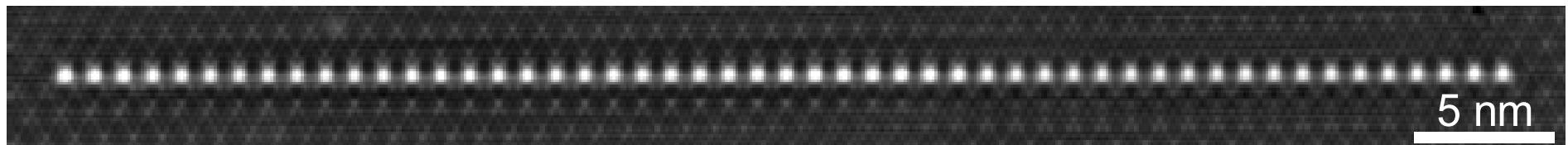




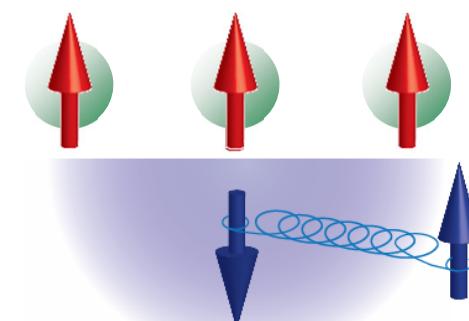
# Building and investigating magnetic adatom chains on superconductors atom by atom

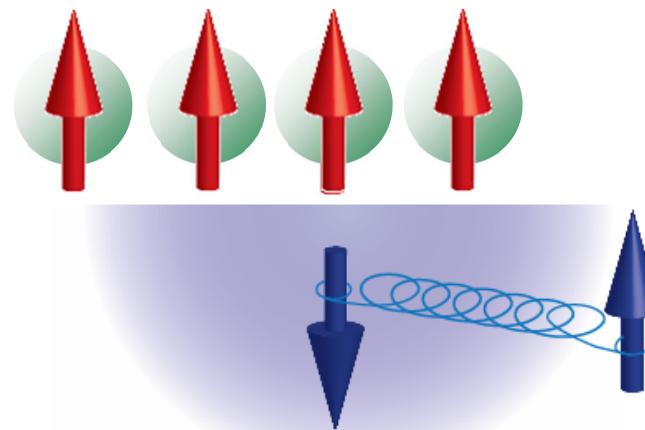


Eva Liebhaber, Lisa Rütten, Gaël Reecht, Katharina Franke

Theory:  
Sergio Acero Gonzalez, Jacob Steiner,  
Felix von Oppen

Samples:  
Sebastian Rohlf, Kai Rossnagel (Kiel)





Fundamentals of local  
spin-superconductor  
interactions

Quantum spin  
magnetism in chains

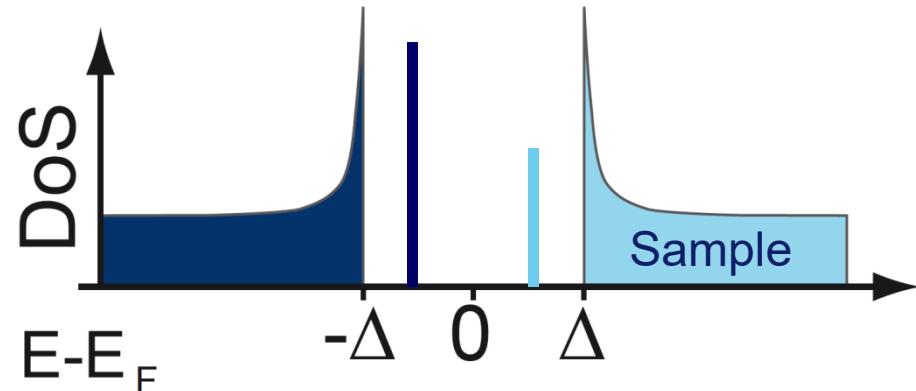
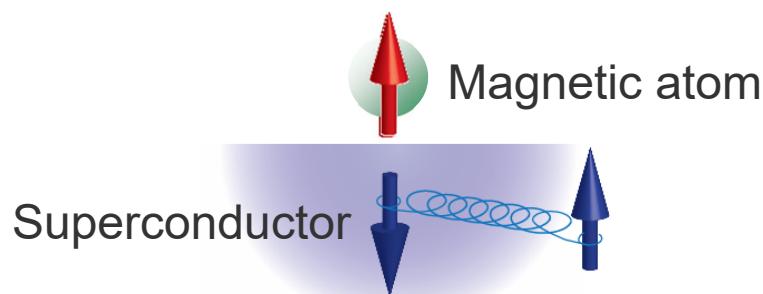
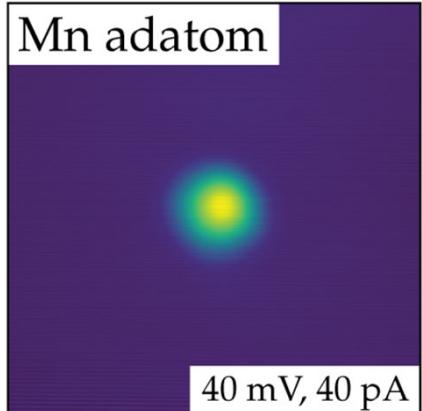
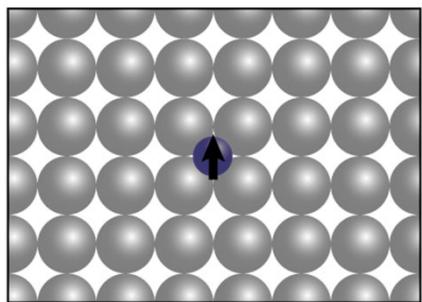
- ▶ Effect of local exchange coupling?
- ▶ Coupling of YSR states?
- ▶ Topological superconductivity?

# Yu-Shiba-Rusinov states: classical model

Freie Universität



Berlin



► Spins exchange scatter  
at a magnetic center:

$$\epsilon_S = \Delta \frac{1 - A^2 + B^2}{\sqrt{4A^2 + (1 - A^2 + B^2)}}$$

$$A = \frac{\pi}{2} J S \nu_0$$

$$B = \pi K \nu_0$$

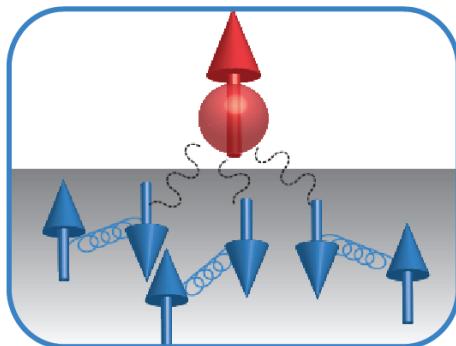
► Yu-Shiba-Rusinov states

- L. Yu, Acta Phys. Sin. 21, 75 (1965)  
H. Shiba, Prog. Theor. Phys. 40, 435 (1968)  
A.I. Rusinov, JETP Lett. 9, 85 (1969)

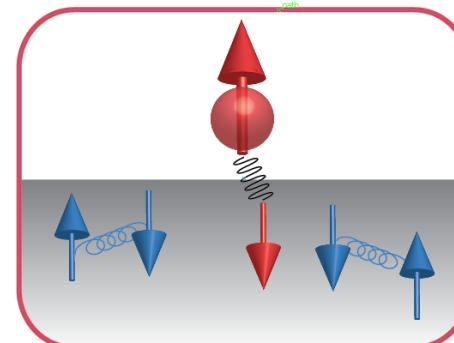
# Yu-Shiba-Rusinov State



- Competition with superconducting order: two possible ground states



Critical coupling:  
Ground state undergoes a  
Quantum Phase Transition

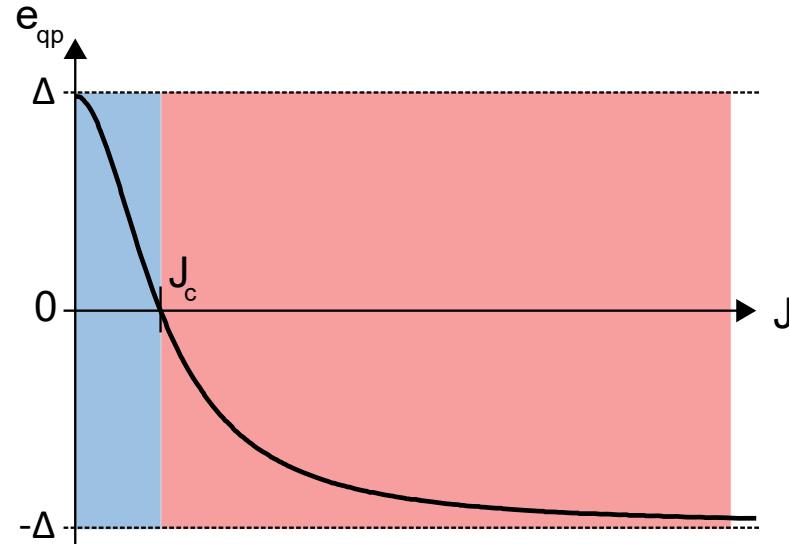


- Energy dependence of  $e_{qp}$  (quasi-particle energy)

$$e_{qp} = \Delta \frac{1 - (\pi \nu_0 J S)^2}{1 + (\pi \nu_0 J S)^2}$$

- $e_{qp} > 0$  : weak coupling
- $e_{qp} < 0$  : bound quasiparticle

L. Yu, *Acta Phys. Sin.* **21**, 75 (1965)  
H. Shiba, *Prog. Theor. Phys.* **40**, 435 (1968)  
A.I. Rusinov, *JETP Lett.* **9**, 85 (1969)

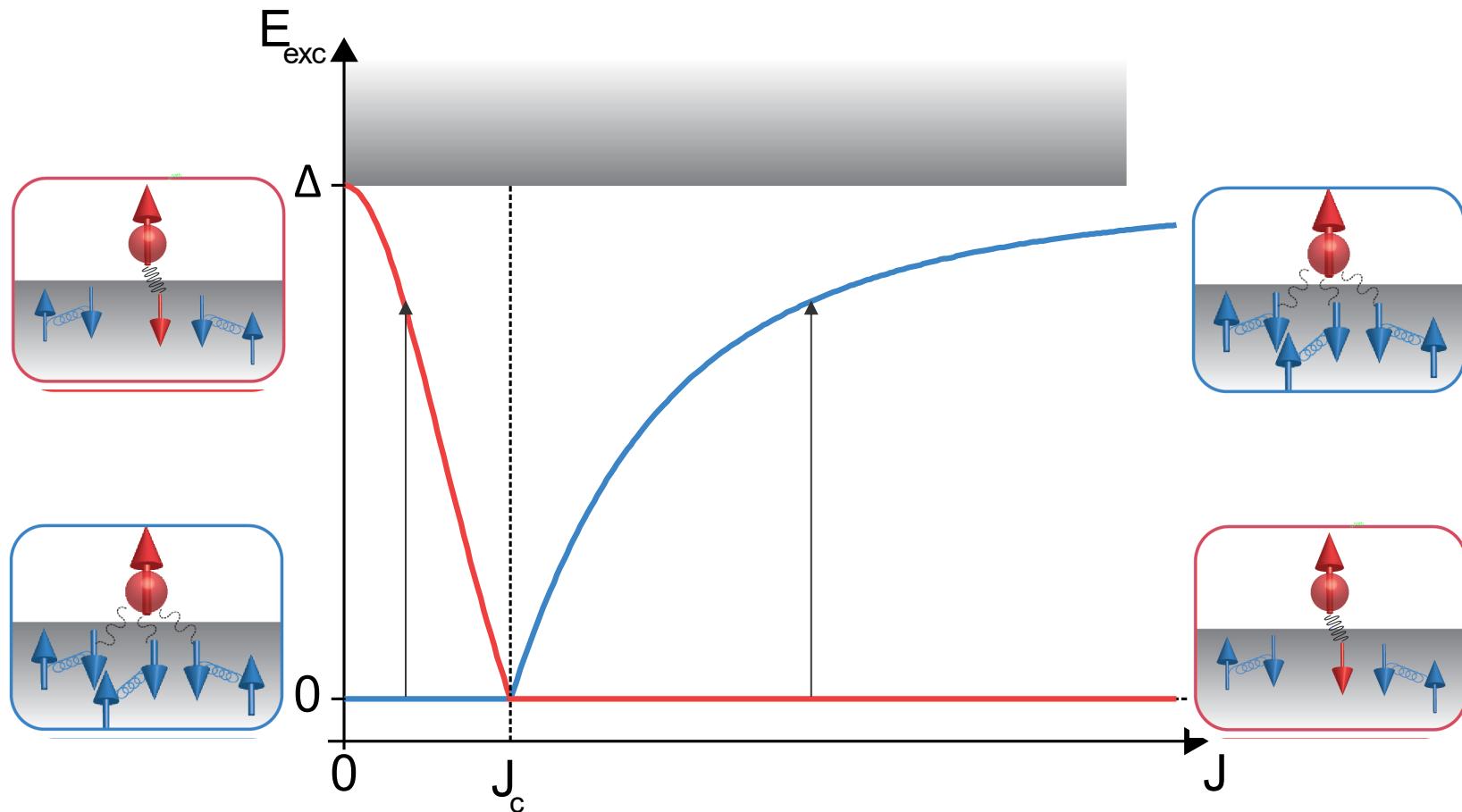


# Yu-Shiba-Rusinov State

Freie Universität Berlin

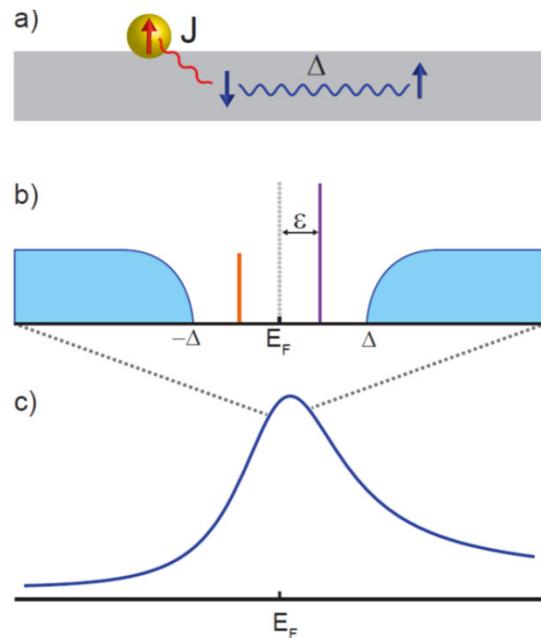


- Energy of the excitation:  $e_b = \pm \Delta \frac{1 - (\pi \nu_0 J S)^2}{1 + (\pi \nu_0 J S)^2}$



# Quantum spins on superconductors

Freie Universität Berlin



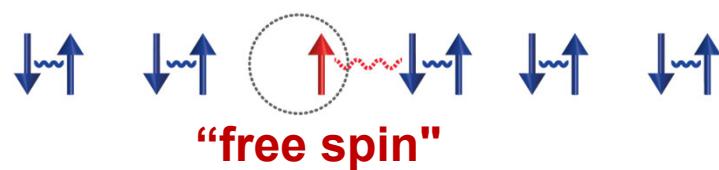
## ► YSR states within superconducting gap

$$\varepsilon = \Delta \frac{1 - a^2}{1 + a^2} \quad a \approx \frac{\pi\Delta}{4k_B T_K} \ln \frac{4k_B T_K}{\pi\Delta} e.$$

- YSR energy scales mainly with  $T_K$
- YSR asymmetry depends on potential scattering

## ► Kondo resonance outside of the gap

ground state:  $S = 1/2$



ground state:  $S = 0$

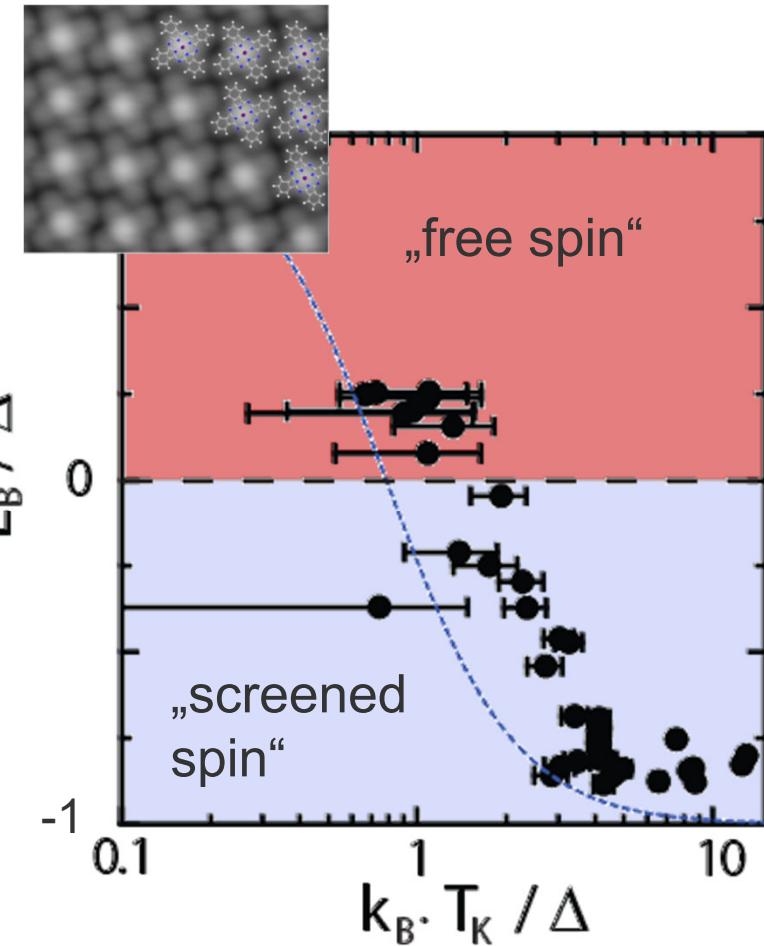
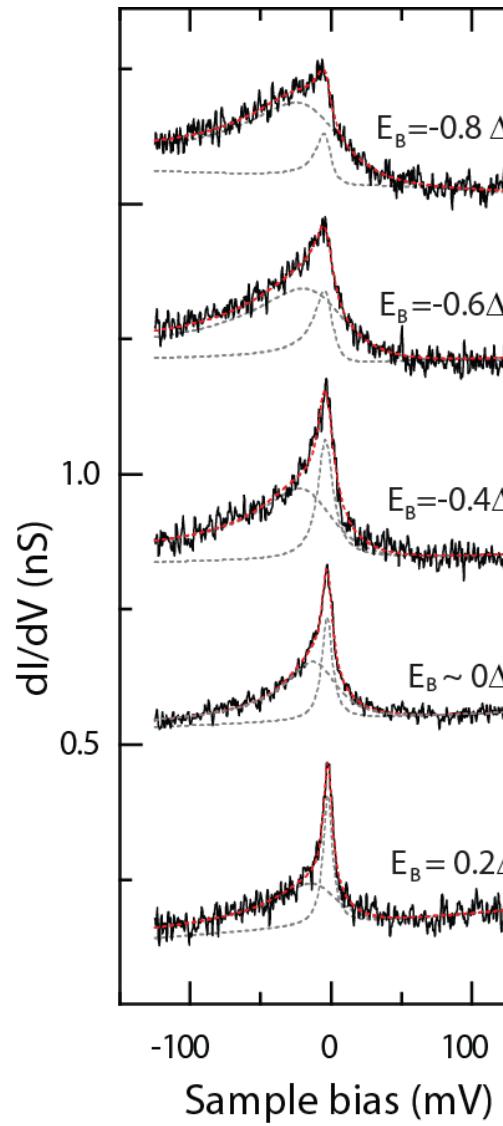
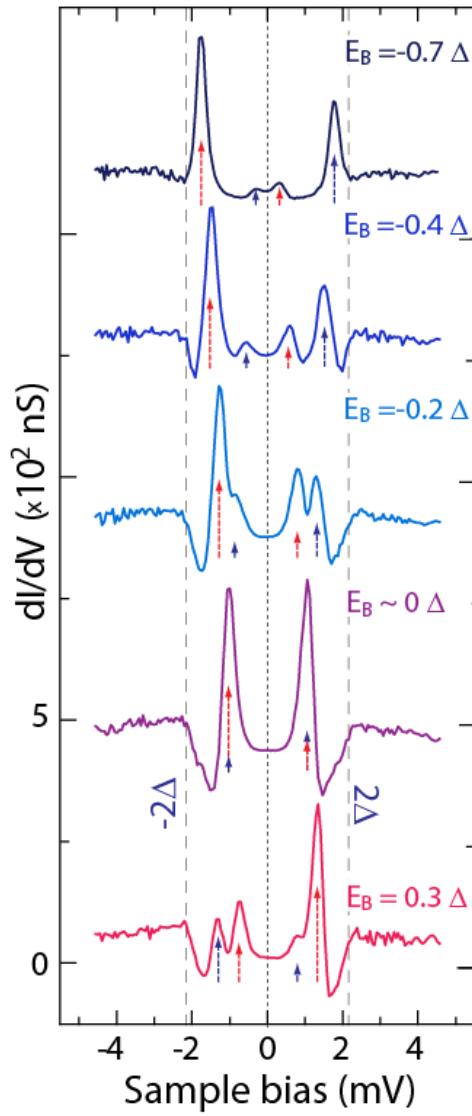


Original YSR papers:

- L. Yu, Acta Phys. Sin. 21, 75 (1965)  
H. Shiba, Prog. Theor. Phys. 40, 435 (1968)  
A.I. Rusinov, JETP Lett. 9, 85 (1969)

# Kondo screening vs. YSR bound states

Freie Universität Berlin



- ▶ Correlation of YSR and Kondo
- ▶ Quantum phase transition

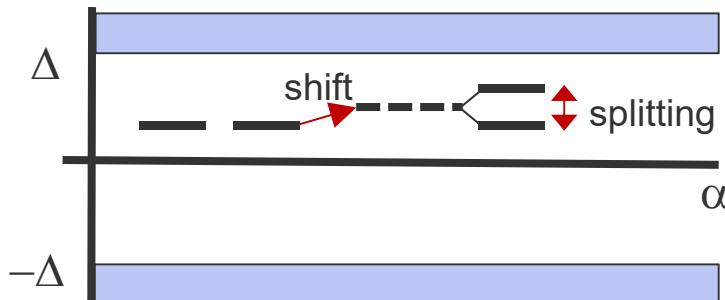
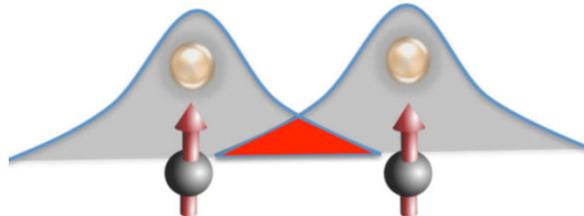
Franke, Schulze, Pascual, Science, 332, 940 (2011)

T. Matsuura, Prog. Theor. Phys. 57, 1823 (1977) 7

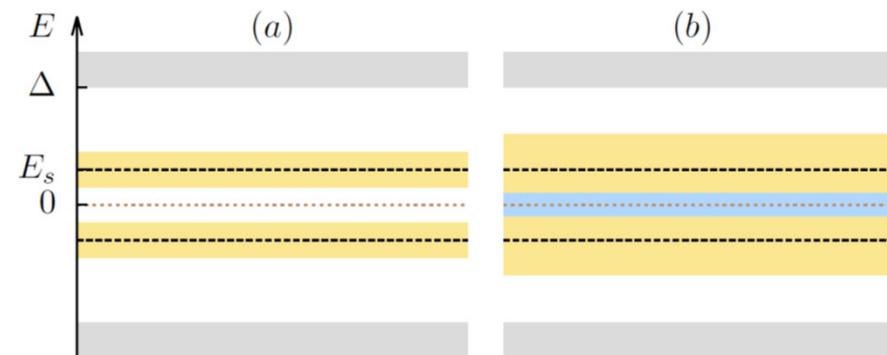
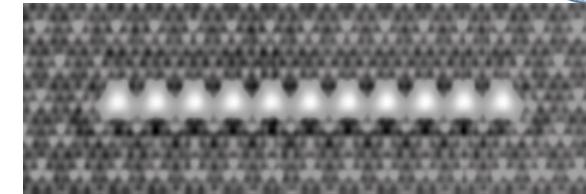
See also: A. Odobesko et al., Phys. Rev. B 102, 174504 (2020)

# Coupling of YSR states

Freie Universität Berlin



Yao et al., PRB 90 (2014), Flatté et al., PRB 61 (2000),  
Yao et al., PRL 113 (2014), Hoffman et al., PRB 92 (2015)

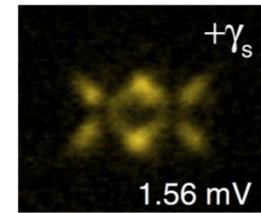
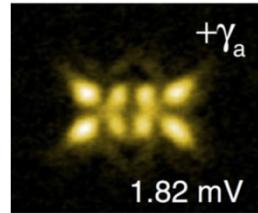


Pientka et al., Phys. Rev. B 88, 155420 (2013)  
Pientka et al., Phys. Scr. 014008 (2015)

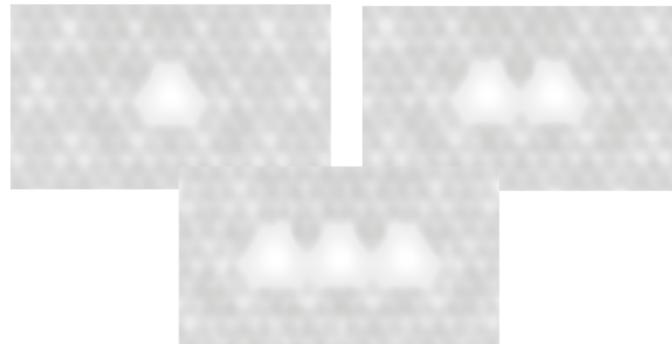
Design from YSR states:

- ▶ Dilute magnetic adatoms
- ▶ Coupling of YSR states
- ▶ YSR bands crossing Fermi level
- ▶ possibly topological gap opening

# Outline



► Hybridisation of YSR states (on Pb)



► Interaction of magnetic atoms on  $\text{NbSe}_2$



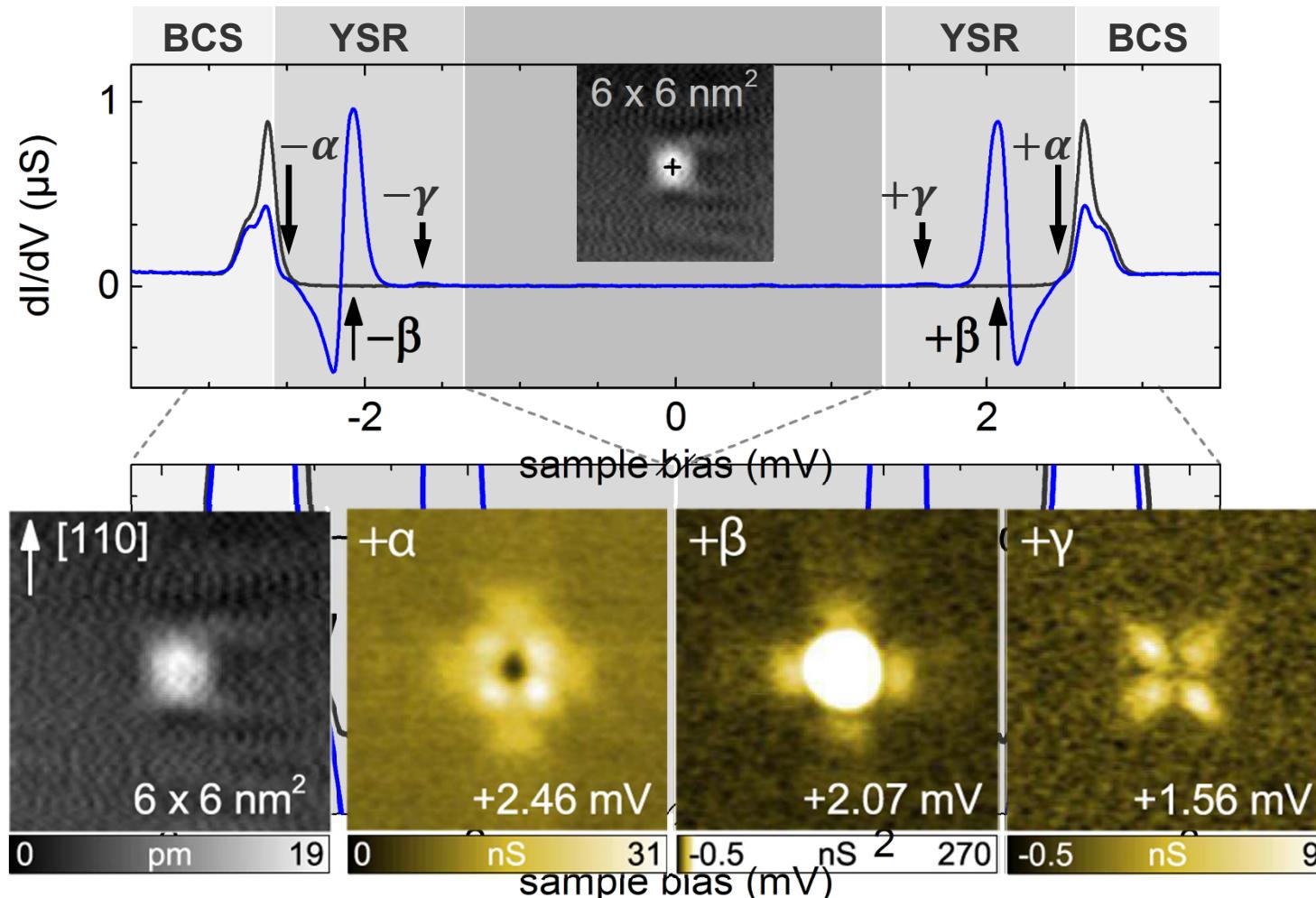
► Evolution of YSR states in atomic chains on  $\text{NbSe}_2$



► Influence of charge-density wave on magnetic chains

# Manganese adatoms on Pb(001)

Freie Universität Berlin

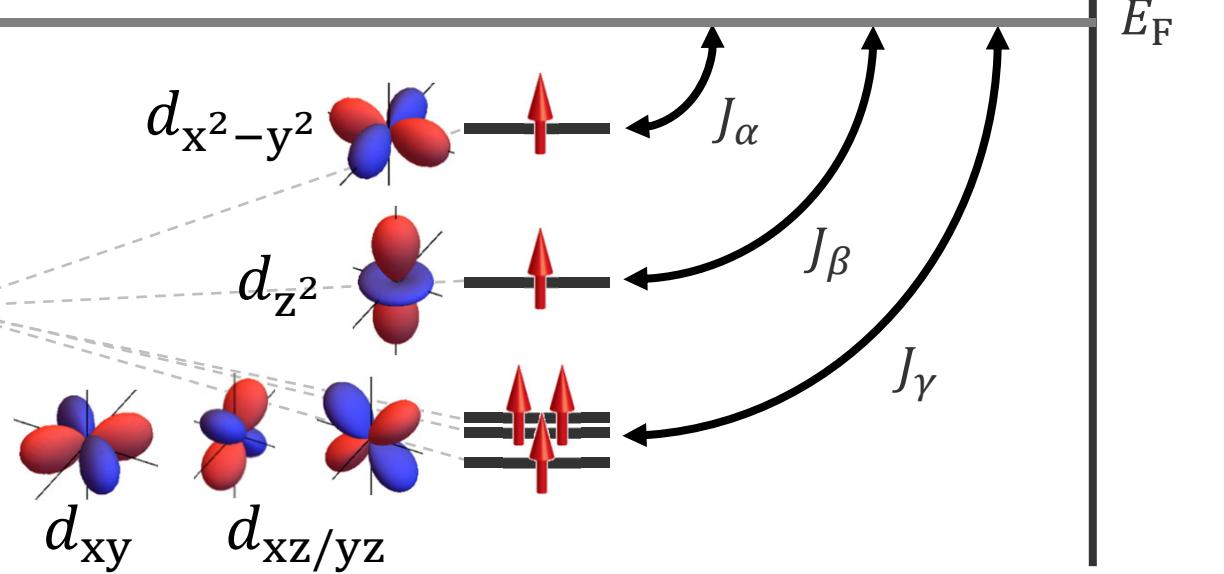
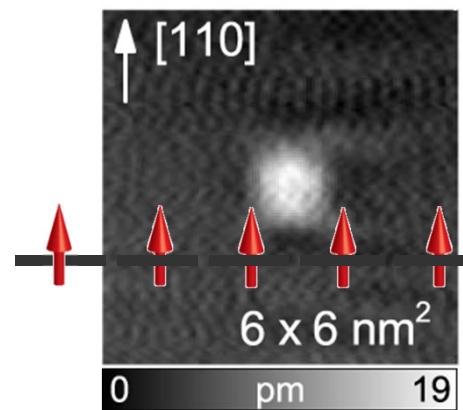
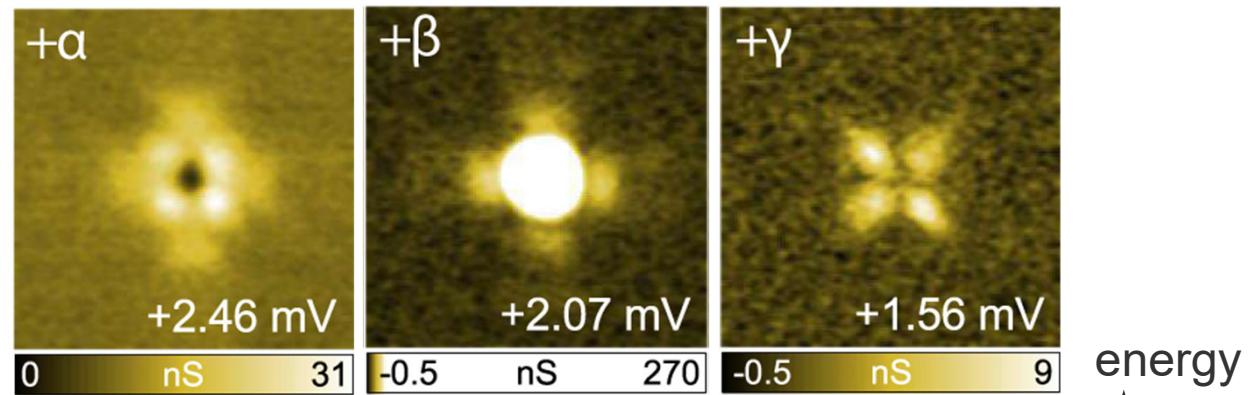
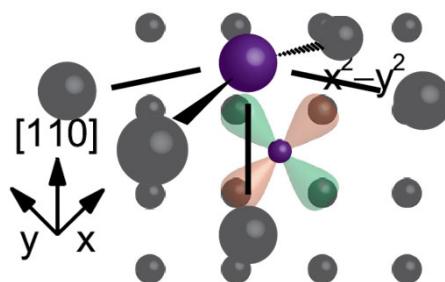


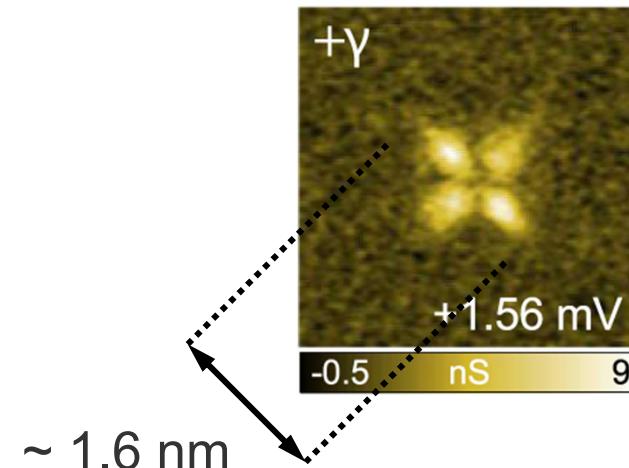
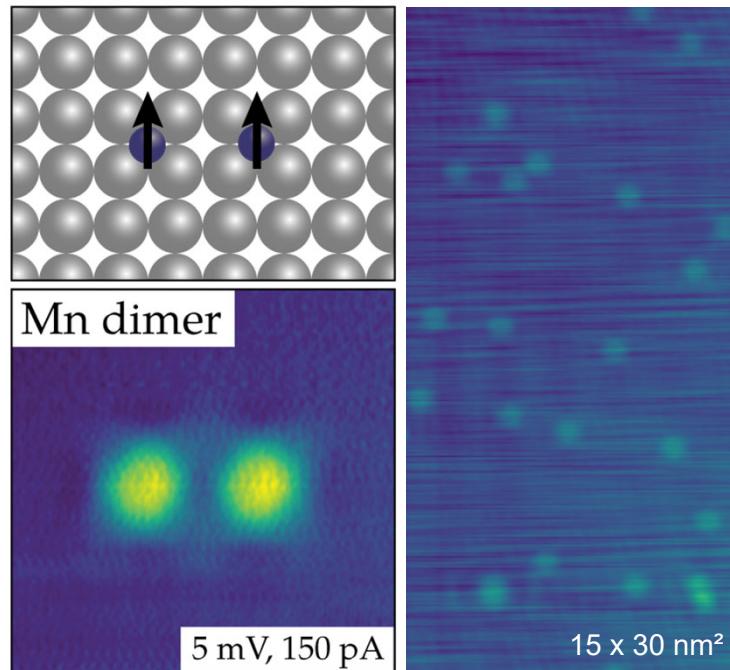
► 3 pairs of YSR resonances

M. Ruby et al., Phys. Rev. Lett. 117, 186801 (2016)

# Pb(001): origin of multiple YSR states

Freie Universität Berlin





► large lateral extension:

$$\Psi(r) \propto \frac{1}{k_F r} e^{-r/\xi}$$

Theoretical predictions of coupled YSR states: Yao *et al.*, PRB **90** (2014), Flatte *et al.*, PRB **61** (2000), Yao *et al.*, PRL **113** (2014)  
Hoffman *et al.*, PRB **92** (2015)

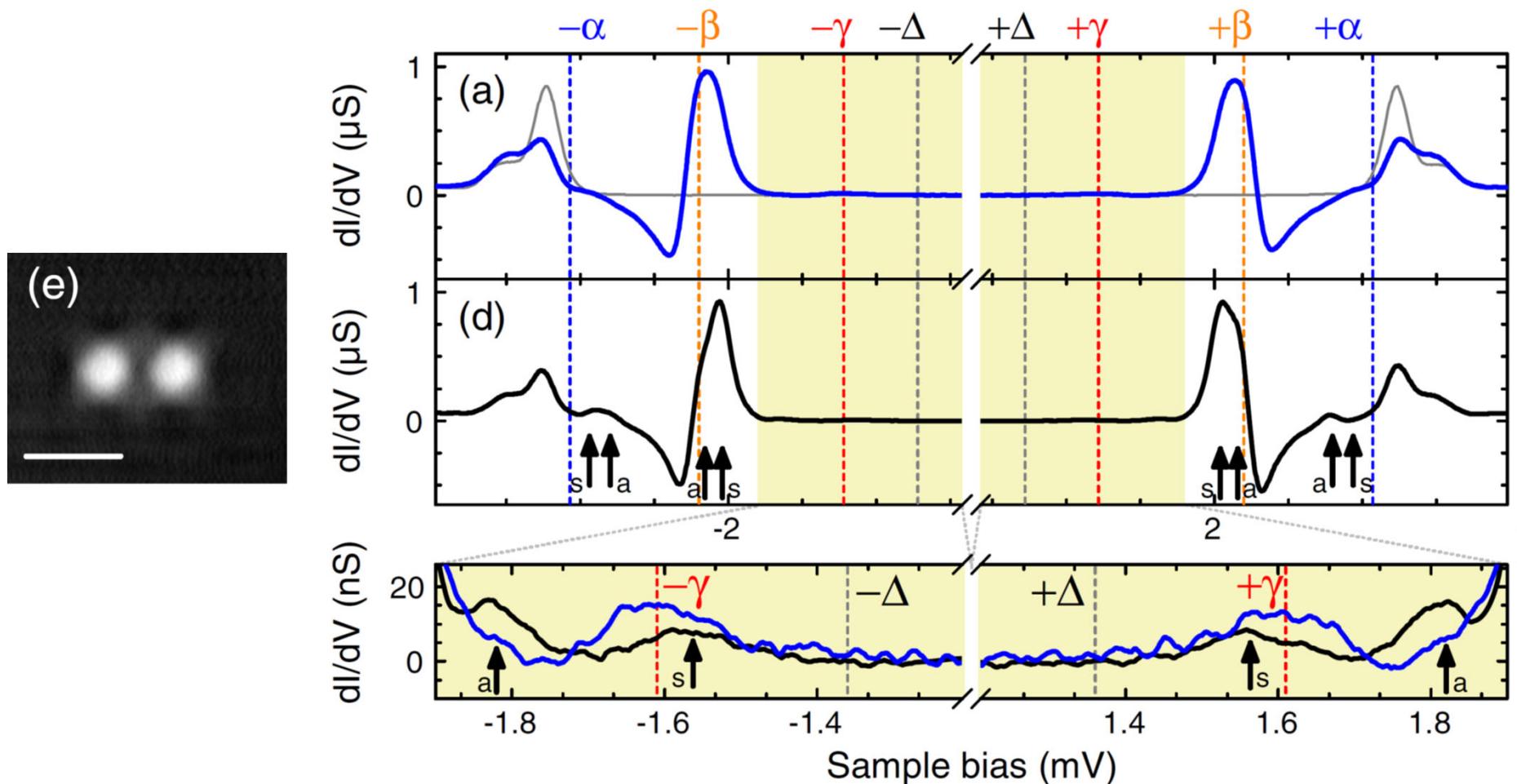
More recent experiments on dimers: Kezilebieke *et al.*, Nano Lett. **18** (2018), Choi, et al., PRL **120** (2018), Beck *et al.*, Nat Comm. **12** (2021), Ding *et al.*, PNAS **118** (2021); QDs: Grove-Rasmussen *et al.*, Nat. Commun. **9** (2018)

# Splitting of YSR states in dimers

Freie Universität Berlin

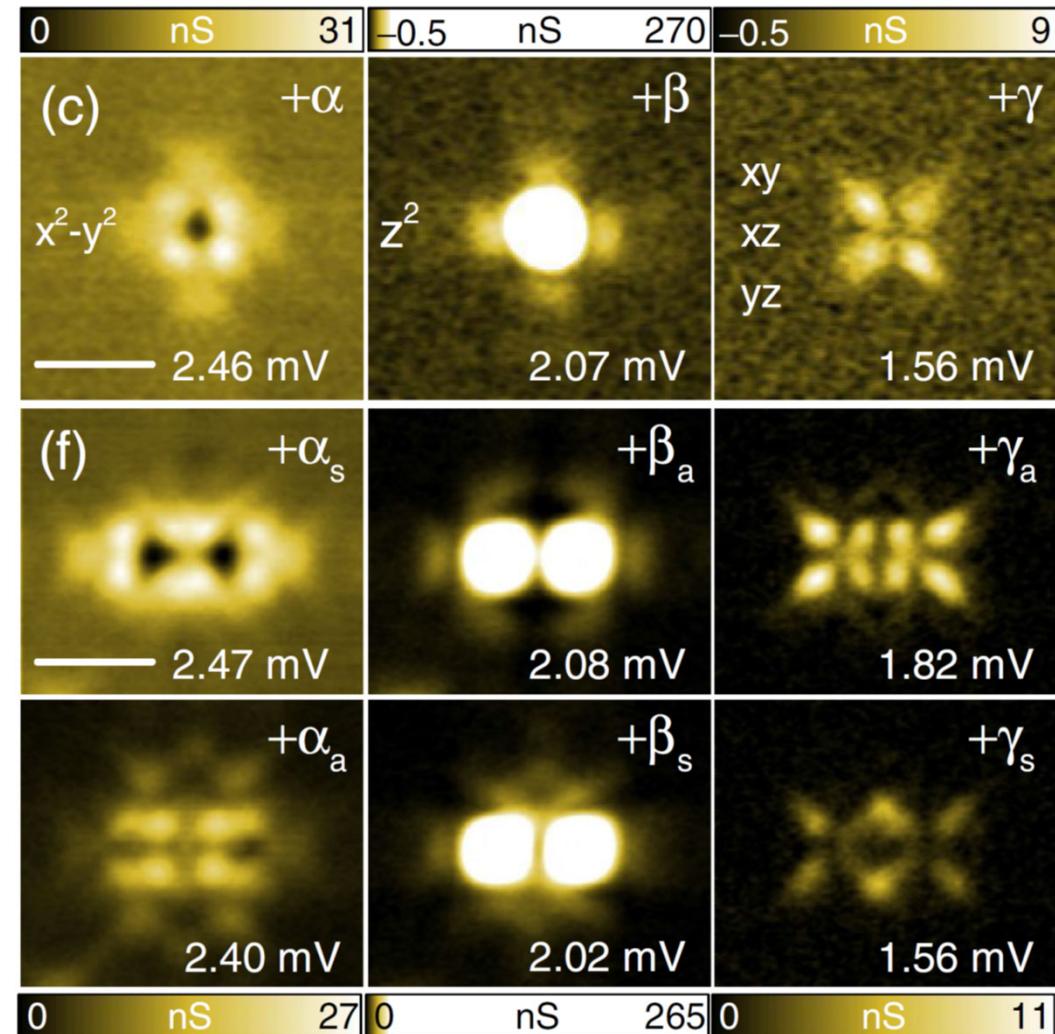
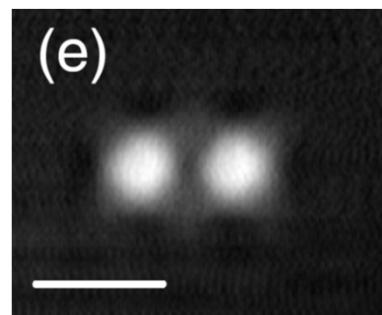


Dimer axis along [110]



# Orbital structure

Dimer axis along [110]

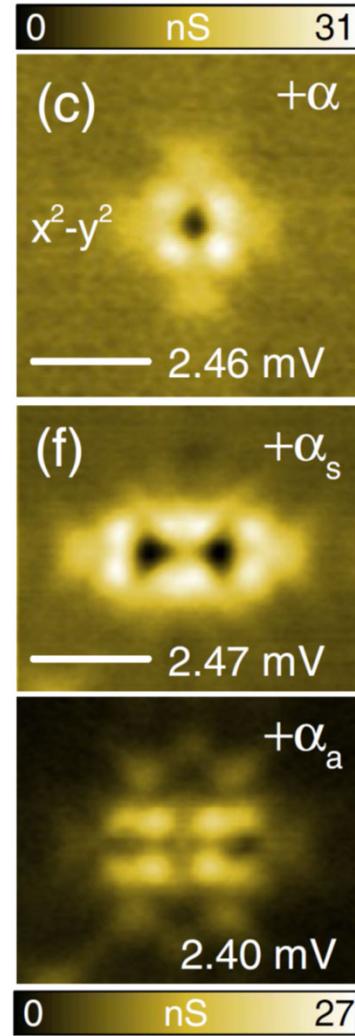
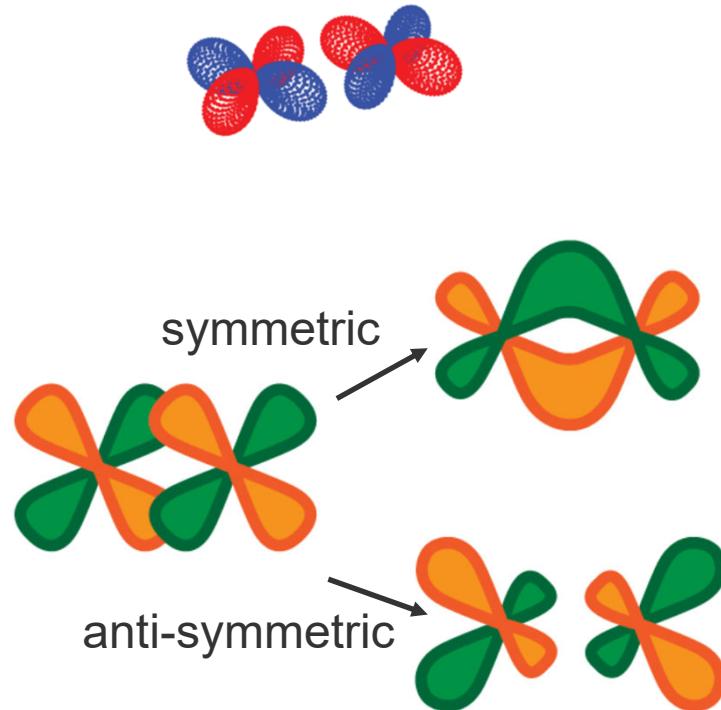


# $\alpha$ resonances



Dimer axis along [110]

$$\alpha: d_{x^2-y^2}$$

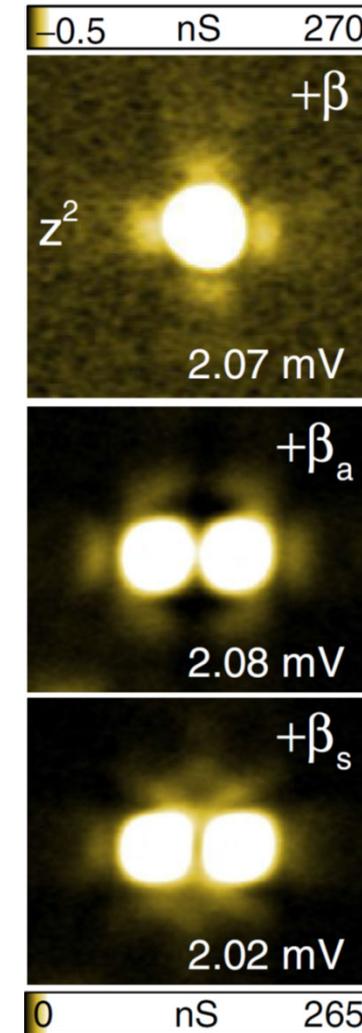
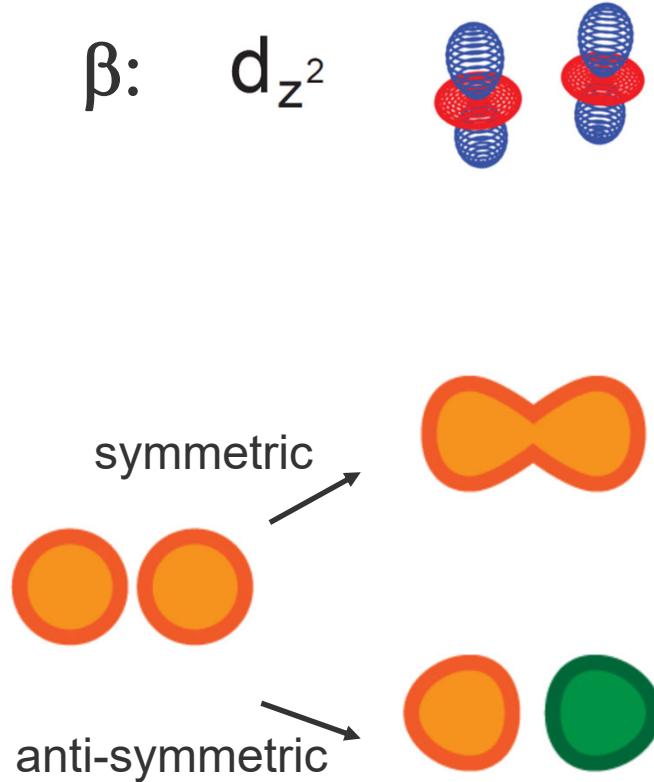


# $\beta$ resonances

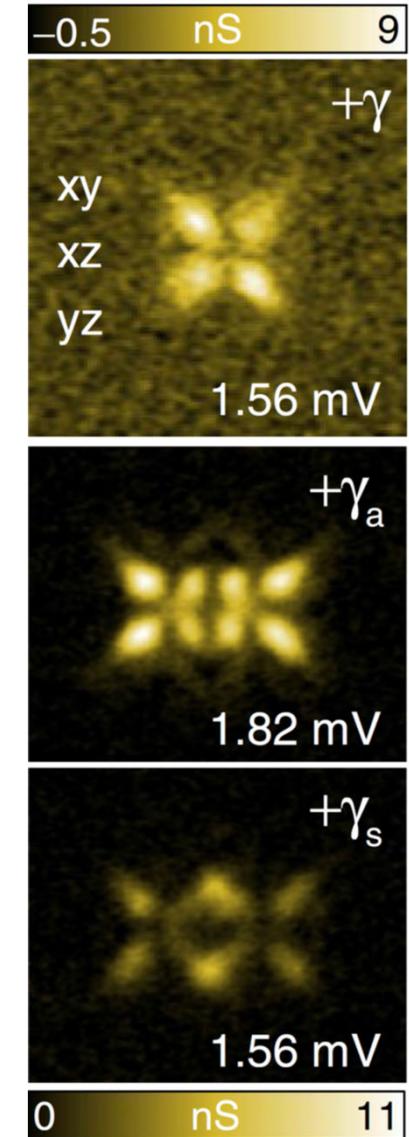
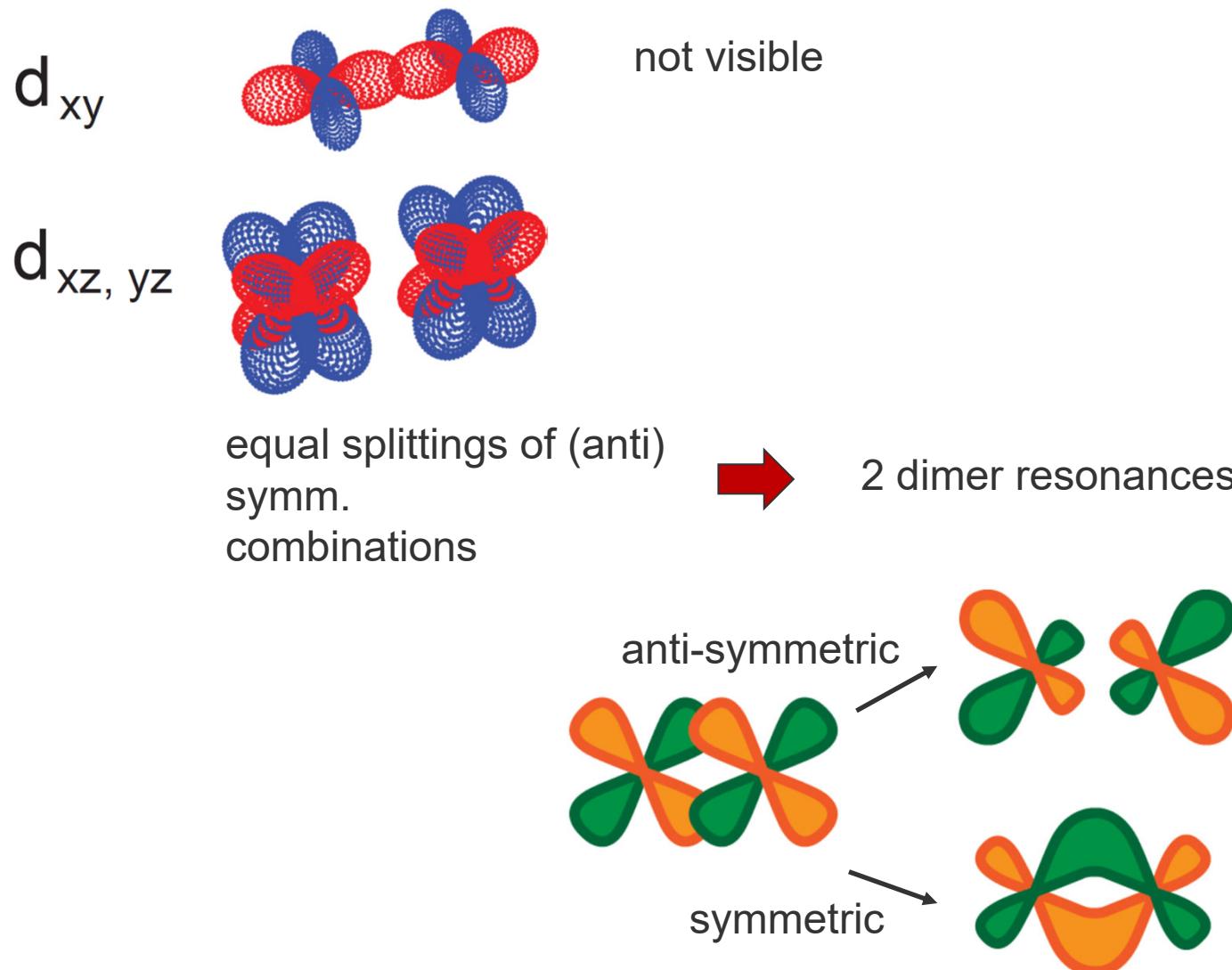
Freie Universität Berlin



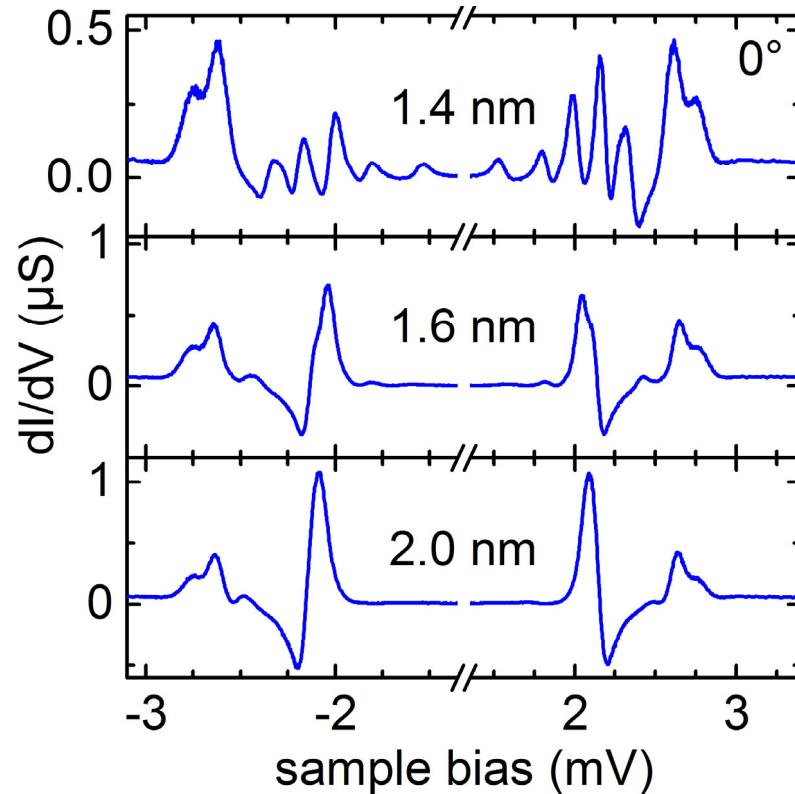
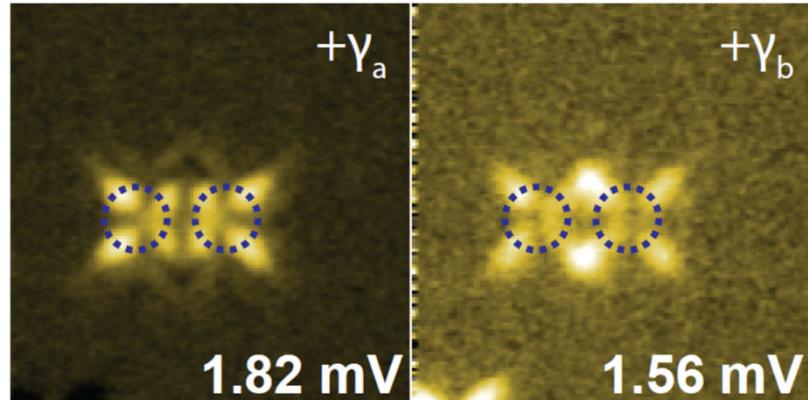
Dimer axis along [110]



# $\gamma$ resonance

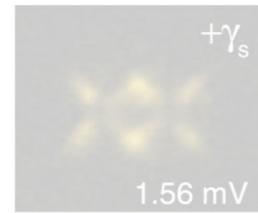


# Hybridization of YSR states in dimers

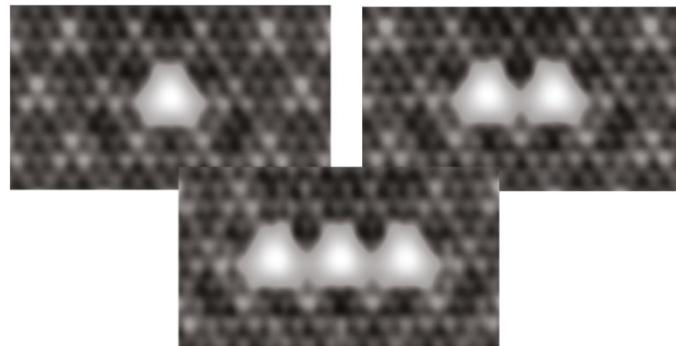


- ▶ symmetric and anti-symmetric character of split Shiba states
- ▶ strength of splitting depends on distance and orientation

# Outline



► Hybridisation of YSR states (on Pb)



► Interaction of magnetic atoms on NbSe<sub>2</sub>



► Evolution of YSR states in atomic chains on NbSe<sub>2</sub>



► Influence of charge-density wave on magnetic chains

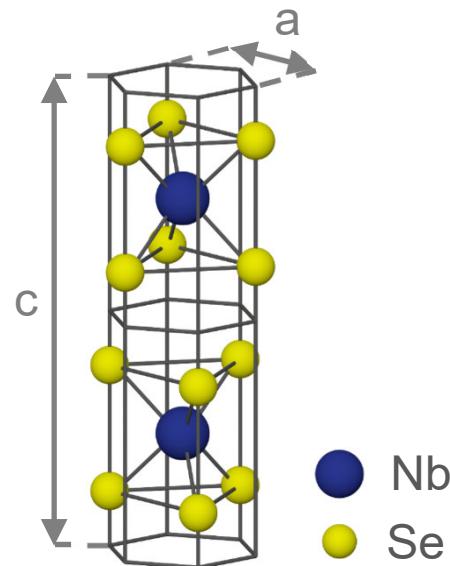
# Quasi-2D superconductor

Freie Universität Berlin



- ▶ van der Waals stacked layers of  $\text{NbSe}_2$

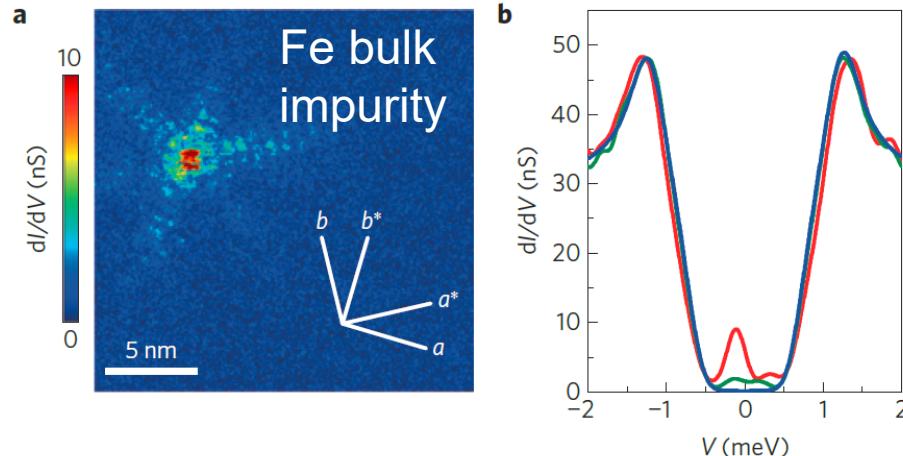
2H stacking order



- ▶ critical temperature:  $T_c=7.2 \text{ K}$

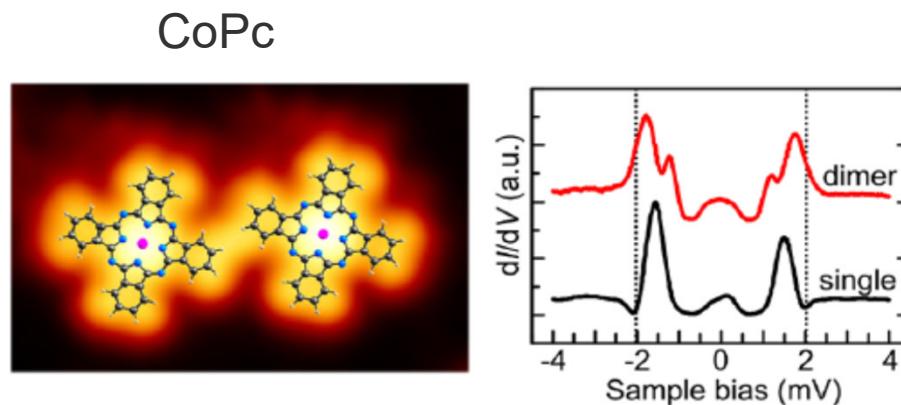
# Previous YSR states in 2H-NbSe<sub>2</sub>

Freie Universität Berlin



Ménard, et al., Nature Phys. 11, 1013 (2015)

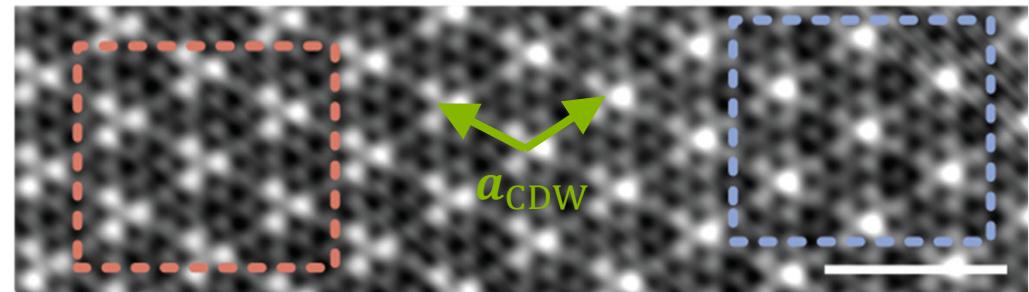
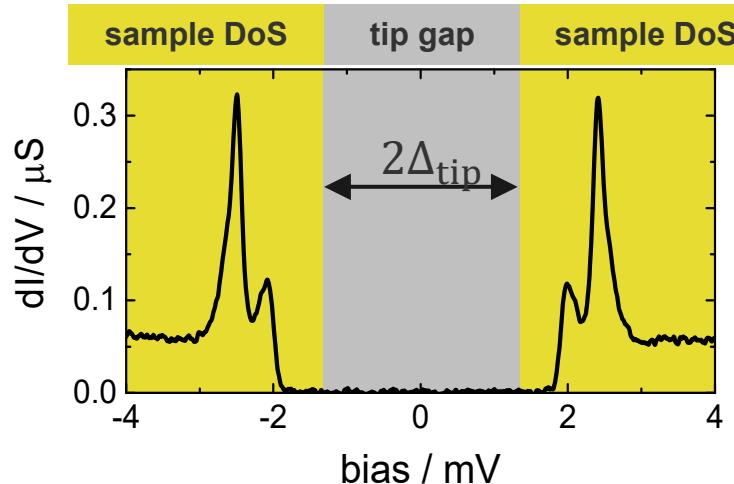
- ▶ Bulk Fe impurities create long-range YSR states
- ▶ Decay with  $1/\sqrt{r}$



Kezilebieke, et al., Nano Lett. 18, 2311 (2018)

- ▶ Magnetic molecules coupled via YSR states

# $2H\text{-NbSe}_2$ - substrate



## superconductivity

- $T_{\text{SC}} \approx 7.2\text{K}$
- multiband/anisotropic superconductor

## charge density wave

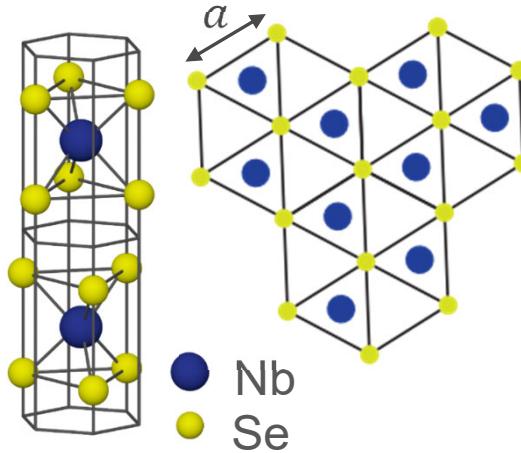
- $T_{\text{CDW}} \approx 33\text{K}$
- **roughly**  $3 \times 3$  – superstructure

STS using superconducting Pb tips at  $T \approx 1.1\text{K}$

↳ energy shifted by  $\pm\Delta_{\text{tip}}$

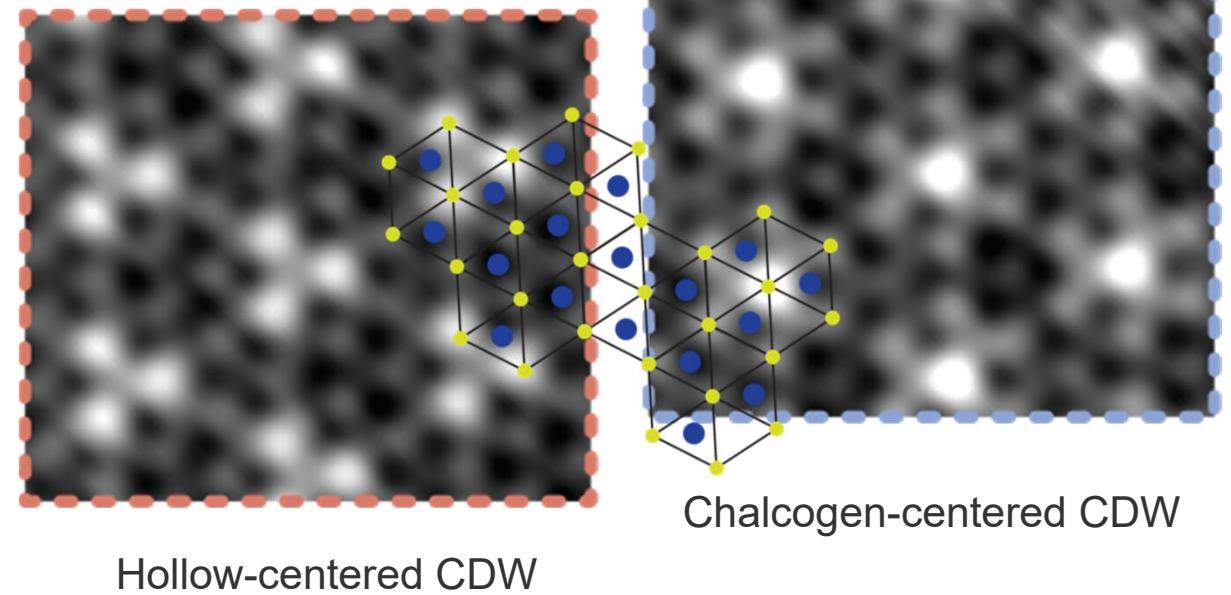
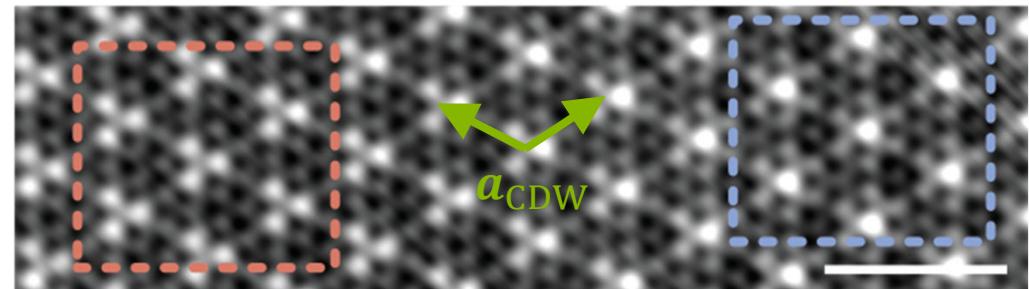
Smooth transition between different regions of CDW

# $2H$ -NbSe<sub>2</sub> – charge density wave



- ▶ Modulation of YSR states?
- ▶ Template for YSR chains?

Smooth transition between different regions of CDW

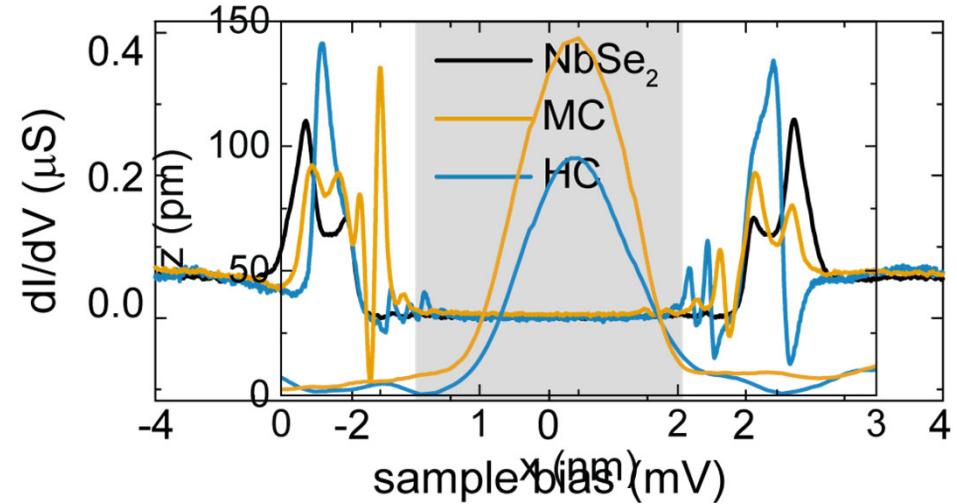
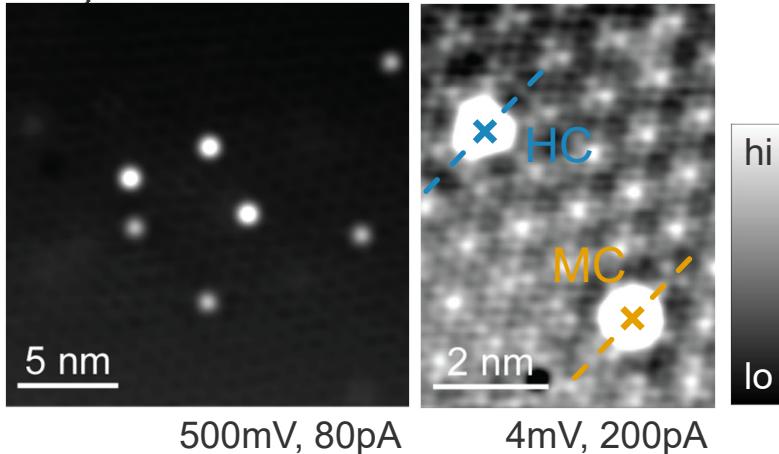


# $2H\text{-NbSe}_2 + \text{Fe atoms}$

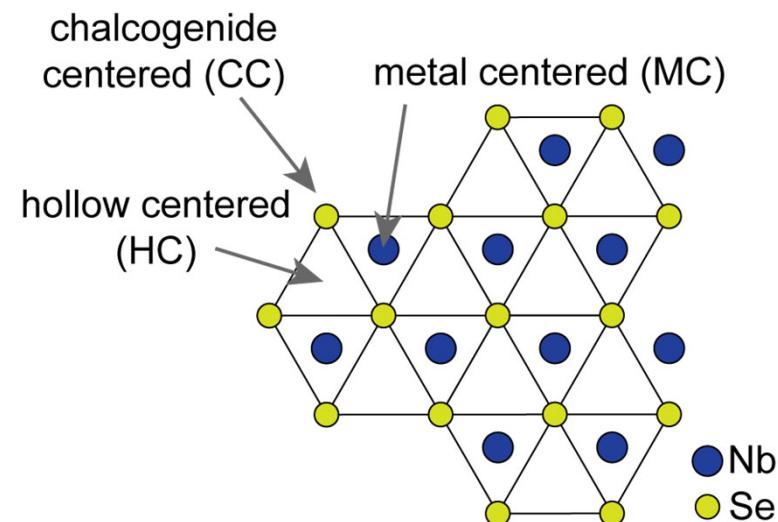
Freie Universität Berlin



Evaporation of Fe atoms at  $T < 12\text{K}$

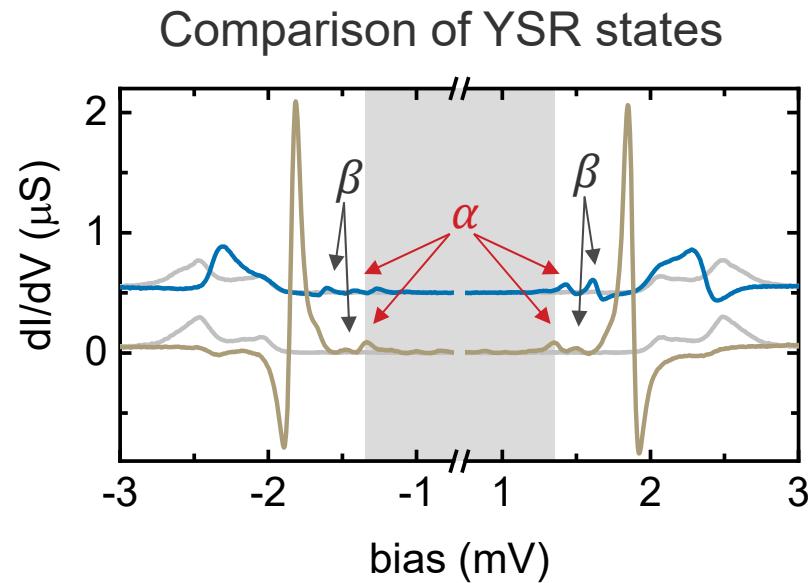


- ▶ different apparent heights
- ▶ adsorption in HC and MC
- ▶ both species show several YSR states  
(unpaired  $e^-$  in  $d$ -orbitals)

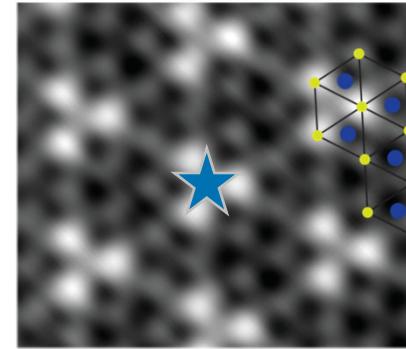


# YSR states on 2H-NbSe<sub>2</sub>

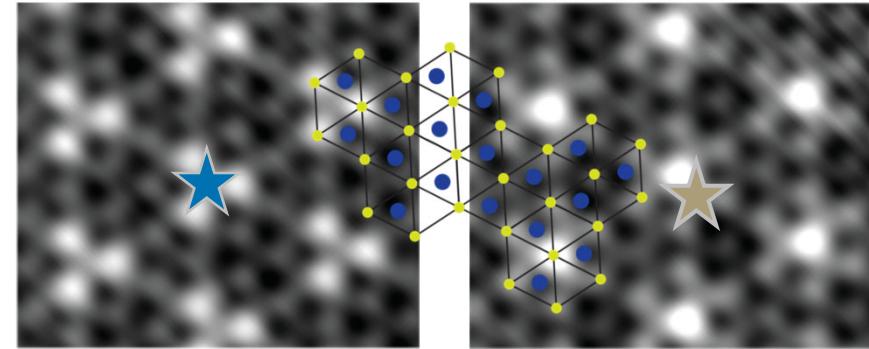
Freie Universität Berlin



Fe atom on  
CDW max



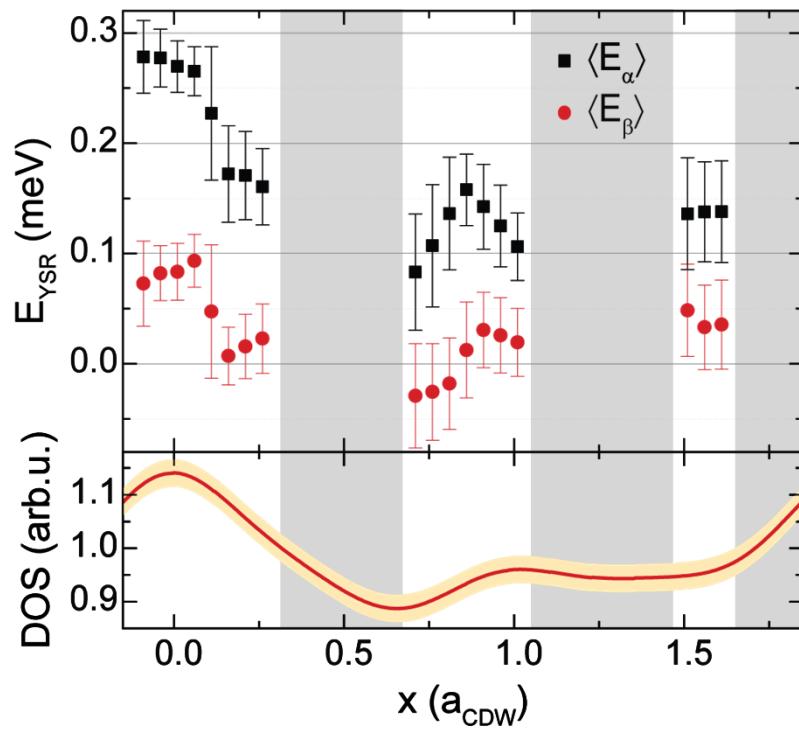
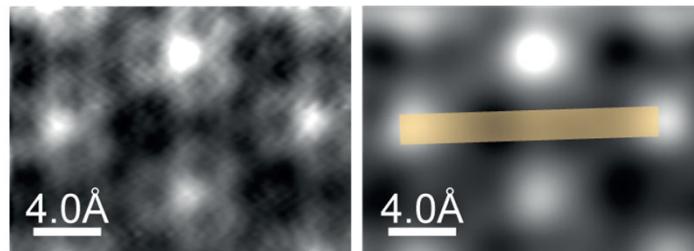
Fe atom near  
CDW max



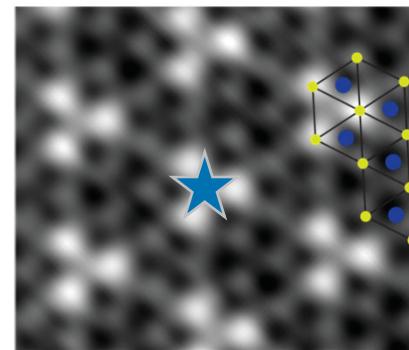
- YSR states depend on CDW

# Tracking the energy shift

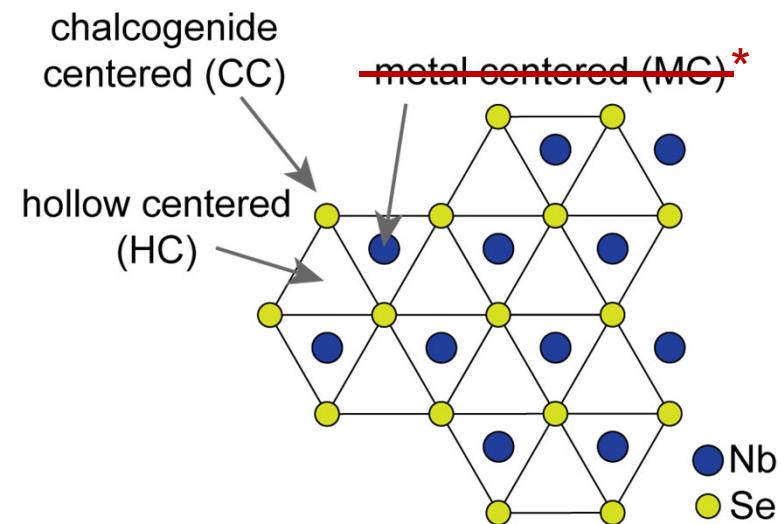
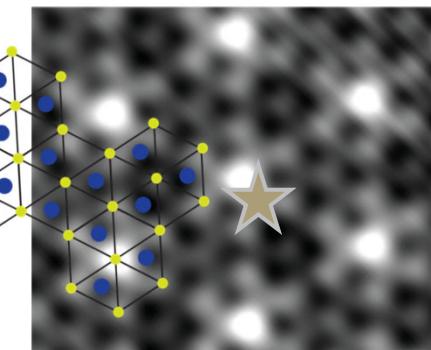
Const. Z dI/dV map ( $V_{\text{bias}} = 0$ ,  $T = 8 \text{ K}$ )



Fe atom on CDW max



Fe atom near CDW max



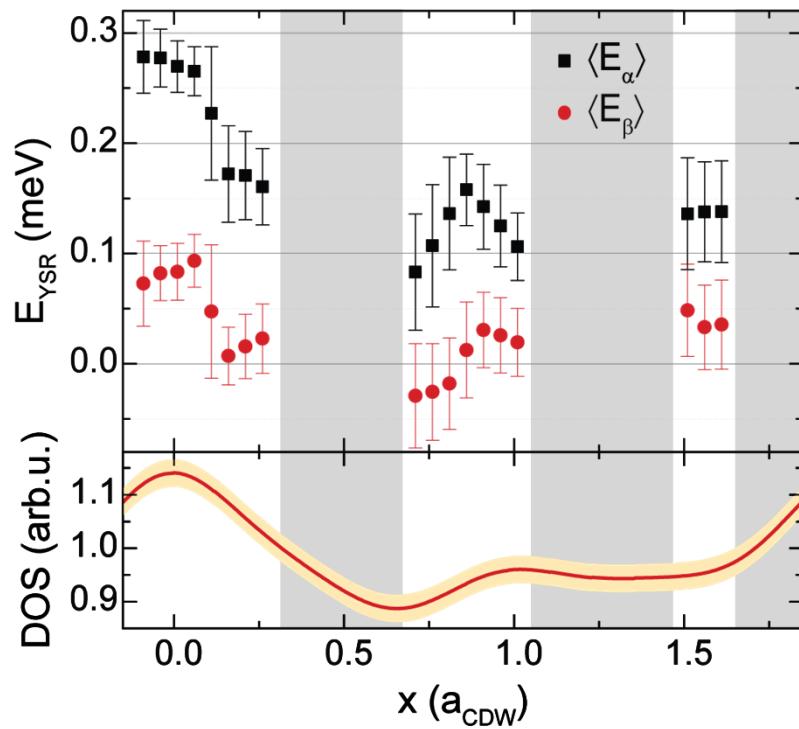
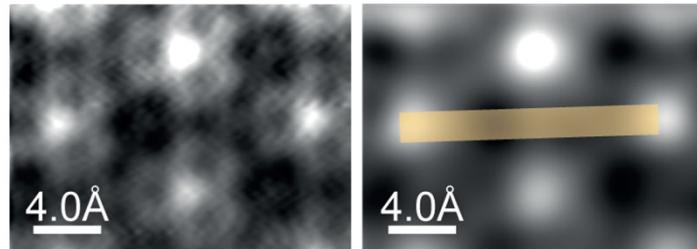
\*Gye, Oh, Yeom – *Phys. Rev. Lett.* **122**, 016403 (2019)

# Tracking the energy shift

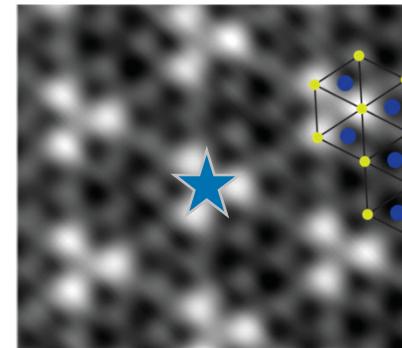
Freie Universität Berlin



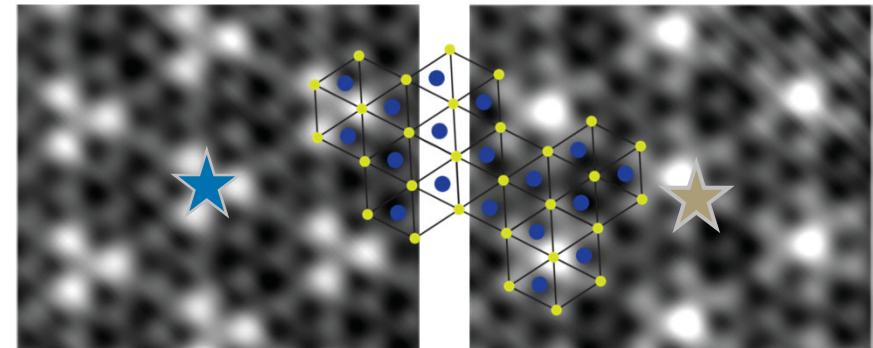
Const. Z dI/dV map ( $V_{\text{bias}} = 0$ ,  $T = 8$  K)



Fe atom on  
CDW max



Fe atom near  
CDW max



Yu-Shiba-Rusinov bound states:

$$\epsilon_S = \Delta \frac{1 - A^2 + B^2}{\sqrt{4A^2 + (1 - A^2 + B^2)}}$$

$$A = \frac{\pi}{2} JS \nu_0$$
$$B = \pi K \nu_0$$

- YSR energy scales with CDW

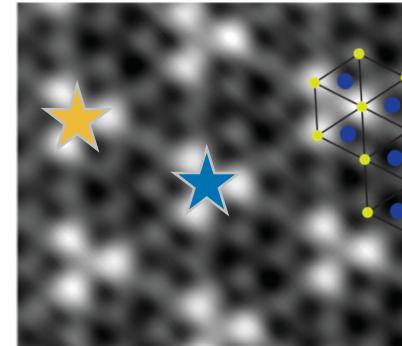
# Choice of adsorption sites for chains

Freie Universität Berlin

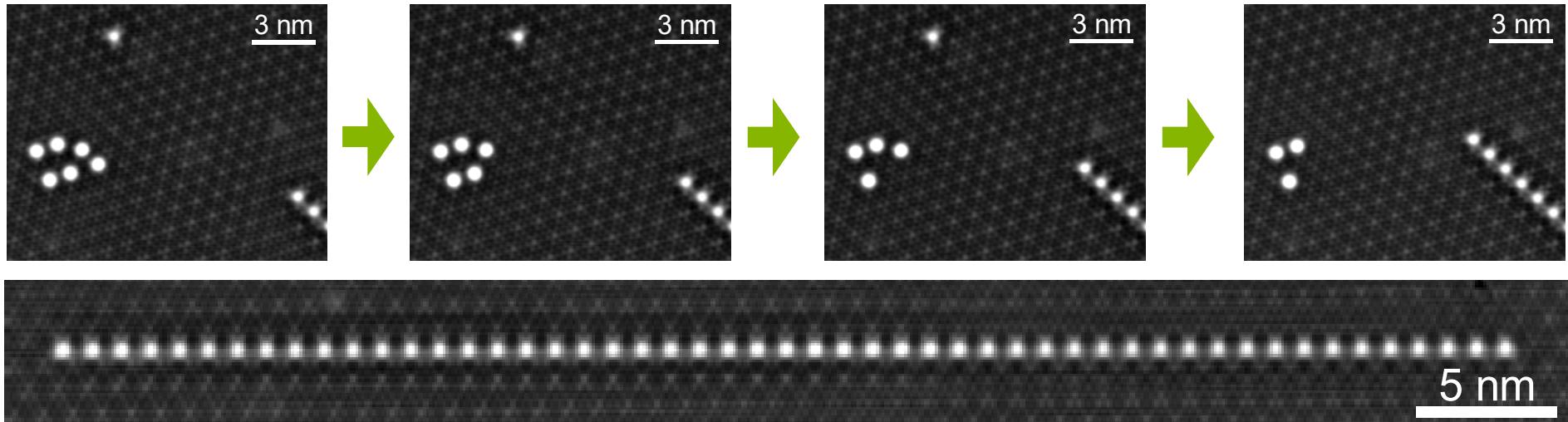
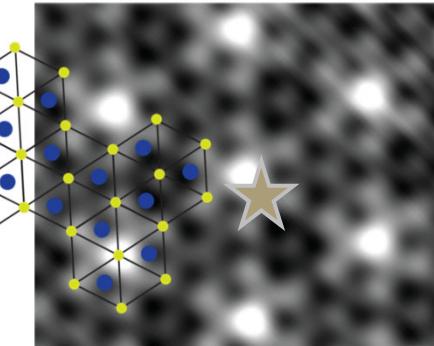


- ▶ Assembly of chains by atom manipulation
- ▶ Placement of atoms in 3a distance: similar w.r.t. CDW
- ▶ Dilute limit of coupling

Fe atom on  
CDW max



Fe atom near  
CDW max



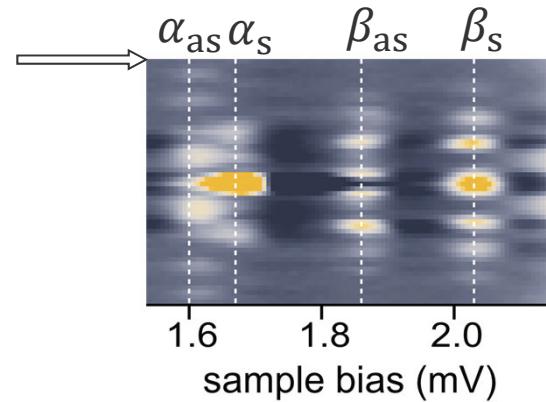
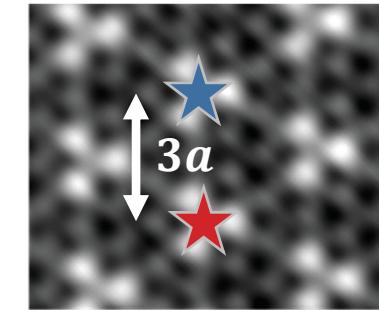
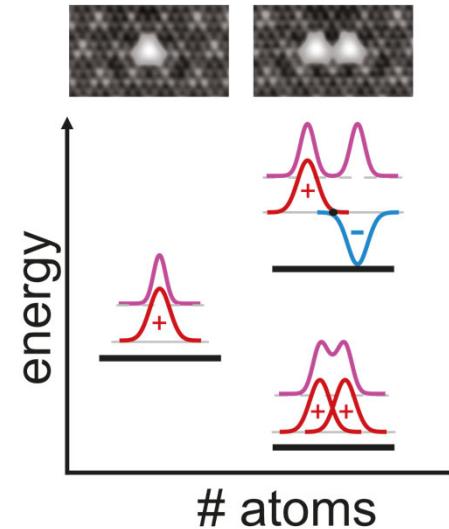
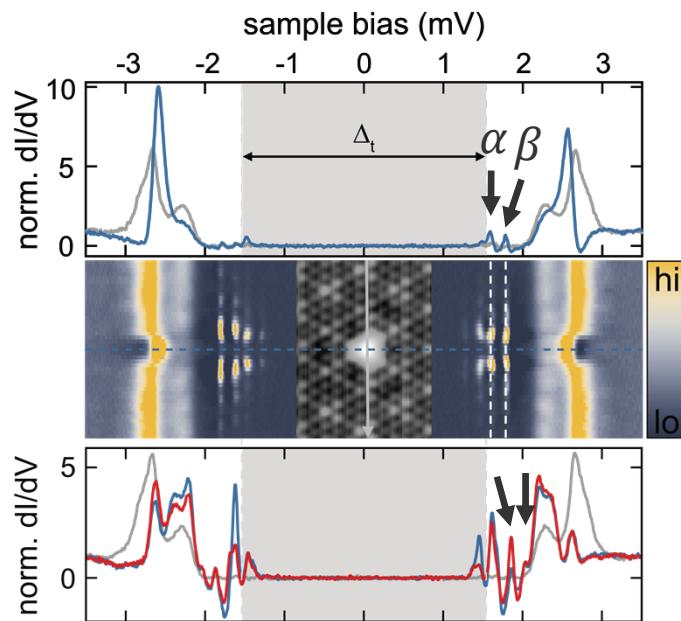
Theoretical predictions of coupled YSR states: Yao *et al.*, PRB **90** (2014), Flatte *et al.*, PRB **61** (2000), Yao *et al.*, PRL **113** (2014)  
Hoffman *et al.*, PRB **92** (2015)

Theoretical YSR chains: Pientka, *et al.*, Phys. Rev. B **88** (2013), Phys. Scr. 014008 (2015)

Experiments on dimers: Kezilebieke *et al.*, Nano Lett. **18** (2018), Choi, *et al.*, PRL **120** (2018), Beck *et al.*, Nat Comm. **12** (2021), Ding *et al.*, PNAS **118** (2021); QDs: Grove-Rasmussen *et al.*, Nat. Commun. **9** (2018)

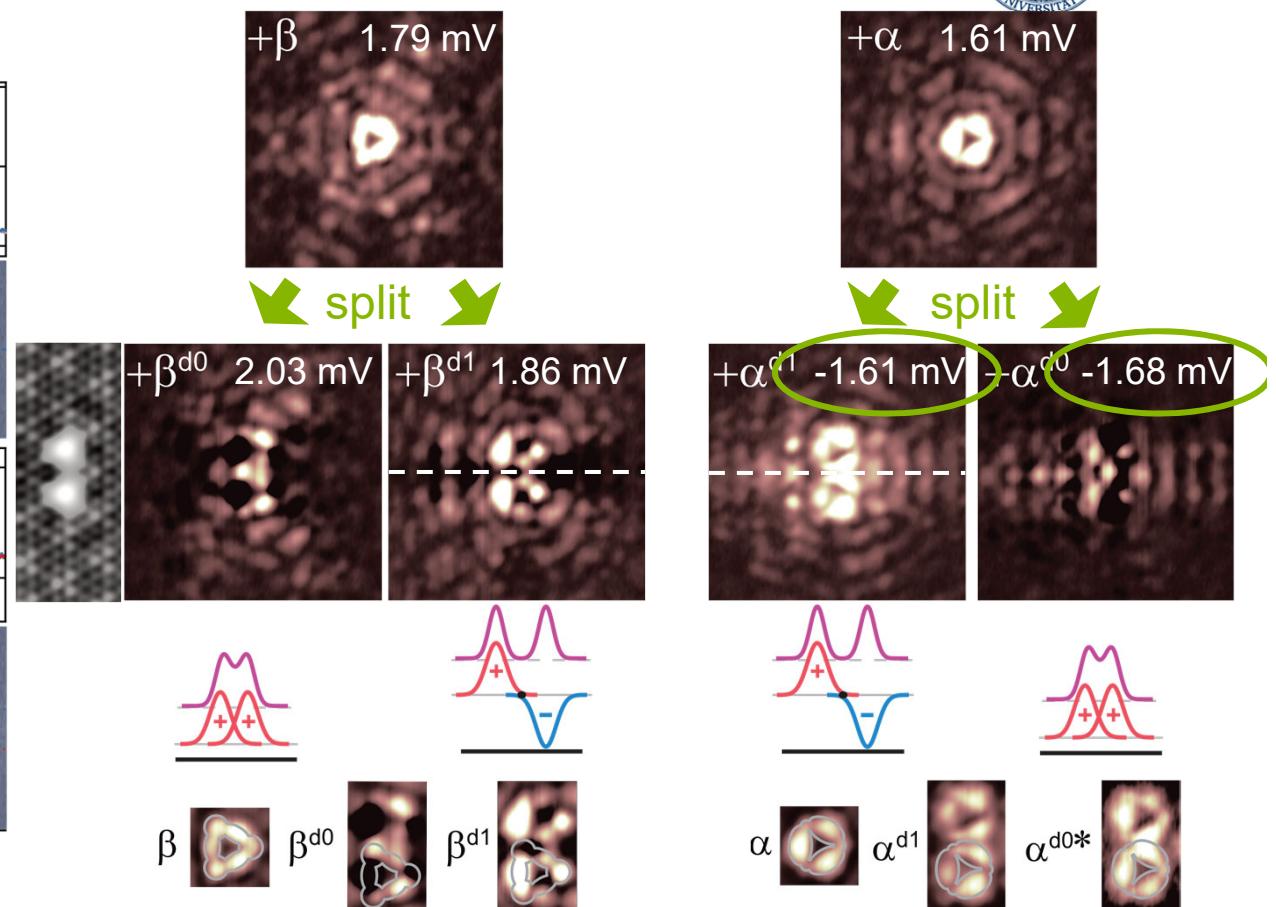
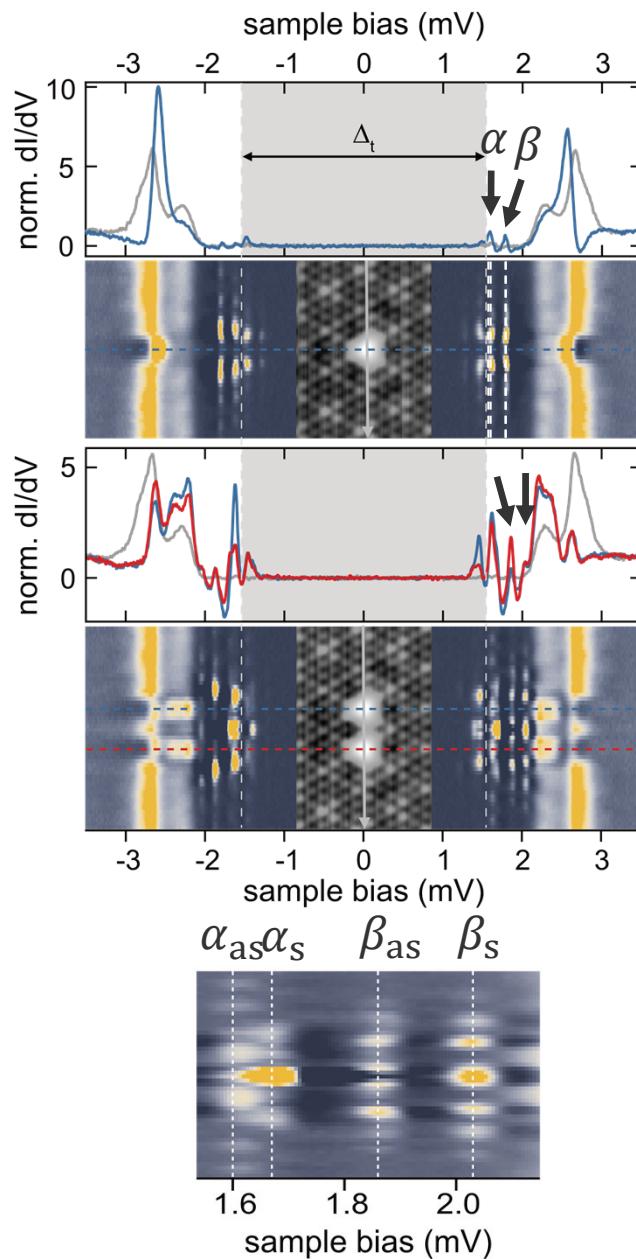
# YSR hybridization

Freie Universität Berlin



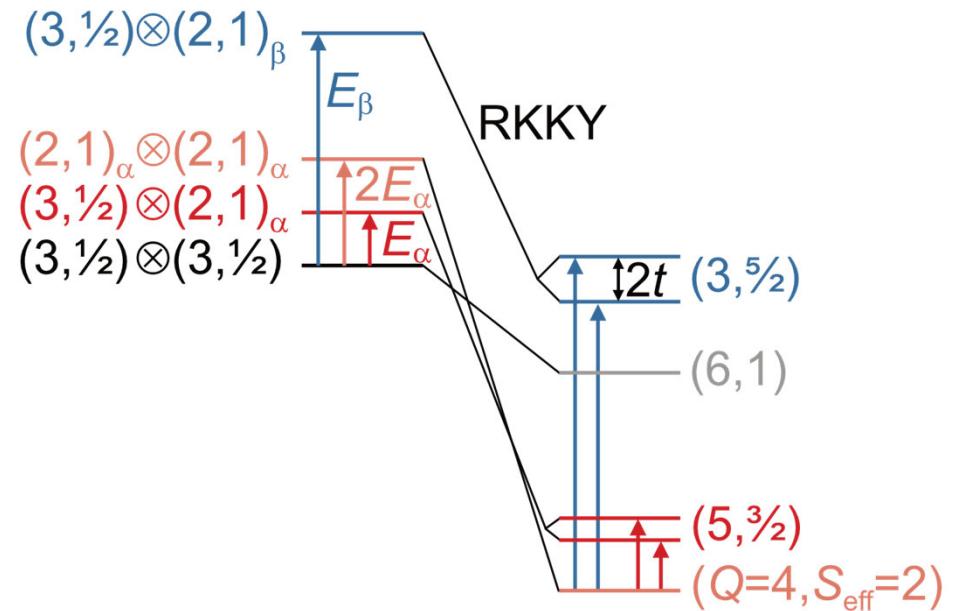
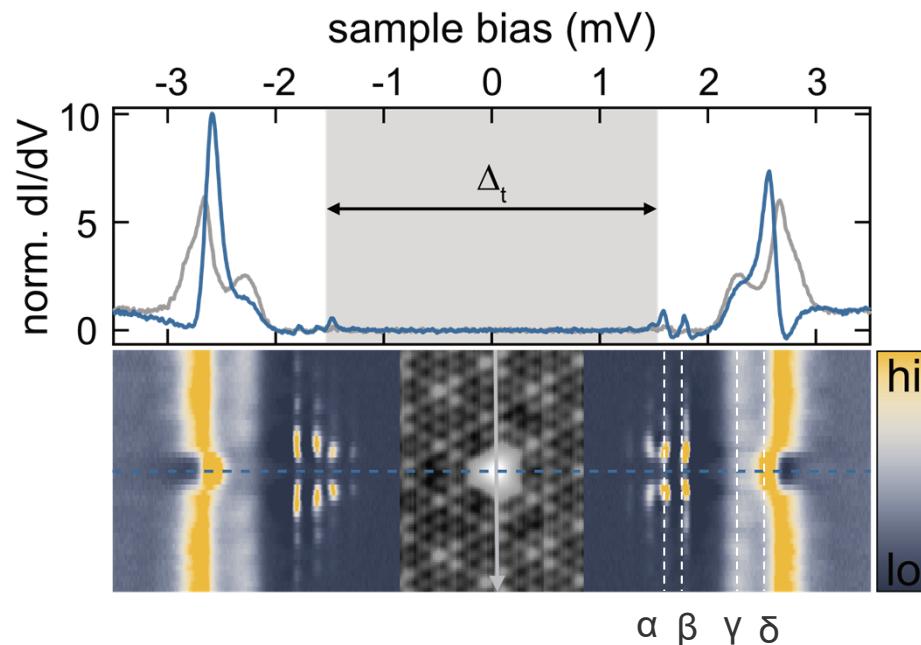
- Doubling of YSR resonances
- Hybridization in dimers with spacing of  $3a \approx a_{CDW}$

# YSR hybridization



- Symmetric and antisymmetric wave functions  
→ Hybridization in dimers with spacing of  $3a \approx a_{CDW}$
- Shift of a resonance through Fermi level  
→ Quantum phase transition

# RKKY interaction



Fe monomer:

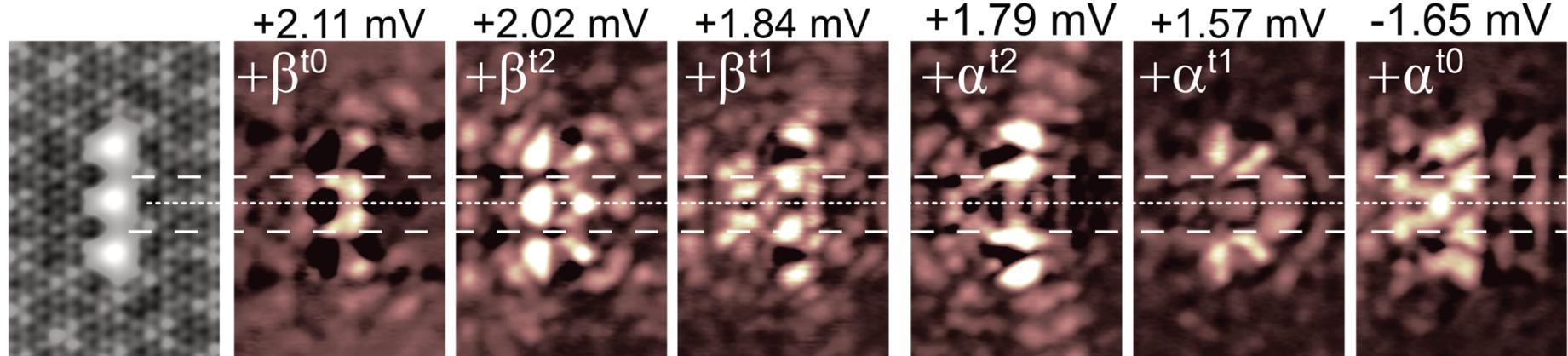
- Four YSR resonances
- $S=2$
- $\alpha$ -,  $\beta$ -,  $\gamma$ - resonances screened
- $\delta$ -resonance unscreened
- $S=1/2$

Fe dimer:

- $\alpha$ -resonance undergoes QPT to unscreened state due to RKKY
- Coupling of two  $S=1$  atoms
- $S=2$  dimer

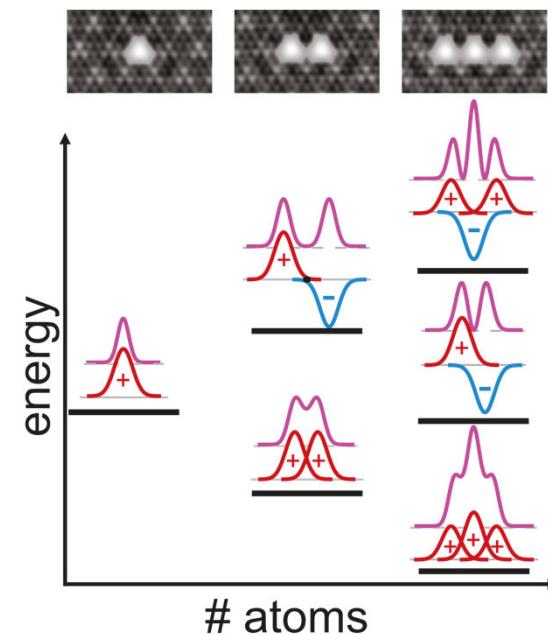
# Fe trimer

Freie Universität Berlin



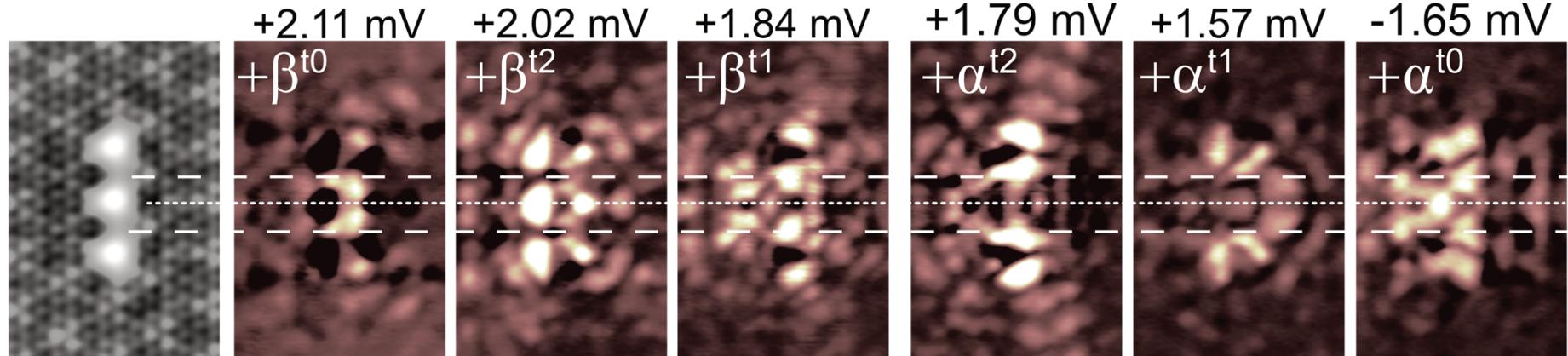
Fe trimer:

- Each YSR resonance splits into three



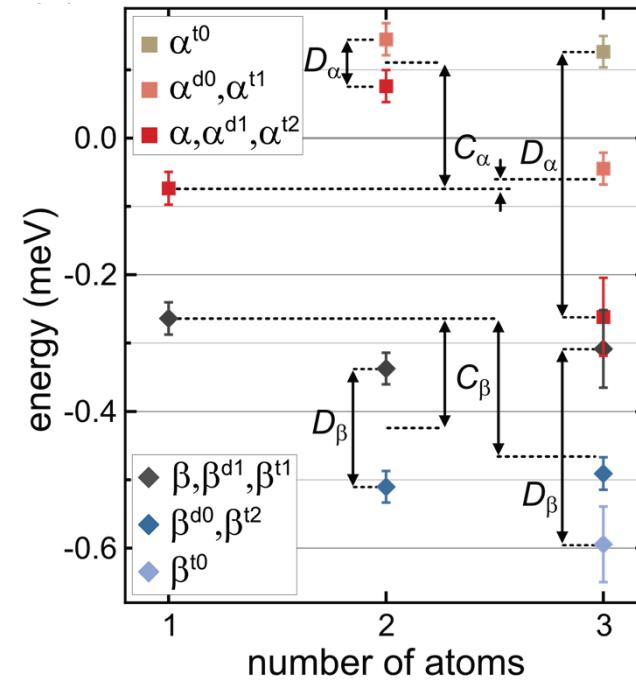
# Fe trimer

Freie Universität Berlin

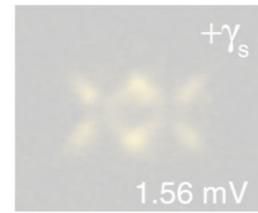


Fe trimer:

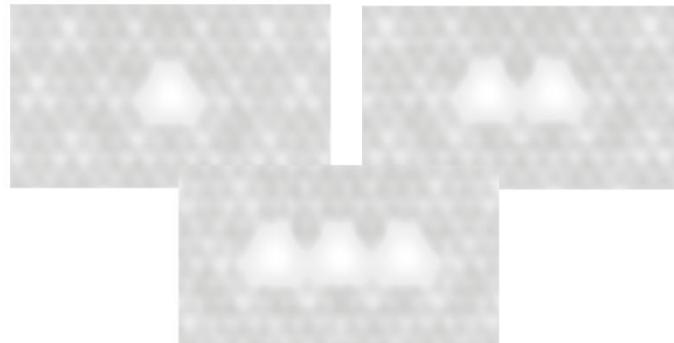
- Each YSR resonance splits into three
- Hybrid  $\alpha$  states overlap with Fermi level



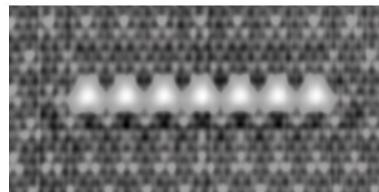
# Outline



► Hybridisation of YSR states (on Pb)



► Interaction of magnetic atoms on NbSe<sub>2</sub>



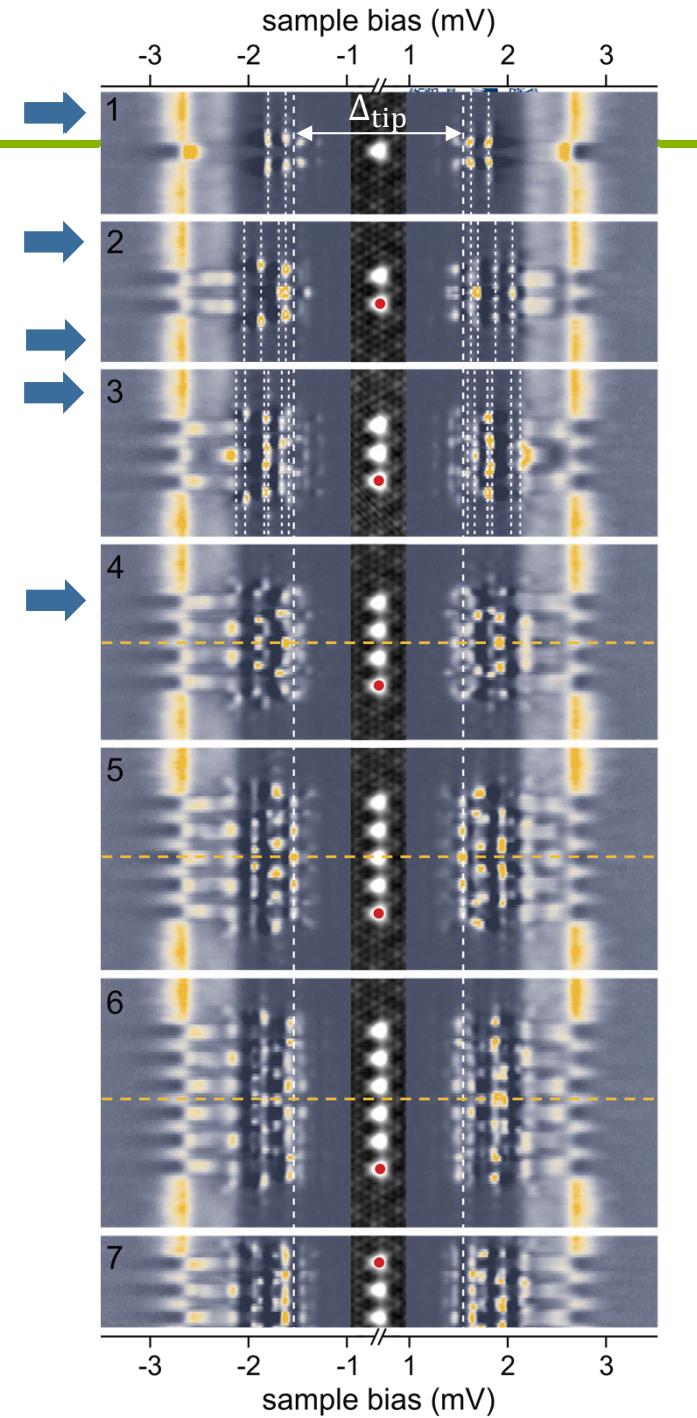
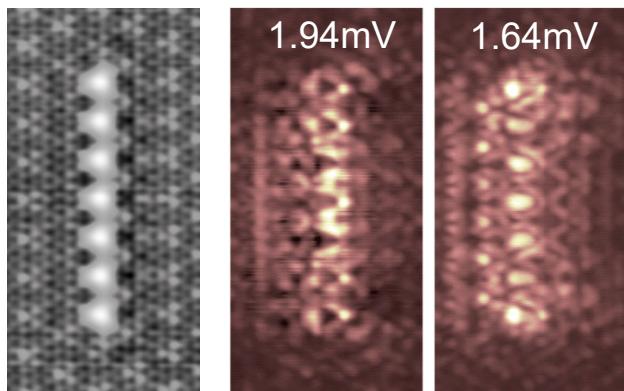
► Evolution of YSR states in atomic chains on NbSe<sub>2</sub>



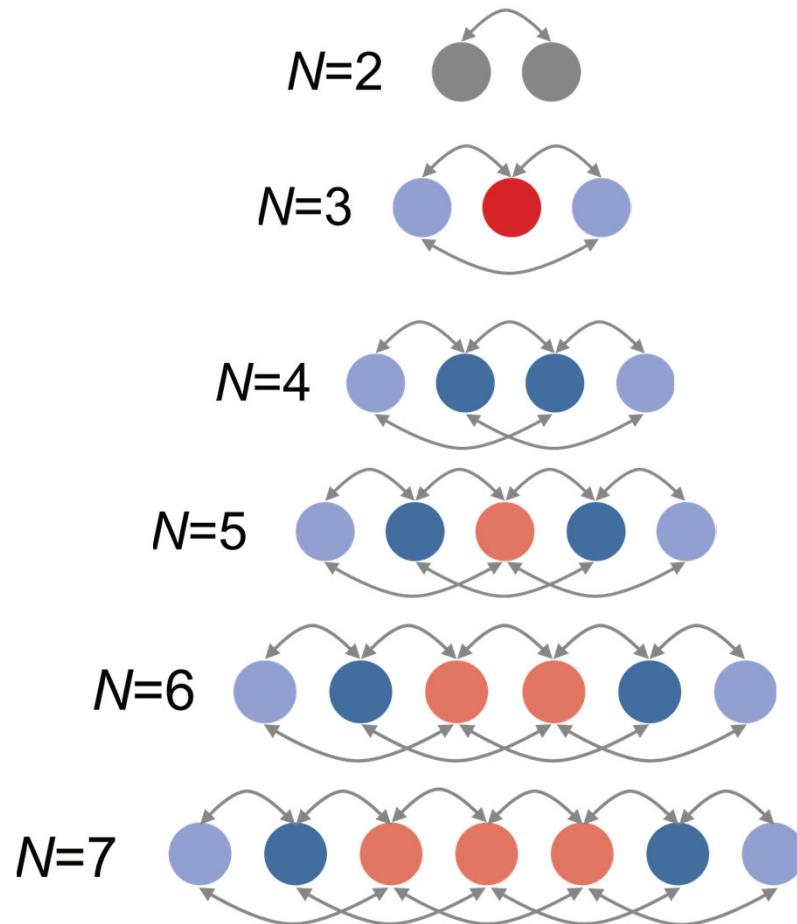
► Influence of charge-density wave on magnetic chains

# Formation of YSR bands

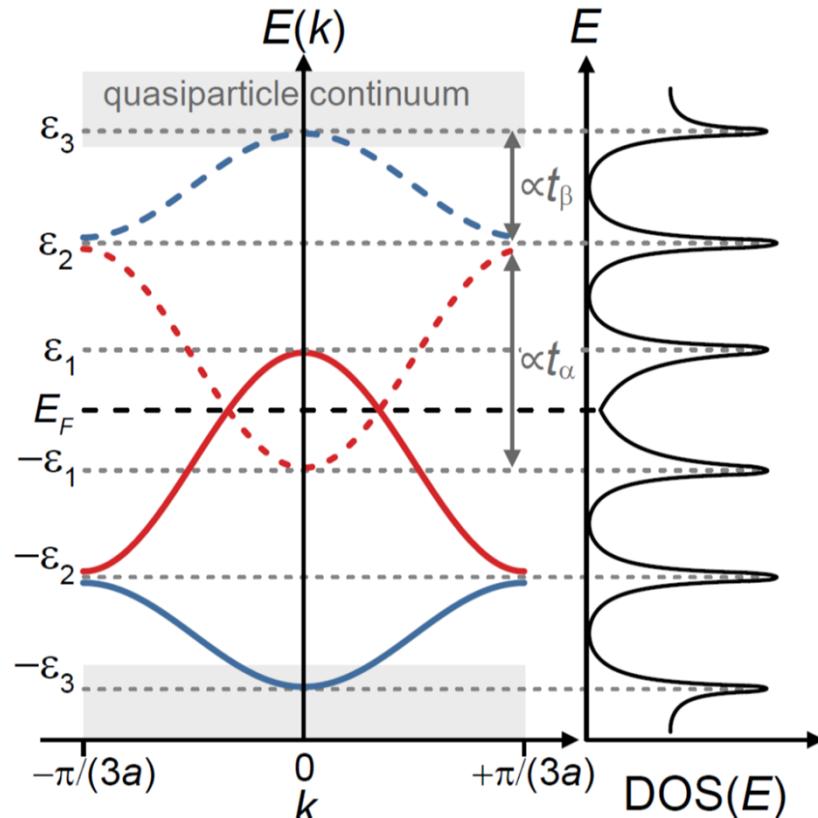
- ▶ all atoms on CDW maximum
- ▶ increasing number of hybrid states
- ▶ horizontal mirror symmetry
- ▶ delocalized over whole chain
- ▶ stabilization at specific energies



# YSR bands



● 1NN	● 2NN	● 1NN, 1NNN
● 2NN, 1NNN	● 2NN, 2NNN	



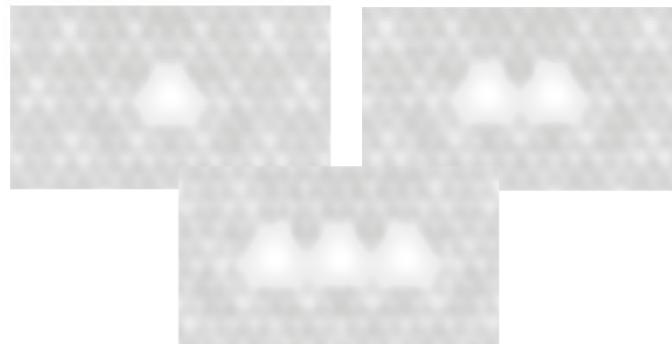
Fe chains:

- Formation of bands
- Van Hove singularities
- $\alpha$  band crossing Fermi level

# Outline



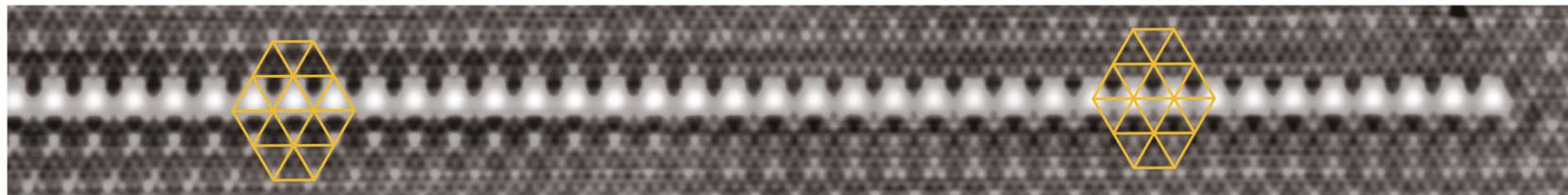
► Hybridisation of YSR states (on Pb)



► Interaction of magnetic atoms on  $\text{NbSe}_2$

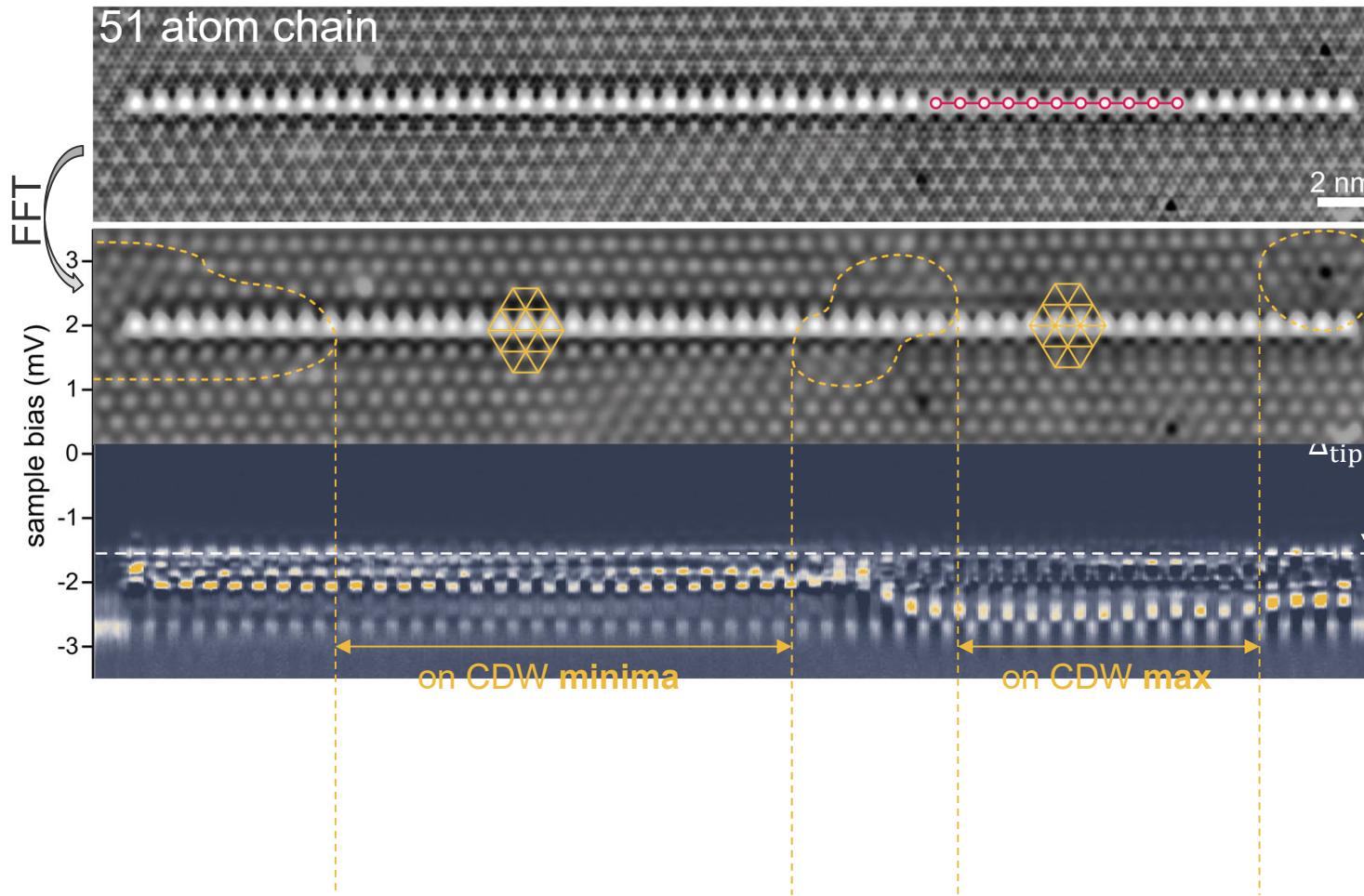


► Evolution of YSR states in atomic chains on  $\text{NbSe}_2$



► Influence of charge-density wave on magnetic chains

# Structures $\gg a_{\text{CDW}}$



- rich energy variations along chain
- CDW smoothly transforms in background

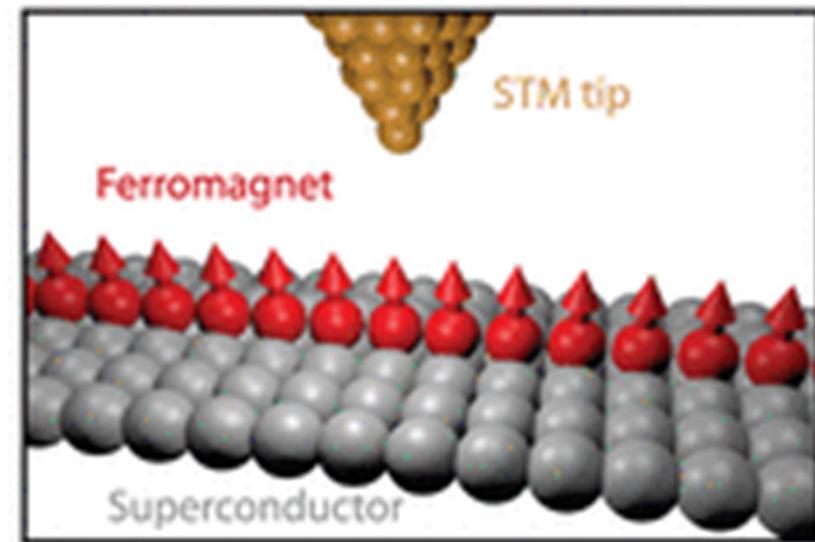
- ▶ band bending along chain due to incommensurate CDW
- ▶ two independent subchains



# Magnetic chains on superconductors

Versatile platform for:

- ▶ Topological superconductivity
- ▶ Majorana states



Nadj-Perge *et al.*, Science 346, 6209 (2014)

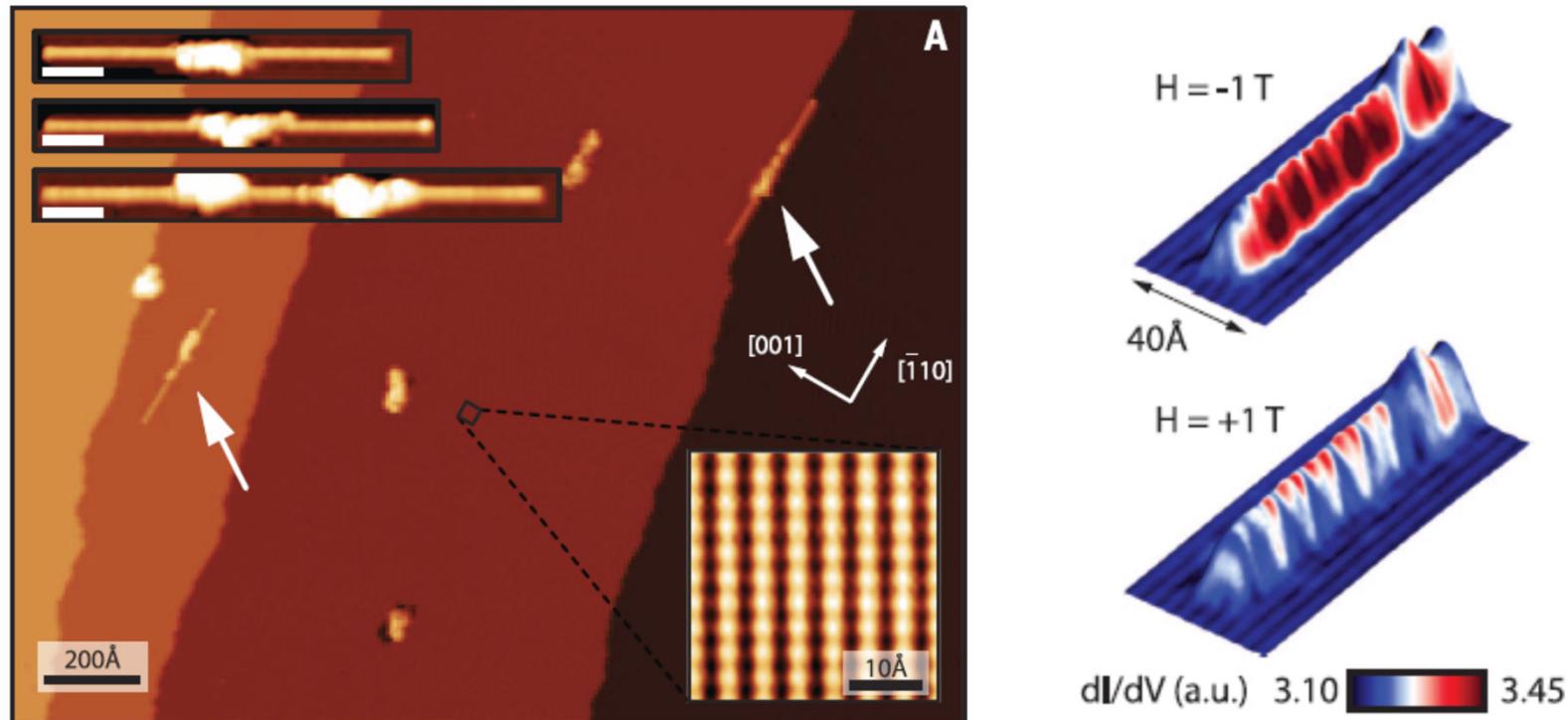
# Magnetism of Fe chain on Pb(110)

Freie Universität Berlin



Nadj-Perge *et al.*, Science 346, 6209 (2014):

**Observation of Majorana fermions in ferromagnetic atomic chains on a superconductor**



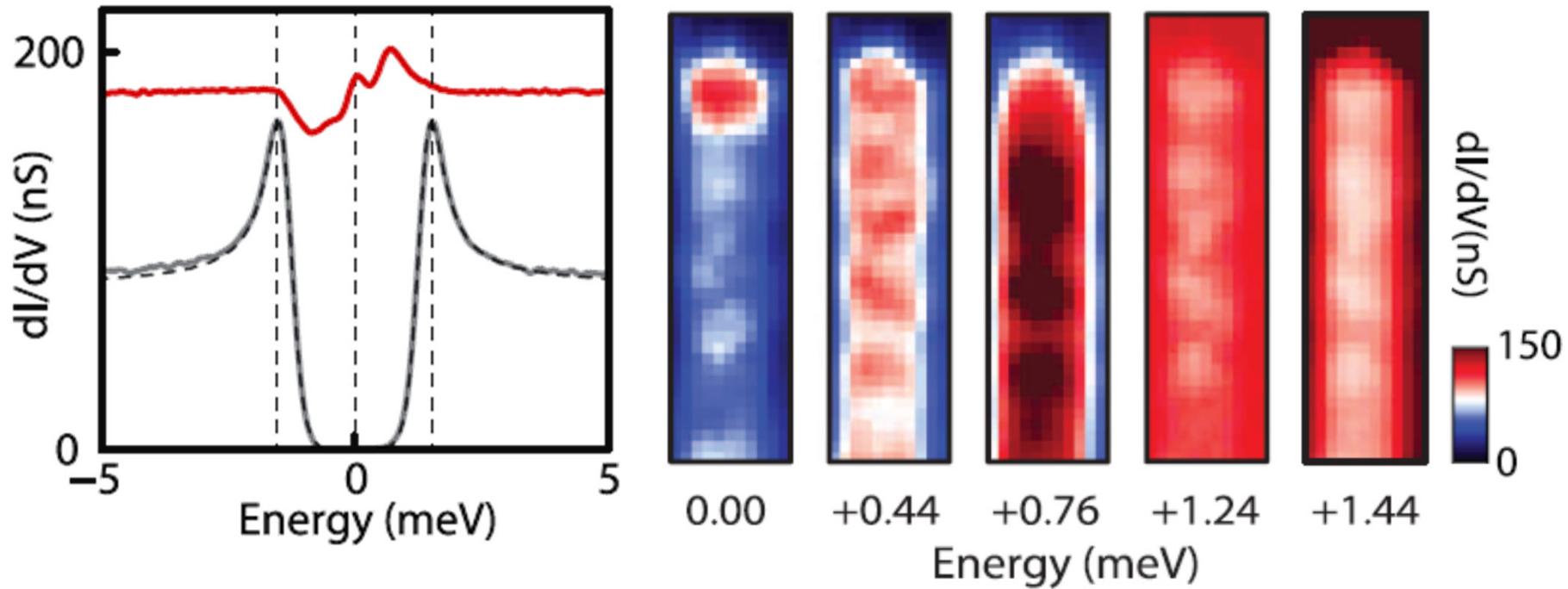
- ▶ Ferromagnetic spin structure along Fe chain

# Sub-gap structure in Fe chains

Freie Universität Berlin



Nadj-Perge *et al.*, Science 346, 6209 (2014):



- ▶ Peak at zero bias
- ▶ Localized at chain end
- ▶ Interpretation as Majorana states
- ▶ Topological gap 200-300  $\mu\text{eV}$

See also:

Exp:

- Ruby, et al., Phys. Rev. Lett. 115, 197204 (2015)  
Pawlak, et al., npj Quantum Information 2, 16035 (2016)  
Kim, et al., Sciences Adv. 4, eaar5251 (2018)

...

See also:

Theory:

- Nadj Perge, et al., Phys. Rev. B 88, 020407 (2013)  
Pientka, et al., Phys. Rev. B 88, 155420 (2013)  
Li, et al., Phys. Rev. B 90, 235422 (2014)

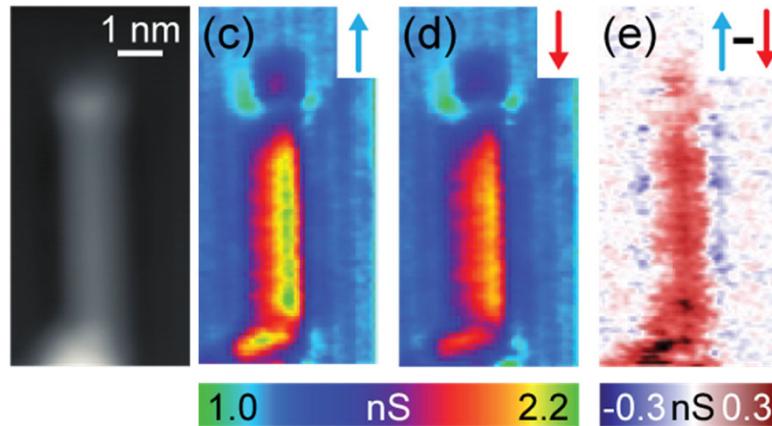
...

# Co chains on Pb(110): no Majorana modes

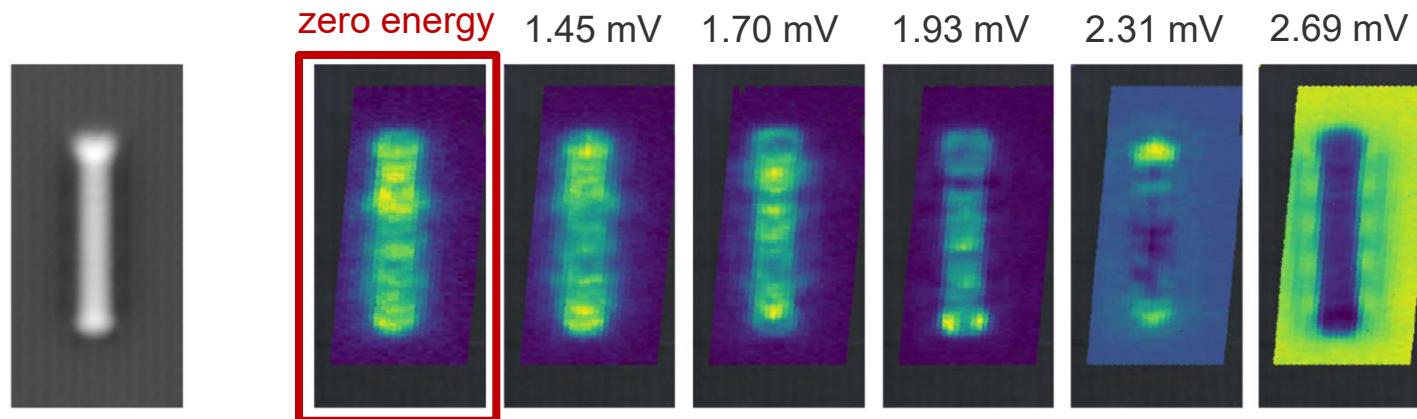
Freie Universität Berlin



- ▶ Ferromagnetic Co chain

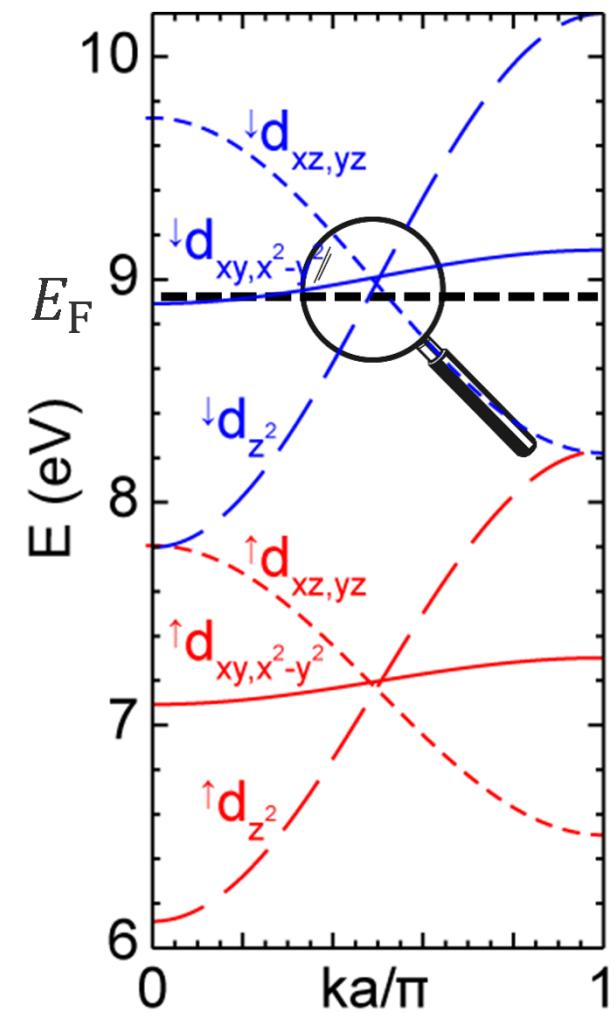
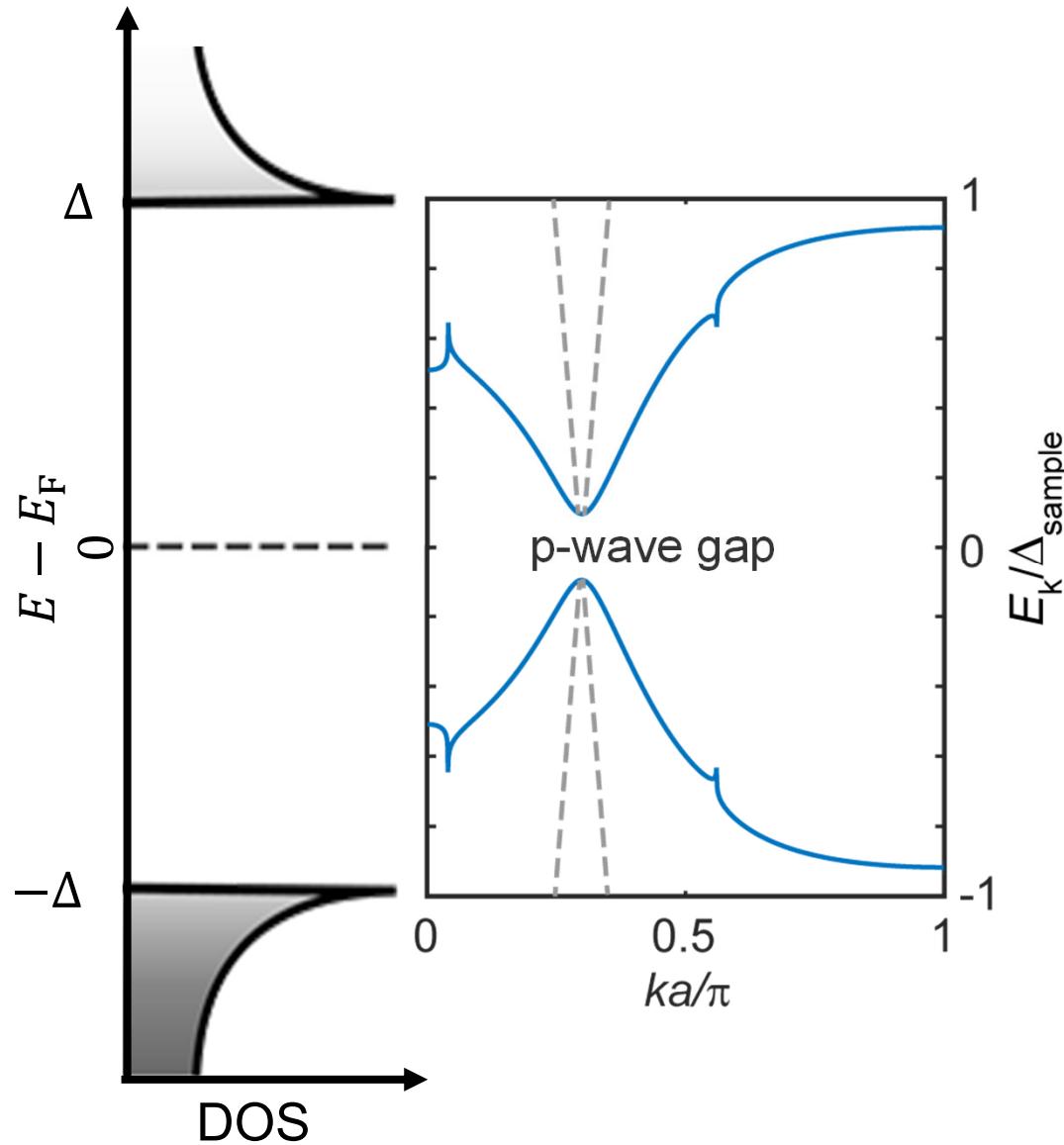


- ▶ Sub-gap signal



- ▶ No signature of Majorana modes

# Transition metal chains on Pb(110)



# Design of topological states

Freie Universität Berlin



So far:

- ▶ Majorana states in ferromagnetic chains with direct exchange coupling

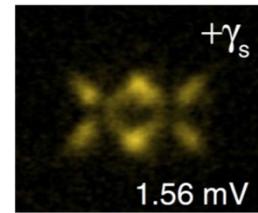
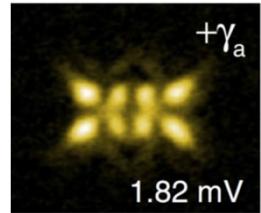


Pientka, et al., Phys. Rev. B 88, 155420 (2013)  
Pientka, et al., Phys. Scr. 014008 (2015)

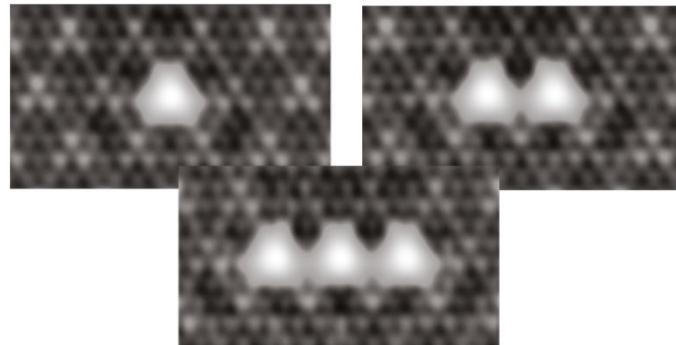
Design from YSR states:

- ▶ Dilute magnetic adatoms
- ▶ Coupling of YSR states
- ▶ YSR bands crossing Fermi level
- ▶ possibly topological gap opening, but not detected for Fe on NbSe<sub>2</sub>

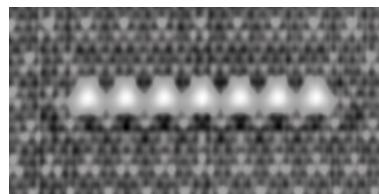
# Conclusions



- ▶ Hybridisation of YSR states:
  - ▶ Symmetric and anti-symmetric linear combination



- ▶ RKKY interactions and hybridisation
  - ▶ Inducing quantum phase transition
  - ▶ Quantum spin description of interactions



- ▶ Formation of YSR bands



- ▶ Band bending and different domains due to charge-density wave

# Thanks!

Freie Universität Berlin



Sergio Acero Gonzalez

Jacob Steiner

Felix von Oppen

Nils Bogdanoff, Giada Franceschi, Jennifer Hartfiel, **Eva Liebhaber**, Christian Lotze, Bharti Mahendru, **Gaël Reecht**, **Lisa Rütten**, Rika Simon, Idan Tamir, Martina Trahms, Sergey Trishin, Ali Yazdani, Asieh Yousofnejad

Sergio Acero Gonzalez, Jacob Steiner, Christophe Mora, Felix von Oppen

Sebastian Rohlf, Kai Rossnagel (Kiel)