Biao Lian | Zhida Song | Nicolas Regnault | Jonah H. Arbeitman | Fang Xie | Aaron Chew | Ali Yazdani | Dmitri K. Efetov | Emanuel Tutuc

*Yonglong Xie, Kevin Nuckolls, Myungchul Oh, Xiaobo Lu, Will Burg, Ipsita Das, Zhijun Wang*

*Also thanks to T Neupert, S Huber, V Peri, Yuanfeng Xu, Hongming Weng, XI Dai, B Wieder, M Verngiory, L. Elcoro*

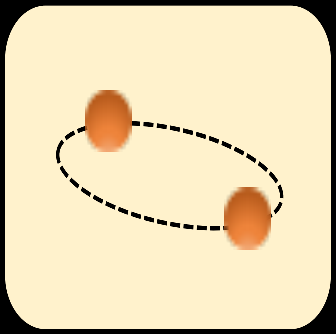
**Main refs: TBG I – VI (arXiv, Sep. 2020)**

B. Andrei Bernevig

# Symmetries, Insulator States and Excitations of Twisted Bilayer Graphene with Coulomb Interaction

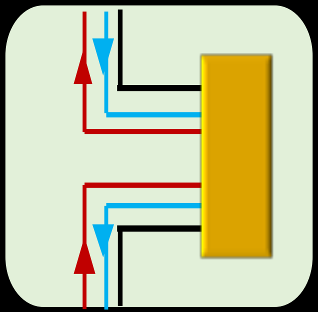
# Quantum Condensed Matter

Correlations



Mott Insulators
Spin liquids
High-Tc Superconductivity
....

Topology



Quantum Hall
Fractional Quantum Hall
(also correlated)

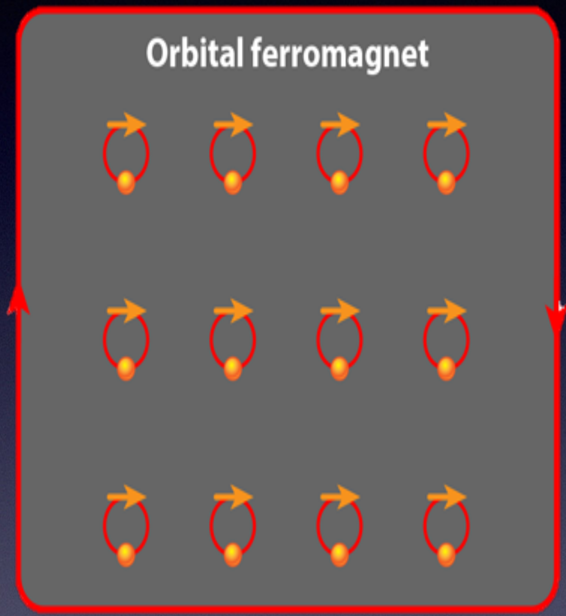
Topological Insulators, semimetals, and superconductors
....

# A New Kind of Magnetism

## Spin Ferromagnetism
Overwhelmingly Common Case


- <u>Spontaneous alignment of spins</u> that break time-reversal symmetry, while preserving the translational symmetries of the system
- Small magnetization contribution from orbital currents <u>only</u> in the presence of SOC

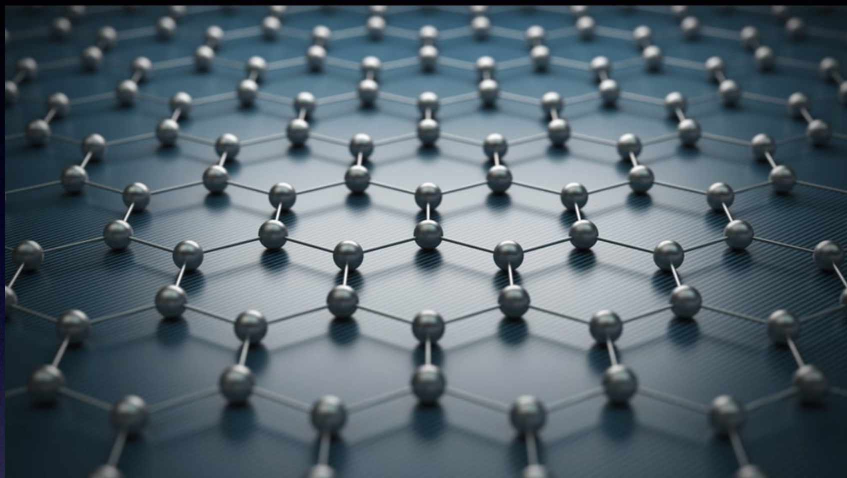
## Orbital Ferromagnetism
Exceedingly Rare Case

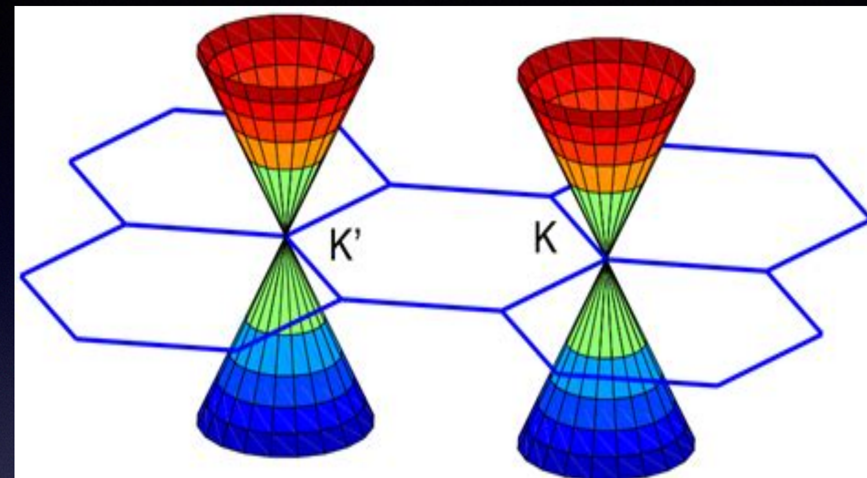


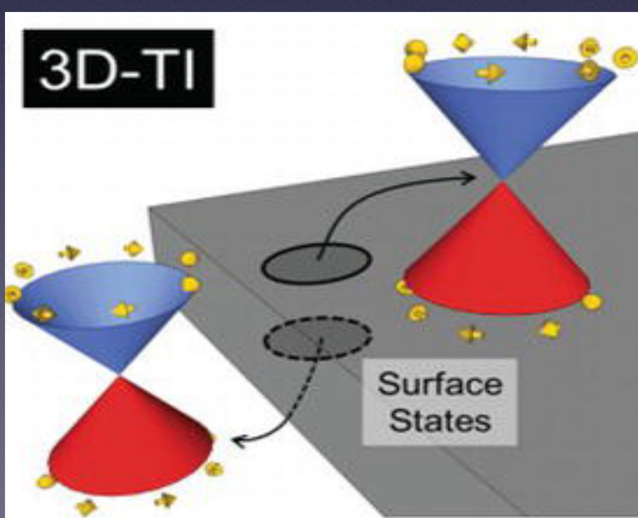

- <u>Spontaneous persistent current loops</u> that break time-reversal symmetry, while preserving the translational symmetries of the system
- At the boundary, the persistent current loops sum to form robust chiral edge states

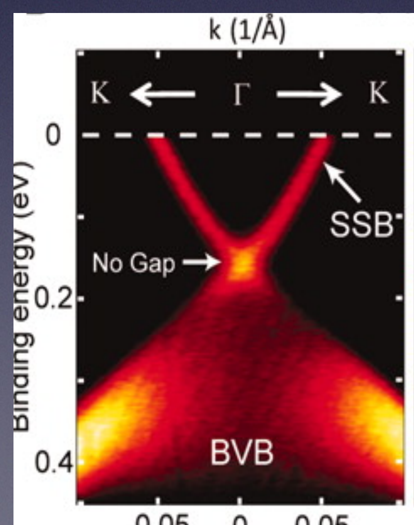
A. MacDonald Physics 12, 12 (2019).

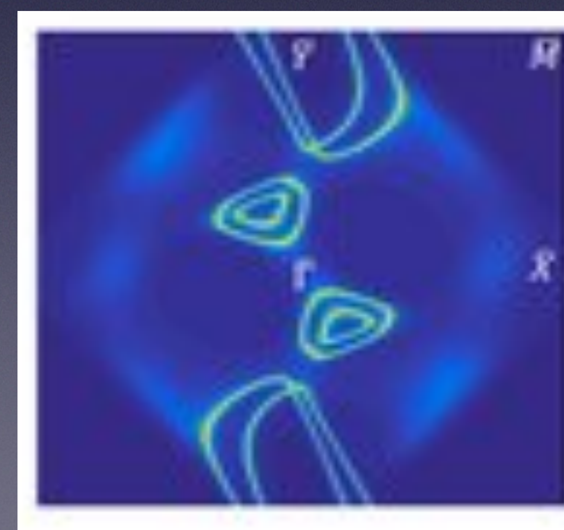
# Topology and New Excitations





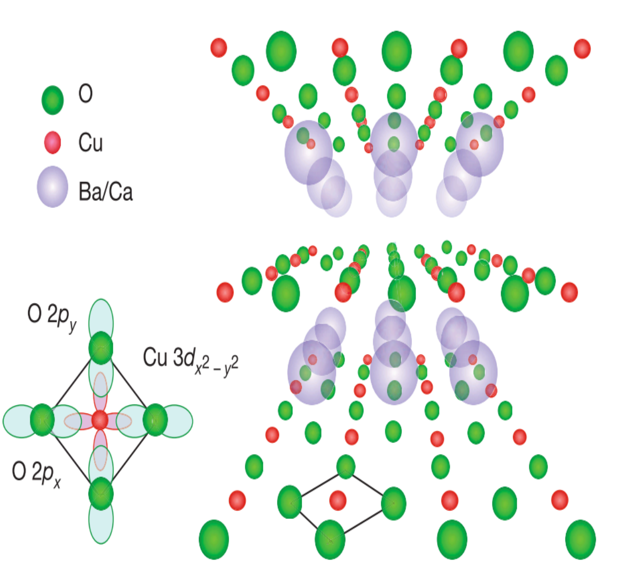



# Engineering Correlations

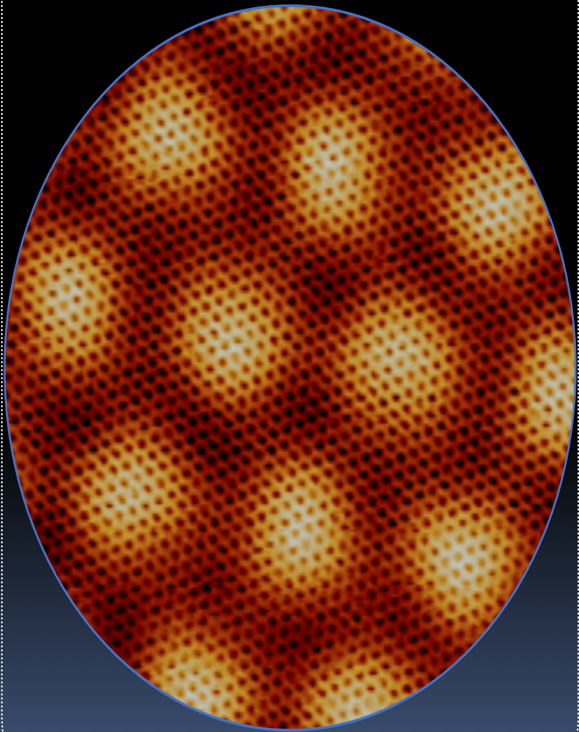
Material Chemistry

few Å, eVs




Moiré superlattices from 2D Materials

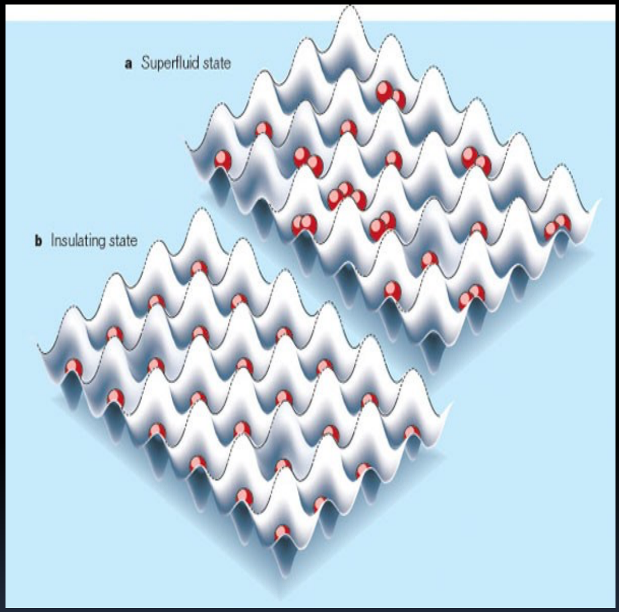
**~ 10 nm**

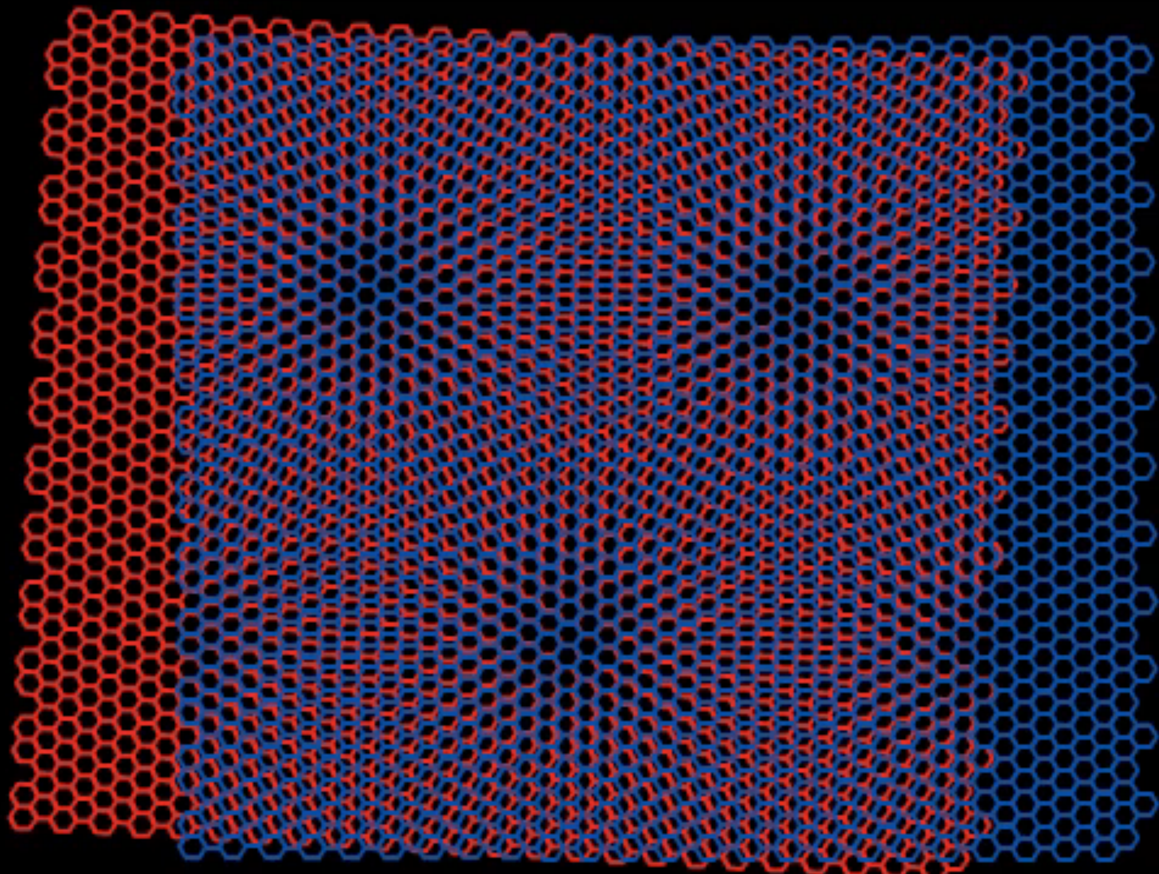


New types of band-structure engineering

Cold atoms & Optical lattices

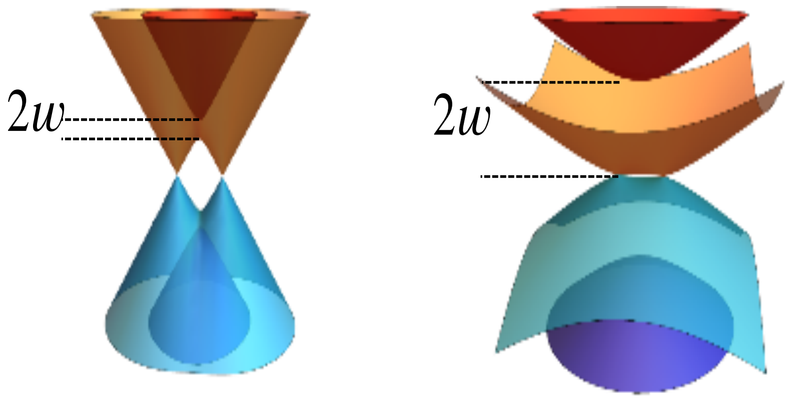
~ 1 micron





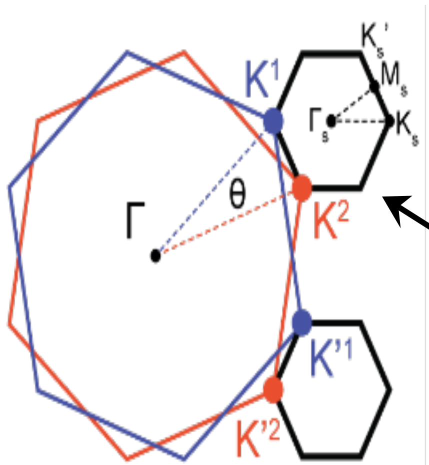
P. Jarillo-Herrero
Efetov, Young, Yazdani, Andrei, Tutuc, Goldhaber Gordon, Dean, groups

# Magic Angle: Twisting to Flatness




$2w \ll v_{F0}k_\theta$ $\quad$ $2w \approx v_{F0}k_\theta$

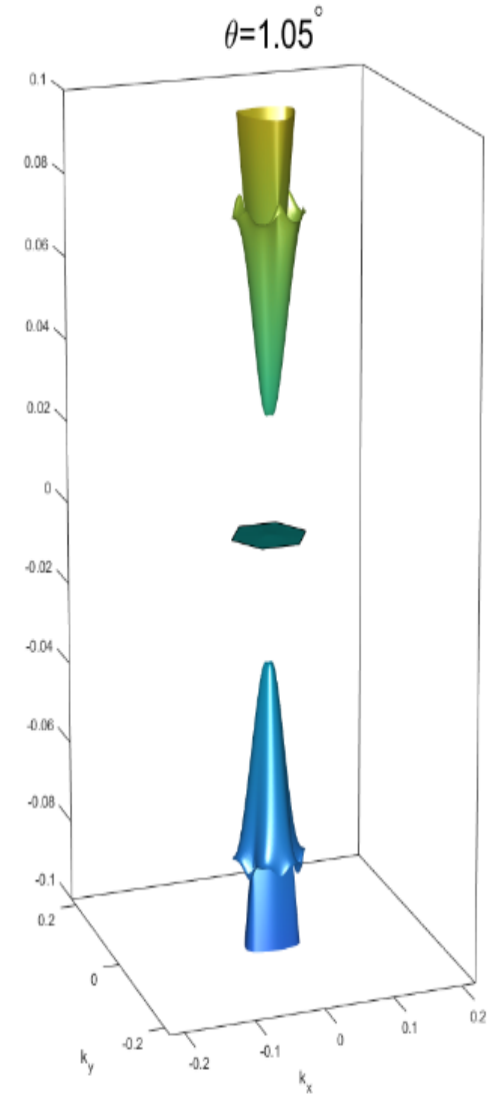
*Decreasing Twist Angle*




Superlattice Brillouin zone
(changes size w/ rotation angle)
Two Valleys K and K'

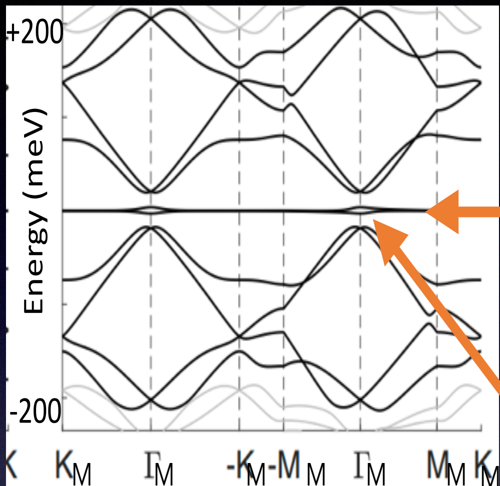
Suarez-Morell *et al. PRB* (2010)
Bistritzer & MacDonald, *PNAS* (2011)
Cao, *et al. Nature* (2018)

## Including lattice relaxation



# Flat Bands in Magic Angle Graphene Bilayers

k-space

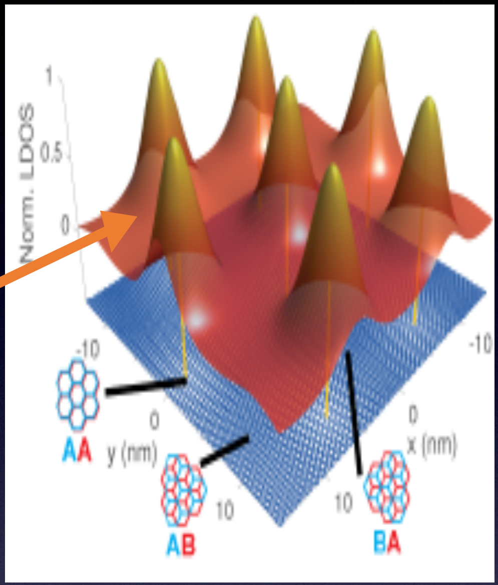



Theory: Flat bands at magic ~1°
Including relaxation: Koshino et al. (2018)

The two flat bands around charge neutrality are 4 fold degenerate: 2 spin and 2 valley

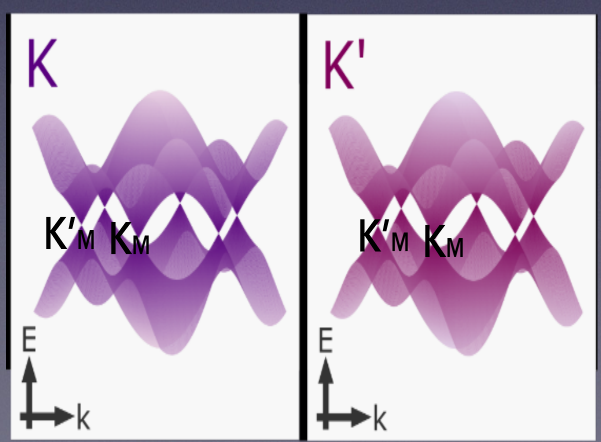
Electron filling of the flat bands $\nu$=+/- 4 electrons per moiré site relative to neutrality

Real Space

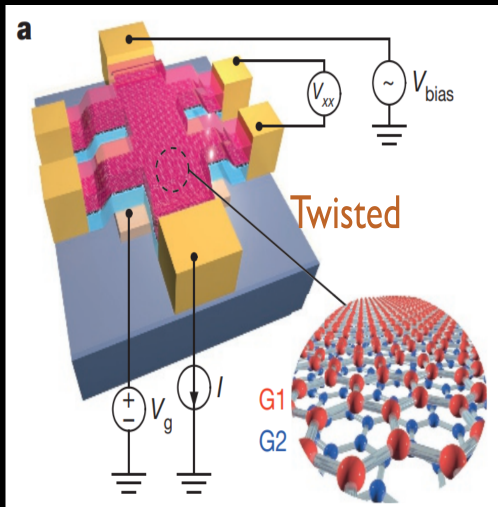



Density of States for the Flat Bands moiré superlattice (from Cao et al. 2018)

Flat Bands inherent the Dirac points from graphene; From valley K in graphene, we have two Moire $K_M$ and $K'_M$ with same chirality

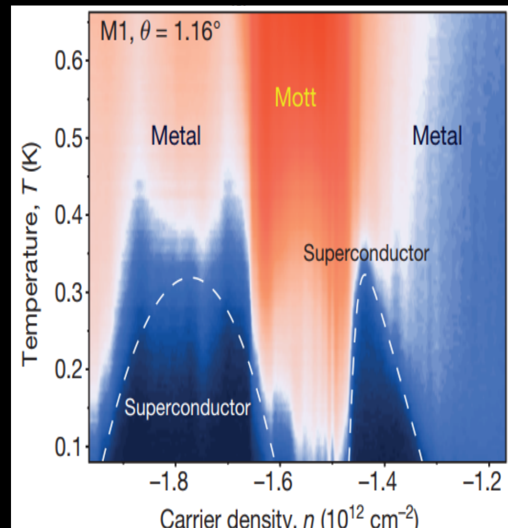


# Discovery of Correlated Insulators & Superconductivity

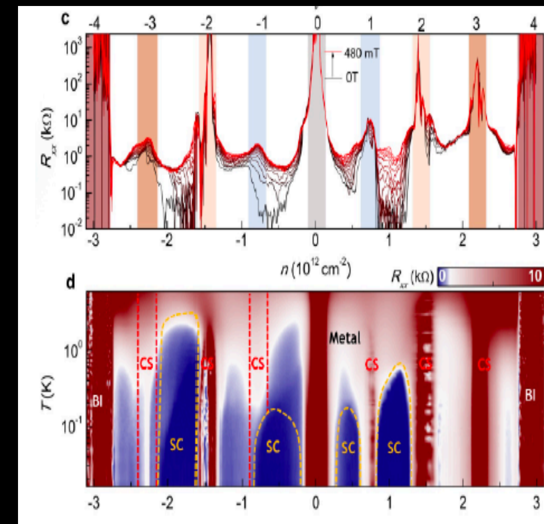



P. Jarillo-Herrero Group MIT

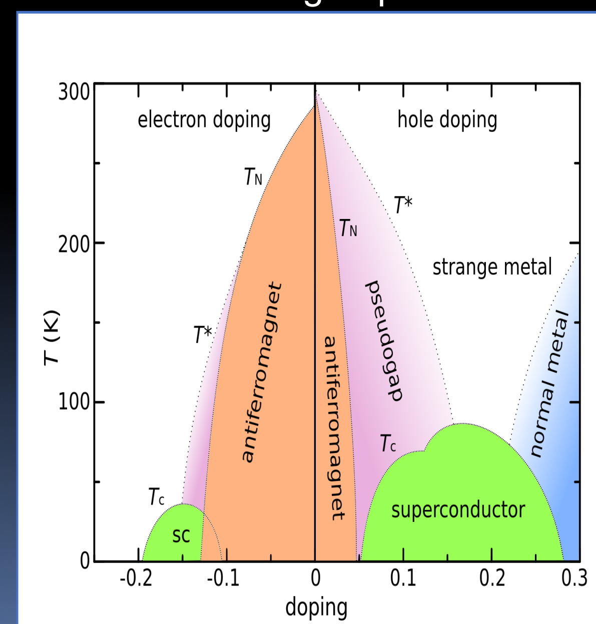
Cao et al, Nature **556,** 43 (2018)
Cao et al, Nature **556,** 80 (2018)
Yankowitz et al, Science **363**, 1059 (2019)
Liu et al, Nature 574, 653 (2019)




$\nu$=-2




Efetov group 2019

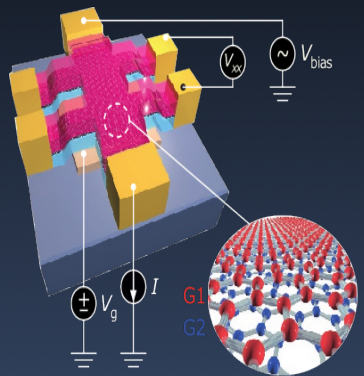



Insulator was conjectured to be a Mott insulator, occur at 1/2 filling of the flat bands, sensitivity to field and resemblance to cuprates.

# Increasing Number of Moiré Materials

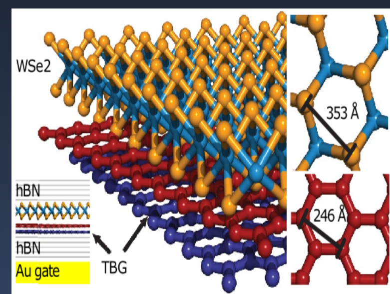
## Magic-Angle Twisted Bilayer Graphene (MATBG)

Jarillo-Herrero, Dean, Young, Efetov



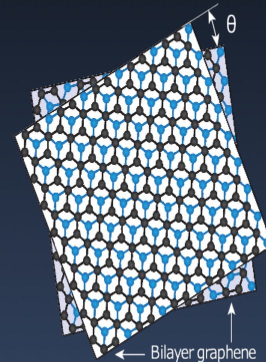

## Engineered MATBG: Aligned, Gate-Screened, and Proximitized Devices

Goldhaber-Gordon, Efetov, Young, Nadj-Perge




## Twisted Double Bilayer Graphene (TDBG)

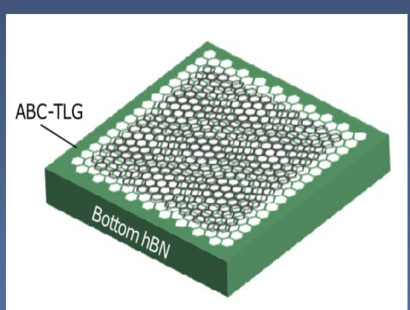
Zhang, Kim, Jarillo-Herrero




Y. Cao et al. Nature 556, 43-50 (2018).
Y. Cao et al. Nature 556, 80-84 (2018).
M. Yankowitz et al. Science 363, 1059-1064 (2019).
X. Lu et al. Nature 574, 653-657 (2019).
A. Sharpe et al. Science 365, 605-608 (2019).
M. Serlin et al. Science 367, 900-903 (2020).
P. Stepanov et al. Nature 583, 375-378 (2020).
Y. Saito et al. Nat. Phys. 16, 926-930 (2020).
H. Arora et al. Nature 583, 379-384 (2020).
C. Shen et al. Nat. Phys. 16, 520-525 (2020).
X. Liu et al. Nature 583, 221-225 (2020).
Y. Cao et al. Nature 583, 215-220 (2020).
G. Chen et al. Nat. Phys. 15, 237-241 (2019).
S. Chen et al. arXiv preprint arXiv:2004.11340 (2020
H. Polshyn et al. arXiv preprint arXiv:2004.11353 (20
L. Wang et al. arXiv preprint arXiv:1910.12147 (2020
E. C. Regan et al. Nature 579, 359-363 (2020).
Y. Tang et al. Nature 579, 353-358 (2020).

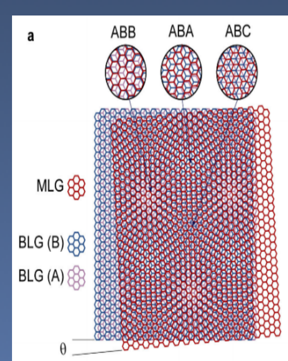
## ABC Trilayer Graphene Aligned with hBN

Wang



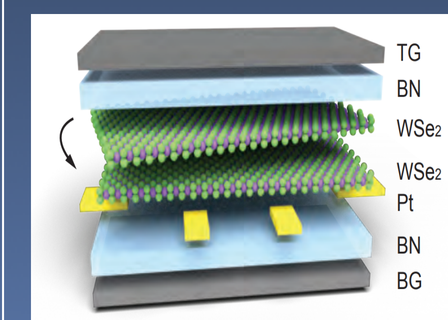

## Twisted Monolayer-Bilayer Graphene (TMBG)

Xu, Dean, Yankowitz, Young



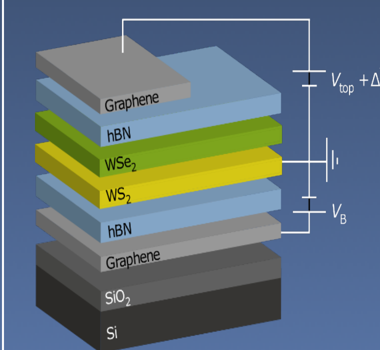

## Twisted Bilayer $WSe_2$ ($TWSe_2$)

Dean




## Aligned $WSe_2$ / $WS_2$

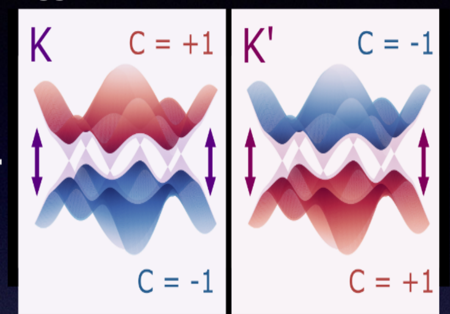
Shan, Mak, Wang

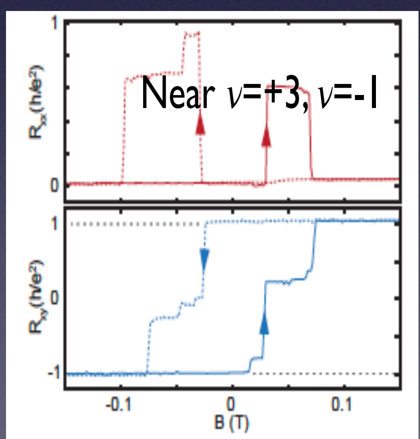


# Many-Body Correlated Chern Insulator States

## Broken $C_2$-Symmetry

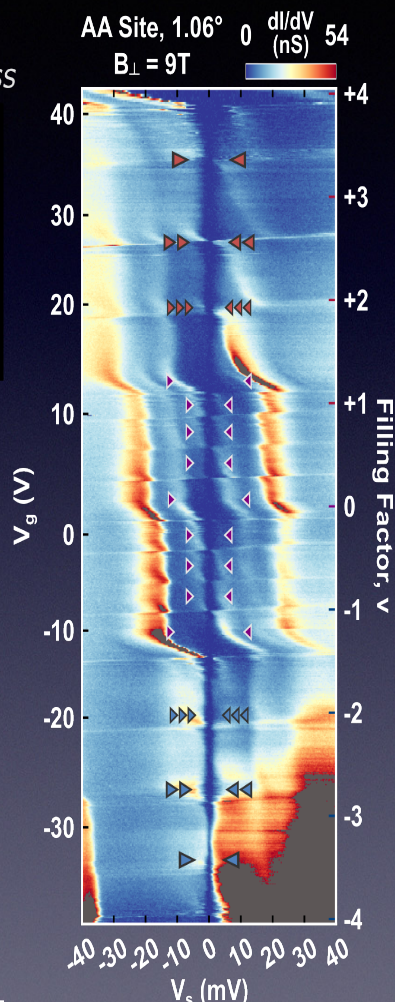
*Staggered Sublattice Potential Mass*




MATBG Aligned to hBN

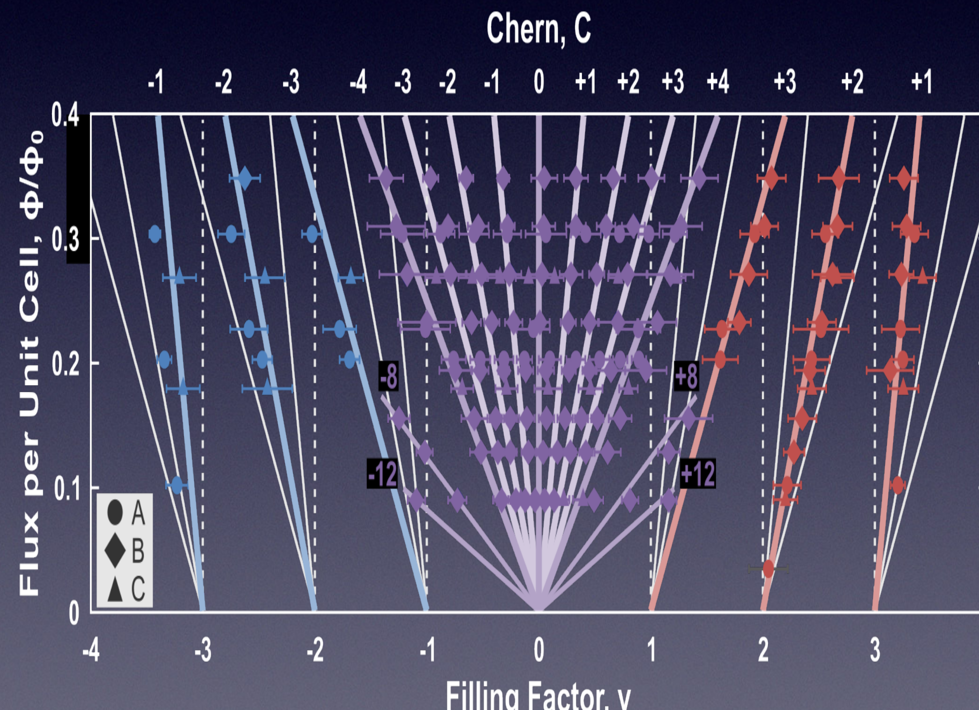
Sharpe et al. Science 365, 605 (2019)
Liu et al, Nature 574, 653 (2019)
Serlin, et al. Science 367 (2020)




Non-Aligned hBN

Streda's Formula: "The charge density of a Chern insulating phase changes with magnetic field at a rate equal to its quantized Hall conductance."

$$\frac{dn}{dB} = \frac{\sigma_{xy}}{e} = \frac{C}{\Phi_0}$$

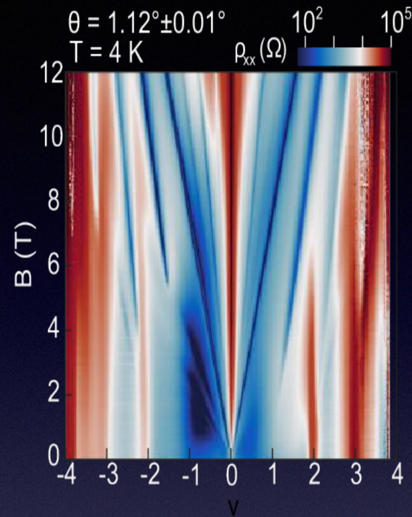


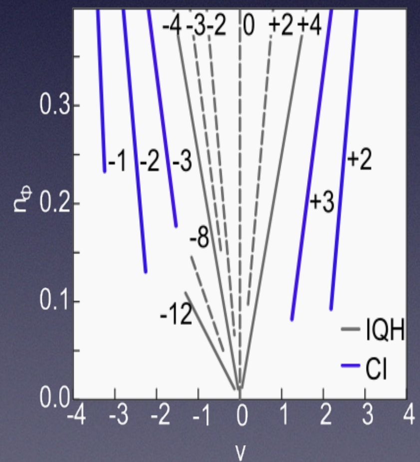

Chern Numbers $= \pm (4 - \nu)$

K. P. Nuckolls*, M. Oh*, D. Wong*, Lian, BAB, Yazdani. arXiv:2007.03810 to appear in Nature (2020).

# Transport experiments also see Chern insulators

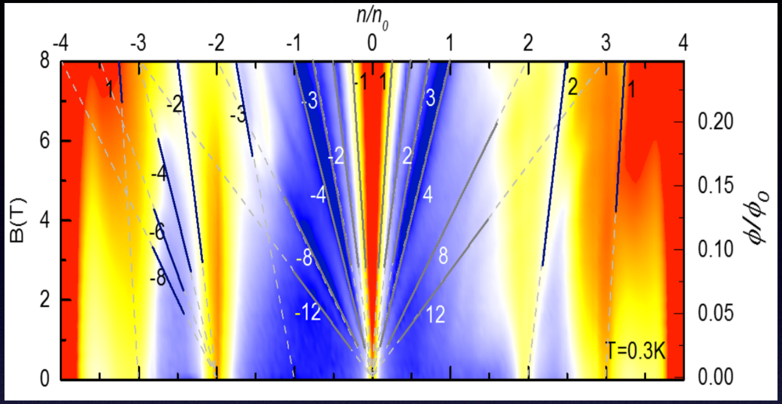
Transport can not rule out single particle gapped states but spectroscopy does



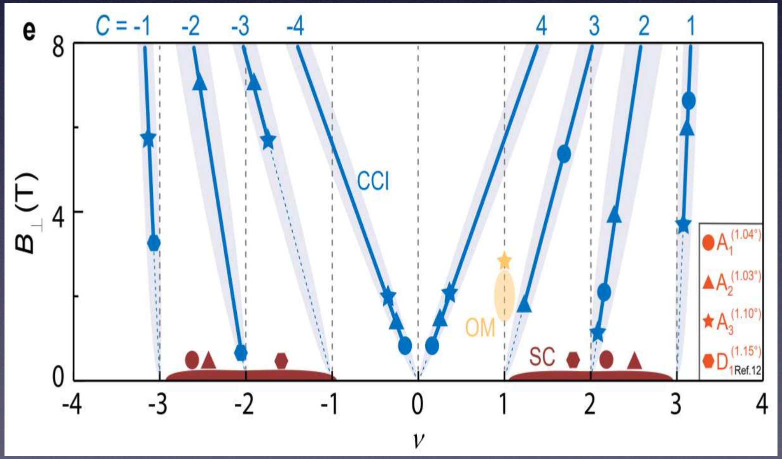




Young Group (2020)
UC Santa Barbara

Andrei Group (2020)
Rutgers University




Efetov Group (2020)
ICFO




S. Wu et al. arXiv preprint arXiv:2007.03735 (2020).

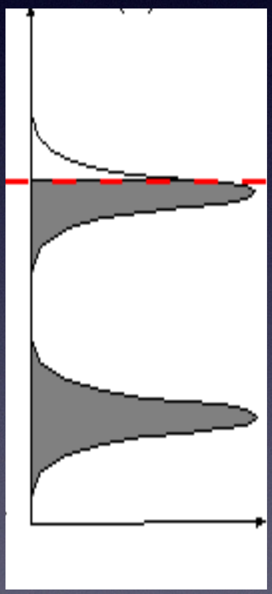
Y. Saito et al. arXiv preprint arXiv:2007.06115 (2020).

Das et al. arXiv preprint arXiv:2007.13390 (2020). Y. Choi et al. arXiv preprint arXiv:2008.11746 (2020). J. M. Park et al. arXiv preprint arXiv:2008.12296 (2020

# New Platform for Correlations & Topology

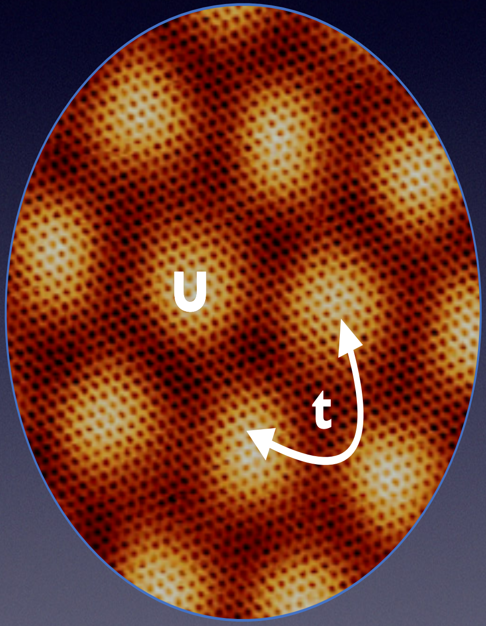
Correlations are strong when interactions > kinetic energy
(U>t)

Flat Landau level at partial filling



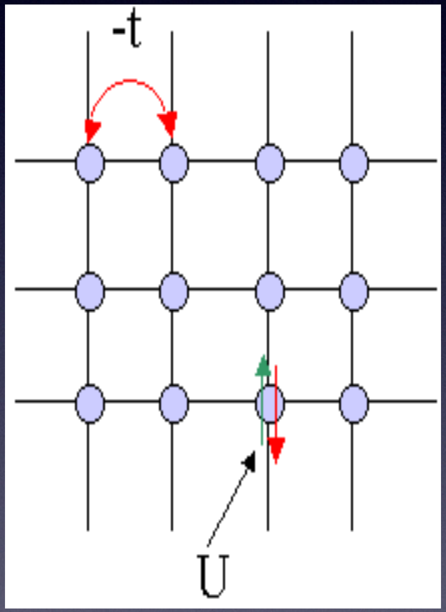
Landau Levels

$$\psi_n = z^n e^{-\frac{|z|^2}{4}}$$




Moire flat bands have elements of both systems and are correlated & topological

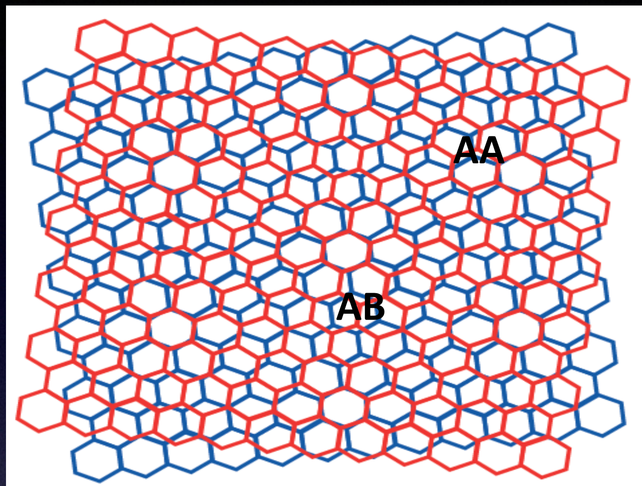
Correlated Materials



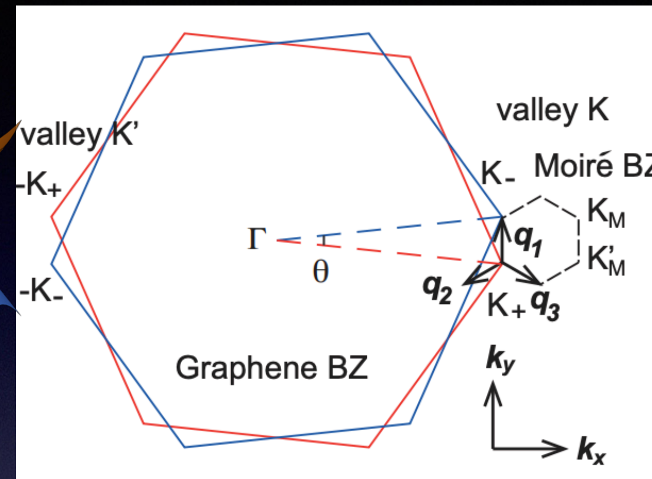

Hubbard model
(Tight binding models t
Wannier functions U )

# TBGI: Origin of Flat Bands and Single Particle Perturbation Theory

BAB, Song, Regnault Lian, *TBG I* (2020)








$$H^K(\boldsymbol{r}) = \begin{pmatrix} -iv_F\boldsymbol{\sigma}\cdot\nabla & T(\boldsymbol{r}) \\ T^\dagger(\boldsymbol{r}) & -iv_F\boldsymbol{\sigma}\cdot\nabla \end{pmatrix}$$

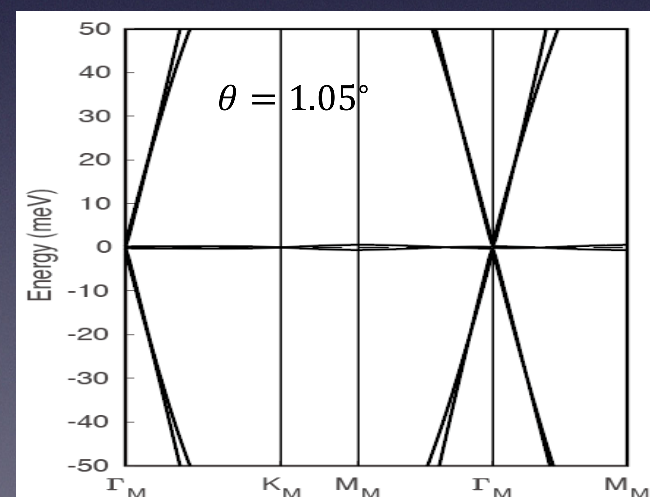
Interlayer hopping: $T(\boldsymbol{r}) = \sum_{j=1}^{3} T_j e^{i\boldsymbol{q}_j\cdot\boldsymbol{r}}$ ,

$$T_j = w_0\sigma_0 + w_1\left[\sigma_x \cos\frac{2\pi(j-1)}{3} + \sigma_y \sin\frac{2\pi(j-1)}{3}\right].$$

$w_0$: AA hoping $\leq$ $w_1$: AB/BA hoping

$\eta = \pm$: valley K, K'

$s = \uparrow, \downarrow$: spin




Bistritzer, Macdonald (2011)

# Topology In TBG Is Stable, Not Fragile

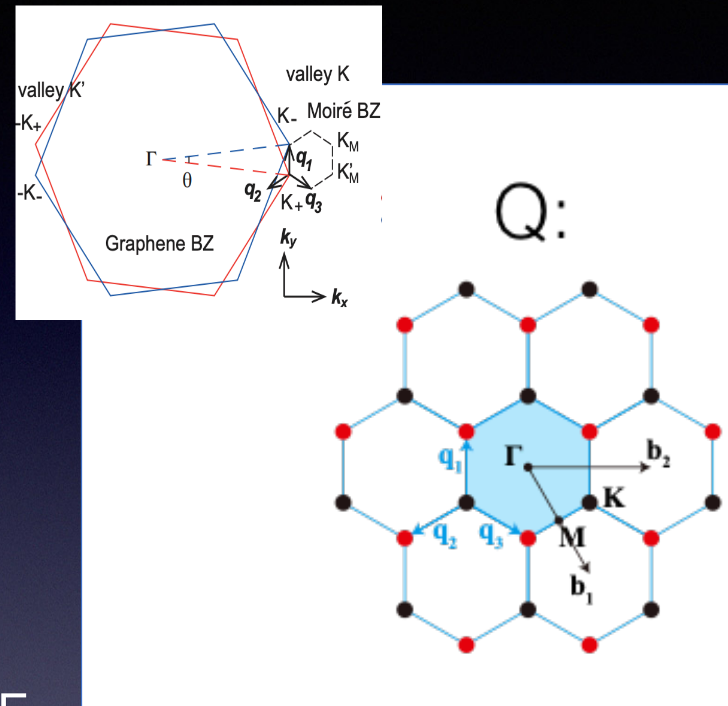
Zhida Song et al, 2018; Song et al TBGII, 2020

Unitary Particle Hole, Squares to -1

$$H\left(\mathbf{k}\right) = -D^{\dagger}\left(P\right) H\left(-\mathbf{k}\right) D\left(P\right)$$

$$D_{\mathbf{Q}',\mathbf{Q}}\left(P\right) = \delta_{\mathbf{Q}',-\mathbf{Q}}\zeta_{\mathbf{Q}}$$

$$\boxed{\{P, \bar{C}_{2x}\} = 0}$$




PH is very good at low energies

Comes from PH of Dirac in Graphene at low E

PH used in 2018 to show state must be topological (thought to be fragile)

PH used in 2019, 2020 to obtain enlarged interacting U(4), U(4)XU(4) symmetries

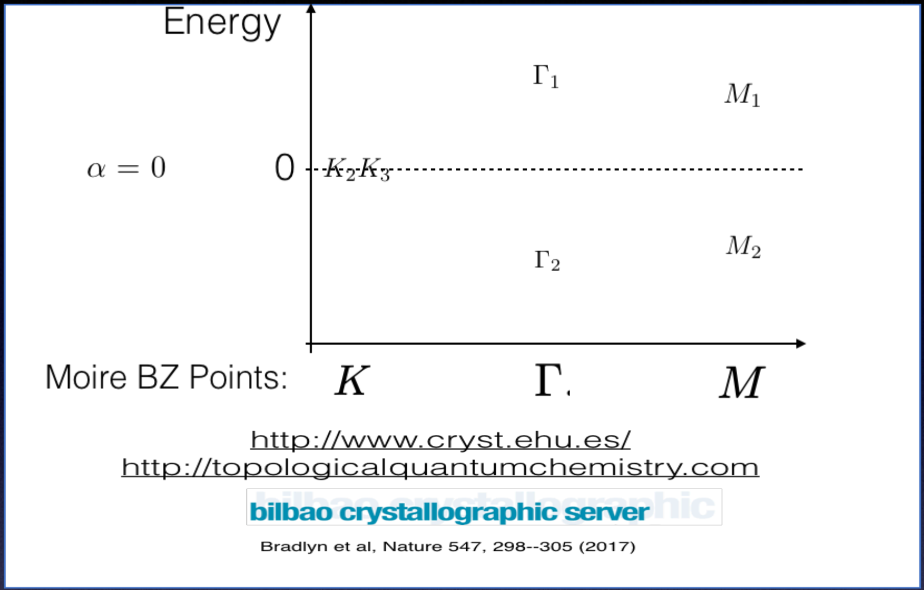
2020: PH provides stable topology, TBG one valley is topological anomalous for any number of bands

# Using Particle-Hole for Model Independent Theorems in TBG:

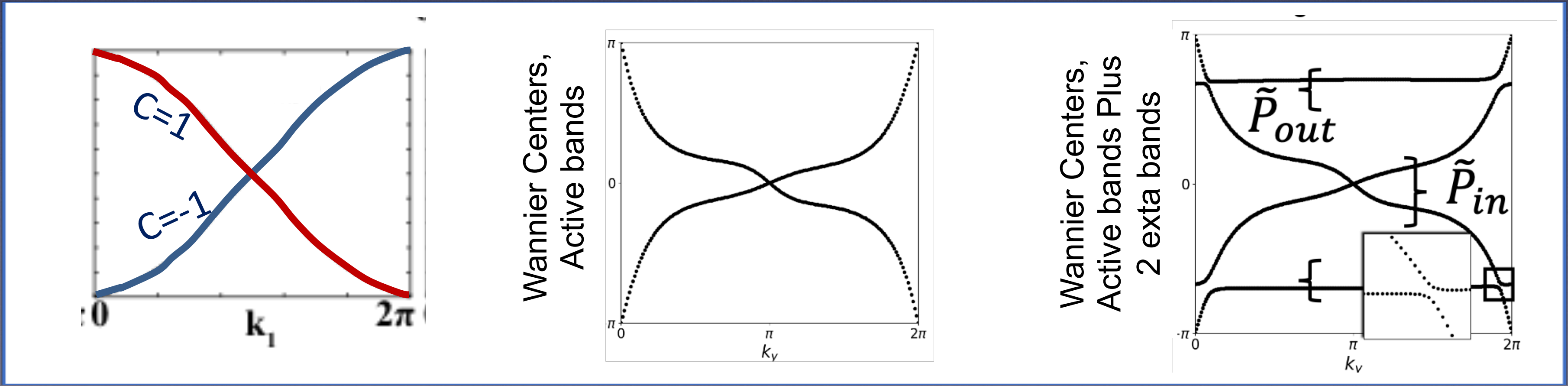
Zhida Song et al, 2018:

1. Representations of active bands fixed by particle-hole; topological at ***all*** angles

2. Topology thought to be “fragile”: adding atomic bands would trivialize the active bands




**Topology from Other Types of Pictures: H. C. Po, L. Zou, A. Vishwanath, and T. Senthil Phys. Rev. X 8, 031089 (2018), Kang, Vafek 2018, L. Zou, H. C. Po, A.. Vishwanath, and T. Senthil 2018; Ahn, S. Park, and B.-J. Yang 2018, Xi Dai et al**

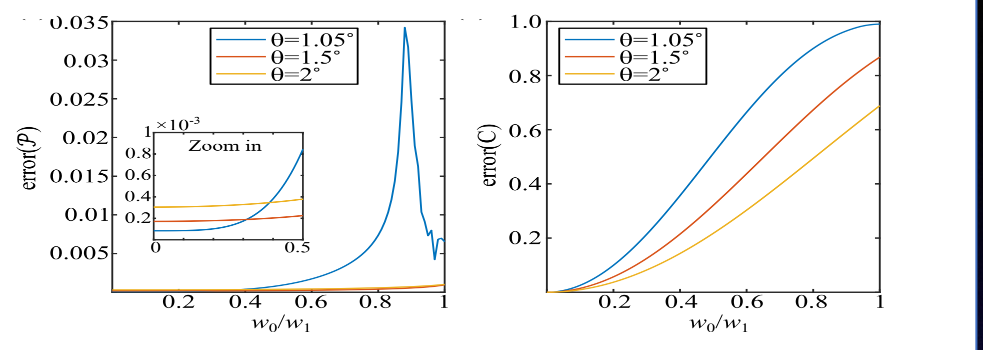


# One Valley TBG: Inconsistent With Any Lattice Model, Anomalous, Stable

Song, Lian, Regnault, BAB *TBG II* (2020)

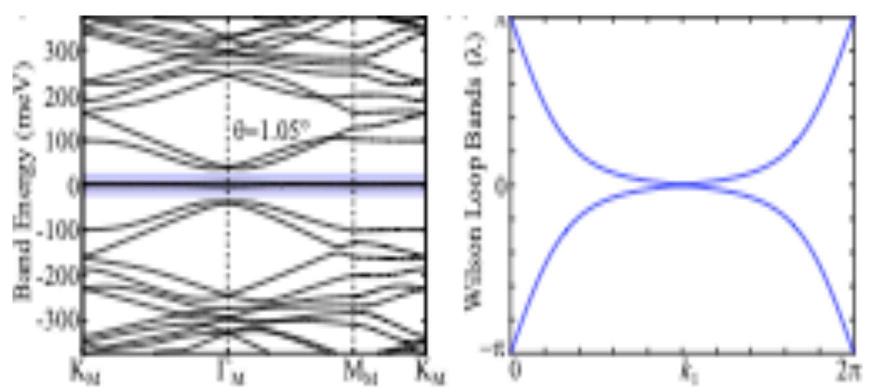
PH: extremely good symmetry ***of the wavefunctions***, from the chiral limit to the isotropic limit (unlike chiral symmetry)

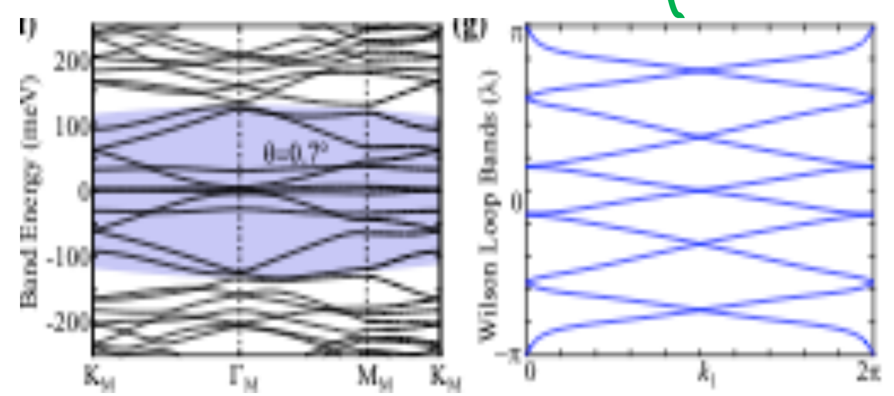
Kang and Vafek: good symmetry even under RG




$$\text{error}(\mathcal{P}) = 1 - \int \frac{d^2\mathbf{k}}{(2\pi)^2} |\langle u_{1,-\mathbf{k}}|\mathcal{P}|u_{-1,\mathbf{k}}\rangle|^2$$

Our PH protects Kramers-like degeneracies in the Wannier spectrum
Always spectral flow if Diracs of same helicity at zero energy

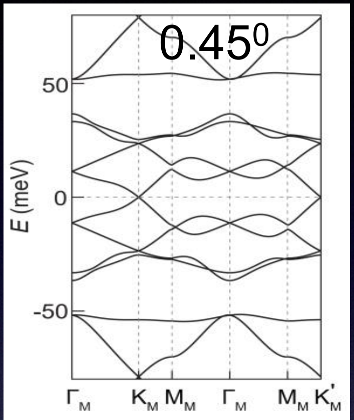


# Theorem: One Valley TBG is Anomalous For Any Number of Bands

Song, Lian, Regnault, BAB *TBG II* (2020)

Any single valley model respecting $C_{2z}T$ and PH not compatible with a lattice




*So what?*

1. Important for lower "magic" angles where more bands connected
2. In TBGIII, U(4) and U(4)xU(4) interacting symmetries valid for *any* number of bands, incompatible with any lattice
3. All lattice models (10 band model of Po et al, 2018), break a symmetry (usually particle-hole, see Pixley, 2020)
4. Important theorem for superconductivity:

**S-wave induced superconductivity in TBG is topological**

When an anomalous metal is gapped by SC, topological SC appears (remember Fu-Kane)
Stevan Nadj-Perge , (2020)

We can prove: Any weak pairing term preserving spin-SU(2), valley-U(1), time-reversal, C2zT, and PH must drive the system into a higher order topological superconducting phase. C2zT-protected Majorana corner states are bound to C2x-invariant corners of the sample

A. Chew, Y. Wang, BAB, Z. Song, to appear

# Theorem: Weak Pairing Superconductivity in TBG is topological

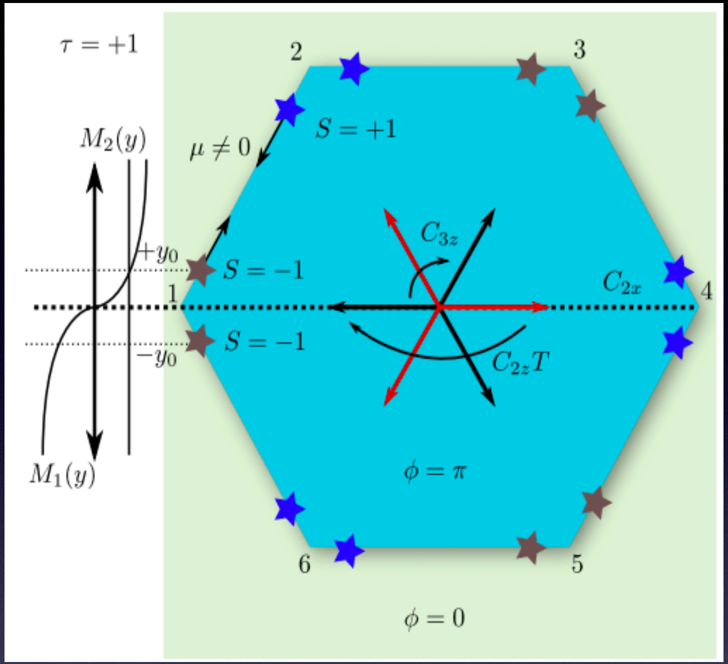
A. Chew, Y. Wang, BAB, Z. Song, to appear

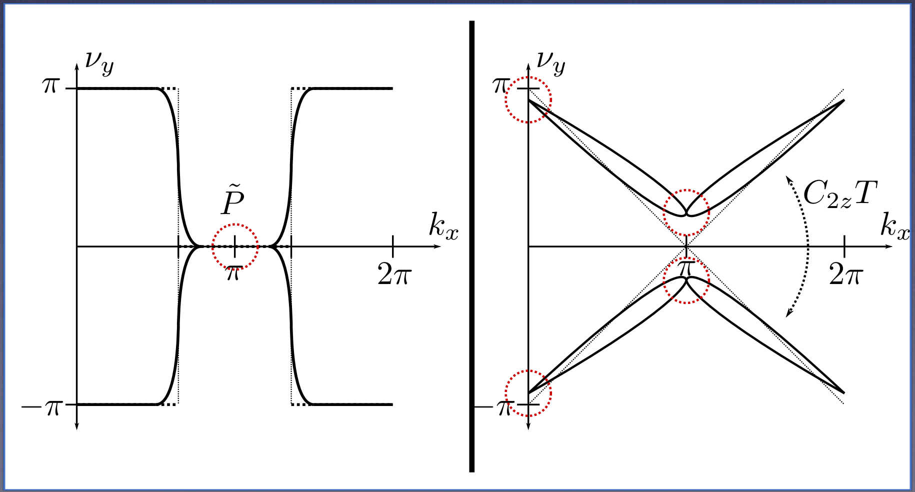
—TBG-TSC captures corner modes at domain walls

—Single valley four Majorana zero modes are bound to $C_{2x}$ invariant points

—Breaking U(1) valley: Majorana modes hybridize

—Breaking particle-hole P: zero modes to move along edge




—Left panel: Wilson loop spectrum in a single valley. Jumps occur at Dirac points and are smoothed out by pairing, resulting in nontrivial Dirac flow.

-Right panel: If different valleys are allowed to hybridize then spectrum can be made trivial.

# Interaction Hamiltonian

Kang, Vafek (2018,2019), Bultinck et al. (2020)
BAB, Song, Regnault, Lian, *TBG III* (2020), Lian, Song, Regnault, Efetov, Yazdani, BAB, TBG IV (2020)

$\delta\rho_q$:electron density.
$\boldsymbol{G}$: moiré k vectors

$$\hat{H}_I = \frac{1}{2\Omega_{\rm tot}} \sum_{\mathbf{G}\in\mathcal{Q}_0} \sum_{\mathbf{q}\in\mathrm{MBZ}} V(\mathbf{q}+\mathbf{G})\delta\rho_{-\mathbf{q}-\mathbf{G}}\delta\rho_{\mathbf{q}+\mathbf{G}} \ ,$$

Flat-band projection to lowest $8n_{\rm max}$ bands:

$$H_I = \frac{1}{2\Omega_{\rm tot}} \sum_{\mathbf{q}\in\mathrm{MBZ}} \sum_{\mathbf{G}\in\mathcal{Q}_0} O_{-\mathbf{q},-\mathbf{G}} O_{\mathbf{q},\mathbf{G}} \ ,$$

$$O_{\mathbf{q},\mathbf{G}} = \sum_{\mathbf{k}\eta s} \sum_{|m|,|n|\le n_{\max}} \sqrt{V(\mathbf{q}+\mathbf{G})} \underbrace{M^{(\eta)}_{m,n}(\mathbf{k},\mathbf{q}+\mathbf{G})}_{\text{form factor}} \left(\rho^{\eta}_{\mathbf{k},\mathbf{q},m,n,s} - \frac{1}{2}\delta_{\mathbf{q},\mathbf{0}}\delta_{m,n}\right)$$

$\boldsymbol{\eta = \pm}$**: valley K, K'** $\boldsymbol{s = \uparrow,\downarrow}$**: spin** $\boldsymbol{n = \pm 1, \pm 2, \cdots}$ **band**

$H_I \geq 0$ positive semi-definite Hamiltonian (**PSDH**).

Kang, Vafek (2019), Huber, 2017, TBGIII (2020)

# Projected Interaction Symmetries In Limits

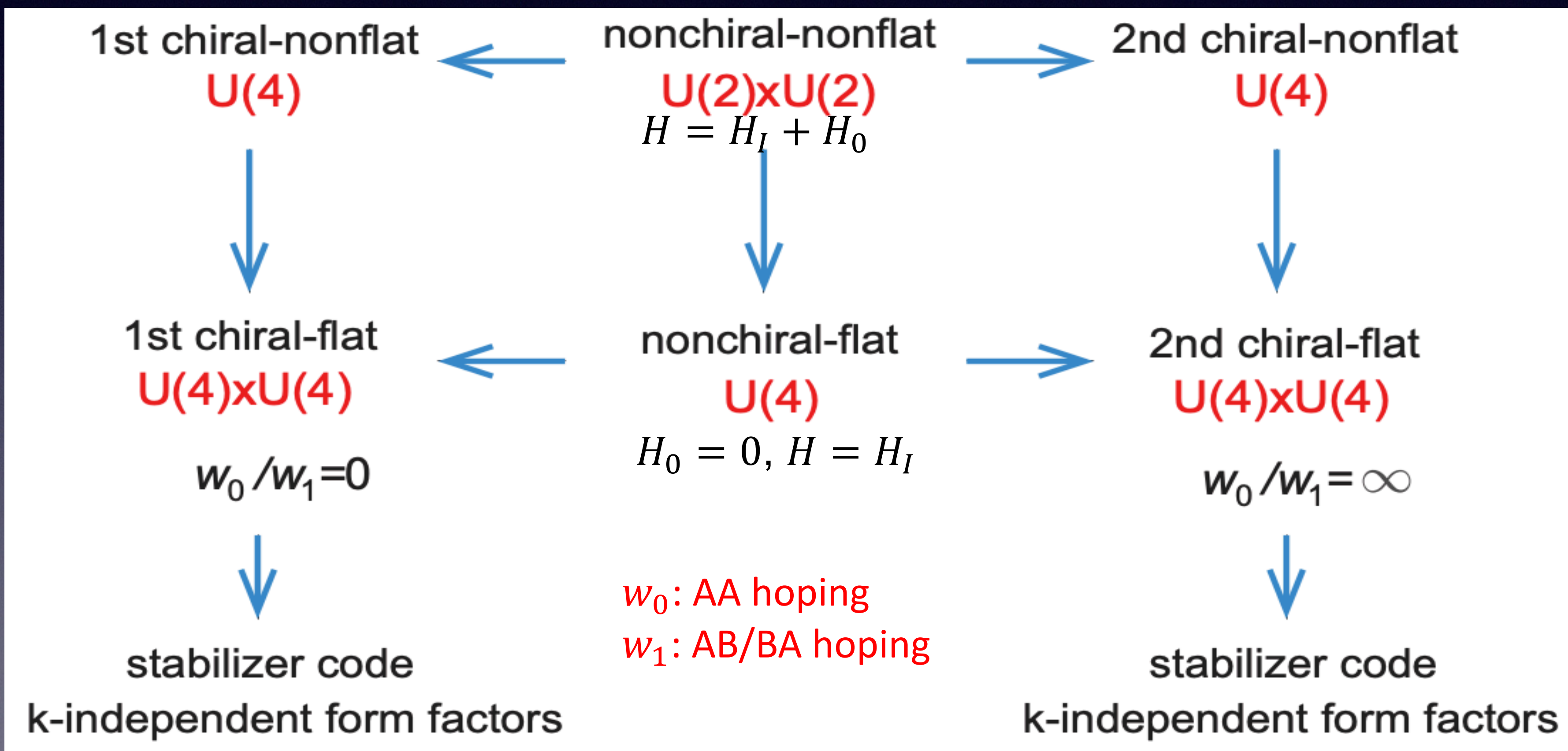
BAB, Song, Regnault, Lian, *TBG III* (2020), Lian et al, TBG IV (2020), Kang, Vafek (2018,2019), Bultinck et al. (2020), Hejazi, Chen, Balents (2020)

PH symmetry $P$ combined with $C_{2z}$

$H_0$ = Bistritzer Macdonald Hamiltonian

Extra Chiral symmetries in some limits

$$\{C_{2z}P, H_0\} = 0 \;, \quad [C_{2z}P, H_I] = 0 \;.$$



# Strategy: Start In Exact Limits, Find Perturbations

Lian, **S**ong, Regnault, Efetov, Yazdani, BAB, *TBG IV* (2020), Vafek Kang (2019), Bultinck et al. (2020

$N_M$: # of moire cells
$A_G$: constants

- $|\Psi\rangle$ will be an **eigenstate** of $H_I$ if (for some $A_G$):

$$(O_{\mathbf{q},\mathbf{G}} - A_{\mathbf{G}} N_M \delta_{\mathbf{q},0})|\Psi\rangle = 0$$

$$H_I = \frac{1}{2\Omega_{\text{tot}}} \sum_{\mathbf{q}\in\text{MBZ}} \sum_{\mathbf{G}\in\mathcal{Q}_0} O_{-\mathbf{q},-\mathbf{G}} O_{\mathbf{q},\mathbf{G}} \,,$$

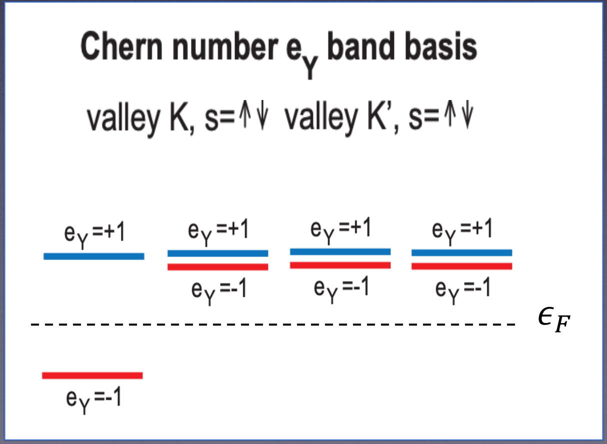
- Flat *(first) chiral limit* $w_0 = 0 < w_1$, under the ***Chern basis***:

$$O_{\mathbf{q},\mathbf{G}} = O^0_{\mathbf{q},\mathbf{G}} = \sum_{\mathbf{k},e_Y,\eta,s} \sqrt{V(\mathbf{k}+\mathbf{G})} M_{e_Y}(\mathbf{k},\mathbf{q}+\mathbf{G}) \left( d^\dagger_{\mathbf{k}+\mathbf{q},e_Y,\eta,s} d_{\mathbf{k},e_Y,\eta,s} - \frac{1}{2}\delta_{\mathbf{q},\mathbf{0}} \right) \,.$$

Eigenstates ***Ground state with Flat Metric Condition:***

$$|\Psi^{\nu_+,\nu_-}_{\nu}\rangle = \prod_{\mathbf{k}} \prod_{j_1=1}^{\nu_+} d^\dagger_{\mathbf{k},+1,\eta_{j_1},s_{j_1}} \prod_{j_2=1}^{\nu_-} d^\dagger_{\mathbf{k},-1,\eta'_{j_2},s'_{j_2}} |0\rangle$$

- Filling $\nu = \nu_+ + \nu_- - 4$,
- Chern number $C = \nu_+ - \nu_- = 4 - |\nu|, 2 - |\nu|, \cdots, |\nu| - 4$ (all degenerate in the chiral limit).



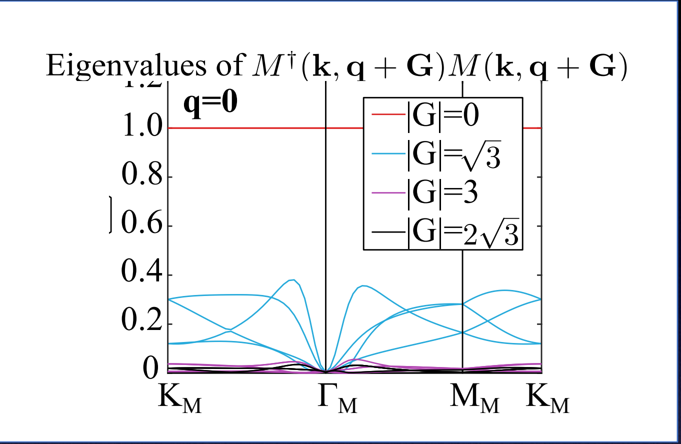

Also, Ming and MacDonald, 2019,2020

# Flat Metric Condition (FMC) and Ground-States

Lian, **S**ong, Regnault, Efetov, Yazdani, BAB, *TBG IV* (2020), Vafek Kang (2018, 2019)

$$M_{m,n}^{(\eta)}(\mathbf{k},\mathbf{q}+\mathbf{G}) = \sum_{\alpha}\sum_{\mathbf{Q}\in\mathcal{Q}_\pm} u^*_{\mathbf{Q}-\mathbf{G},\alpha;m\eta}(\mathbf{k}+\mathbf{q})\, u_{\mathbf{Q},\alpha;n\eta}(\mathbf{k})$$

**FMC:** $M_{m,n}^{(\eta)}(\mathbf{k},\mathbf{G}) = \xi(\mathbf{G})\delta_{m,n}$

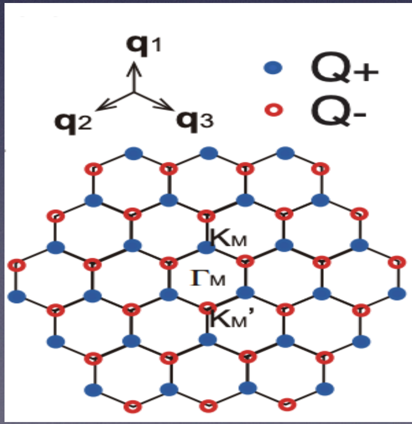



- If form factors satisfy ***FMC,*** $|\Psi\rangle$ is a **ground state** in the flat-band limit ($H_0 = 0$).

$u_{\mathbf{Q},\alpha;n\eta}(\mathbf{k})$ decays exponentially from center, for large plane waves

See TBGI for proof $u_Q \cong Q\, u_{Q+1}$

FMC satisfied, for all G with exception of |G|=1




Remarkably, we found that Kang and Vafek Wannier basis Ham satisfies exactly the FMC (see TBG II)

# TBGIV: NonChiral-NonFlat Ground-States

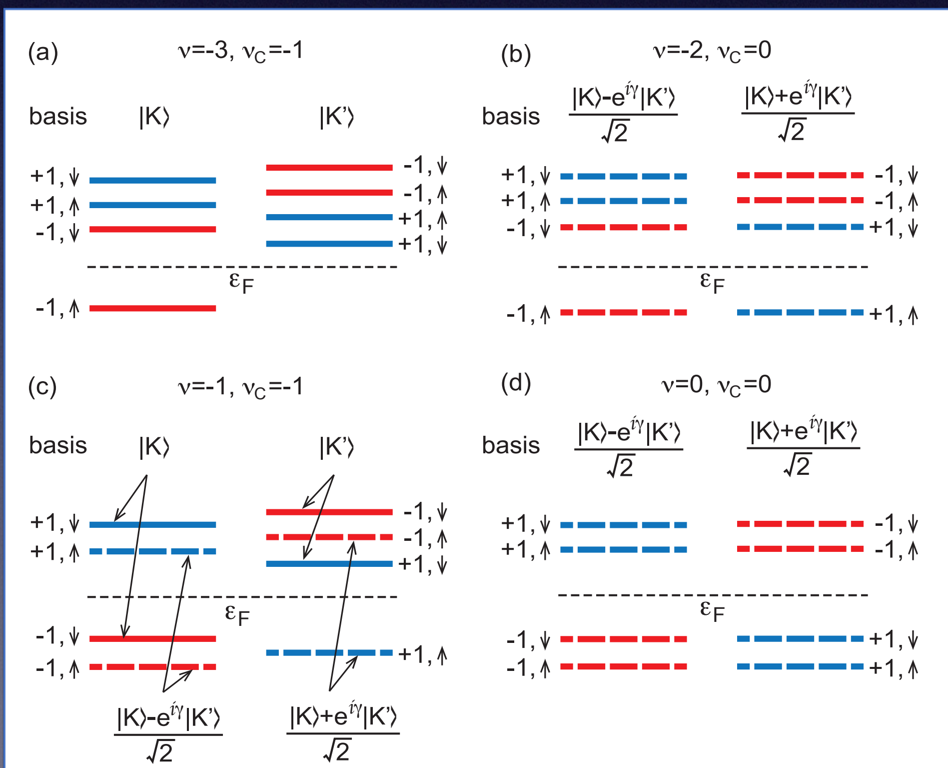
Lian, Song, Regnault, Efetov, Yazdani, BAB, *TBG IV* (2020)

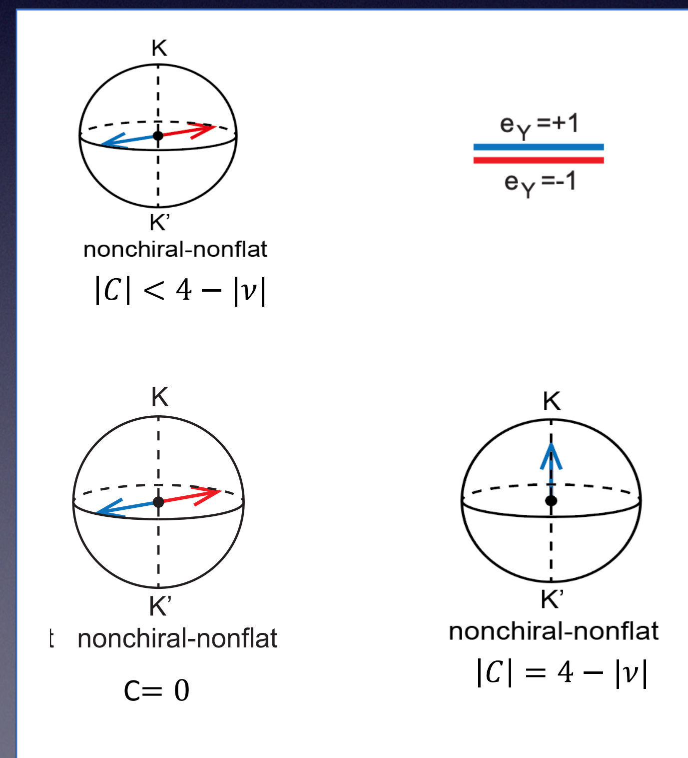
For even fillings, see also Bultnick et al, 2019, 2020 and Kang-Vafek, 2019;
Also, Ming and MacDonald, 2019,2020

- $0<|C| < 4-|\nu|$: partially intervalley coherent.
- $|C| = 4-|\nu|$: valley polarized
- |C|=0; inter-valley coherent

Lowest C are ground-states

$\nu = 0$, Exact GS, no FMC needed
$|\nu| = 2$, Exact State, GS only with FMC
$|\nu| = 2$, Perturbative GS



# TBGIV: Agreement and Predictions

Lian, Song, Regnault, Efetov, Yazdani, BAB, *TBG IV* (2020) Xie, Cowsik, Song, Lian, BAB, Regnault, *TBG VI* (2020)

- GS at Filling +/-2 exhibits C=0 in B=0 and |C|=2 in B>0.5-1T

  (explains Yazdani, Andrei, Young, Efetov, Nadj-Perge experiments, 2020)

- GS at Filling +/-1 exhibits |C|=1 at B=0 and |C|=3 in B>1T

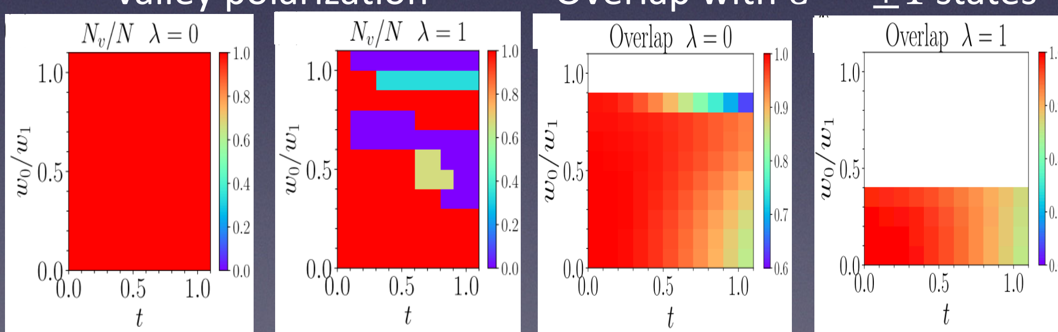
  (explains B>1T Yazdani, Andrei, Young, Efetov, Nadj-Perge experiments, 2020)
  (**Prediction**: |C|=1 GS at B=0)

- GS at Filling +/-3 is predicted by perturbation theory |C|=1 but numerics and analytics show excitation gap closing, and nematic/CDW order developing

$\nu = -3$ **Valley polarized**
**Chern number C=** $\pm 1$
$w_0 < 0.9 w_1 / 0.4 w_1$ w/wo FMC

Large $w_0/w_1$: **gap closing transition**.
Ground state at $\Gamma_M$ (nematic) or $K_M, M_M$ (CDW),

valley polarization | Overlap with $C = \pm 1$ states




with FMC | without FMC | with FMC | without FMC

(see also recent DMRG, Kang, Vafek (2020), Zaletel (2020))

# TBGV+ VI: Exact Wavefunctions and Energies of Charge +/-1 Excitations

BAB, Lian, Cowsik, Xie, Regnault, Song, *TBG V* (2020). Xie, Cowsik, Song, Lian, BAB, Regnault, *TBG VI* (2020) Vafek and Kang, 2020, same week on arxiv

**Chiral-flat / nonchiral-flat** limits, exact charge $\pm 1$ excitations form a 2×2 Hamiltonian:

$$\left[H_I - \mu N, c^\dagger_{\mathbf{k},n,\eta,s}\right]|\Psi\rangle = \frac{1}{2\Omega_{\text{tot}}}\sum_m R^\eta_{mn}(\mathbf{k}) c^\dagger_{\mathbf{k},m,\eta,s}|\Psi\rangle ,$$

$$R^\eta_{mn}(\mathbf{k}) = \sum_{\mathbf{G}\mathbf{q}m'} V(\mathbf{G}+\mathbf{q}) \underbrace{M^{(\eta)*}_{m'm}(\mathbf{k},\mathbf{q}+\mathbf{G})\, M^{(\eta)}_{m'n}(\mathbf{k},\mathbf{q}+\mathbf{G})}_{\text{Finite q Generalization of the Fubini Study Metric}}$$

Remember The Form Factors?

Dispersion and eigenstates of the exact excitations related to the FS metric

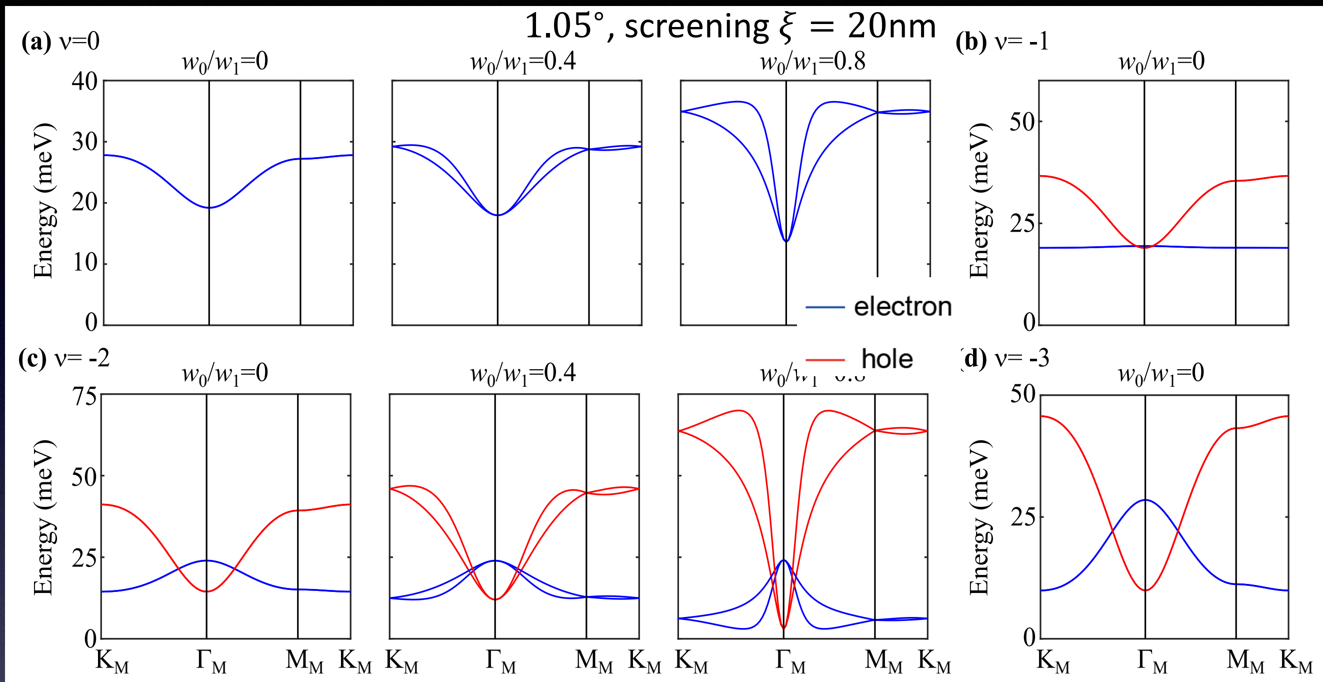
(for more on the FB metric see Paivi et al, 2017, Huber et al, 2017, Rossi et al, 2019, Fang et al, 2019)

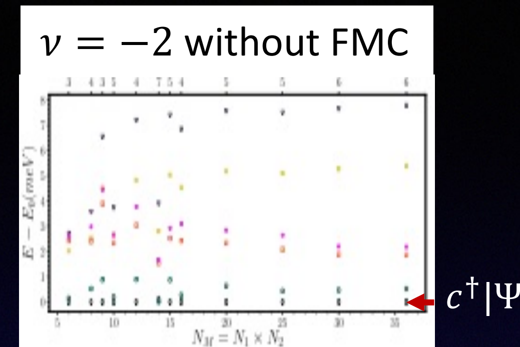
At charge neutrality, where the FMC not needed, or at any filling, with FMC, gapped, positive semidefinite excitations

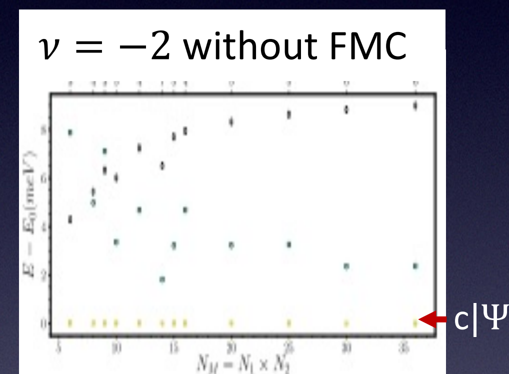
Without FMC, slightly more complicated exact expression obtainable

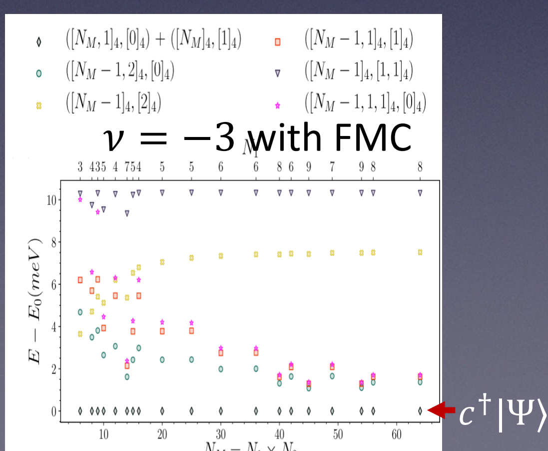
# TBGV+ VI: Exact Wavefunctions and Energies of Charge +/-1 Excitations

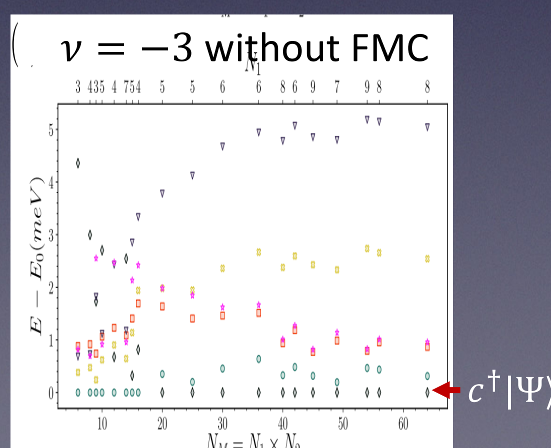
BAB, Lian, Cowsik, Xie, Regnault, Song, *TBG V* (2020). Xie, Cowsik, Song, Lian, BAB, Regnault, *TBG VI* (2020) Vafek and Kang, 2020, same week on arxiv

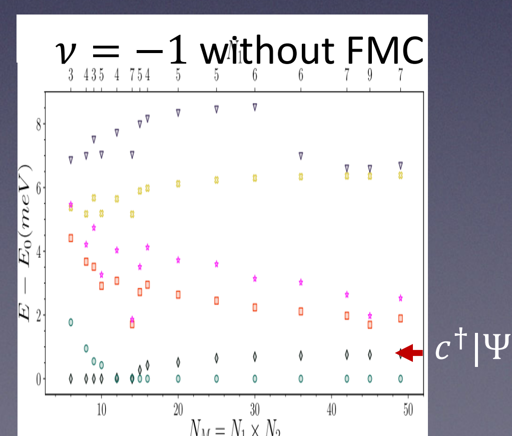


# TBGV: Exact Neutral and Goldstone Excitations Expressions

BAB, Lian, Cowsik, Xie, Regnault, Song, *TBG V* (2020). Xie, Cowsik, Song, Lian, BAB, Regnault, *TBG VI* (2020) Vafek and Kang, 2020, same week on arxiv

Exact charge neutral excitations (including Goldstones) obtained exactly, solving a 1-body problem

$$\left[H_I - \mu N, c^\dagger_{\mathbf{k}+\mathbf{p},m_2,\eta_2,s_2} c_{\mathbf{k},m_1,\eta_1,s_1}\right] |\Psi_\nu\rangle = \frac{1}{2\Omega_{\text{tot}}} \sum_{m,m'} \sum_{\mathbf{q}} S^{(\eta_2,\eta_1)}_{m,m';m_2,m_1}(\mathbf{k}+\mathbf{q},\mathbf{k};\mathbf{p})\, c^\dagger_{\mathbf{k}+\mathbf{p}+\mathbf{q},m,\eta_2,s_2} c_{\mathbf{k}+\mathbf{q},m',\eta_1,s_1} |\Psi_\nu\rangle$$

Exact zero modes can be proved analytically

U(4) and U(4)xU(4) Goldstone counting comes out naturally from the wavefunctions

Goldstone stiffness can be obtained $E_{\text{Goldstone}}(\mathbf{p}) = \frac{1}{2} m_{ij} p_i p_j$

## Chiral limit:

| Little group | Number of GMs | Ground states |
|---|---|---|
| $\mathrm{U}(4)\times\mathrm{U}(4)$ | 0 | $\lvert\Psi_0^{4,0}\rangle$ |
| $\mathrm{U}(1)\times\mathrm{U}(3)\times\mathrm{U}(4)$ | 3 | $\lvert\Psi_{-3}^{1,0}\rangle$, $\lvert\Psi_{-1}^{3,0}\rangle$ |
| $\mathrm{U}(2)\times\mathrm{U}(2)\times\mathrm{U}(4)$ | 4 | $\lvert\Psi_{-2}^{2,0}\rangle$ |
| $\mathrm{U}(1)\times\mathrm{U}(3)\times\mathrm{U}(1)\times\mathrm{U}(3)$ | 6 | $\lvert\Psi_{-2}^{1,1}\rangle$, $\lvert\Psi_0^{3,1}\rangle$ |
| $\mathrm{U}(2)\times\mathrm{U}(2)\times\mathrm{U}(1)\times\mathrm{U}(3)$ | 7 | $\lvert\Psi_{-1}^{2,1}\rangle$ |
| $\mathrm{U}(2)\times\mathrm{U}(2)\times\mathrm{U}(2)\times\mathrm{U}(2)$ | 8 | $\lvert\Psi_0^{2,2}\rangle$ |

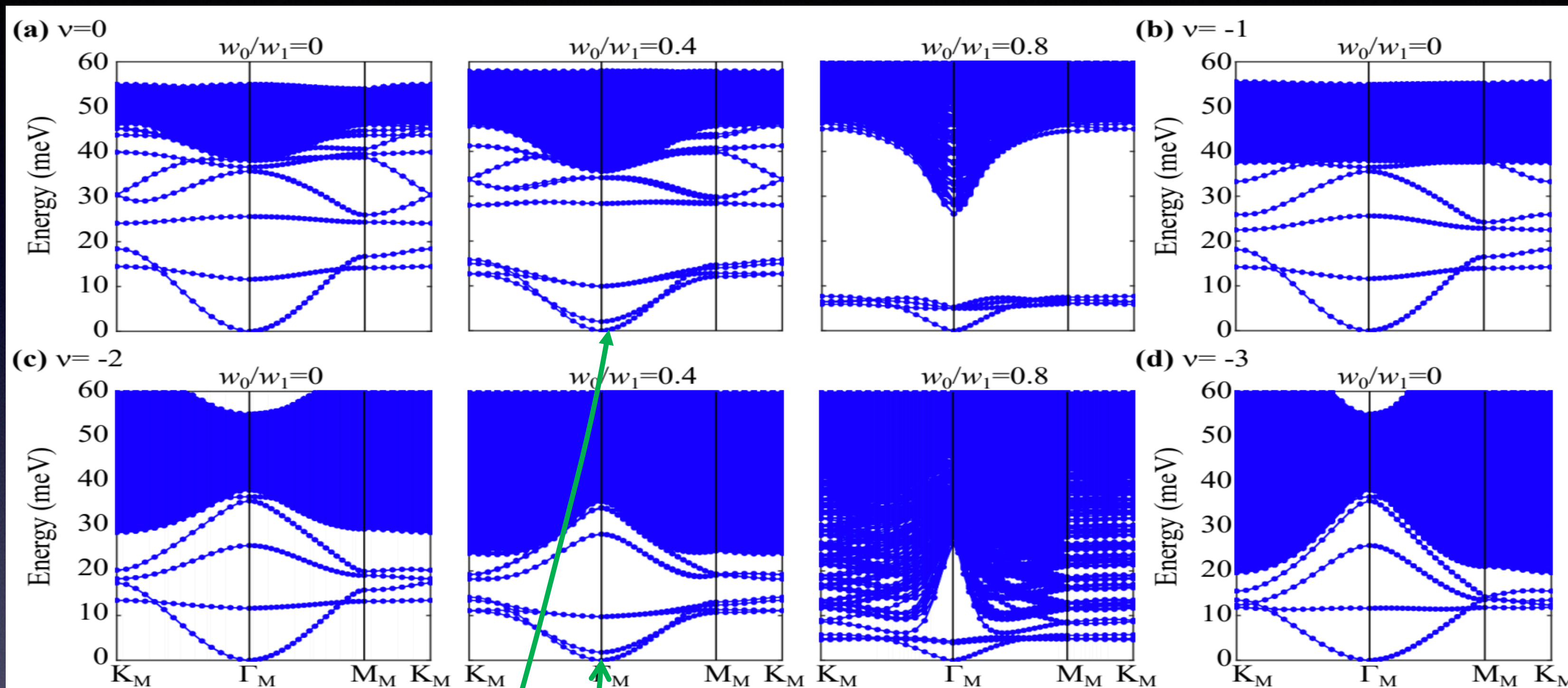
## Non-Chiral limit:

| Little group | Number of GMs | Ground states |
|---|---|---|
| $\mathrm{U}(1)\times\mathrm{U}(3)$ | 3 | $\lvert\Psi_{-2}\rangle$, $\lvert\Psi_2\rangle$ |
| $\mathrm{U}(2)\times\mathrm{U}(2)$ | 4 | $\lvert\Psi_0\rangle$ |

See also further work onGoldstone modes from Hartree Fock:
Kumar, Ajesh; Xie, Ming; MacDonald, A. H. 2020
Khalaf, et al, 2020

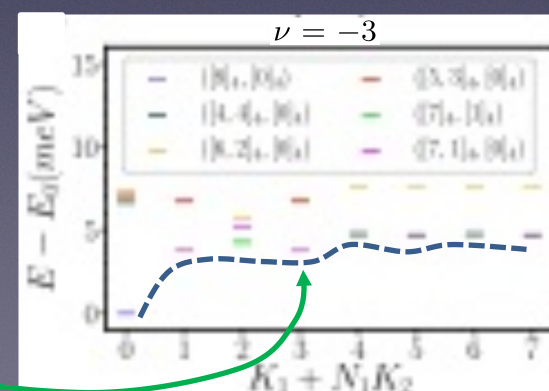
# TBGV: Exact Neutral and Goldstone Excitations Dispersion and ED

BAB, Lian, Cowsik, Xie, Regnault, Song, *TBG V* (2020). Xie, Cowsik, Song, Lian, BAB, Regnault, *TBG VI* (2020) Vafek and Kang, 2020, same week on arxiv




Softening branch of excitation modes, as higher symmetry chiral limit (left) is approached

ED Goldstone Branch, matches exact eigenstates



# TBGV: Exact Charge +/- 2 Excitations Expressions

BAB, Lian, Cowsik, Xie, Regnault, Song, *TBG V* (2020). Xie, Cowsik, Song, Lian, BAB, Regnault, *TBG VI* (2020)

Exact charge +/- excitations can be obtained exactly, solving a 1-body problem

$$\left[H_I - \mu N, c^\dagger_{\mathbf{k}+\mathbf{p},m_2,\eta_2,s_2} c^\dagger_{-\mathbf{k},m_1,\eta_1,s_1}\right]|\Psi\rangle = \frac{1}{2\Omega_{\text{tot}}}\sum_{m,m',\mathbf{q}} T^{(\eta_2,\eta_1)}_{m,m';m_2,m_1}(\mathbf{k}+\mathbf{q},\mathbf{k};\mathbf{p})\, c^\dagger_{\mathbf{k}+\mathbf{q}+\mathbf{p},m,\eta_2,s_2} c^\dagger_{-\mathbf{k}-\mathbf{q},m',\eta_1,s_1}|\Psi\rangle$$

Scattering matrix T has the identical terms of the Neutral modes, with **one crucial** sign difference

**For the Goldstone mode, this is a – (minus) sign, everything else is identical**

$$T_{e_{Y2};e_{Y1}}(\mathbf{k}+\mathbf{q},\mathbf{k};\mathbf{p}) = \delta_{\mathbf{q},0}(R_0(\mathbf{k}+\mathbf{p}) + R_0(-\mathbf{k})) + 2\sum_{\mathbf{G}} V(\mathbf{G}+\mathbf{q}) M_{e_{Y2}}(\mathbf{k}+\mathbf{p},\mathbf{q}+\mathbf{G}) M_{e_{Y1}}(-\mathbf{k},-\mathbf{q}-\mathbf{G})$$

Charge +1 energy at momentum **k+p**

Charge +1 energy at momentum **-k**

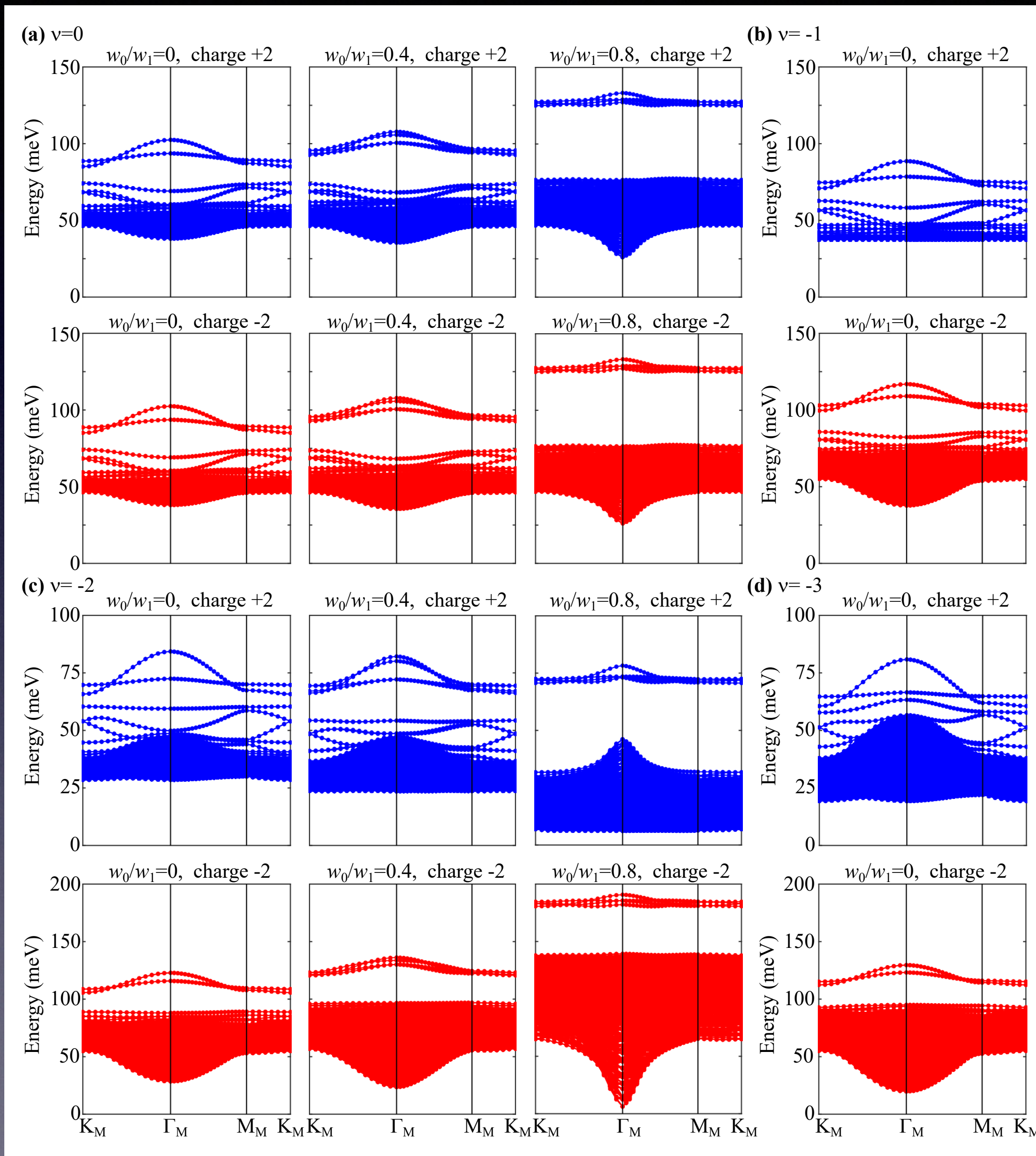
Sum of two non-interacting charge +1 excitations total momentum **p**

Interaction term between the two charge +1 excitations.

Depends on the form factors.

# TBGV: Exact Charge +/- 2 Excitations Plots

BAB, Lian, Cowsik, Xie, Regnault, Song, *TBG V* (2020). Xie, Cowsik, Song, Lian, BAB, Regnault, *TBG VI* (2020)




Gapped charge $\pm 2$ excitations without FMC ($\nu = 0$ equivalent to with FMC), 1.05°, screening $\xi = 20$nm

**Gapped excitations, with no bound states visible below the continuum!**

Bound states above the continuum, like an "inverse" Goldstone spectrum; this is due to the sign difference in the scattering matrix.

# TBGV: No (Luttinger-Kohn) Superconductivity Theorem For Flat Bands

BAB, Lian, Cowsik, Xie, Regnault, Song, *TBG V* (2020).

***Richardson criterion***: Richardson (1963, 1964): Absence of SC if

$$\Delta(N+2) = E(N+2) + E(N) - 2E(N+1) \geq 0$$

Lowest charge 2 excitation     Lowest charge 1 excitation

Cooper pairing energy in TBG flat-band limit at integer $\nu$:

$$T_{k+q,k;p} = \delta_{q,0}\left(R_{-k} + R_{k+p}\right) + T''_{k+q,k;p}$$

$T''$: interaction between pair of electrons

$T$: charge +2 Hamiltonian, $R$: charge +1 Hamiltonian

In TBGV we have showed that $T'' \geq 0$ analytically for these types of FB Coulomb projected H

*Weyl's inequality*: $\min(T) \geq 2\min(R) + \min(T'')$

→ pairing energy $\Delta(N+2) = \min(T) - 2\min(R) \geq \min(T'') \geq 0$ .

**No Cooper pairing** from Coulomb interaction in flat bands at integer fillings $\nu$ !

Berry phases cannot save you if you only have Coulomb.

# TBGV: Implications for Superconductivity

If our exact charge 2 are the lowest energy excitations, no pair binding w coulomb

**Superconductivity: requires kinetic energy - nonflat bands, or other pairing glue (e.g., phonon)**
Wu, Macdonald, Martin (2018), Lian, Wang, BAB (2018), Xie, Song, Lian, BAB, (2018), Peri, Song, BAB, Huber (2020)

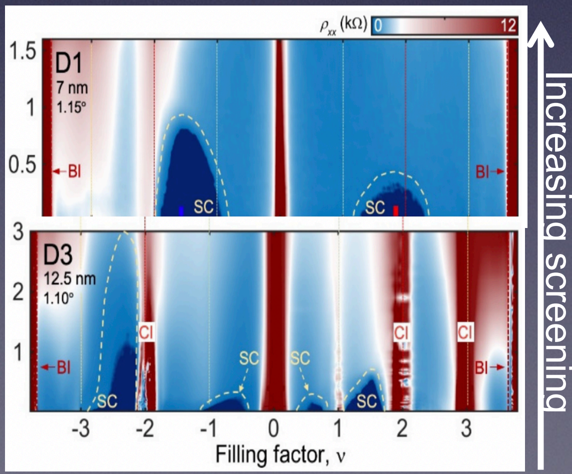
Since insulating states well-described by Coulomb, this suggests competition

Experiments proving correlations rather than causation; we know SC "seems" related to the insulating state, because you have to kill the insulating state to go to SC

But the question is does it compete with or is it helped by Coulomb interactions.

"At face value, it appears that the superconductivity can be decoupled from the correlated insulator… On the face of it these results points to superconductivity that is more robust than the correlated insulator and complicates attempts to interpret the superconductor as the result of doping a correlated insulator. " T. Senthil

Stepanov, Das, Lu, Fahimniya, Watanabe, Taniguchi, Koppens, Lischner, Levitov, Efetov, arXiv:1911.09198
Saito, Ge, Watanabe, Taniguchi, and Young, arXiv:1911.13302
Liu, Wang, Watanabe, Taniguchi, Vafek, and Li, arXiv:2003.11072
Arora, Polski, Zhang, Thomson, Choi, Kim, Lin, Wilson, Xu, Chu, Watanabe, Taniguchi, Alicea, Nadj-Perge, arXiv:2002.03003




Screening effect

# Conclusions

- Perturbation theory on the TBG model
- The TBG flat bands are proved to be stably topological.
- In combinations of chiral & flat limits, the Coulomb interacting Hamiltonian exhibit enhanced U(4) or U(4)×U(4) symmetries.
- Exact/perturbative (Chern) insulators are derived at integer fillings $\nu$ and confirmed in ED. Chern number transitions are predicted in magnetic field.
- Charge $0, \pm 1, \pm 2$ excitations can be exactly calculated.
- Outlook: superconductivity not from Coulomb? Phonons?
- Experimental consequences of Valley polarized/Coherent states?