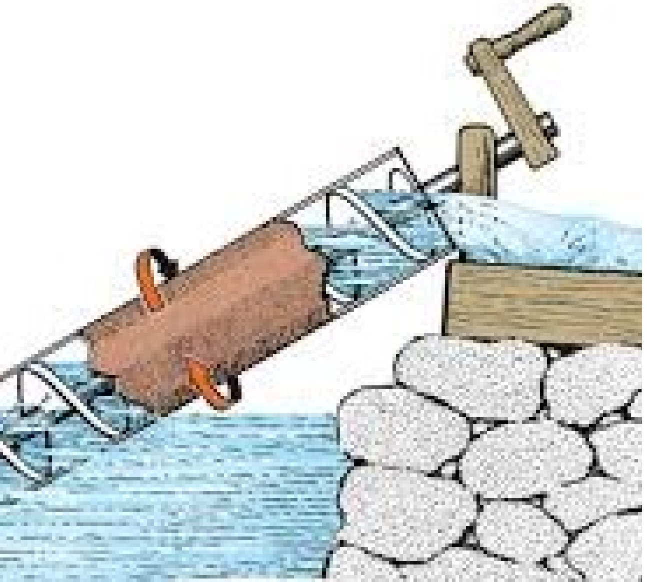
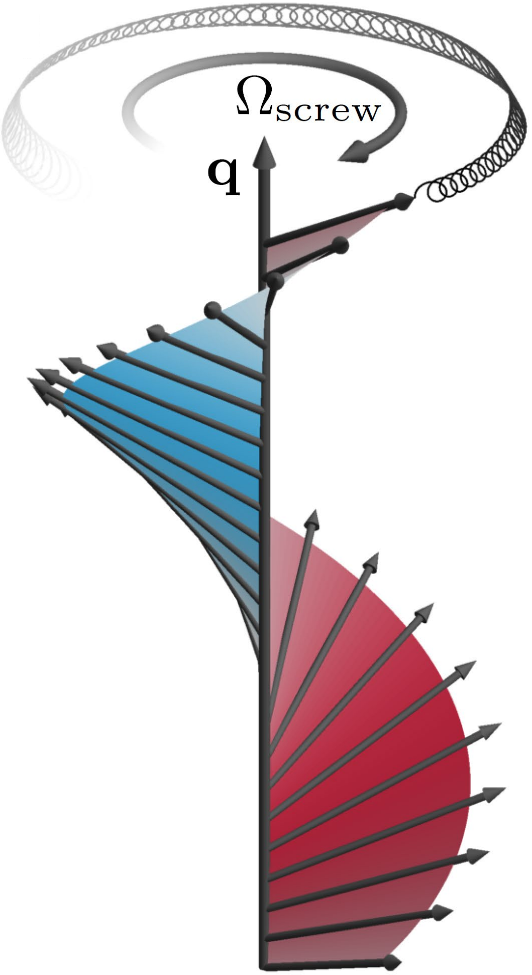
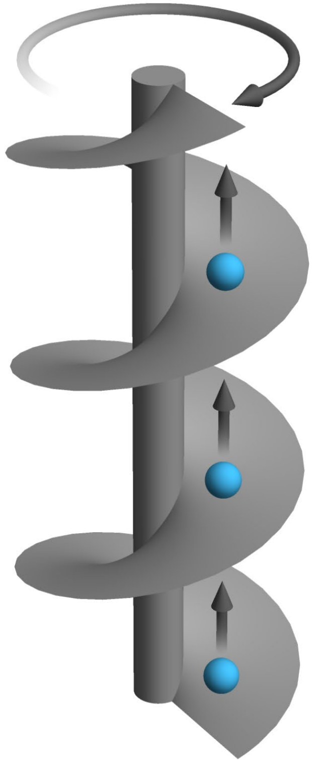
Archimedean screw and time quasi-crystals in driven chiral magnets

Nina del Ser, Lukas Heinen, Achim Rosch
University of Cologne, Germany

- driving helical magnets by GHz radiation
- Archimedean-screw like motion
- time-quasicrystals



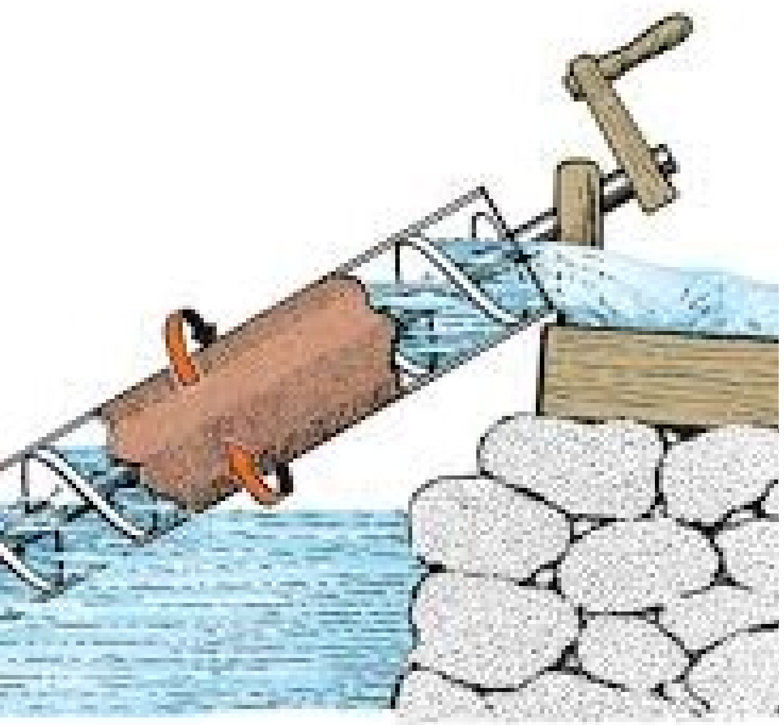


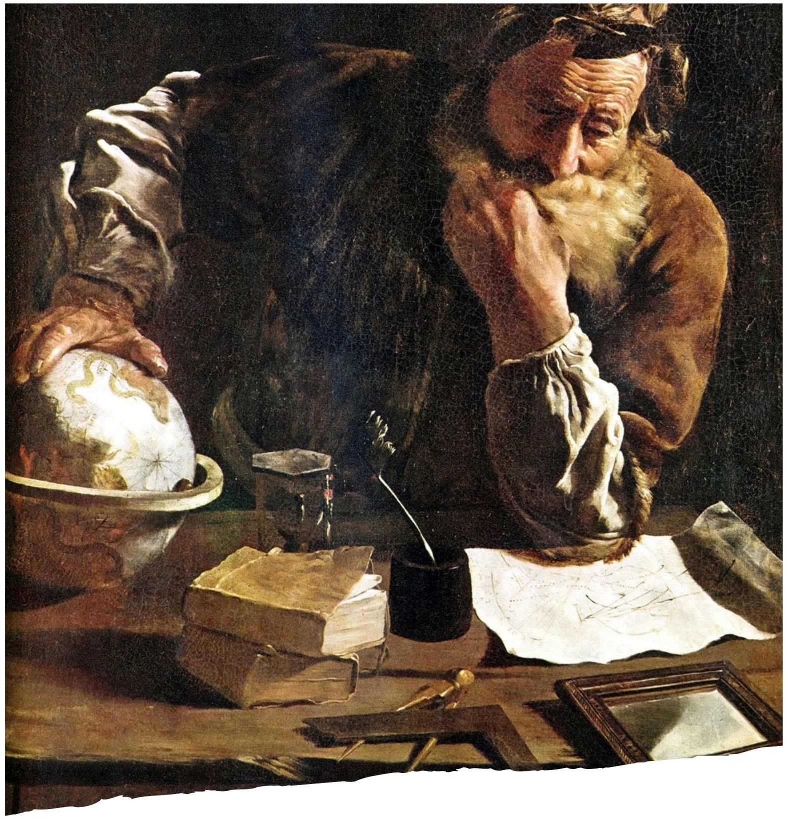


Can we build some small machines using driven quantum matter?

- to explore physical mechanisms & fundamental concepts
- to create something useful?

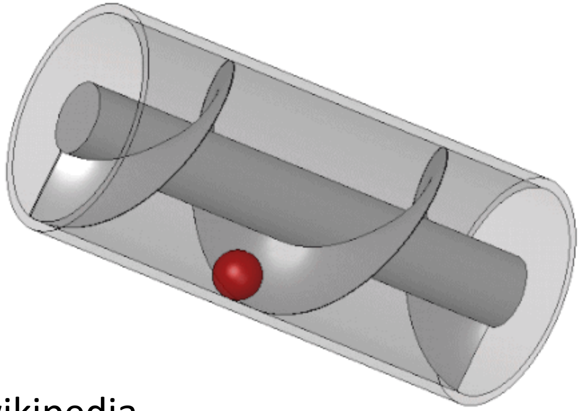
one of the oldest machines: Archimedean screw





Archimedes of Syracuse *267BC
mathematician, inventor, engineer, physicist

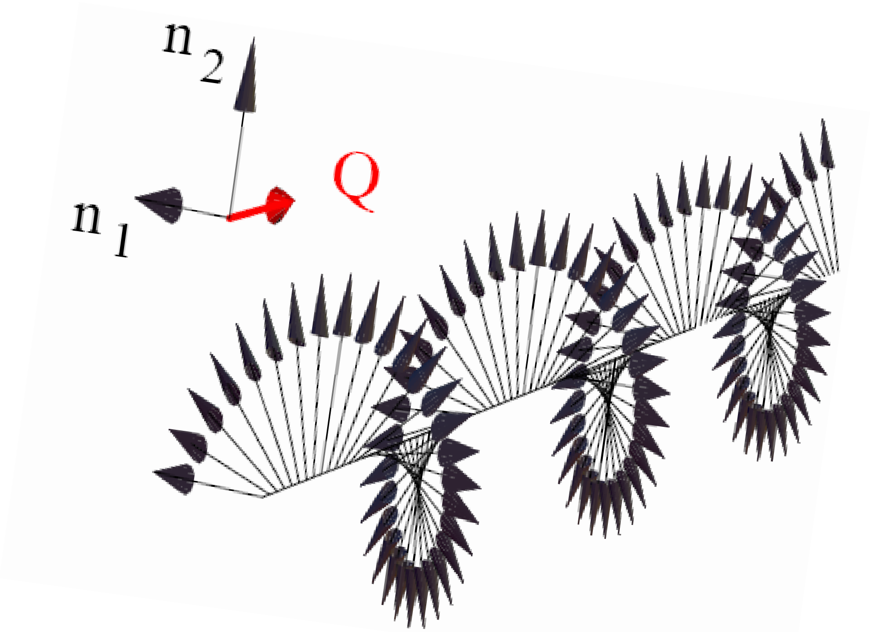
use screw to drain ships



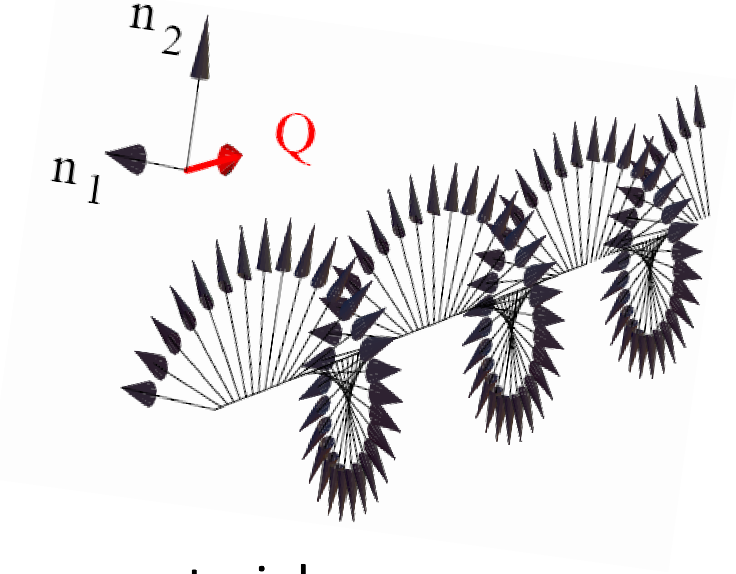
wikipedia

screw principle:
rotation implies translation

our screw:
helical (or conical) phase of chiral magnet

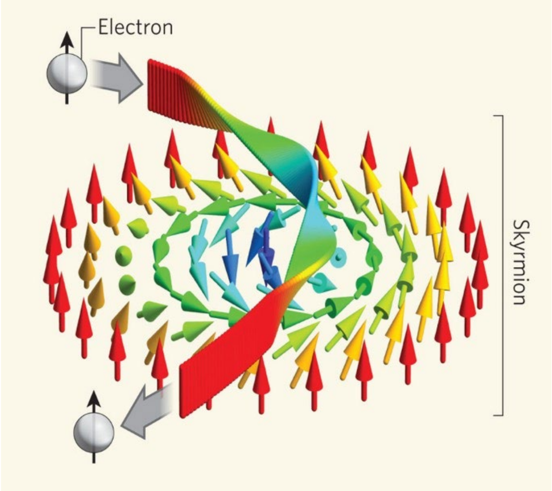


symmetry: combination of spin-rotation and translation
locked by chiral spin-orbit interaction



materials:

- **O(100) different systems** with pitch of helix ranging from nm to μm
- metals, insulators, semiconductors,…
  both at room temperatures and low temperatures
- mainly studied because of magnetic skyrmion phases
- simplest class: cubic magnets without inversion symmetry where
  DMI interactions twist ferromagnet into a helix
- most famous: MnSi (very clean metal), Cu_2OSeO_3 (insulator)



The model:

chiral magnet in its helical or conical phase (including dipolar interactions) in a **small oscillating external magnetic field**

$$\boldsymbol{B}_{\text{ext}} = \begin{pmatrix} \boldsymbol{B_\perp^x} \cos(\boldsymbol{\omega t}) \\ 0 \\ B_0^z \end{pmatrix}$$

$$F = \int \mathrm{d}^3 r \left[-\frac{J}{2} \hat{\mathbf{M}} \cdot \nabla^2 \hat{\mathbf{M}} + D \hat{\mathbf{M}} \cdot (\boldsymbol{\nabla} \times \hat{\mathbf{M}}) - \mathbf{M} \cdot \mathbf{B}_{\text{ext}} \right] + F_{\text{demag}}[\mathbf{M}]$$

$$\dot{\mathbf{M}} = \gamma \mathbf{M} \times \mathbf{B}_{\text{eff}} - \frac{\gamma}{|\gamma|} \alpha \hat{\mathbf{M}} \times \dot{\mathbf{M}}$$

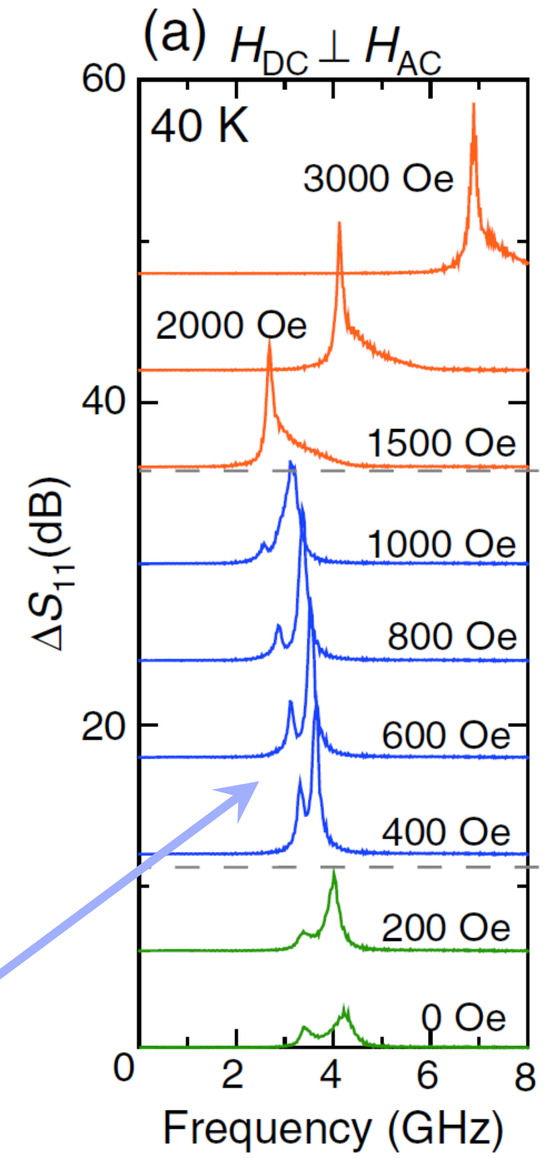
$$\boldsymbol{B}_{\text{ext}} = \begin{pmatrix} \boldsymbol{B}_\perp^x \cos(\boldsymbol{\omega t}) \\ 0 \\ B_0^z \end{pmatrix}$$

$O(B_\perp^0)$ no perturbation: conical state $\boldsymbol{M} = M_0 \begin{pmatrix} \sin\theta_0 \cos qz \\ \sin\theta_0 \sin qz \\ \cos\theta_0 \end{pmatrix}$

$O(B_\perp^1)$ linear response: oscillation at frequency $\boldsymbol{\omega}$

resonantly enhanced at k=0 magnon frequencies
measured via microwave absorption:

Onose, Okamura, Seki, Ishiwata,Tokura, PRL (2012)
Schwarze et al., Nature Materials (2015)



two resonances in
conical phase split
by dipolar interactions

$$\boldsymbol{B}_{\text{ext}} = \begin{pmatrix} \boldsymbol{B_\perp^x} \cos(\boldsymbol{\omega t}) \\ 0 \\ B_0^z \end{pmatrix}$$

$O(B_\perp^0)$ no perturbation: conical state

$$\boldsymbol{M} = M_0 \begin{pmatrix} \sin\theta_0 \cos qz \\ \sin\theta_0 \sin qz \\ \cos\theta_0 \end{pmatrix}$$

$O(B_\perp^1)$ linear response: oscillation at frequency $\boldsymbol{\omega}$

$O(B_\perp^2)$ quadratic response at frequencies 2ω and 0

pumping into the Goldstone mode

allowed by symmetries for arbitrary “perpendicular“ pumping

Analytics

expand direction of magnetization on powers of oscillating field
useful: use angles to parametrize magnetization

$$\theta = \theta_0 + \theta_1(z,t) + \theta_2(z,t) + O\left(B_{1,\perp}^3\right)$$

$$\phi = \phi_0 + \phi_1(z,t) + \phi_2(z,t) + O\left(B_{1,\perp}^3\right)$$

solve Landau-Lifshitz-Gilbert equation order by order

linear order full analytic solution possible

resonance frequencies:
(including dipolar
interactions)

$$\omega_{\mathrm{res}}^{\pm} = \frac{1}{2}\sqrt{\Big[c^2\left(\delta^2(2N_xN_y - N_x - N_y) - 4 - 4\delta\right) + (\delta+2)(\delta(N_x+N_y)+4) \pm \sqrt{\left(\left(c^2\left(\delta^2(2N_xN_y - N_x - N_y) - 4 - 4\delta\right) + (\delta+2)(\delta(N_x+N_y)+4)\right)^2 - 4\left(c^2\left(2\delta+\delta^2N_x+2\right) - (\delta+2)(\delta N_x+2)\right)\left(c^2\left(2\delta+\delta^2N_y+2\right) - (\delta+2)(\delta N_y+2)\right)\right)}\Big]}$$

Analytics

quadratic order shown below: formulas without dipolar interactions

$$\mathrm{sgn}(\gamma)\dot{\theta}_2 - \alpha(s\dot{\phi}_2 + c\theta_1\dot{\phi}_1) = -2c\theta_1'\phi_1' - c\theta_1\phi_1'' - s\phi_2'' + \phi_1\left(b_x(t)\cos(z) + b_y(t)\sin(z)\right) \tag{7}$$

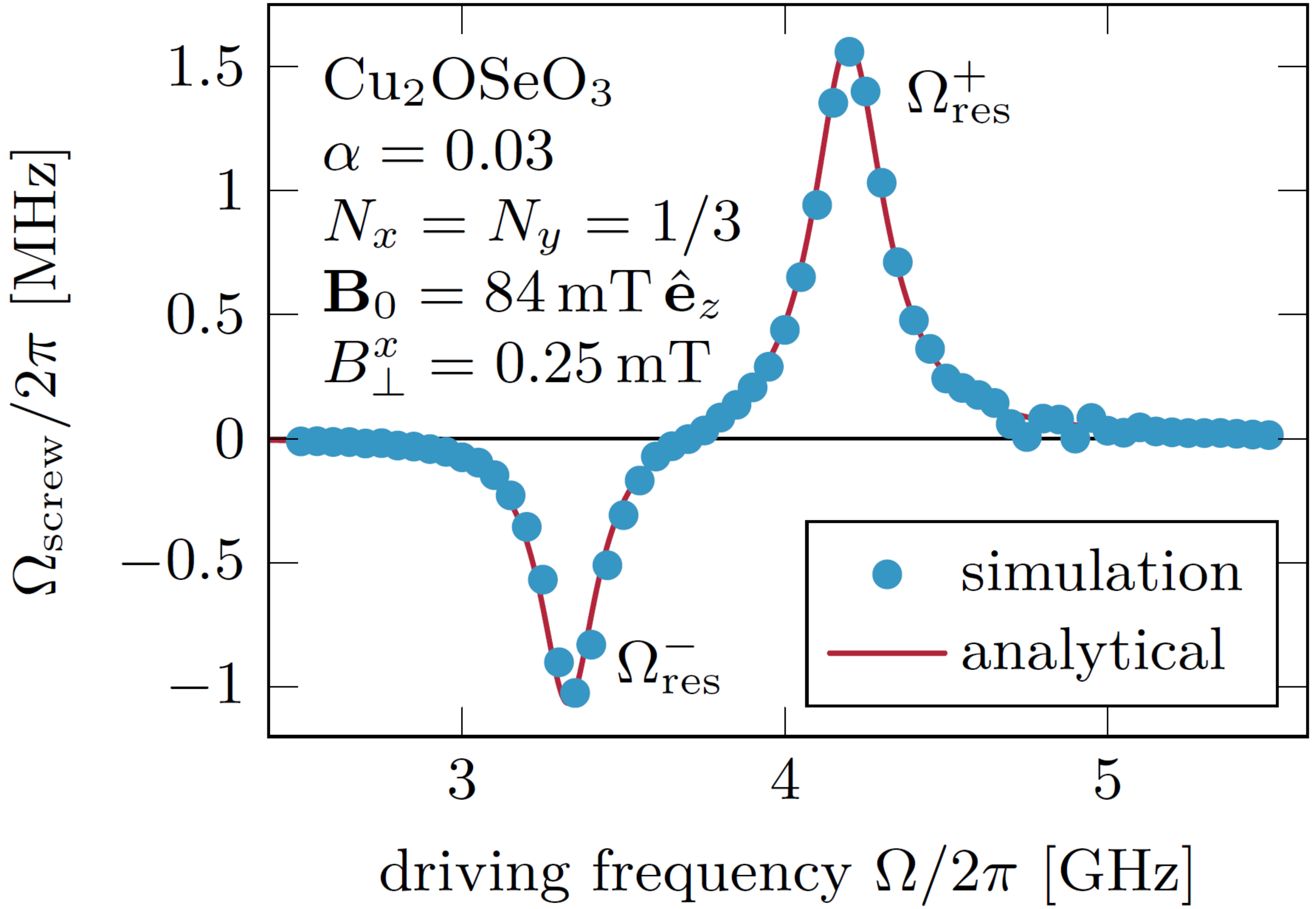
$$\begin{aligned}\mathrm{sgn}(\gamma)s(2c\theta_1\dot{\phi}_1 + s\dot{\phi}_2) + \alpha(c\theta_1\dot{\theta}_1 + s\dot{\theta}_2) =& s\theta_2'' + c\theta_1\theta_1'' - s^2c\phi_1'^2 - \frac{5}{2}cs^2\theta_1^2 - s^3\theta_2 \\ &+ (c^2 - s^2)\theta_1\left[b_x(t)\cos(z) + b_y(t)\sin(z)\right] + sc\phi_1\left[b_y(t)\cos(z) + b_x(t)\sin(z)\right]\end{aligned}$$

Fourier-transformation: equations do **not** have a solution at $\omega = 0$, $k=0$

solution:

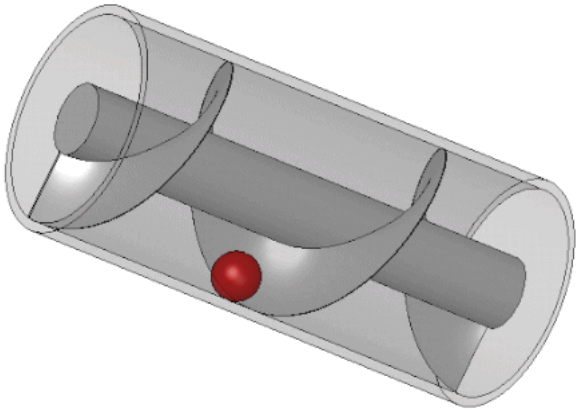
$$\phi_2(t) = \Omega_{\mathrm{screw}} t + \ldots \qquad \Omega_{\mathrm{screw}} \propto B_\perp^2$$

angle grows linear in time: **screw-like motion**
rotations & translation of helix with constant (angular) velocity



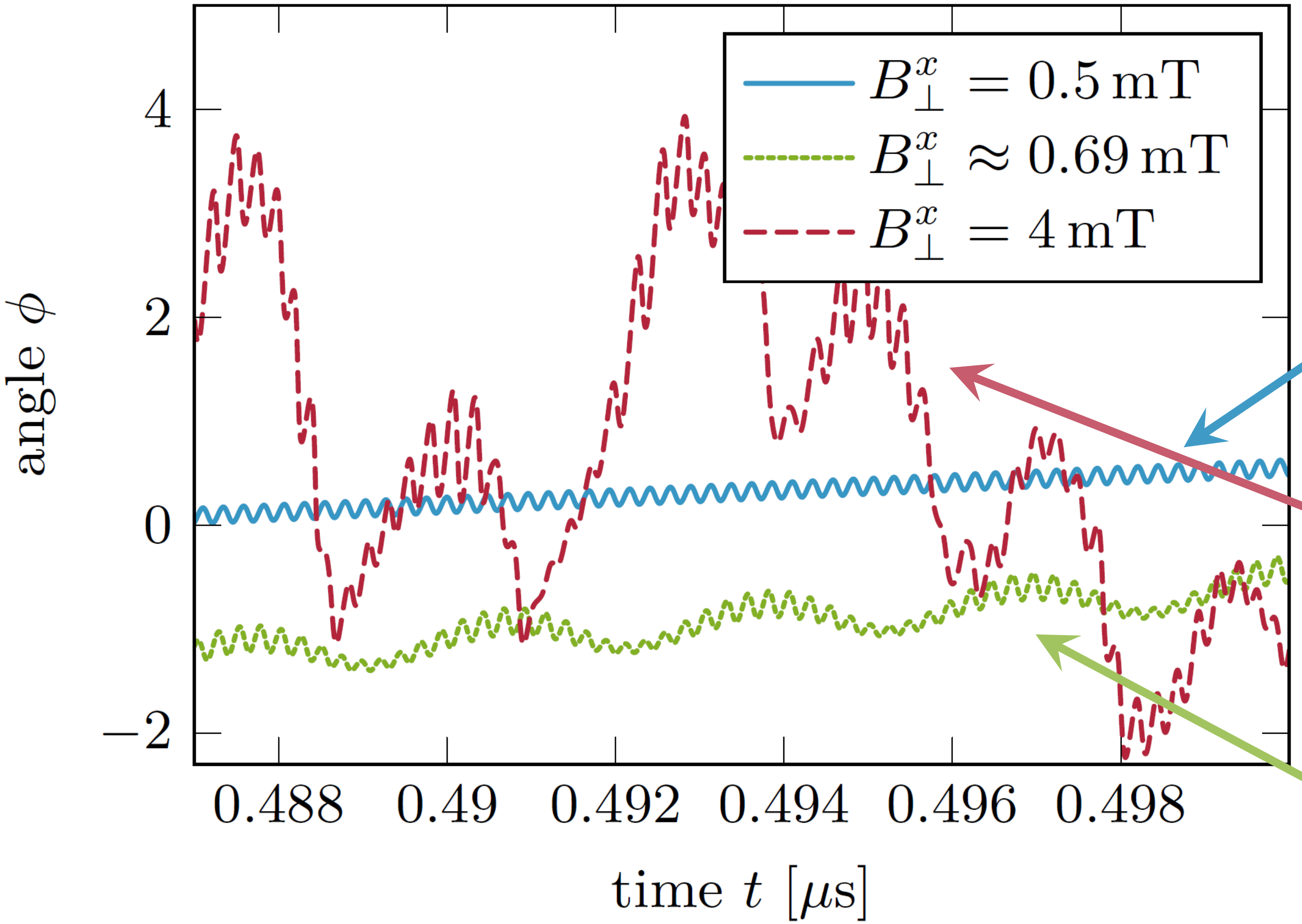
parameters: Cu_2OSeO_3 (α too large)

Archimedean screw solution found



stability?

numerics: track a single spin in a large unit cell

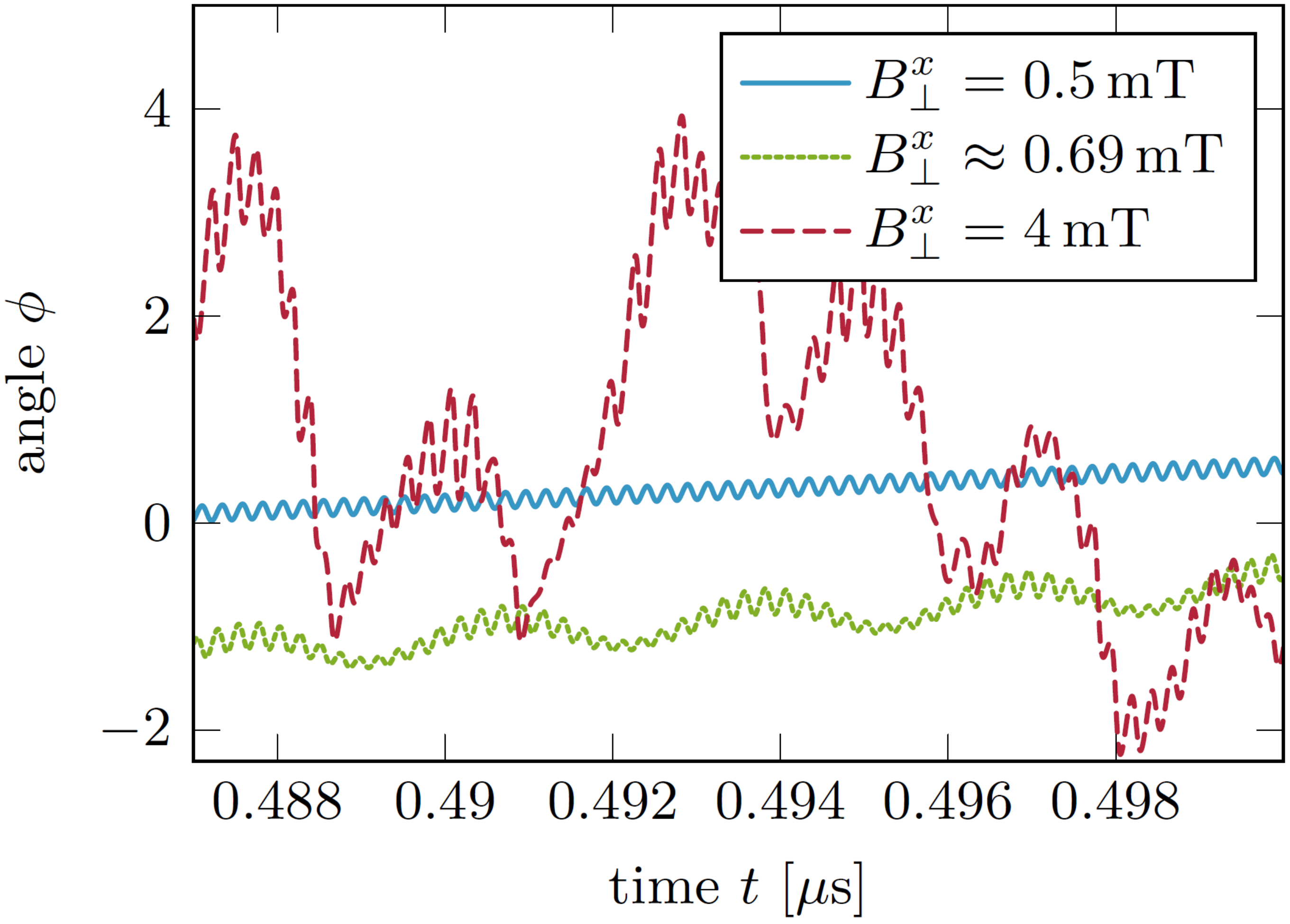


Archimedean screw solution

chaotic solution

extra period (space & time)
time-quasi-crystal forms spontaneously

numerics: track a single spin in a large unit cell



numerics alone: **not reliable**

depends strongly on size of simulated region

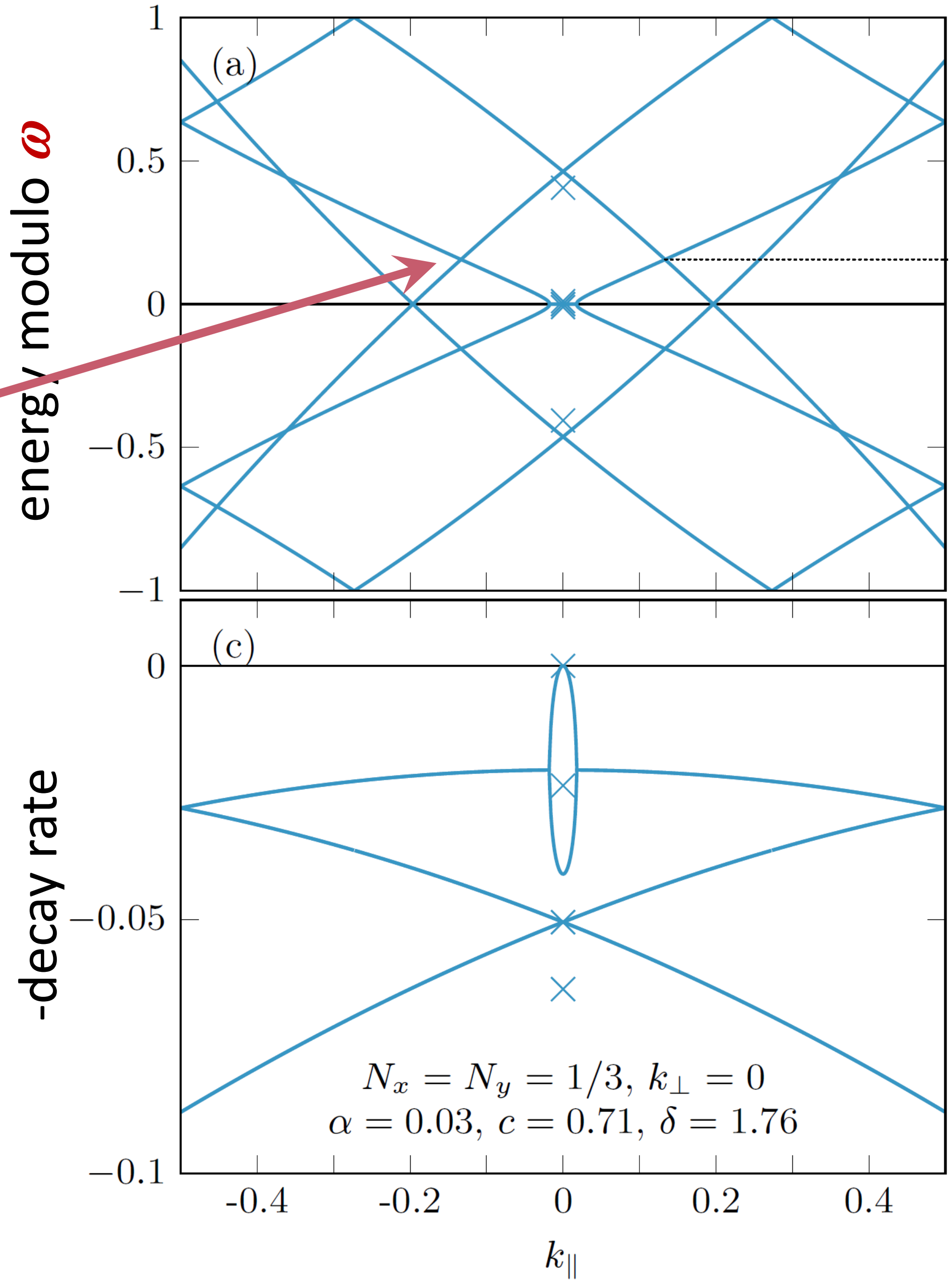
Stability of Archimedean screw: analytics

Bogolioubov-Floquet spin-wave theory
including damping terms

unavoidable:
crossing points describing **resonant creation of a magnon-pair**

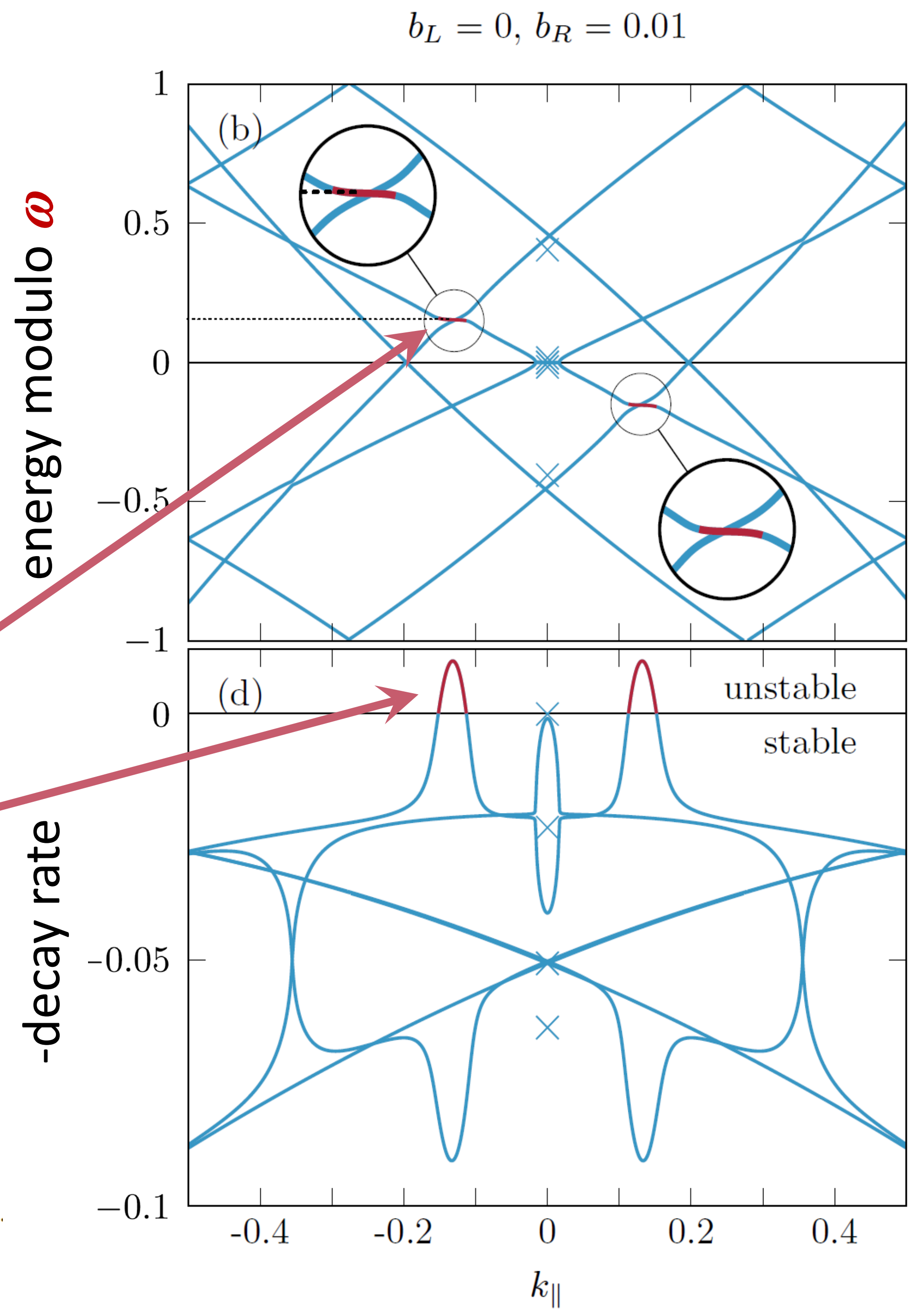
$$\epsilon_{i,\boldsymbol{k}} + \epsilon_{j,-\boldsymbol{k}} = \boldsymbol{\omega}$$

unperturbed system



Stability of Archimedean screw: analytics

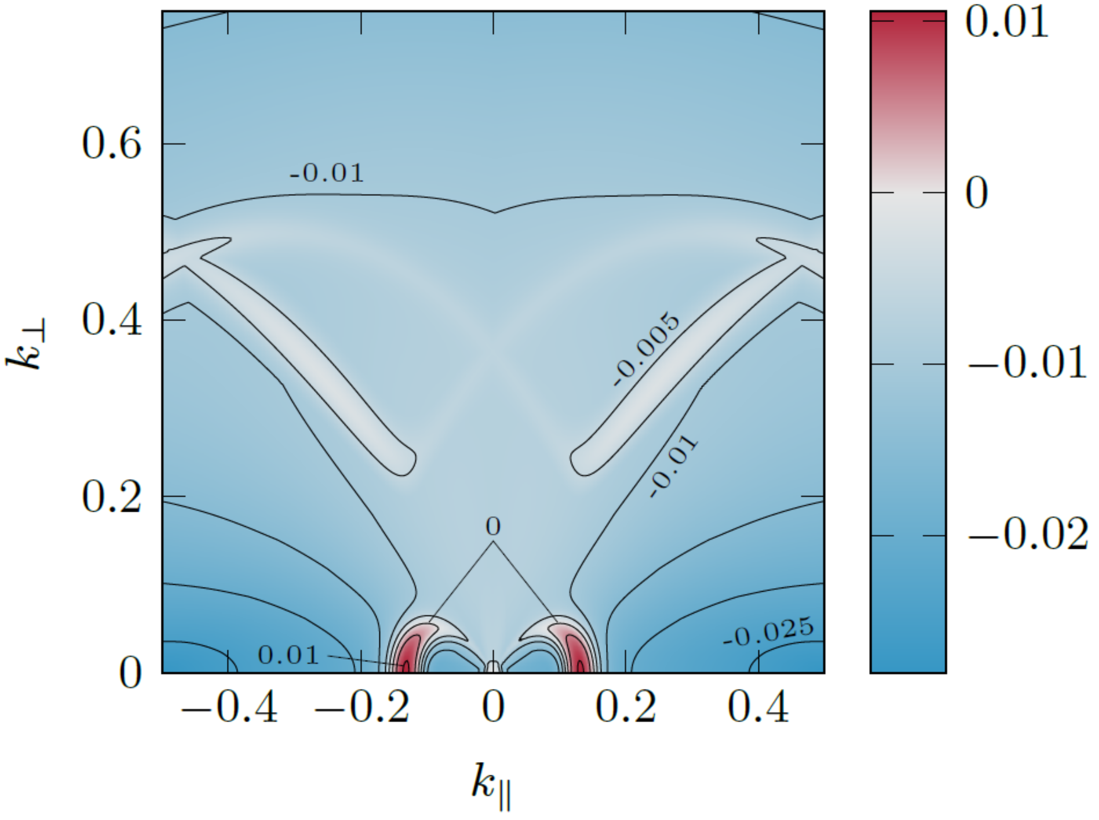
Bogolioubov-Floquet spin-wave theory including damping terms



instability when driving sufficiently large close to resonant condition

$$\epsilon_{i,\boldsymbol{k}} + \epsilon_{j,-\boldsymbol{k}} = \omega$$

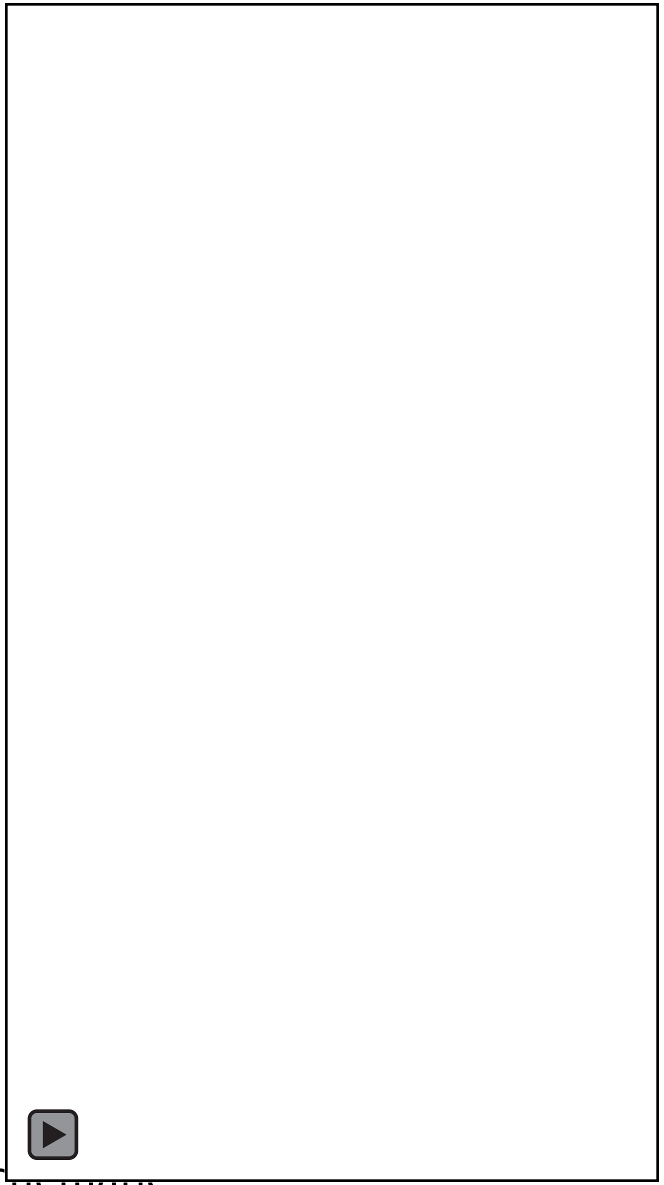
$$\lambda^{\pm}_{\mathrm{res}} = \epsilon^{0}_{i,\mathbf{k}} - i\alpha\frac{\Gamma_1+\Gamma_2}{2} \pm i\sqrt{\mu^{(1)}_{\omega}\mu^{(2)}_{\omega} + \alpha^2\left(\frac{\Gamma_1-\Gamma_2}{2}\right)^2}.$$


0.6
0.4
0.2
0
$k_\perp$
−0.4
−0.2
0
0.2
0.4
$k_\parallel$
-0.01
-0.005
-0.01
0
0.01
-0.025
0.01
0
−0.01
−0.02

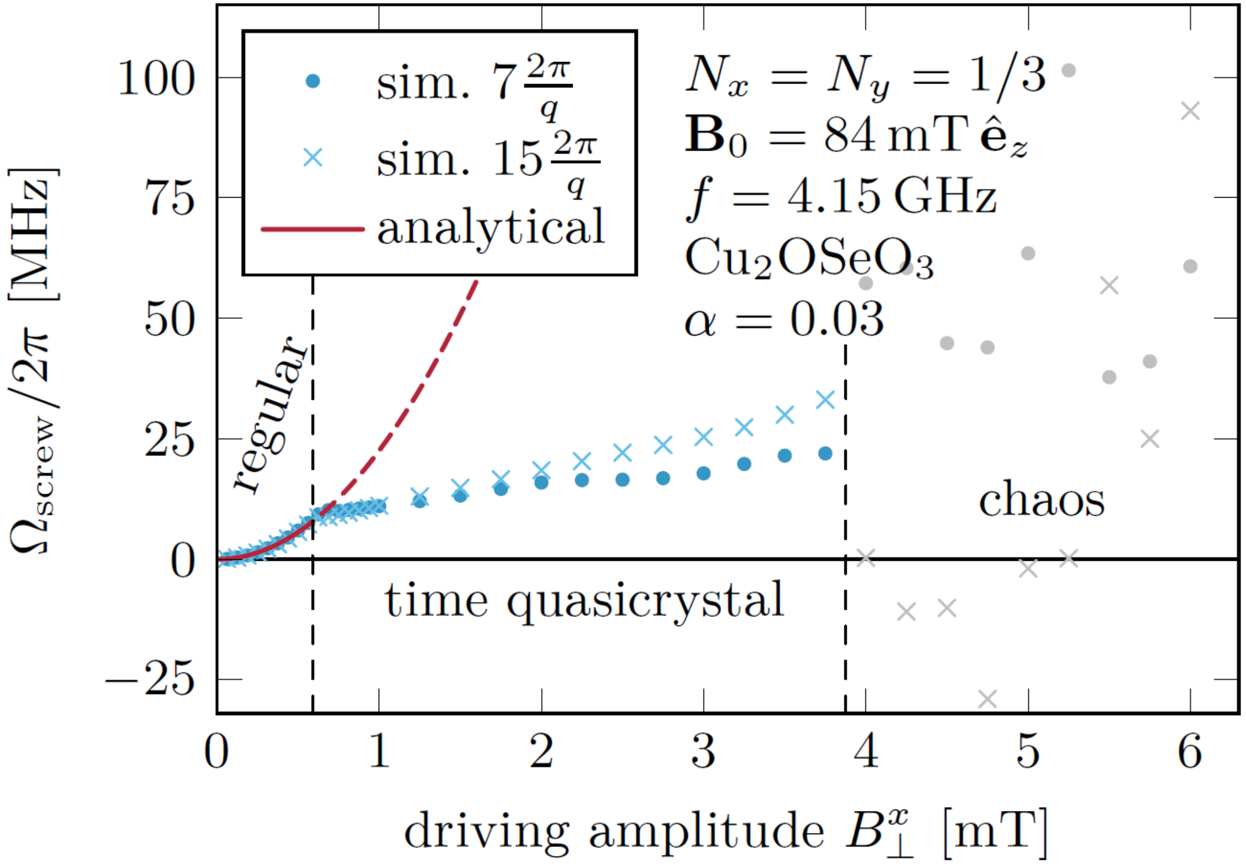
gain

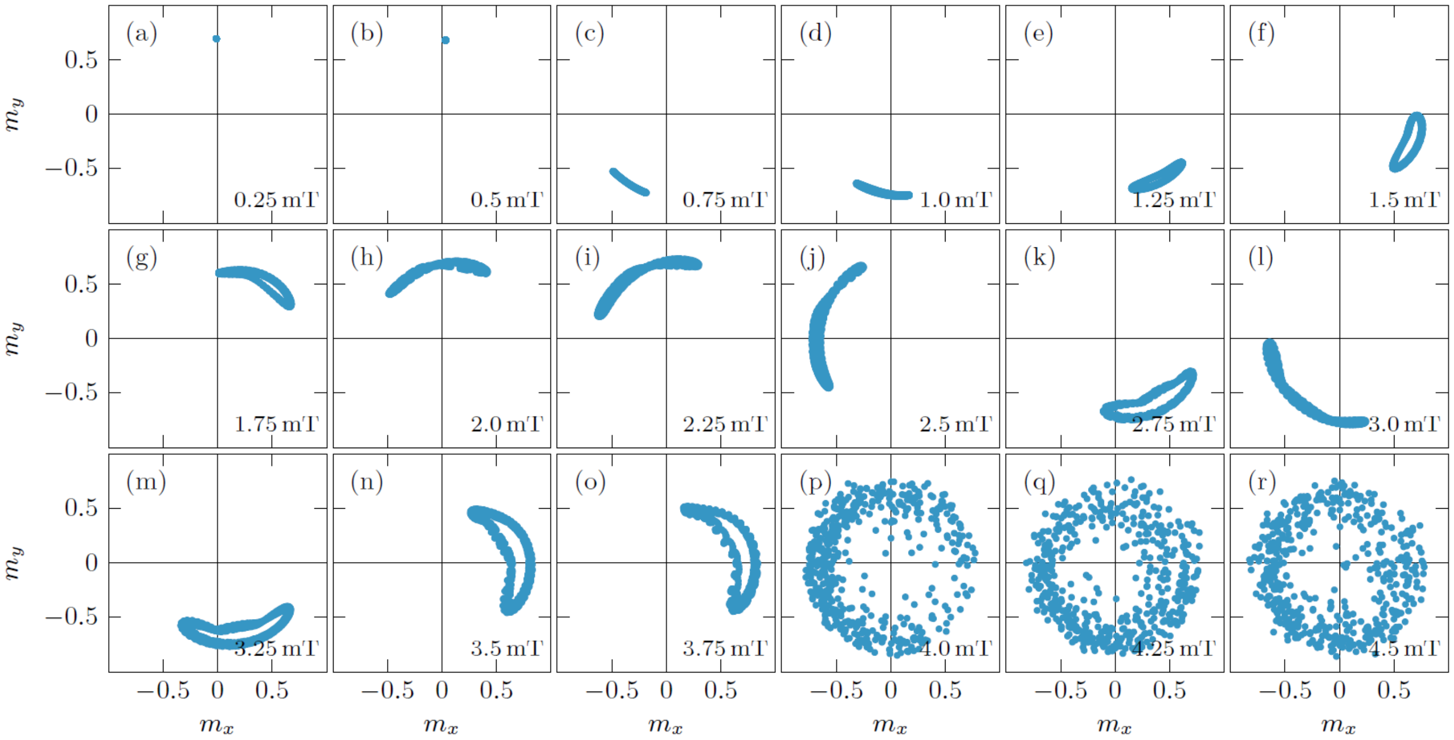
loss

magnon laser = time quasi crystal



schematic





stroboscopic time evolution

generic properties of magnonic systems driven by GHz/THz B-fields

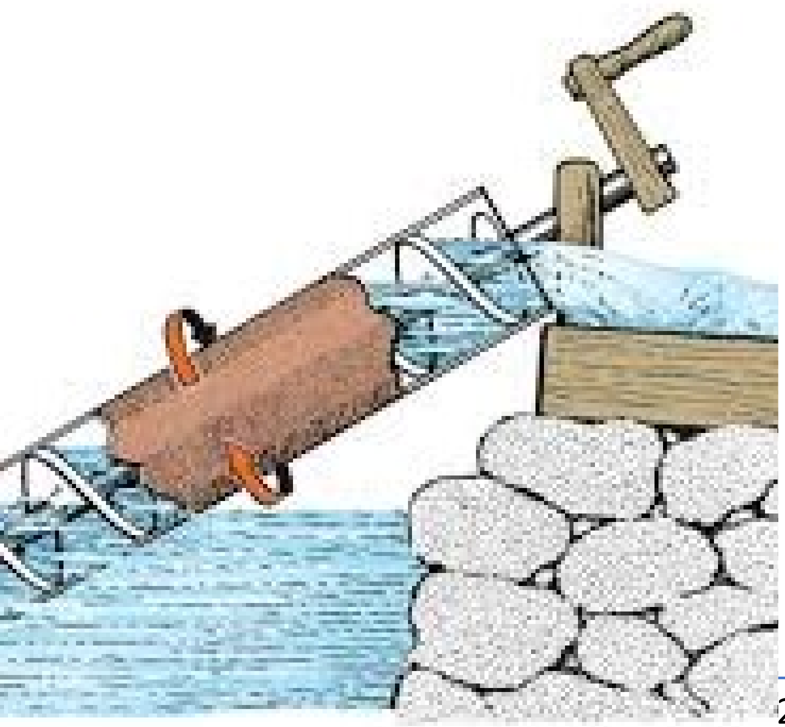
- for incommensurate magnetic order & if symmetry-allowed: translational **Goldstone mode activated** for arbitrariy weak driving (in absence of pinning) $\Omega_{\text{screw}} \propto B_\perp^2$
- next leading instability: **resonant creation of magnon pairs** $\epsilon_{i,\boldsymbol{k}_0} + \epsilon_{j,-\boldsymbol{k}_0} = \omega$
- **stabilization** of periodically driven phases only due **to magnon damping** (extrinsic or due to magnon-magnon interactions)
- consequence of secondary instability:

magnon laser = oscillating texture with momentum $\boldsymbol{k}_0$ and frequency $\epsilon_{i,\boldsymbol{k}_0}$
= **time quasi crystal**

back to Archimedes

Can we pump something ?
charge – heat – spin

now: charge pump



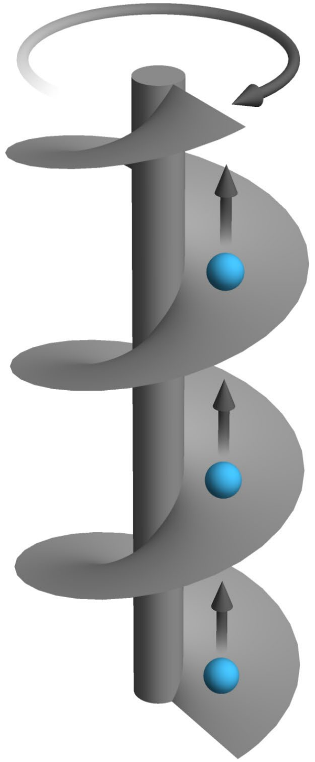
kinetic energy

$$H = \int \boldsymbol{c}^\dagger \left(\frac{p^2}{2m} + \lambda\, \boldsymbol{p} \cdot \boldsymbol{\sigma} + J_H\, (\boldsymbol{n}(\boldsymbol{r}, t) \cdot \sigma) + V_{\text{dis}}(\boldsymbol{r}) \right) \boldsymbol{c} + H_{\text{int}}$$

spin-orbit coupling
(needed !!)

disorder potential

Archimedean screw coupled by exchange field



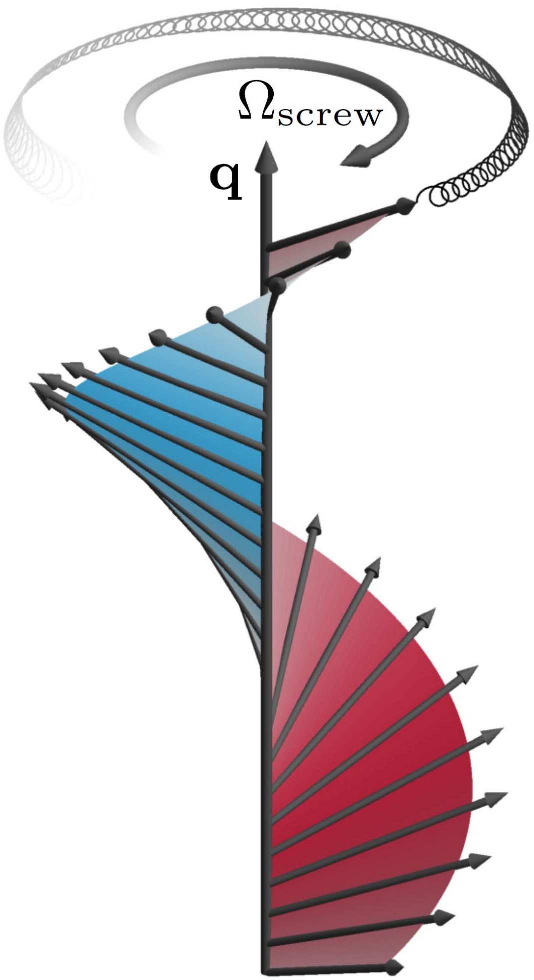
clean limit

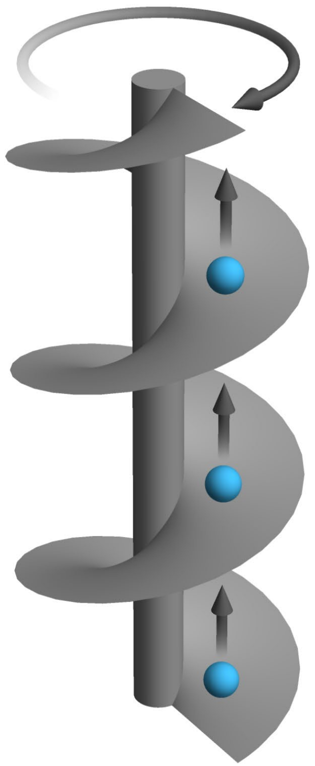
no disorder, no Umklapp scattering from atomic lattice

transformation to comoving coordinate system

$$\boldsymbol{r} \to \boldsymbol{r} - \boldsymbol{v}_{\text{screw}} t, \qquad \boldsymbol{v}_{\text{screw}} = \Omega_{\text{screw}} \lambda_{\text{helix}}$$

➡ electric current density $j = e n_e v_{\text{screw}}$
highly efficient pump (later)





dirty limit

different transformation: spin-quantization axis parallel to local magnetization

$$\tilde{H} \approx \sum_{\sigma,\mathbf{k}} \epsilon_{\sigma,\mathbf{k}} d^\dagger_{\sigma,\mathbf{k}} d_{\sigma,\mathbf{k}} + H_1(t) + H_{\text{dis}}$$

$$H_1(t) = \sum_{\sigma,\mathbf{k}} \frac{\hbar s k_\perp \lambda}{2} (d^\dagger_{\sigma,\mathbf{k}} d_{\sigma,\mathbf{k}+\mathbf{q}} e^{-i\omega_{\text{screw}} t} + h.c.)$$

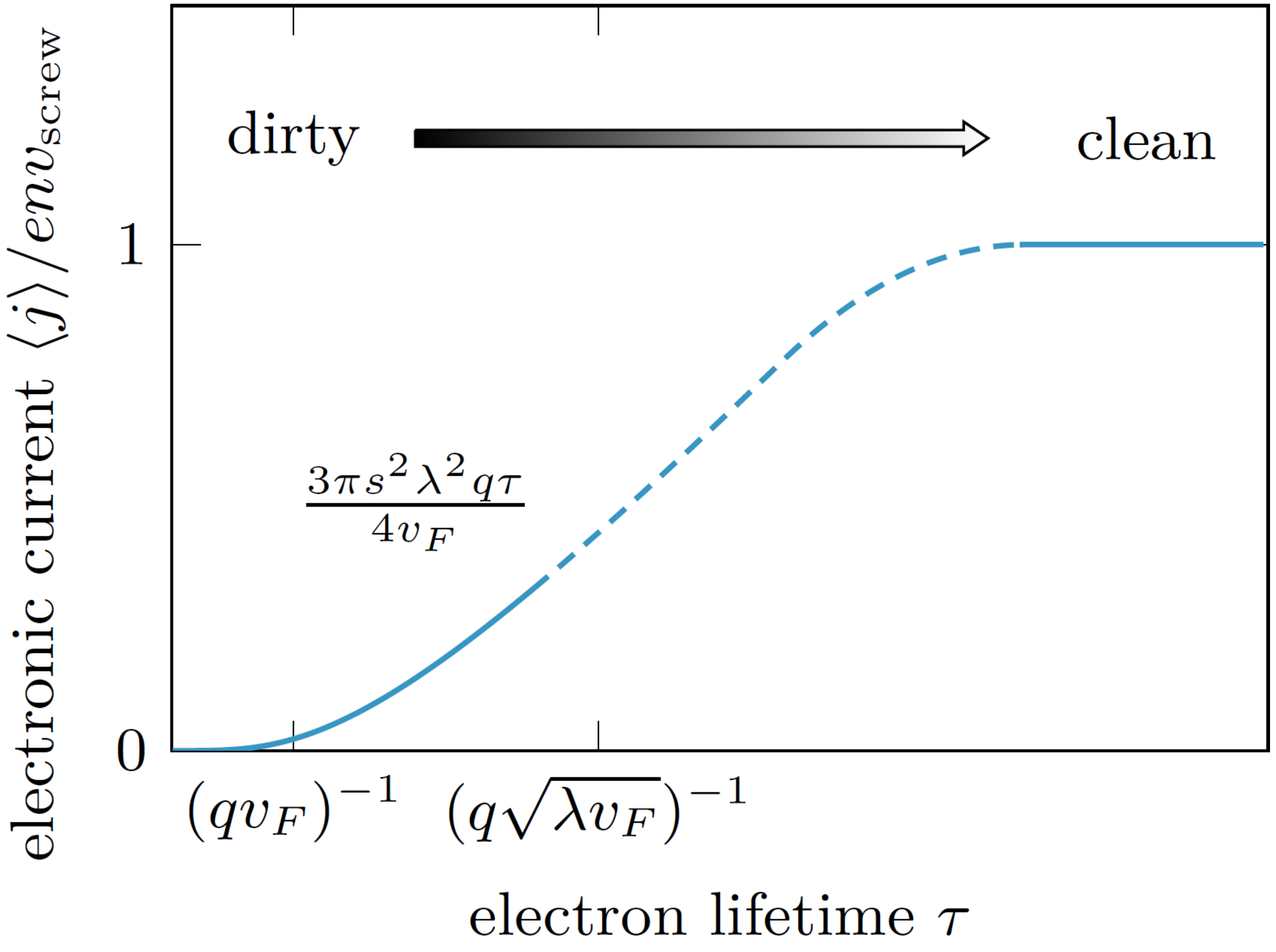
driving only due to spin-orbit interactions

$$\epsilon_{\uparrow/\downarrow,\mathbf{k}} \approx \frac{\hbar^2}{2m} \left((k_\parallel \mp k_0)^2 + k_\perp^2\right) \mp J_H$$

to do: 2nd order Keldysh-PT in oscillating term
(ignoring disorder-induced vertex corrections)

$$\langle J_\parallel \rangle = J_0 \sum_{\sigma,\mathbf{k}} \frac{k_\perp^2 (k_\parallel - \sigma k_0)(n_{\sigma,\mathbf{k}} - n_{\sigma,\mathbf{k}+\mathbf{q}})(\epsilon_{\sigma,\mathbf{k}} - \epsilon_{\sigma,\mathbf{k}+\mathbf{q}})}{\left((\epsilon_{\sigma,\mathbf{k}+\mathbf{q}} - \epsilon_{\sigma,\mathbf{k}})^2 + (\hbar\tau^{-1})^2\right)^2}$$

$$J_0 = \frac{2\lambda^2 s^2 e \hbar^4 q v_{\text{screw}}}{m}$$



experimental regime
e.g. bulk MnSi, $\ell_s \sim 1000\,\text{Å}$ at T=0

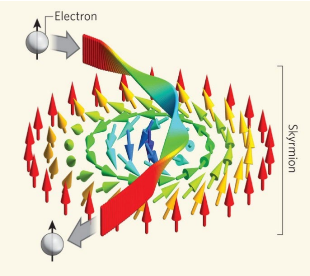
numbers

$$\Omega_{\text{screw}} \sim 1\,\text{MHz}, \quad \lambda_h \sim 200\text{Å} \qquad\qquad v_{\text{screw}} \sim 20\,\text{mm/s}$$

the biggest enemy: **pinning** of the helix by **disorder**

compare to depinning of skyrmions in MnSi
(similar pinning forces expected

$$v_{\text{skyrmion}} \sim 0.2\,\text{mm/s}$$



→ pinning most likely **not a problem** in clean systems like bulk MnSi

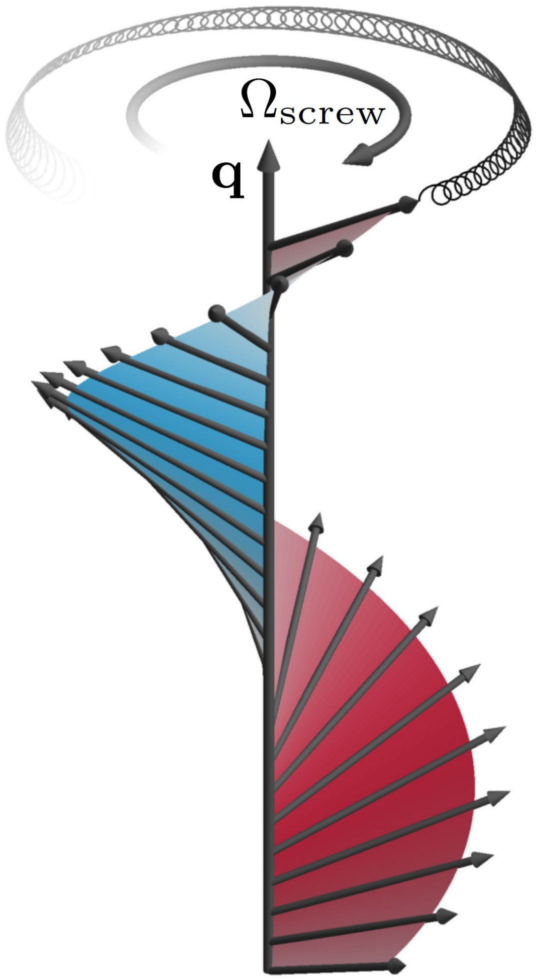
order-of-magnitude estimate of
achievable current densities (MnSi type parameters)
voltage drop easily measurable

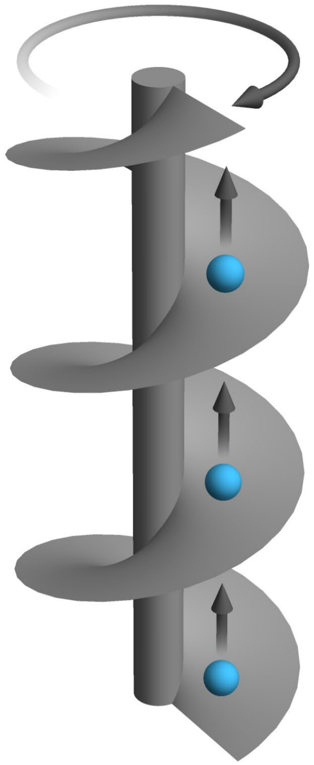
$$j \sim 10^{3\ldots5}\,\text{A/m}^2$$

possible issue: sample thinner than GHz penetration depth (μm)
high surface quality

conclusions

- activation of Goldstone mode
- screw like motion in helical magnets realizes Archimedean screw
- generic instability of driven system:
  resonant magnon emission
  magnon laser may form (time quasi crystal)
- efficient pumping of charge
- promising for experimental observations





Nina del Ser, Lukas Heinen, Achim Rosch, arXiv:2012.11548 (2020)