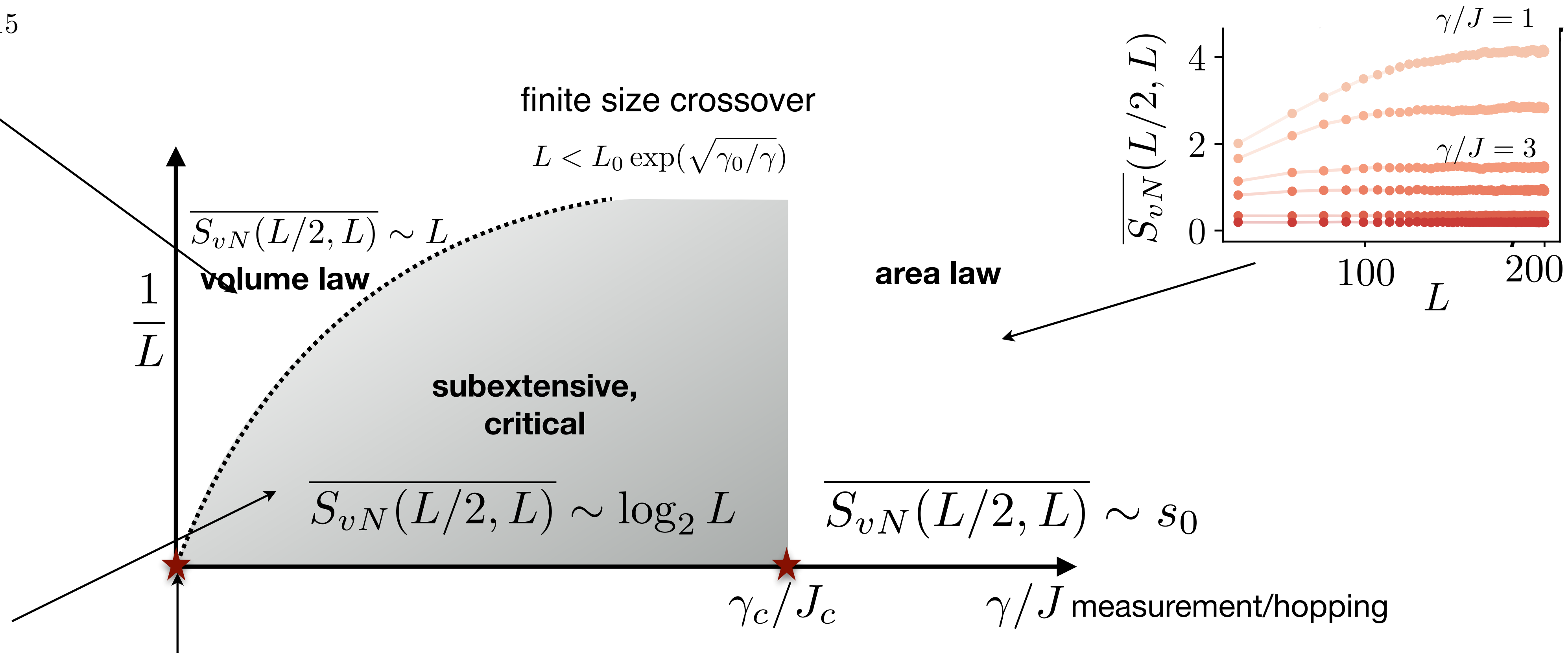
critical phase but no transition for non-unitary circuits: Chen, Li, Fisher, Lucas PRR (2020)

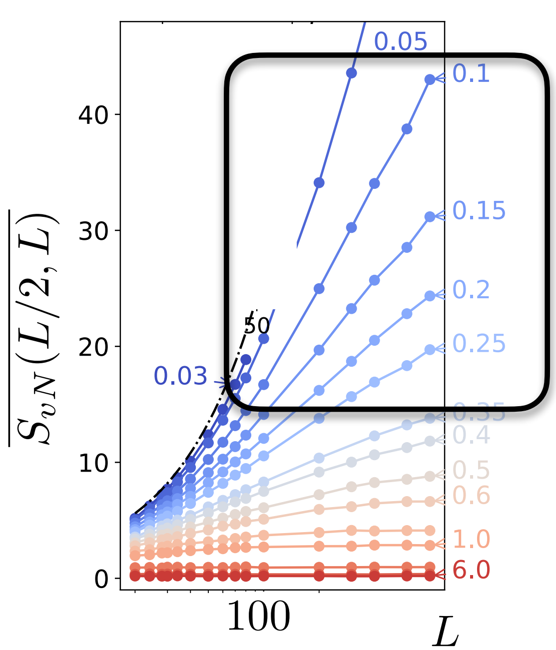
Trajectory Ensemble Phase Diagram



- weak monitoring
- strong monitoring







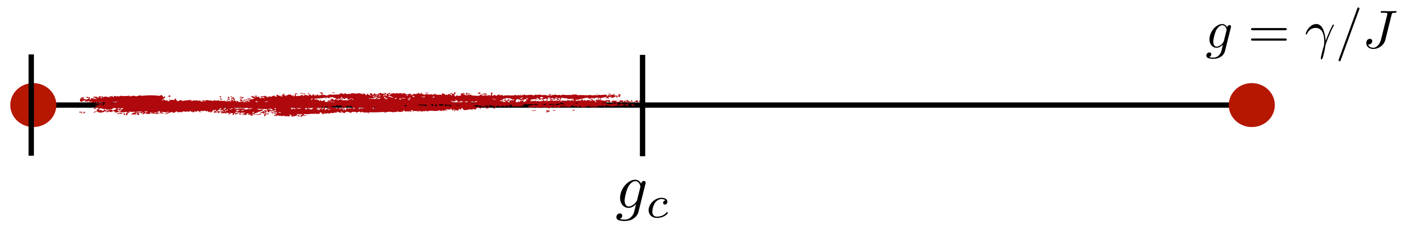
purely unitary case not smoothly connected

➡ extended subextensive, critical phase at intermediate monitoring

➡ new phase transition for physical measurement protocol

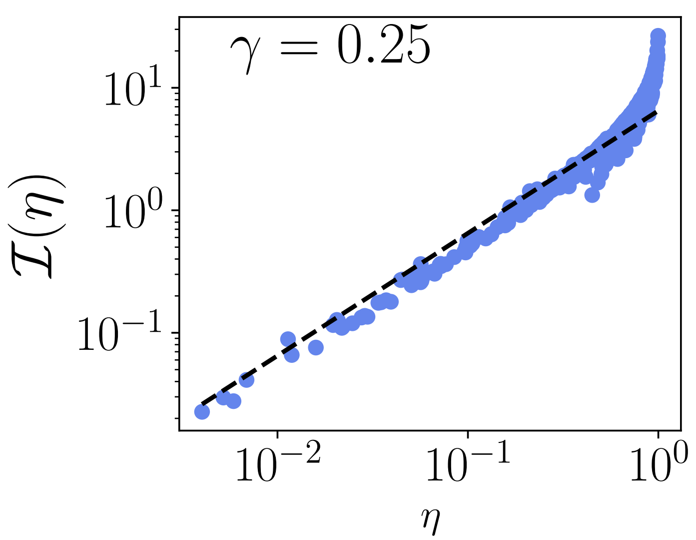
critical phase but no transition for non-unitary circuits: Chen, Li, Fisher, Lucas PRR (2020)

Characterizing the Weak Monitoring Phase & Phase Transition



- Mutual information

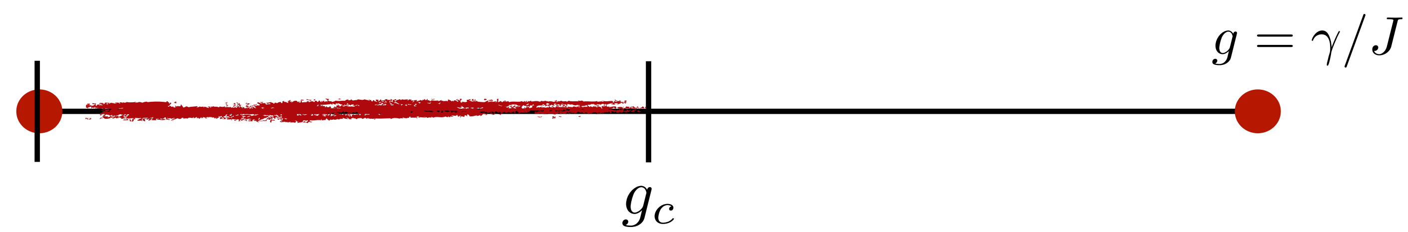
$$\mathcal{I}(A,B) = \overline{S_{vN}(A)} + \overline{S_{vN}(A)} + \overline{S_{vN}(A \cup B)}$$



conformally invariant critical point:
Nahum et al. PRX (2019); Li Chen Fisher PRB (2019); Jian et al. PRB (2020);

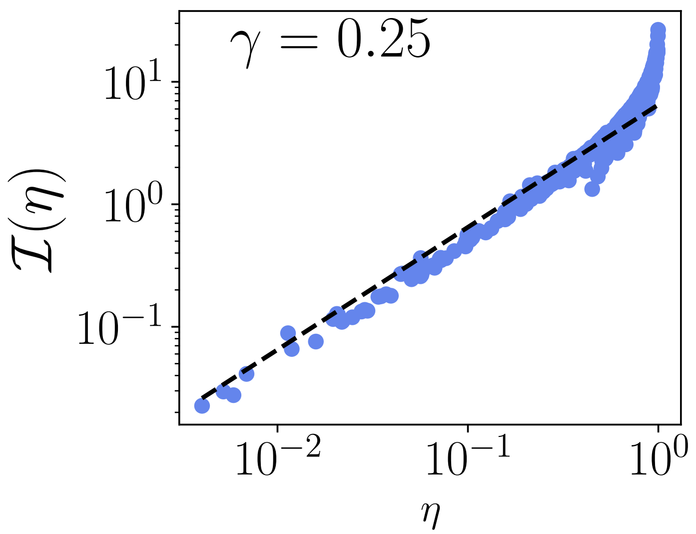
➡ emergent conformality

Characterizing the Weak Monitoring Phase & Phase Transition



- Mutual information

$$\mathcal{I}(A,B) = \overline{S_{vN}(A)} + \overline{S_{vN}(A)} + \overline{S_{vN}(A \cup B)}$$

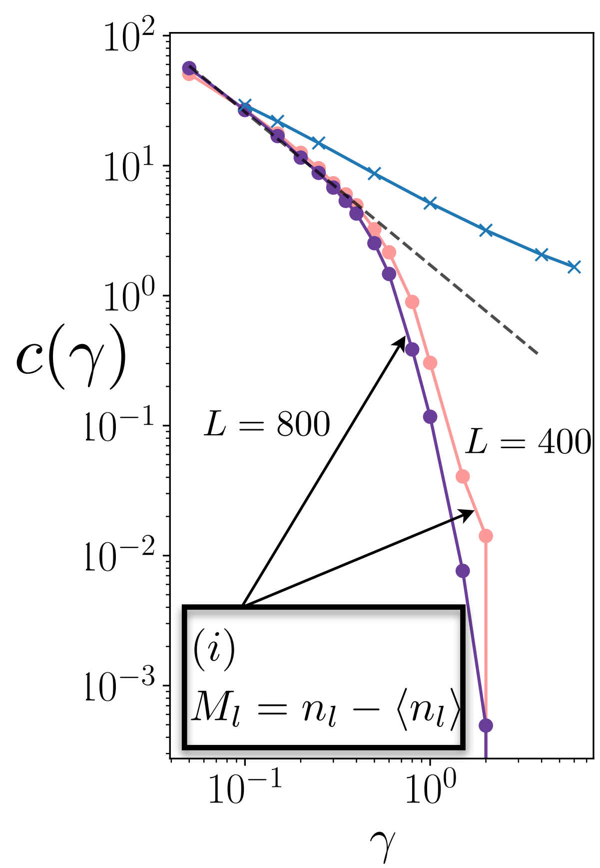


conformally invariant critical point: Nahum et al. PRX (2019); Li Chen Fisher PRB (2019); Jian et al. PRB (2020);

➡ emergent conformality

- effective central charge $c(\gamma)$

$$\overline{S_{vN}(l,L)} = \frac{c(\gamma)}{3} \log_2 \left[\frac{L}{\pi} \sin\left(\frac{\pi l}{L}\right)\right] + s(\gamma)$$

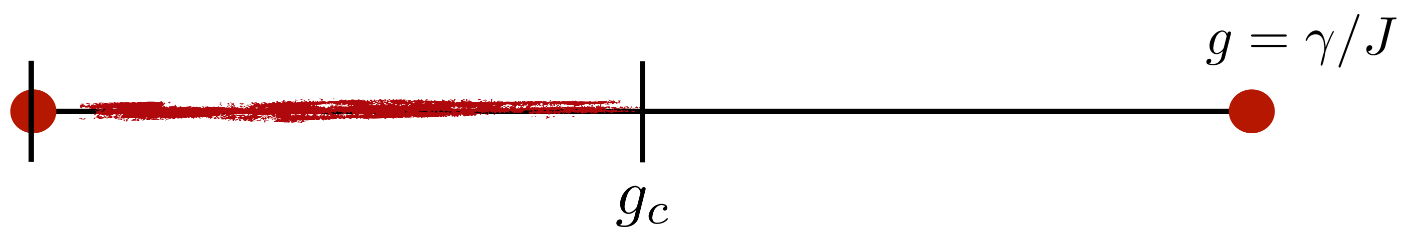


parameter dependent c

random systems: Cardy Jacobsen PRL (1997); Refael, Moore PRL (2004)

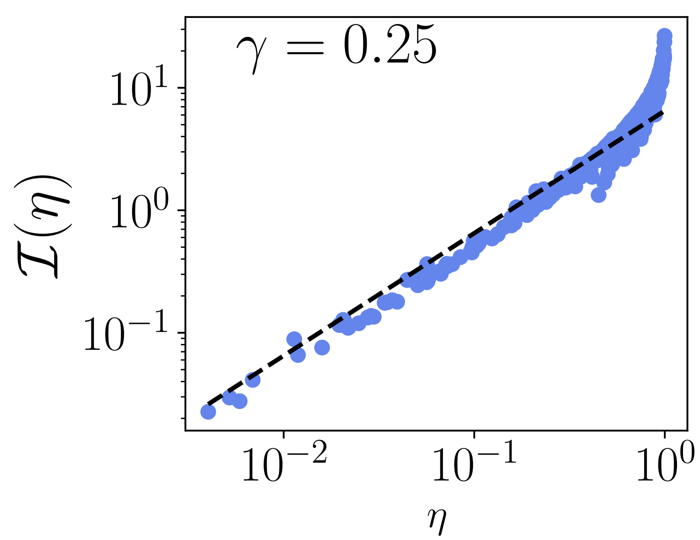
➡ sudden jump reminiscent of BKT

Characterizing the Weak Monitoring Phase & Phase Transition



- Mutual information

$$\mathcal{I}(A,B) = \overline{S_{vN}(A)} + \overline{S_{vN}(A)} + \overline{S_{vN}(A\cup B)}$$

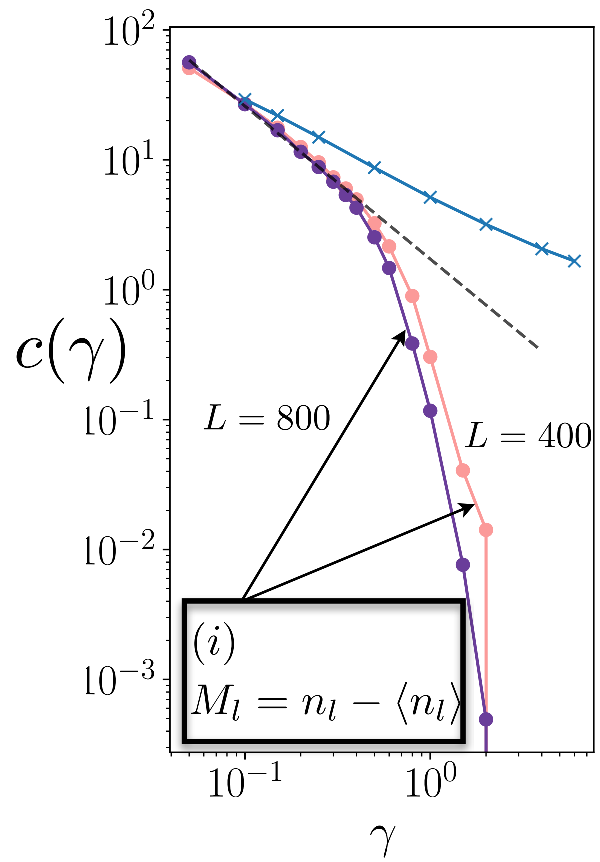


conformally invariant critical point: Nahum et al. PRX (2019); Li Chen Fisher PRB (2019); Jian et al. PRB (2020);

➡ emergent conformality

- effective central charge $c(\gamma)$

$$\overline{S_{vN}(l,L)} = \frac{c(\gamma)}{3}\log_2\left[\frac{L}{\pi}\sin\left(\frac{\pi l}{L}\right)\right] + s(\gamma)$$



parameter dependent c

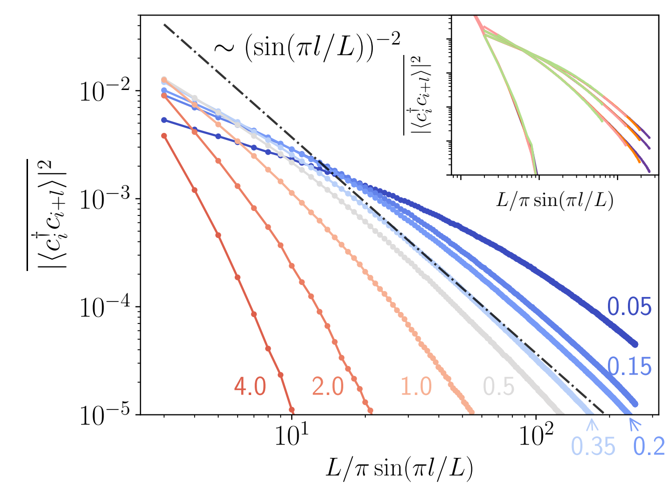
random systems: Cardy Jacobsen PRL (1997); Refael, Moore PRL (2004)

➡ sudden jump reminiscent of BKT

- extended criticality: Connected correlation function

$$C_{i,i+l} = \overline{\langle \hat{n}_i \rangle \langle \hat{n}_{i+l} \rangle} - \overline{\langle \hat{n}_i \hat{n}_{i+l} \rangle}$$

$$\sim l^{-2}$$

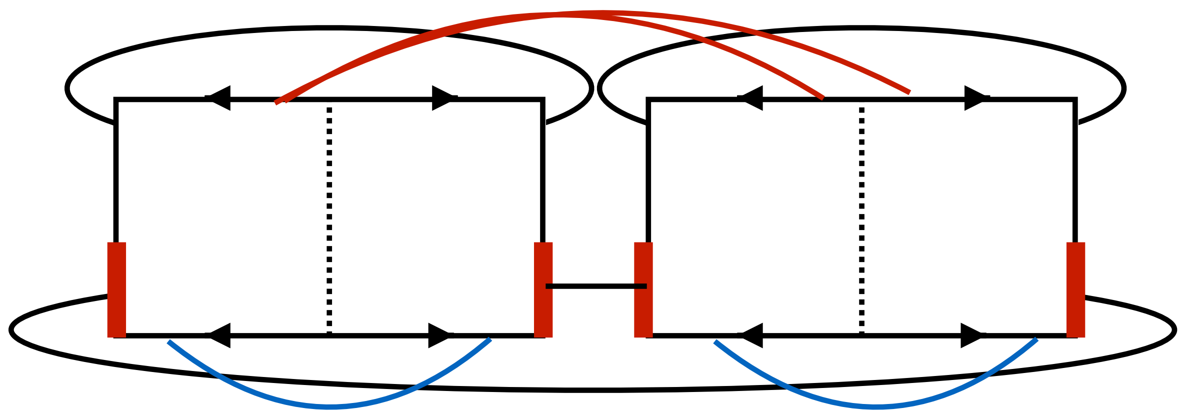


$$C_{i,i+l} \sim \begin{cases} 0 & \text{for } H = 0 \\ \exp(-l/\xi) & \text{for } \gamma \gg J \\ l^{-2} & \text{for } \gamma \ll J \\ l^{-1} & \text{for } \gamma = 0 \end{cases}$$

➡ correlation functions equally characterize the transition

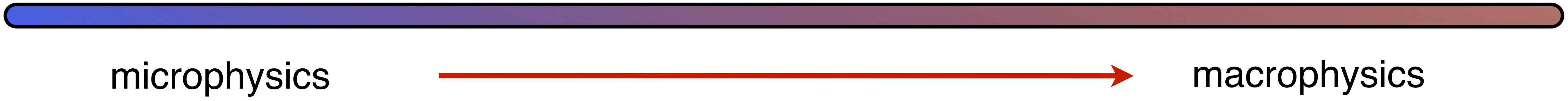
➡ further: measurement protocol dependence, trajectory entanglement distribution as probe of transition…

Replica Field Theory Approach
to
Measurement Induced Phase Transitions

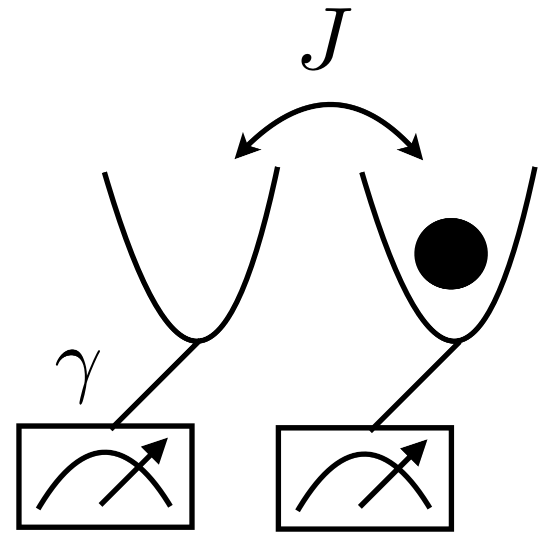


M. Buchhold, Y. Minoguchi, A. Altland, SD, arxiv:2102.08381

related work (random circuits): Y. Bao, S. Choi, and E. Altman, arXiv:2102.09164



Pinning picture: Toy model



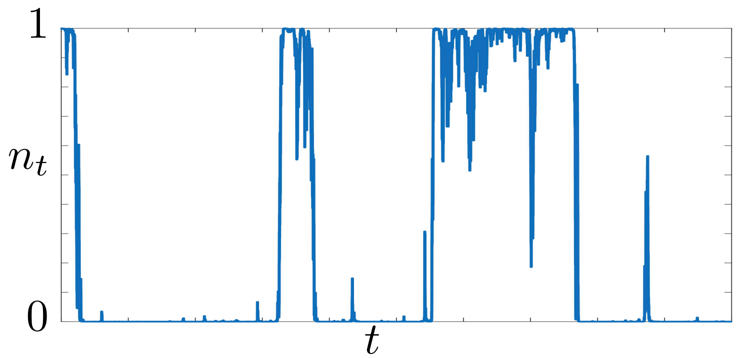
- toy model: trajectory evolution of single fermion on two sites

$$|\psi_{t+dt}\rangle = |\psi_t\rangle - idt\hat{H}_{\text{eff}}|\psi_t\rangle + \sum_{l=1}^{2} dW_l \left(\hat{n}_l - \langle \hat{n}_l \rangle_t\right)|\psi_t\rangle$$

$$\hat{H}_{\text{eff}} = \hat{H} - i\hat{K} \qquad \hat{H} = -J\left(c_1^\dagger c_2 + h.c.\right) \qquad \hat{K} = \frac{\gamma}{2}\sum_{l=1}^{2}\left(\hat{n}_l - \langle \hat{n}_l \rangle_t\right)^2$$

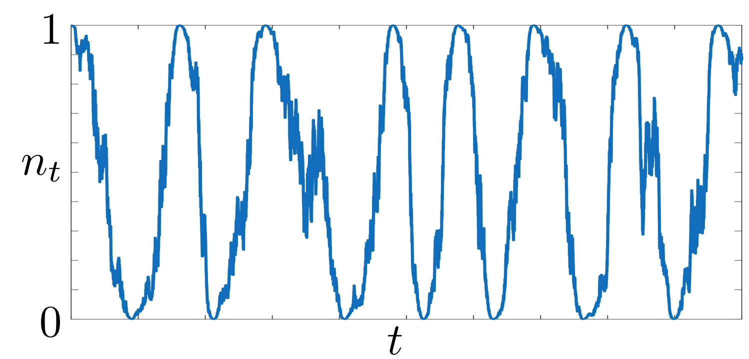
➡ H=0: collapse into dark state at long times $\hat{n}_l|\psi_t\rangle = \langle \hat{n}_l \rangle|\psi_t\rangle \Longrightarrow n_l = 0, 1$

- strong monitoring $J/\gamma \ll 1$



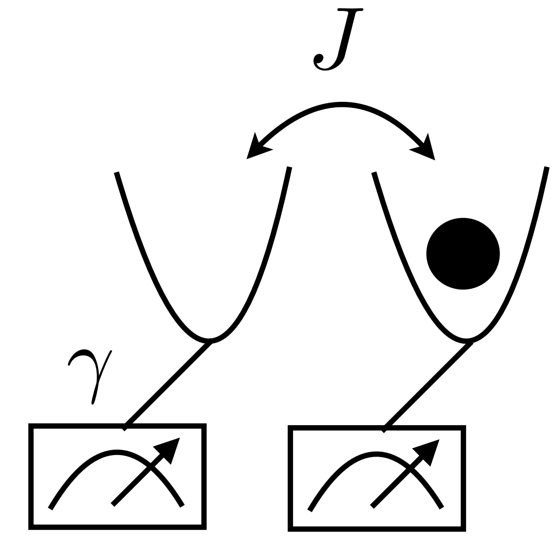
➡ pinning to measurement eigenstate

- weak monitoring $J/\gamma \gg 1$



➡ vanishing time spent in eigenstate

Pinning picture: Toy model



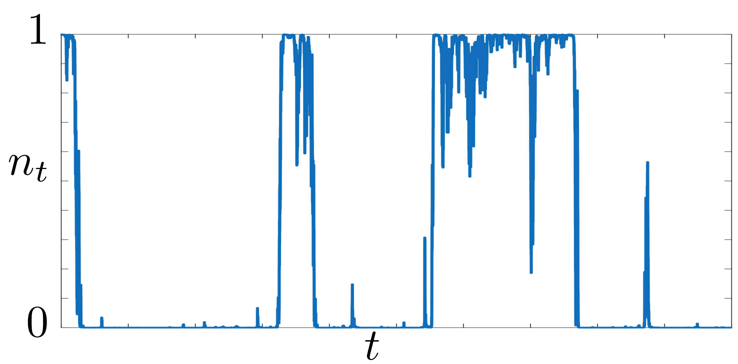
- toy model: trajectory evolution of single fermion on two sites

$$|\psi_{t+dt}\rangle = |\psi_t\rangle - idt\hat{H}_{\text{eff}}|\psi_t\rangle + \sum_{l=1}^{2} dW_l\left(\hat{n}_l - \langle\hat{n}_l\rangle_t\right)|\psi_t\rangle$$

$$\hat{H}_{\text{eff}} = \hat{H} - i\hat{K} \qquad \hat{H} = -J\left(c_1^\dagger c_2 + h.c.\right) \qquad \hat{K} = \frac{\gamma}{2}\sum_{l=1}^{2}\left(\hat{n}_l - \langle\hat{n}_l\rangle_t\right)^2$$

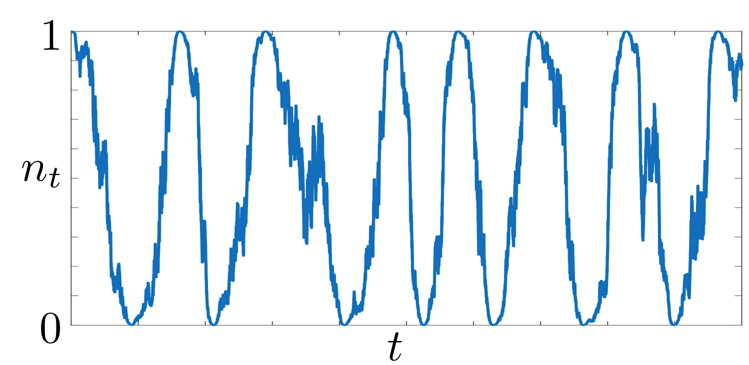
➡ H=0: collapse into dark state at long times $\hat{n}_l|\psi_t\rangle = \langle\hat{n}_l\rangle|\psi_t\rangle \Longrightarrow n_l = 0, 1$

- strong monitoring $J/\gamma \ll 1$



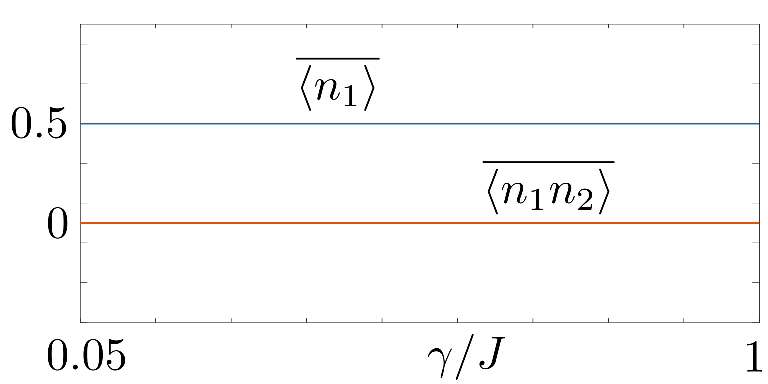
➡ pinning to measurement eigenstate

- weak monitoring $J/\gamma \gg 1$

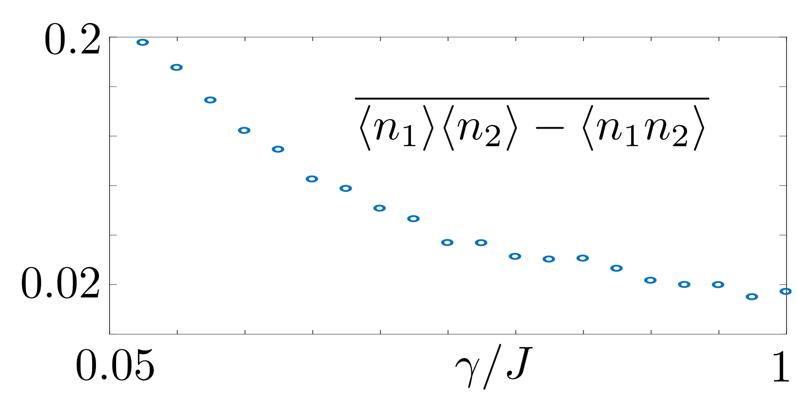


➡ vanishing time spent in eigenstate

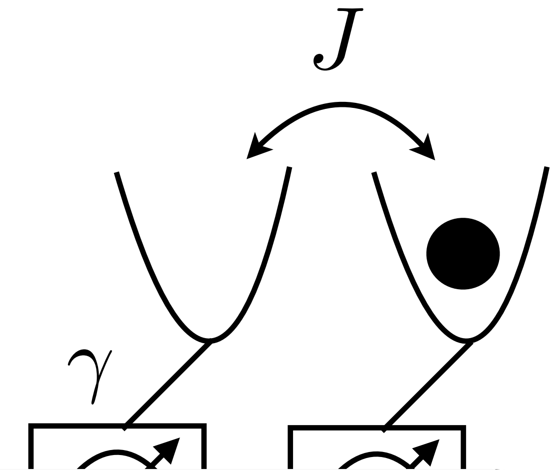
- invisible in linear averages



- seen in averaged trajectory covariance matrix



Pinning picture: Toy model



- toy model: trajectory evolution of single fermion on two sites

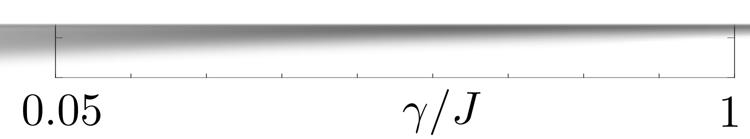
$$|\psi_{t+dt}\rangle = |\psi_t\rangle - idt\hat{H}_{\text{eff}}|\psi_t\rangle + \sum^{2} dW_l\left(\hat{n}_l - \langle \hat{n}_l\rangle_t\right)|\psi_t\rangle$$

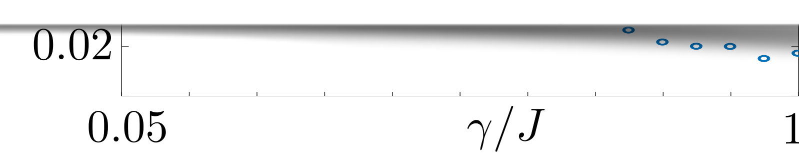
guiding picture and practical approach:

- thermodynamic limit: pinning quantum phase transition may happen at sharply defined point
  - ➡ Minimal continuum model in (1+1) dimensions)
- signalled in state dependent 'observable', like the covariance matrix
  - ➡ Replica construction (no disorder here!)

main insight:

➡ Replica degrees of freedom host non-Hermitian Sine-Gordon model => Pinning transition in BKT universality class





Continuum (1+1) dimensional Model

- preface: model obtains from naive continuum limit and bosonization of lattice fermion model

fermionic variant ⟶ bosonized variant

- Hamiltonian: massless Dirac fermions $\hat{\Psi}_x = (\hat{\psi}_{R,x}, \hat{\psi}_{L,x})^T$ — Luttinger liquid

$$\hat{H} = iv \int_x \hat{\Psi}_x^\dagger \sigma_z \partial_x \hat{\Psi}_x \longrightarrow \hat{H} = \frac{v}{2\pi} \int_x [(\partial_x \hat{\theta}_x)^2 + (\partial_x \hat{\phi}_x)^2]$$

phase density

Continuum (1+1) dimensional Model

- preface: model obtains from naive continuum limit and bosonization of lattice fermion model

fermionic variant $\longrightarrow$ bosonized variant

- Hamiltonian: massless Dirac fermions $\hat{\Psi}_x = (\hat{\psi}_{R,x}, \hat{\psi}_{L,x})^T$ — Luttinger liquid

$$\hat{H} = iv \int_x \hat{\Psi}_x^\dagger \sigma_z \partial_x \hat{\Psi}_x \longrightarrow \hat{H} = \frac{v}{2\pi} \int_x [(\partial_x \hat{\theta}_x)^2 + (\partial_x \hat{\phi}_x)^2]$$

phase density

- measurement operators: current and vertex operators

rate γ_1 : $\hat{O}_{1,x} = \Psi_x^\dagger \Psi_x = \hat{J}_x^{(0)} \longrightarrow \hat{O}_{1,x} = -\frac{1}{\pi}\partial_x \hat{\phi}_x$ linear gapless

rate γ_2 : $\hat{O}_{2,x} = \Psi_x^\dagger \sigma_x \Psi_x \longrightarrow \hat{O}_{2,x} = m \cos(2\hat{\phi}_x)$ nonlinear

m: $\mathcal{O}(1)$

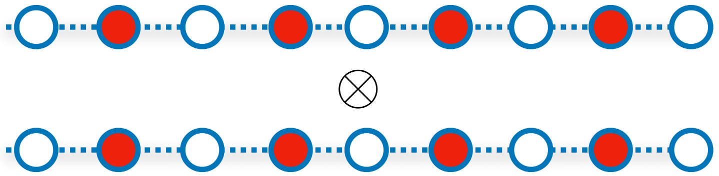
common eigenstates: $\hat{\phi}_x |\Psi_D\rangle = \phi_x |\Psi_D\rangle$

- stabilize product dark states: exactly local
- realize competition: do not commute with H (phase fluctuations)

Replica Approach (n=2)

- Access state-dependent observables, e.g. covariance matrix $C_{xy} = \overline{\langle \hat{n}_x \hat{n}_y \rangle} - \overline{\langle \hat{n}_x \rangle \langle \hat{n}_y \rangle}$

- Introduce replicas in Hilbert space $|\Psi_t\rangle = |\psi_t^{(1)}\rangle \otimes |\psi_t^{(2)}\rangle =$



- All quadratic-in-state observables encoded in

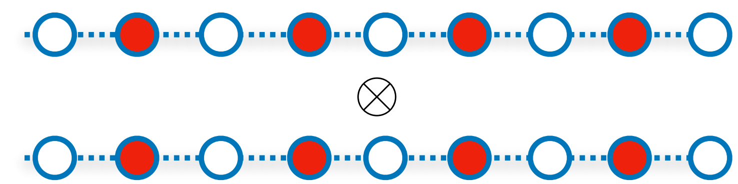
$$\rho^{2\mathrm{R}} = \overline{|\Psi_t\rangle\langle\Psi_t|}$$

➡ linear statistical average of replica density matrix

Replica Approach (n=2)

- Access state-dependent observables, e.g. covariance matrix $C_{xy} = \overline{\langle \hat{n}_x \hat{n}_y \rangle} - \overline{\langle \hat{n}_x \rangle \langle \hat{n}_y \rangle}$

- Introduce replicas in Hilbert space $|\Psi_t\rangle = |\psi_t^{(1)}\rangle \otimes |\psi_t^{(2)}\rangle =$



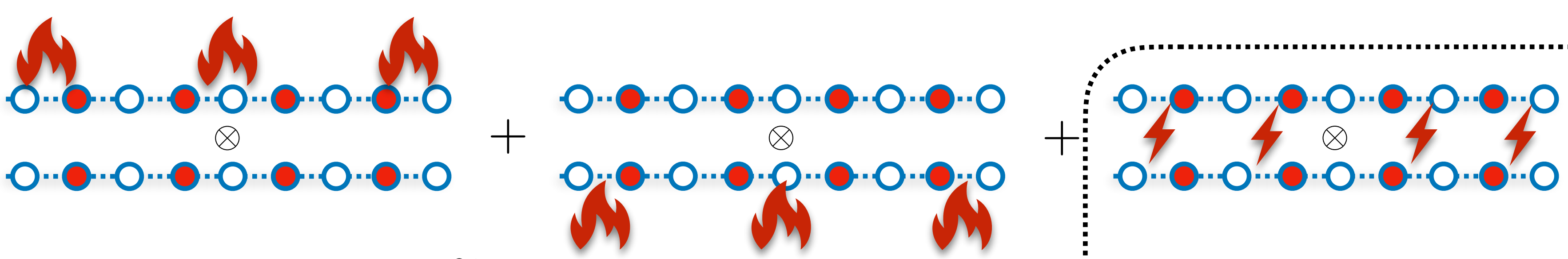
- All quadratic-in-state observables encoded in

$$\rho^{2\mathrm{R}} = \overline{|\Psi_t\rangle\langle\Psi_t|}$$

➡ linear statistical average of replica density matrix

- Quantum master equation (truncate coupling to ρ^{3R})

$$\partial_t \rho^{2\mathrm{R}} =$$



$$i[\rho^{2R}, H^{(\alpha)}] - \frac{\gamma}{2}[\hat{M}_x^{(\alpha)}, [\hat{M}_x^{(\alpha)}, \rho^{2R}]] \quad + \quad \gamma\{\hat{M}_x^{(1)}, \{\hat{M}_x^{(2)}, \rho^{2R}\}\}$$

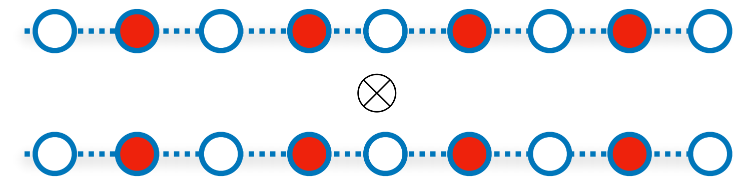
individual heating Lindbladians

replica coupling

Replica Approach (n=2)

- Access state-dependent observables, e.g. covariance matrix $C_{xy} = \overline{\langle \hat{n}_x \hat{n}_y \rangle} - \overline{\langle \hat{n}_x \rangle \langle \hat{n}_y \rangle}$

- Introduce replicas in Hilbert space $|\Psi_t\rangle = |\psi_t^{(1)}\rangle \otimes |\psi_t^{(2)}\rangle =$



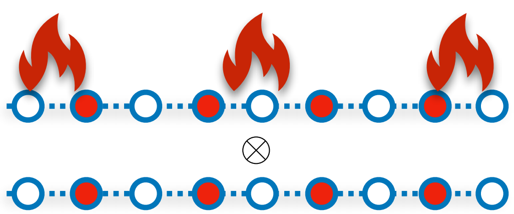
- All quadratic-in-state observables encoded in

$$\rho^{2\mathrm{R}} = \overline{|\Psi_t\rangle\langle\Psi_t|}$$

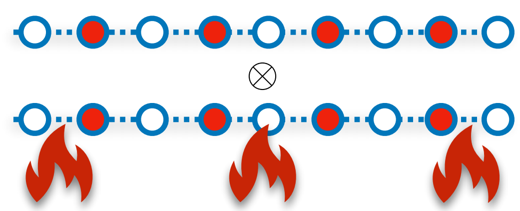
➡ linear statistical average of replica density matrix

- Quantum master equation (truncate coupling to ρ^{3R})

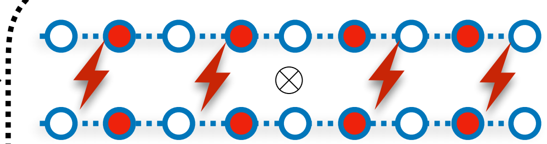
$$\partial_t \rho^{2\mathrm{R}} =$$



$+$



$+$



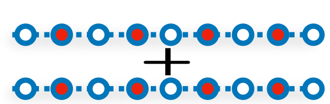
$$i[\rho^{2R}, H^{(\alpha)}] - \frac{\gamma}{2}[\hat{M}_x^{(\alpha)}, [\hat{M}_x^{(\alpha)}, \rho^{2R}]]$$

individual heating Lindbladians

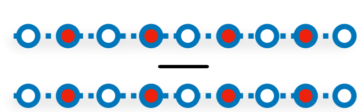
$$\gamma\{\hat{M}_x^{(1)}, \{\hat{M}_x^{(2)}, \rho^{2R}\}\}$$

replica coupling

- New degrees of freedom



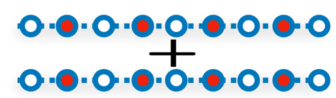
$: \hat{\phi}^{(a)} = \hat{\phi}^{(1)} + \hat{\phi}^{(2)}$ average coordinate



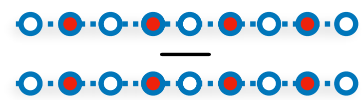
$: \hat{\phi}^{(r)} = \hat{\phi}^{(1)} - \hat{\phi}^{(2)}$ replica fluctuations

Boson Replica Quantum Master Equation

- New degrees of freedom



$: \hat{\phi}^{(a)} = \hat{\phi}^{(1)} + \hat{\phi}^{(2)}$ average coordinate



$: \hat{\phi}^{(r)} = \hat{\phi}^{(1)} - \hat{\phi}^{(2)}$ replica fluctuations

➡ Master equation becomes separable (exact for Gaussian dynamics, useful more generally)

- Average coordinate: heating to infinite temperature

$$\partial_t \rho^{(a)} = i[\rho^{(a)}, H^{(a)}] + \frac{2\gamma}{\pi} \sum_l \left(\partial_x \hat{\phi}^{(a)} - \overline{\langle \partial_x \hat{\phi}^{(a)} \rangle} \right) \rho^{(a)} \left(\partial_x \hat{\phi}^{(a)} - \overline{\langle \partial_x \hat{\phi}^{(a)} \rangle} \right)$$

← only jump term!

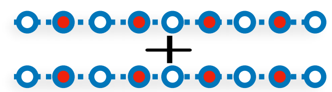
- Relative coordinate: cooling/damping into dark state

$$\partial_t \rho^{(r)} = i[\rho^{(r)}, H^{(r)}] - \frac{\gamma}{\pi} \sum_l \left\{ (\partial_x \hat{\phi}^{(r)})^2, \rho^{(r)} \right\}$$

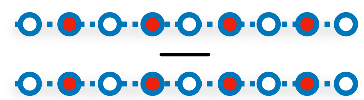
← no jump term!

Boson Replica Quantum Master Equation

- New degrees of freedom



: $\hat{\phi}^{(a)} = \hat{\phi}^{(1)} + \hat{\phi}^{(2)}$ average coordinate



: $\hat{\phi}^{(r)} = \hat{\phi}^{(1)} - \hat{\phi}^{(2)}$ replica fluctuations

➡ Master equation becomes separable (exact for Gaussian dynamics, useful more generally)

- Average coordinate: heating to infinite temperature

$$\partial_t \rho^{(a)} = i[\rho^{(a)}, H^{(a)}] + \frac{2\gamma}{\pi} \sum_l \left(\partial_x \hat{\phi}^{(a)} - \overline{\langle \partial_x \hat{\phi}^{(a)} \rangle} \right) \rho^{(a)} \left(\partial_x \hat{\phi}^{(a)} - \overline{\langle \partial_x \hat{\phi}^{(a)} \rangle} \right)$$

⟵ only jump term!

- Relative coordinate: cooling/damping into dark state

$$\partial_t \rho^{(r)} = i[\rho^{(r)}, H^{(r)}] - \frac{\gamma}{\pi} \sum_l \left\{ (\partial_x \hat{\phi}^{(r)})^2, \rho^{(r)} \right\}$$

⟵ no jump term!

- Non-Hermitian Schrödinger equation for relative coordinate

$$\partial_t |\psi_t^{(r)}\rangle = -i H_{\text{eff}} |\psi_t^{(r)}\rangle$$

➡ cooling into dark state

$$H_{\text{eff}} = \frac{\nu}{2\pi} \int_x (\partial_x \hat{\theta})^2 + (1 - i\eta^2)(\partial_x \hat{\phi})^2 \; - i \frac{\gamma m}{\pi} \int_x [1 - \cos(\sqrt{8}\hat{\phi}_x)]$$

effect of non-linearity

➡ non-Hermitian Sine-Gordon: pinning via cos term, depinning via theta term

➡ extract physics in path integral approach

Phase diagram

➡ Sine-Gordon action with complex coefficients

Fendley, Saleur, Zamolodchikov,
International Journal of Modern Physics (1993)

$$S = \int_{t,x} \left\{ \frac{K}{16\pi} \left[\frac{1}{\eta} (\partial_t \phi)^2 - \eta (\partial_x \phi)^2 \right] - i\lambda \cos(\phi) \right\}$$

- 'Wick rotation' brings free part to standard Euclidean (2+0) dimensional form $(x,t) \to (\eta^{\frac{1}{2}} x, i\eta^{-\frac{1}{2}} t)$

➡ RG flow: standard KT flow with complex K, λ

- UV flow modified

➡ shift of phase border

- IR flow reaches standard KT flow

➡ same long wavelength properties

Phase diagram

➡ Sine-Gordon action with complex coefficients

Fendley, Saleur, Zamolodchikov, International Journal of Modern Physics (1993)

$$S = \int_{t,x} \left\{ \frac{K}{16\pi} \left[\frac{1}{\eta} (\partial_t \phi)^2 - \eta (\partial_x \phi)^2 \right] - i\lambda \cos(\phi) \right\}$$

- ‘Wick rotation’ brings free part to standard Euclidean (2+0) dimensional form $(x,t) \to (\eta^{\frac{1}{2}} x, i\eta^{-\frac{1}{2}} t)$

➡ RG flow: standard KT flow with complex K, λ

- UV flow modified

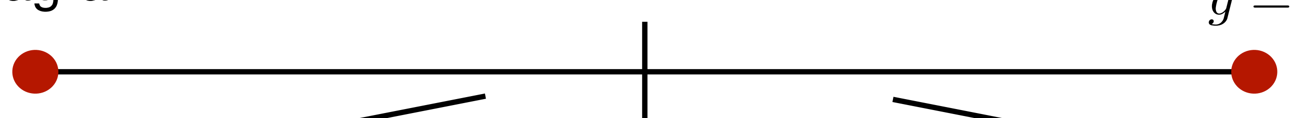
➡ shift of phase border

- IR flow reaches standard KT flow

➡ same long wavelength properties

- resulting phase diagram:

$g = \gamma/J$



g_c

$$C_{xy} = \frac{C(\gamma)}{|x-y|^2}$$

➡ algebraic decay (generalized CFT)

➡ depinned phase

$$C_{xy} \sim e^{-\frac{|x-y|}{\xi}} \qquad \xi^{-1} = \log(2\pi\gamma/v)$$

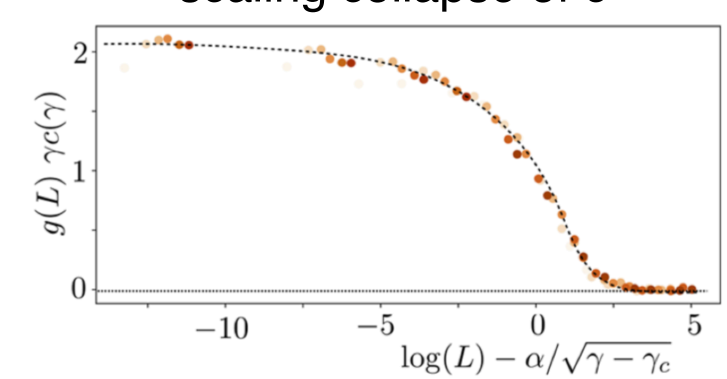
➡ exponential decay

➡ pinned phase

➡ phase transition in the BKT universality class

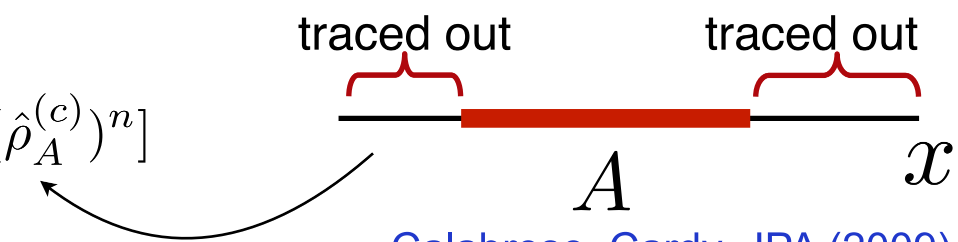
➡ all in line with numerics for lattice fermions

scaling collapse of c



Entanglement Entropies: n-Replica Keldysh approach

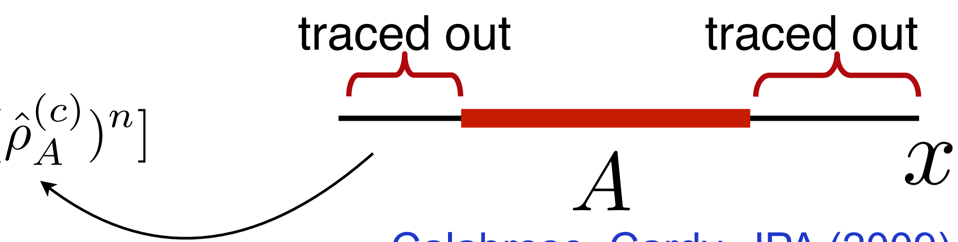
- Rényi entropy $S_n(L) = \frac{1}{1-n}\overline{\log Z_A(n,\{dW\})},\ Z_A(n,\{dW\}) \equiv \mathrm{tr}[(\hat{\rho}_A^{(c)})^n]$
- von Neumann entropy: $n \to 1$



Calabrese, Cardy, JPA (2009)

Entanglement Entropies: n-Replica Keldysh approach

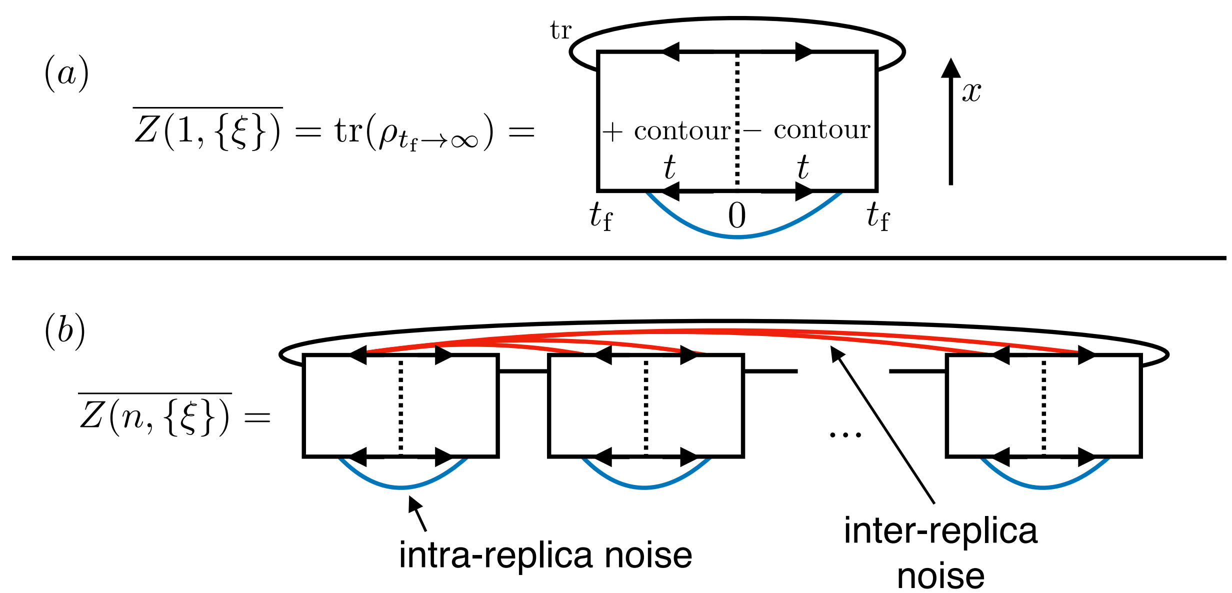
- Rényi entropy $S_n(L) = \frac{1}{1-n}\overline{\log Z_A(n,\{dW\})},\ Z_A(n,\{dW\}) \equiv \mathrm{tr}[(\hat{\rho}_A^{(c)})^n]$
- von Neumann entropy: $n \to 1$



Calabrese, Cardy, JPA (2009)

- Approach: Keldysh replica field theory for n replicas

$$Z(n) = \overline{Z(n,dW)} = \int \mathcal{D}[\{\bar{\Psi}_X, \Psi_X\}] e^{i\mathcal{S}_n[\Psi]}$$

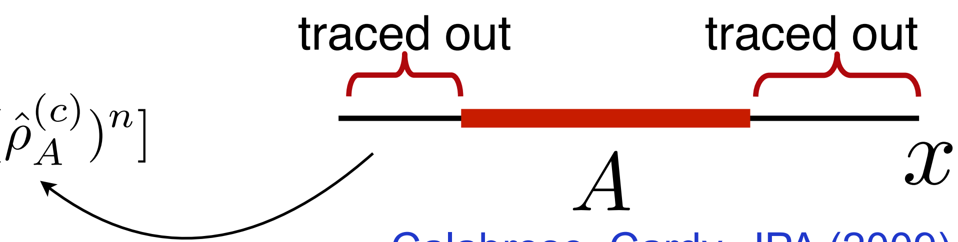


- 1 mode heats up (noisy)
- n-1 modes cool down (noiseless)

Replica-Keldysh in other contexts:
Aleiner, Faoro, Ioffe, AoP (2016);
Ansari, Nazarov, JETP (2016)

Entanglement Entropies: n-Replica Keldysh approach

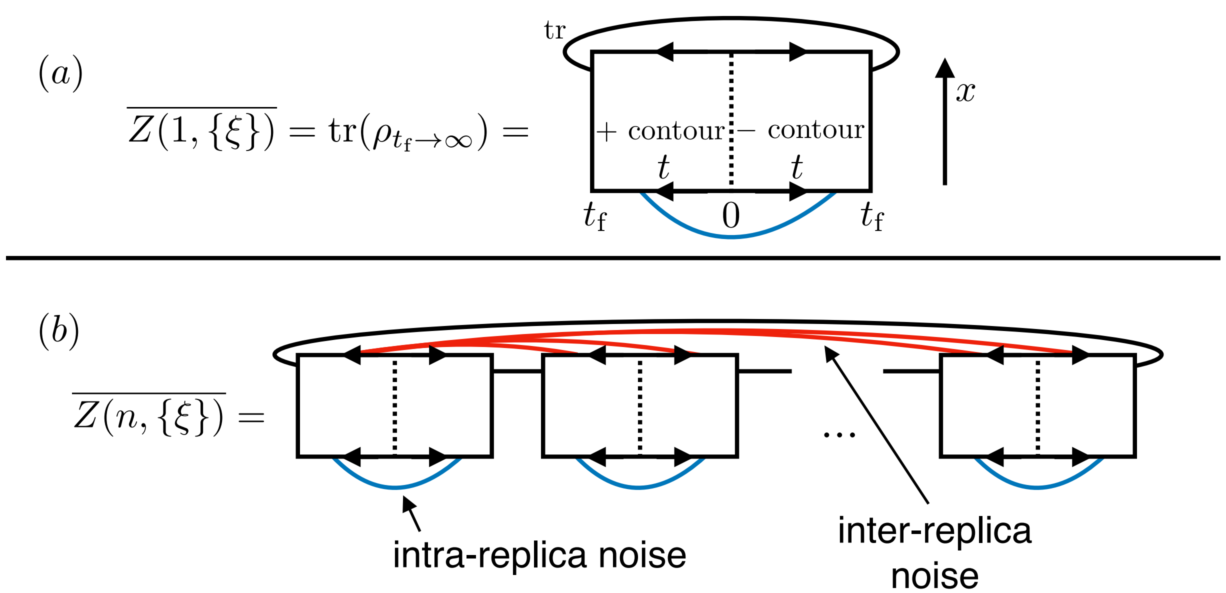
- Rényi entropy $S_n(L) = \frac{1}{1-n}\overline{\log Z_A(n,\{dW\})},\ Z_A(n,\{dW\}) \equiv \mathrm{tr}[(\hat{\rho}_A^{(c)})^n]$
- von Neumann entropy: $n \to 1$



Calabrese, Cardy, JPA (2009)

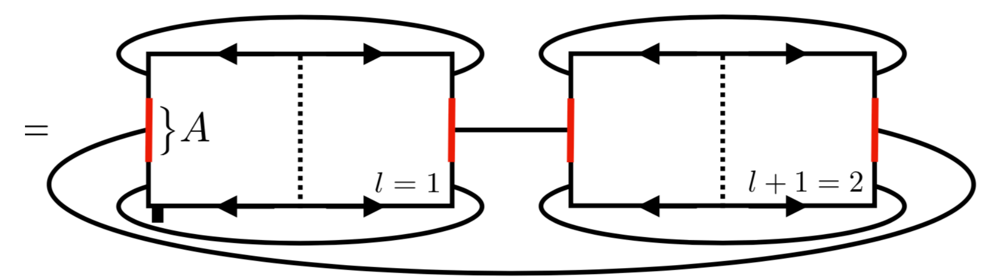
- Approach: Keldysh replica field theory for n replicas

$$Z(n) = \overline{Z(n,dW)} = \int \mathcal{D}[\{\bar{\Psi}_X, \Psi_X\}] e^{i\mathcal{S}_n[\Psi]}$$



- For entropies: modified boundary conditions

e.g. $Z_A(2) = \overline{\mathrm{tr}\rho_A^2}$



- 1 mode heats up (noisy)
- n-1 modes cool down (noiseless)

→

- noisy contribution A independent
- all A dependence in noiseless modes!

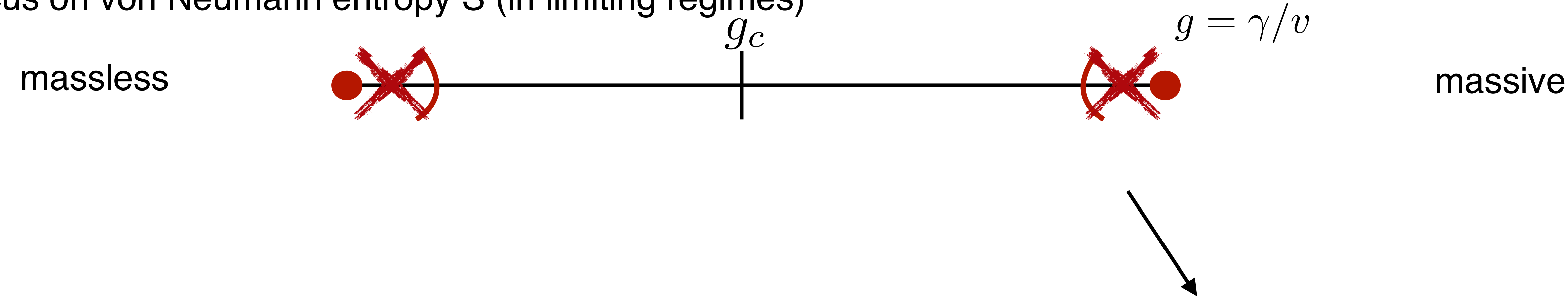
Replica-Keldysh in other contexts:
Aleiner, Faoro, Ioffe, AoP (2016);
Ansari, Nazarov, JETP (2016)

➡ Rényi entropy calculation as for ground states

e.g. Peschel, Eisler, JPA (2009)

Entanglement Transition from Replica Approach

- focus on von Neumann entropy S (in limiting regimes)

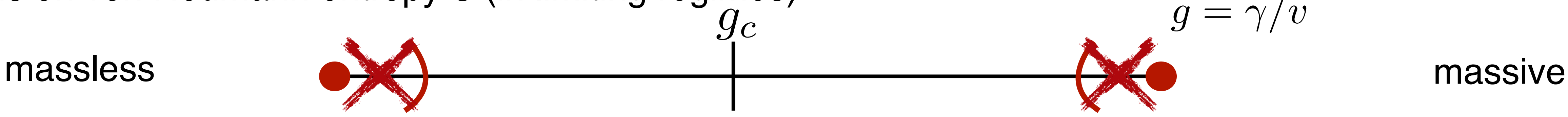


$$S = \frac{1}{3}c(\gamma)\log(m^{-1}) \sim L^0$$

- saturation to area law

Entanglement Transition from Replica Approach

- focus on von Neumann entropy S (in limiting regimes)



$$S = \frac{1}{3}c(\gamma)\log(L)$$

- sub-volume log-law

- $c(\gamma \to 0) \to 1$

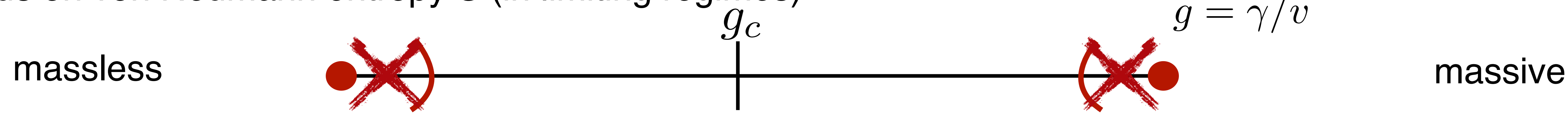
➡ ground state entropy of massless Dirac

$$S = \tfrac{1}{3}c(\gamma)\log(m^{-1}) \sim L^0$$

- saturation to area law

Entanglement Transition from Replica Approach

- focus on von Neumann entropy S (in limiting regimes)



$$S = \frac{1}{3} c(\gamma) \log(L)$$

- sub-volume log-law

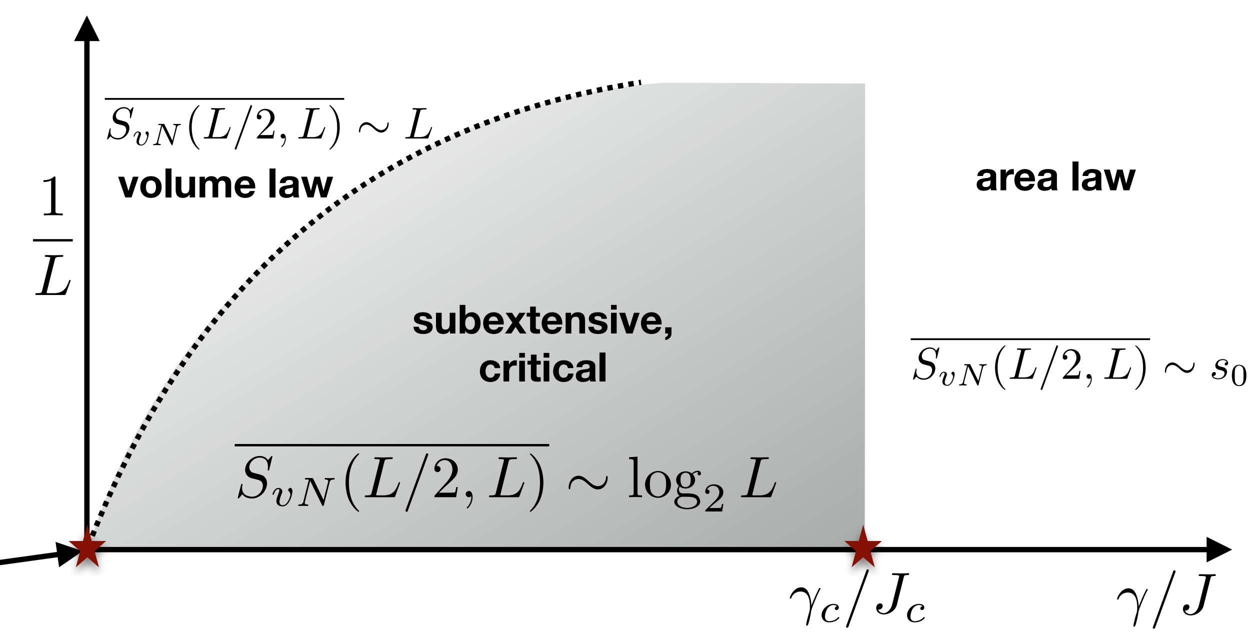
- $c(\gamma \to 0) \to 1$

➡ ground state entropy of massless Dirac

- non-commuting limit: $\gamma = 0$
  finite temperature initial state

$$S \sim L$$

➡ volume law <—> finite temperature massless Dirac



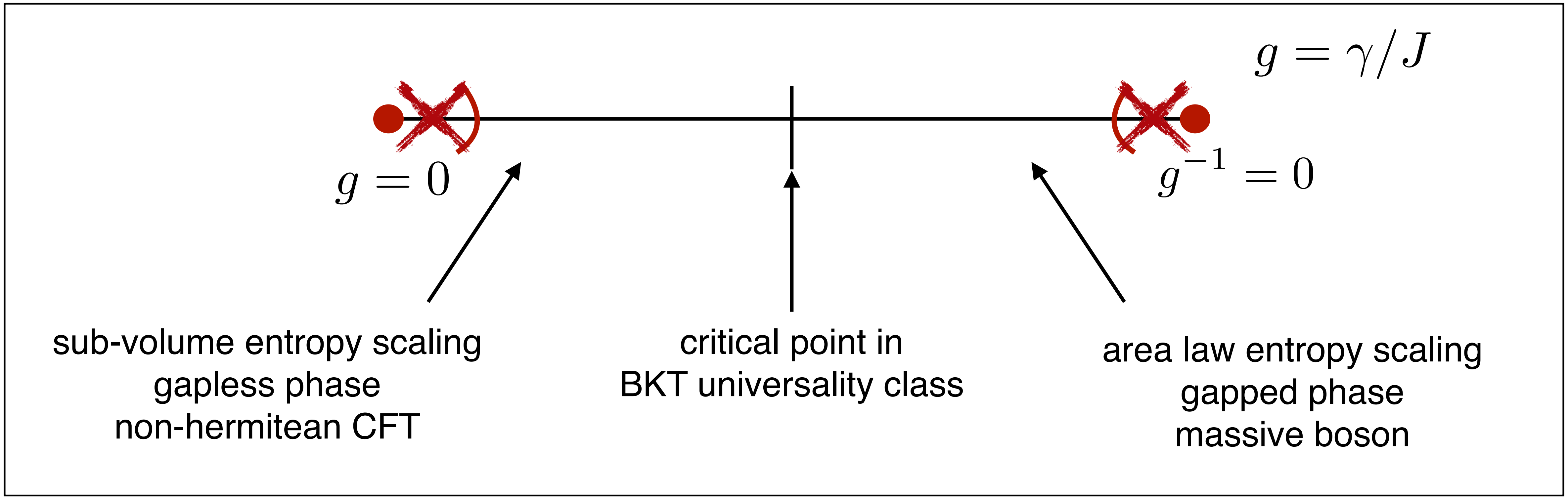
➡ underpins entanglement transition at finite critical g

➡ picture qualitatively (not fully quantitatively) in line with numerics

Conclusions & Outlook

O.Alberton, M.Buchhold, SD, PRL 126, 170602 (2021)
M. Buchhold, Y. Minoguchi, A. Altland, SD, arXiv:2102.08381

- monitored fermions: new type of measurement induced phase transition



- quantum phase transition in trajectory wavefunction witnessed by state-dependent 'observables' beyond entanglement entropy

- 'hot' and 'cold' modes as relevant degrees of freedom for the transition

- physical picture: transition induced by pinning/localization into measurement operator eigenstates

Even more directions:

- area-to-volume law transitions as incomplete decoupling of 'hot' and 'cold' modes? integrability vs. non-integrability: O. Lunt, A. Pal, PRR (2020)

- relation to no-click evolutions Biella, Schiro, arxiv:2011.11620 (2020); Gopalakrishnan Gullans, arxiv:2012.01435 (2020) Turkeshi et al. arxiv:2103.09138 (2021)