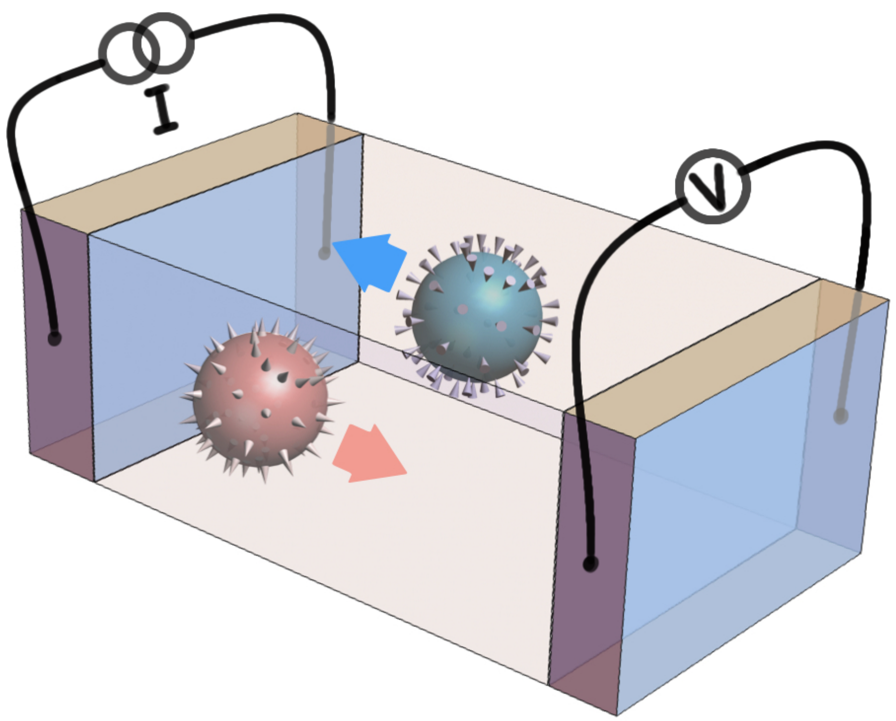
Topological transport of deconfined hedgehogs in magnets

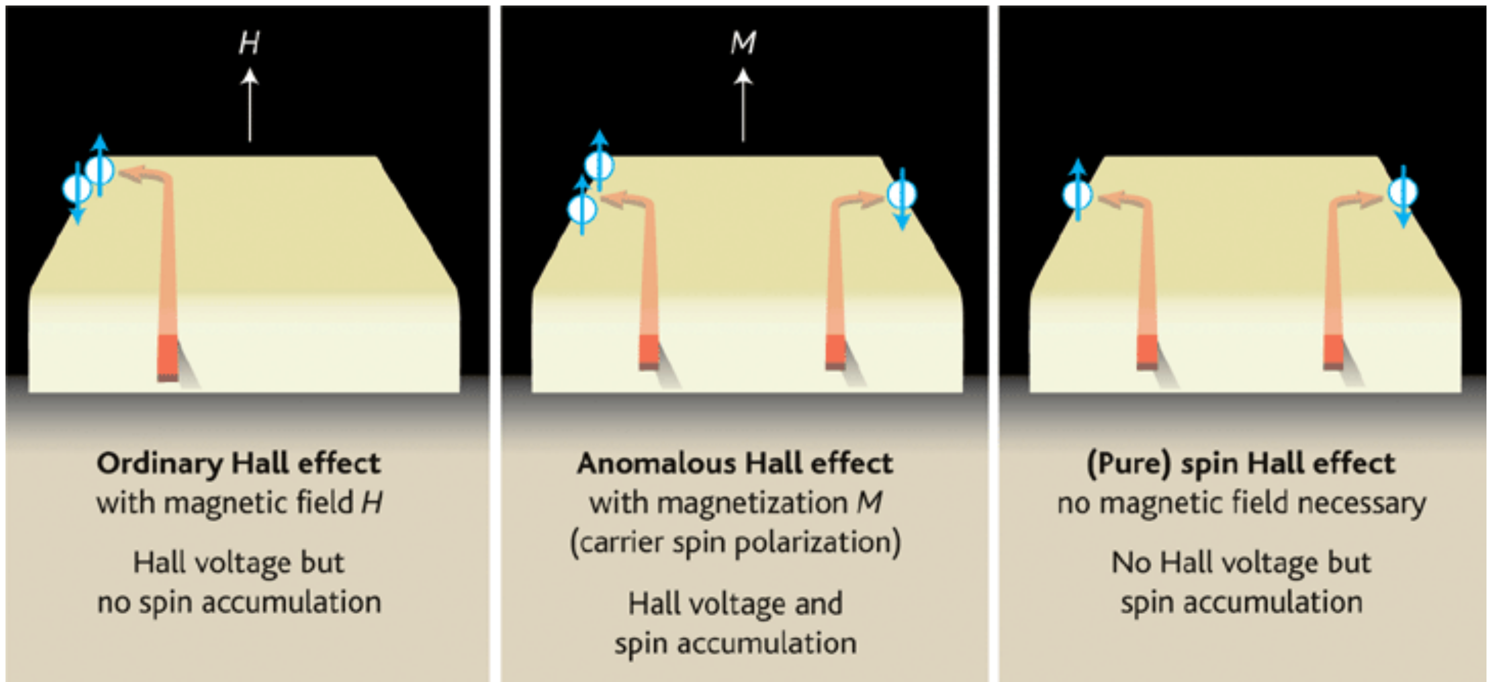
disordered magnets as (classical and quantum) conduits of topological neutral transport and simulators of strongly-correlated media

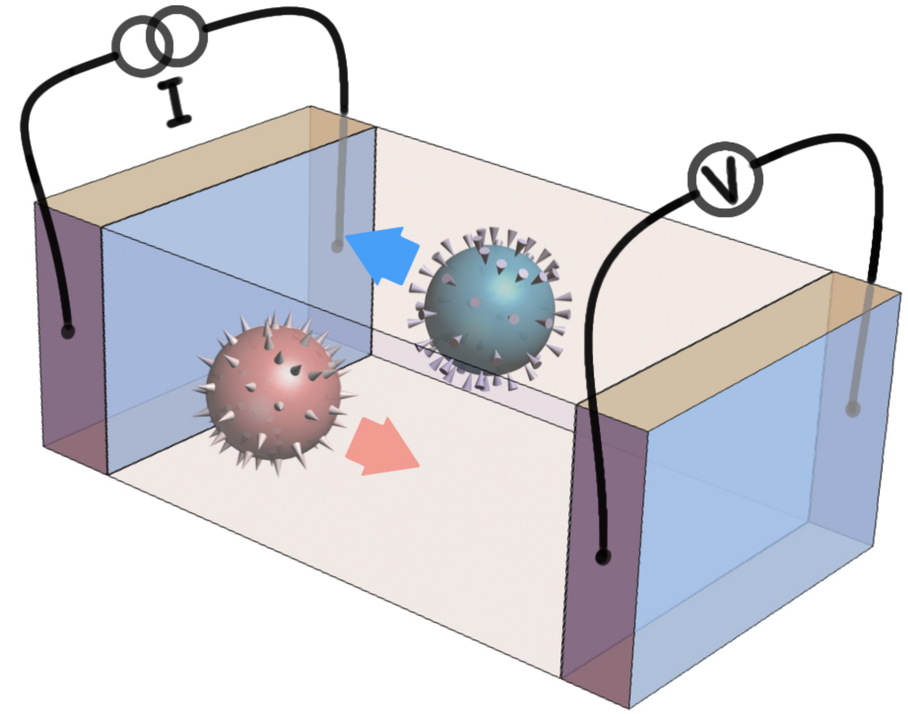


Yaroslav Tserkovnyak

w/ Ji Zou and Suzy Zhang

Motivation and open questions





- Using spin Hall transport as a conceptual blueprint, we wish to explore *neutral nonentropic* transport phenomena in condensed matter

- One broad theme is provided by *topological* conservation laws in insulators

- *Can the associated correlated phenomena and their transport signatures offer new opportunities for applications and fundamental studies of nonequilibrium properties of classical and quantum systems?*

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Editors' Suggestion

Topological Transport of Deconfined Hedgehogs in Magnets

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We theoretically investigate the dynamics of magnetic hedgehogs, which are three-dimensional topological spin textures that exist in common magnets, focusing on their transport properties and connections to spintronics. We show that fictitious magnetic monopoles carried by hedgehog textures obey a topological conservation law, based on which a hydrodynamic theory is developed. We propose a nonlocal transport measurement in the disordered phase, where the conservation of the hedgehog flow results in a nonlocal signal decaying inversely proportional to the distance. The bulk-edge correspondence between the hedgehog number and skyrmion number, the fictitious electric charges arising from magnetic dynamics, and the analogy between bound states of hedgehogs in ordered phase and the quark confinement in quantum chromodynamics are also discussed. Our study points to a practical potential in utilizing hedgehog flows for long-range neutral signal propagation or manipulation of skyrmion textures in three-dimensional magnetic materials.

Stokes theorem for continuous topological flows

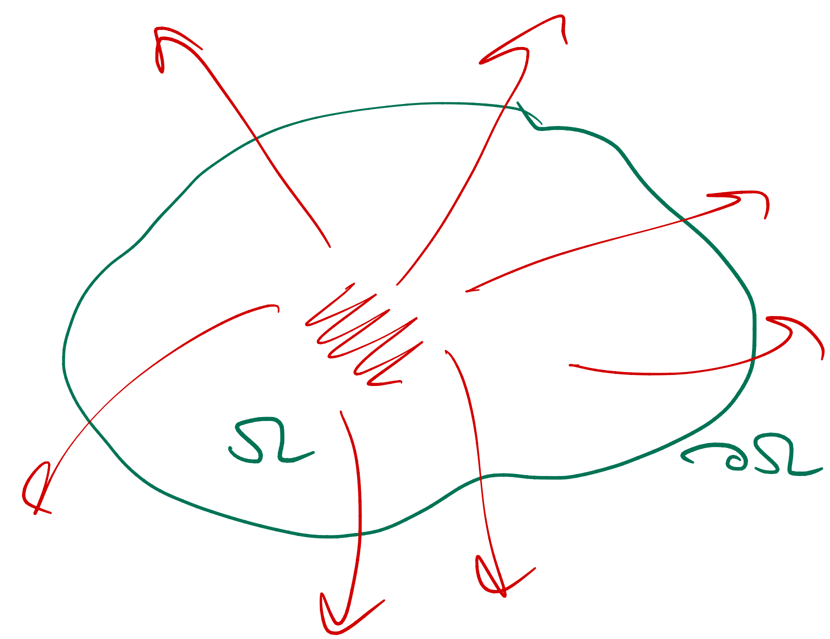
- For a smooth dynamic field $\mathbf{n}(\mathbf{r},t) : \mathbb{R}^3 \to \mathbb{R}^3$, we can define a *4-current*

$$j^\mu = \epsilon^{\mu\nu\alpha\beta}\partial_\nu \mathbf{n} \cdot (\partial_\alpha \mathbf{n} \times \partial_\beta \mathbf{n})/8\pi$$

which obeys a *topological conservation law:*

$$\partial_\mu j^\mu = 0 \qquad\qquad \mu = 0,1,2,3 \leftrightarrow t,x,y,z$$

that does not rely on any symmetries or any other special properties of the Hamiltonian



- The conserved (topological) charge is related to the *skyrmion number* on the boundary:

$$\mathcal{Q} \equiv \int_\Omega dx dy dz\, j^0 = \frac{1}{8\pi}\int_{\partial\Omega} dx^j \wedge dx^k\, \mathbf{n}\cdot(\partial_j \mathbf{n} \times \partial_k \mathbf{n})$$

according to the *generalized Stokes' theorem*

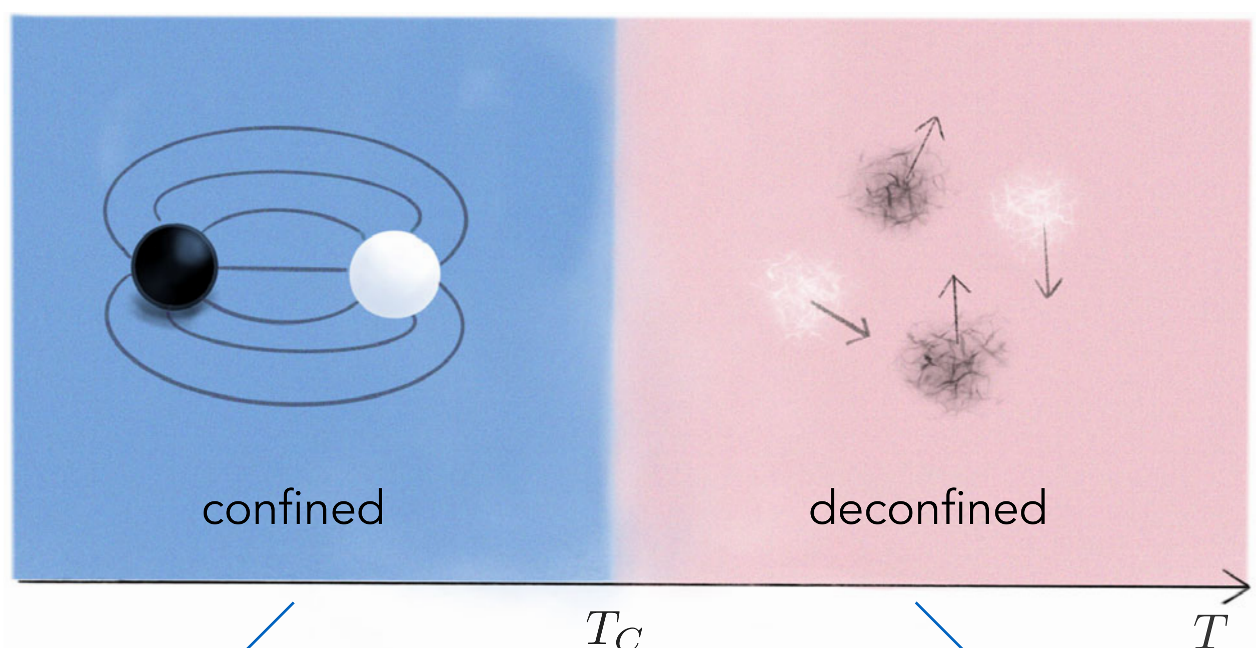
both of these are generally nonquantized at high T

High-T deconfinement

$$\mathcal{U} = \int d^3\vec{r} \left[\frac{\mathcal{A}}{2} (\vec{\nabla}\mathbf{n})^2 + \frac{\mathcal{C}}{2} (\partial_i \mathbf{n} \times \partial_j \mathbf{n})^2 \right] \sim \mathcal{A}R - \mathcal{C}/R$$

the magnetic exchange stiffness results in confinement of hedgehogs in the ordered phase

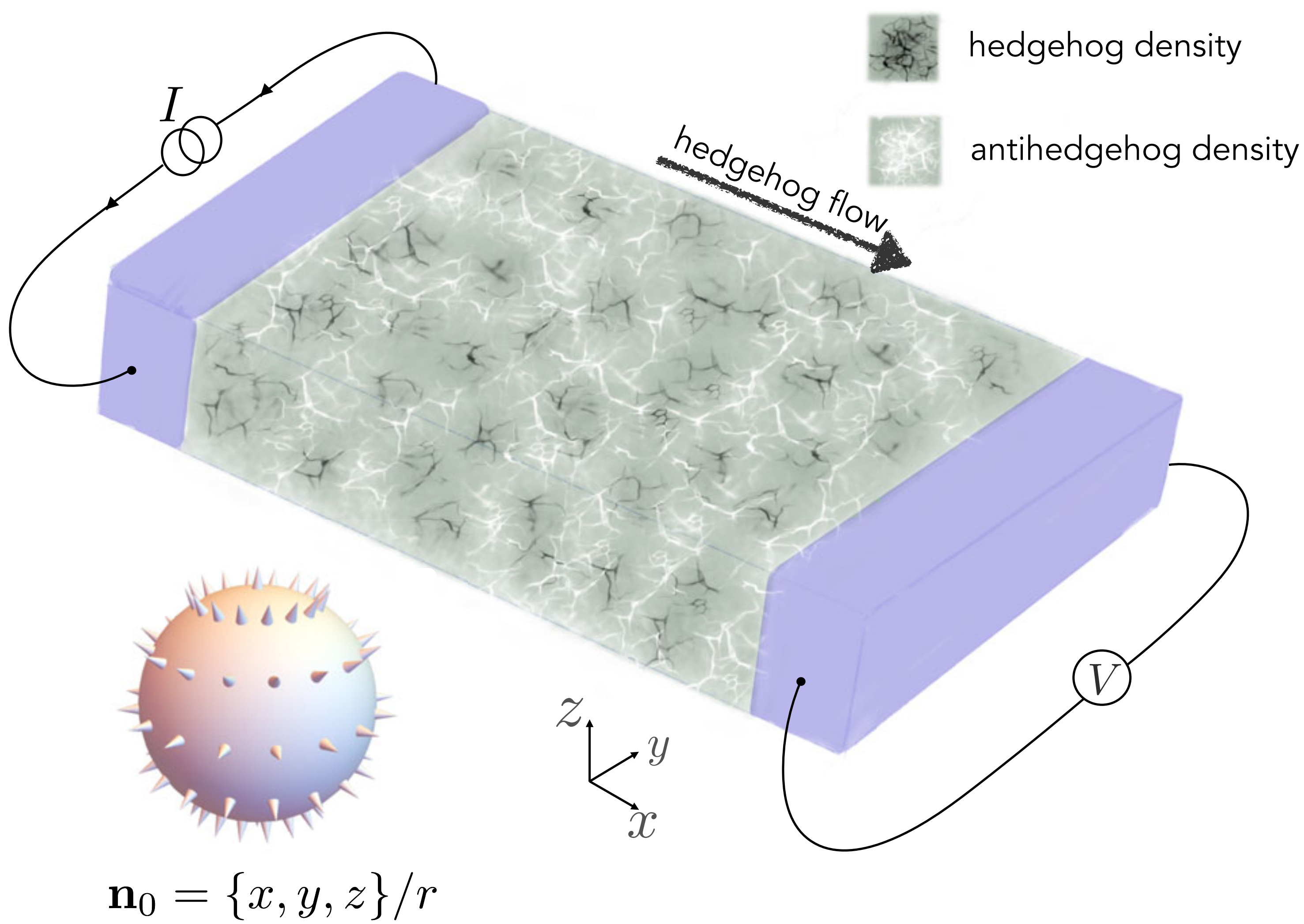
nonlocal coupling that results in Coulomb-type interaction at short distances



cf. quark confinement in QCD

quark-gluon plasma

Electrical biasing of hedgehog flows

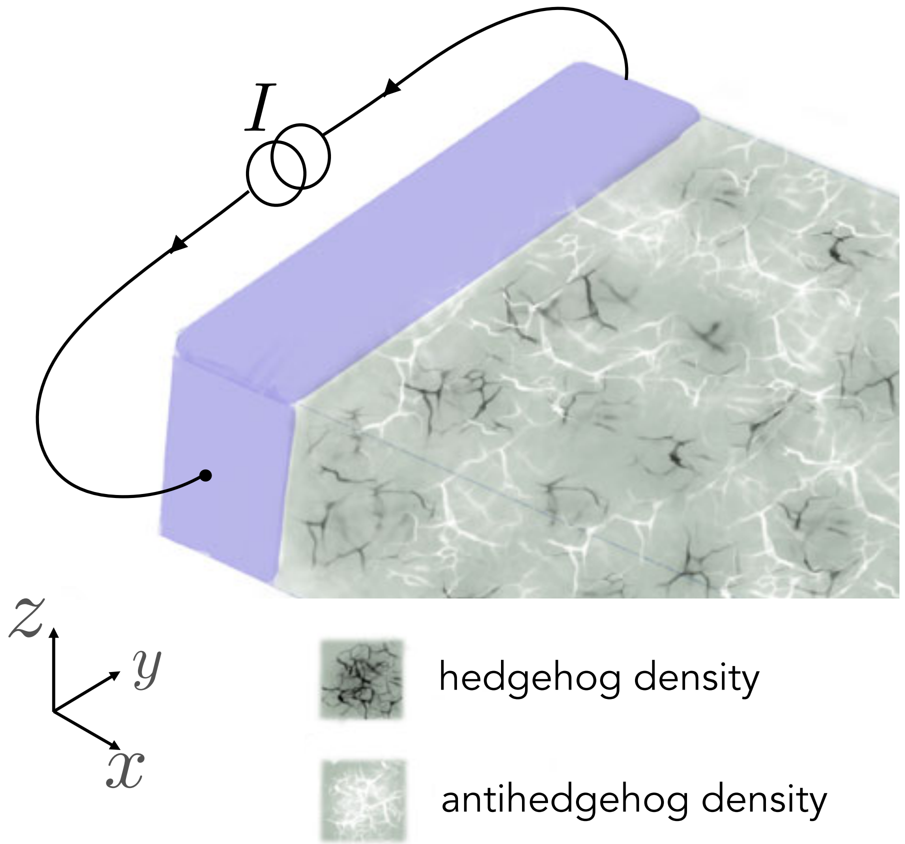


- The main idea, which has been utilized before in the context of electrically-driven spin superfluidity, is to generate a Magnus force on the hedgehog transport near the metal-insulator interface

- As an illustration, we treat this based on the energetics, using Landau phenomenology near the transition temperature

Takei and YT, *PRL* (2014); Kim, Takei, and YT, *PRB* (2015)

Interfacial spin transfer



$$\mathfrak{J} = \frac{3\hbar\zeta}{8e\pi}(\vec{\mathcal{J}}\cdot\vec{\nabla}\mathbf{n})\times(\vec{z}\cdot\vec{\nabla}\mathbf{n})$$

current-induced spin transfer

- reflection symmetry broken along z
- no spin-orbit interaction

$$\partial_t\mathbf{n} = -\Gamma\mathbf{H} + \mathfrak{J}/s \qquad \mathbf{H}\equiv\delta_{\mathbf{n}}F[\mathbf{n}]$$

Rate of change of the Landau free energy F :

$$P \equiv \partial_t\mathbf{n}\cdot\mathbf{H} = \mathfrak{J}\cdot\partial_t\mathbf{n}/s\Gamma - (\partial_t\mathbf{n})^2/\Gamma$$

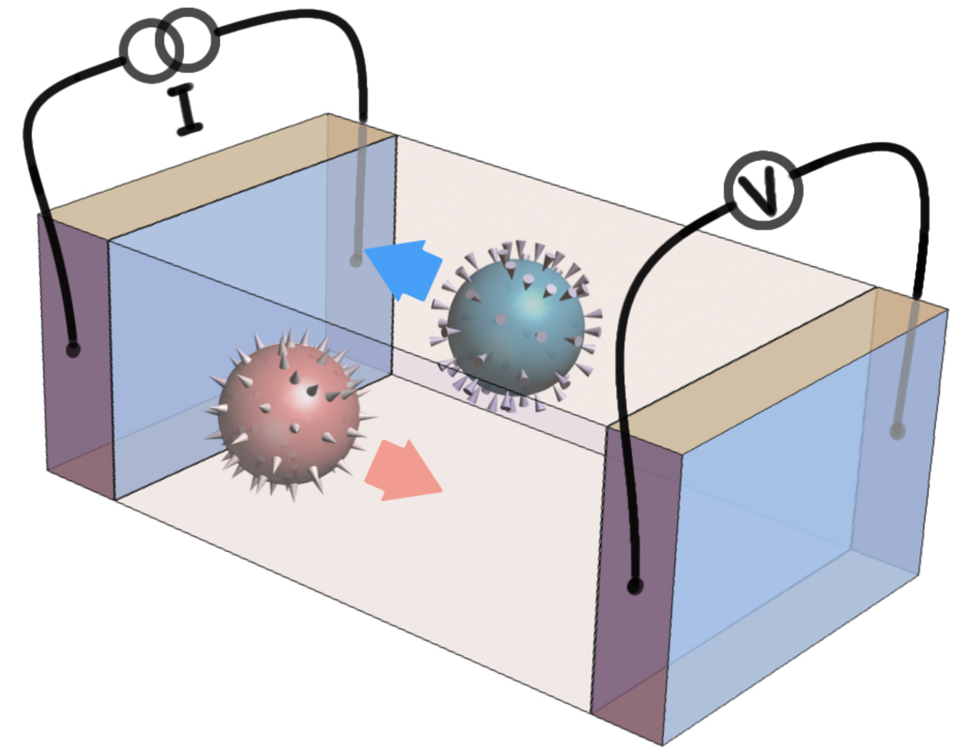
work by the bias

Rayleigh dissipation

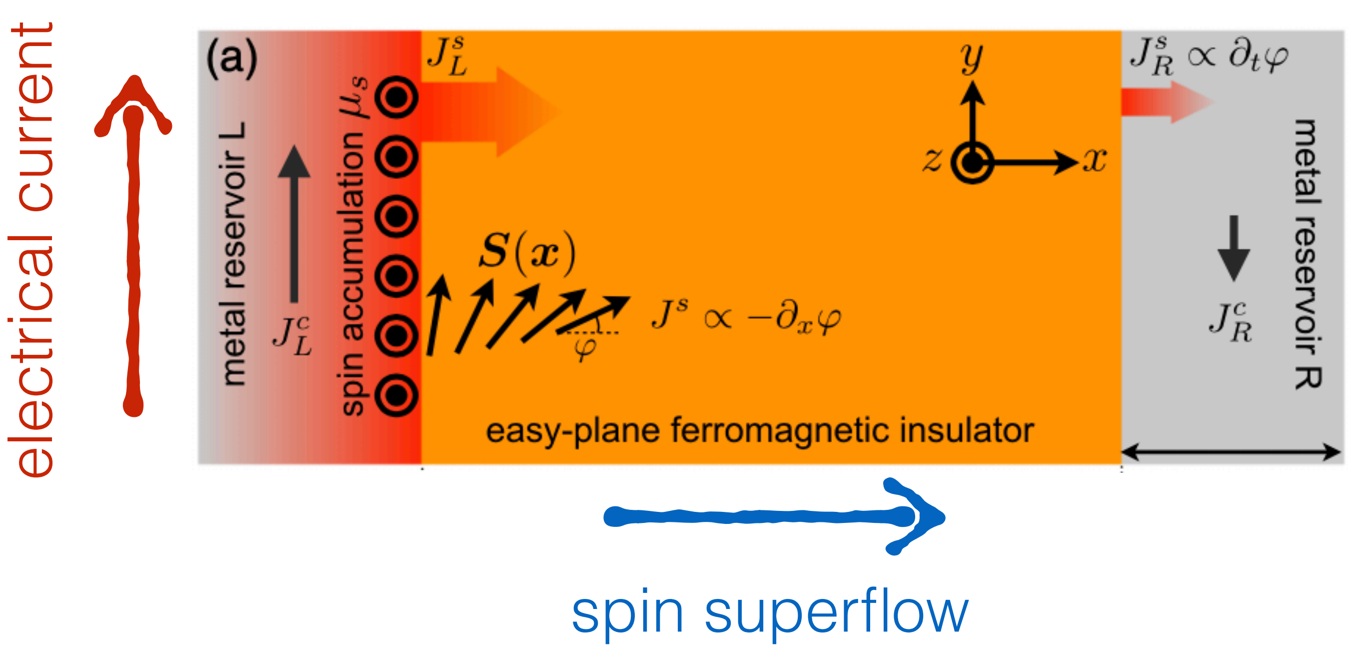
$$W = \int dydzdt\frac{\mathfrak{J}\cdot\partial_t\mathbf{n}}{s\Gamma} = \frac{\hbar\zeta\mathcal{J}}{2es\Gamma}\mathcal{Q} \longrightarrow \boxed{\mu \equiv \frac{\delta W}{\delta\mathcal{Q}} = \frac{\hbar\zeta}{2es\Gamma}\mathcal{J}}$$

hedgehog chemical potential

Plugging hedgehogs into electrical circuits



The Magnus cross coupling between the electric and hedgehog flows allows to think of electronic devices



An immediate consequence is algebraically-scaling (nonlocal) transconductances as in typical spin (superflow) Hall measurements

Takei and YT, *PRL* (2014)

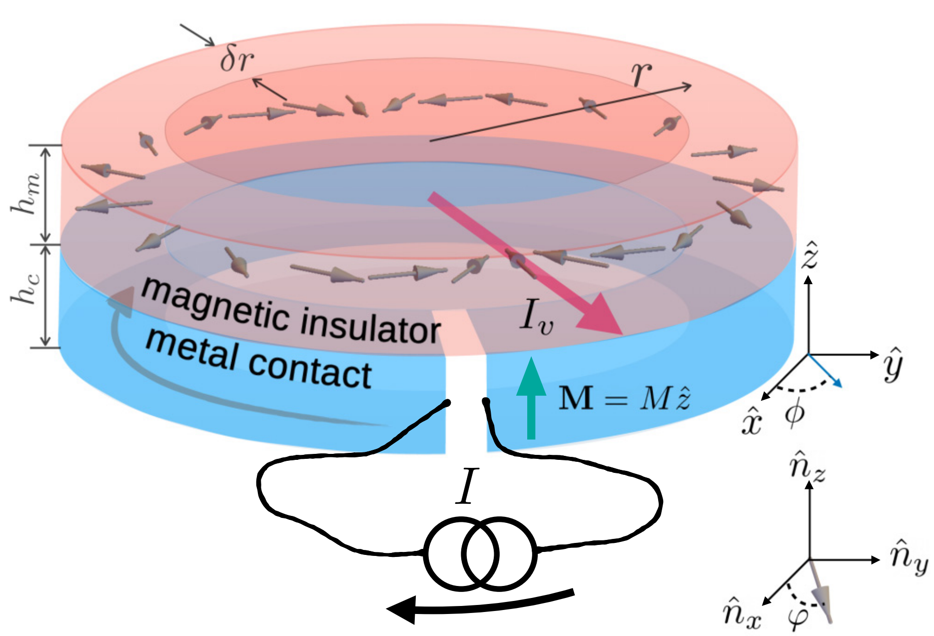
Hedgehog flow: Versatile and tunable transport

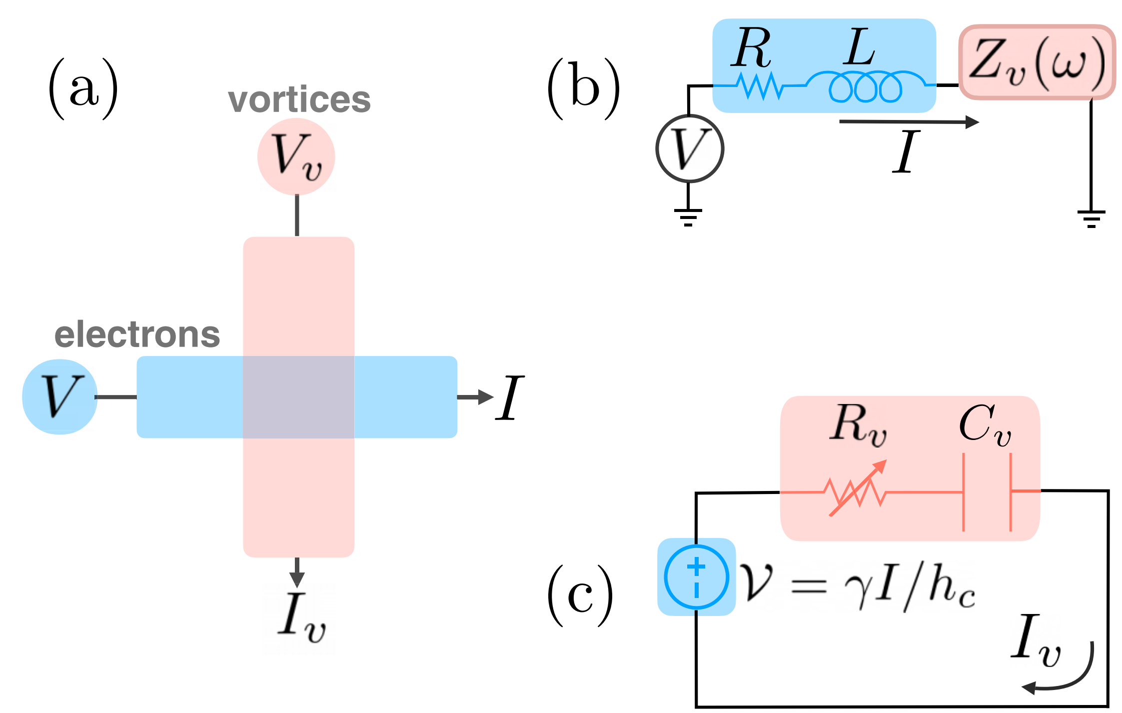
The 3D hedgehog flow with skyrmionic bulk/boundary correspondence is closely analogous to the 2D vorticity flow with winding bulk/boundary correspondence we studied previously

Jones, Zou, Zhang, and YT, *PRB* (2020)

skyrmion buildup (capacitance)

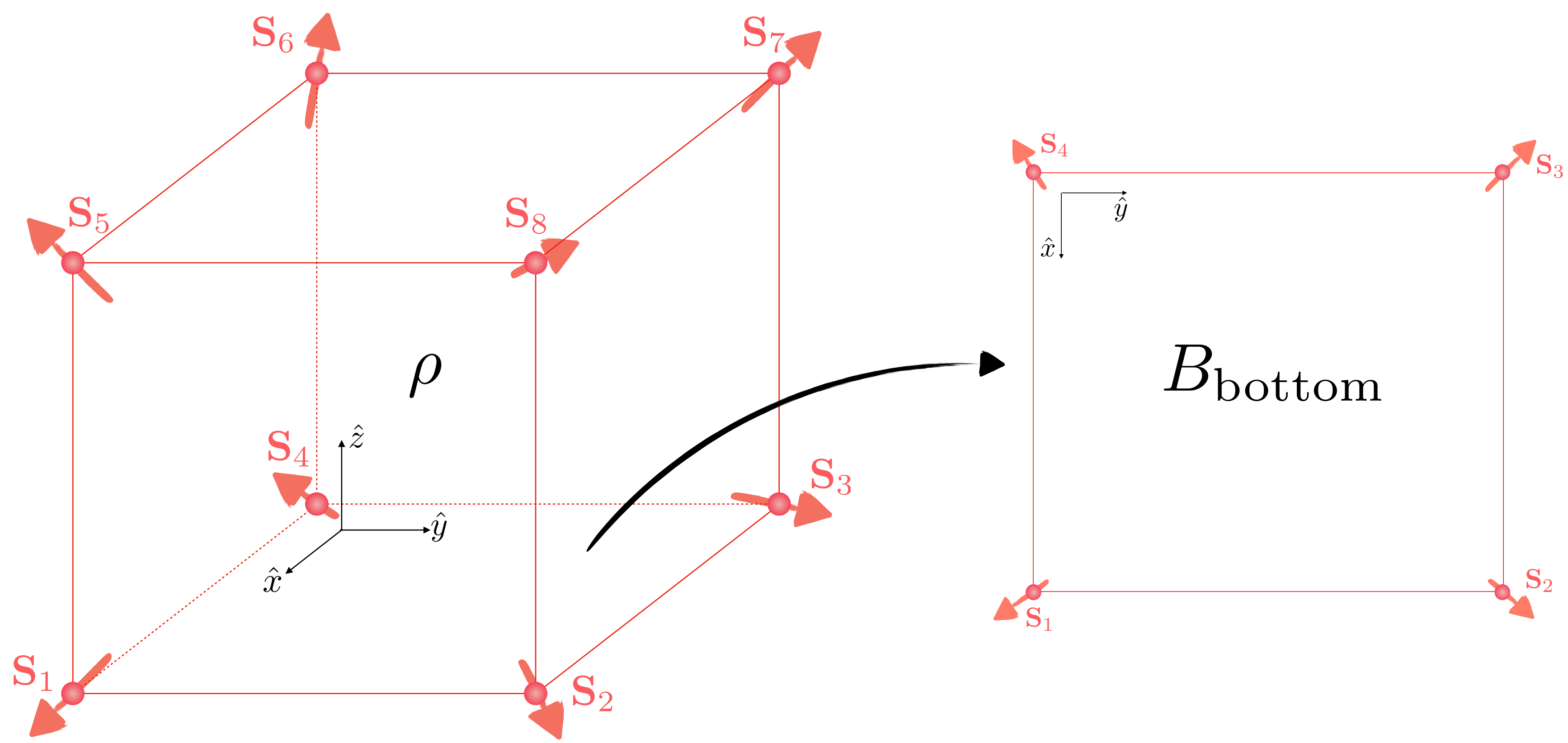
hedgehog flux (conductance)





One can think of devices where a highly-tunable hedgehog conductivity (e.g., associated with their quark-like deconfinement) is utilized as a switch

Atomistic/quantum limit of the Stokes' theorem



$$\rho = \frac{B_{\text{top}} - B_{\text{bottom}} + B_{\text{right}} - B_{\text{left}} + B_{\text{front}} - B_{\text{back}}}{4\pi a} \to \frac{3}{4\pi} \partial_x \mathbf{n} \cdot \partial_y \mathbf{n} \times \partial_z \mathbf{n}$$

quantized hedgehog density

$$B_{\text{bottom}} = \frac{c_{123} + c_{134} + c_{234} + c_{124}}{4a^2}, \quad c_{ijk} \equiv \frac{\mathbf{S}_i \cdot \mathbf{S}_j \times \mathbf{S}_k}{S^3}$$

triangulated skyrmion density (scalar chirality) on a plaquette

Quantum continuity equation

Heisenberg-evolving the individual spins, $\partial_t \mathbf{S} = \frac{1}{i\hbar}[\mathbf{S}, H]$,

we get atomistic/quantum continuity equation:

$$\partial_t \rho + \frac{j^z_{\mathrm{top}} - j^z_{\mathrm{bottom}} + j^y_{\mathrm{right}} - j^y_{\mathrm{left}} + j^x_{\mathrm{front}} - j^x_{\mathrm{back}}}{a} = 0$$

where the hedgehog fluxes reduce to the previous expressions in the continuous classical limit

There is an underlying quantum description, with a robust conservation law, which retains full quantum and thermal spin fluctuations and discrete atomistic character (in contrast to, e.g., skyrmion or hopfion dynamics)

Kubo formula for intrinsic bulk hedgehog conductivity

PHYSICAL REVIEW RESEARCH **1**, 033071 (2019)

Editors' Suggestion

Quantum hydrodynamics of vorticity

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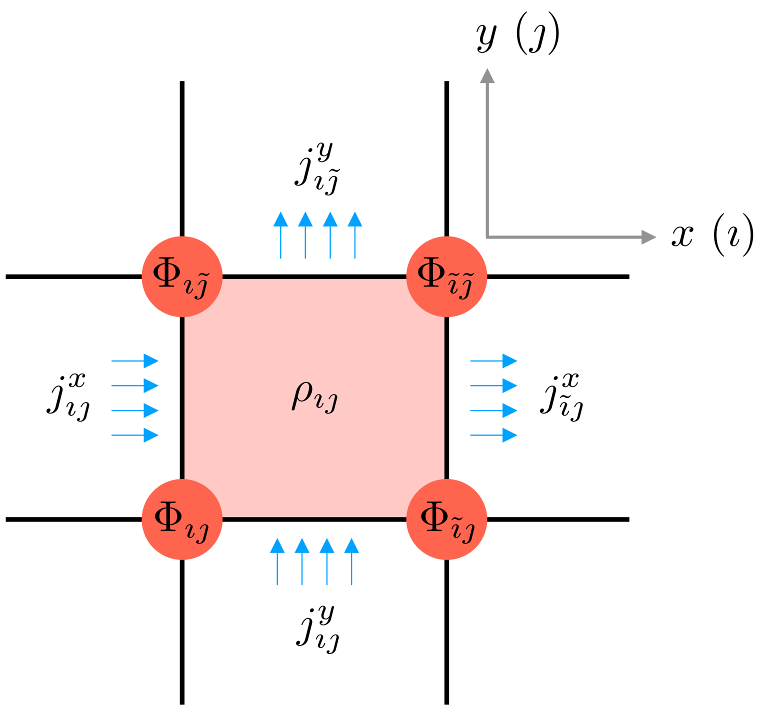
We formulate a quantum theory of vorticity (hydro)dynamics on a general two-dimensional bosonic lattice. In the classical limit of a bosonic condensate, it reduces to conserved plasma-like vortex-antivortex dynamics. The nonlocal topological character of the vorticity flows is reflected in the bulk-edge correspondence dictated by the Stokes theorem. This is exploited to establish physical boundary conditions that realize, in the coarse-grained thermodynamic limit, an effective chemical-potential bias of vorticity. A Kubo formula is derived for the vorticity conductivity, which could be measured in a suggested practical device, in terms of quantum vorticity-flux correlators of the original lattice model. As an illustrative example, we discuss the superfluidity of vorticity, exploiting the particle-vortex duality at a bosonic superfluid-insulator transition.



Hedgehog conductivity in 3D is analogous to the vorticity conductivity in 2D, which was proposed to investigate the particle-vortex duality

$$\sigma_{ij}(\mathbf{k},\omega) = \frac{i}{\omega}\chi_{ij}(\mathbf{k},\omega)$$

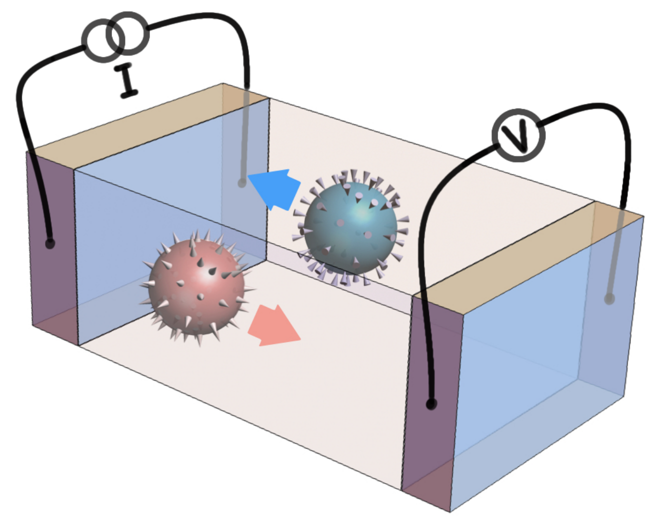
$$\chi_{ij}(\mathbf{r}-\mathbf{r}',t-t') \equiv -i\theta(t-t')[j_i(\mathbf{r},t),j_j(\mathbf{r}',t')] + \delta(t-t')p_{ij}(\mathbf{r}-\mathbf{r}')$$

Some take-home messages

- Hedgehog textures in generic bulk magnetic materials can provide a new topological transport medium for fundamental studies of correlated phenomena and novel applications

- Strong quark-like confinement in the ordered phase and no stringent symmetry requirements for inducing electrical Magnus force may make them experimentally attractive

- Microscopic robustness of the conservation law allows for direct field-theoretic investigation, e.g., using Kubo formula for hedgehog conductivity



Thank you!