Critical behavior near the many-body localization transition in driven open systems

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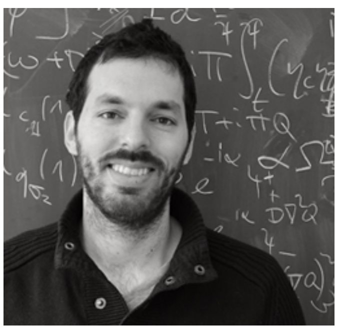
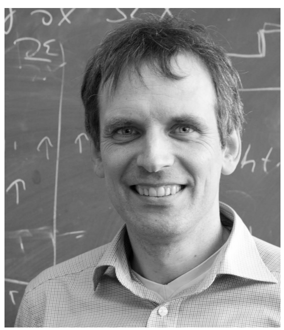
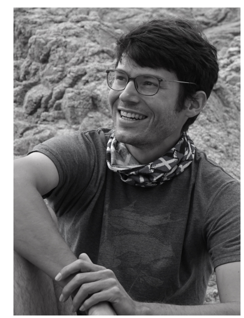


Ori Alberton



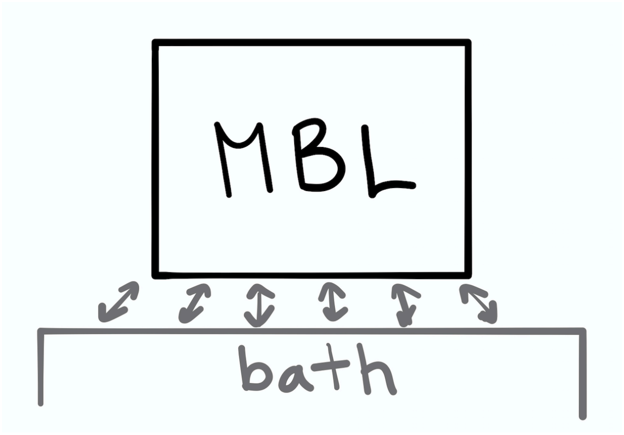
Achim Rosch

Ehud Altman



Markus Schmitt

- **Can we profit from studying MBL in open systems?**
  - Observe signatures of MBL in solid state experiment?
  - New numerical approach $\rightarrow$ go around limitations of ED



ZL, Altman, Rosch, PRL 121, 267603 (2018),
ZL, Alberton, Rosch, Altman, PRL 125, 116601 (2020)

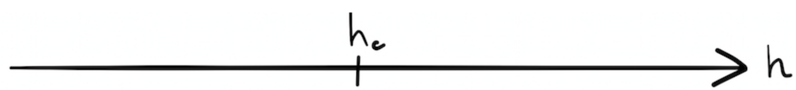
Many-body localization transition

- Many-body generalization of Anderson localization
  Basko et al '06, Gorny et al '05, Abanin et al RMP '19'

- Disordered interacting systems

$$H_{xxzV} = \sum_j V_j S_j^z + J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y) + \Delta S_j^z S_{j+1}^z, \;\; V_j \in [-h, h]$$

- Critical disorder strength h_c already in 1D

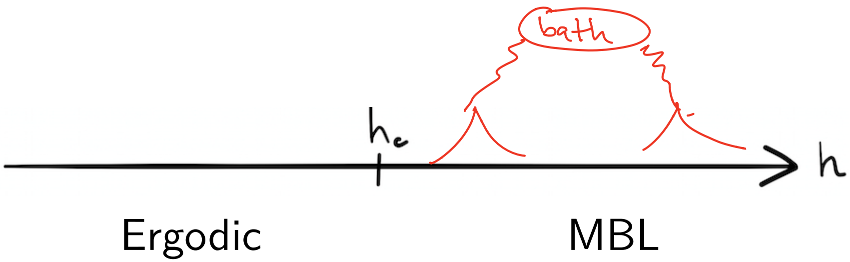


Ergodic MBL

- MBL transition: physics exactly at level spacing!

  Need for numerical approaches beyond exact-diagonalization:
  Šuntajs et al, PRE 102, 062144 (2020),
  Abanin et al, arXiv:1911.04501 (2019),
  Panda et al, EPL 128 67003 (2019),
  Sels et al, arXiv:2009.04501 (2020)

Many-body localization transition



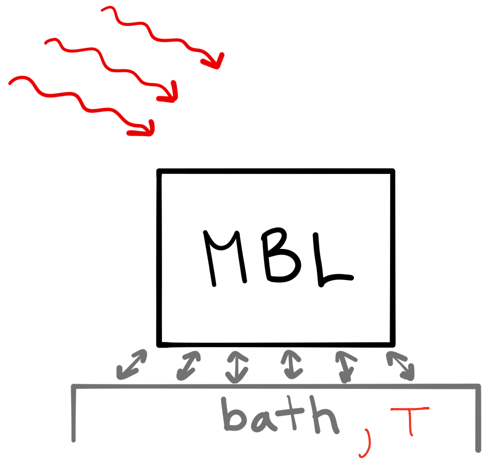
- **Ergodic**: a few conservation laws H, S^z
- **MBL**: Macroscopically many local conservation laws $[\tau_i^z, H] = 0$

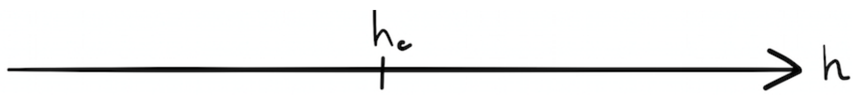
$$\tau_i^z = S_i^z + \ldots,$$
$$H = \sum_j V_j S_j^z + J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y) + \Delta S_j^z S_{j+1}^z$$

  - τ_i^z: **localized in space** $\rightarrow$ no transport

    Vosk and Altman '13, Serbyn and Abanin '13, Ros et al. '15, ...
  - **MBL lost when coupled to a bath.**

Open driven setup





Ergodic

→ only H

→ **thermal** steady-state

MBL

→ more conservation laws C_i

→ **non-thermal** steady-state

Level spacing out of question!

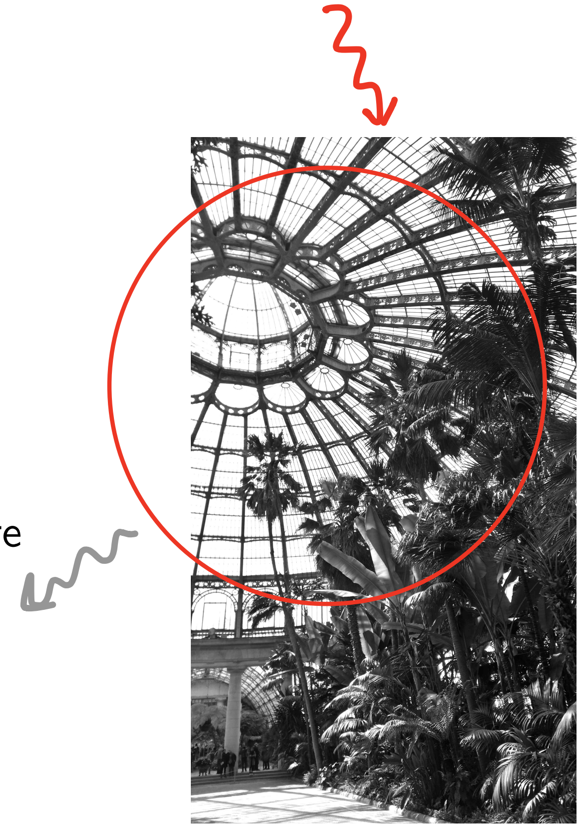
Greenhouse: thermal state

- Temperature from rate Eq.

$$\partial_t \langle H \rangle = \epsilon\theta \text{ (gain)} + \epsilon \text{ (loss)} = 0$$

- Driven setup, still can use temperature

$$\rho \sim e^{-\beta H}, \quad \beta \approx \beta\left(\frac{\epsilon\theta}{\epsilon}\right)$$



Generalized greenhouse: non-thermal state

- More conservation law, $[H, C_i] = 0$

$$\partial_t \langle C_i \rangle = \epsilon\theta \ (\text{gain}) + \epsilon \ (\text{loss}) = 0$$

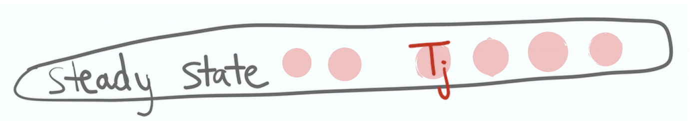
- Need additional parameters λ_i,

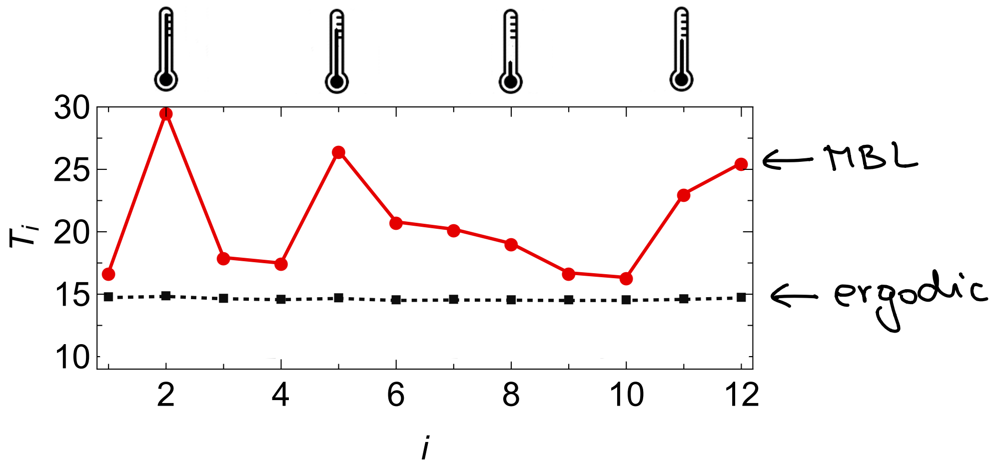
$$\rho \sim e^{-\sum_j \lambda_j C_j}, \quad \lambda_i \approx \lambda_i \left(\frac{\epsilon\theta}{\epsilon} \right)$$

Integrable systems:

Phys. Rev. B 97, 165138 (2018), Nat. Comm. 8, 15767 (2017)

Measure local temperatures

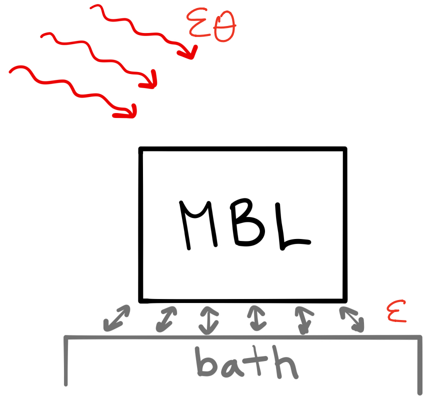




Order parameter:

$$\frac{\delta T}{\bar{T}} = \frac{\sqrt{\langle\langle \mathrm{Var}(T_i) \rangle\rangle}}{\langle\langle \mathbb{E}(T_i) \rangle\rangle}.$$

Lenarčič, Altman, Rosch, PRL 121, 267603 (2018)



Ergodic:

- only H
- **thermal** steady-state

$$\rho \sim \frac{1}{Z} e^{-\beta(\theta)H} + \delta\rho(\epsilon, \theta)$$

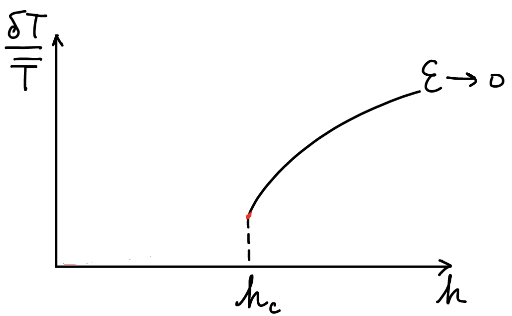
MBL:

- more conservation laws τ_i^z
- **non-thermal** steady-state

$$\rho \sim \frac{1}{Z} e^{\Sigma_i \lambda_i(\theta)\tau_i^z + \dots} + \delta\rho(\epsilon, \theta)$$

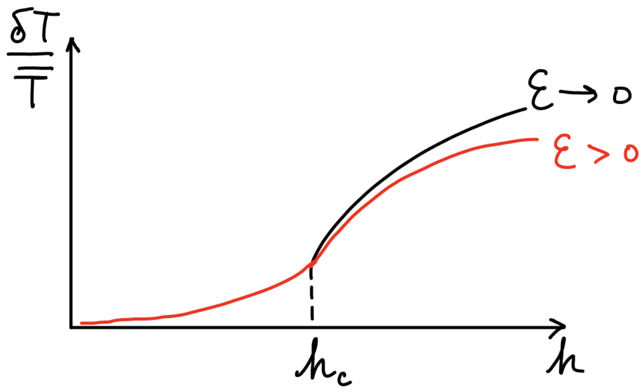
Order parameter:

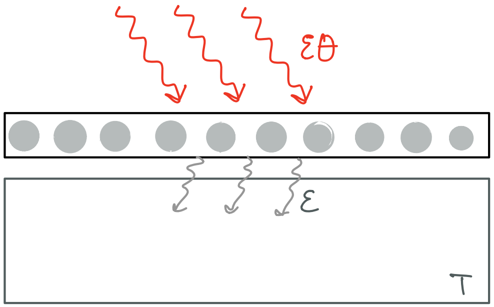
$$\frac{\delta T}{\bar{T}} = \frac{\sqrt{\langle\langle \mathrm{Var}(T_i) \rangle\rangle}}{\langle\langle \mathbb{E}(T_i) \rangle\rangle}.$$



PRL 121, 267603 (2018), PRL 125, 116601 (2020)

Finite coupling to the environment





- Ergodic phase: $T(\boldsymbol{r},\epsilon) = \bar{T} + \delta T(\boldsymbol{r},\epsilon)$.
- **Hydrodynamic theory** based on approximate energy conservation.

$$\partial_t e - \nabla(\kappa(\boldsymbol{r})\nabla T(\boldsymbol{r})) = -\epsilon g_c(\boldsymbol{r})(T(\boldsymbol{r}) - T_0) + \epsilon\theta g_d(\boldsymbol{r})$$

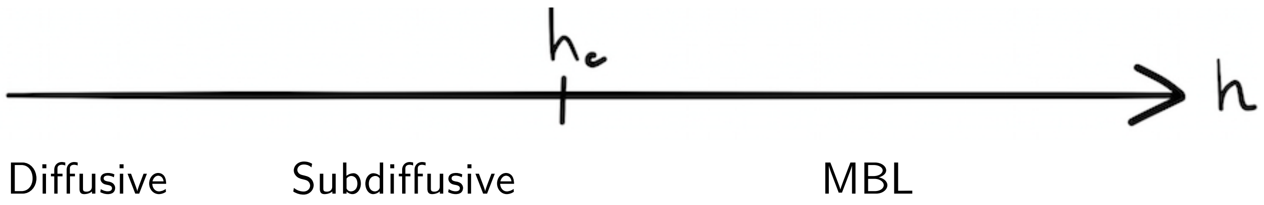
disorder sink source

- Fluctuations

$$\boxed{\delta T = \sqrt{Var(T(\boldsymbol{r}))} \sim \epsilon^{1/4}}$$

Finite coupling to the environment

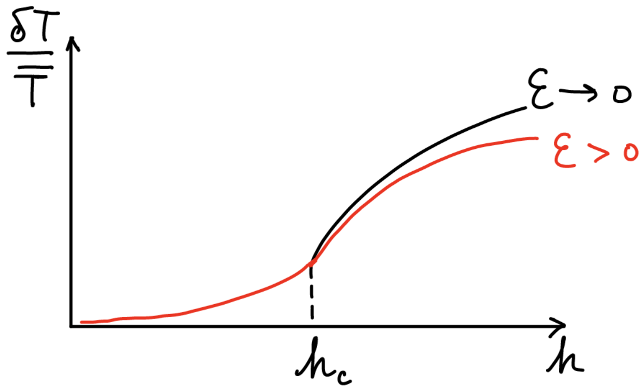
Subdiffusive transport precedes MBL

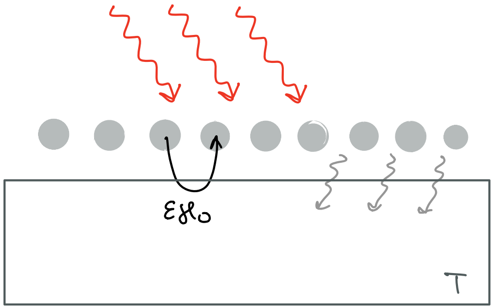


- Fractional derivative: $\nabla^2 \to \nabla^z$, z-dynamical exponent
  - $z = 2$: diffusion
  - $z > 2$: subdiffusion
- Fluctuations

$$\boxed{\delta T \sim \epsilon^{1/2z}}$$

Finite coupling to the environment: MBL side





- MBL: finite conductivity only via perturbations: $\kappa = \epsilon\kappa_0$
- **Hydrodynamic theory** based on approximate energy conservation.

$$\partial_t e - \nabla(\epsilon\kappa_0(\boldsymbol{r})\nabla T(\boldsymbol{r})) = -\epsilon g_c(\boldsymbol{r})(T(\boldsymbol{r}) - T_0) + \epsilon\theta g_d(\boldsymbol{r})$$

disorder sink source

- Fluctuations

$$\boxed{\delta T \sim \mathcal{O}(1)}$$

TEBD calculations for coupling to Markovian baths

Steady state

$$\hat{\mathcal{L}}\rho_\infty = -i[H, \rho_\infty] + \epsilon\hat{\mathcal{D}}\rho_\infty = 0$$

Dominant Hamiltonian part

$$H = \sum_i S_i \cdot S_{i+1} + h(\alpha_i^z S_i^z + \alpha_i^x S_i^x), \quad \alpha_i^{x,z} \in [-1, 1]$$

Coupling to Markovian baths (bulk)

$$\hat{\mathcal{D}} = \sum_\alpha \hat{\mathcal{D}}^{(\alpha)}, \quad \hat{\mathcal{D}}^{(\alpha)}\rho = \sum_i L_i^{(\alpha)}\rho(L_i^{(\alpha)})^\dagger - \frac{1}{2}\{(L_i^{(\alpha)})^\dagger L_i^{(\alpha)}, \rho\}$$

Lindblad operators

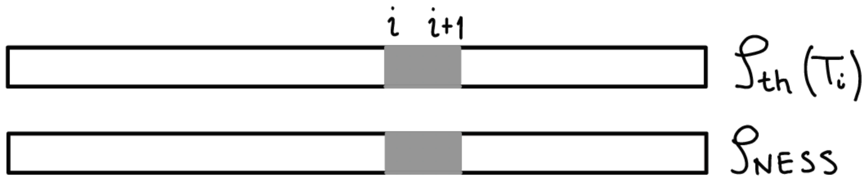
$$L_i^{(1a)} = S_i^+\left(\frac{1}{2}\mathbb{1}_{i+1} - S_{i+1}^z\right), \; L_i^{(1b)} = \left(\frac{1}{2}\mathbb{1}_i - S_i^z\right)S_{i+1}^+,$$

$$L_i^{(2a)} = S_i^-\left(\frac{1}{2}\mathbb{1}_{i+1} + S_{i+1}^z\right), \; L_i^{(2b)} = \left(\frac{1}{2}\mathbb{1}_i + S_i^z\right)S_{i+1}^-,$$

$$L_i^{(3)} = S_i^z$$

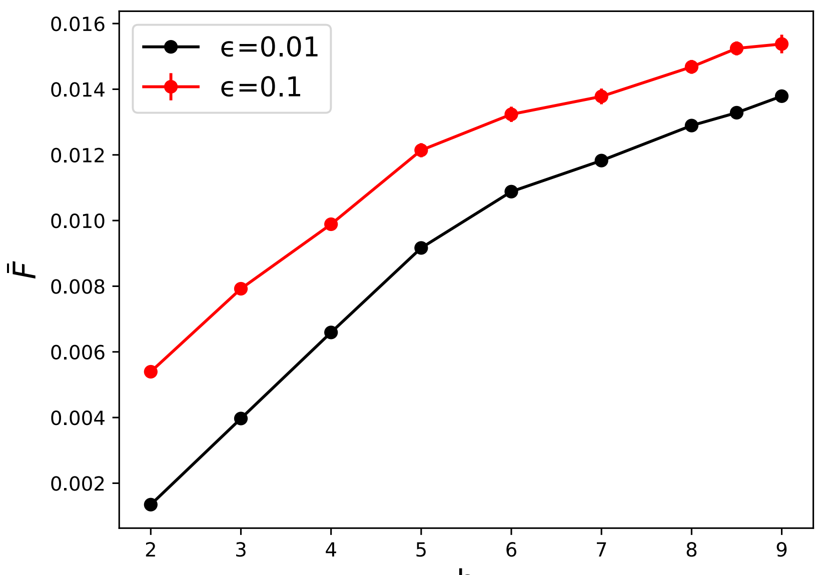
Perform TEBD time evolution

Condition for local temperature T_i



Minimize Frobenium norm with respect to T_i

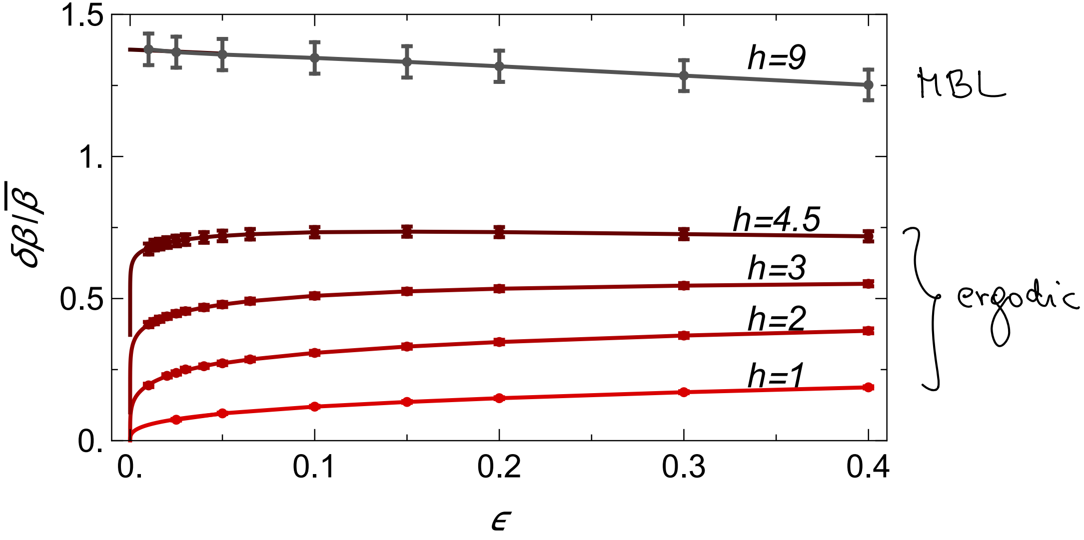
$$F[T_i] = \mathrm{tr}\left[\left(\rho_{\infty}^{(i,i+1)} - \rho_{\mathrm{th}}^{(i,i+1)}(T_i)\right)^2\right]$$



Detect the underlying transition from ϵ dependence

- Two regimes

$$\frac{\delta\beta}{\bar{\beta}}(\epsilon) \sim \begin{cases} 0 \quad + \quad \epsilon^{1/2z}, & \text{ergodic} : h < h_c, \\ \frac{\delta\beta}{\bar{\beta}}\big|_{\epsilon\to 0} - b\,\epsilon + \mathcal{O}(\epsilon^2), & \text{MBL} : h \geq h_c \end{cases}$$

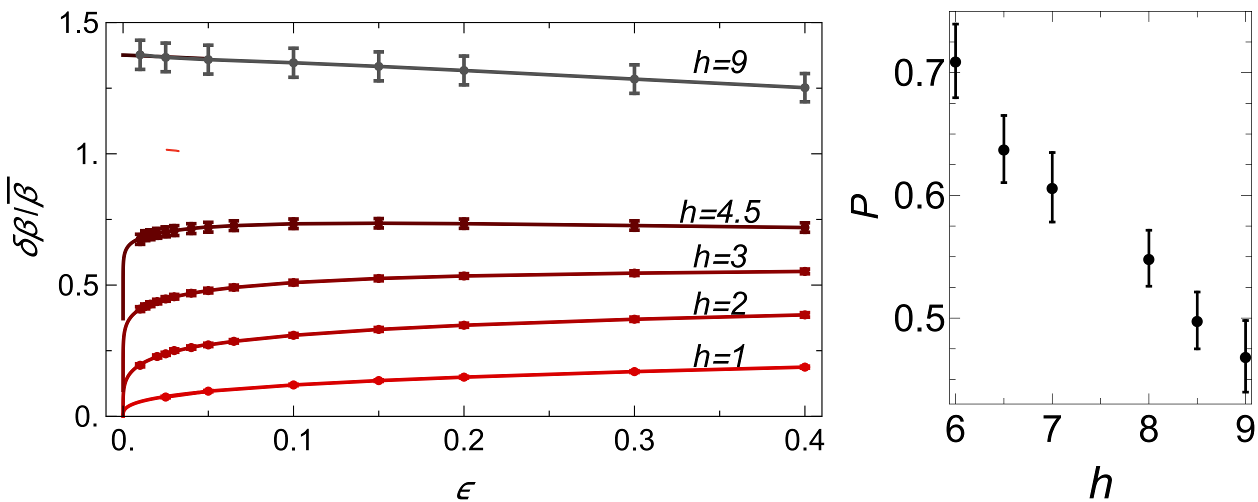


Critical disorder strength

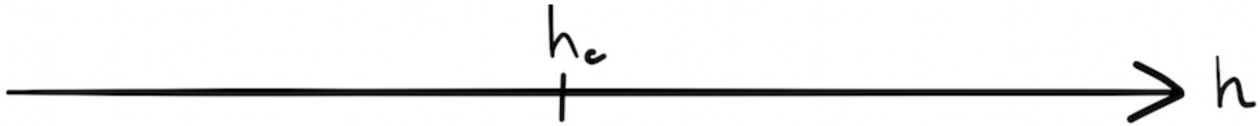
- New criterion for transition

$$\frac{\delta\beta}{\bar{\beta}}(\epsilon) \sim \begin{cases} 0 \quad + \quad \epsilon^{1/2z}, & h < h_c, \\ \frac{\delta\beta}{\bar{\beta}}\Big|_{\epsilon\to 0} - b\,\epsilon + \mathcal{O}(\epsilon^2), & h \geq h_c \end{cases}$$

- Critical disorder strength: $h_c \gtrsim 8.75 \pm 0.5$



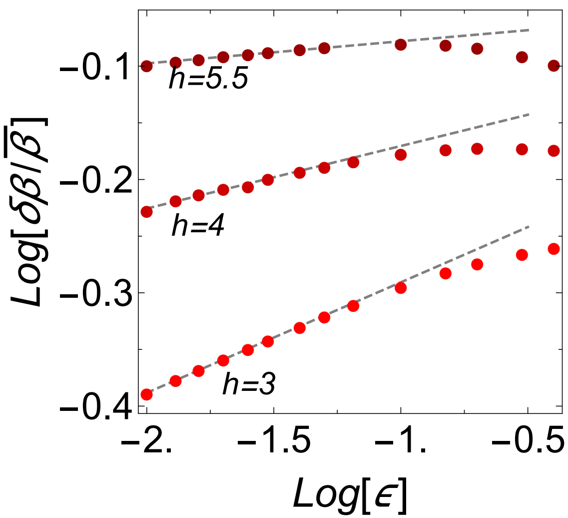
Divergence of dynamical exponent

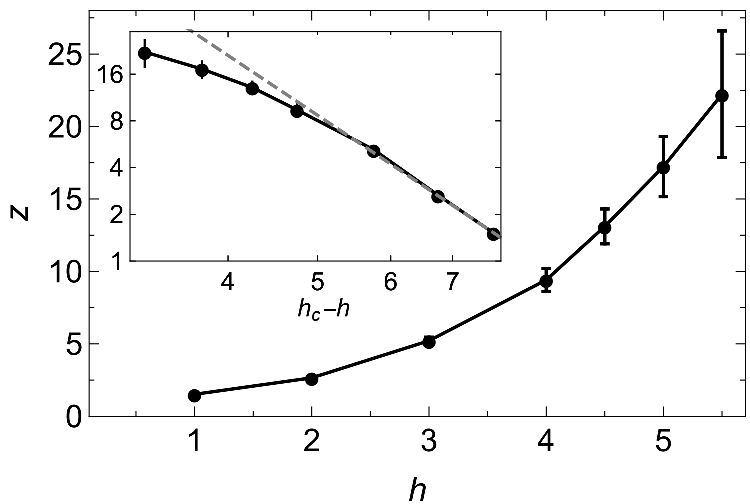


Diffusive Subdiffusive MBL

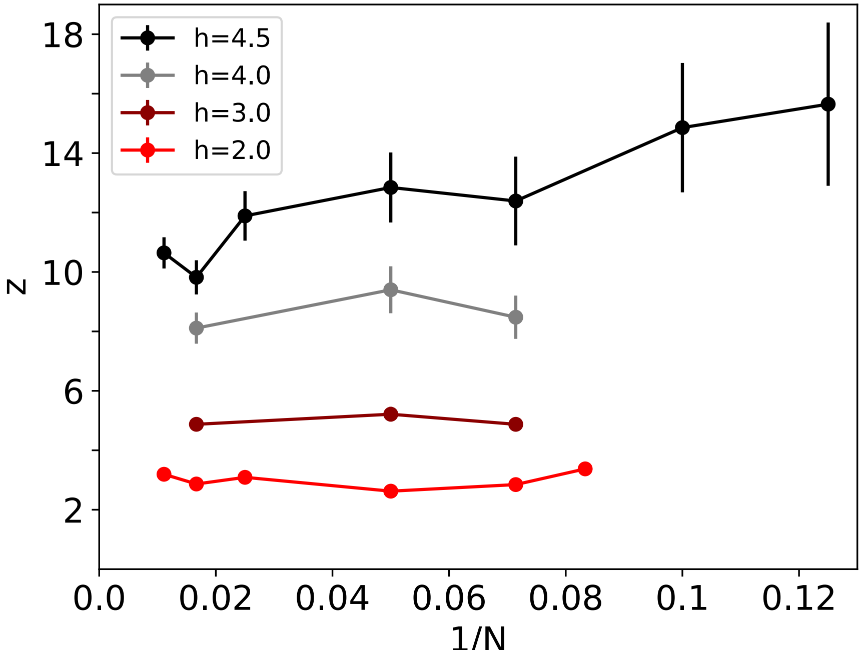
Direct bulk measurement of dynamical exponent $z(h)$

$$\frac{\delta\beta}{\bar{\beta}} \sim \epsilon^{1/2z} \quad \rightarrow \quad z \sim \xi \sim (h_c - h)^{-\nu}, \quad \nu = 4 \pm 0.9$$





System size analysis for dynamical exponent



- The main limitation is not finite system, but finite ϵ.
- Can get deeper into MBL than with boundary driving
  - M. Žnidarič et al, PRL 117, 040601 (2016), ...

Comparison with ED

Critical exponent ν

$$z \sim \xi \sim (h_c - h)^{-\nu}, \quad \nu = 4 \pm 0.9$$

- Obeys Harris bound: $\nu > 2/d$
- Exact diagonalization: $\nu \approx 1$
  Luitz, Laflorencie, Alet, PRB '16
- Cannot be distringuished from KT

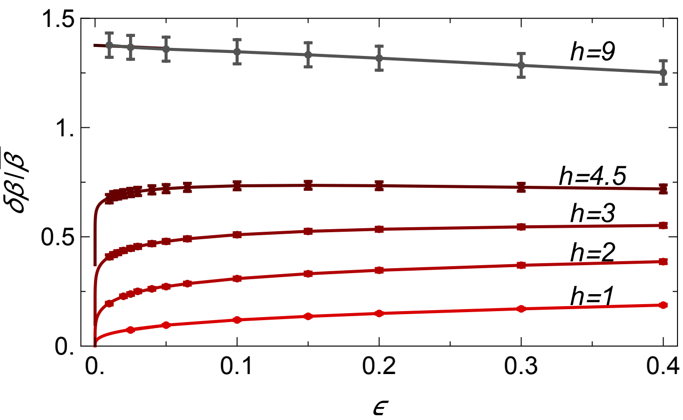
$$z \sim e^{c/\sqrt{h_c - h}}$$

  Goremykina, Vasseur, Serbyn, PRL '19
  Dumitrescu et al, PRB '19
- KT-like, $\nu \sim \ln\ln(|h - h_c|)$
  Morningstar, Huse, Imbrie, PRB '20

Critical disorder strength

$$h_c >= 8.75$$

- Larger then ED, $h_c \in [2, 7]$
  Geraedts et el, New J. Phys. '17



Experimental detection

- Solid state experiment:
  - Measure local temperatures with tip-enhanced local Raman spectroscopy

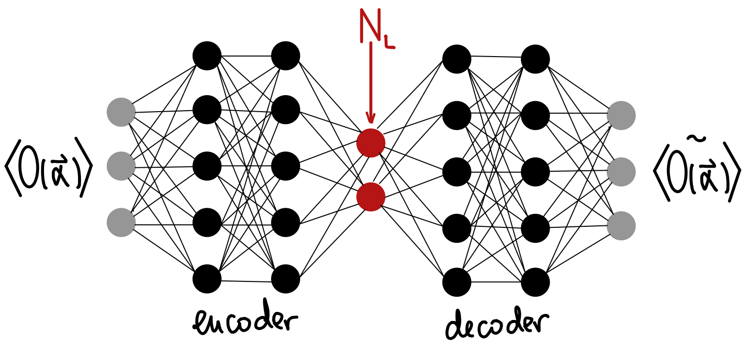
$$\frac{\delta T}{\bar{T}}(\epsilon) \sim \begin{cases} \epsilon^{1/2z}, & h < h_c, \\ \frac{\delta T}{\bar{T}}\Big|_{\epsilon\to 0} - b\,\epsilon + \mathcal{O}(\epsilon^2), & h \geq h_c \end{cases}$$

  - Measure current through disordered media

$$J(\epsilon) \sim \left(\frac{\ln \epsilon^{-1}}{\epsilon}\right)^{1/z}$$

- Quantum simulators
  - local observables $\rightarrow$ unsupervised learning

How many parameters needed → autoencoders



Input x: expectation values $\mathrm{tr}[O(\boldsymbol{\alpha})\rho]$ of local operators

$$O(\boldsymbol{\alpha}) = \sigma_1^{\alpha_1} \dots \sigma_{|\mathcal{S}|}^{\alpha_{|\mathcal{S}|}}, \qquad \boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_{|\mathcal{S}|}) \in \{0, x, y, z\}^{|\mathcal{S}|}$$

Bootleneck: N_L neurons

Output $f_\theta(x)$: network reproduction of local observables

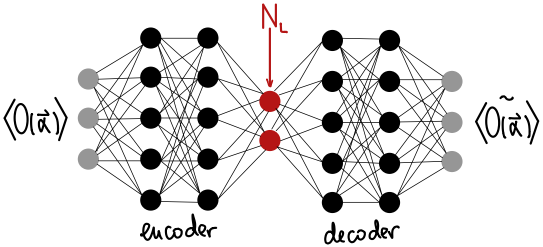
1. Initialize the network by training on a subset of realizations

$$\mathcal{L}_{\mathcal{D}_T}(\theta) = \frac{1}{|\mathcal{D}_T|} \sum_{x \in \mathcal{D}_T} \big(f_\theta(x) - x\big)^2$$

2. How well unseen $\langle O(\boldsymbol{\alpha}) \rangle$ can be reproduced by the network: Test error

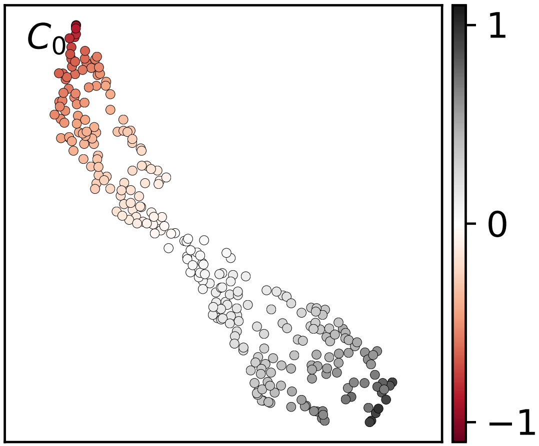
Steady states for coupling to environment

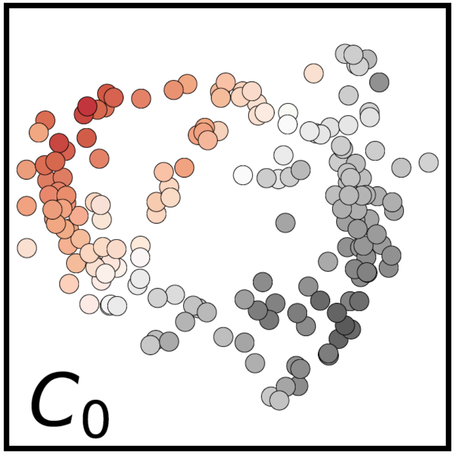
What is learned?
Do t-SNE projection of latent space



Chaotic $H \to \rho \approx \frac{1}{Z} e^{-\beta H} + \delta\rho,$

Integrable $H \to \rho \approx \frac{1}{Z} e^{-\sum_i \lambda_i C_i} + \delta\rho$

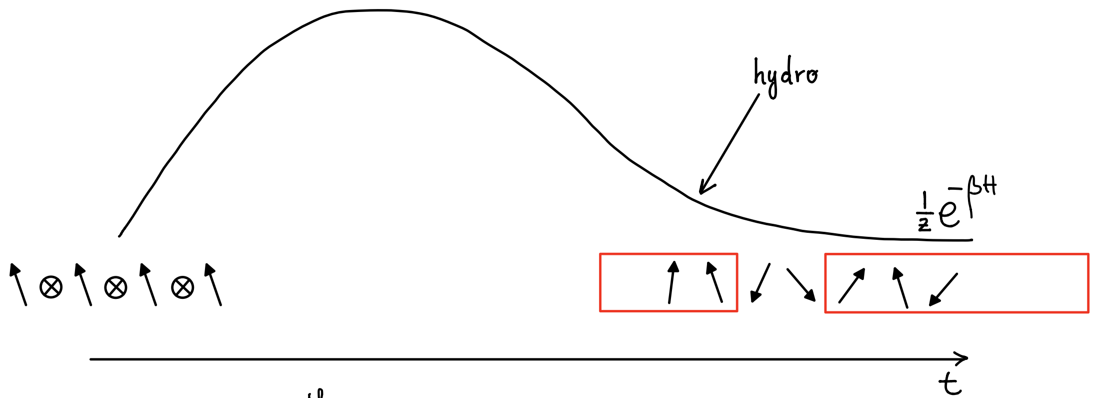


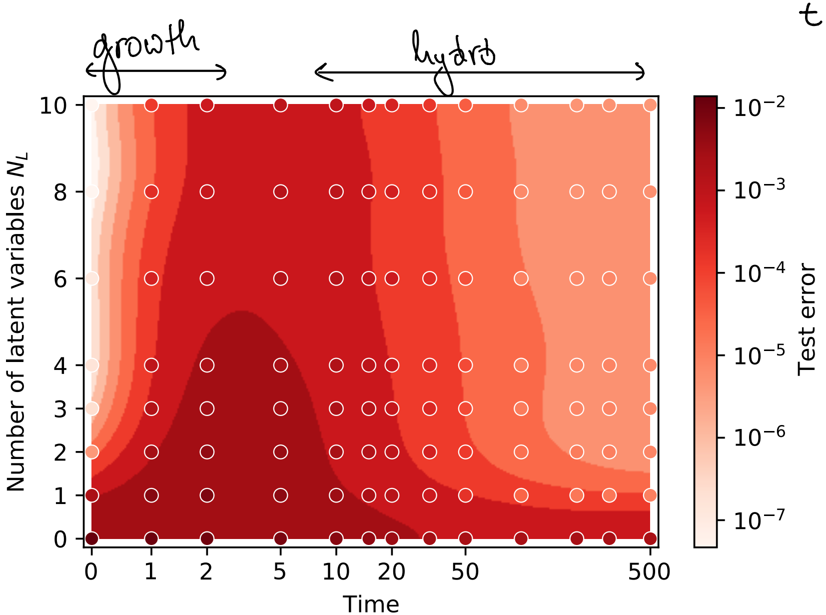


→ H reconstruction
→ noise reconstruction

→ C_i reconstruction

Complexity growth and decay

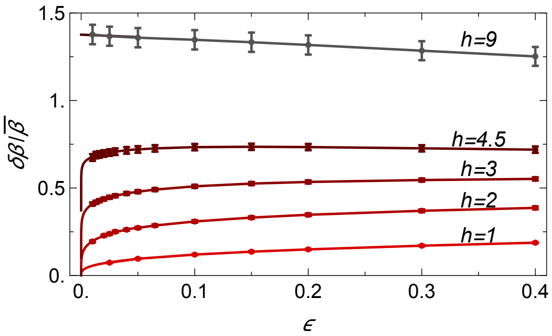




M. Schmitt and Z. Lenarčič, arXiv:2102.11328.

Conclusions

- New, experimentally relevant order parameter $\frac{\delta T}{\bar{\bar{T}}}$.
- New numerical approach, which goes beyond limitation of ED
- Measurement of dynamical exponent $z(h)$ and critical disorder



$$\frac{\delta T}{\bar{\bar{T}}}(\epsilon) \sim \begin{cases} \epsilon^{1/2z}, & h < h_c, \\ \frac{\delta T}{\bar{\bar{T}}}\Big|_{\epsilon\to 0} - b\,\epsilon + \mathcal{O}(\epsilon^2), & h \geq h_c \end{cases}$$

$\epsilon \to 0$: Lenarčič, Altman, Rosch, PRL 121, 267603 (2018),
$\epsilon > 0$: Lenarčič, Alberton, Rosch, Altman, PRL 125, 116601 (2020)
M. Schmitt and Z. Lenarčič, arXiv:2102.11328.