Unveiling new phenomena in magnetic systems

Benedetta Flebus
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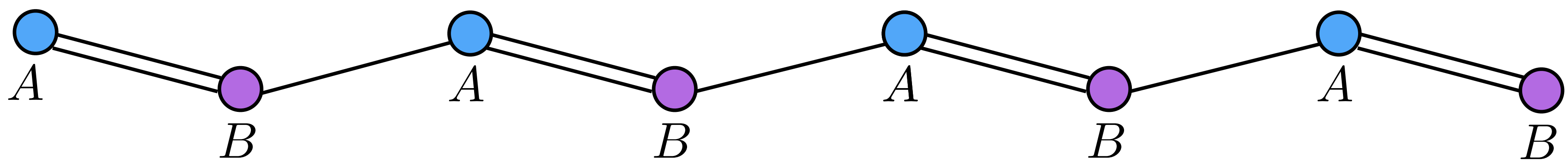
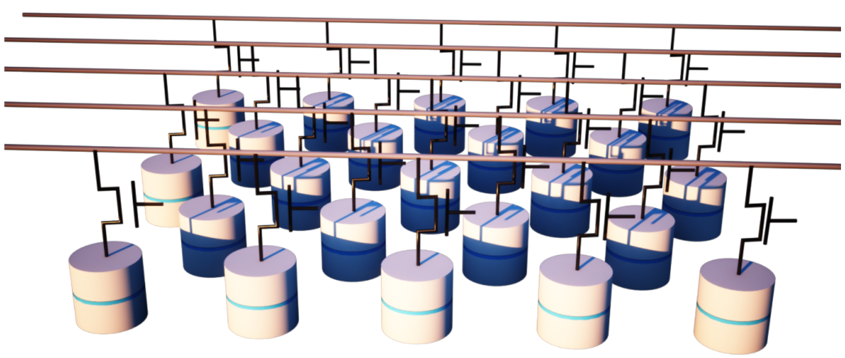
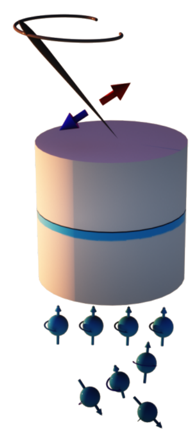
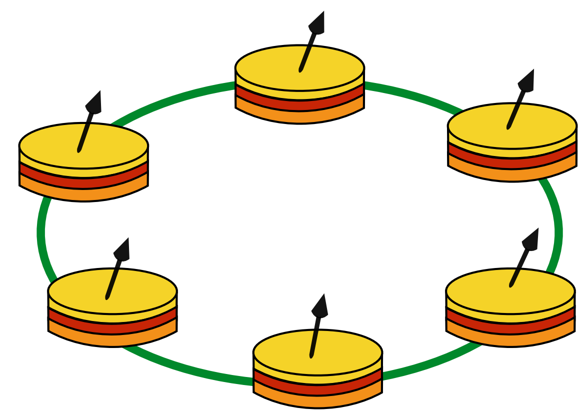








P. Gunnink, **B.F.** *et al.*, in preparation

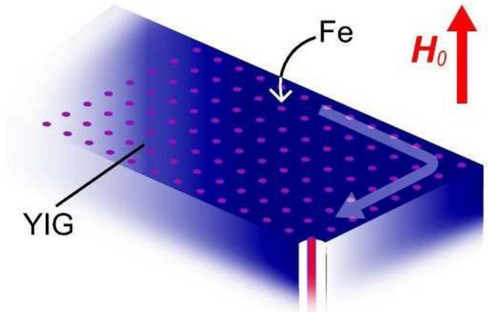
B.F. *et al.*, PRB Rapids (2020)

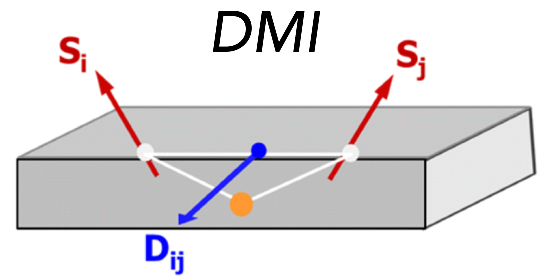
K. Deng and **B.F.**, in preparation

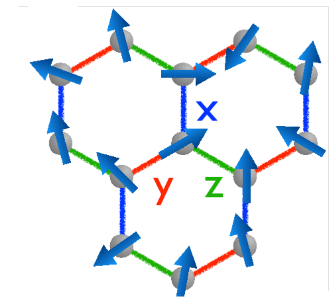
Topology and dissipation in magnetic systems

- Earlier proposals of magnetic topological phases based on Hermitian models

S.K. Kim *et al.*, PRL (2016)
K.H. Lee *et al.*, PRB (2018)
K. Nakata *et al.*, PRB (2017)
N. Okuma *et al.*, PRL (2017)
And many more…

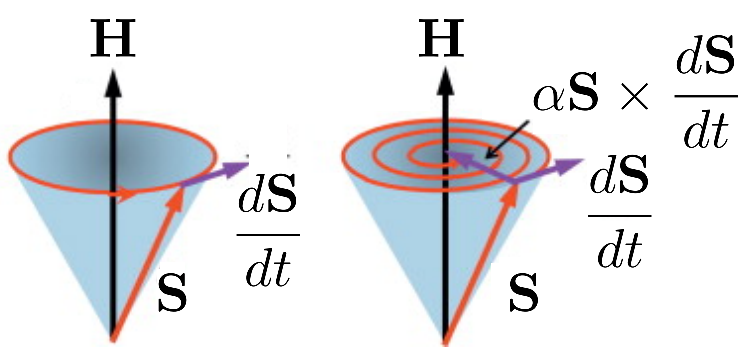






Magnon spin non-conserving interactions with the crystalline lattice
(and electrons, impurities, magnons, etc) are ubiquitous

- Let's consider the simplest picture of magnetic dissipation (FM macrospin, long wavelength limit)



LLG equation: $\frac{d\mathbf{S}}{dt} = -\mathbf{S}\times\mathbf{H} + \alpha\mathbf{S}\times\frac{d\mathbf{S}}{dt}$

\+ linearization and Holstein-Primakoff

Non-Hermitian Hamiltonian

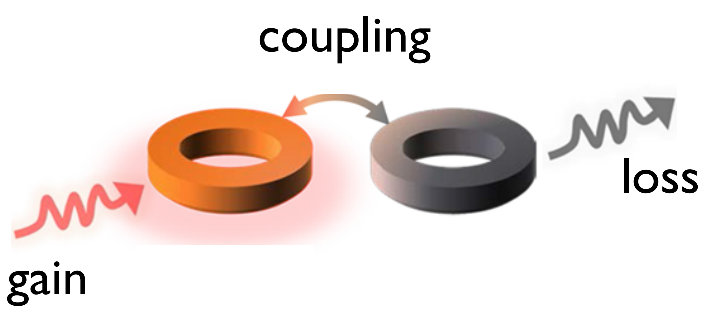
$$\mathcal{H} \propto -i\alpha\, a^\dagger a$$

Non-Hermitian topological theories

Unique non-Hermitian topological phases with no Hermitian counterpart

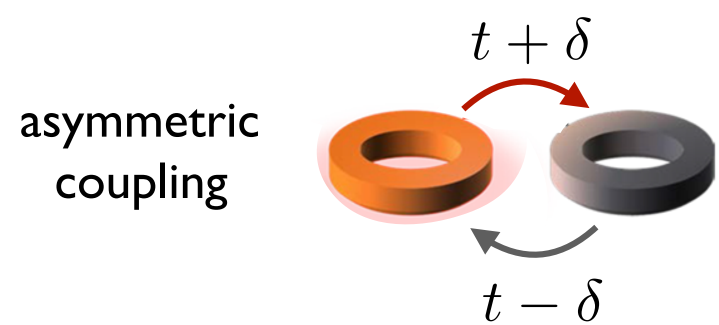
Lasing edge states

- Purely real bulk spectrum Im(E)=0
- Edge states as complex conjugate pairs (Im(E)>0 lasing and Im(E)<0 lossy)



Skin effect

- Breakdown of the bulk-edge correspondence (Bloch's theorem does not hold)
- Macroscopic number of bulk states accumulate at a boundary



Experimentally realized in (mostly one-dimensional) meta-materials and photonic systems

Solid state platform is lacking

Non-Hermitian topology of magnetic systems

Key ingredients:

- Loss (intrinsic to any magnetic system)
- Gain (achievable with established experimental techniques, e.g., spin injection, AC fields…)

To serve as nearly ideal solid-state platform for unconventional phenomena

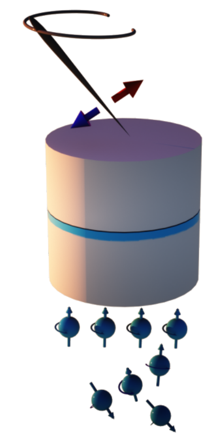
Magnetic systems

Non-Hermitian topology

To shed light on the long sought-after magnon topological edge states

Outline

- Non-Hermitian topological phase and lasing edge modes in an array of spin torque oscillators

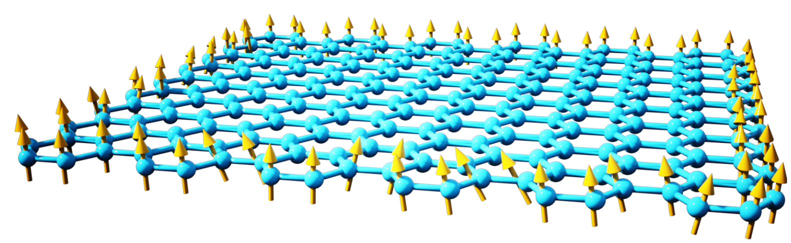


B.F. *et al.*, PRB Rapids (2020)

P. Gunnink, **B.F.** *et al.*, in preparation

Analysis of linearized dynamics (Hamiltonian) and of non-linear effects

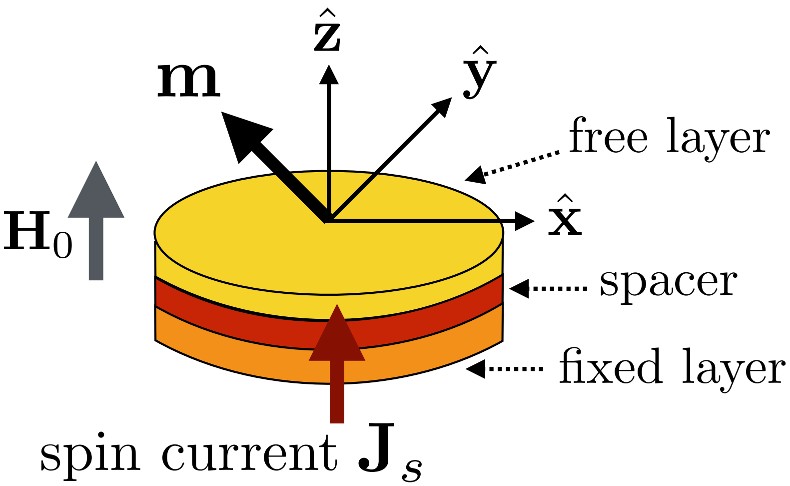
- Emergence of skin effect in a spin-orbit-coupled vdW ferromagnet

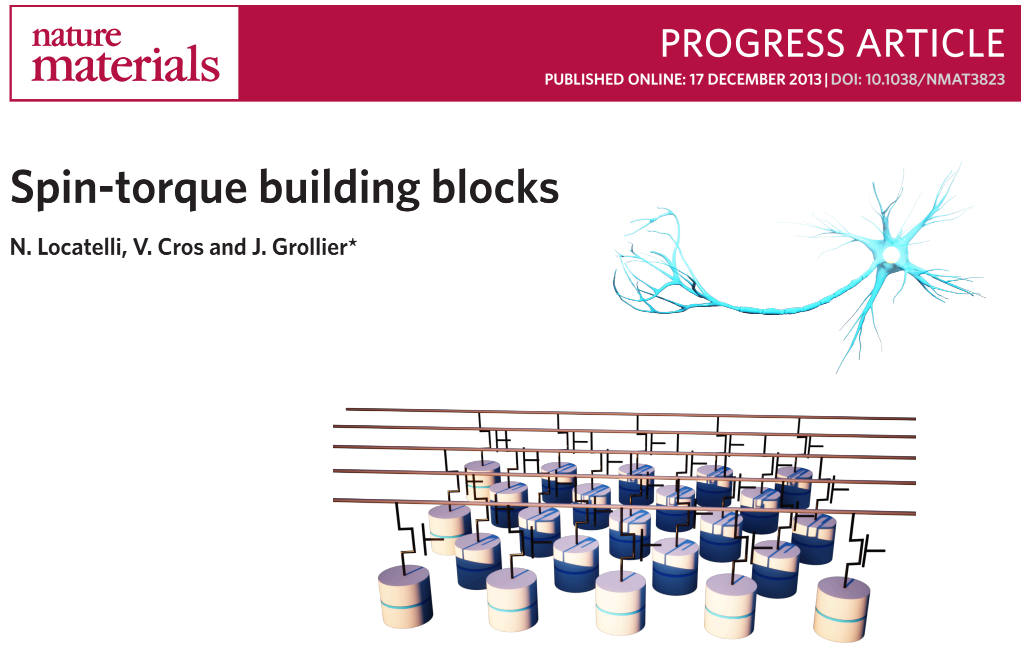


K. Deng and **B.F.**, in preparation

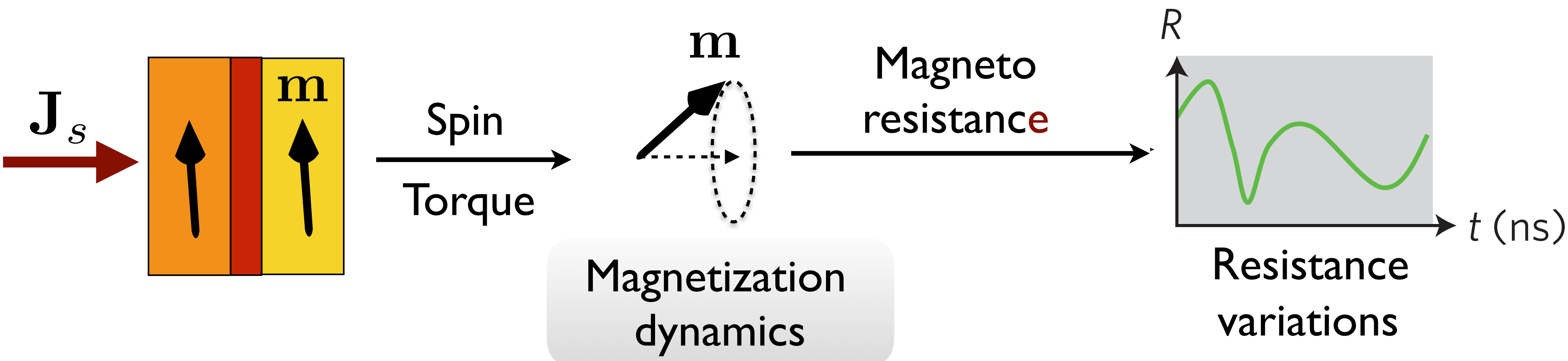
New phenomenological approach to magnetic dissipation in a lattice model

Spin-torque oscillator as building block





Spin-torque oscillator (STO)



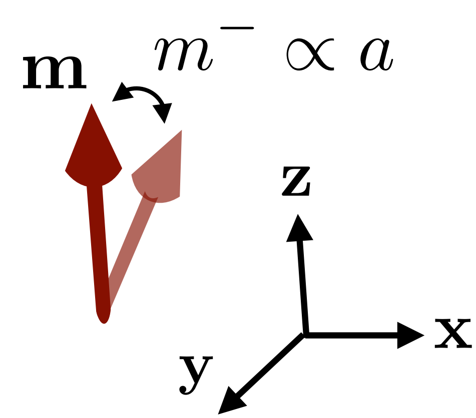
Magnetization dynamics

$$\frac{d\mathbf{m}}{dt} = \omega_0 \hat{\mathbf{z}} \times \mathbf{m} + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt} + J_s \mathbf{m} \times (\mathbf{m} \times \hat{\mathbf{z}})$$

FMR frequency — damping — spin torque (Berger, Slonczewski)

- Linear regime: $\mathbf{m} \simeq (\delta m_x, \delta m_y, 1)$
- Holstein-Primakoff transformation:

$$2m_+ = m_x - i m_y \propto a$$



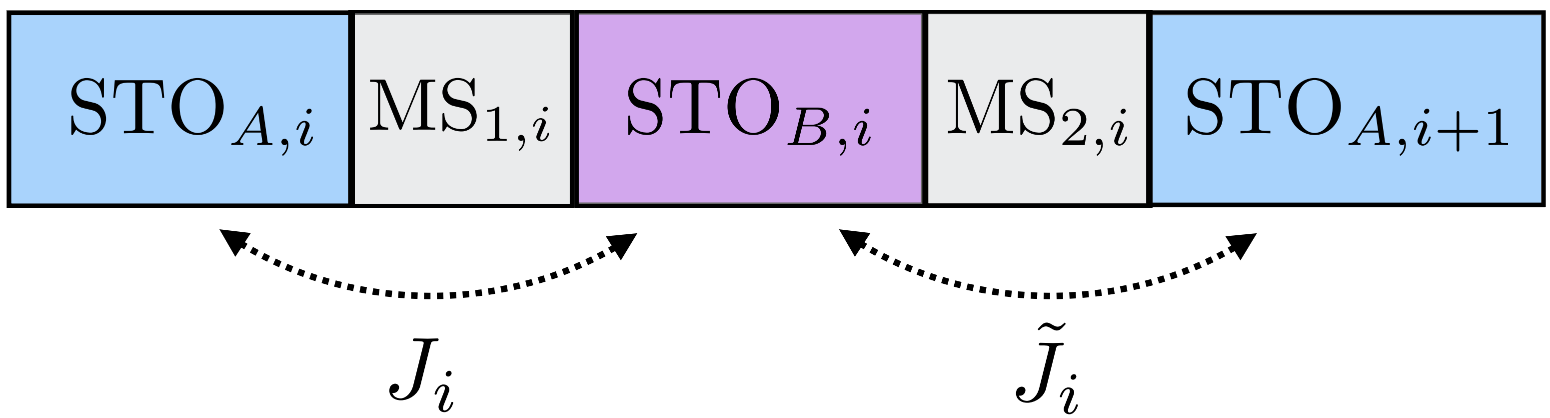
Non-Hermitian Hamiltonian

$$\mathcal{H} = \omega_0 a^\dagger a + i(J_s - \alpha\omega_0) a^\dagger a$$

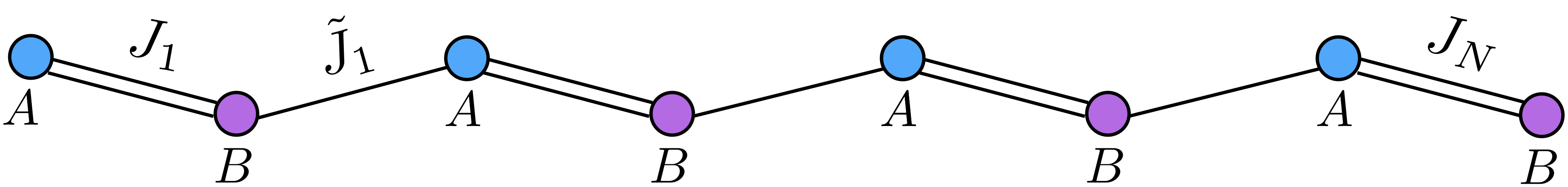
Gain/loss can by tuned via spin current

Coupling spin-torque oscillators

$2N$ spin torque oscillators arranged in N unit cells

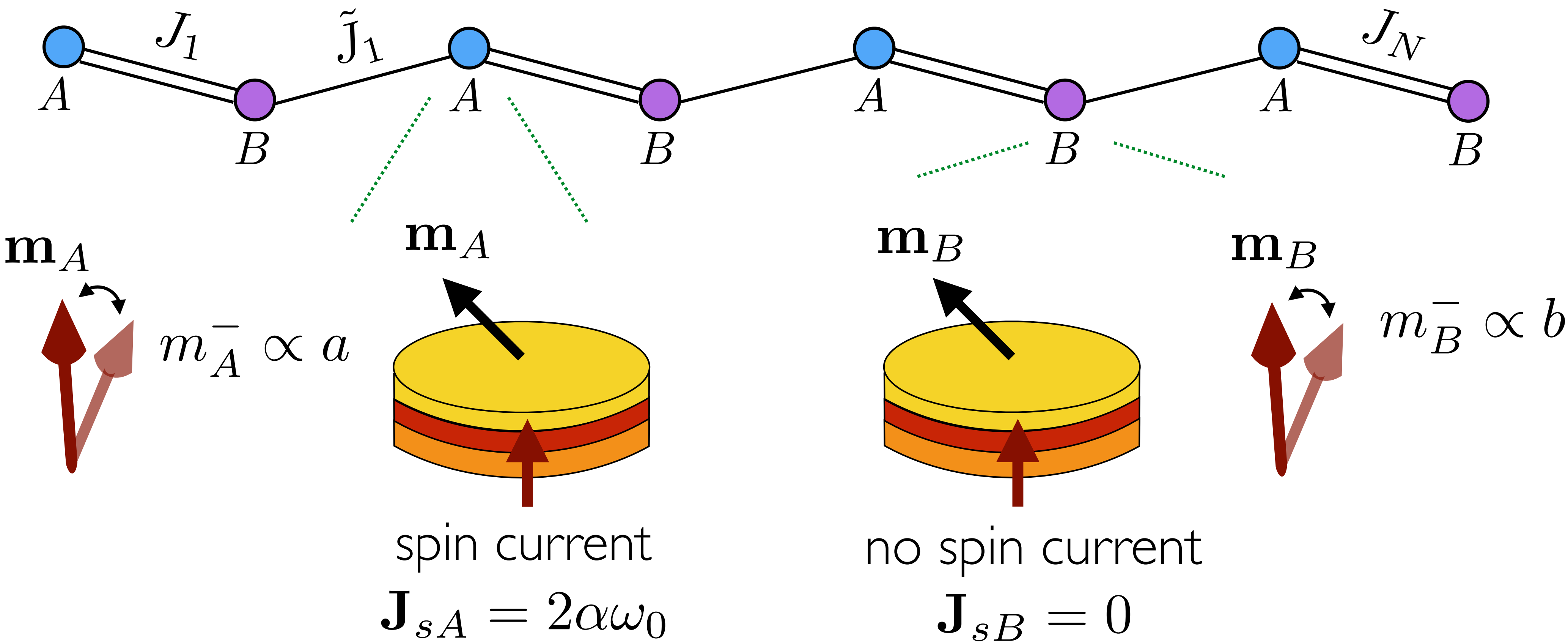


metallic spacers mediating a nearest-neighbor RKKY-like coupling $\propto J, \tilde{J}$ between spin-torque oscillators



non-Hermitian SSH model

Gain and losses



Gain on the A sites

$$\mathcal{H} \propto i\alpha\omega_0 a^\dagger a$$

Loss on the B sites

$$\mathcal{H} \propto -i\alpha\omega_0 b^\dagger b$$

PT-symmetric SSH model: balanced gain/loss

Bulk-edge correspondence

Can we straightforwardly define a topological invariant that predicts the number of edge states?

Not guaranteed in non-Hermitian systems
(skin effect, discrepancy between OBC and PBC spectra)

PBC Hamiltonian

$$\mathcal{H}_k = -\begin{pmatrix} -i\alpha\omega & J + \tilde{J}e^{-ik} \\ J + \tilde{J}e^{ik} & i\alpha\omega \end{pmatrix}$$

Parity-time (PT) symmetry

$$\sigma_x \mathcal{H}_k^* \sigma_x = \mathcal{H}_k$$

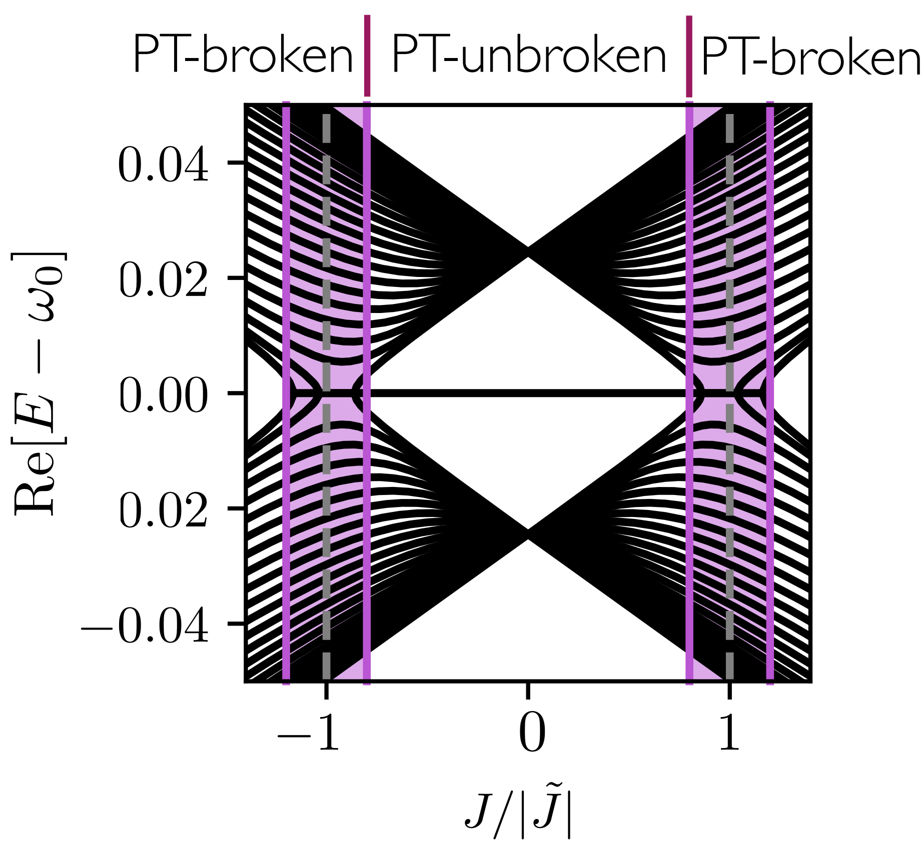
PT symmetry + real PBC spectrum

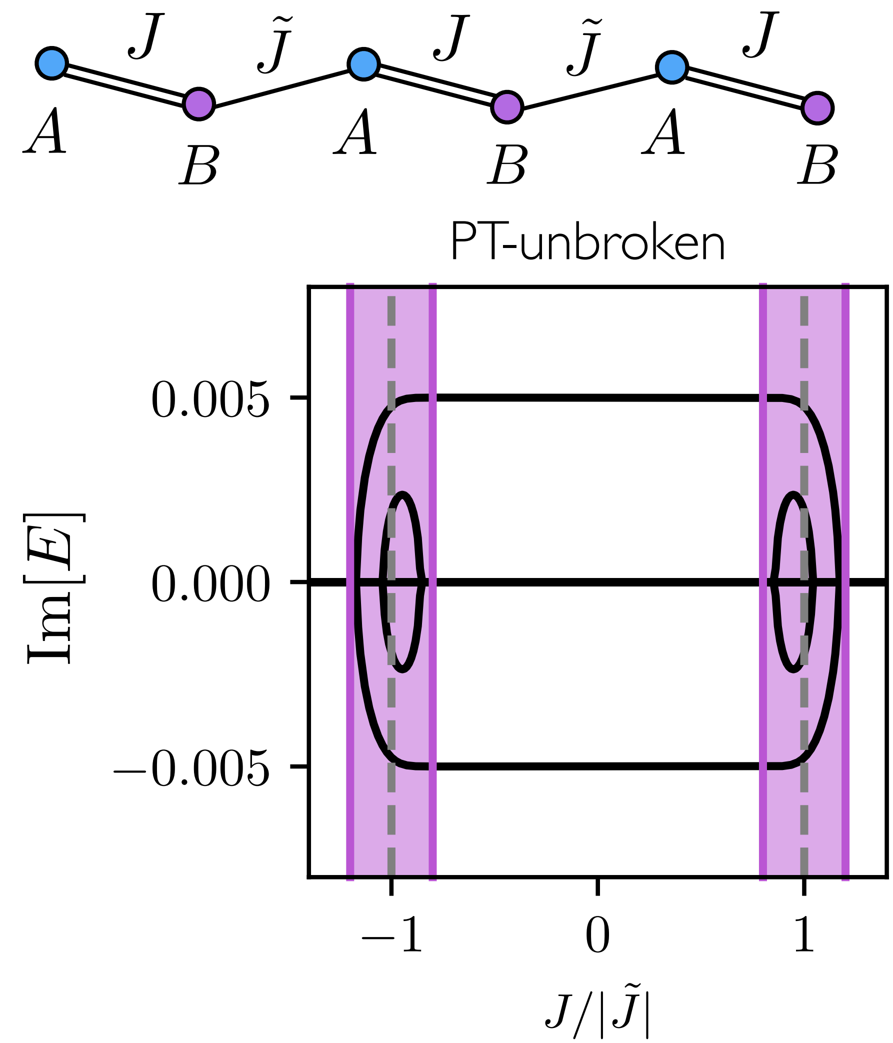
→ **bulk-edge correspondence holds**

K. Kawabata et al., PRX (2019)

PT-symmetry and edge states

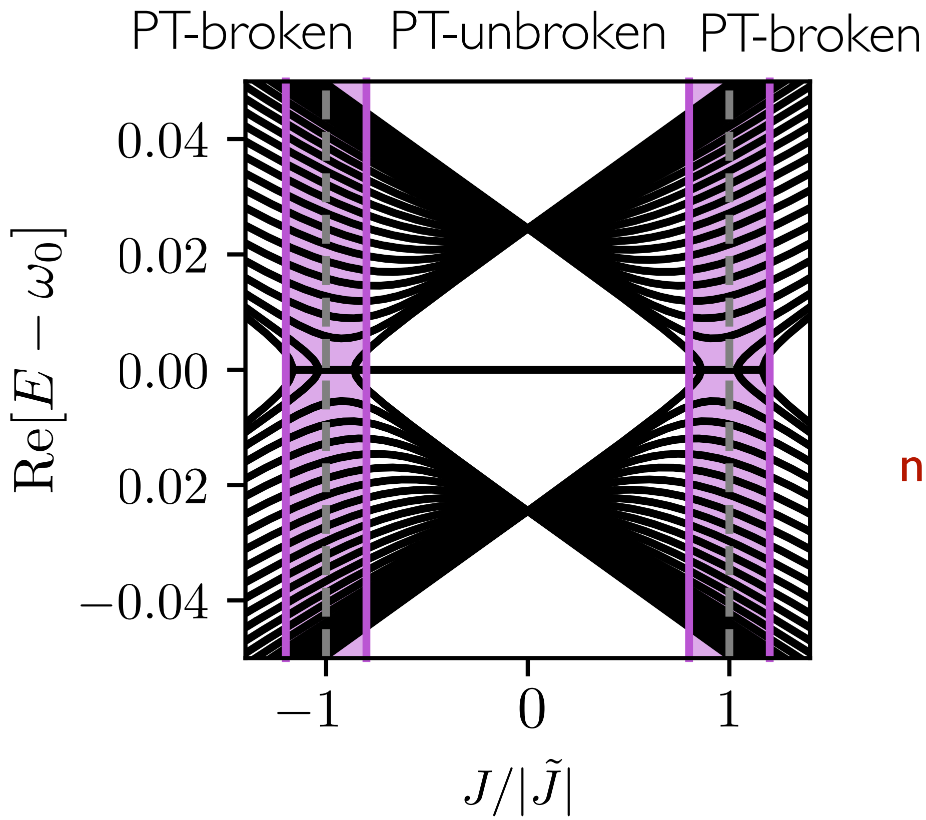
- Open boundary conditions





- Bulk states can be: purely real (PT-unbroken) or complex conjugated pairs (PT-broken)
- Two complex conjugated topological edge states with $\mathrm{Re}[E] = \omega_0, \ \mathrm{Im}[E] = \pm\alpha\omega_0$

Topological invariant



Global Berry phase

$$Q^c = i \sum_{j=\pm} \int_{BZ} \langle \phi_k^j | \partial_k | \psi_k^j \rangle dk$$

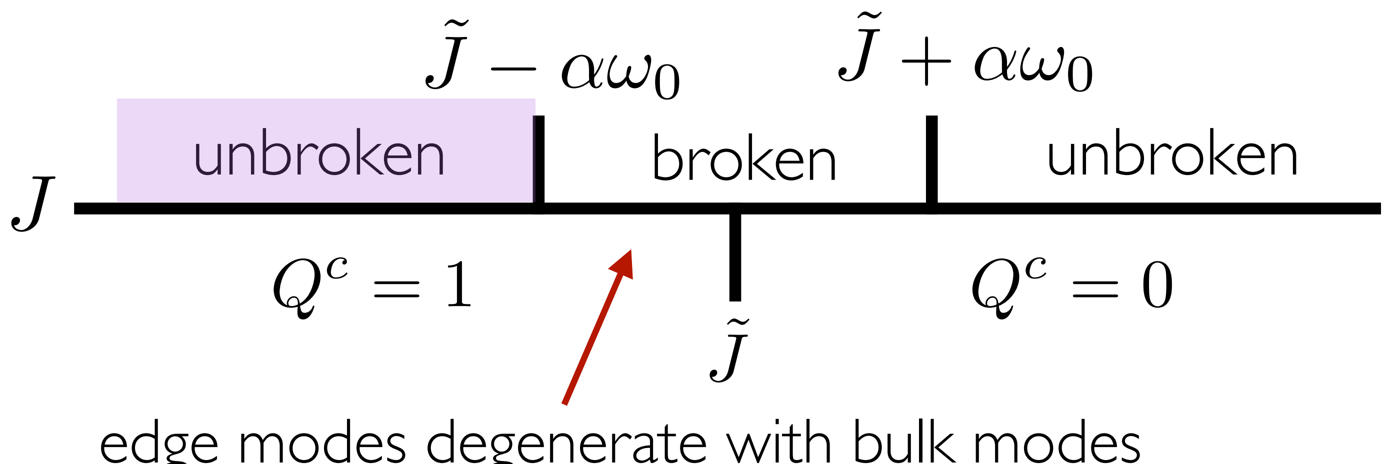
$|\psi_k\rangle, \langle \phi_k|$ right and left eigenvectors of $\mathcal{H}$

number of pairs of gapless-real-energy edge modes

$$Q^c = 1, \quad |J| < |\tilde{J}|$$

S. Liang *et al.*, PRA (2013)

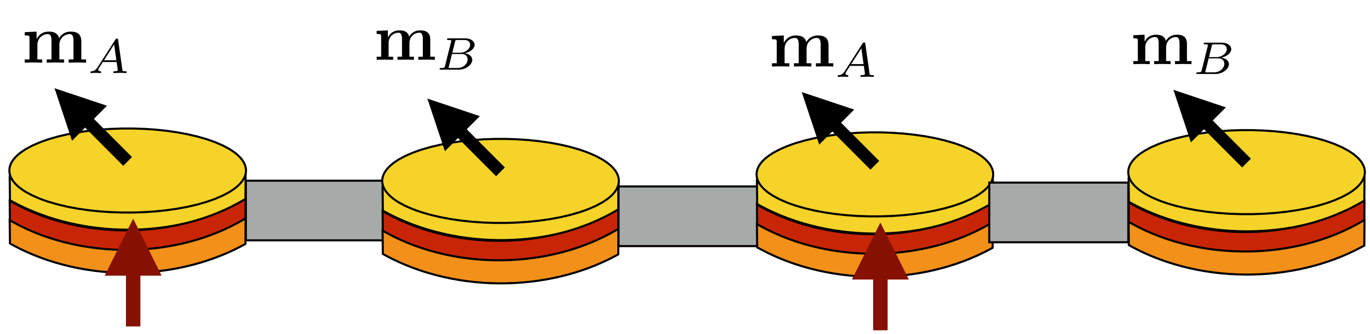
S. Lieu *et al.*, PRB (2018)



Going beyond the linear regime

Linearized equations of motion yield Hamiltonian that displays a topological phase with a lossy and a lasing magnon edge states + purely real bulk spectrum

Consequences on the non-linear LLG classical dynamics?



spin current
$\mathbf{J}_{sA} = 2\alpha\omega_0$

Naively, we would expect all type A STOs to quickly undergo precession (B STOs follow due to coupling)

Magnetization simulations

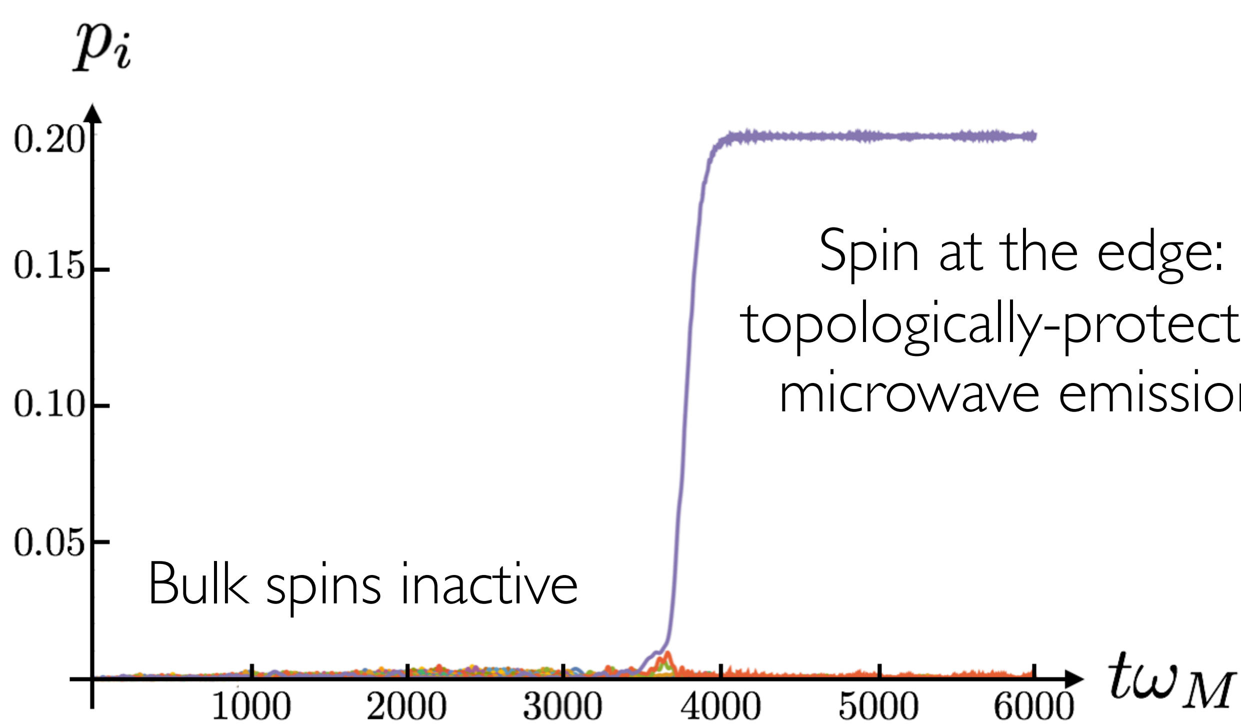
Simulations of the magnetization dynamics of 20 coupled STOs

$$\mathbf{m}_i = \begin{pmatrix} 2\sqrt{p_i(1-p_i)}\sin\phi_i \\ 2\sqrt{p_i(1-p_i)}\cos\phi_i \\ 1-2p_i \end{pmatrix}$$

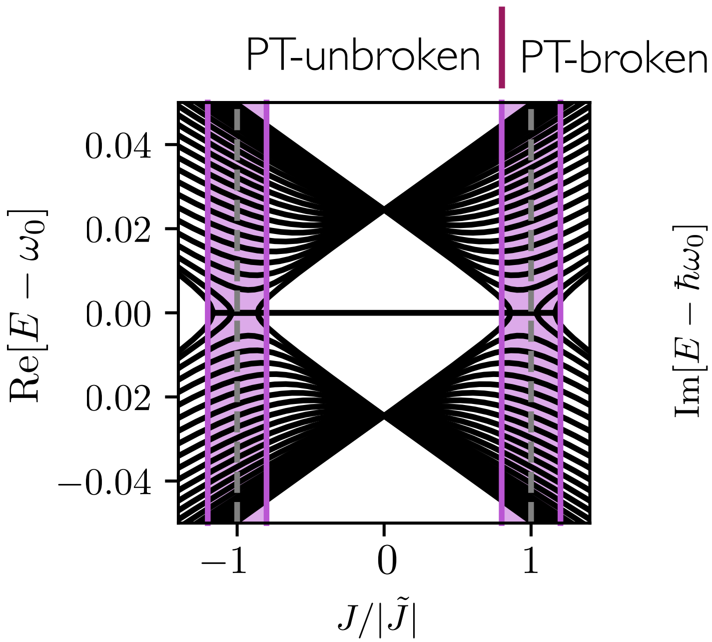
power of the ith oscillator p_i
(experimental microwave power)

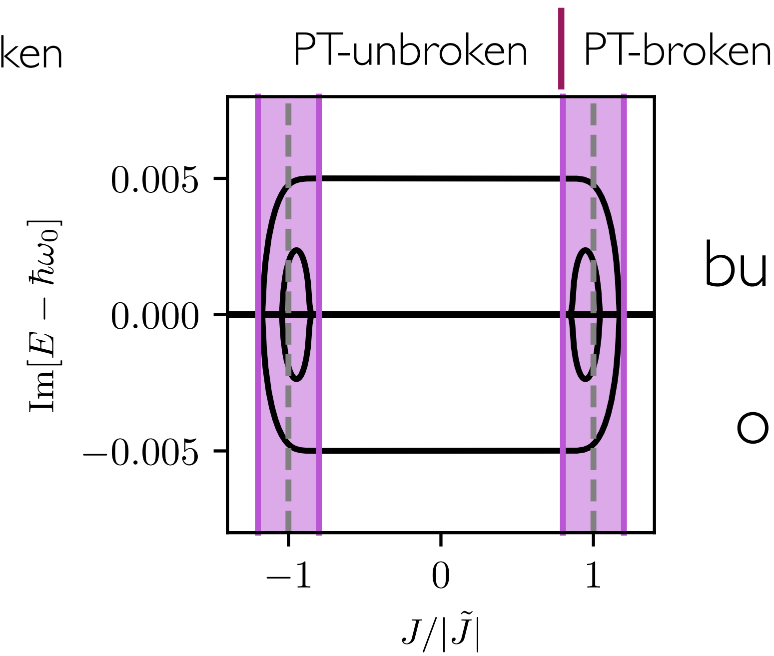
precession angle ϕ_i

PT-unbroken phase: purely real bulk spectrum, one lasing edge state

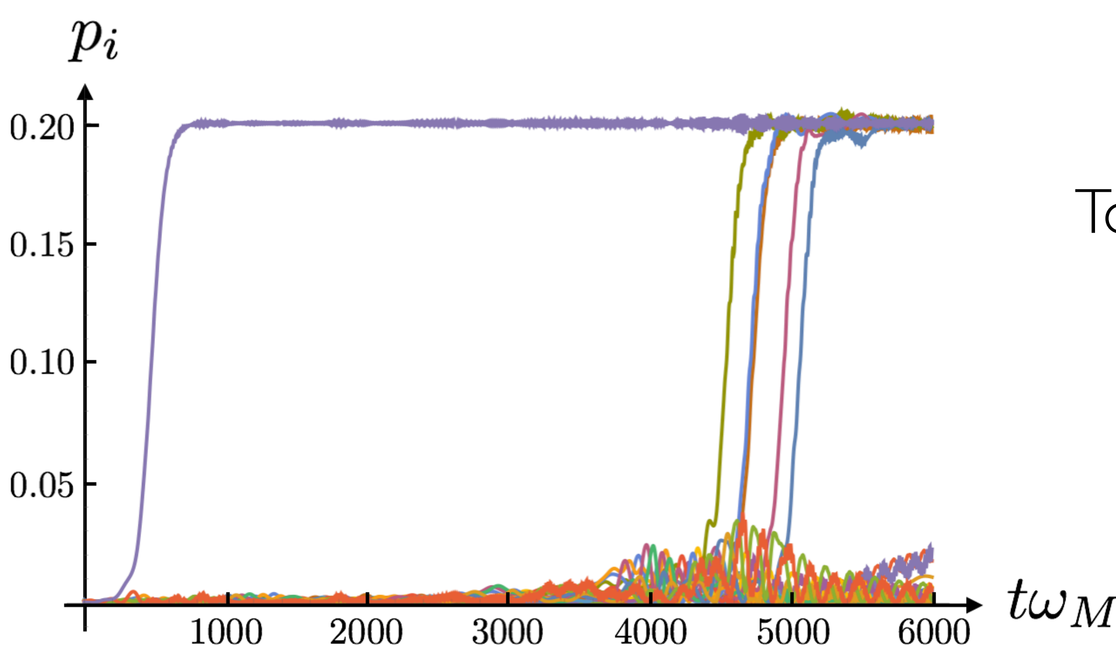


Magnetization simulations





PT-broken phase:
bulk spectrum acquires imaginary energy,
one lasing edge state



Spin at the edge:
Topologically microwave emission

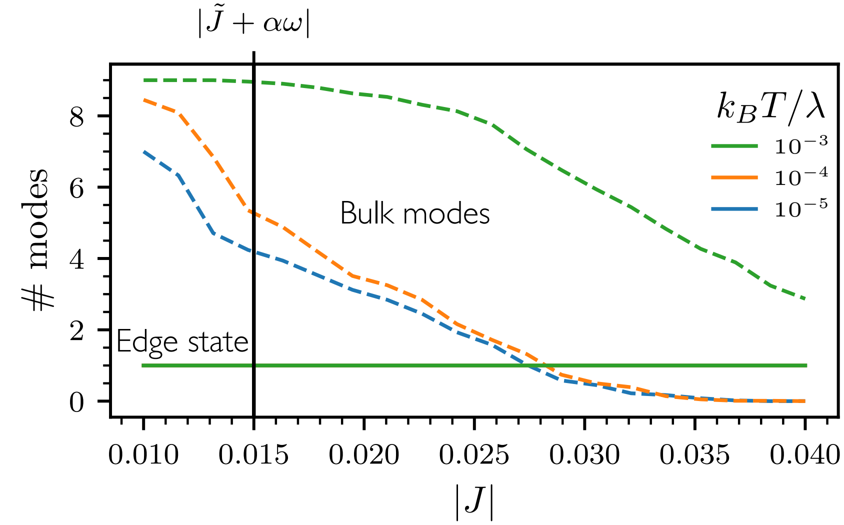
Shortly after: bulk spins start precessing as well

Robustness against perturbations

P. Gunnink, **B.F.** *et al.*, in preparation

Perturbations +
Non-linear thermal dynamics

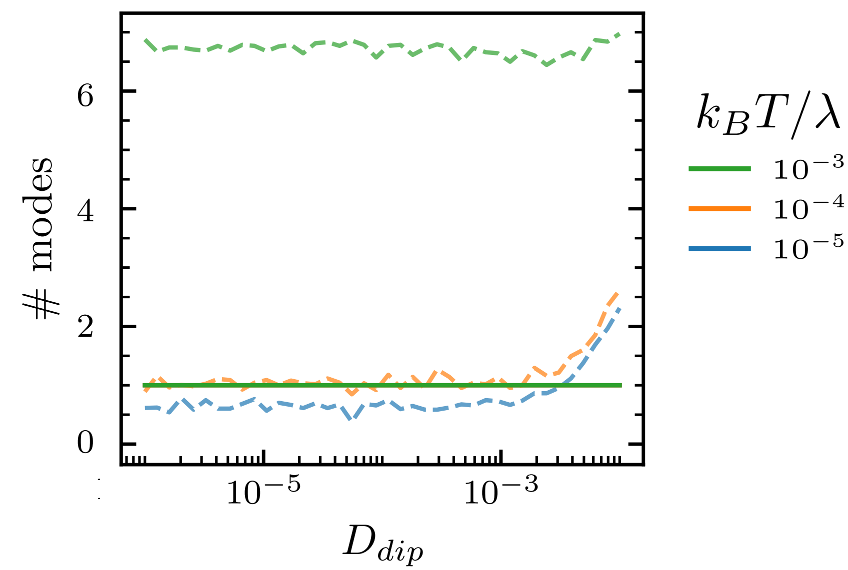
- At higher temperatures bulk can start lasing in the PT-unbroken phase



- Nonlocal damping/spin pumping through via metallic spacers $J \to J - iG\omega_0$

Topological protection survives,
bulk can start lasing

- Dipolar interactions: above a critical value bulk can start lasing



Summing up …

- **Linearized** dynamics of a coupled STOs array with gain/loss can realize a topological non-Hermitian SSH model
- Our model has a PT-unbroken phase with purely real bulk modes and magnon lasing and loss edge states
- Simulations of **non-linear** dynamics show classical spin at the lasing edge emitting topologically-protected microwave power
- We have investigated experimentally-relevant perturbations
- Numerous potential applications

nature materials | PROGRESS ARTICLE
PUBLISHED ONLINE: 17 DECEMBER 2013 | DOI: 10.1038/NMAT3823

Spin-torque building blocks

N. Locatelli, V. Cros and J. Grollier*

memory devices,
spintronics neural networks,
spin-wave waveguides…

Inclusion of magnetic dissipation can unveil new phenomena

Skin effect

- For decades, the application of topology to condensed matter has relied on **Bloch theory** and the **bulk-edge correspondence**

These principles hold for Hermitian systems,
not always for non-Hermitian systems

Consequence: Skin effect

macroscopic accumulation of bulk states at a boundary of the system

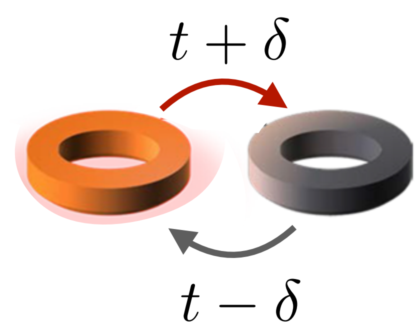
~ Recall bulk wave functions are delocalized in Bloch's theory

- Understood and experimentally realized (in meta-materials and photonic systems) in the context of the one-dimensional asymmetric SSH model

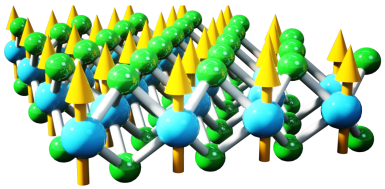
  ~ pile-up of bulk modes due to imbalance hopping in the left and right directions

- Higher dimensions?
- Naturally-occurring solid state systems?

asymmetric coupling



Our model



$$\mathcal{H} = -J\sum_{\langle i,j\rangle}\mathbf{S}_i\cdot\mathbf{S}_j - J_2\sum_{\langle\langle i,j\rangle\rangle}\mathbf{S}_i\cdot\mathbf{S}_j + B\sum_i S_i^z + D\sum_{\langle\langle i,j\rangle\rangle}\nu_{ij}\hat{\mathbf{z}}\cdot(\mathbf{S}_i\times\mathbf{S}_j)$$

NNN DMI allowed by symmetry in many van der Waals magnets (e.g. Chromium trihalides)

Dissipation in a lattice model valid over the first BZ?

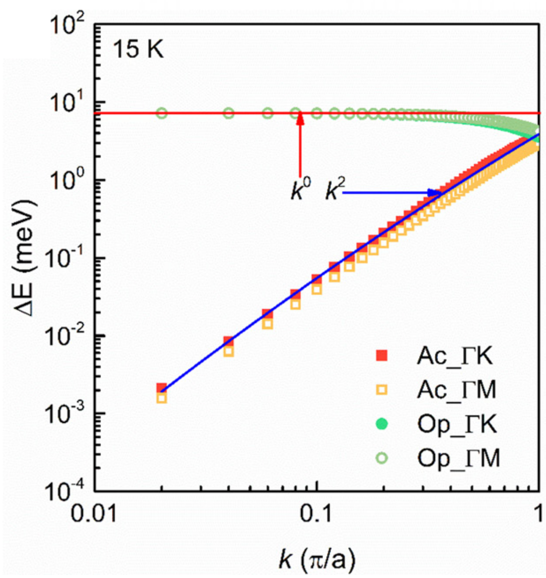
- Many sources (magnon-magnon, magnon-phonon, magnon-impurity scattering)
  A microscopic comprehensive description is a very challenging (yet untackled) task
- Usually treated (LLG phenomenology or perturbation theories) in the long-wavelength limit
  Not appropriate for a lattice model defined over the whole first Brillouin zone
- *Ab initio* calculations provide continuum expressions over the first BZ, e.g., $\propto k^2$
  Can not be readily incorporated in the lattice model since it does not respect its symmetries

Phenomenological approach that respects lattice symmetries
and reproduces *ab initio* and/or experimental results

Phenomenological approach to magnetic dissipation

Applied Physics Letters ARTICLE scitatio

Magnon relaxation time in ferromagnetic $Cr_2Ge_2Te_6$ monolayer governed by magnon-phonon interaction



$$\Delta E_{\mathrm{ac}} \propto k^2 \qquad \Delta E_{\mathrm{op}} \propto constant$$

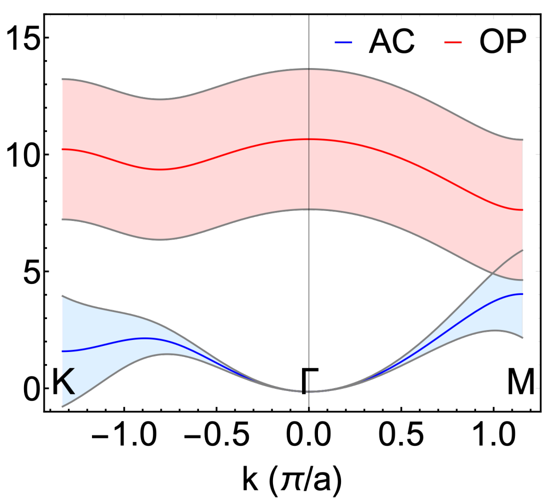
Can not be readily incorporated in the lattice model since it does not respect its symmetries

Ansatz consistent with translational and C3 symmetry

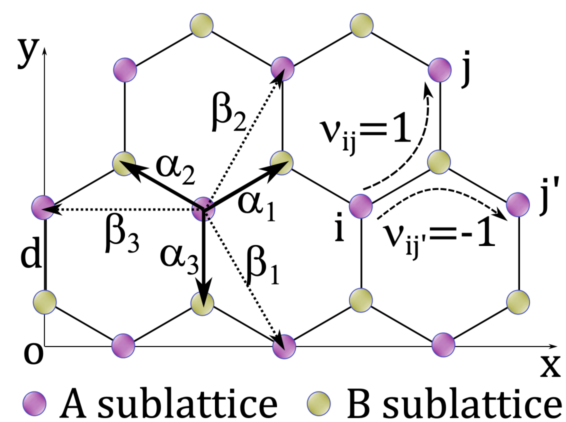
$$\Delta E_{\mathrm{ac}} = -i\chi_{11}(3 - \sum_j \cos \boldsymbol{k} \cdot \boldsymbol{\alpha}_j) - i\chi_{12}(3 - \sum_j \cos \boldsymbol{k} \cdot \boldsymbol{\beta}_j)$$

$$\Delta E_{\mathrm{op}} = -i\chi_2$$

$\chi_{11}, \chi_{12}, \chi_2$: fitting parameters



Next:
Conversion from eigenstate basis to lattice space basis



Phenomenological approach to magnetic dissipation

$$\mathcal{H}_{\mathrm{nh}} = [3JS + 6J_2S + B - \frac{i}{2}(\chi_2 + 3\chi_{11} + 3\chi_{12})] \times \sum_i (a_i^\dagger a_i + b_i^\dagger b_i)$$

$$+(-J_2S + \frac{i\chi_{12}}{4}) \sum_{\langle\langle i,j\rangle\rangle} (a_i^\dagger a_j + a_j^\dagger a_i + b_i^\dagger b_j + b_j^\dagger b_i)$$

$$-JS \sum_{\langle i,j\rangle} \left[1 - \frac{i(\chi_2 - 3\chi_{11} - 2\chi_{12})}{2S\sqrt{3(J^2+2D^2)}}\right] (a_i^\dagger b_j + b_j^\dagger a_i)$$

$$-DS \sum_{\langle\langle i,j\rangle\rangle} \nu_{ij} \left[i + \frac{\chi_2 - 3\chi_{11} - 2\chi_{12}}{2S\sqrt{3(J^2+2D^2)}}\right] \times (a_i^\dagger a_j - a_j^\dagger a_i + b_i^\dagger b_j - b_j^\dagger b_i) + \ldots$$

Local dissipation

Gilbert damping - like

Reminiscent of nonlocal spin pumping terms (electron-magnon interactions)

$$J \to J - iG\omega_0$$

B. Heinrich *et al.*, PRL (2003);
Y. Tserkovnyak *et al.*, PRB (2003)

Nonlocal terms must be here for a k-dependent dissipation (microscopic theory needed)

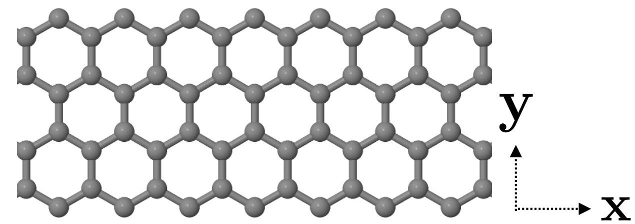
Dissipative higher-order-nearest-neighbor terms
(do not affect qualitatively our results — unclear microscopic origin)

Results

- Exact diagonalization on a 60x60 nano-ribbon

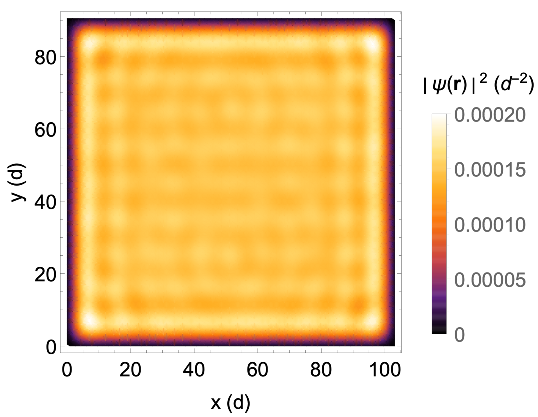
zig-zag terminations

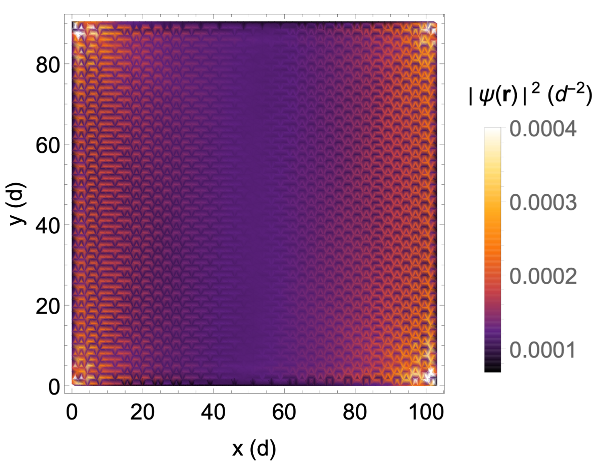
armchair terminations



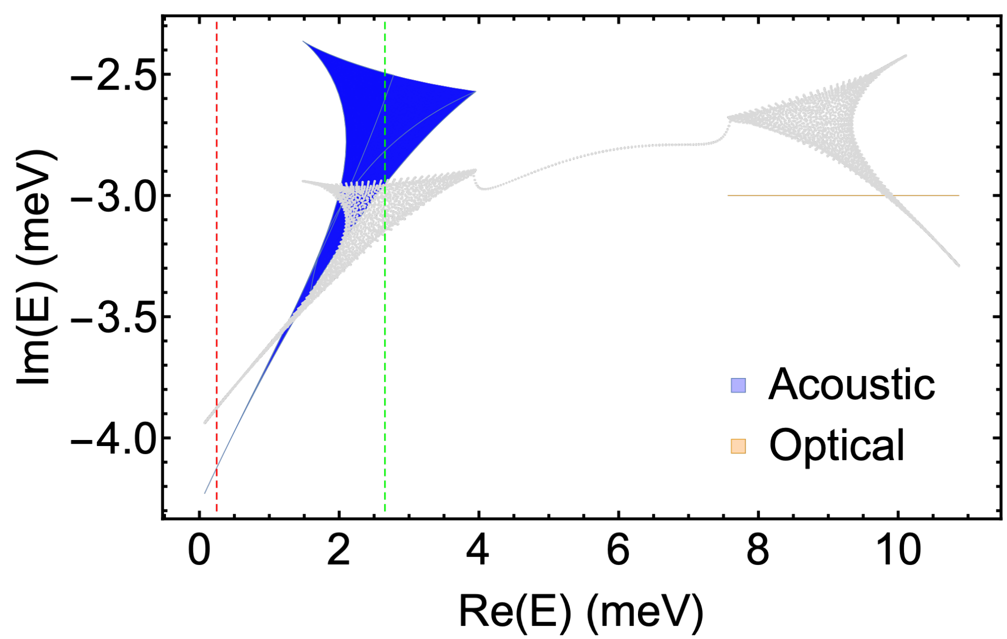
- **Skin effect:** $|\psi(\mathbf{r})|^2 = \frac{1}{N}\sum_{n=1}^{N} |\phi_n^R(\mathbf{r})|^2$

increasing temperature

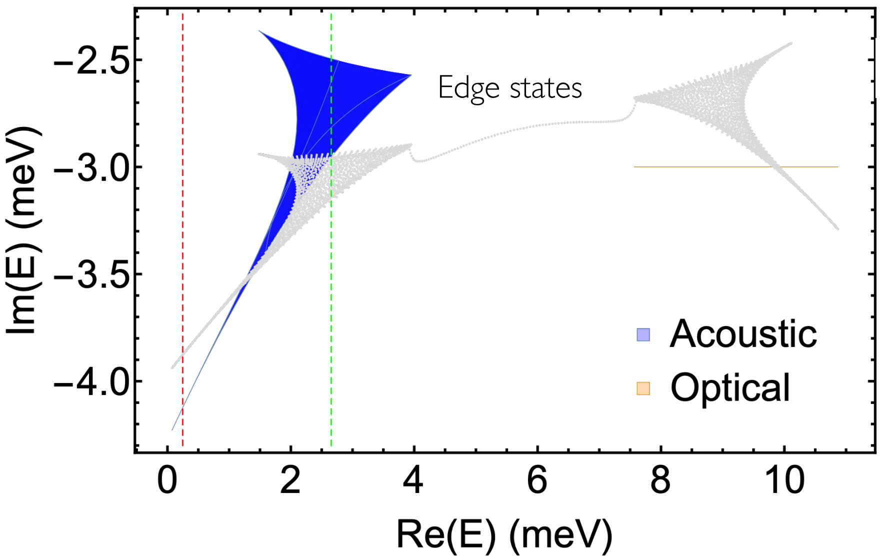




- **Broken bulk-edge correspondence** : discrepancy between PBC and OBC spectra

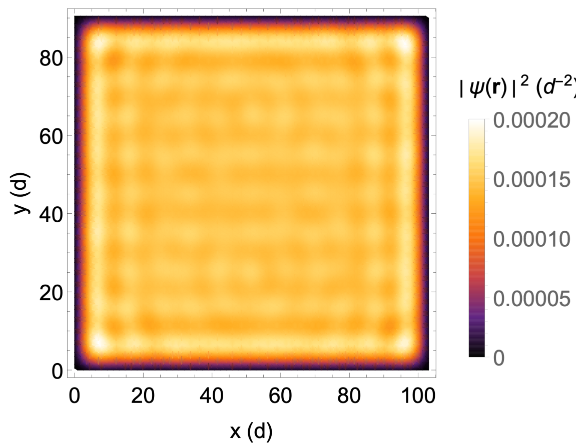


Spectral area and the emergence of the skin effect



Low frequency

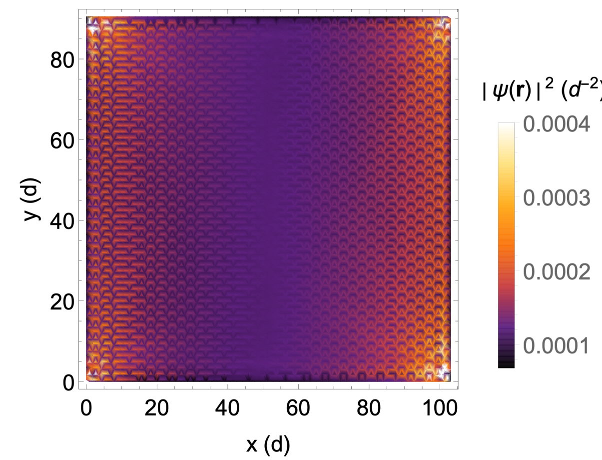
(PBC spectrum is a line)



No skin effect - Bloch behavior
Agreement with long-wavelength
LLG phenomenology

Higher frequency

(PBC spectrum has a finite area)



Skin effect appears!

In correspondence of arc or line (finite effective spectral area) in complex energy space, the mapping from momenta to energy is 2d to 1d (2d): for a wave impinging at the boundary there are infinite (finite) reflection channels, and an open boundary eigenstate can (can not) be described as superposition of Bloch waves.

Similar "area law" argument in arXiv:2102.05059 (2021)

However, there is a caveat …

"Effective area law"

$$\Delta E_{\text{ac}} = -\, i\chi_{11}(3 - \sum_j \cos \boldsymbol{k}\cdot\boldsymbol{\alpha}_j) - i\chi_{12}(3 - \sum_j \cos \boldsymbol{k}\cdot\boldsymbol{\beta}_j)$$

$$\Delta E_{\text{op}} = -\, i\chi_2$$

The dependence on the reciprocal vector leads to a **finite spectral area** for the acoustic mode

However: Inverse Fourier transformation

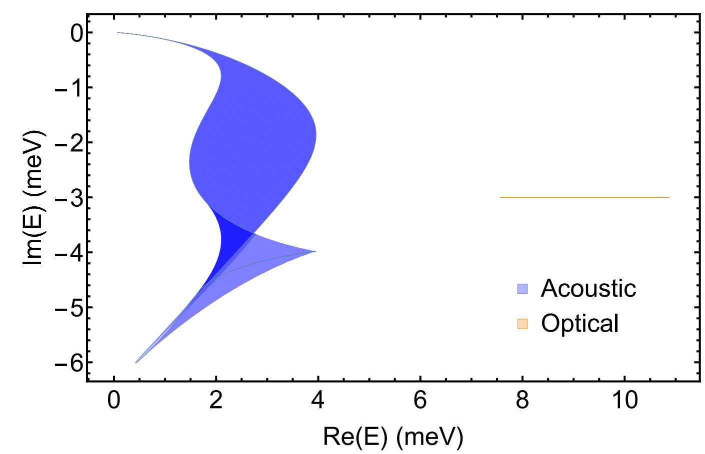
$$\chi_{12}\sum_{\boldsymbol{k}}\sum_{i,j}\sum_{n}\left\{e^{i\boldsymbol{k}\cdot[\boldsymbol{r}_i-(\boldsymbol{r}_j+\boldsymbol{\alpha}_n)]}+e^{i\boldsymbol{k}\cdot[\boldsymbol{r}_i-(\boldsymbol{r}_j-\boldsymbol{\alpha}_n)]}\right\}a_i^\dagger a_j$$

$= 0$ $\quad \boldsymbol{\alpha}_n$ connects A and B sites, ij labels AA or BB

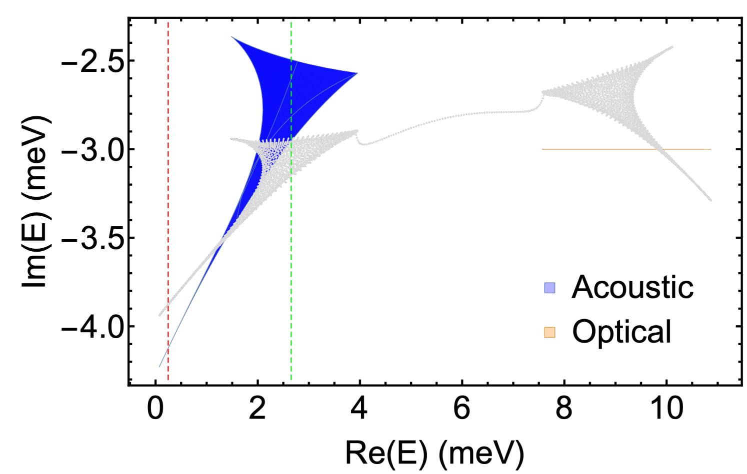
Ineffective term: No skin effect at any T

Skin effect requires **k**-dependent dissipation (non-local damping) that **survives in the real lattice representation**

Total area



Effective area = Total area - ineffective term

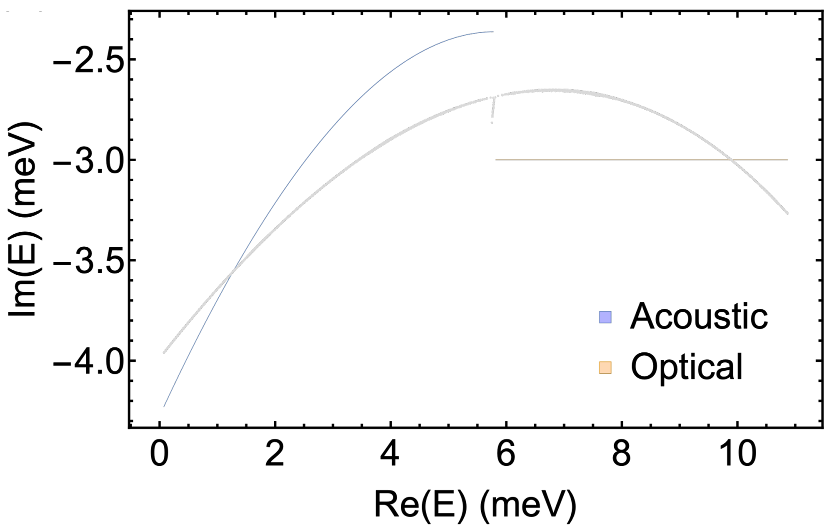


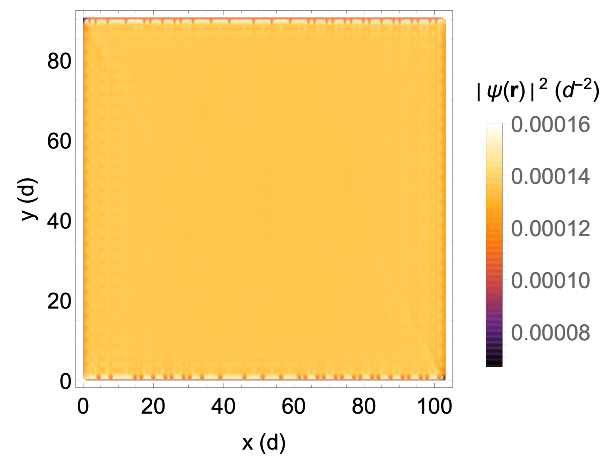
Key ingredients

- ***k*-dependent dissipation (non-local damping) that survives in the real lattice representation**

Is it enough?

No, if we switch DMI: 1) finite spectral area vanishes;
2) skin effect disappears.

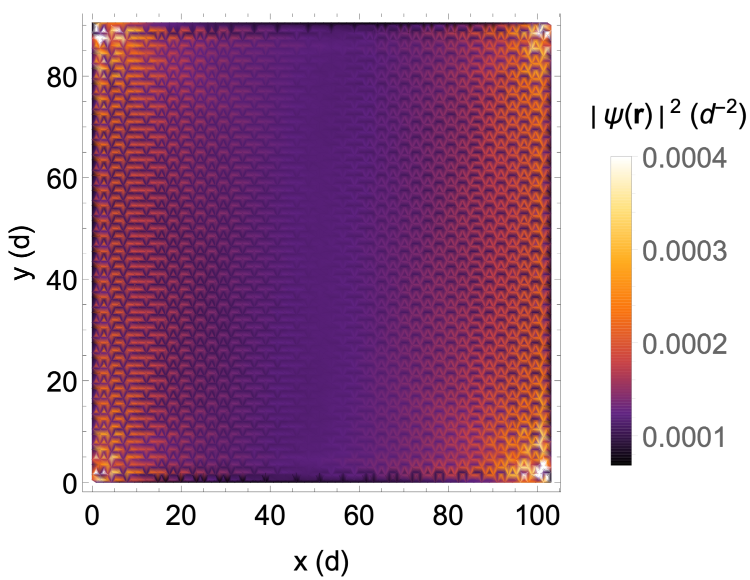




Interference between Dzyaloshinskii-Moriya interaction (DMI) and nonlocal magnetic dissipation plays a key role in the accumulation of bulk states at the boundaries

Summary and outlook

- Phenomenological treatment of magnetic dissipation within a lattice model
- Emergence of the skin effect due to non local dissipation and non-reciprocal interactions, understood via the effective spectral area law
- Future research should address :
  - Ideal platforms
  - Microscopic theories
  - Experimental observables



≠ Density of states (bi-orthogonal real space basis requires care)

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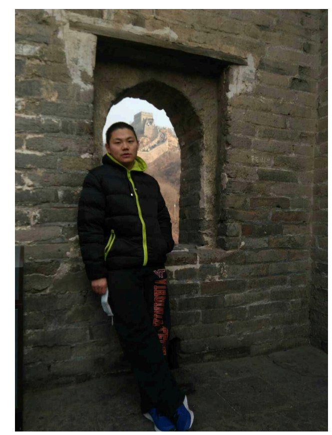

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Thanks for the attention!