Spatio-Temporal Dynamics of Magnon Bose-Einstein Condensates

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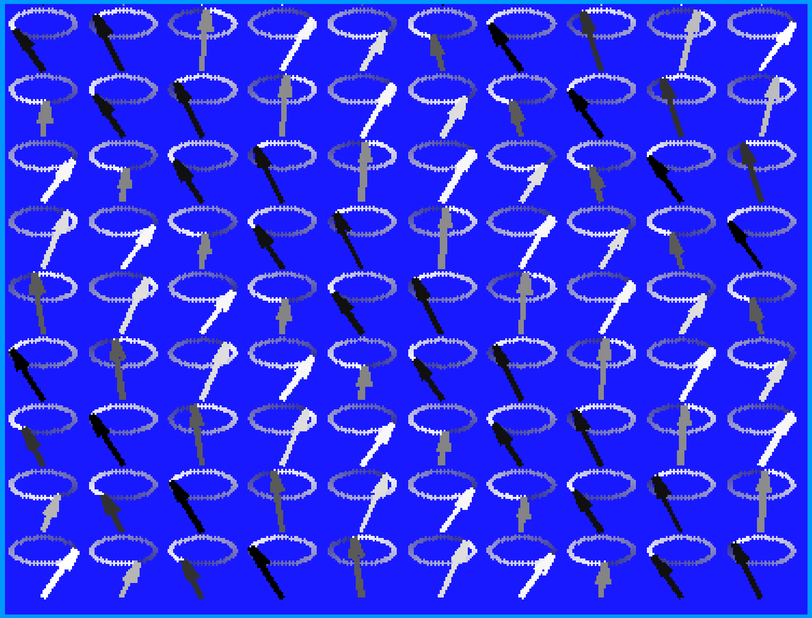


Courtesy: Prof. C. Patton

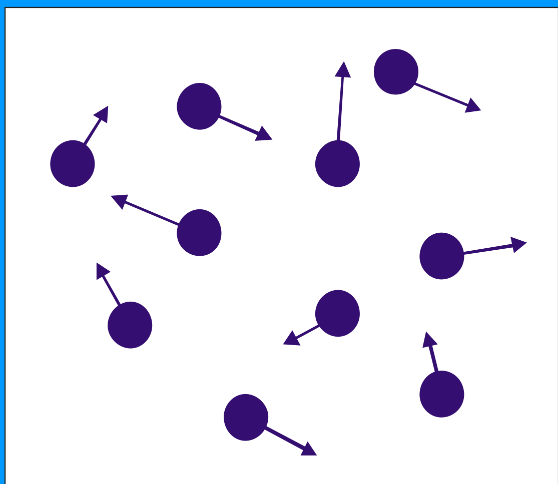
Magnons???
Ground state of a FM:
$S_z = Ns_z$
Excited states:
$S_z = Ns_z - 1$, $S_z = Ns_z - 2$, $S_z = Ns_z - 3$, …
1 magnon, 2 magnons, 3 magnons,…
⇒magnons are Bose-particles

Carry transverse magnetization

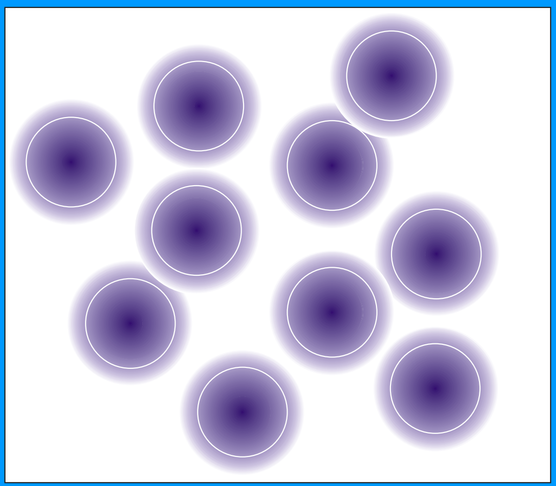
Group of NonLinear Magnetic Dynamics

Bose-Einstein-Condensation of atoms

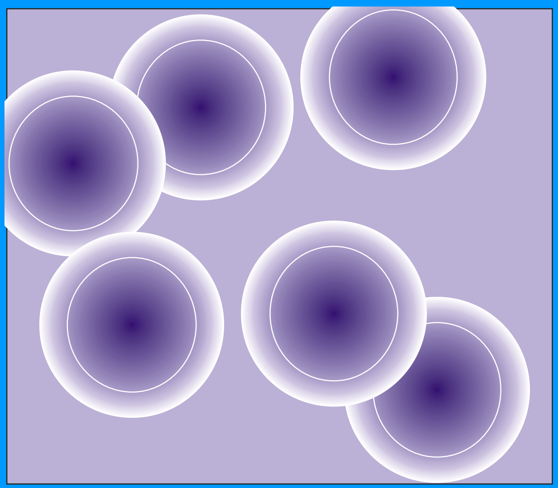
classical gas



quantum gas



BEC



Condition of BEC transition:

$$kT_c = 3.31\frac{\hbar^2}{m}N^{2/3}$$

$$N_c^{2/3} = kT\frac{m}{3.31\hbar^2}$$

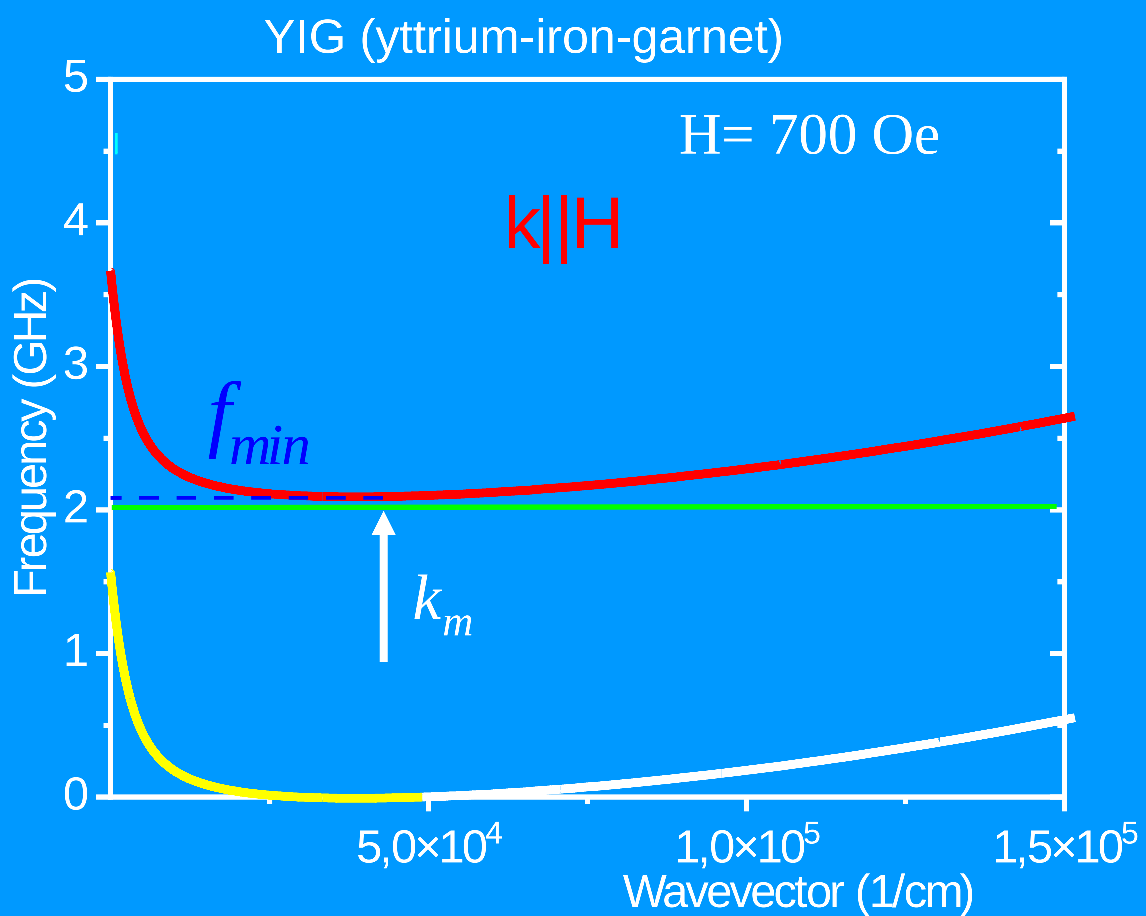
$$\langle\lambda\rangle \approx \sqrt{\langle r^2\rangle} = N^{-1/3}$$

Thermodynamics of BEC:

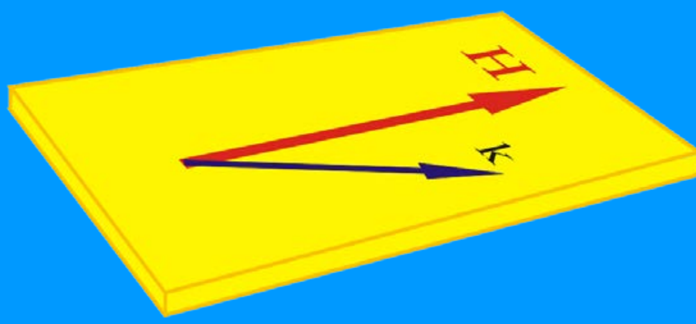
$$n = \frac{1}{\exp\left(\frac{E-\mu}{kT}\right)-1}$$

$$\mu(T_c, N_c) = E_{\min}$$

Magnons in ferromagnetic films



Transparent ferromagnet
Films 5-10 μm thick
No domains



Three contributions to the magnon energy: Zeeman, exchange, and dipole-dipole (long-range, anisotropic)

$@k_m$:

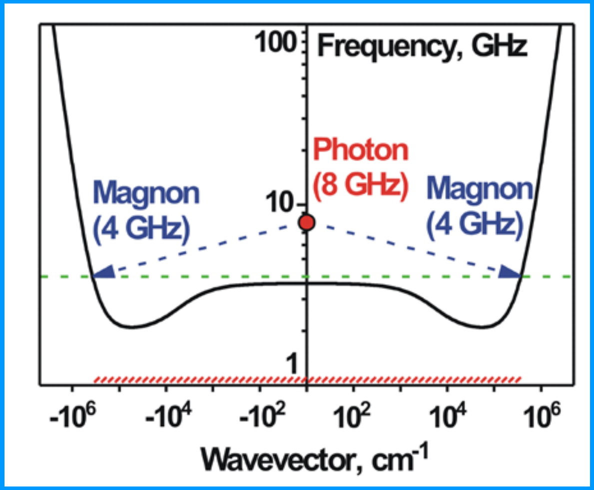
Zeeman ≈ 2 GHz
Dipole-dipole ≈ 150 MHz
Exchange ≈ 15 MHz

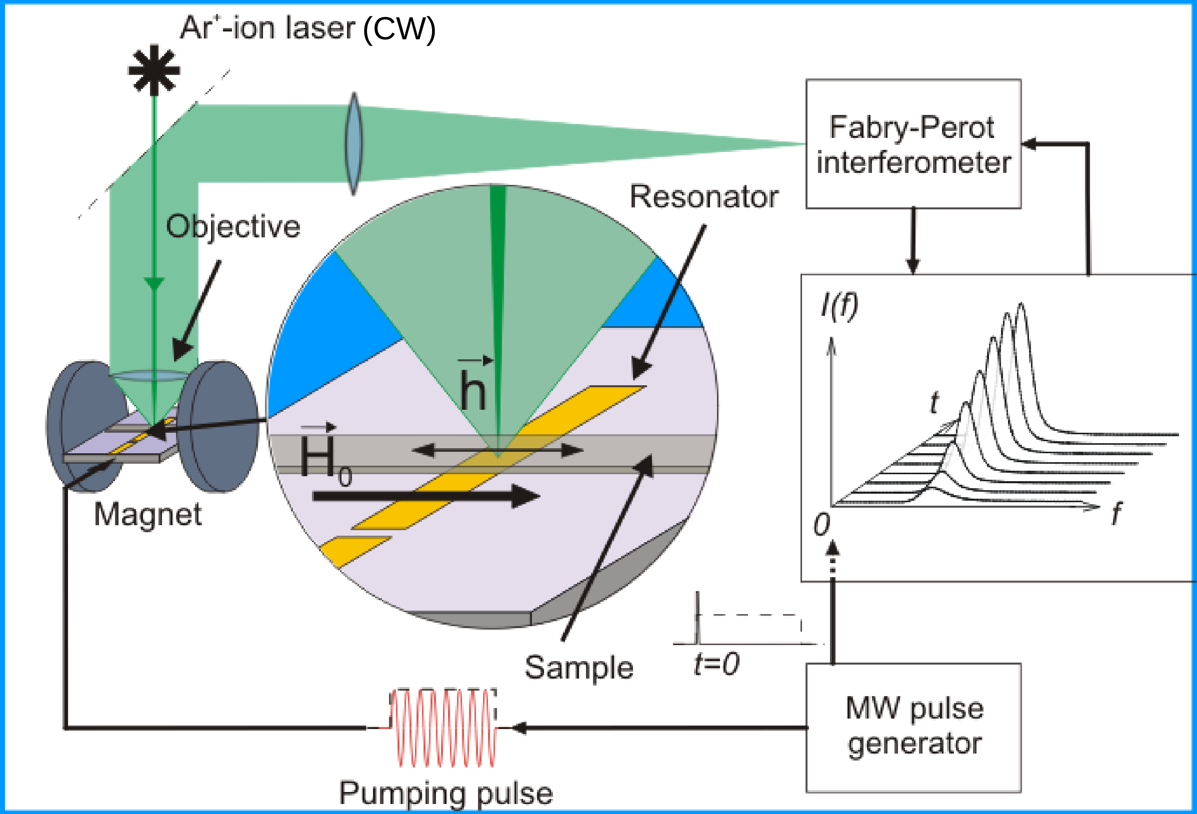
Magnons at $@k_m$ are dipole-dominated

NLMD

Experimental setup for BEC observation

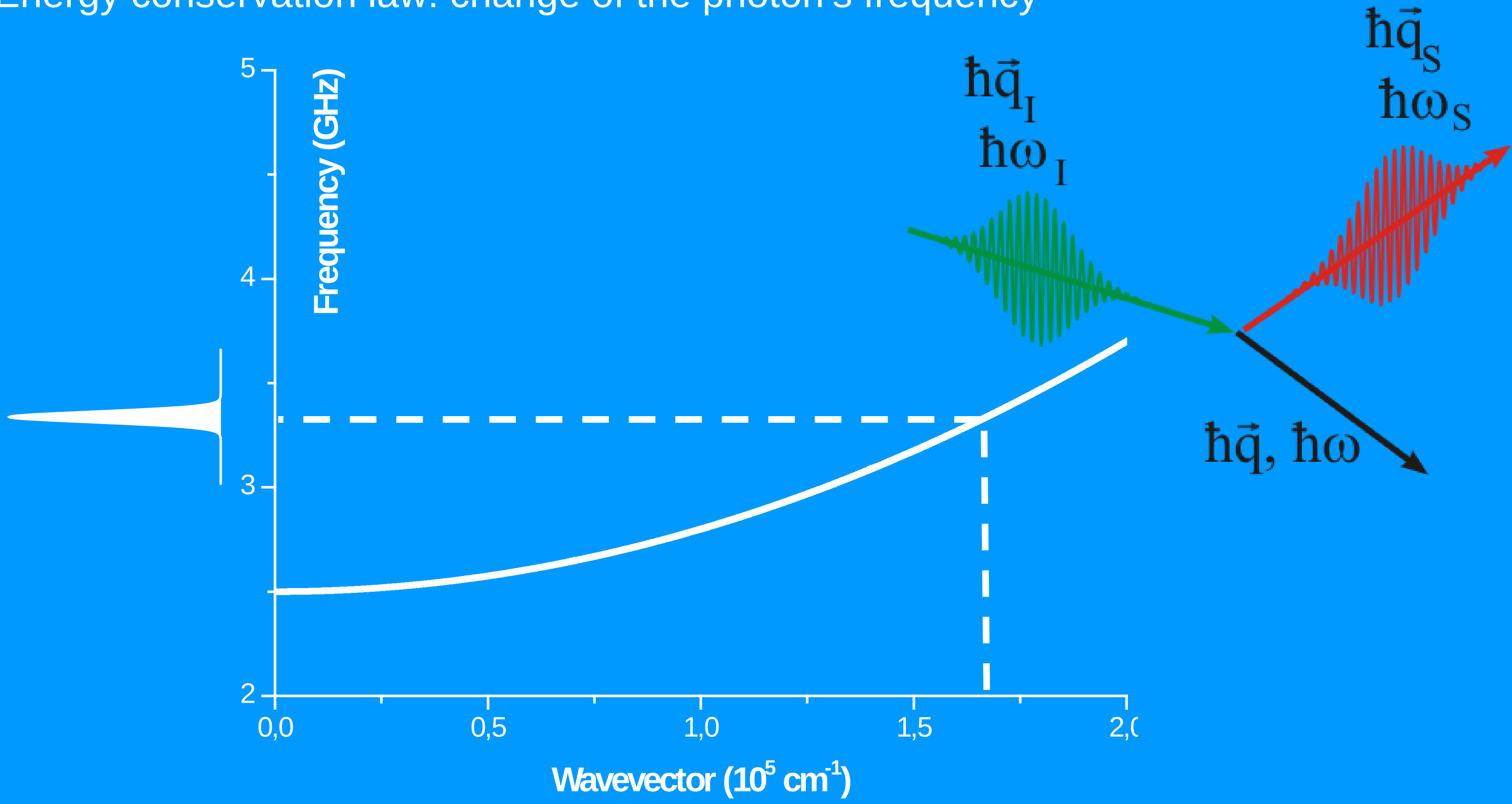
Magnons created by microwaves and detected by light scattering with time and space resolution





Brillouin Light Scattering

Momentum conservation law: the geometry defines the spin-wave wavevector
Energy conservation law: change of the photon’s frequency



Brillouin Light Scattering

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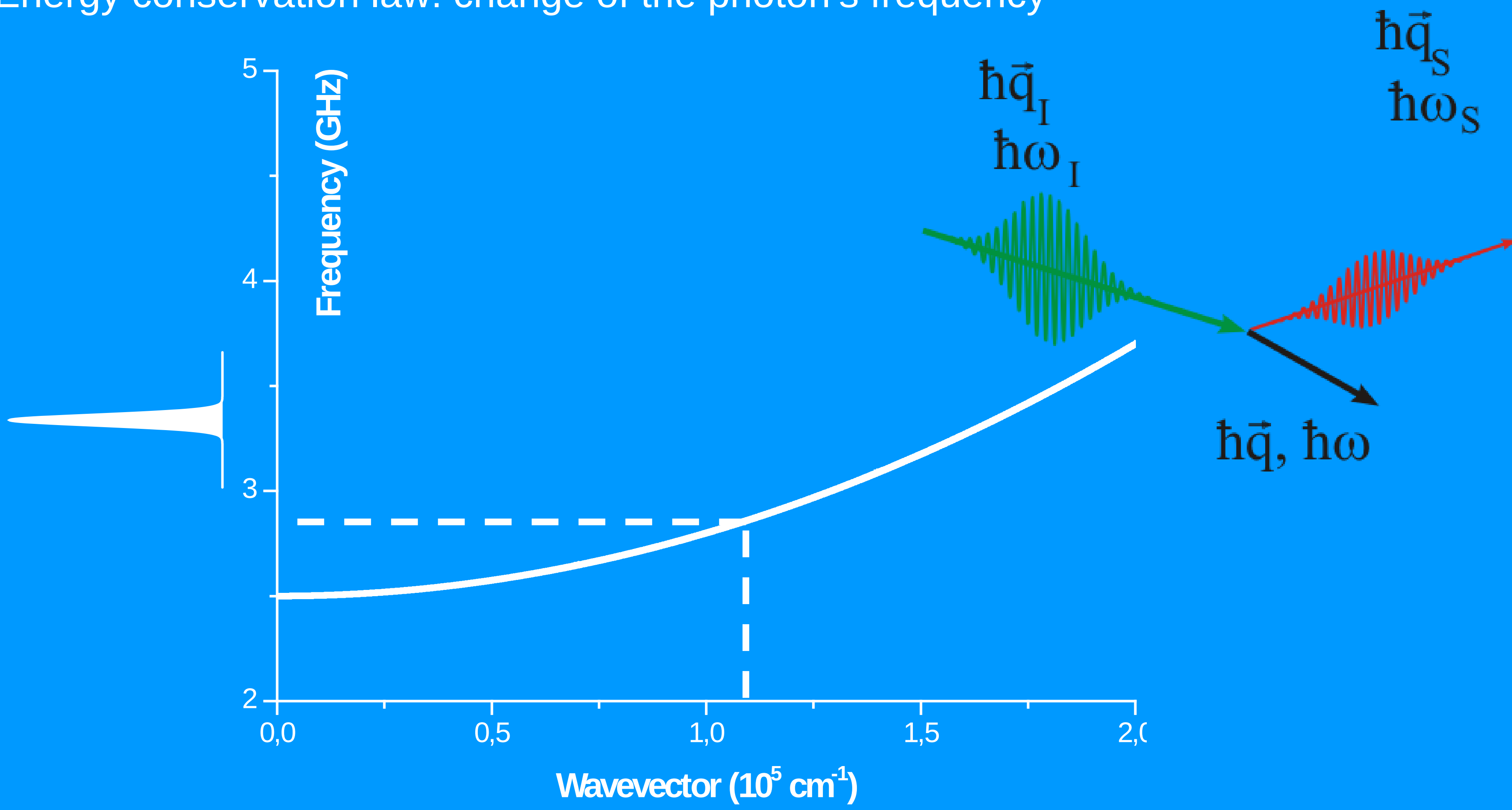
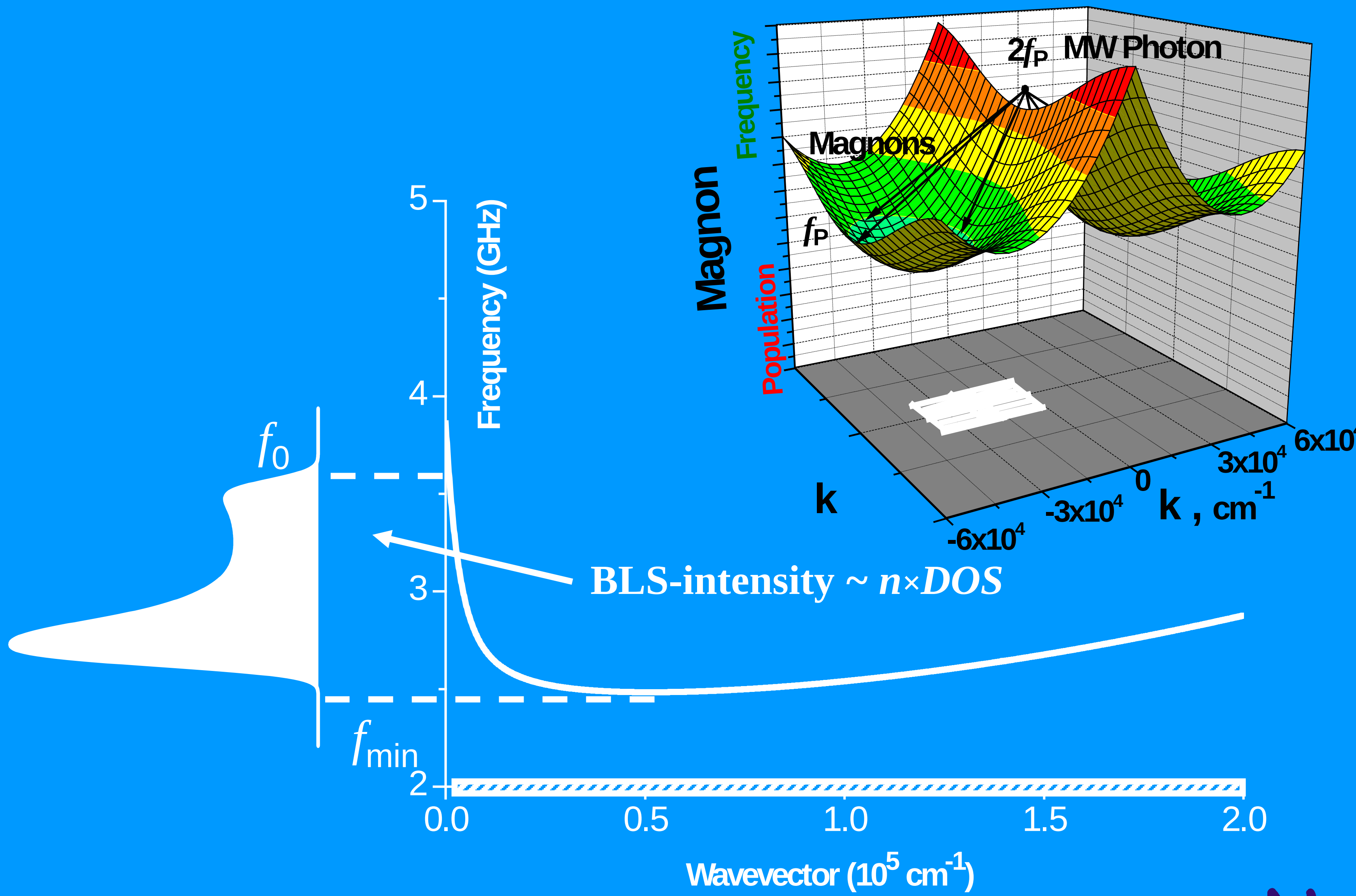
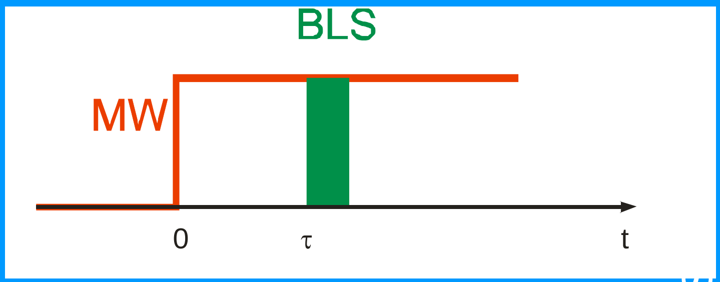


BLS spectroscopy



Creation of the condensate



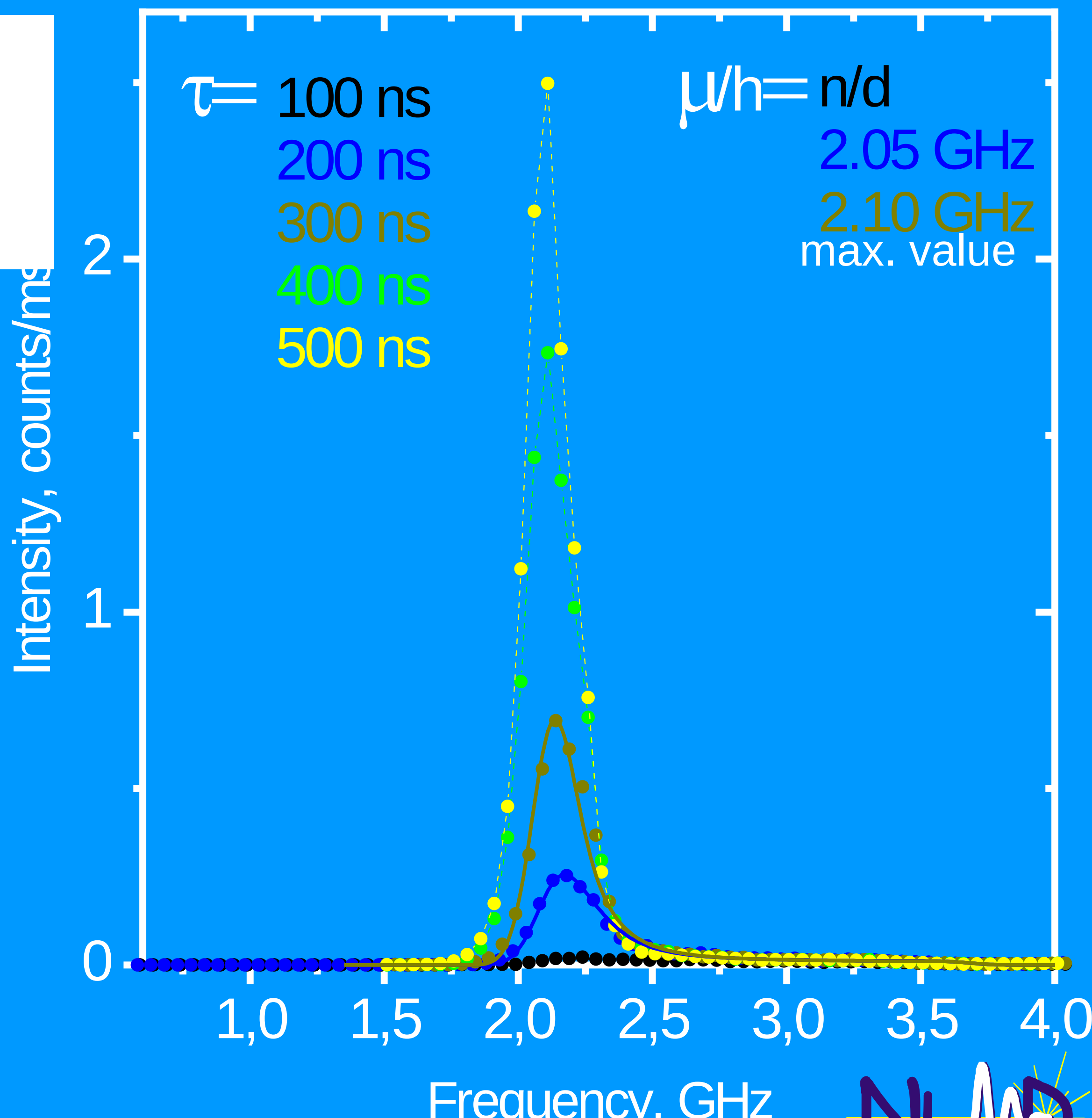
P = 5.9 W

Time development of magnon distribution

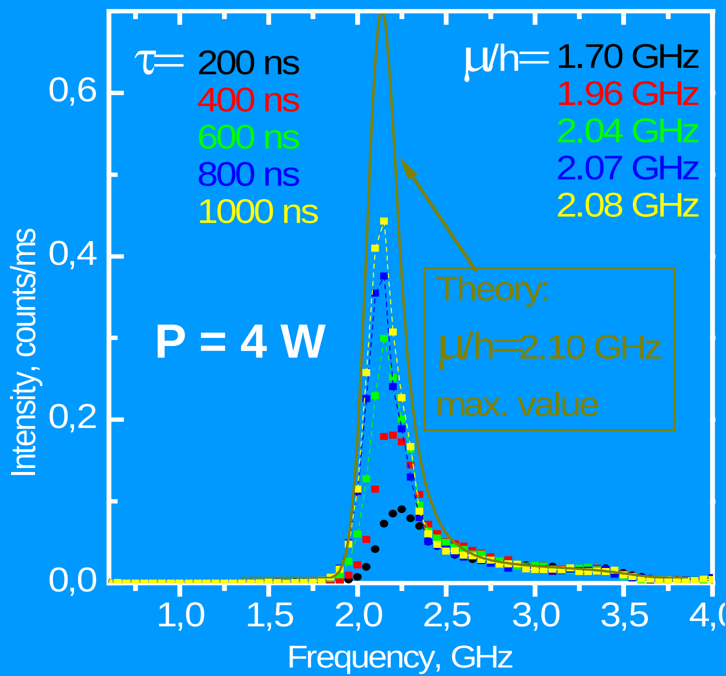
Known DOS: $n(\omega)$ fit with μ

Bose-statistics.
At 300 ns critical density:

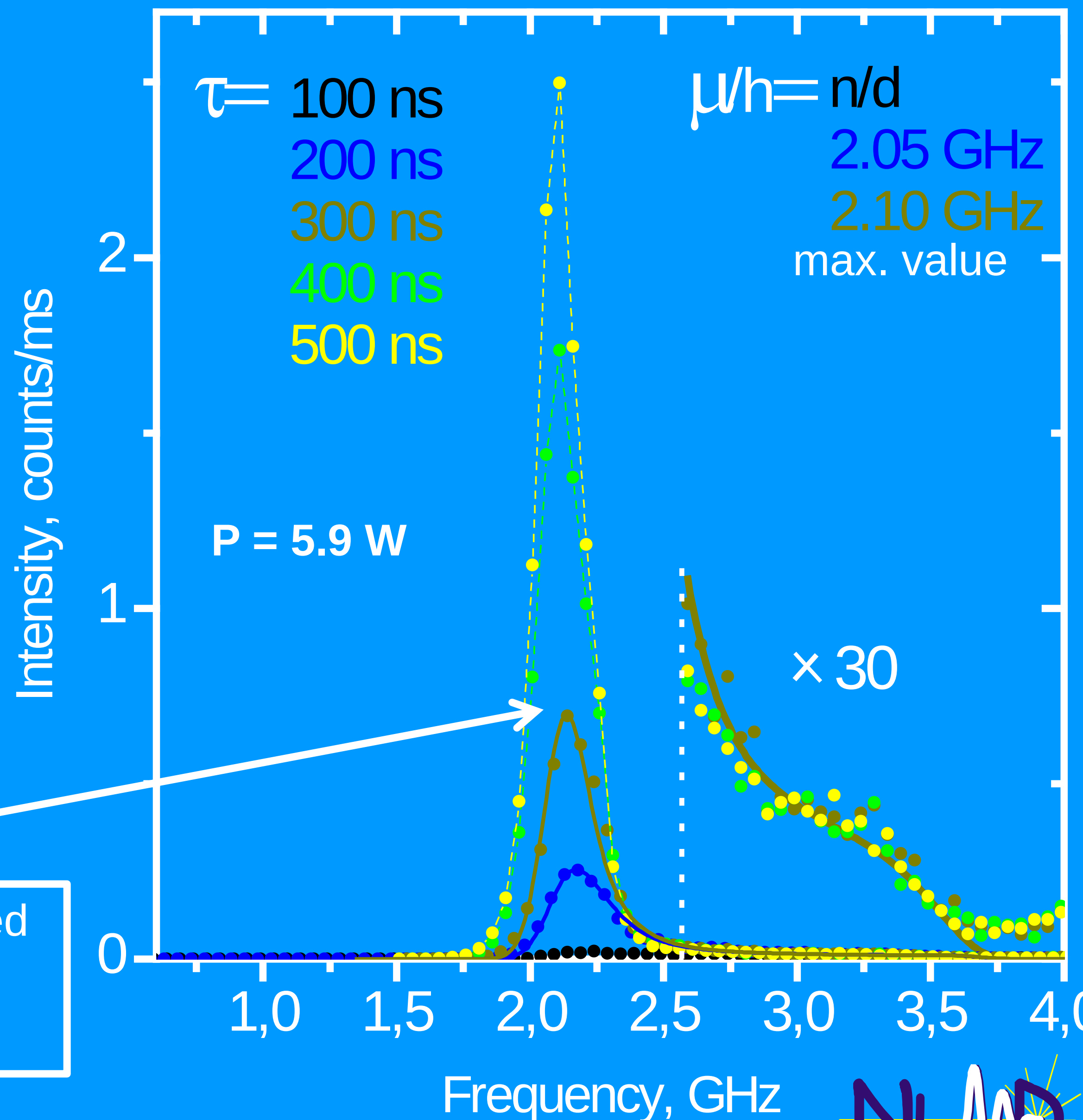
$$\mu \approx \mu_{max}$$



Creation of the condensate

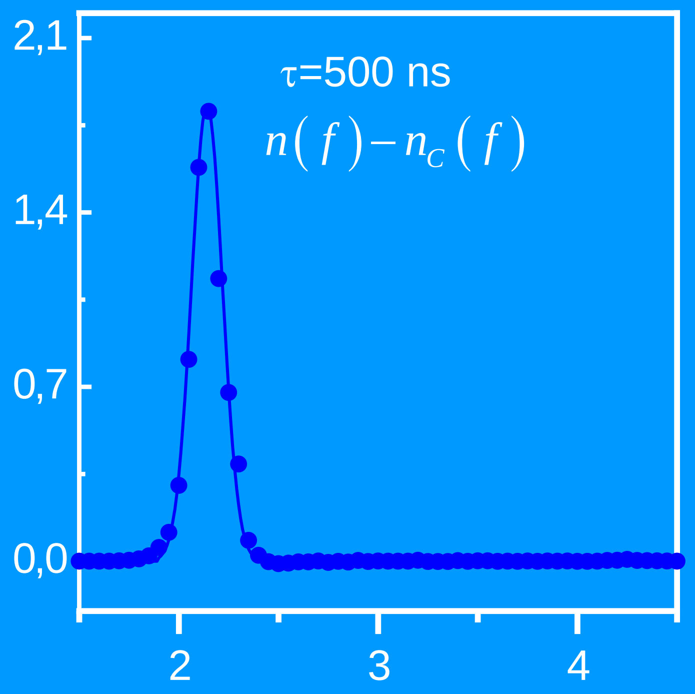


At 300 ns we reach the critical density: $\mu \approx \mu_{\max}$



After 300 ns most of the injected magnons are collected at the minimum freqeuncy

Discovery of RT magnon condensate



A condensate is created!

Condensate: a lot of spins precess in phase.

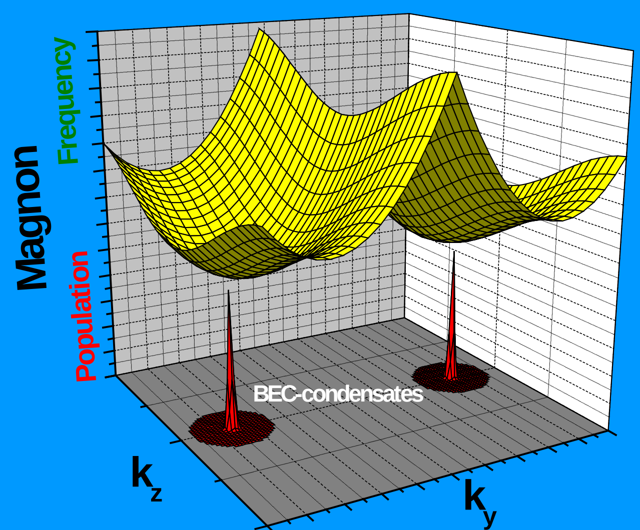
E.g.: synchronization of the transverse magnetization

nature Vol 443|28 September 2006|doi:10.1038/nature05117

LETTERS

Bose–Einstein condensation of quasi-equilibrium magnons at room temperature under pumping

S. O. Demokritov[1], V. E. Demidov[1], O. Dzyapko[1], G. A. Melkov[2], A. A. Serga[3], B. Hillebrands[3] & A. N. Slavin[4]



NATURE|Vol 443|28 September 2006

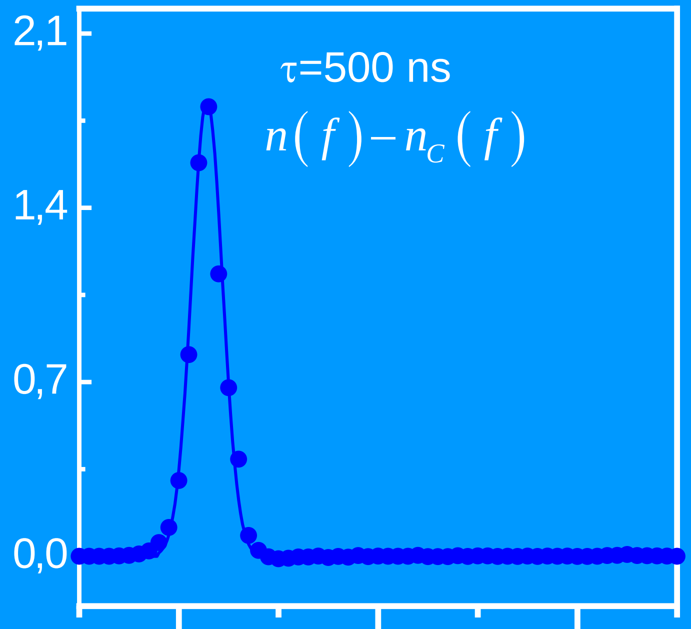
CONDENSED-MATTER PHYSICS

Coherent questions

David Snoke

Bose–Einstein condensation occurs when many particles enter into the same, coherent quantum state, and is now claimed to occur in various systems of 'quasiparticles' in solids. But is it the right term to use here?

Discovery of RT magnon condensate



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nature | Vol 443 | 28 September 2006 | doi:10.1038/nature05117

LETTERS

Bose–Einstein condensation of quasi-equilibrium magnons at room temperature under pumping

PRL **100,** 257202 (2008) PHYSICAL REVIEW LETTERS week ending 27 JUNE 2008

Stability of Bose-Einstein Condensates of Hot Magnons in Yttrium Iron Garnet Films

I. S. Tupitsyn,[1] P. C. E. Stamp,[1] and A. L. Burin[2]

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[2]*Department of Chemistry, Tulane University, New Orleans, Louisiana 70118, USA*

(Received 9 November 2007; revised manuscript received 27 March 2008; published 24 June 2008)

We investigate the stability of the recently discovered room-temperature Bose-Einstein condensate (BEC) of magnons in yttrium iron garnet films. We show that magnon-magnon interactions depend strongly on the external field orientation, and that the BEC in current experiments is actually metastable—

Questions and challenges

Direct evidence of spontaneous coherence of quasiparticles in solids has been hard to come by. Demokritov et al. demonstrate a build-up of a population of magnons near the ground state, but present no direct test of coherence. (D. Snoke)

1. What is the temporal coherence of the condensate?
2. What is the spatial coherence of the condensate?

That the coherence should be spontaneous is essential. Many systems can be coherent if driven by another coherent source: any loud-speaker, for example, generates a coherent state of long-wavelength phonons . (D. Snoke)

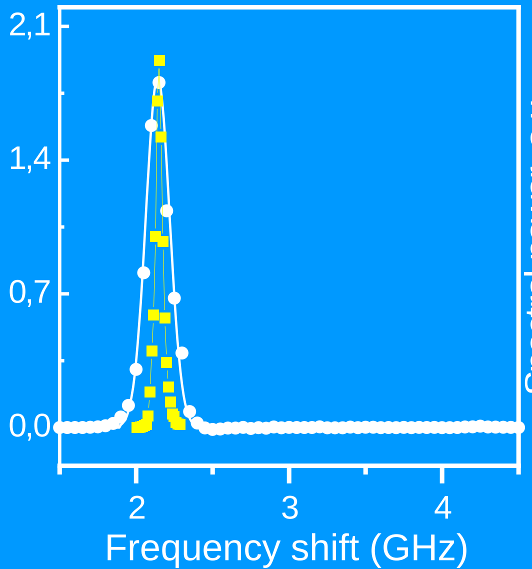
3. Is the coherence spontaneous? Is the coherent condensate created as a result of thermalization?

A rather basic question about the stability of the system needs to be answered. We find the rather startling result that in experiments reported so far, the intermagnon interactions were actually attractive: the BEC should have been unstable. (Tupitsyn et al.)

4. What is the mechanism of stabilization of the condensate?

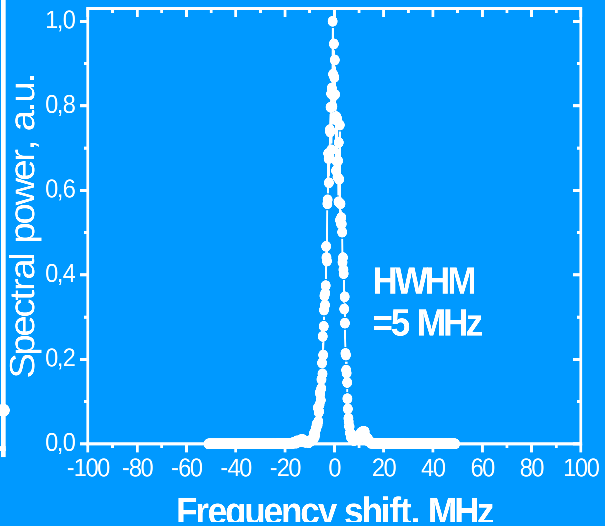
1. What is the temporal coherence of the condensate?

@2006 < 50 MHz



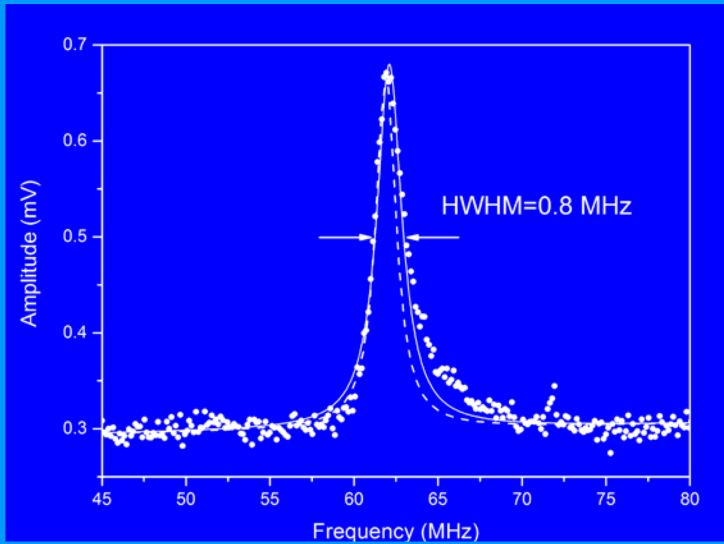
Nature ***443*** *430* '06

@2008 < 5 MHz



Appl. Phys. Lett. **92** 162510 08'

@2016 = 0.8 MHz

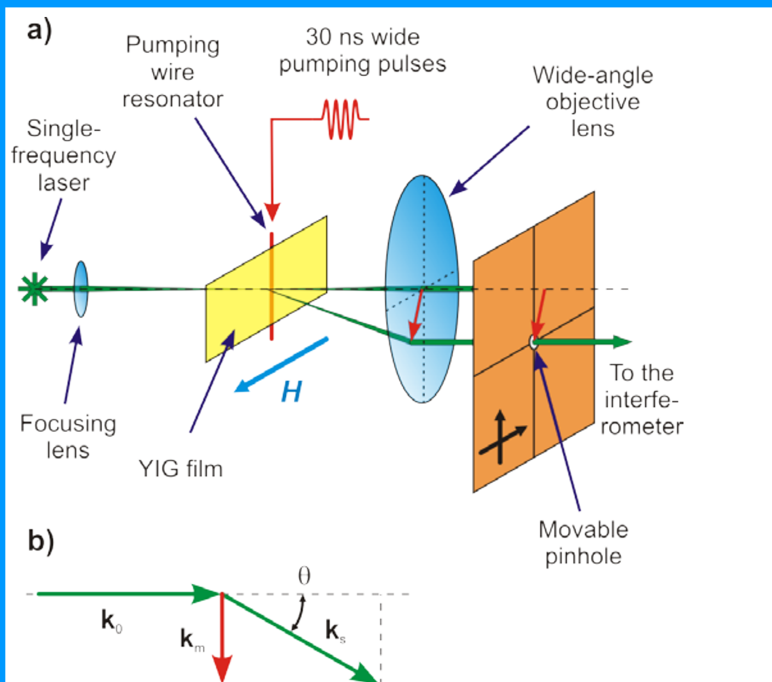


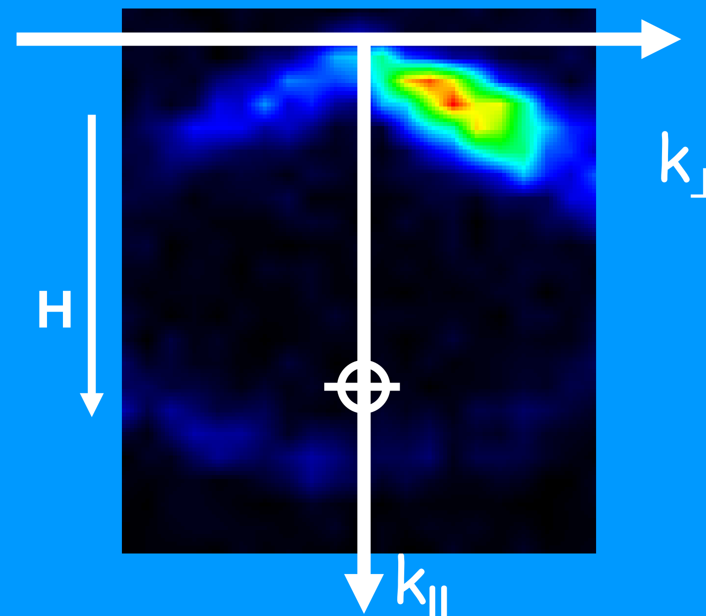
IEEE Magnetic Lett., **7** (2016) 350180

By taking advantage of a novel heterodyne spectroscopy (frequency resolution 200 Hz) we were able to determine the intrinsic linewidth of the condensate and to confirm its high temporal coherence

2. What is the spatial coherence of the condensate?

Instead of integration of the signal over ($k_{||}$, $k_{\perp}$),
k-resolved measurements are performed.



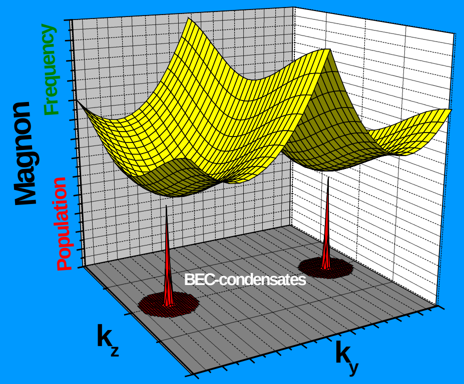


Magnon distribution in the k-space becomes much narrower with time.

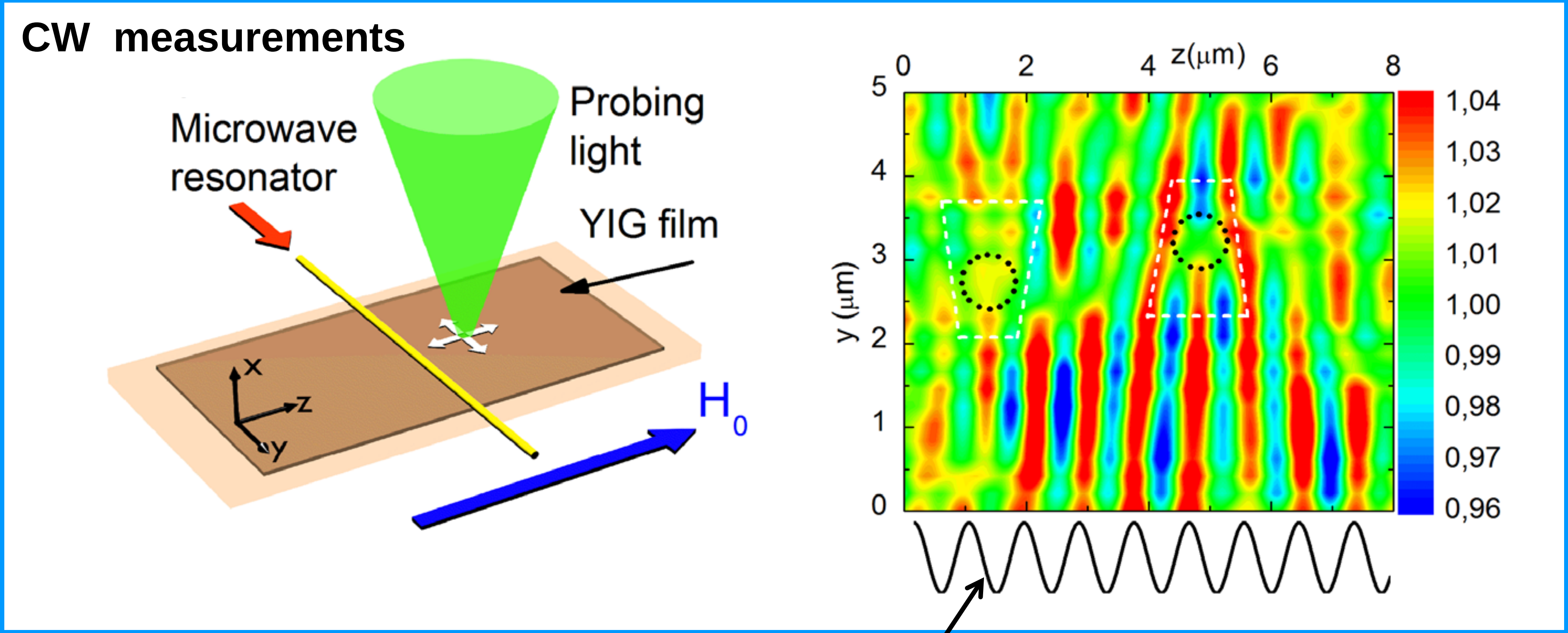
Phys. Rev. Lett. **101** 257201 ´08.

2. What is the spatial coherence of the condensate?

We took advantage of interference bertween two components the condensate.



The coherence length is about 40 μm



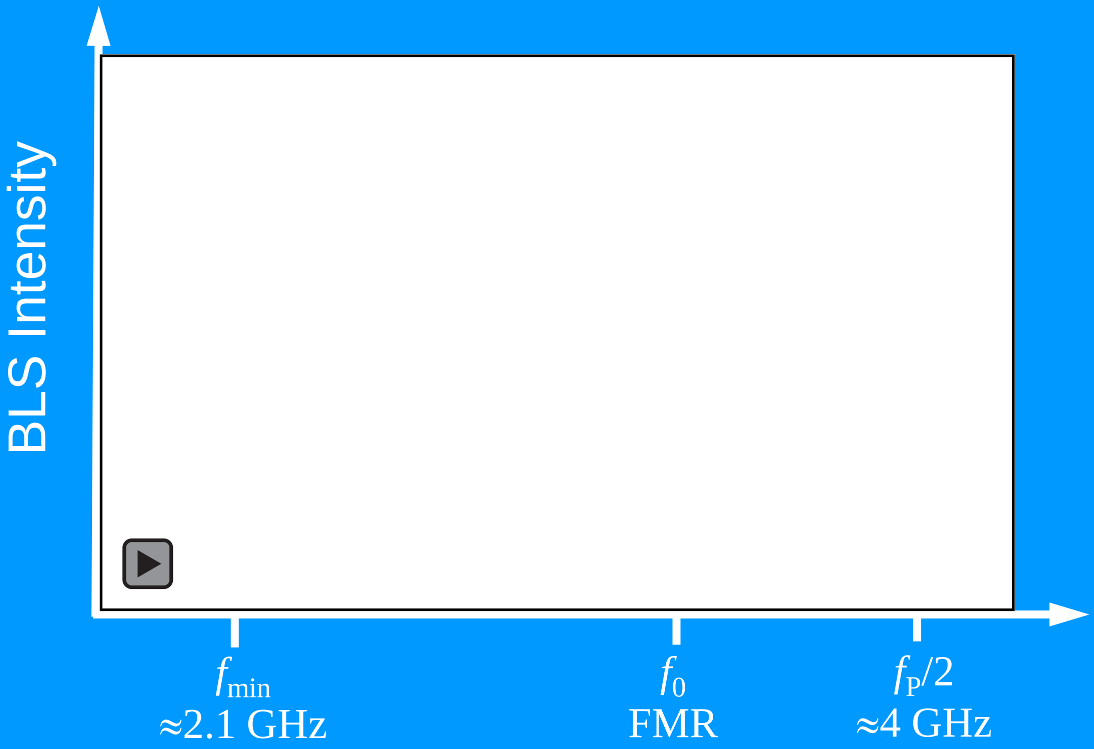
$$\propto \left|\psi(z,t)\right|^2 = \cos(2k_{BEC}z)$$

Scientific Reports **2** (2012) 482

3. Is the coherence spontaneous?

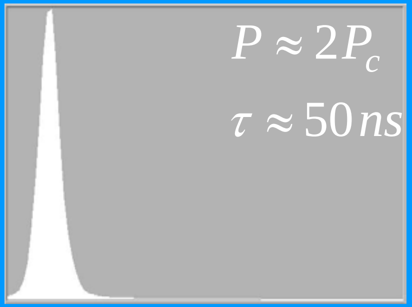
Is the coherent condensate created as a result of thermalization?

Continous pumping



Wave-like redistribution of magnons over the spectrum.

Thermalization of the injected magnons happens trough multiple magnon-magnon scattering events.



With increasing pumping power:

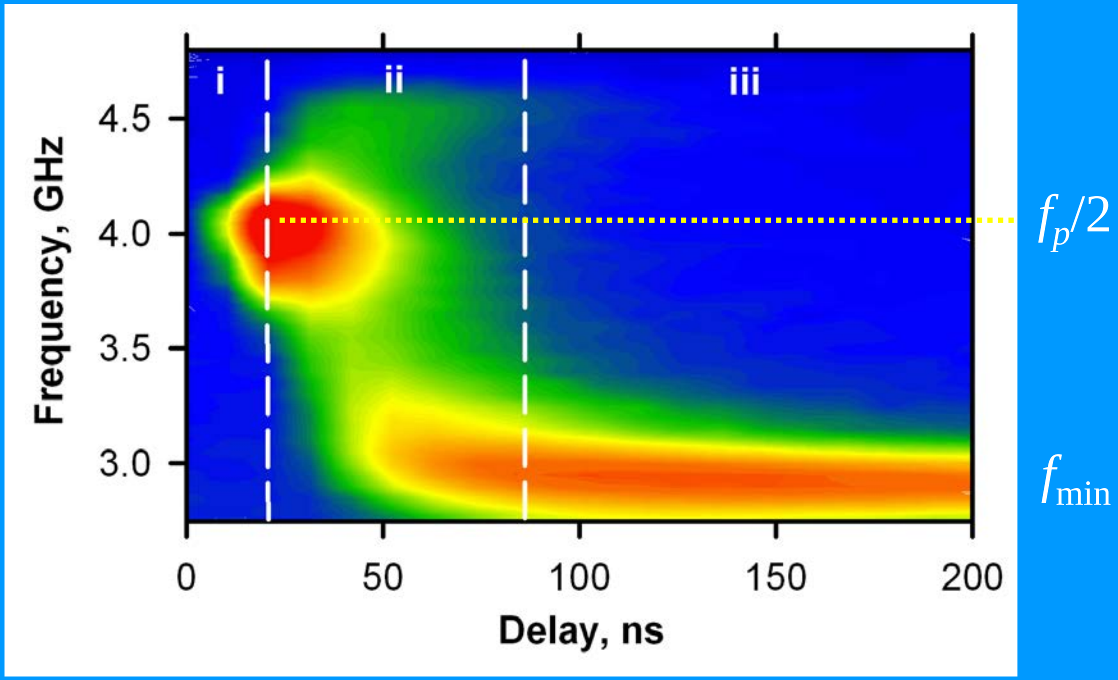
1. Thermalization process becomes faster.
2. The lowest-energy magnon state becomes strongly overpopulated

Phys. Rev. Lett. **99** (2007) 037205

3. Is the coherence spontaneous?

Is the coherent condensate created as a result of thermalization?

Pulsed pumping, duration = 20 ns



The peak at $f_{\min}$ is formed after the pumping is switched off.

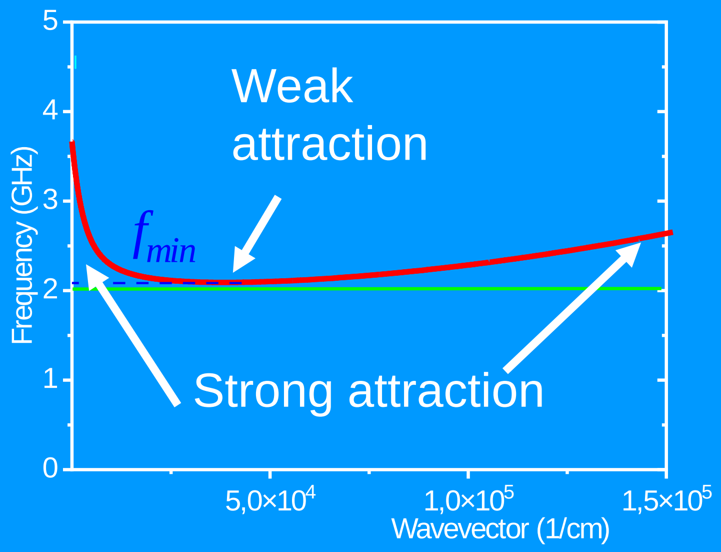
It is caused by internal interactions in the magnon gas.

A clear narrowing of the linewidth with time is observed

Phys. Rev. Lett. **100** (2008) 047205

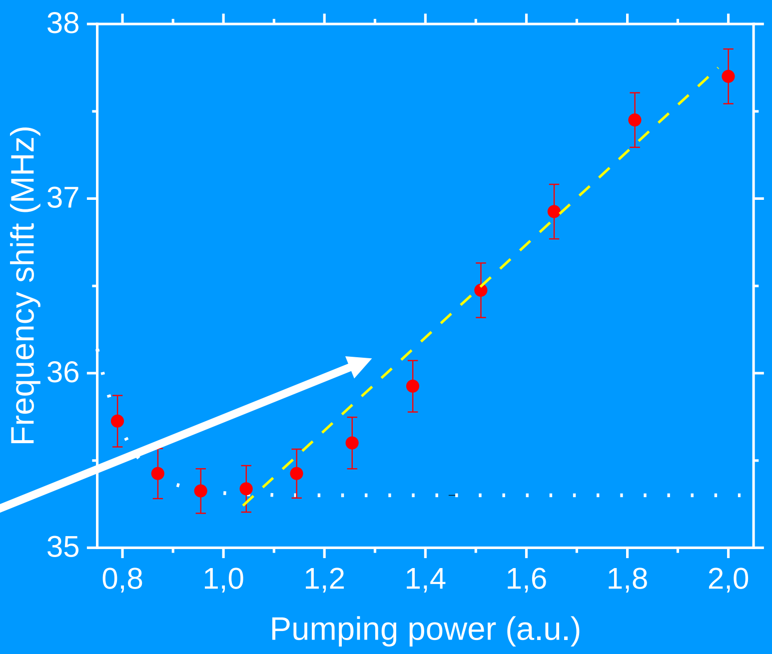
4. What is the mechanism of stabilization of the condensate?

Standard theory, considering self-interaction of magnons



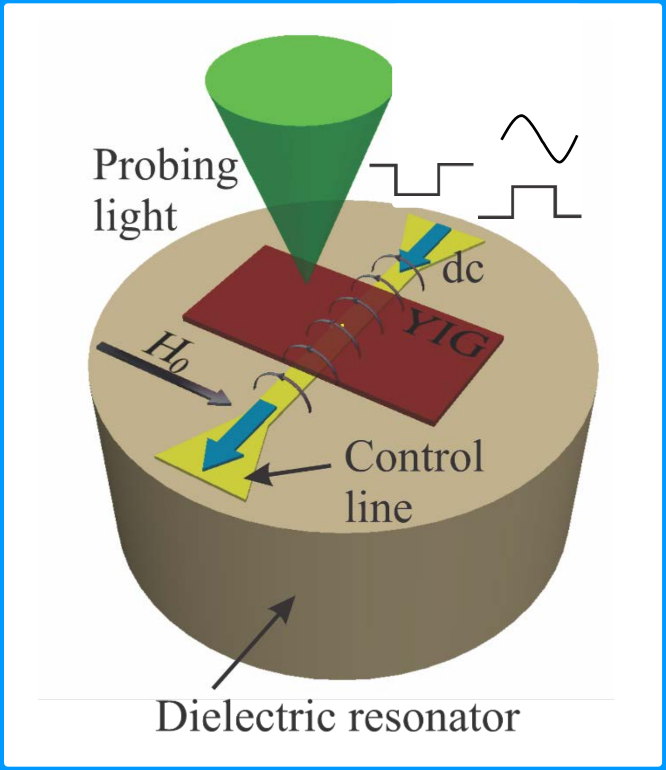
Consideration of the cross interaction of magnons belonging to different spectral minima $\vec{k}_m, -\vec{k}_m \rightarrow \vec{k}_m, -\vec{k}_m$ results in repulsion.

The experiment also demonstrates a weak repulsion



*Phys. Rev. B **96** (2017) 064438*

Control of the condensate by inhomogeneous field

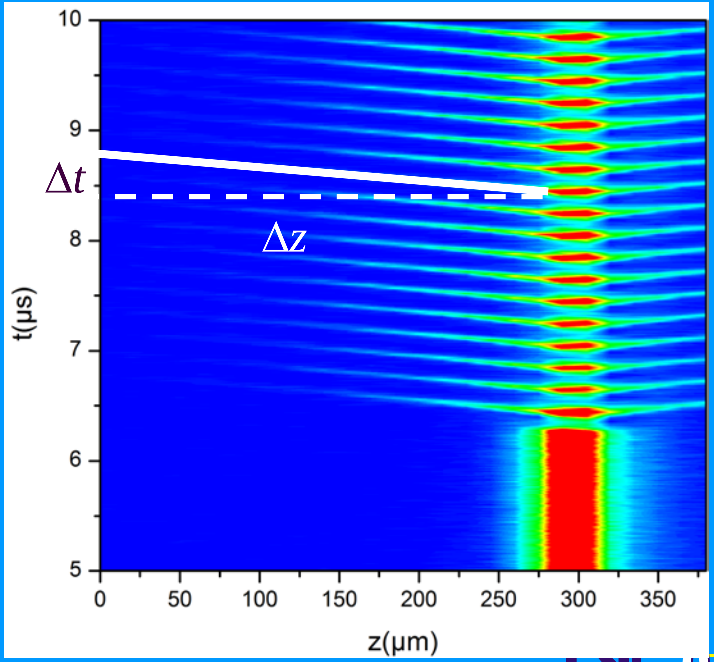


Pumped magnon gas is created by the microwave pumping via dielectric resonator

It is locally perturbed by a magnetic field created by a narrow stripe

The temporal and spatial dynamics of the gas density is measured by BLS

A new powerful experimental tool for magnon BEC studies



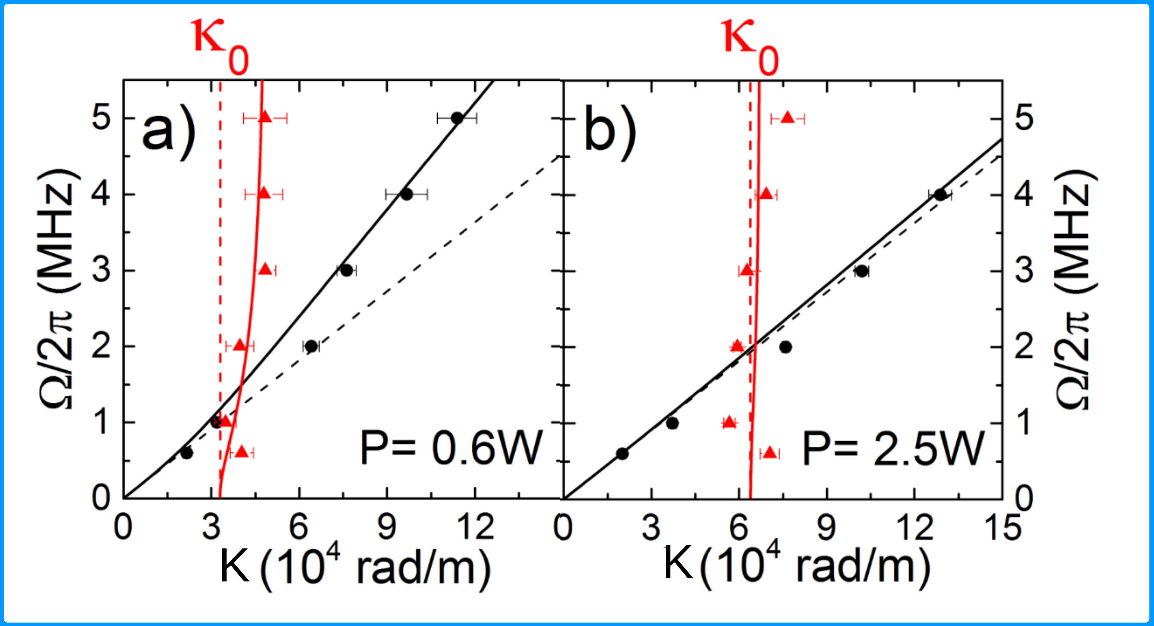
Control line allows local pertrubation of the field/chem.potential of the condensate.
Here: RF-modulation

Second sound wave is excited

$$v_{ph} = \Omega / K = \Delta z / \Delta t$$

Second sound: dispersion and dissipation

For real frequency Ω_K, wavevector K is complex $k = K + i\kappa$, waves are dissipative



Low pumping power (below BEC threshold)

High pumping power (above BEC threshold)

The speed of sound almost independent of the gas density, but depends on the applied magnetic field in agreement with the theory

The theory connects the wave properties with the existence of two relaxation rates:

$$\frac{\partial e}{\partial t} = -\Gamma_e e \qquad \frac{\partial p}{\partial t} = -\Gamma_p p$$

Energy Momentum

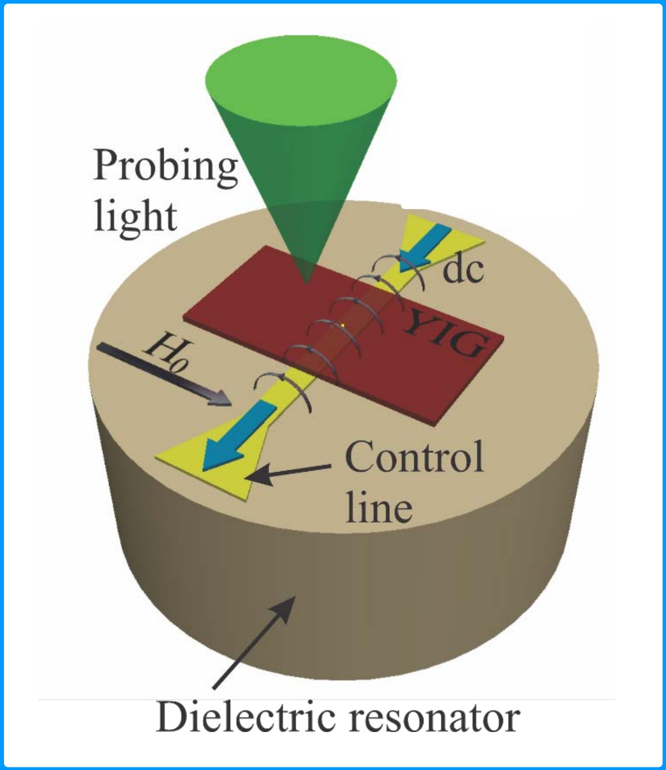
$$\kappa_0 \propto \sqrt{\Gamma_p \Gamma_e}$$

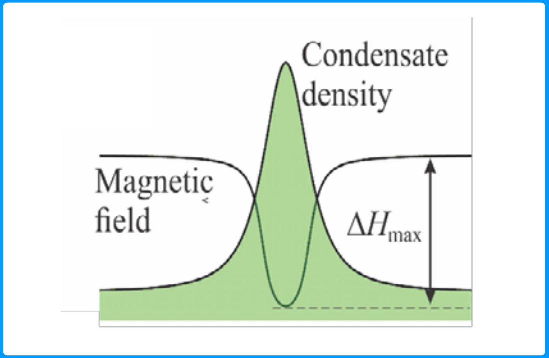
$$q = 2\sqrt{\Gamma_p \Gamma_e} \Big/ (\Gamma_p + \Gamma_e)$$

$$\Omega_K = VK\sqrt{\frac{1+(K/\kappa_0)^2}{1+q^2(K/\kappa_0)^2}}$$

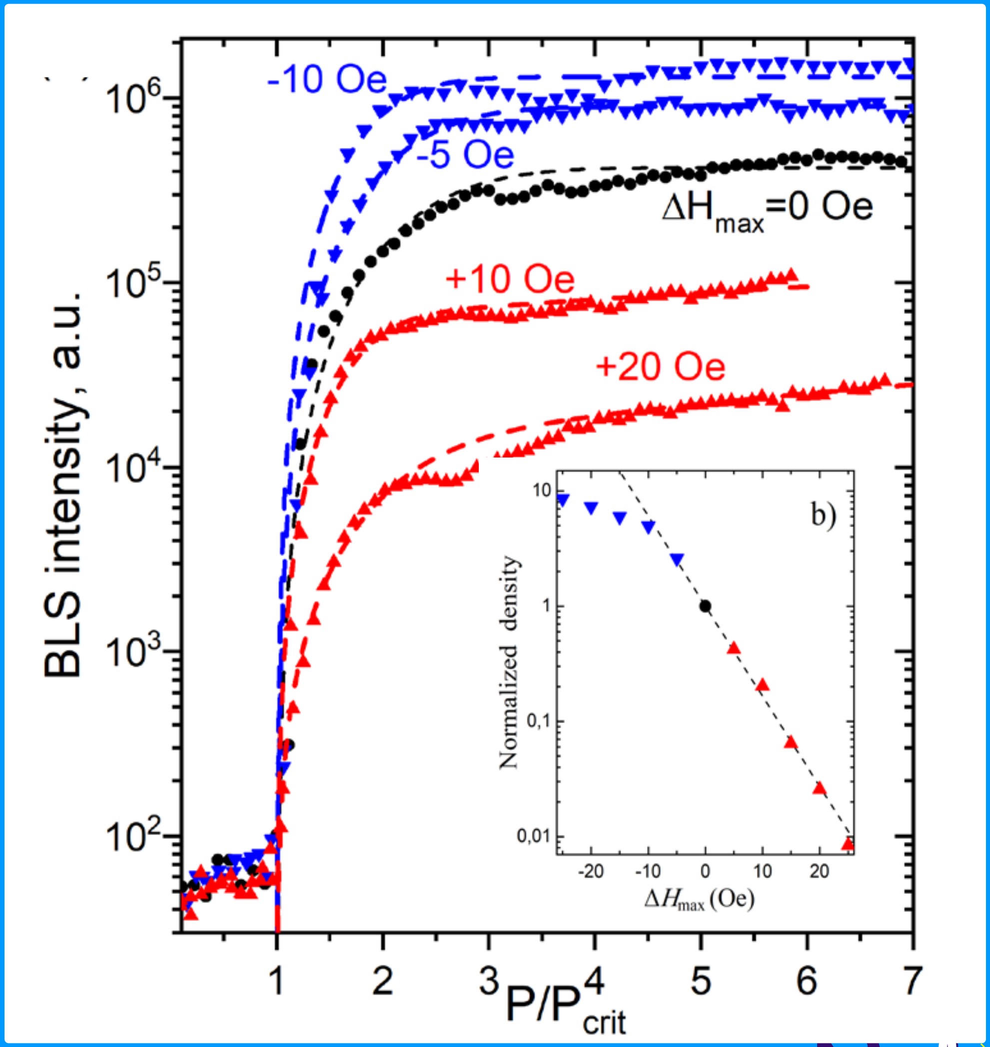
Sci. Rep. **9** (2019) 9063

Lateral transport in inhomogeneous field

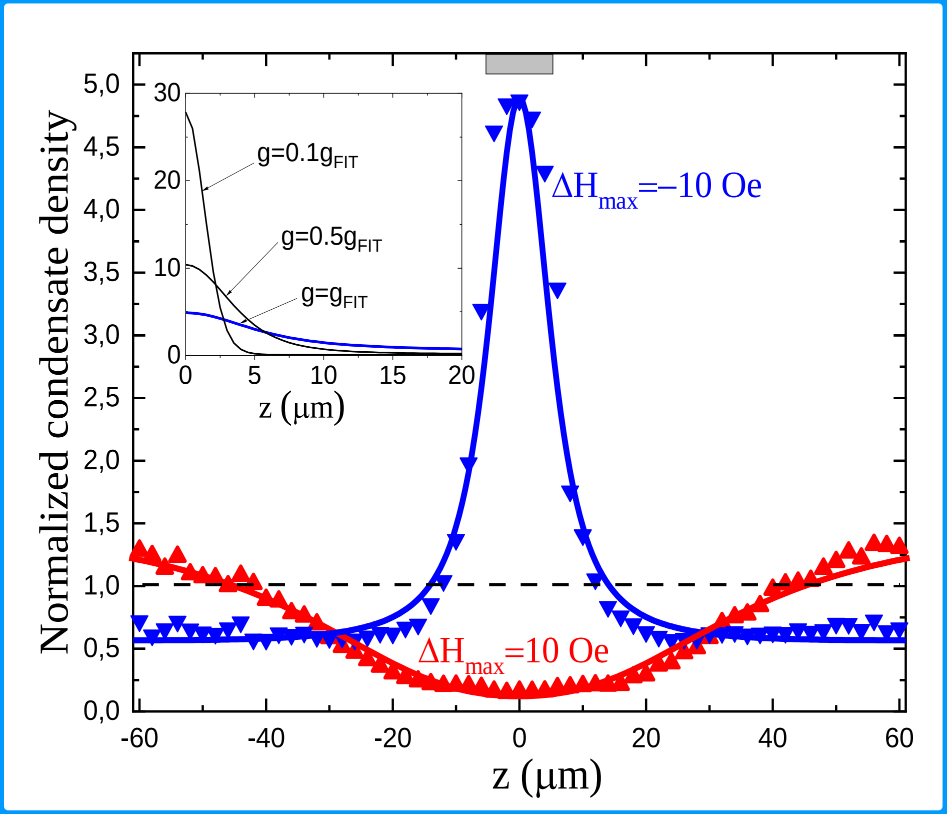




Both the inhomogeneous field and the pumping are steady



Profile of the condensate density



Gross-Pitaevskii equation – dynamic, no dissipation

M-M interaction

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + U(z) + g|\psi|^2 - \mu_C\right)$$

Quasi-classical flow equation with dissipation

Mobility

Relaxation

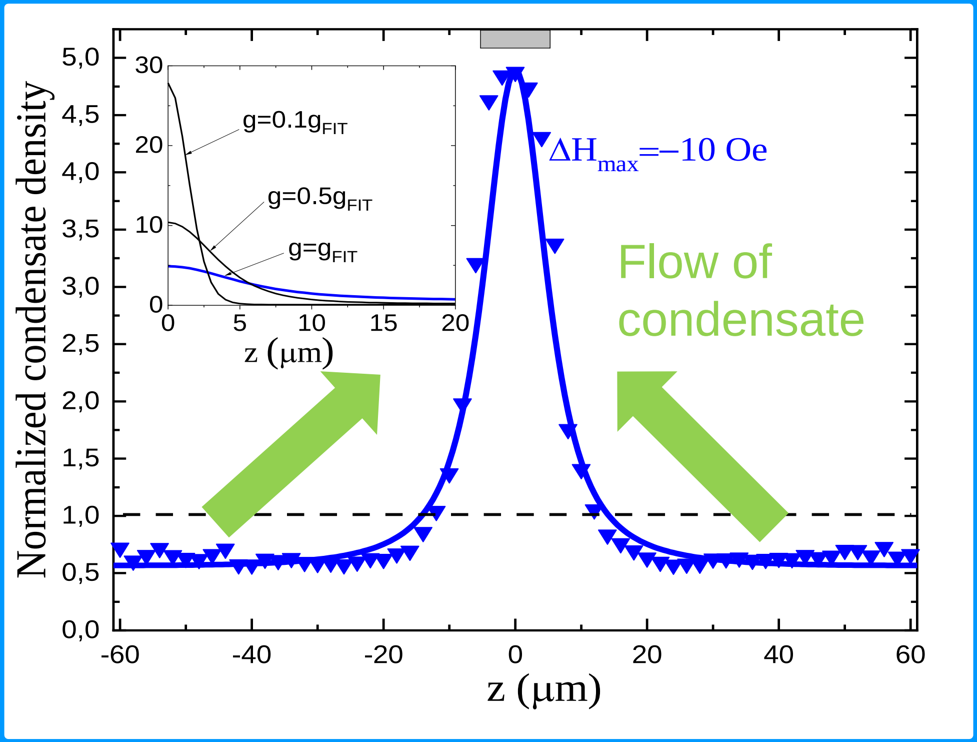
$$\eta\frac{\partial}{\partial z}\left[n\frac{\partial(U+gn)}{\partial z}\right] - \frac{n-n_0}{\tau} = 0,$$

Theoretical model nicely describes the recorded profiles of the condensate density

Only two fit parameters, since τ = 130 ns was be obtained idependently from relaxatión of free condensate

Nat. Comm. 11 (2020) 1691

Profile of the condensate density



Gross-Pitaevskii equation – dynamic, no dissipation

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + U(z) + g|\psi|^2 - \mu_C\right)$$

Quasi-static flow equation with dissipation

Mobility

Relaxation

$$\eta\frac{\partial}{\partial z}\left[n\frac{\partial(U+gn)}{\partial z}\right] - \frac{n-n_0}{\tau} = 0,$$

Theoretical model nicely describes the recorded profiles of the condensate density

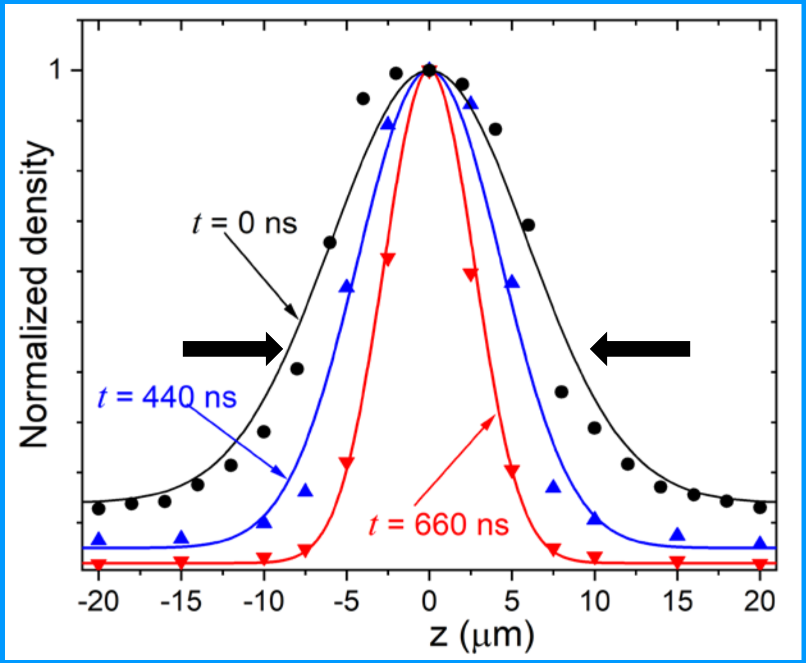
Only two fit parameters, since $\tau = 130$ ns was obtained idependently from relaxatión of free condensate

From the fit value of η

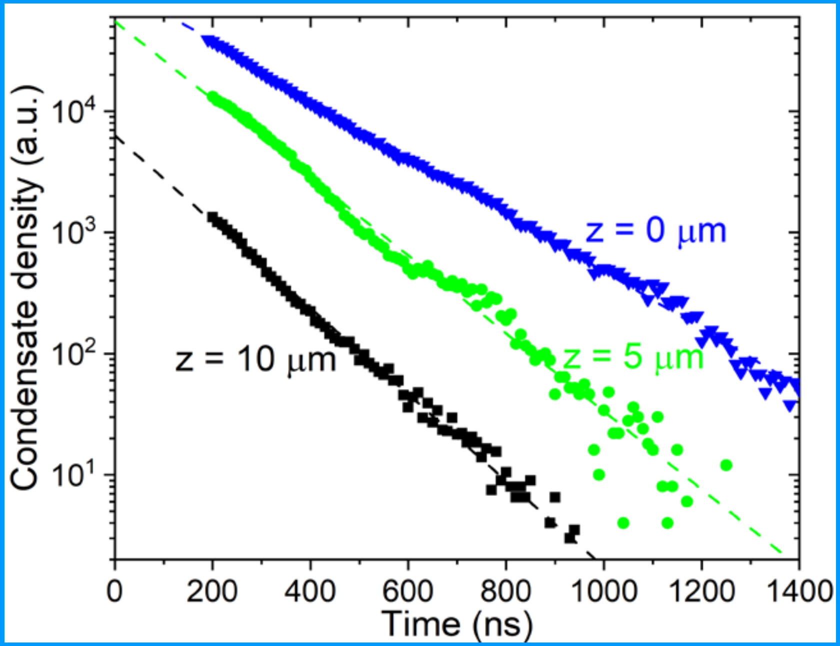
The corresponding relaxation time is10000 ns!

Time evolution of a free condensate

Time evolution of the condensate density in a potential well after turning the microwave pumping off



The density profile becomes narrower with decreasing condensate density

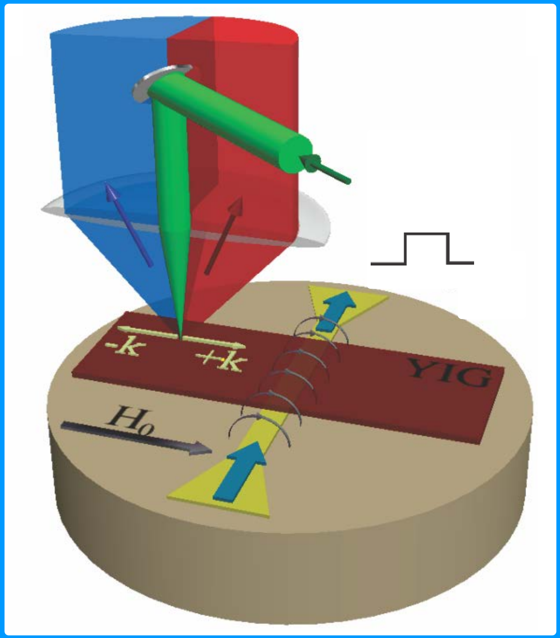


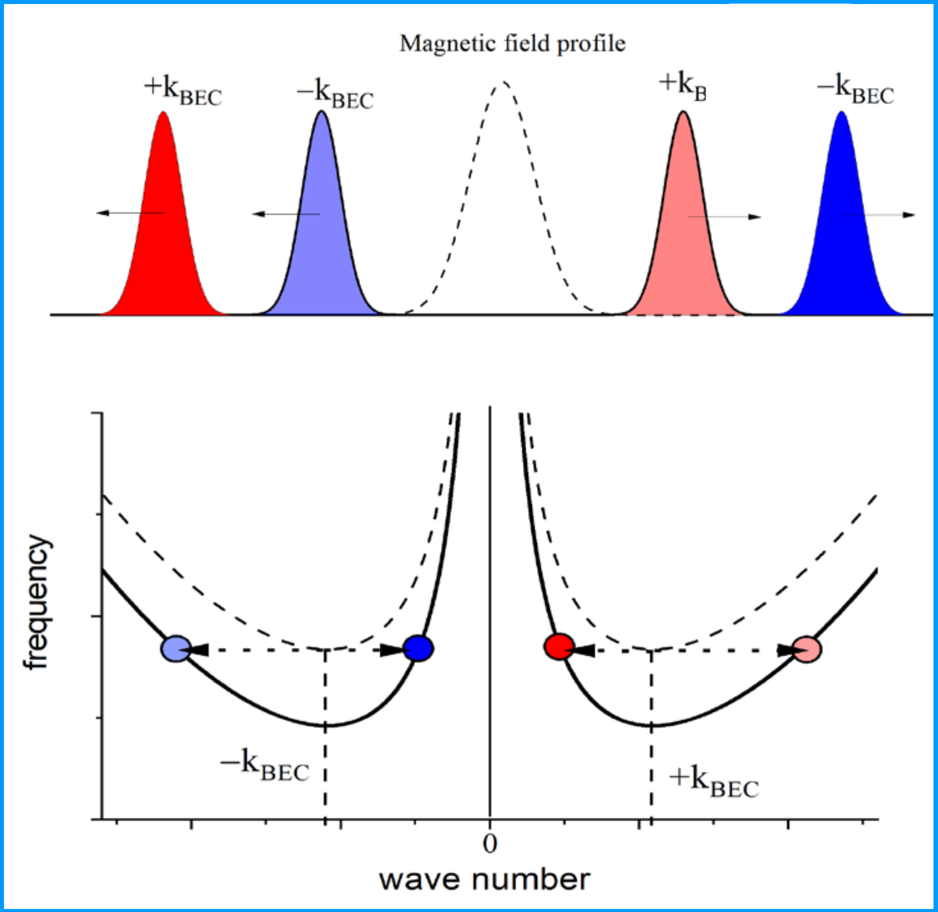
The density decays slower in the center of the trap than outside the gap

At higher densities the condensate is extruded out from the trap. A pictorial evidence of the repulsive interaction, which in this case has the dipole origin.

Spatial separation of two condensates

Time evolution of the condensate density after a potential hill is turnined on



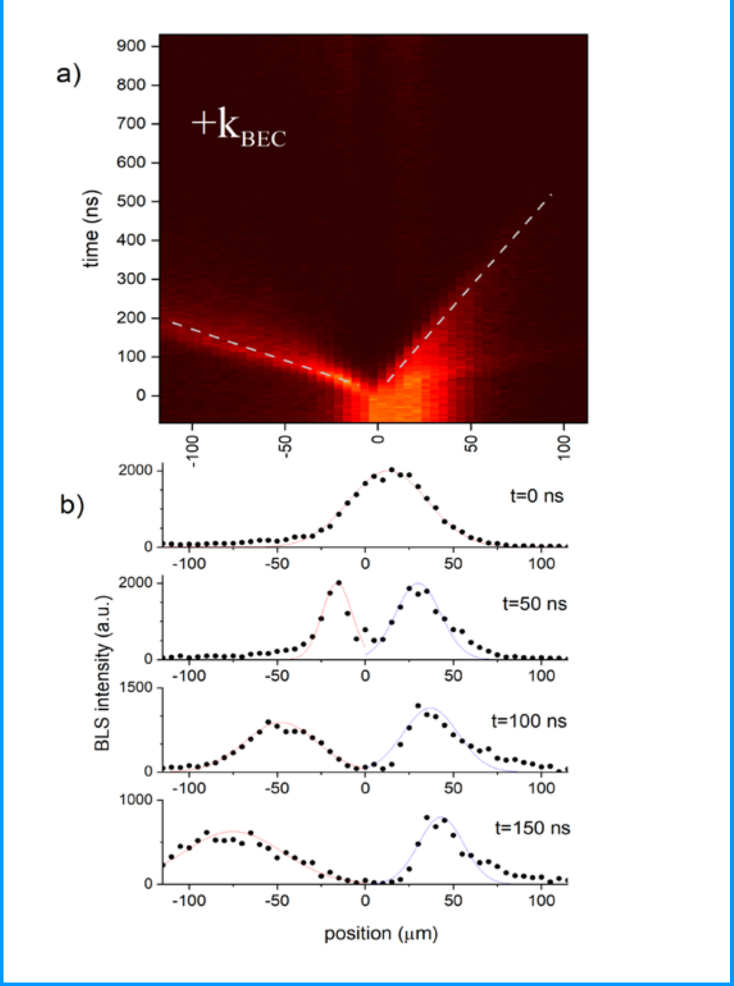


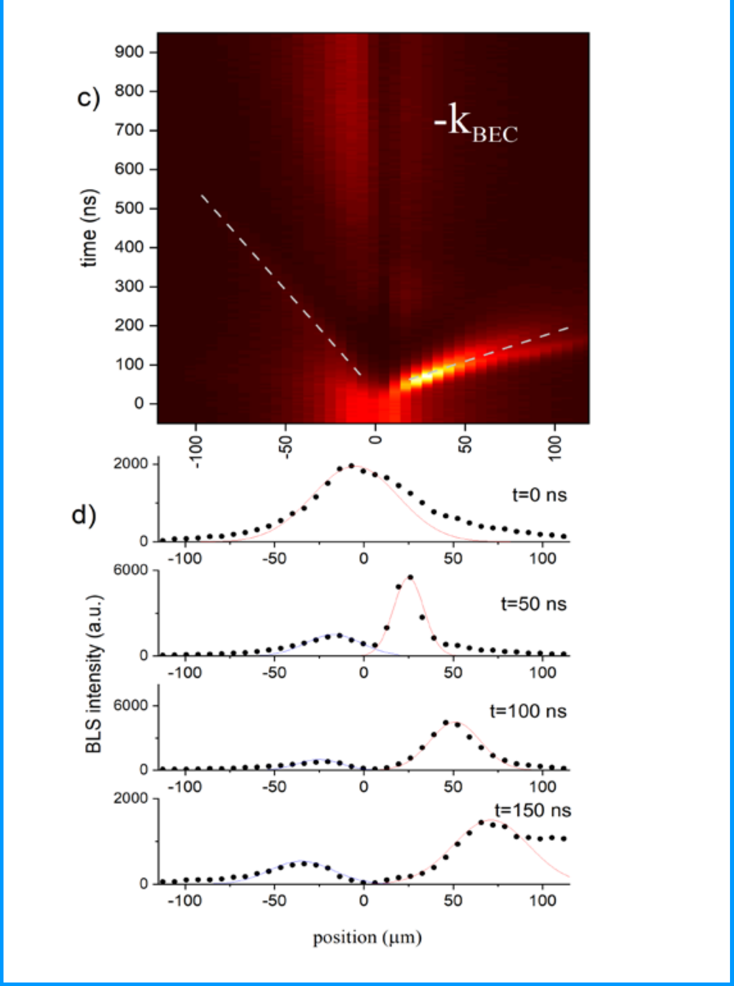
Using selectivity of BLS to wavevector one can addres the condensates separately

Under influence of pulsed field condensates should move in the opposite directions

Sci Rep **10**, 14881 (2020).

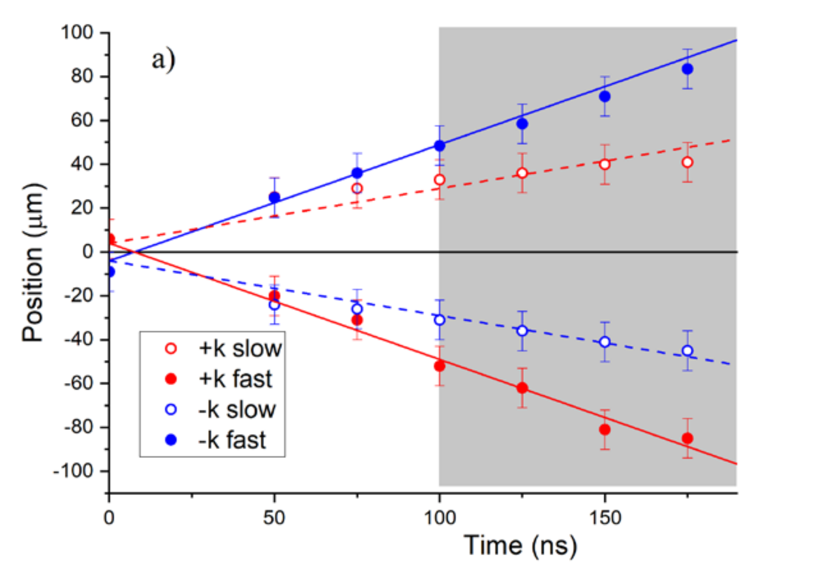
Motion of two condensates



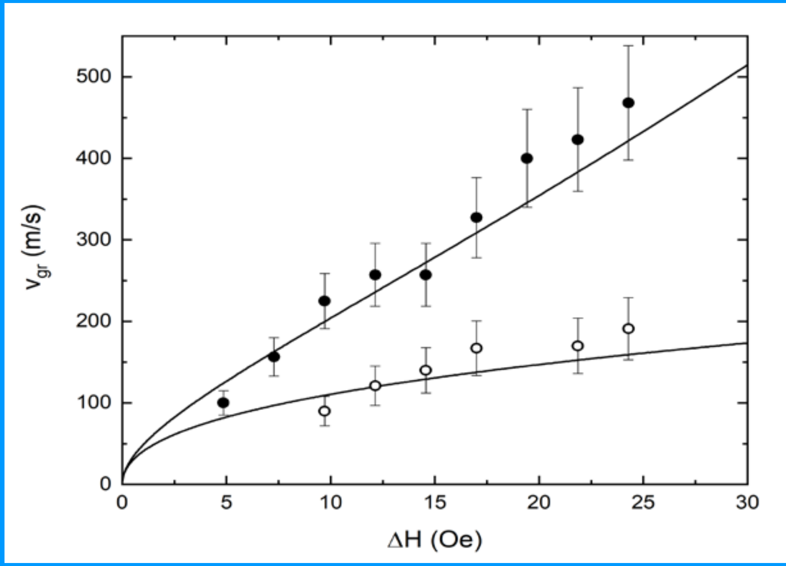


Under influence of pulsed field the condensates move in the opposite directions

Motion of two condensates



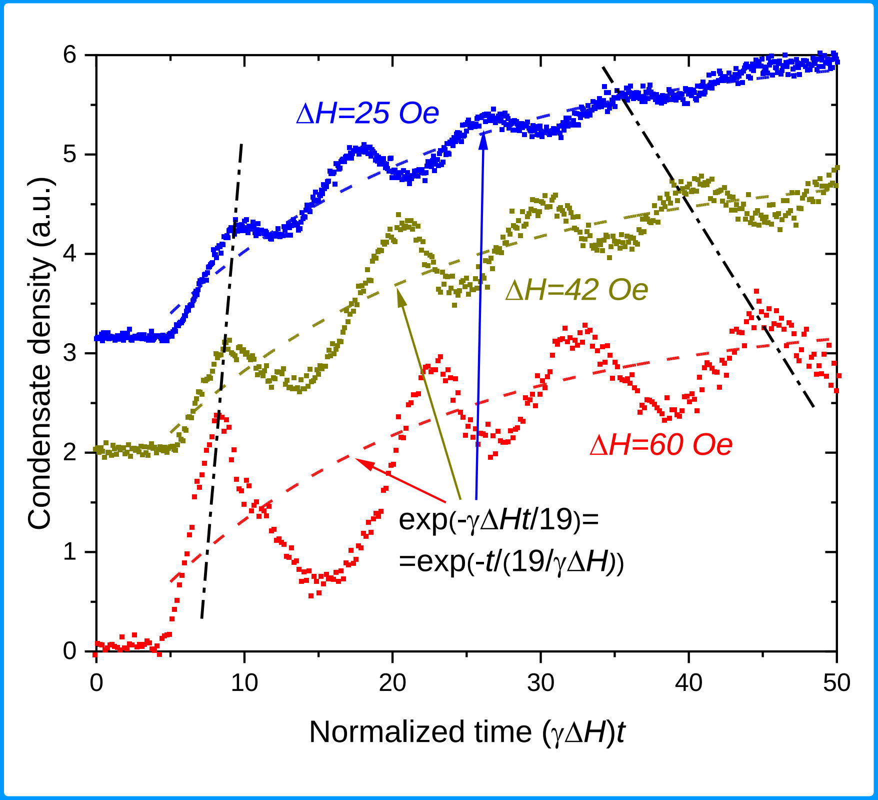
Note the same speeds for both condensate and a symmetry connecting the real space motion and the wavevectors of the condensates.



Velocities of moving condensate clouds versus the applied localized magnetic field. Lines are calculated based on the shape of the magnon spectrum close to its bottom

Oscillations of the condensate density in the trap

Time evolution of the condensate density after a
potential trap is turnined on



After the trap is formed, we observe damped oscillations on top of monotonic exponential growth of the density.

There are 3 time/frequency parameters:

1. Time of monotonic exponential growth

   $\tau \propto 1/\Delta H$

2. Frequency of the oscillations
   5 - 20 MHz

   $f \propto \sqrt{\Delta H}$

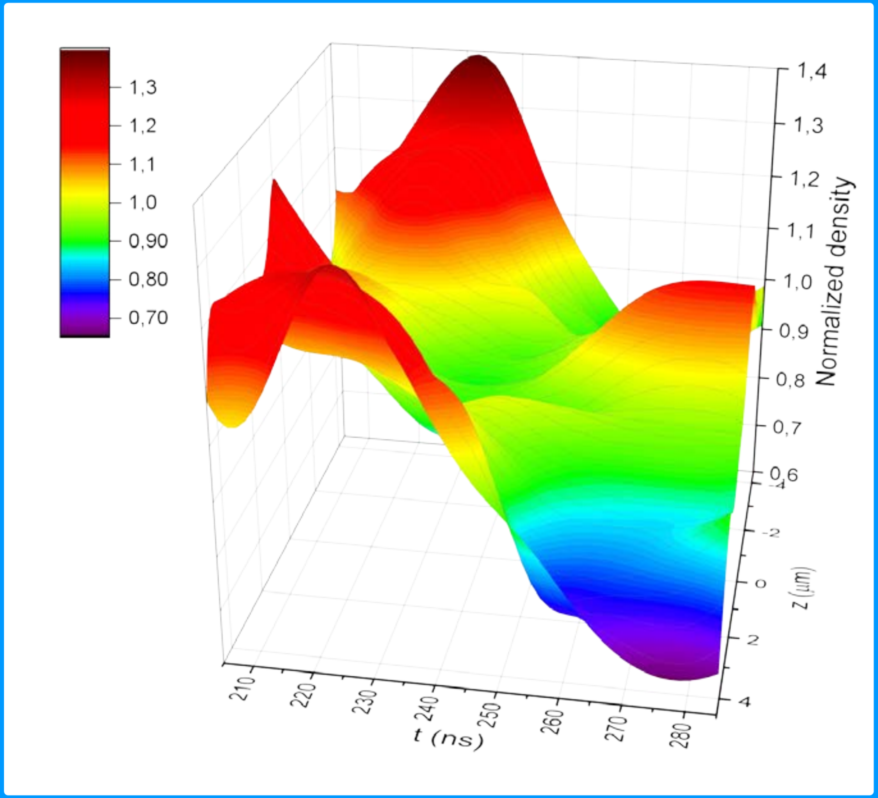
3. Damping of the oscillations

   $\Gamma \approx const$

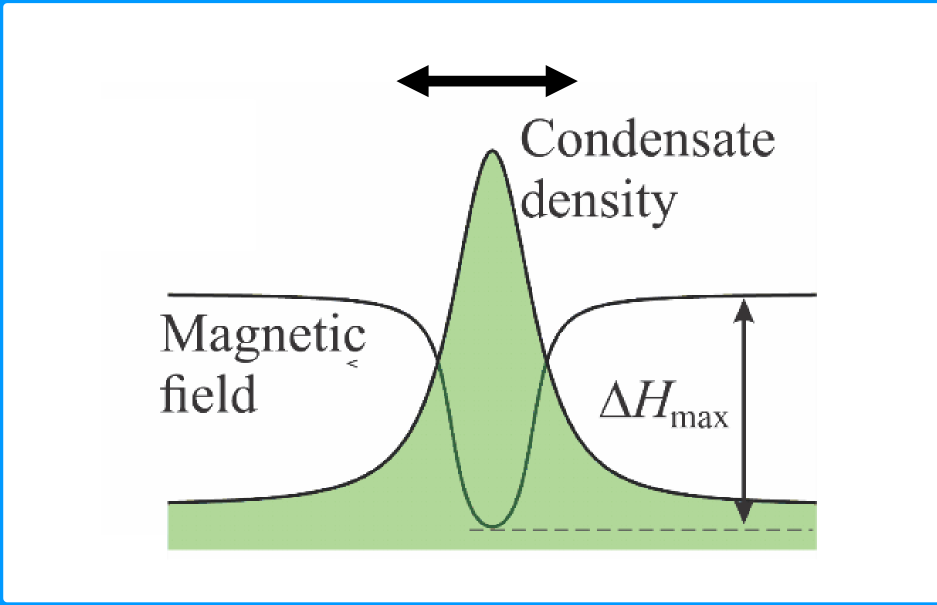
A well-defined oscillating transient process is observed

What kind of oscillations are observed?

We map the oscillation over the trap with spatial resolution.



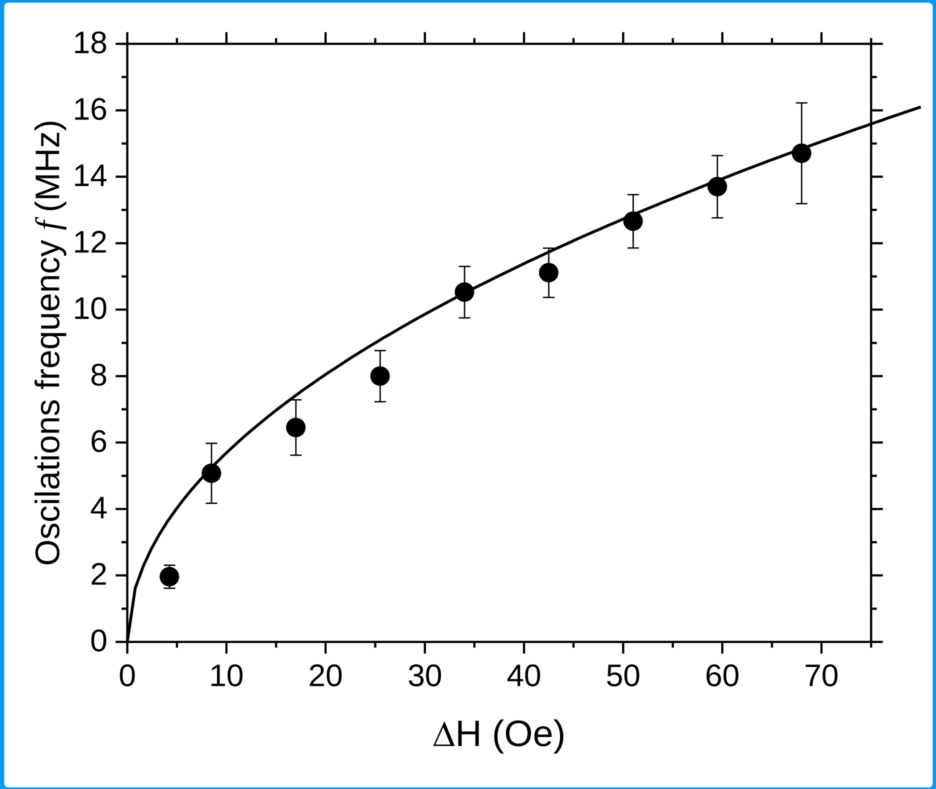
The oscillations correspond to redistribution of the condensate density within the trap



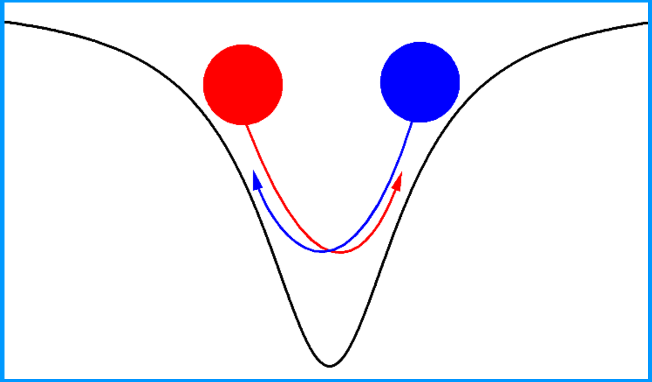
Condensate demonstrates properties of magnon liquid.

What kind of oscillations are observed?

By changing the trap depth we also change the curvature of the bed of the trap.



The oscillations correspond to redistribution of the condensate density within the trap



Model: two harmonic oscillators

Two components oscillate out of phase

Theory for description of magnonic liquid is needed.

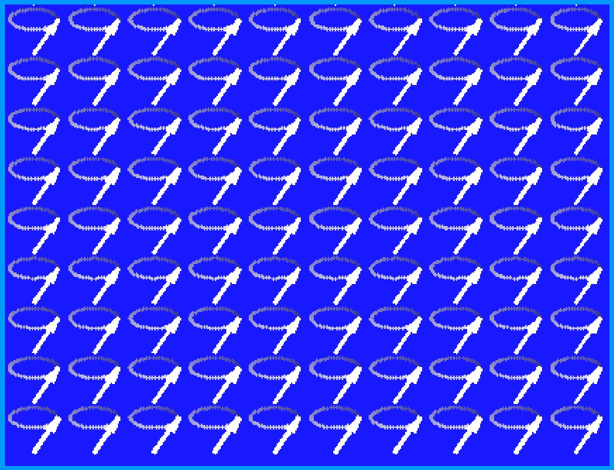
Magnon BEC versus atomic one

$$\left|\Psi,t\right\rangle = e^{-i\hat{H}t/\hbar}\left|\Psi,0\right\rangle = e^{-iEt/\hbar}\left|\Psi,0\right\rangle \Rightarrow -Et/\hbar$$ - QM-phase, is not observable

Spin 1/2 up/down states: $\left|+,t\right\rangle = e^{i\gamma Bt/2\hbar}\left|+,0\right\rangle \quad \left|-,t\right\rangle = e^{-i\gamma Bt/2\hbar}\left|-,0\right\rangle$

Arbitrary spin state: $\left|\Psi,0\right\rangle = \cos\frac{\theta}{2}\left|+,0\right\rangle + \sin\frac{\theta}{2}e^{i\varphi_0}\left|-,0\right\rangle$

Precessing spin: $\left|\Psi,t\right\rangle = e^{+i\gamma Bt/2\hbar}\left(\cos\frac{\theta}{2}\left|+,0\right\rangle + \sin\frac{\theta}{2}e^{i(\varphi_0-\gamma Bt/\hbar)}\left|-,0\right\rangle\right)$



Courtesy: Prof. C. Patton

The frequency and the phase of precessing spins is determined by the phase difference of two pure QM-states.

The phase of magnon condensate is observable and, in principle, it can be controlled by external stimuli

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Spin current is a correct approach for spin systems
with dominating exchange interaction

Landau-Lifshitz equation

$$\frac{1}{\gamma}\frac{d\vec{M}\left(\vec{r}\right)}{dt}=\vec{T}_{eff}\left(\vec{M},\nabla\vec{M},\vec{r}\right)=$$

$$=-\vec{M}\times\vec{H}_{eff}\left(\vec{M},\nabla\vec{M},\vec{r}\right)$$

Application of spin current

$$\frac{1}{\gamma}\frac{d\vec{M}\left(\vec{r}\right)}{dt}=-\nabla j\left(\vec{M},\nabla\vec{M},\vec{r}\right)$$

Is it always possible? No!

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Transverse spin diffusion in ferromagnets

Yaroslav Tserkovnyak,[1] E. M. Hankiewicz,[2] and Giovanni Vignale[3]

$$\partial_t \mathbf{S} = D\nabla^2\mathbf{S}$$

$$\mathbf{j}_i' = -A\mathbf{m}\times\partial_i\mathbf{m}$$

$$\partial_t \mathbf{S} = -\sum_{i=x,y,z}\partial_i \mathbf{j}_i$$

$$\vec{j}\propto-\vec{M}\times\frac{d\vec{M}}{dx}$$

Purely exchange model

Conservation law,
continuity equation,
and current:

$$\frac{dQ}{dt}+\nabla\vec{j}=0$$

One cannot introduce a current,
if the corresponding quantity is not conserved

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The angular momentum in the spin subsystem is not concerved

PHYSICAL REVIEW VOLUME 114, NUMBER 2 APRIL 15, 1959

Cross-Relaxation in Spin Systems*

N. Bloembergen, S. Shapiro, P. S. Pershan,† and J. O. Artman‡
Gordon McKay Laboratory, Harvard University, Cambridge, Massachusetts

(Received November 20, 1958)

Consider the spin Hamiltonian

$$\mathcal{H}=\mathcal{H}_m+\mathcal{H}_{cr}+\mathcal{H}_{int}, \quad (1)$$

$\mathcal{H}_{cr}$ is the sum of the crystalline field couplings of the individual ions or the quadrupole couplings of the nuclei. The Zeeman energy in the applied field is given by

$$\mathcal{H}_m=-\sum_i \beta \mathbf{H}\cdot\mathbf{g}_i\cdot\mathbf{S}_i.$$

The interaction between the spins consists of dipolar, pseudodipolar, and exchange terms:

$$W_{dip}=\frac{\vec{M}_1\cdot\vec{M}_2}{r^3}-\frac{3\left(\vec{M}_1\cdot\vec{r}\right)\left(\vec{M}_2\cdot\vec{r}\right)}{r^5}$$

Replace ϕ_{ij} in Eqs. (2) by $\phi_{ij}+\Phi$. The angular momentum associated with the rotation of the crystal is represented by the operator $(\hbar/i)(\partial/\partial\Phi)$. The total angular momentum around the z-axis is given by

$$J_z=\sum_j S_{zj}+\frac{\hbar}{i}\frac{\partial}{\partial\Phi}.$$

It can readily be verified that this operator commutes with the Hamiltonian (1):

$$[J_z,\mathcal{H}]=0.$$

The spin angular momentum $\sum_j S_{zj}$ alone does not commute with the dipolar interaction, but total angular momentum is indeed conserved. If the spin system

Magnetic dipole interaction results in spin-lattice coupling.

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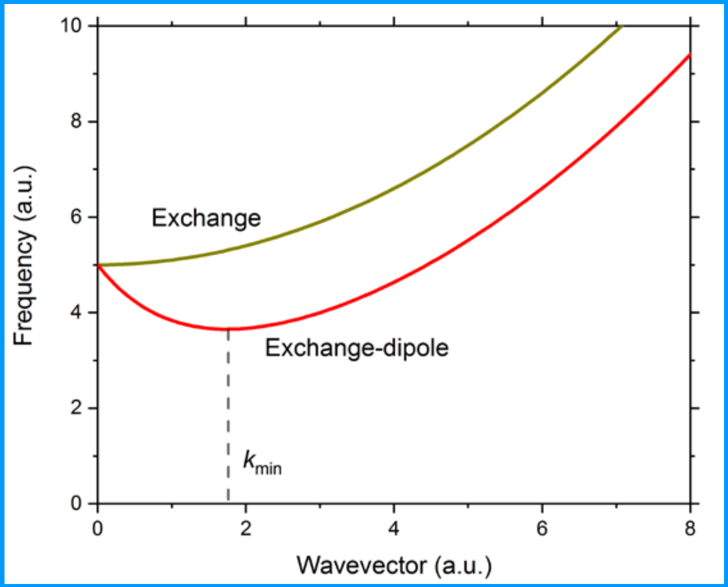
The angular momentum in the spin subsystem is not concerved

For a plane wave: $\vec{j} \propto -\vec{M} \times \frac{d\vec{M}}{dx} \propto k_{\min}$



At k_{min} the description based on the spin current definition derived for purely exchange model obviously fails.

Spin waves at k_{min} do not transfer any angular momentum, since their group velocity is zero.

Created cloud of such spin waves is at rest.

$@k_m$:

Zeeman ≈ 2 GHz
Dipole-dipole ≈ 150 MHz
Exchange ≈ 15 MHz

Spin waves at k_{min} are dipole-dominated

Summary

- Discovery of RT magnon BEC opened several fundamental questions. And it takes >10 years to answer them
- Coherence properties of the condensate are now well investigated
- Second sound is observed and studied
- Inroduction of local inhomogeneous magnetic field as a stimulus opened a lot of opportunities for investigation of spatio-temporal dynamics of the condenste, which show properties of magnon fluid
- Since magnons of the condensate are dipole dominated, the attempts to describe the condensate using spin-current approach are questionable

http://www.uni-muenster.de/Physik/AP/Demokritov/