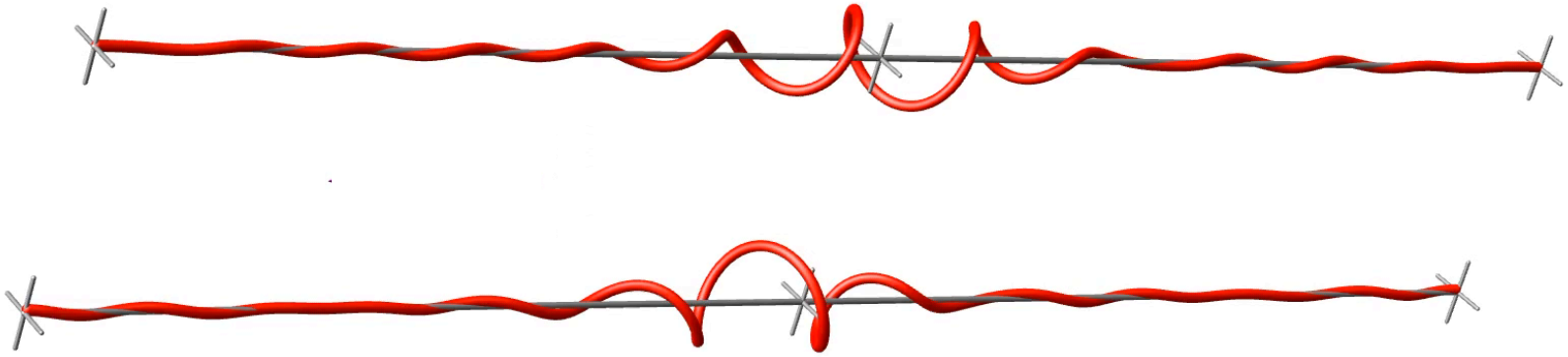
Karlsruher Institut für Technologie

Magnetic skyrmion strings: how they bend, twist and vibrate

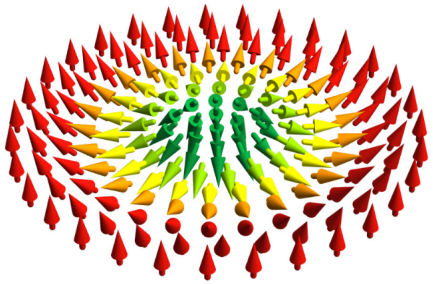
Markus Garst



Outline

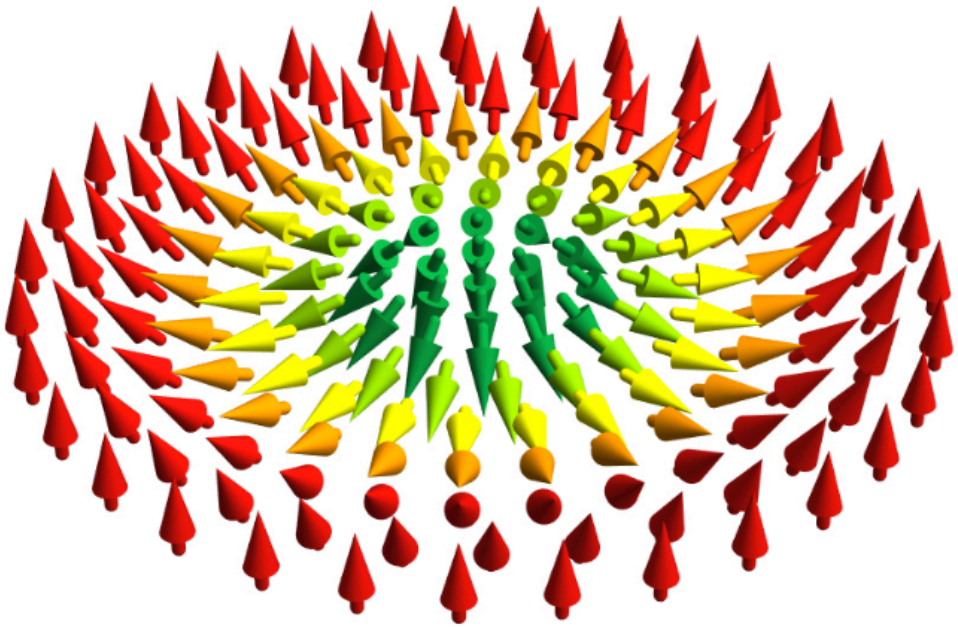
- Introduction: magnetic skyrmion strings
- Overview: Linear spin-wave dynamics of skyrmion strings
- Low-energy theory of a skyrmion string & solitary waves
- Current-driven instability of skyrmion strings

Introduction:
magnetic skyrmion strings



Magnetic skyrmion

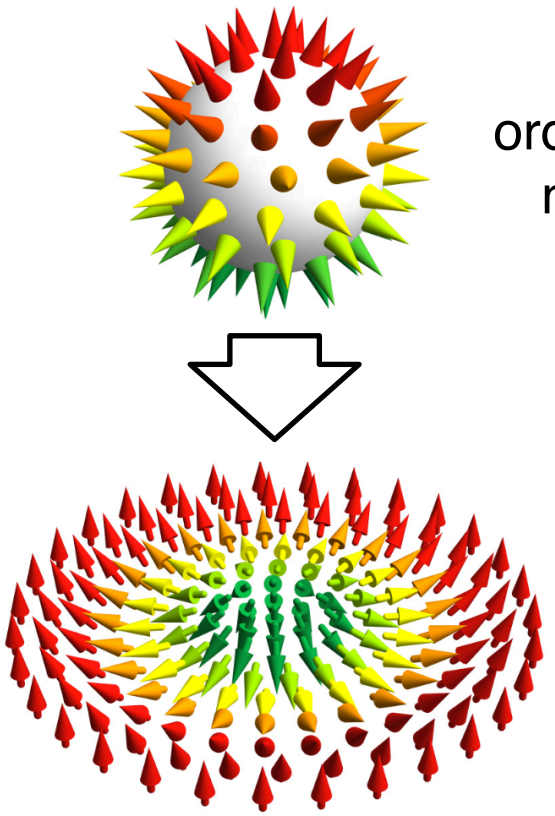
Heisenberg magnet in 2d:



topological texture with integer winding number W

Magnetic skyrmion

topologically non-trivial mapping from sphere S^2 to 2d plane: $\pi_2(S^2) = \mathbb{Z}$



order parameter manifold: S^2

2d plane

topological skyrmion density

$$\rho_{\text{top}} = \frac{1}{4\pi}\hat{M}(\partial_x \hat{M} \times \partial_y \hat{M})$$

orientation of magnetization:

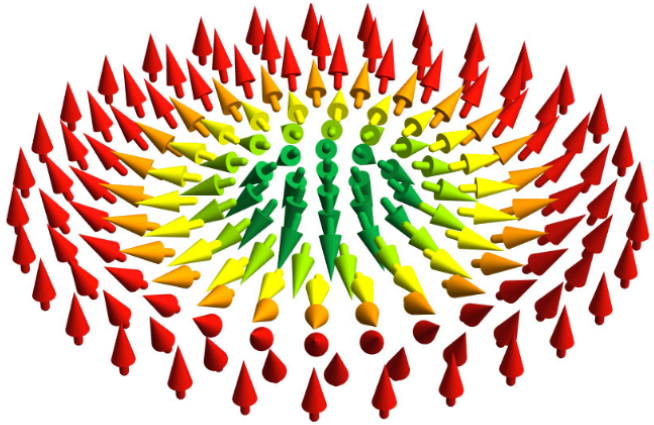
unit vector field $\hat{M}$

winding number

$$W = \int d^2\mathbf{r}\, \rho_{\text{top}}$$

Magnetic skyrmion string

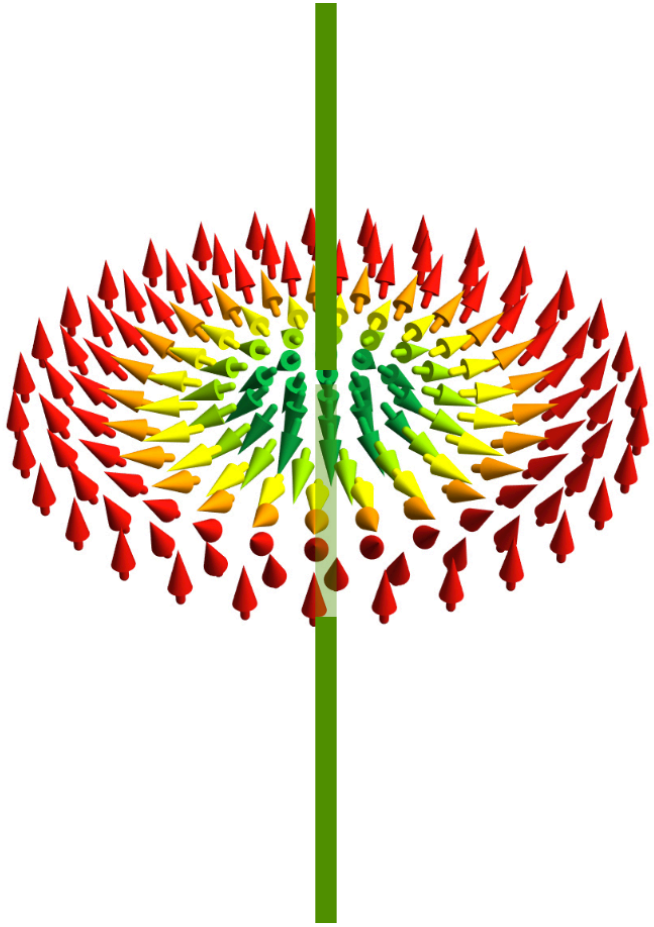
Heisenberg magnet in 3d:



1d string with skyrmion texture
in each 2d cross section

Magnetic skyrmion string

Heisenberg magnet in 3d:

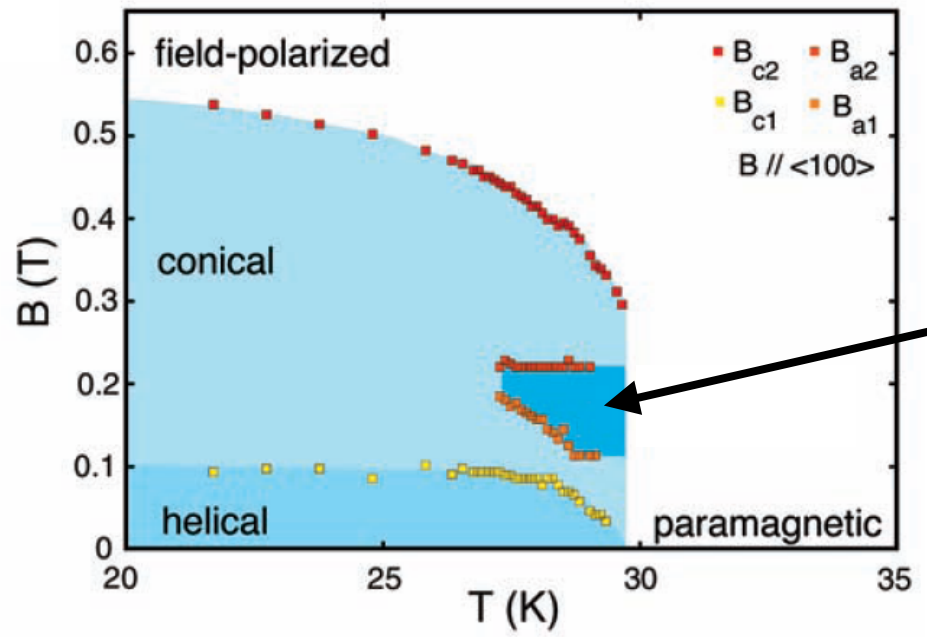


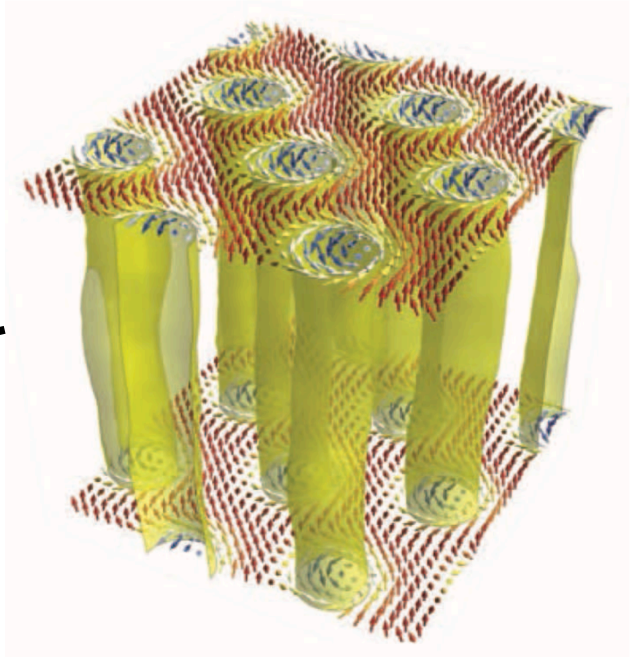
1d string with skyrmion texture
in each 2d cross section

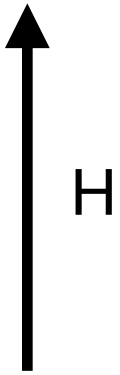
Where can they be found?

cubic chiral magnets: MnSi, FeGe, Cu_2OSeO_3, …

universal phase diagram for small spin-orbit coupling:







S. Mühlbauer *et al.* Science (2009)
P. Milde *et al.* Science (2013)

skyrmion strings aligned with the magnetic field form a hexagonal lattice

Cubic chiral magnets

universal theory for small spin-orbit coupling:

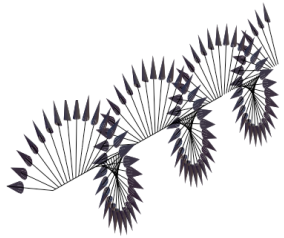
$$\mathcal{V} = A(\nabla_i \hat{M}_j)^2 + D\hat{M}(\nabla \times \hat{M}) - \mu_0 M_S \hat{M}\vec{H}$$

exchange interaction | Dzyaloshinskii-Moryia interaction (DMI) | Zeeman energy

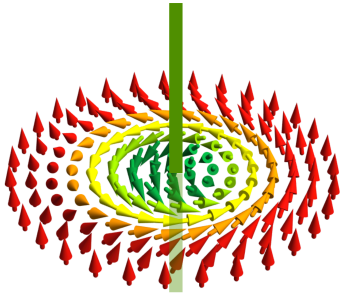
+ dipolar interactions (+ magnetocrystalline anisotropies)

⇨ magnetic texture with typical spatial wavelength $\lambda \sim \frac{A}{D}$

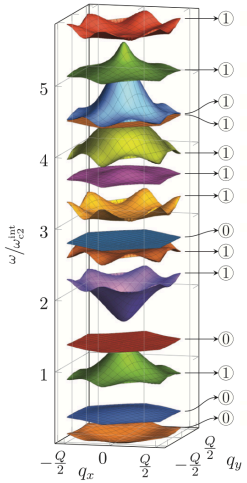
helices (topological trivial)



skyrmion strings (topological non-trivial)



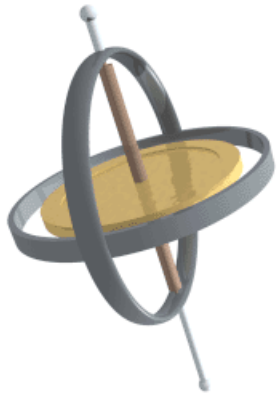
Overview:
Linear spin-wave dynamics of skyrmion strings



Magnetization dynamics

Landau-Lifshitz equation

$$\partial_t \hat{M} = -\gamma \hat{M} \times \vec{B}_{\text{eff}}$$



wikipedia

precession of the magnetization

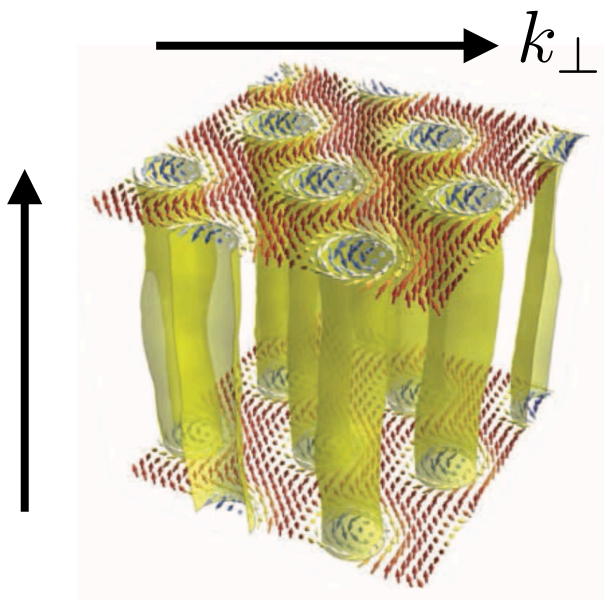
gyromagnetic ratio $\gamma = \dfrac{g\mu_B}{\hbar}$

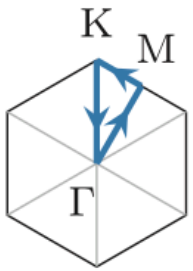
effective field is determined by the magnetic texture: $\vec{B}_{\text{eff}} = -\dfrac{1}{M_s}\dfrac{\delta F}{\delta \hat{M}}$

with the energy functional $F = \displaystyle\int d\vec{r}\,\mathcal{V}(\hat{M})$

Dynamics of the skyrmion-string lattice

for in-plane wavevectors

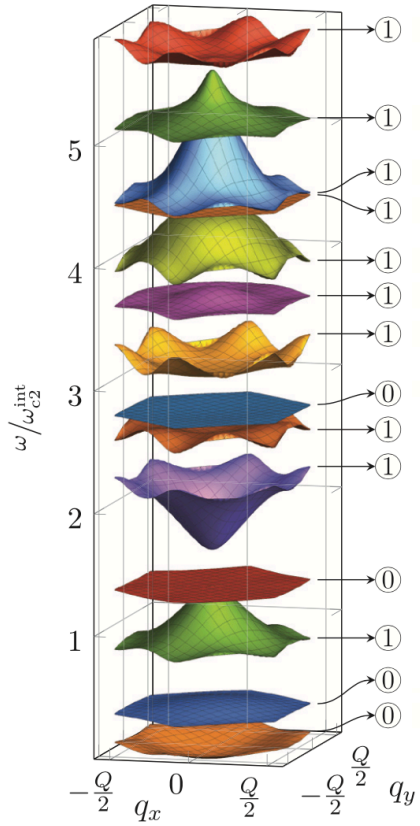




2d hexagonal magnetic Brillouin zone

Bragg scattering off the magnonic crystal:

⇨ magnon band structure



Roldan-Molina et al. New J. Phys. 18, 045015 (2016)
MG, J. Waizner, D. Grundler, J. Phys. D: Appl. Phys. 50, 293002 (2017)

Dynamics of the skyrmion-string lattice

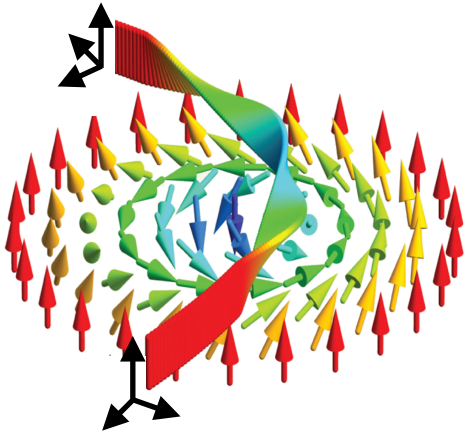
non-trivial topology of skyrmions ⇨ topological magnon band structure

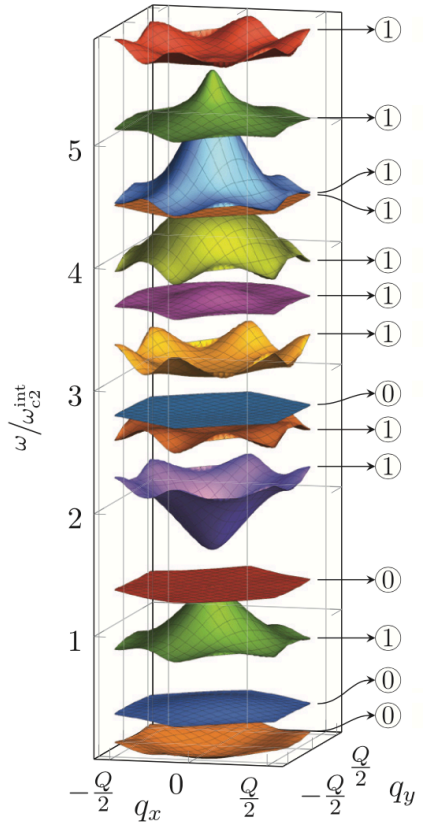
each skyrmion acts like a source of emergent magnetic flux

⇩

emergent magnon electrodynamics

formation of magnon Landau levels





Chern numbers

Roldan-Molina et al. New J. Phys. 18, 045015 (2016)
MG, J. Waizner, D. Grundler, J. Phys. D: Appl. Phys. 50, 293002 (2017)

Dynamics of the skyrmion-string lattice

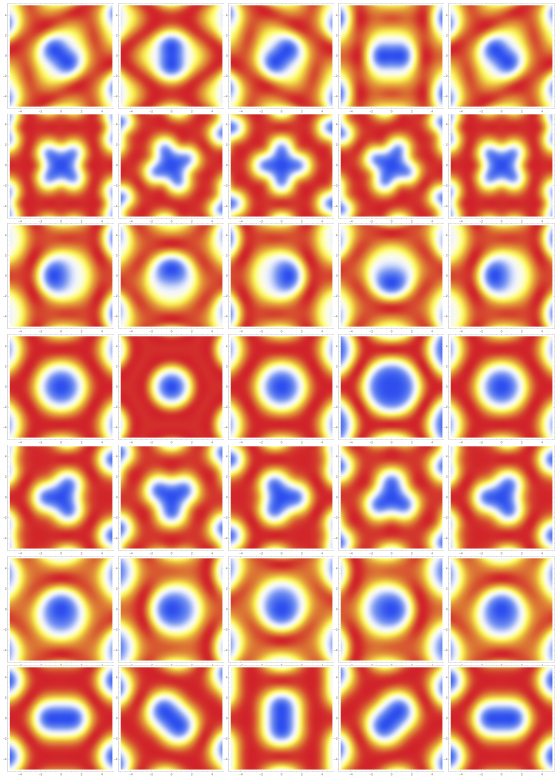
three magnetic resonances:

clockwise →

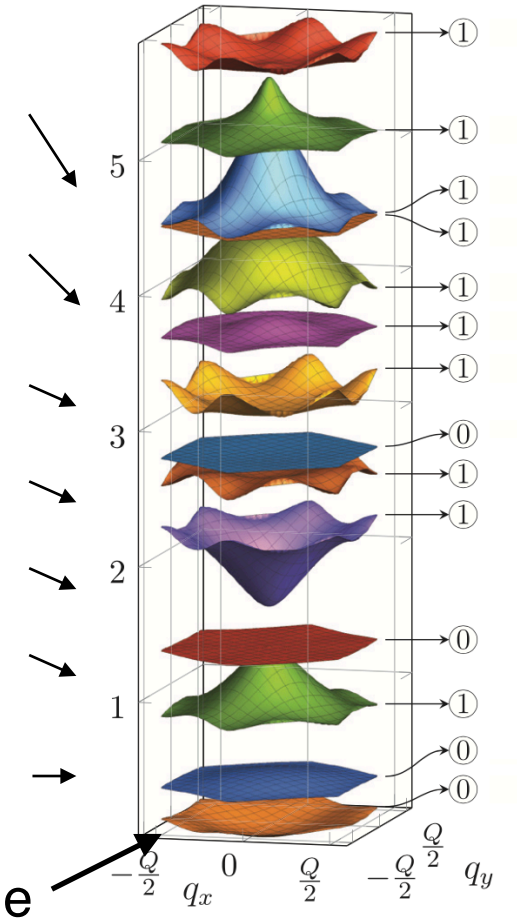
breathing →

counterclockwise →

magnon normal modes



magnon band structure

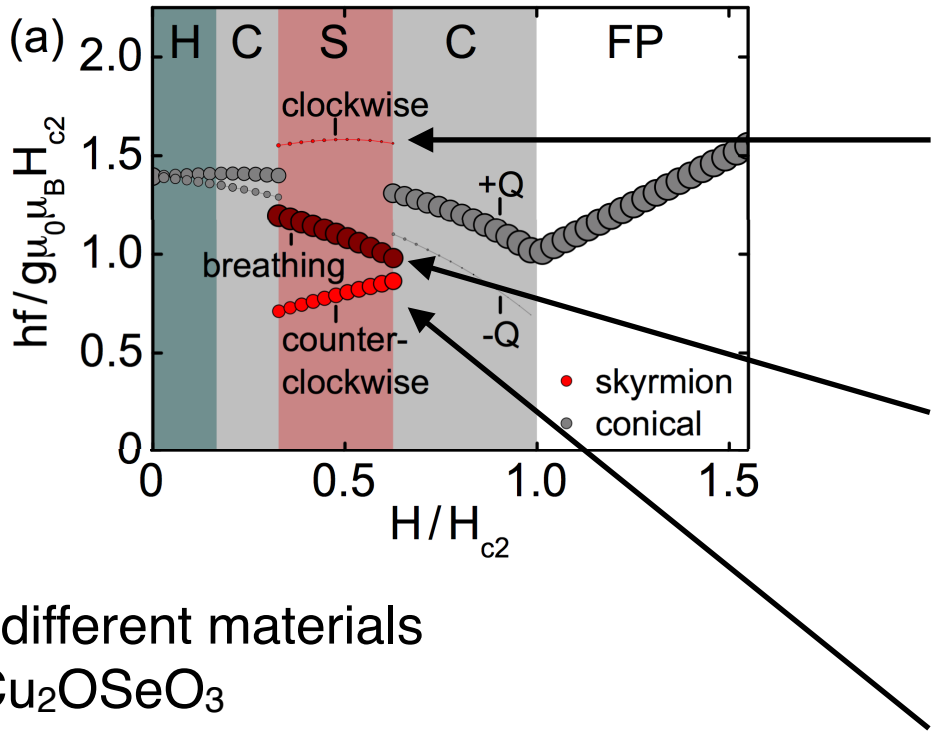


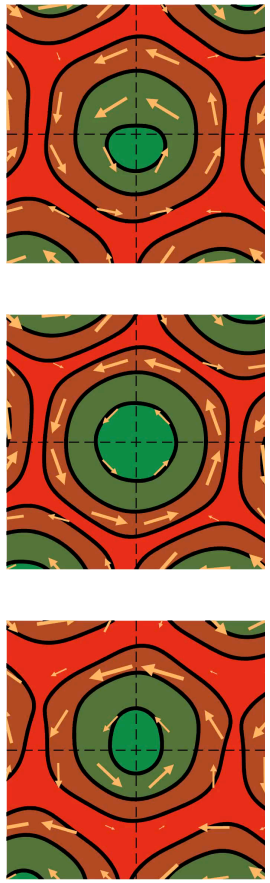
Goldstone mode

Mochizuki PRL (2012),
Onose et al. PRL (2012),
T. Schwarze, *et al.* Nat Mater (2015)

Magnetic skyrmion resonances

field-dependence of the resonance frequencies





clockwise (CW)

breathing

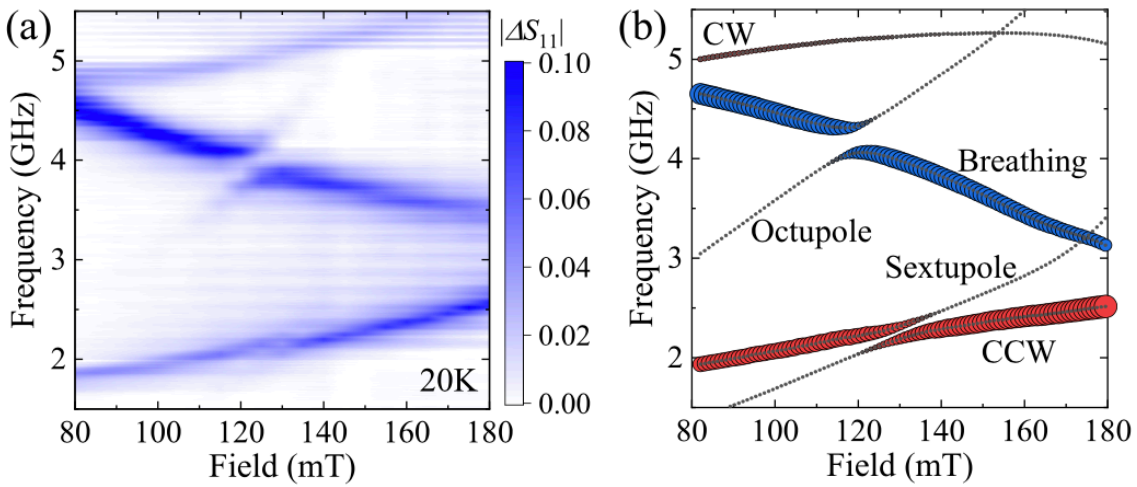
counter-clockwise (CCW)

measurements on three different materials
MnSi, $Fe_{0.8}Co_{0.2}Si$ and Cu_2OSeO_3

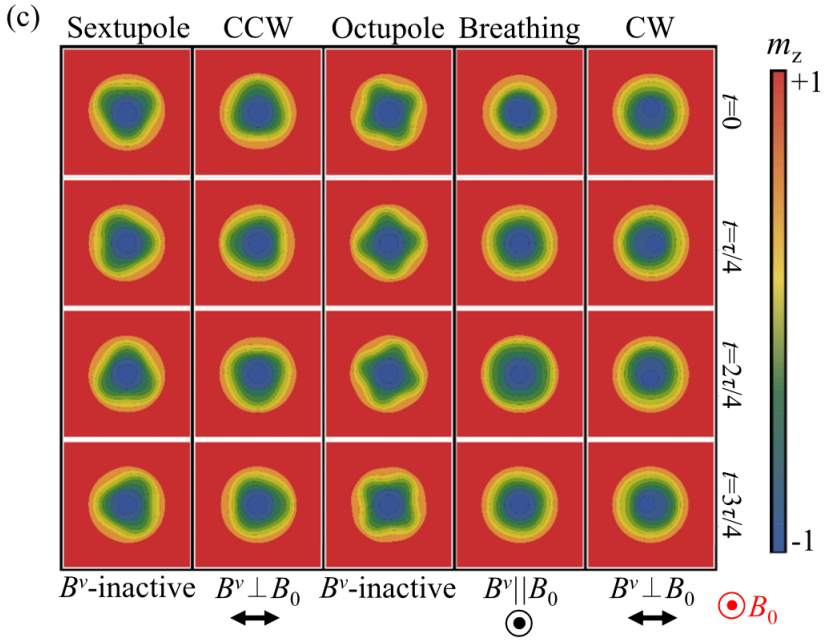
Mochizuki PRL (2012),
Onose et al. PRL (2012),
T. Schwarze, *et al.* Nat Mater (2015)

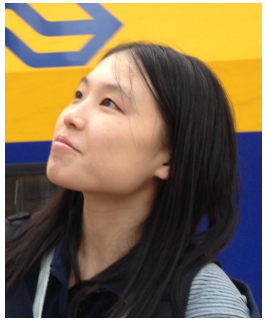
Magnetic skyrmion resonances

hybridization between modes due to weak magnetocrystalline anisotropies



Cu_2OSeO_3



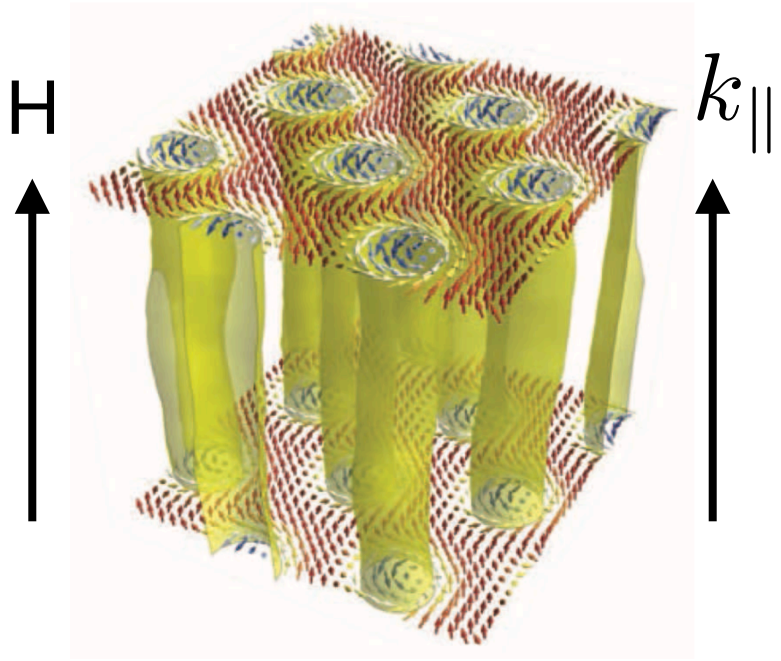


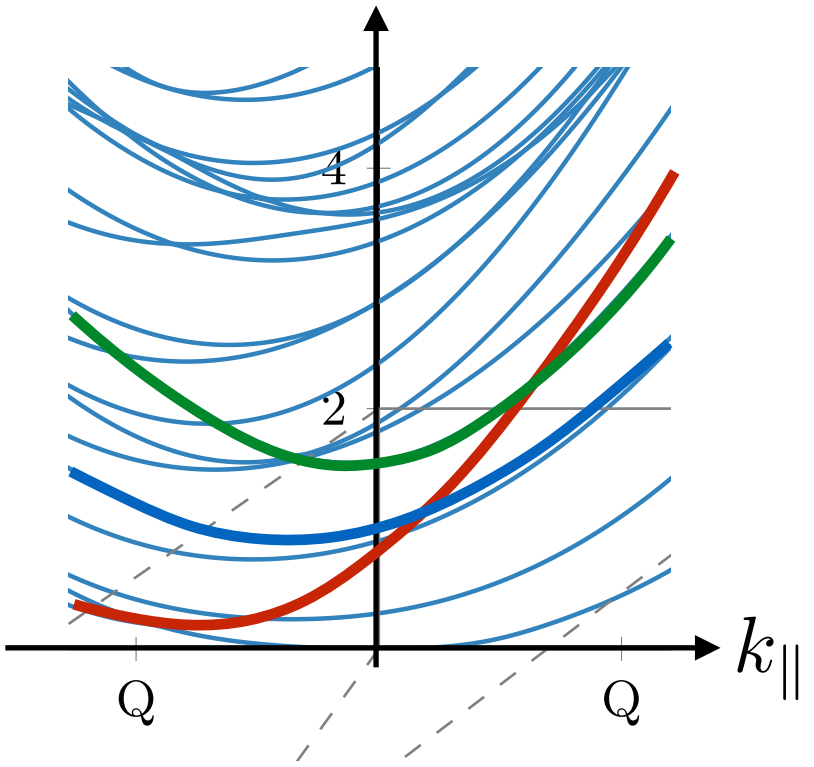
R. Takagi (Tokyo)

R. Takagi et al. Phys. Rev. B **104**, 144410 (2021)

Non-reciprocity of skyrmion-lattice magnons

magnon dispersion for out-of-plane momenta





CW

breathing

CCW

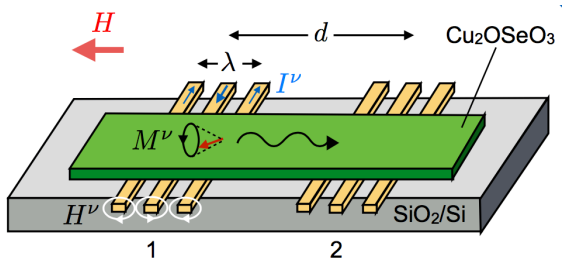
non-reciprocity $\omega(k_{||}) \neq \omega(-k_{||})$

most pronounced for the CCW mode!

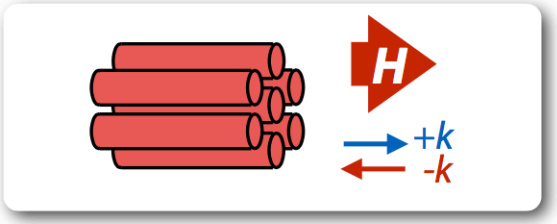
Spinwave spectroscopy on Cu_2OSeO_3

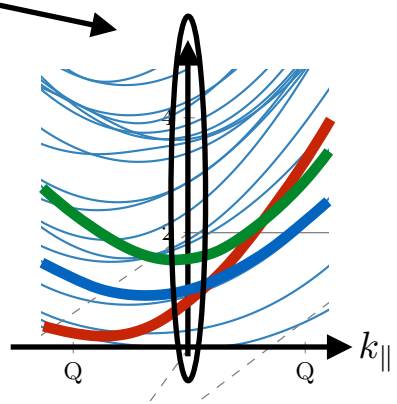
S. Seki
(Tokyo)

probing the spectrum in the magnetostatic regime close to q=0

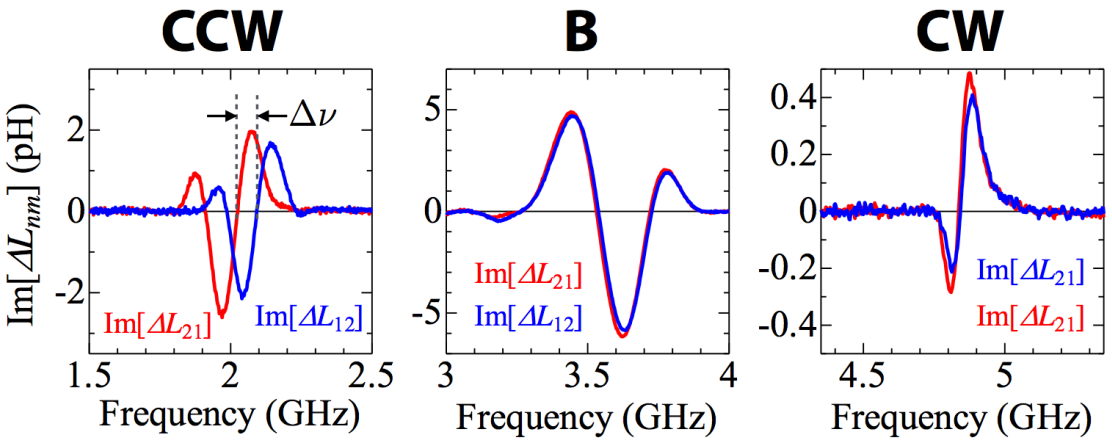


- Wavelength : λ = 12μm (i.e., k = 0.50μm^{-1})
- Sample thickness : t = 1~2 μm
- Propagation distance : d = 20 μm





non-reciprocity most pronounced for the CCW mode!



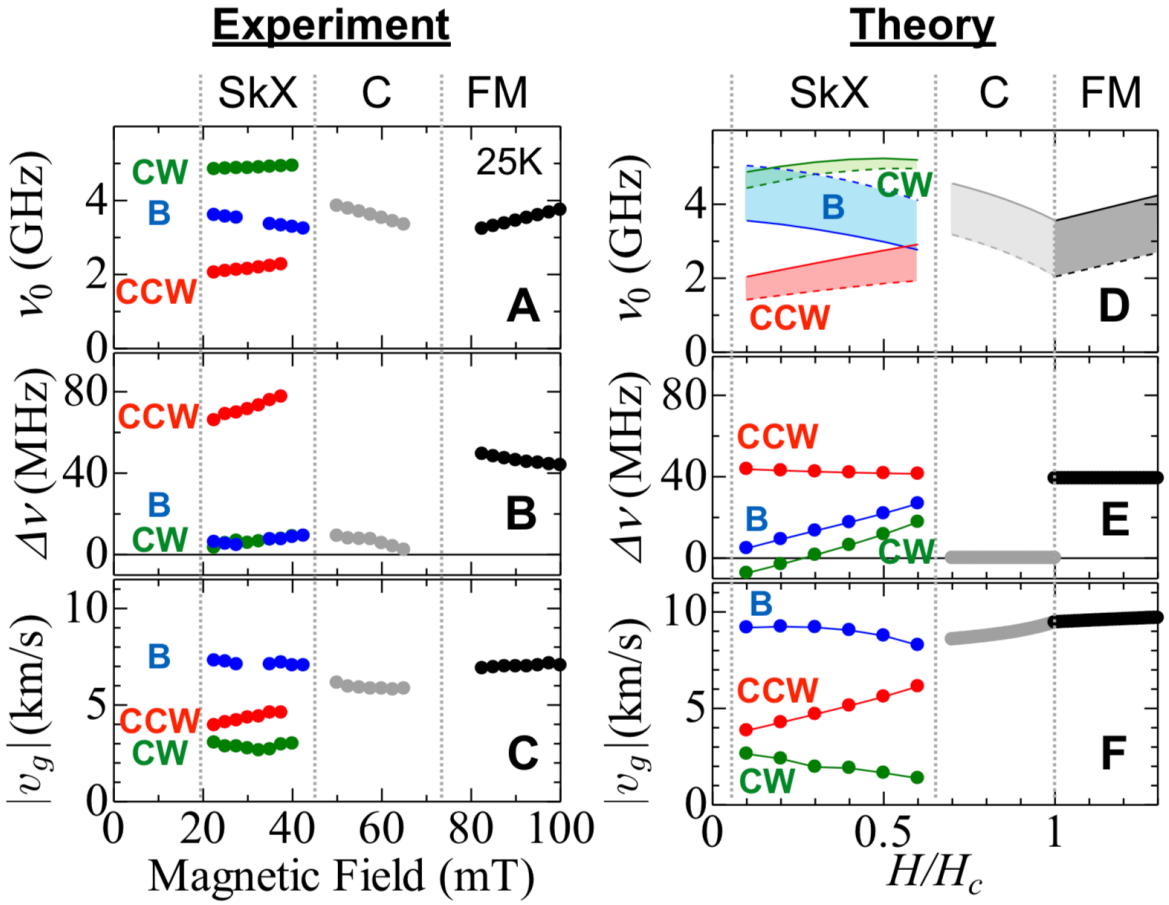
S. Seki, Nat. Commun. **11**, 256 (2020)

Non-reciprocity of skyrmion-lattice magnons

resonance frequency

non-reciprocal shift

group velocity

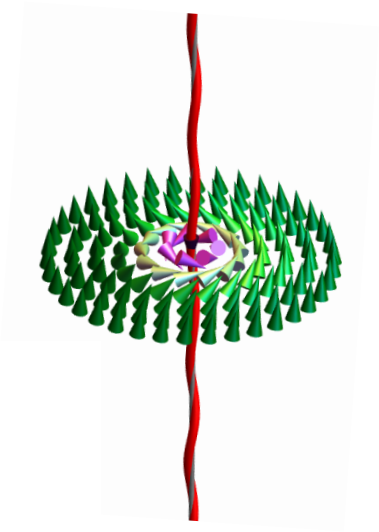


no fitting parameters!

agreement with theory!

S. Seki, Nat. Commun. **11**, 256 (2020)

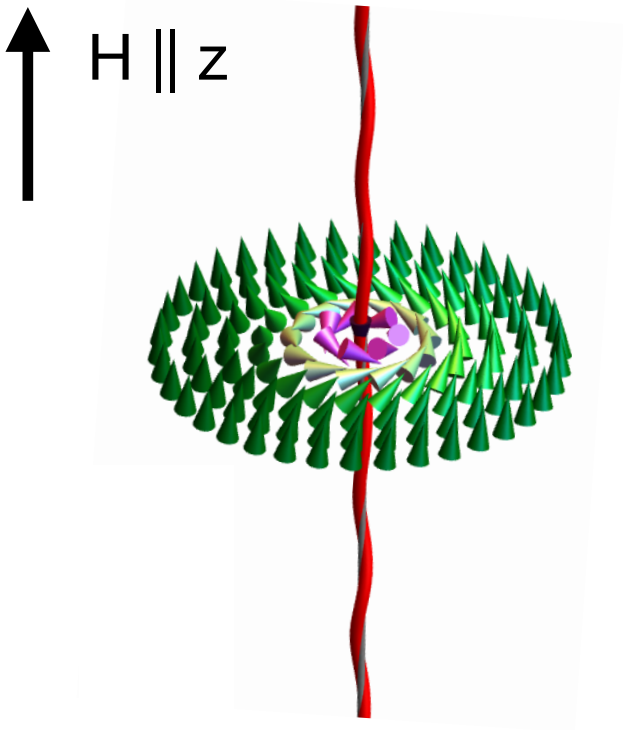
Low-energy theory of a skyrmion string & solitary waves



V. Kravchuk et al. Phys. Rev. B **102**, 220408(R)

Single skyrmion string

skyrmion string excitation of the field-polarized phase of cubic chiral magnets



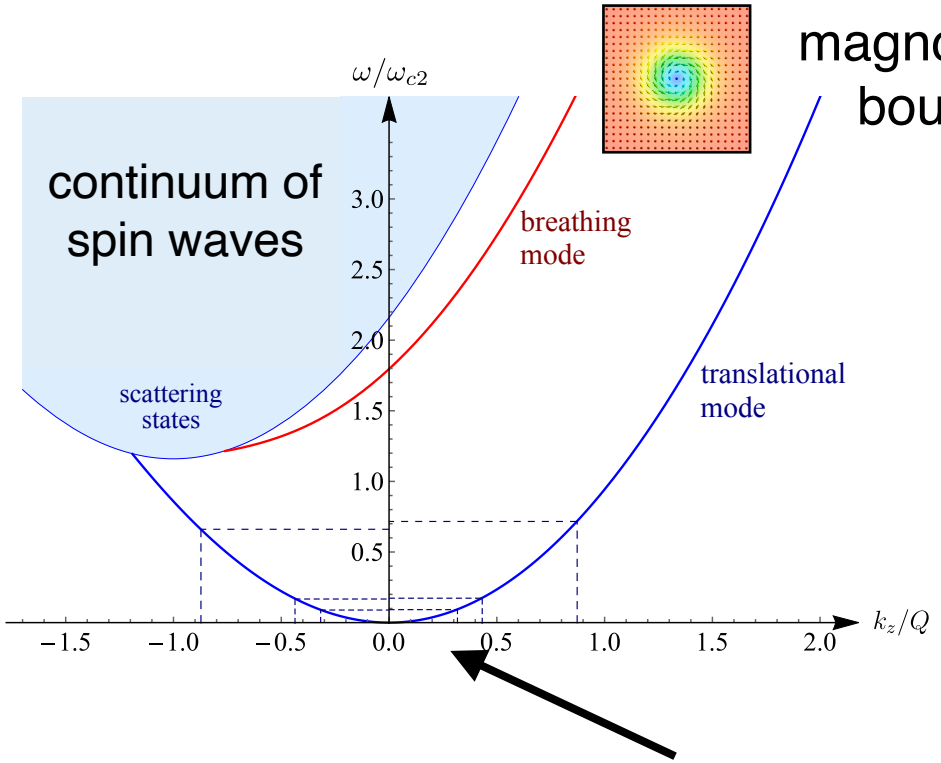
$$\mathcal{V} = A(\nabla_i \hat{M}_j)^2 + D\hat{M}(\nabla \times \hat{M}) - \mu_0 M_S \hat{M}\vec{H}$$

skyrmion texture within the plane⊥H

Bogdanov & Hubert (1994)

Linear dynamics of the skyrmion string

linear spin wave theory:

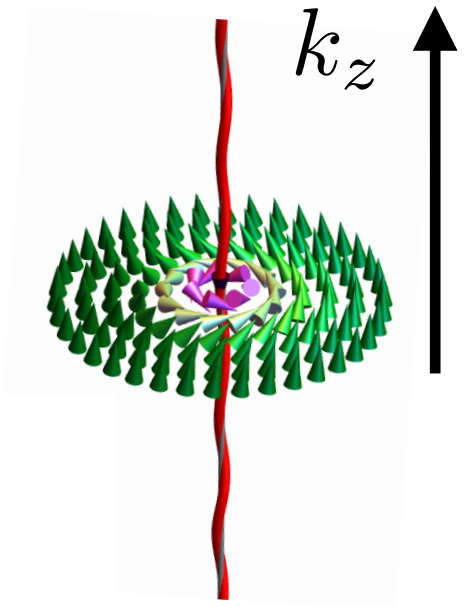


magnon-skyrmion bound states

spin wave dispersion in general non-reciprocal

$$\omega(k_z) \neq \omega(-k_z)$$

typical for a DMI system

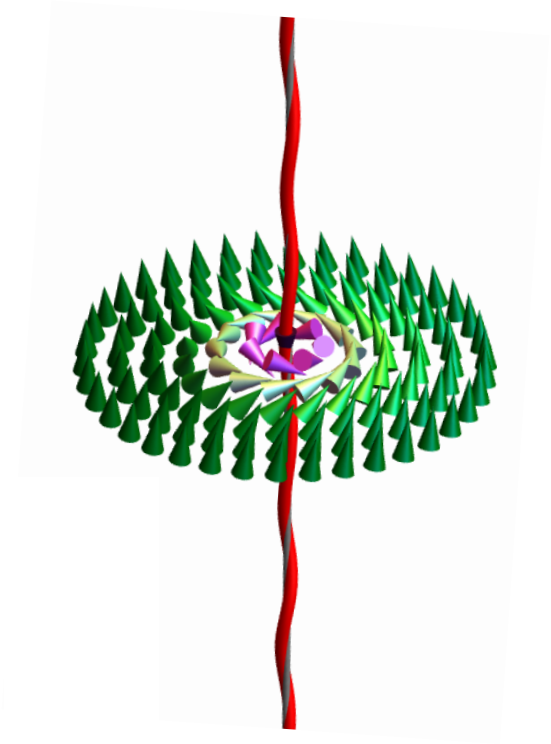


translational Goldstone mode: $\omega(k_z) \approx k_z^2 + a k_z^3$

small non-reciprocity in the low-energy limit

see also
Xing, Zhou, Braun, PR Applied (2020)
Lin, Zhu, Saxena, PRB (2019)

Beyond linear dynamics?



Effective low-energy, hydrodynamic theory?

What are the conserved quantities of a skyrmion string?

Topological conservation law

for a smooth unit vector $\hat{M}$ in 2+1 space-time:

continuity equation

$$\partial_t \rho_{\mathrm{top}} + \partial_i j_{\mathrm{top},i} = 0$$

no hedgehog defects

topological density

$$\rho_{\mathrm{top}} = \frac{1}{4\pi} \hat{M} (\partial_x \hat{M} \times \partial_y \hat{M})$$

topological current

$$j_{\mathrm{top},i} = \frac{1}{4\pi} \epsilon_{ij} \hat{M} (\partial_j \hat{M} \times \partial_t \hat{M})$$

with i = x,y

Kinetic conservation laws

topological current $$j_{\mathrm{top},i} = \frac{1}{4\pi}\epsilon_{ij}\hat{M}(\partial_j\hat{M}\times\partial_t\hat{M})$$

Landau-Lifshitz equation $$\partial_t\hat{M} = -\gamma\hat{M}\times\vec{B}_{\mathrm{eff}}$$

for a translationally invariant system

$$\boxed{j_{\mathrm{top},i} + \frac{\gamma}{4\pi M_s}\epsilon_{ij}\partial_k T_{kj} = 0}$$

anomalous conservation law for linear momentum

see e.g. O. Tchernyshyov, Ann. Phys (2015)

where T is the energy-momentum tensor using:

$$-M_s(\partial_i\hat{M})\vec{B}_{\mathrm{eff}} = \partial_k T_{ki} \quad \text{and} \quad \vec{B}_{\mathrm{eff}} = -\frac{1}{M_s}\frac{\delta F}{\delta\hat{M}}$$

Conservation law for linear momentum

anomalous conservation law for linear momentum $$j_{\mathrm{top},i} + \frac{\gamma}{4\pi M_s}\epsilon_{ij}\partial_k T_{kj} = 0$$

topological conservation law $$\partial_t \rho_{\mathrm{top}} + \partial_i j_{\mathrm{top},i} = 0$$

$$\Rightarrow \quad \boxed{\partial_t \rho_{\mathrm{top}} = \frac{\gamma}{4\pi M_s}\epsilon_{ij}\partial_i\partial_k T_{kj}}$$

Papanikolaou & Tomaras (1991)

not only the topological charge but also its first moment is conserved:

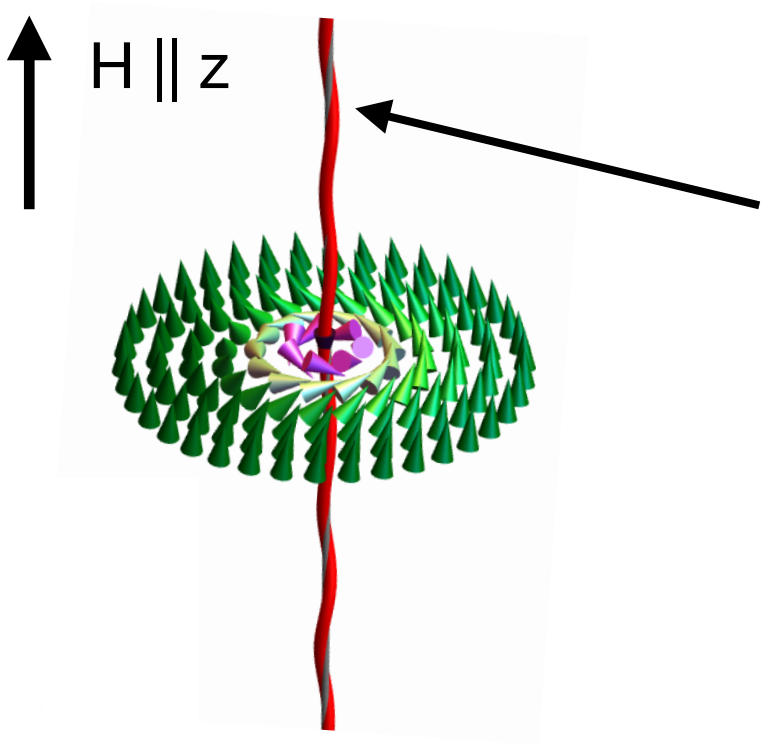
Linear momentum of the texture

$$\boxed{\vec{R} = \frac{\int d^2r\, \vec{r}\rho_{\mathrm{top}}}{\int d^2r\, \rho_{\mathrm{top}}}}$$

with $\dot{\vec{R}} = 0$

(cf. Thiele equation)

Linear momentum of the skyrmion string



linear momentum = first moment of topological charge

$$\vec{R}(z) = \frac{1}{W} \int dxdy \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \rho_{\mathrm{top}}(x,y)$$

with winding number W = -1

collective coordinate identifying the string!

Moreover: $\vec{R}^2$ related to total angular momentum

Papanikolaou & Tomaras (1991)
Schütte & MG (2015)

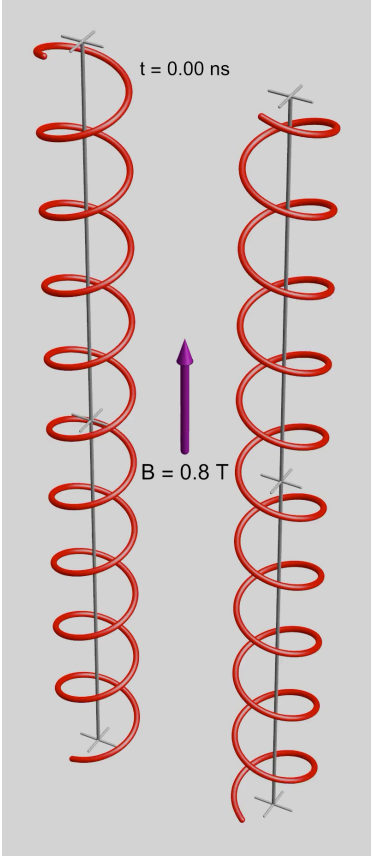
Linear low-energy dynamics

collective coordinate obeys Thiele equation: $$\vec{G} \times \partial_t \vec{R} = K_1 \partial_z^2 \vec{R}$$

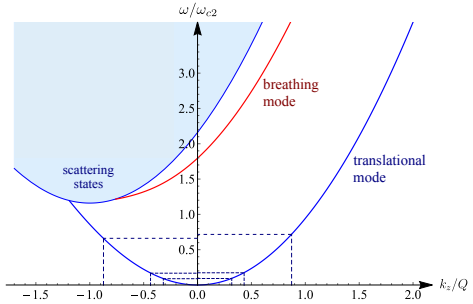
stiffness of the string

gyrocoupling vector $$\vec{G} = \frac{4\pi M_s W}{\gamma} \hat{z}$$

⇨ recovers quadratic Goldstone spectrum!

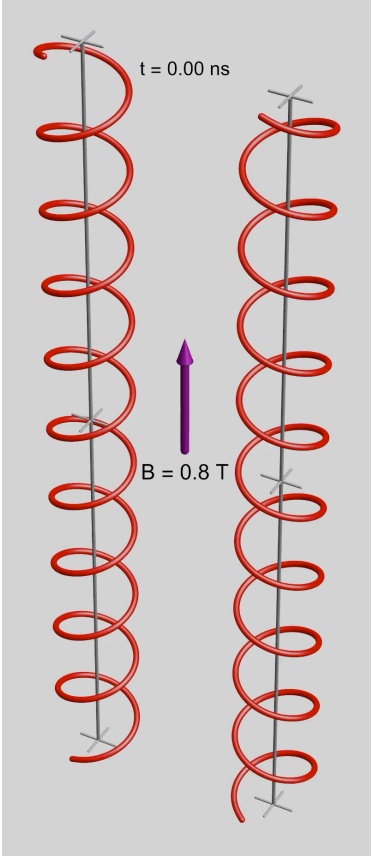


linear spin waves propagating along the string



Mapping to Schrödinger equation

introducing wave function $\psi \sim X + iY$ where $\vec{R} = \begin{pmatrix} X \\ Y \end{pmatrix}$



linear waves described by the Schrödinger Lagrangian in 1+1 space-time

$$\mathcal{L} = i\psi^* \partial_t \psi - \frac{1}{2}|\partial_z \psi|^2$$

Euler-Langrange equation reproduces Thiele equation

Corrections to the Schrödinger theory

corrections to the theory

$$\mathcal{L} = i\psi^* \partial_t \psi - \frac{1}{2}|\partial_z \psi|^2$$

strong constraints due to symmetries

translational invariance: dependence on derivatives only, $\partial_z \psi$ etc.

rotational invariance: U(1) symmetry

non-reciprocity correction $(\partial_z \psi^*)(\partial_z^2 \psi)$

mass correction $|\partial_t \psi|^2$

mixed correction $(\partial_t \psi^*)(\partial_z \psi)$

all irrelevant in the low-energy limit!

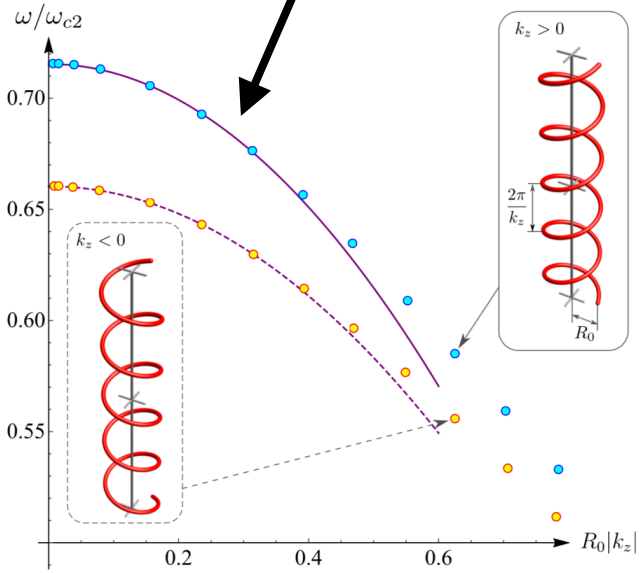
Non-linear Schrödinger theory

consider leading non-linear correction (after proper rescaling)

$$\mathcal{L} = i\psi^* \partial_t \psi - \frac{1}{2}|\partial_z \psi|^2 + \frac{1}{8}|\partial_z \psi|^4$$

attractive interaction!
stabilizes solitary waves!

energy decreases
with amplitude!

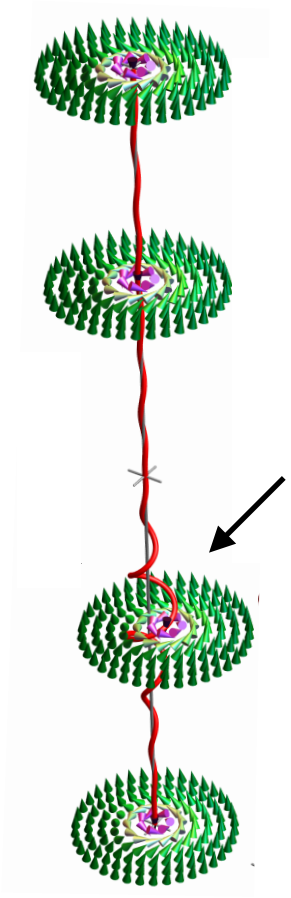


attractive interaction confirmed with micromagnetic simulations
of non-linear waves:

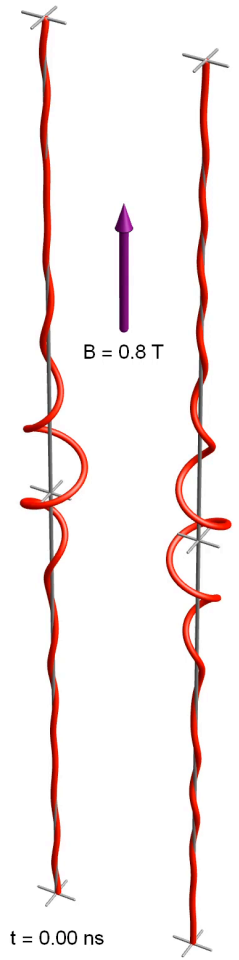
Solitary waves: numerics

existence confirmed by micromagnetic simulations:

skyrmion string



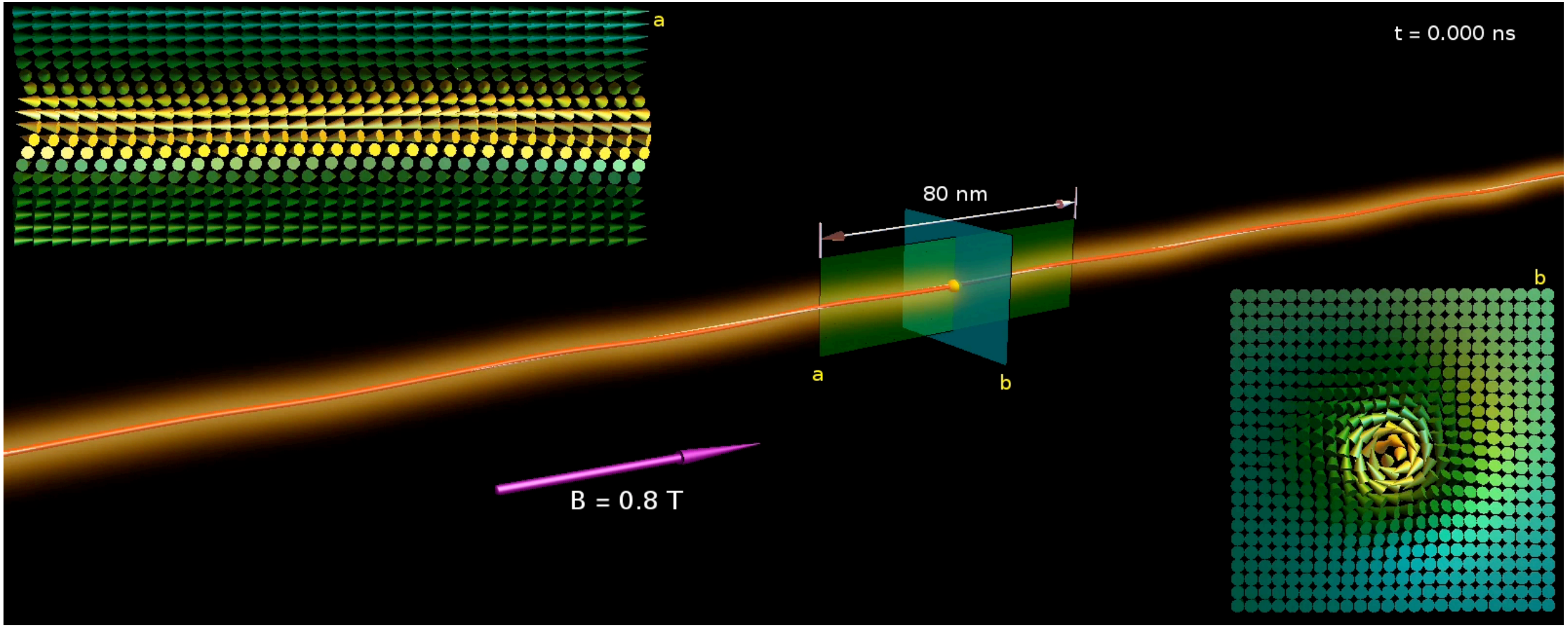
solitary wave



dynamics of solitary waves

Solitary waves: numerics

magnetization profile along the string



magnetization profile ⊥ to the string

Solitary waves: analytics

effective theory
for the skyrmion string:

$$\mathcal{L} = i\psi^* \partial_t \psi - \frac{1}{2}|\partial_z \psi|^2 + \frac{1}{8}|\partial_z \psi|^4$$

solution for solitary waves with the help of elementary conserved quantities:

1. density $\partial_\mu j_\mu = 0$ due to U(1) symmetry = total angular momentum conservation
2. energy-momentum tensor $\partial_\mu T_{\mu\nu} = 0$

Papanicolaou & Tomaras (1991)
Schütte & MG (2014)

Solitary waves: analytics

Ansatz for solitary waves: $\psi(z,t) = \Psi(z - vt)e^{iv(z-vt)-i(\omega-\frac{v^2}{2})t}$

Galilean transformation of wave function Ψ oscillating with frequency ω;
broken Galilean invariance: non-trivial dependence of Ψ on velocity v

wave function $\Psi(\xi) = A(\xi)e^{i\phi(\xi)}$ with $\xi = z - vt$ obeys

$$A(\xi) = \frac{1}{|v|\sqrt{1+\alpha}} f_1(\frac{\phi'}{v})$$

$$A'(\xi) = \frac{1}{\sqrt{1+\alpha}} f_2(\frac{\phi'}{v}, \alpha)$$

⇨

solution:
two-parameter family of solitary waves

velocity v
ratio $\alpha = -2\omega/v^2$ with $0 < \alpha < \frac{1}{8}$

with the functions

$$f_1(x) = \frac{\sqrt{x(2-x)}}{1+x} \qquad f_2(x,\alpha) = \frac{\sqrt{x(2\alpha - x + x^2)}}{\sqrt{1+x}}$$

Solitary waves: analytics

Ansatz for solitary waves: $\psi(z,t) = \Psi(z - vt)e^{iv(z-vt)-i(\omega-\frac{v^2}{2})t}$

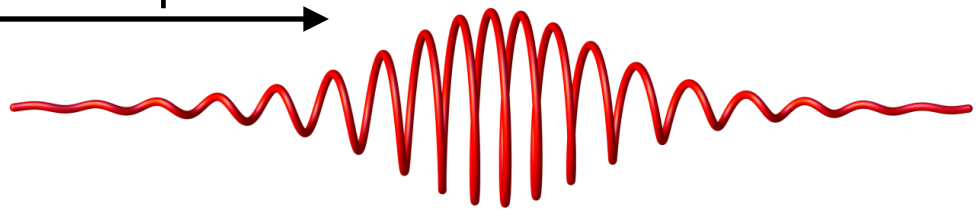
wave function $\Psi(\xi) = A(\xi)e^{i\phi(\xi)}$ with $\xi = z - vt$

Closed-form analytical solution in the limit $\alpha = -2\omega/v^2 \to 0$

$$A(\xi) \approx \frac{2}{|v|}\frac{\sqrt{\alpha}}{\cosh(\sqrt{\alpha}v\xi)}$$

$$\phi(\xi) \approx 2\sqrt{\alpha}\tanh(\sqrt{\alpha}v\xi)$$

envelope



ϕ' ∝ coil density

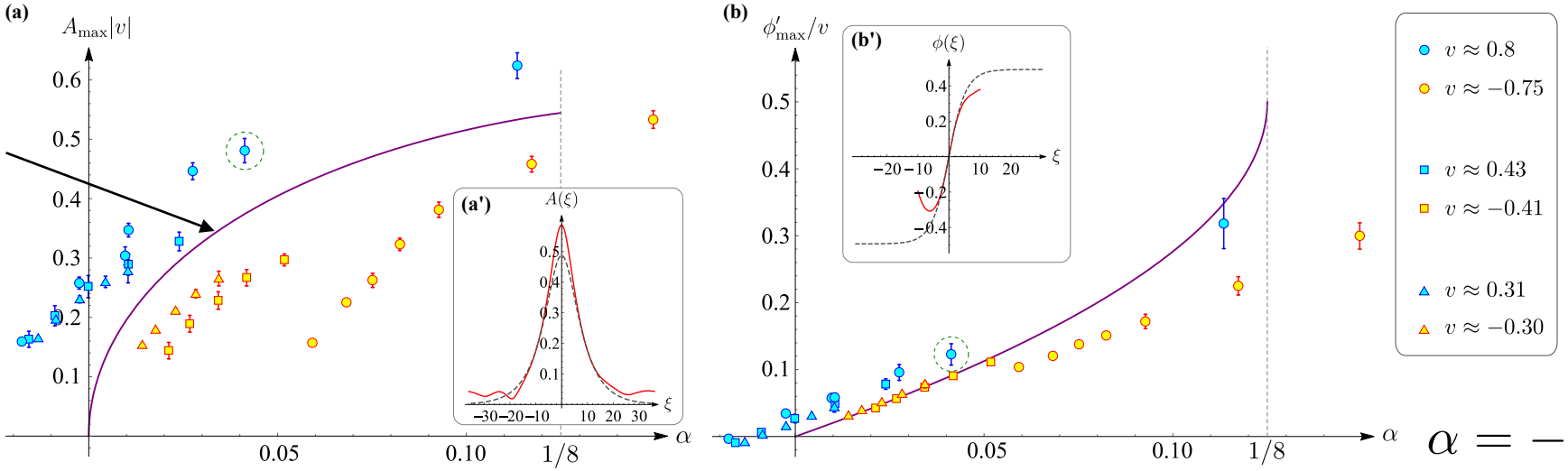
localized excitation in magnitude and phase

Comparison: numerics & analytics

amplitude

maximal derivative of phase

analytics



$$\alpha = -2\omega/v^2$$

finite size effects in the numerics: only relatively large velocities investigated

⇨ non-reciprocal corrections that diminish in the low-energy limit $v \to 0$

agreement between numerics and analytical predictions!

Comparison: vortex filament in (super-)fluids

wikipedia

vortices are singular defects

→ non-local Biot-Savart dynamics

→ non-analytic dispersion of Kelvin waves $\omega(k) \sim k^2 \log k$

enforcing locality: local induction approximation

$$\partial_t \vec{R} = \partial_s \vec{R} \times \partial_s^2 \vec{R}$$

Da Rios (1906)

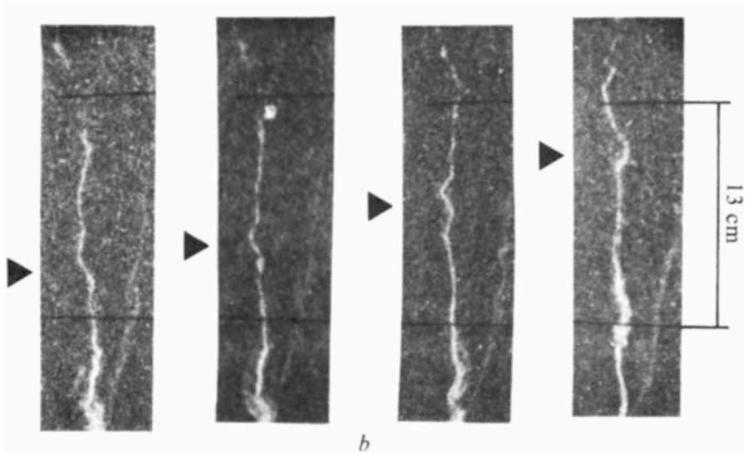
integrable!

with arclength s of the filament

⇨ solitons Hasimoto (1972)

experiment: rotating, turbulent flow

Hopfinger & Browand, Nature (1982)



Approximate integrability

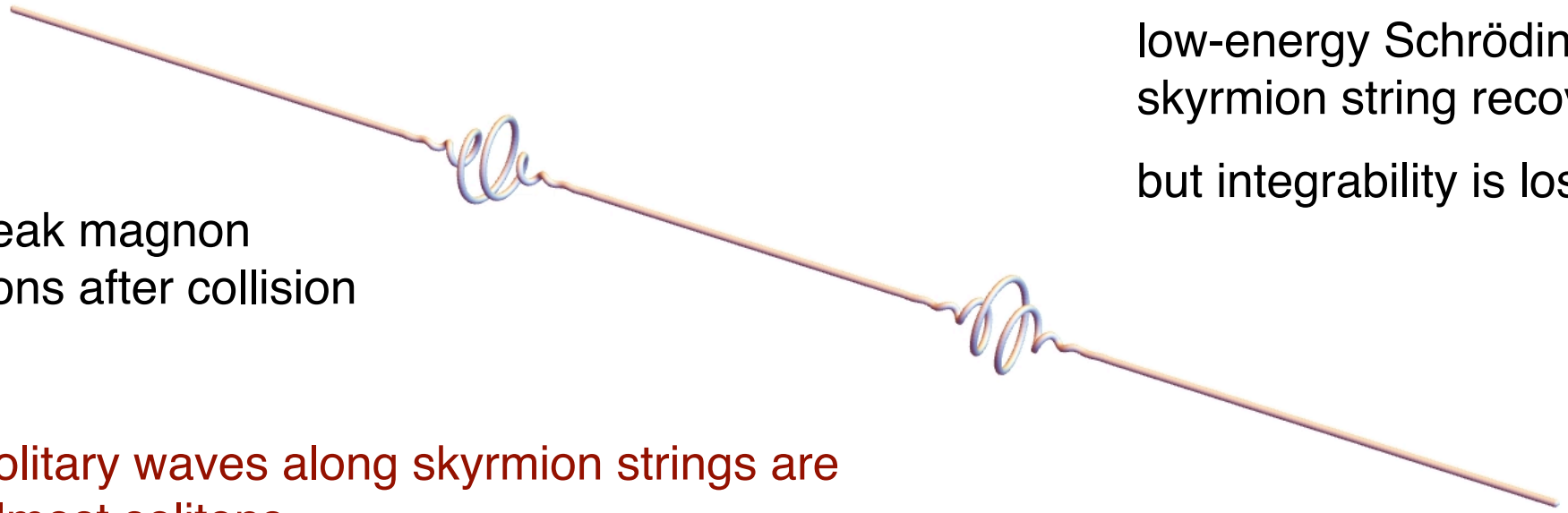
mapping integrable Da Rios equation to non-linear Schrödinger equation

⇨ Hamiltonian $H = \int dz \sqrt{1 + |\partial_z \psi|^2}$ length of the string (stretching neglected!)

for small intrinsic curvature: Taylor expansion $H \approx \int dz (1 + \frac{1}{2}|\partial_z \psi|^2 - \frac{1}{8}|\partial_z \psi|^4)$

low-energy Schrödinger equation for skyrmion string recovered!

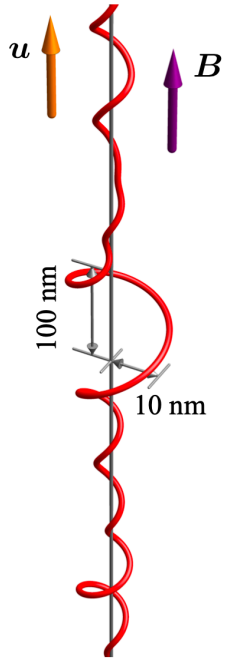
but integrability is lost!



only weak magnon emissions after collision

⇨ solitary waves along skyrmion strings are almost solitons

Current-driven instability of skyrmion strings



Spin-transfer torque on the skyrmion string

Landau-Lifshitz equation in the presence of Gilbert damping & Zhang-Li spin-transfer torque

$$(\partial_t + \vec{v}_s \nabla)\hat{M} = -\gamma \hat{M} \times \vec{B}_{\text{eff}} + \hat{M} \times (\alpha \partial_t + \beta \vec{v}_s \nabla)\hat{M}$$

adiabatic spin-transfer torque

Gilbert damping

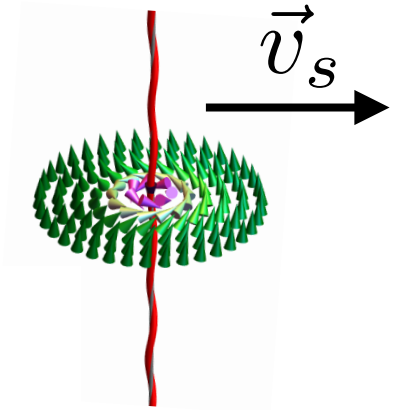
non-adiabatic spin-transfer torque

for a current perpendicular to the skyrmion string:

well-known Thiele equation:

$$\vec{G} \times (\dot{\vec{R}} - \vec{v}_s) = \mathcal{D}(\dot{\vec{R}} - \frac{\beta}{\alpha}\vec{v}_s)$$

dissipative Tensor



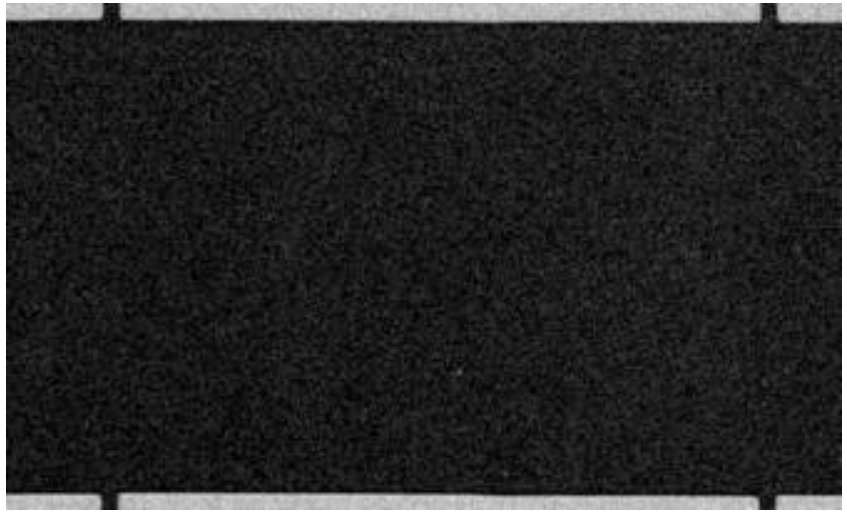
Skyrmion Hall effect

Thiele equation

$$\vec{G} \times (\dot{\vec{R}} - \vec{v}_s) = \mathcal{D}(\dot{\vec{R}} - \frac{\beta}{\alpha}\vec{v}_s)$$

for a topological non-trivial magnetic texture with G > 0: Skyrmion Hall effect

$\vec{v}_s$



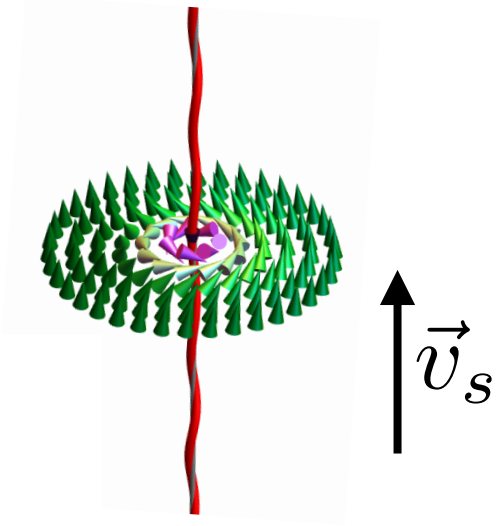
$\dot{\vec{R}} \nparallel \vec{v}_s$

transversal velocity component

magnetic trilayer

Jiang et al. Nat. Phys. (2017)

Current longitudinal to skyrmion string



effective Thiele equation:

$$\vec{G} \times (\partial_t \vec{R} - v_s \partial_z \vec{R}) = \mathcal{D}(\partial_t \vec{R} - \frac{\beta}{\alpha} v_s \partial_z \vec{R}) + K_1 \partial_z^2 \vec{R}$$

Zhang-Li torques generate derivatives!

string stiffness

current-current interaction!

cf. $\hat{M} = \hat{M}(\vec{r} - \vec{R}(z,t))$

immediately destabilizes the spin wave spectrum!

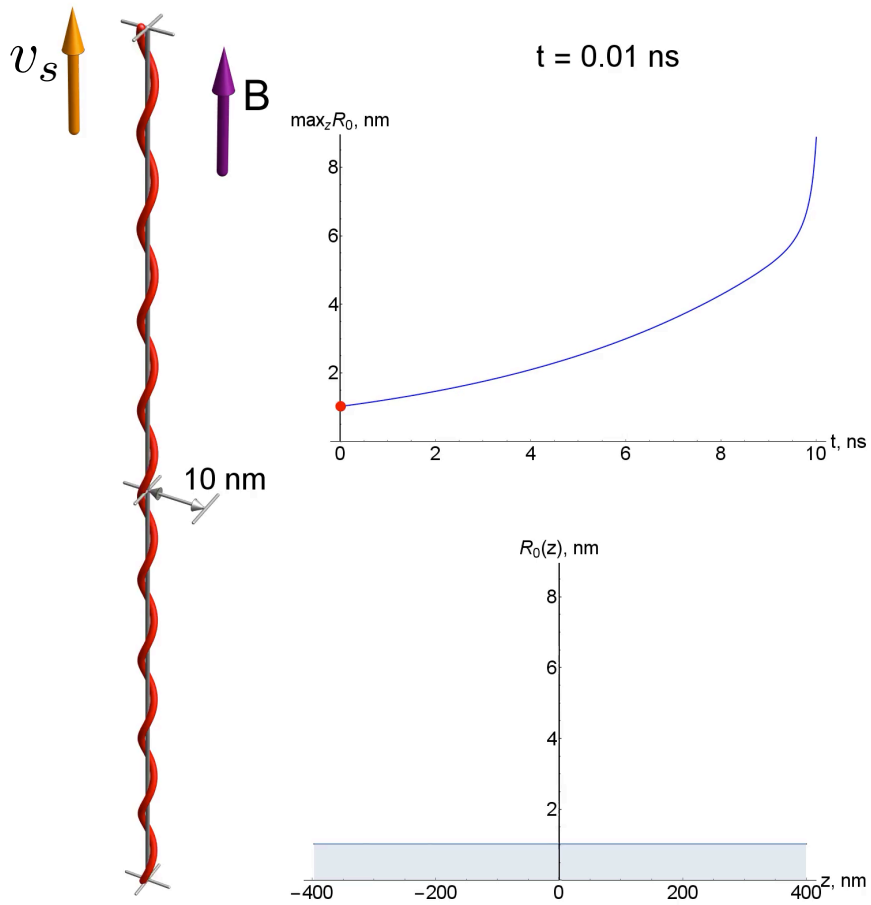
pumping of the Goldstone mode by spin currents!

with disorder: finite threshold current!

Current-driven instability of the skyrmion string

micromagnetic simulations:

similar instability for the skyrmion-string lattice! (not shown)



skyrmion-string unstable for longitudinal spin currents!

Acknowledgments:

theory:

L. Köhler
(KIT)

Jan Masell
(Tokyo)

V. Kravchuk
(KIT)

S. Okumura
(Tokyo)

experimental groups:

C. Pfleiderer
(TUM)

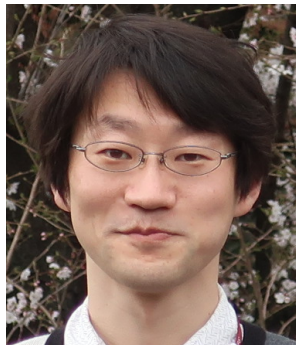
Peter Böni
(TUM)

M. Janoschek
(PSI)

D. Meier
(Trondheim)

D. Grundler
(Lausanne)

C. Back
(TUM)

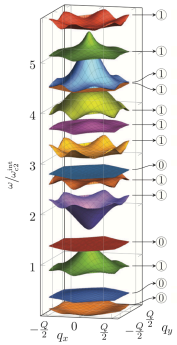


S. Seki
(Tokyo)

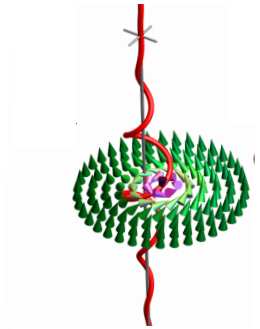
C. Panagopolous
(Singapore)

Summary

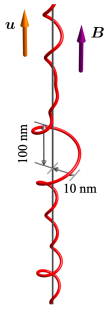
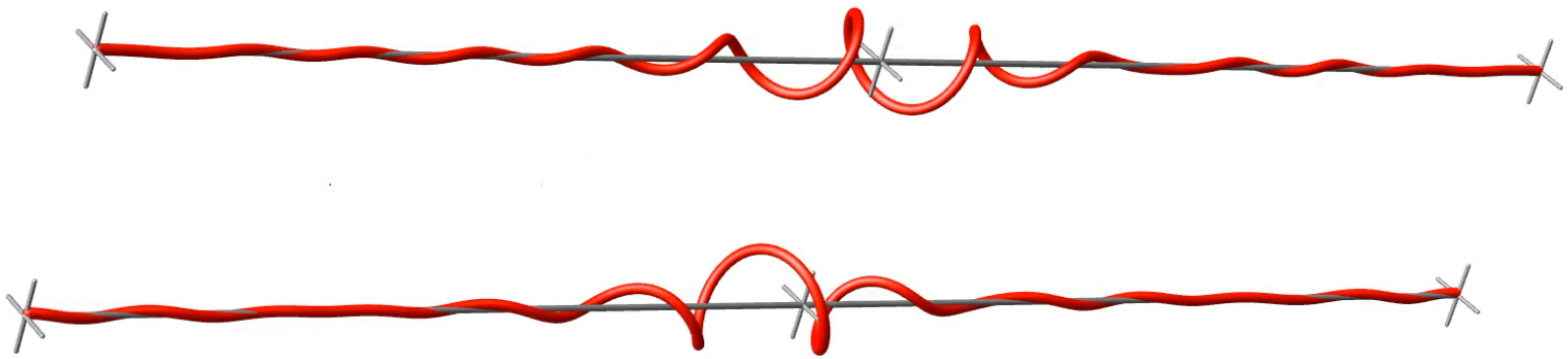
- Linear spin-wave dynamics of skyrmion strings: many experiments with quantitative agreement



- Low-energy theory of a skyrmion string & solitary waves



- Current-driven instability of skyrmion strings



Thank you for your attention!