



The Thermal Chiral Anomaly in ideal field-induced Weyl semimetals.

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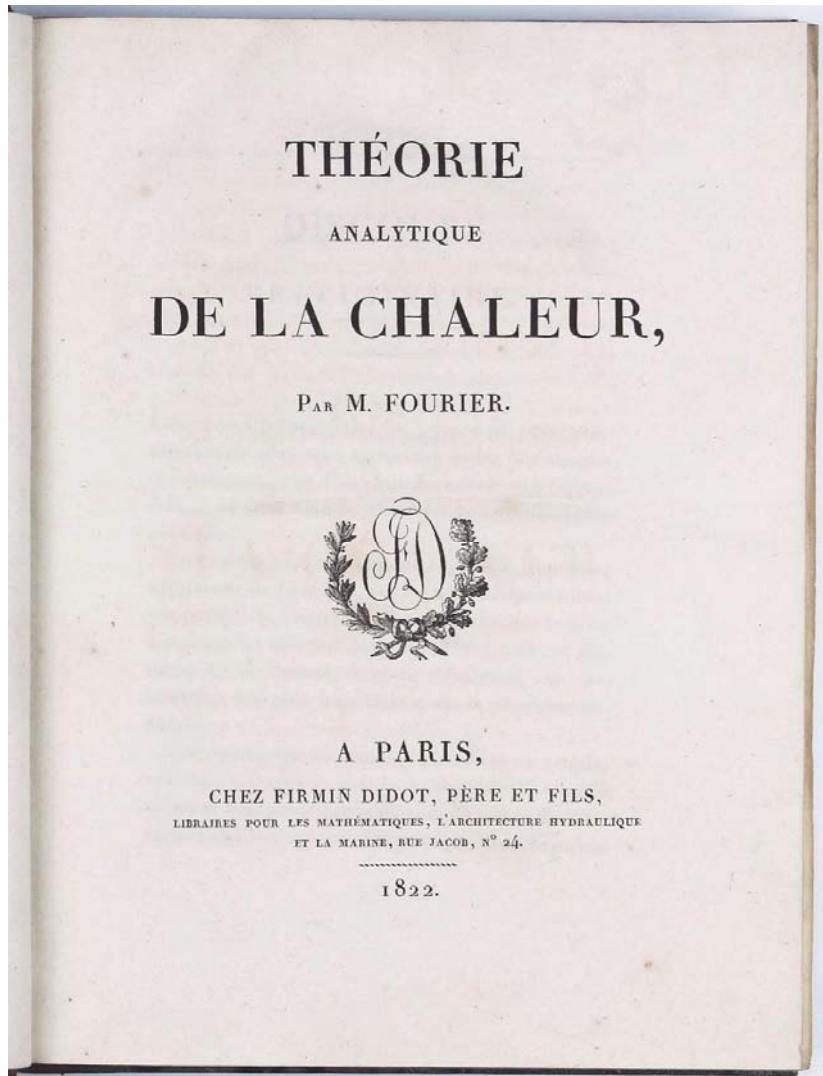


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1822: Fourier's Thermal Conductivity



“La chaleur pénètre, comme la gravité, toutes les parties de l'univers.”

“Heat, like gravity, permeates all parts of the universe.”

+ Thermal transport can be measured on quasi-particles that have neither charge nor spin

- Heat goes everywhere => measurements difficult.

1. Introduction:

- Weyl semimetals: chiral anomaly and thermal conductivity
- Experimental difficulties
- Bi-Sb semiconductors alloys and TI's
- In the ultra-quantum limit: field-induced Weyls

2. Thermal conductivity

- Experimental evidence
- Robustness to phonons and defects
- Decay only via inter-Weyl point scattering
- The Wiedemann-Franz law

Weyl semimetals: 3-dimensional topological solids

$$E(\vec{k}) = \pm v_F \hbar \vec{k}$$

$$\vec{k} = (k_x, k_y, k_z)$$

Fermi level at the Dirac points:

- At $T > 0$, same amount of electrons and holes
- We work at temperatures

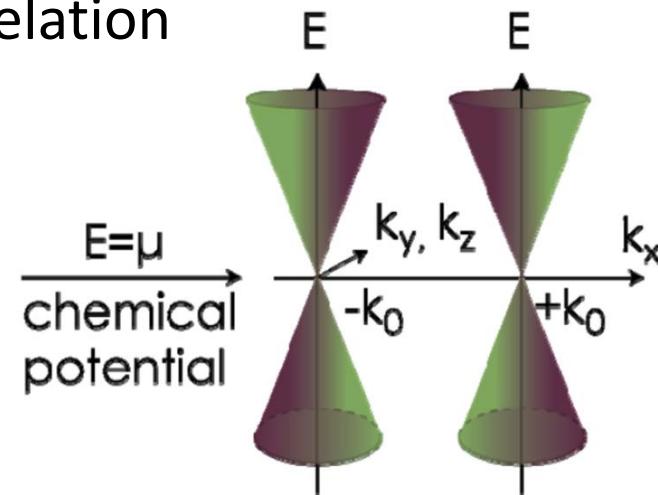
$$\mu \ll k_B T < E_{BW}$$

- Mass is not defined
- Electron has **velocity** v_F
- Electron has **chirality**
 $\chi = 1$ at \vec{k}_0 , $\chi = -1$ at $-\vec{k}_0$
- Berry phase
 $\vec{\Omega}_{\pm}(\vec{k}) = \pm \frac{\vec{k}}{k^3}$
- Equation of motion

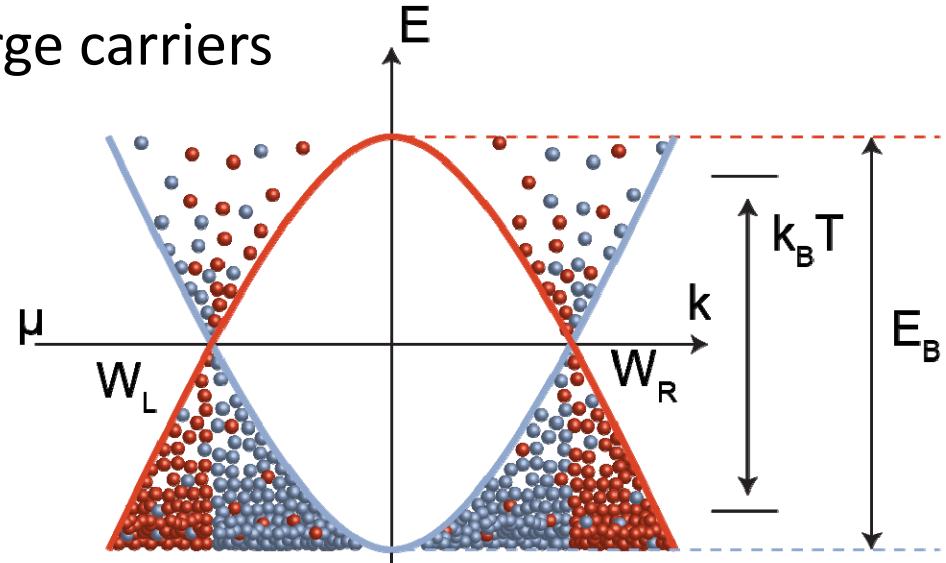
$$\frac{\partial \vec{r}}{\partial t} = \nabla_{\vec{k}} \mathcal{E} + \frac{\partial \vec{k}}{\partial t} \times \vec{\Omega}$$

Anomalous velocity

Dispersion relation

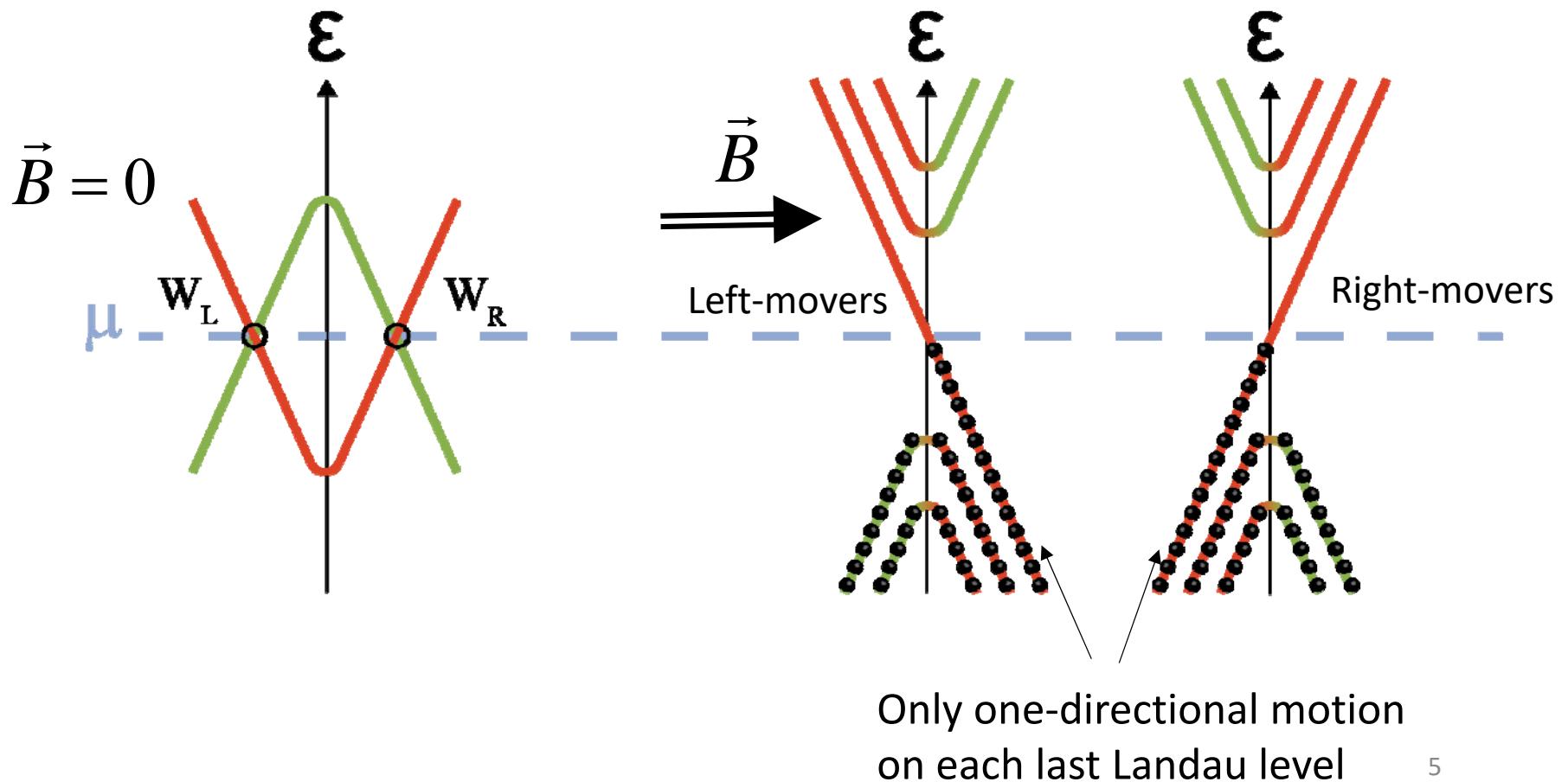


Charge carriers

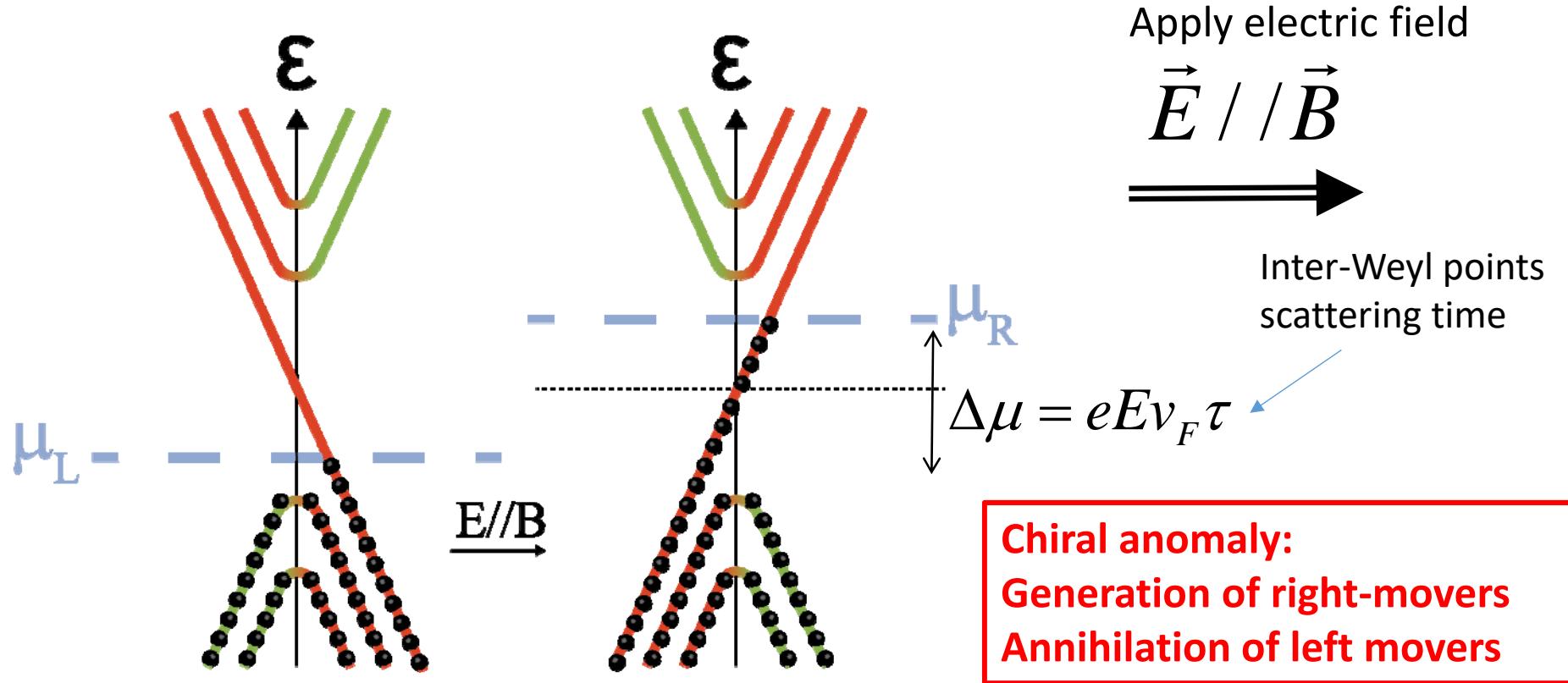


Landau levels, extreme-quantum limit (EQL)

- Apply quantizing magnetic field => Landau levels
- Extreme quantum limit: only last Landau level crosses chemical potential
- Effect of Ω : only one velocity allowed for the last Landau level
- => carriers separate into left-movers and right-movers



The chiral anomaly: apply electric field



Anomalous Current $j_A \propto DOS_{2D}ev_F\tau E \propto \vec{E} \cdot \vec{B}$

Anomalous conductivity: $\sigma_A = \frac{e^2 v_F \tau}{2\pi h \ell_B^2}$

Negative magnetoresistance

Effects of the anomalous velocity on transport

Solve Boltzmann transport equations with both $\mathbf{E} \parallel \mathbf{B}$ & $\nabla_r T \parallel \mathbf{B}$

=> Change in carrier concentration between left and right movers:

$$\delta n_\chi = \frac{\chi e^2 \tau}{4\pi^2 \hbar^2} \left[\vec{B} \cdot \vec{E} \right] C_0 + \frac{\chi e \tau}{4\pi^2 \hbar^2} \left[\vec{B} \cdot \frac{-\nabla_r T}{T} \right] C_1$$

$$C_n = \int_{-\infty}^{\infty} (\varepsilon - \mu)^n \left(-\frac{\partial f_0}{\partial \varepsilon} \right) d\varepsilon; \quad n \in \{0, 1, 2, \dots\}; \quad \chi = \pm 1$$

In an ideal Weyl semimetal $\mu = 0 \Rightarrow C_0 = 1; C_1 = 0$

$$\delta n_\chi = \frac{\chi e^2 \tau}{4\pi^2 \hbar^2} \left[\vec{B} \cdot \vec{E} \right]$$

Anomalous electrical conductivity, negative magnetoresistance

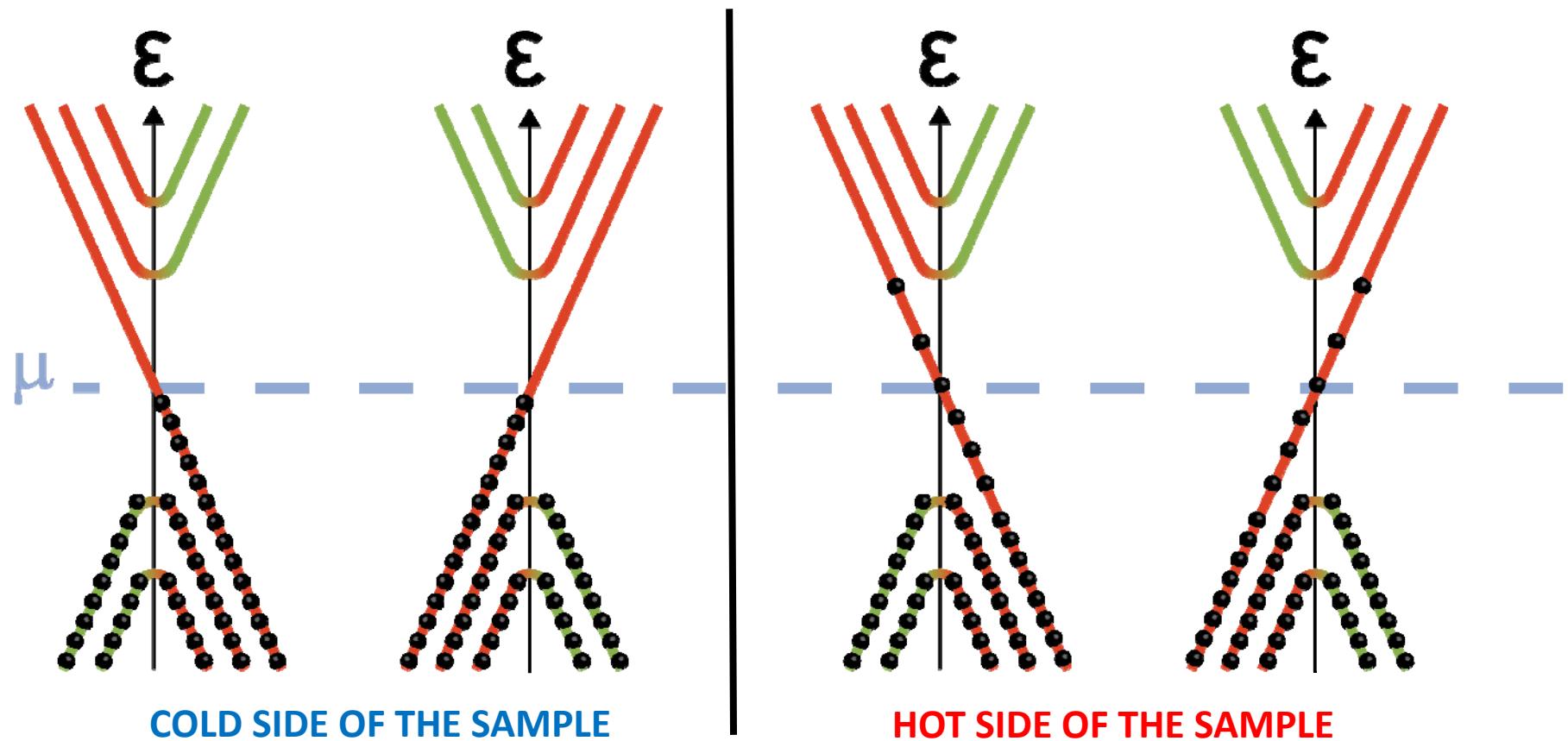
$$\sigma_A = N_w \frac{e^2 v \tau}{4\pi \hbar \ell_B^2} = N_w \frac{e^3 v \tau}{4\pi \hbar^2} B_z$$

N_w = number of degenerate pairs of Weyl points

N. Trivedi and W. Zhang; Spivak, N. Z. and Andreev, A. V., *Phys. Rev. B* **93**, 085107 (2016)]

Apply a temperature gradient $\nabla T \parallel B$

Thermal gradient BY ITSELF does NOT produce an imbalance between left movers and right movers.



The chiral anomaly requires an electric field: anomalous current $j_A \propto \vec{E} \cdot \vec{B}$

Effects of the anomalous velocity on thermal transport

Solve Boltzmann transport equations with both $\mathbf{E} \parallel \mathbf{B}$ & $\nabla_r T \parallel \mathbf{B}$

=> Change in carrier energy between left and right movers:

$$\delta\epsilon_\chi = \frac{\chi e^2 \tau}{4\pi^2 \hbar^2} \left[\vec{B} \cdot \vec{E} \right] (\mu C_0 + C_1) + \frac{\chi e \tau}{4\pi^2 \hbar^2} \left[\vec{B} \cdot \frac{-\nabla_r T}{T} \right] (\mu C_1 + C_2)$$

In an ideal Weyl semimetal: $\mu = 0 \Rightarrow C_0 = 1; C_1 = 0; C_2 = \frac{\pi^2}{3} k_B T$

If $\mathbf{E} = 0$ & $\nabla_r T \parallel \mathbf{B}$

$$\boxed{\delta\epsilon_\chi = -\frac{\pi^2}{3} k_B \frac{\chi e \tau}{4\pi^2 \hbar^2} \left[\vec{B} \cdot \nabla_r T \right]}$$

$$\delta n_\chi = 0$$

Anomalous thermal conductivity, positive magneto-thermal conductivity

$$\kappa_A = N_w \frac{\pi^2}{3} \frac{v \tau k_B^2 T}{4\pi \hbar \ell_B^2} = N_w \frac{\pi^2}{3} \frac{e v \tau k_B^2 T}{4\pi \hbar^2} B_z$$

Summary Ideal Weyl semimetal ($\mu = 0$)

1. Electric field only: $\delta n_\chi = \frac{\chi e^2 \tau}{4\pi^2 \hbar^2} [\vec{B} \cdot \vec{E}]$

$$\delta \epsilon_\chi = 0$$

⇒ **Change in carrier density** between left and right movers

⇒ **No change in carrier energy**

2. Thermal gradient only: $\delta \epsilon_\chi = -\frac{\pi^2}{3} k_B \frac{\chi e \tau}{4\pi^2 \hbar^2} [\vec{B} \cdot \nabla_r T]$

$$\delta n_\chi = 0$$

⇒ **Change in carrier energy** between left and right movers

⇒ **No change in carrier density**

3. Ratio of the two: $\kappa_A = L T \sigma_A \quad L = L_0 = \frac{\pi^2}{3} \left(\frac{k_B}{e} \right)^2$

⇒ **The Wiedemann-Franz law holds with the free electron Lorenz ratio**

if the inelastic scattering rate is dominated by the helicity and $\mu=0$ (ideal Weyl)

Structure of this talk

1. Introduction:

- Weyl semimetals: chiral anomaly and thermal conductivity
- **Experimental difficulties**
- Bi-Sb semiconductors alloys and TI's
- In the ultra-quantum limit: field-induced Weyls

2. Thermal conductivity

- Experimental evidence
- Robustness to phonons and defects
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Problem #1 with magnetoresistance: current jetting

High-mobility materials:

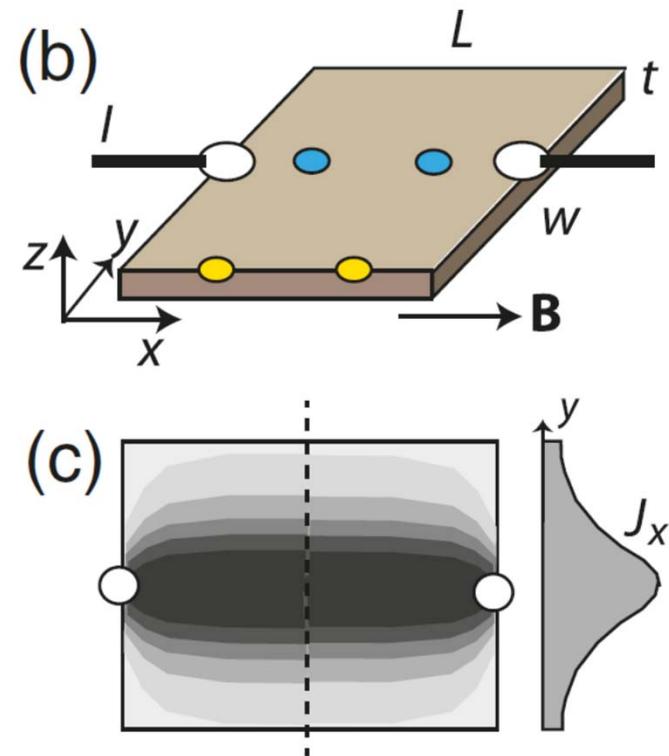
4-probe longitudinal magnetoresistance is OFTEN negative due to the distortions of the current lines in the sample.

Purely geometrical effect

Current bundles up in the middle of the sample

Voltage wires loose contact to the region of the sample with the current

=> looks like negative longitudinal MR, but it isn't.



Problem #2 : Geometrical magnetoresistance

In high-mobility nanoscale materials and transverse magnetic fields: current lines follow the Hall angle.

High field: diameter of the helix < sample size **(b)**

- => surface scattering is suppressed
- => resistivity goes down

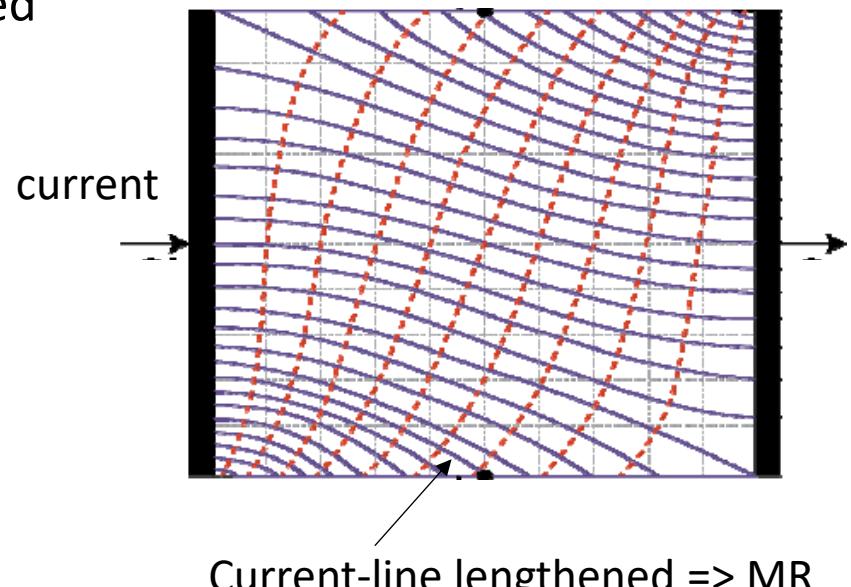
Purely geometrical effect

Apparent positive transverse MR,
But is a side-effect of very slight
Field misalignments

Appears even in 2-wire measurements.

Order of magnitude: $R(B) / R_0 = 1 + \mu^2 B^2$

In our samples, $\mu \sim 10^6 \text{ cm}^2/\text{Vs}$ => 1 degree misalignment 1 Tesla gives 200% MR



Thermal conductivity measurements

Experimentally much easier than magnetoresistance measurements:

1. No electrical contacts => no circulating currents
=> no Lorentz force of the current lines
2. Thermal conductivity has an electronic contribution and a lattice contribution.
The physics we seek to measure arises from the electronic contribution
The lattice thermal conductivity little affect by magnetic field
=> The lattice conductivity redistributes the heat flux lines
3. Summary: thermal measurements much less sensitive to problems.

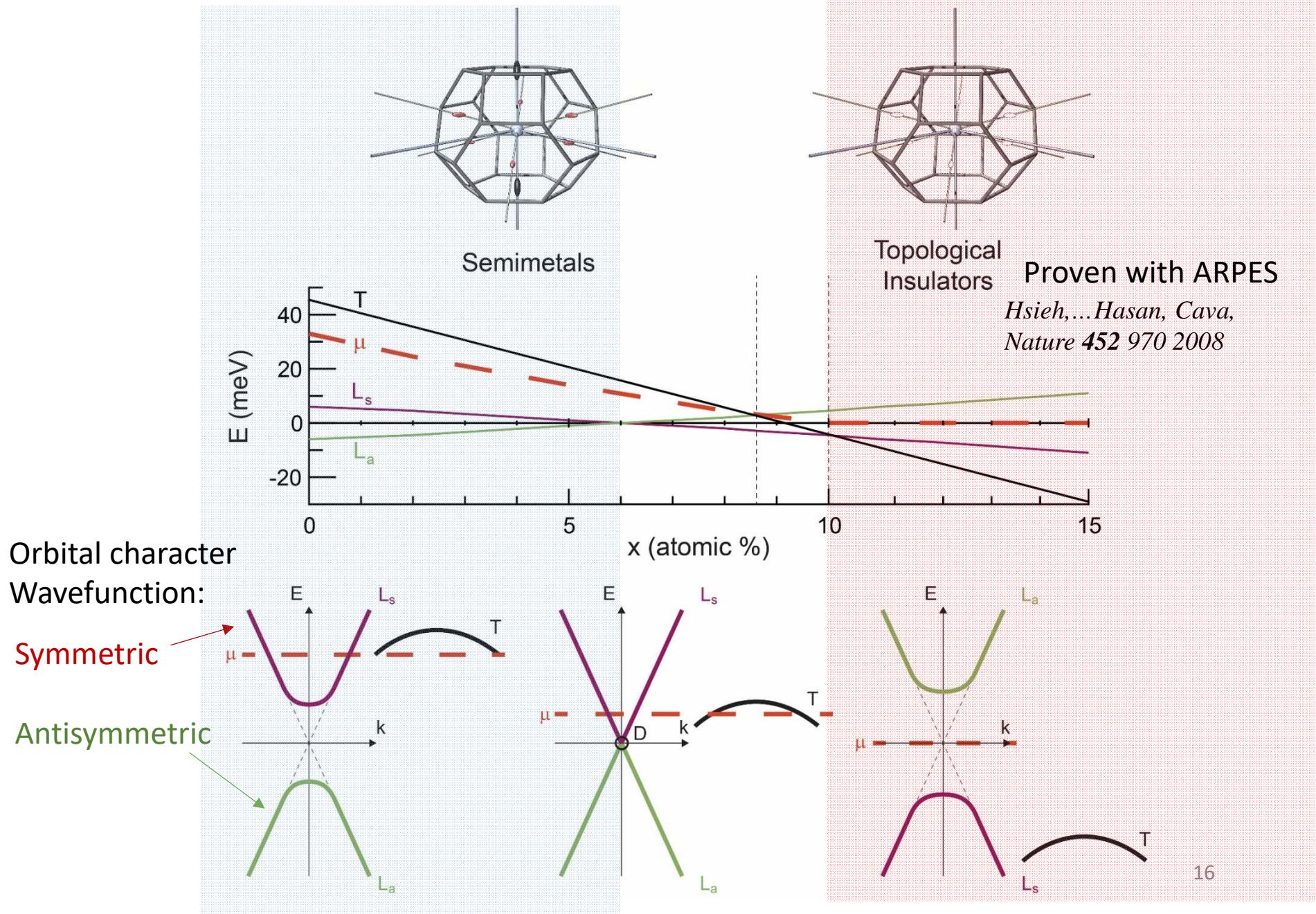
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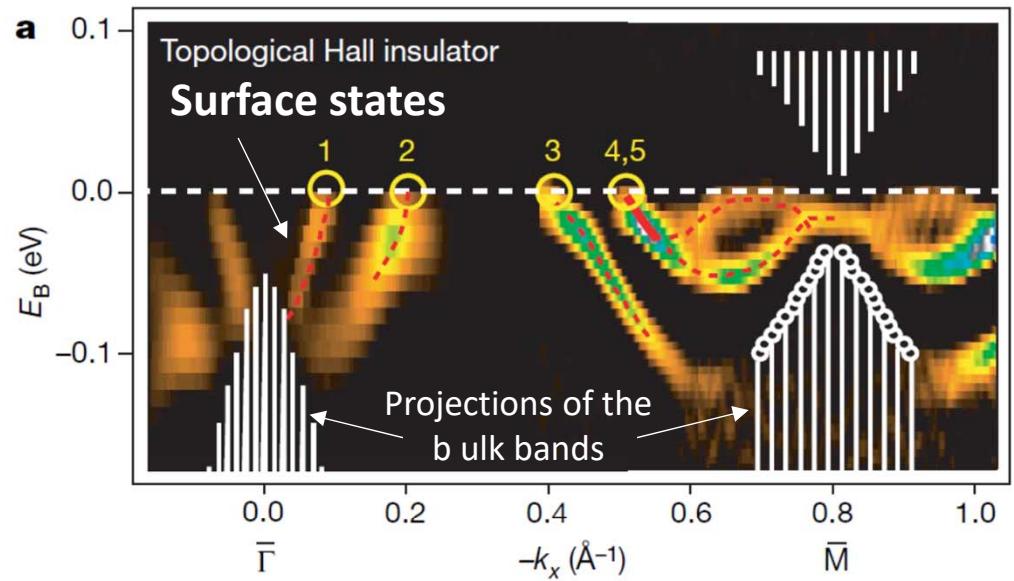
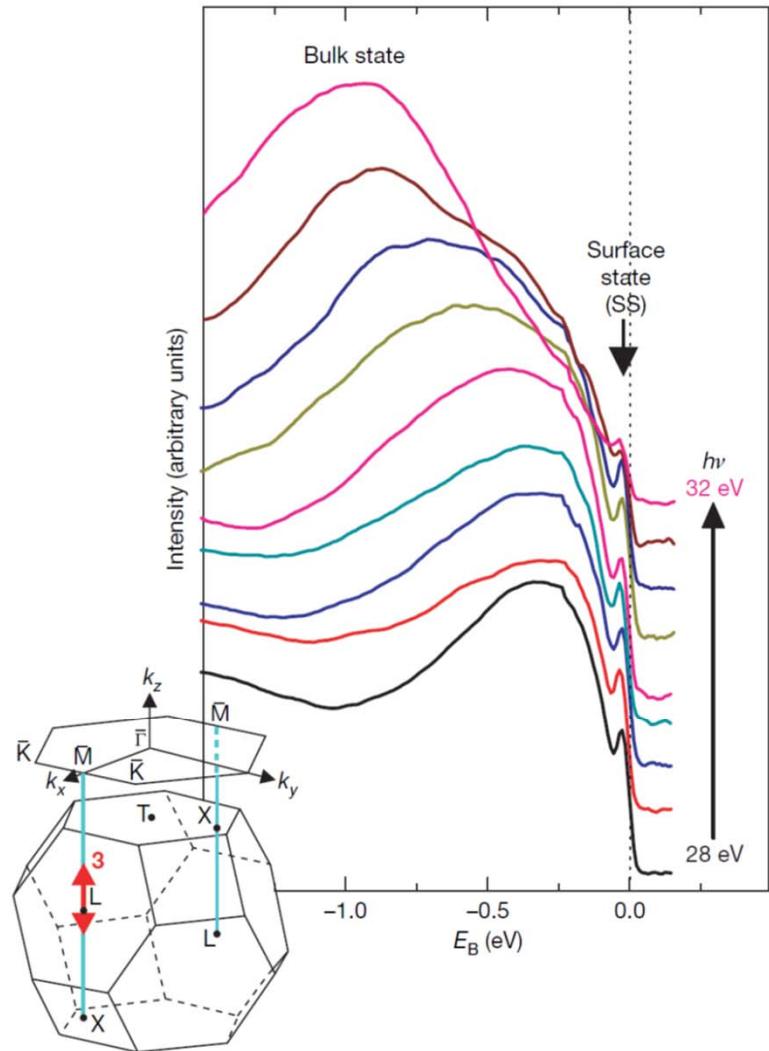
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The $\text{Bi}_{1-x}\text{Sb}_x$ semiconductors and semimetals



ARPES Surface and bulk states in $TlBi_{88}Sb_{12}$



Last Landau level CLOSES the gap at L-point

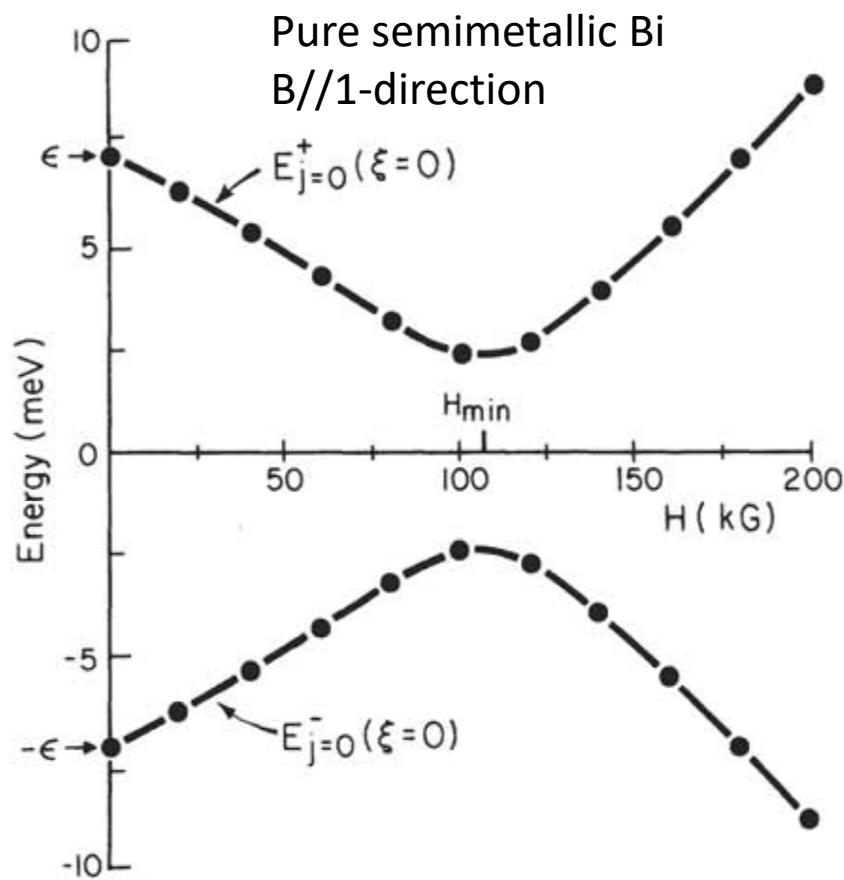


FIG. 2. Magnetic field dependence of the $j = 0$ energy levels at $\xi = 0$ ($k_z = 0$) for the light binary electrons of Bi.

Magnetic field dependence of the Landau level energies at $k_z = 0$

$$E(k_z = 0) = (n + \frac{1}{2})\hbar\omega_C + sg\mu_B B$$

The Landé factors of $\text{Bi}_{1-x}\text{Sb}_x$ are enormous, diamagnetic and anisotropic.

Pure semimetal Bi: **bands closing observed experimentally** with magneto-optics

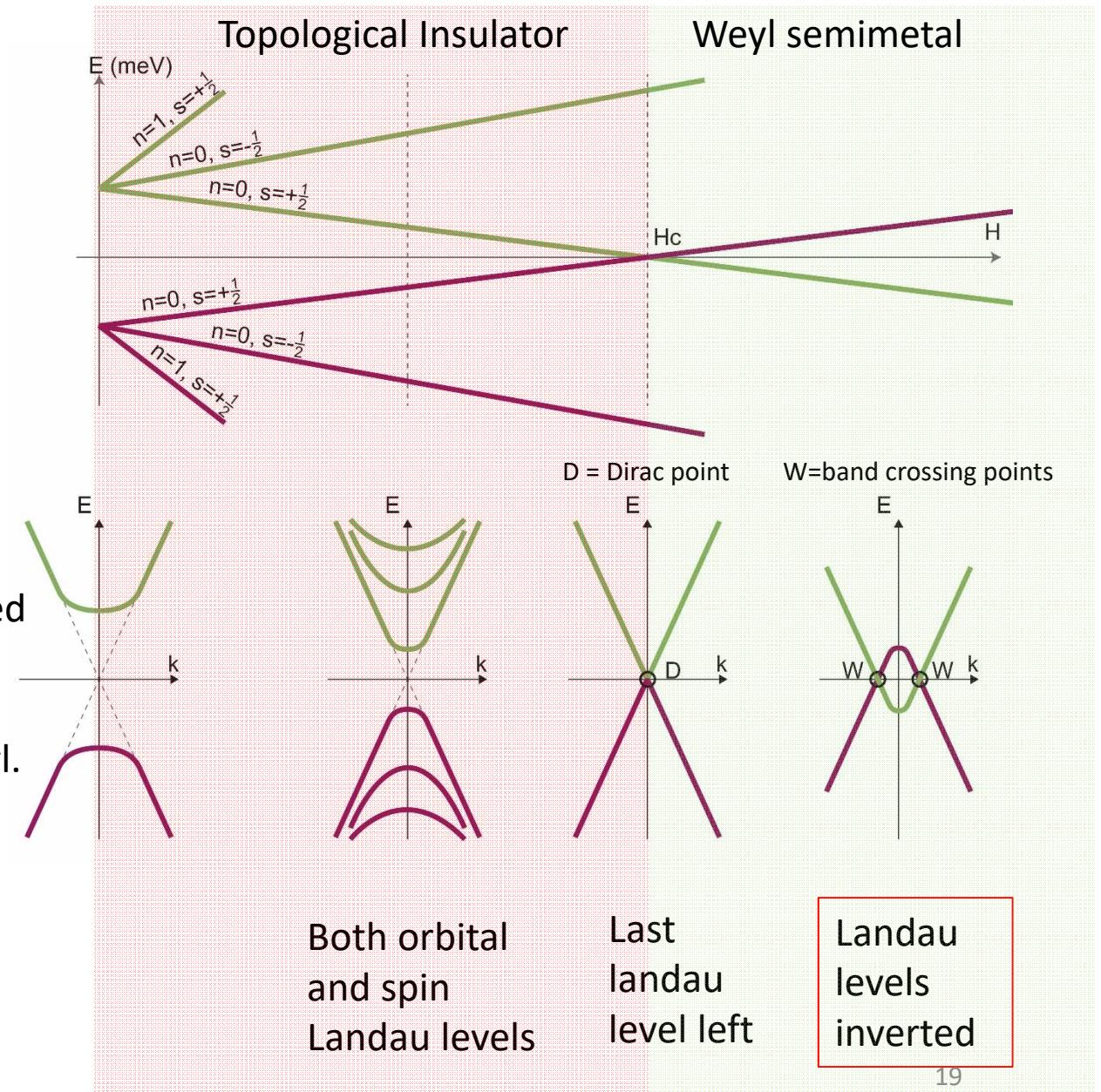
Same effect is calculated for $\text{Bi}_{1-x}\text{Sb}_x$ alloys

Effect small in binary (1) field, much larger in trigonal (3) field => crossing near 1 T.

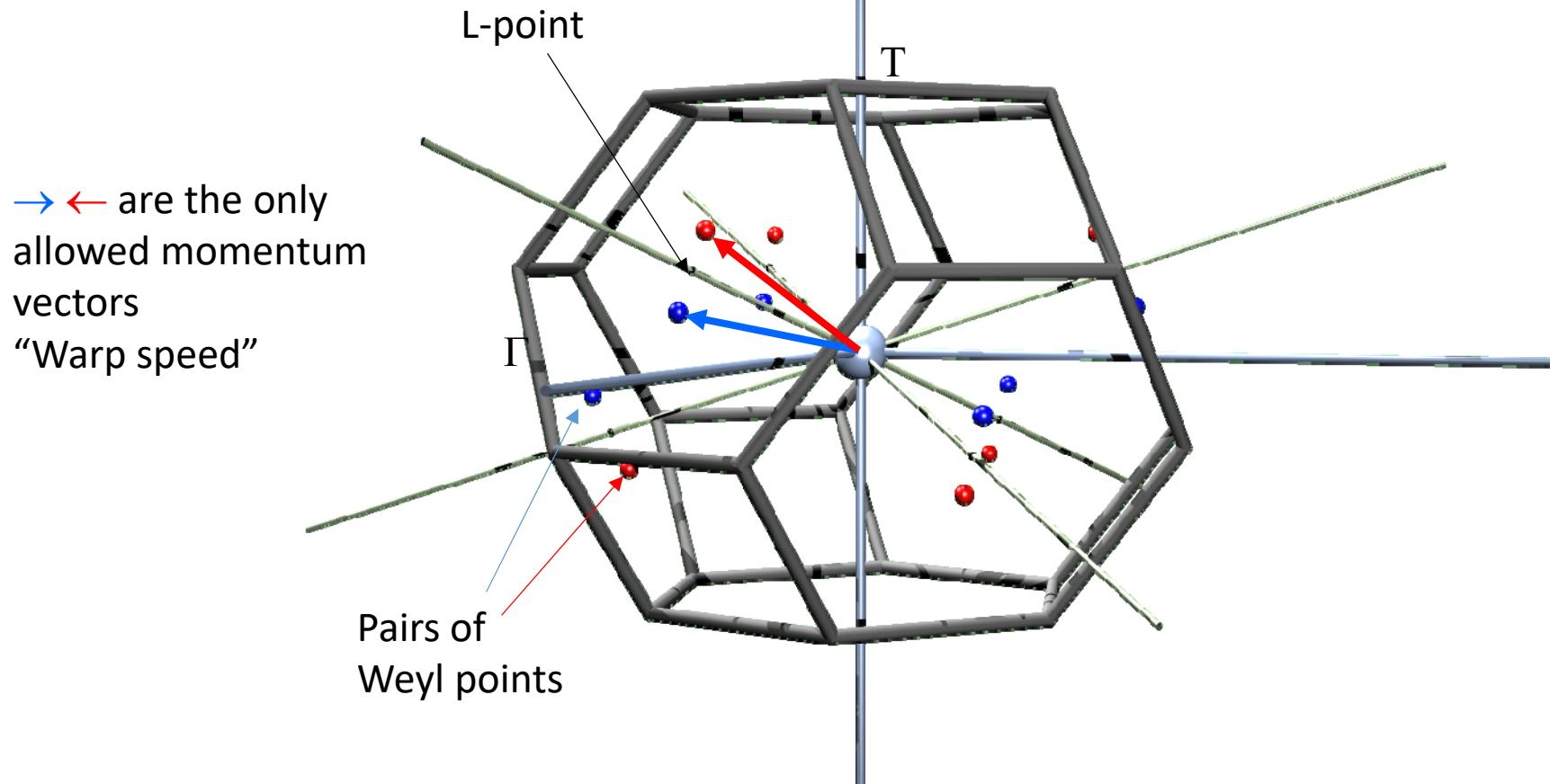
Ultraquantum TI's become field-driven Weyls

The Landé factors of $\text{Bi}_{1-x}\text{Sb}_x$ are enormous, diamagnetic and anisotropic.

- ⇒ The Landau levels can be made to close with field
- ⇒ The bands can be made to invert a second time
- ⇒ These are single spin-polarized Landau levels
- ⇒ Second inversion gives a Weyl.



Weyl points in trigonal magnetic field



- Painstakingly identified, calculation in supplemental slides
- No trivial pockets to the Fermi surface.
- 3 degenerate pairs of Weyl points per unit cell
- If we can pin the electrochemical potential to the Weyl points we have the ideal Weyl

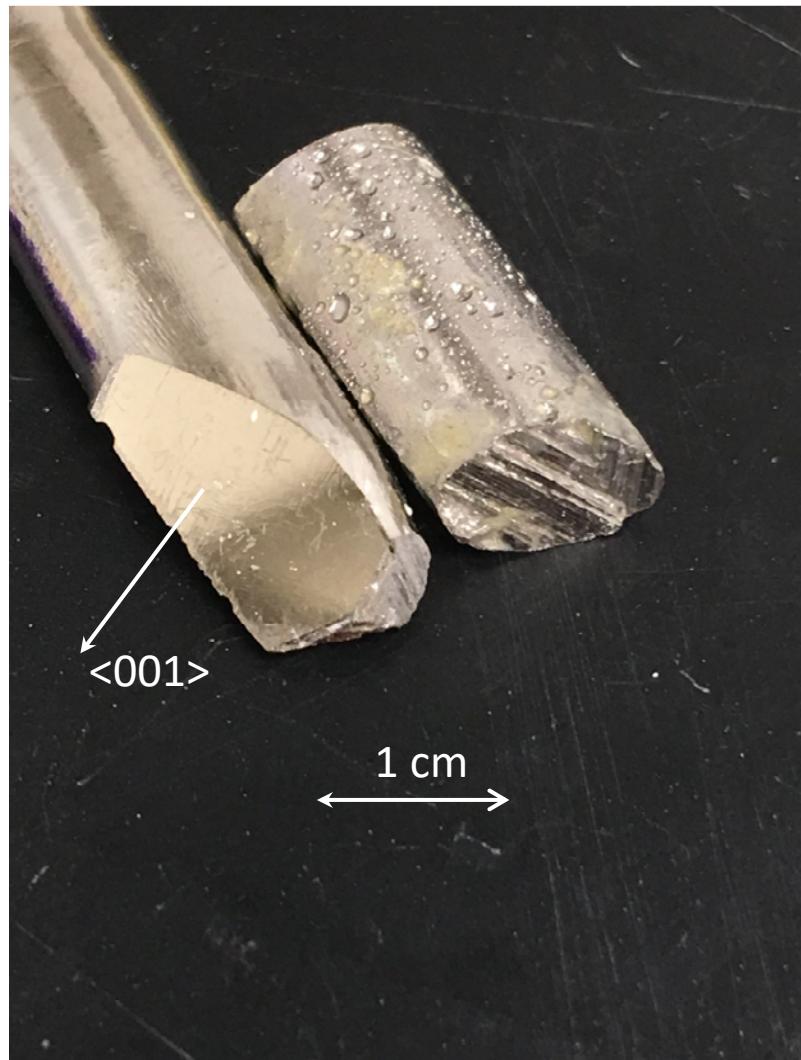
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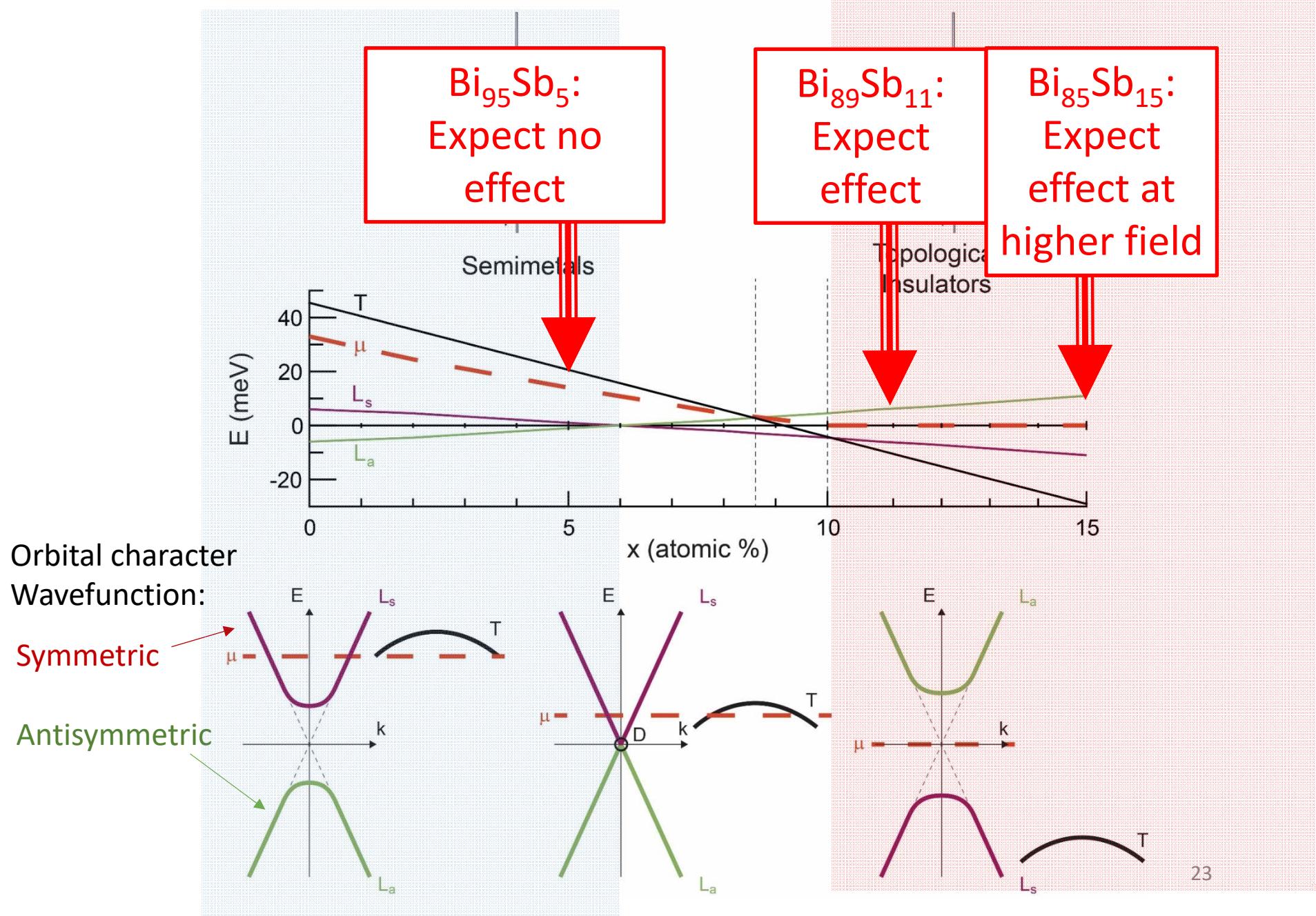
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Samples

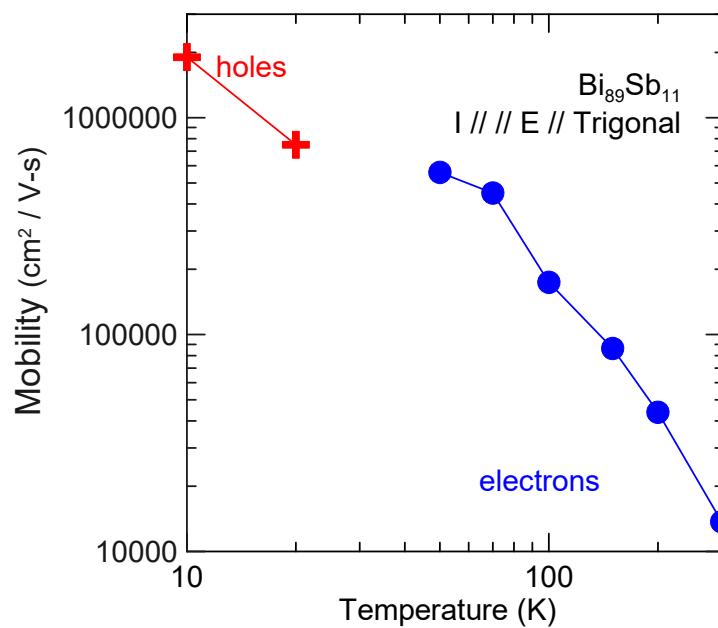
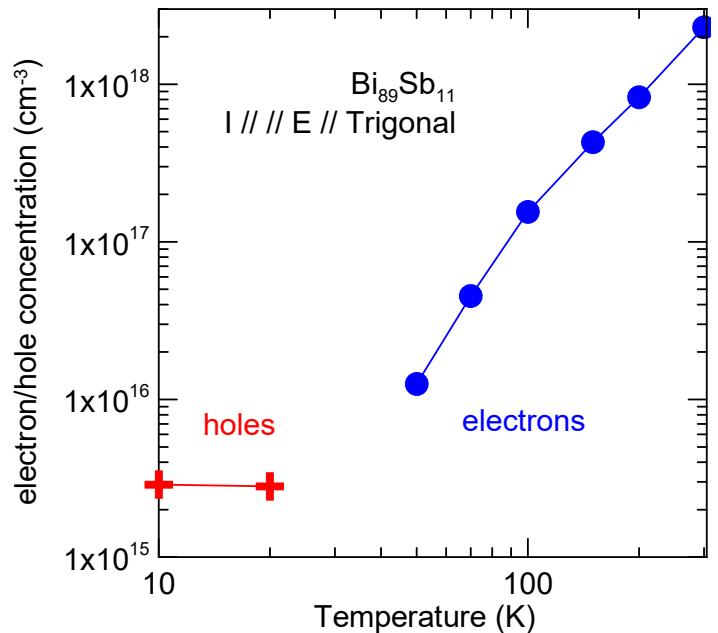


- Bi and Sb are isoelectronic
=> no problem with stoichiometry control
- Full solid solution $\text{Bi}_{1-x}\text{Sb}_x$ ($0 \geq x \geq 1$)
⇒ Can be prepared with exquisite purity and perfection
⇒ => Extraordinary mobility (> 10,000,000 $\text{cm}^2/\text{V s}$ at 4K for pure Bi)
- Starting material must be purified in-house by zone-melting
- Crystals grown in-house
- Measured 6 samples $x=11\%$ with consistent results, and $x=15$, $x=5\%$.
- Uniformity $x \sim 1\%$ (nominal) checked by XRD and XRF
- Extremely low carrier concentrations => electrochemical potential at Weyl points

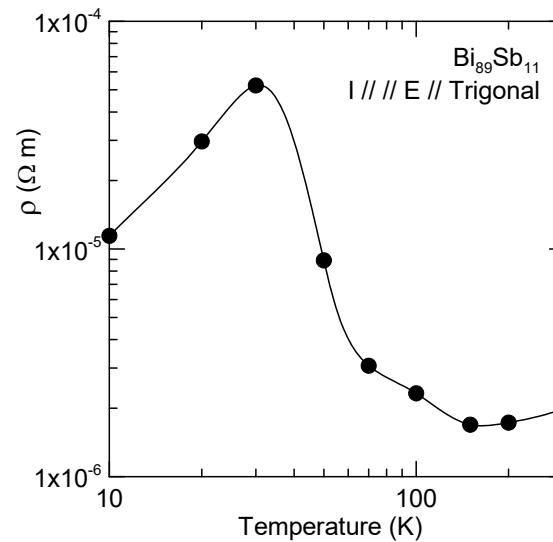
Samples for this study



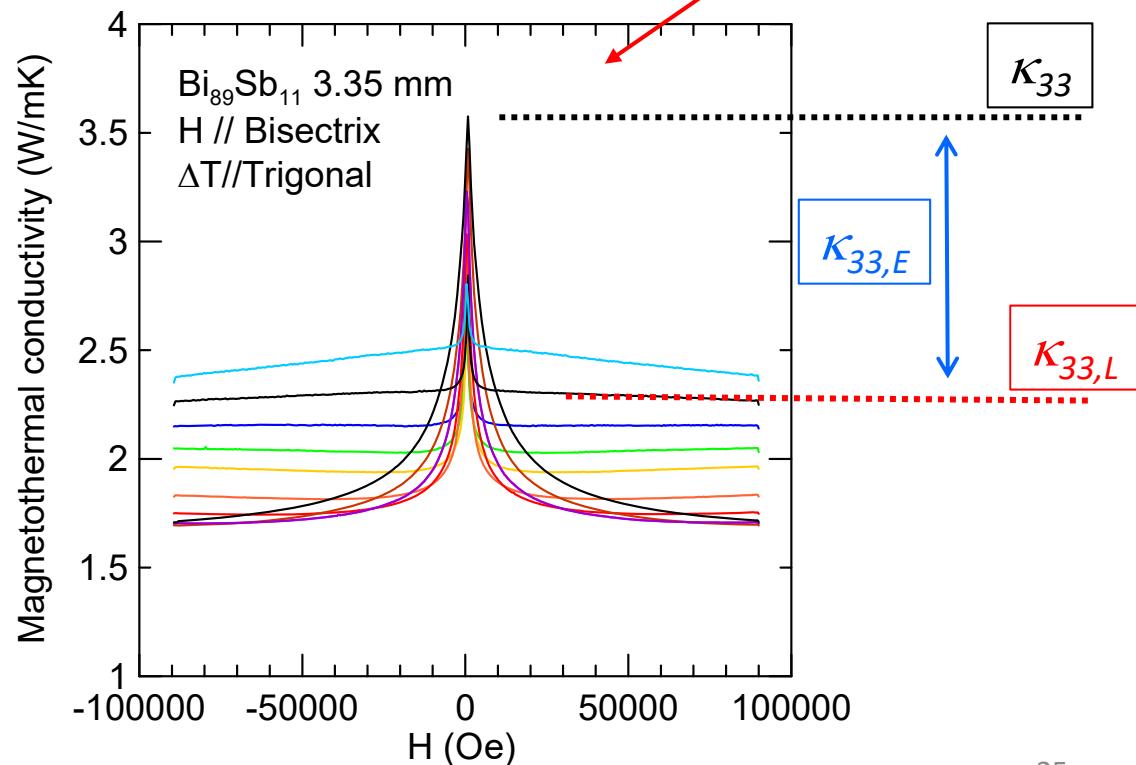
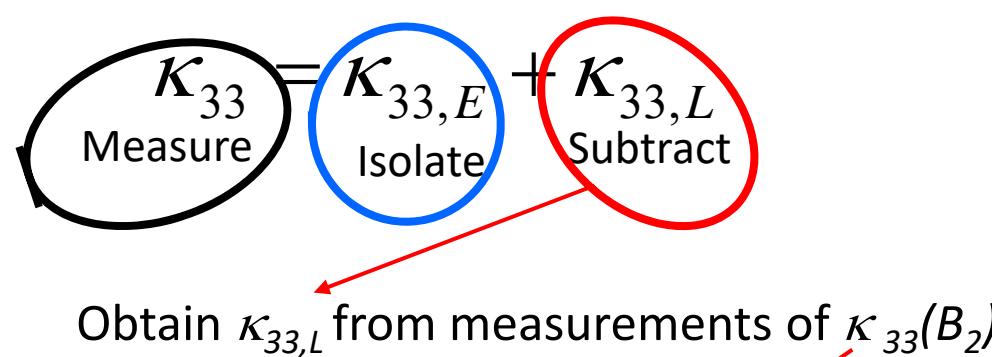
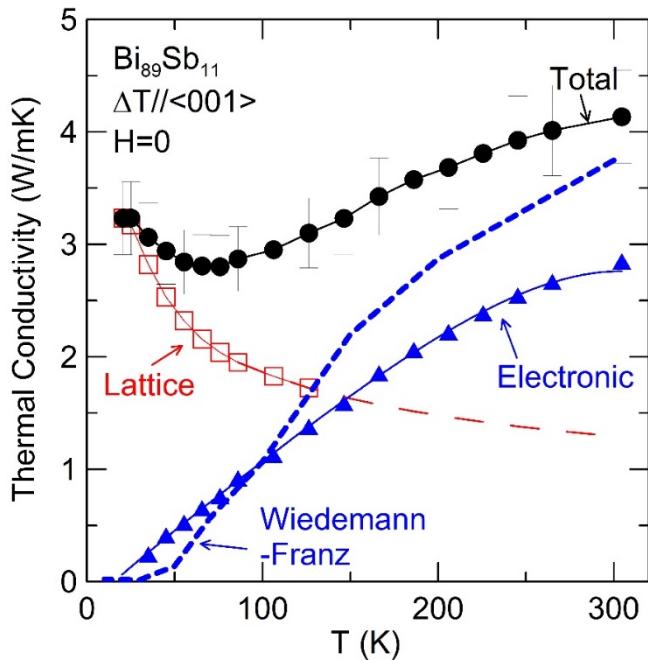
Superb single crystals



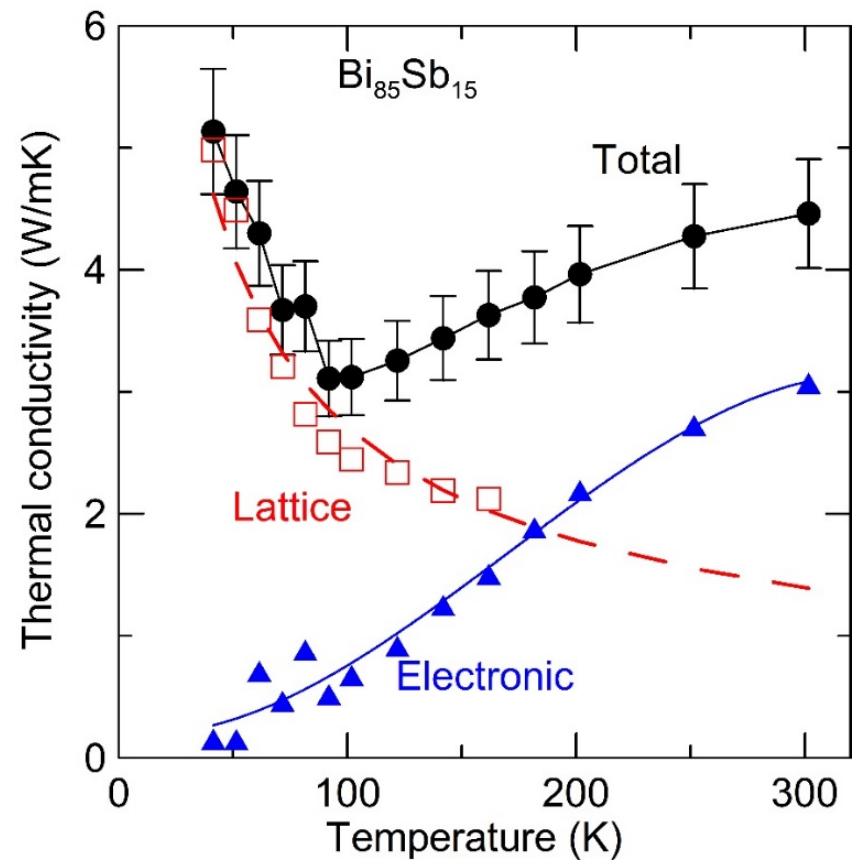
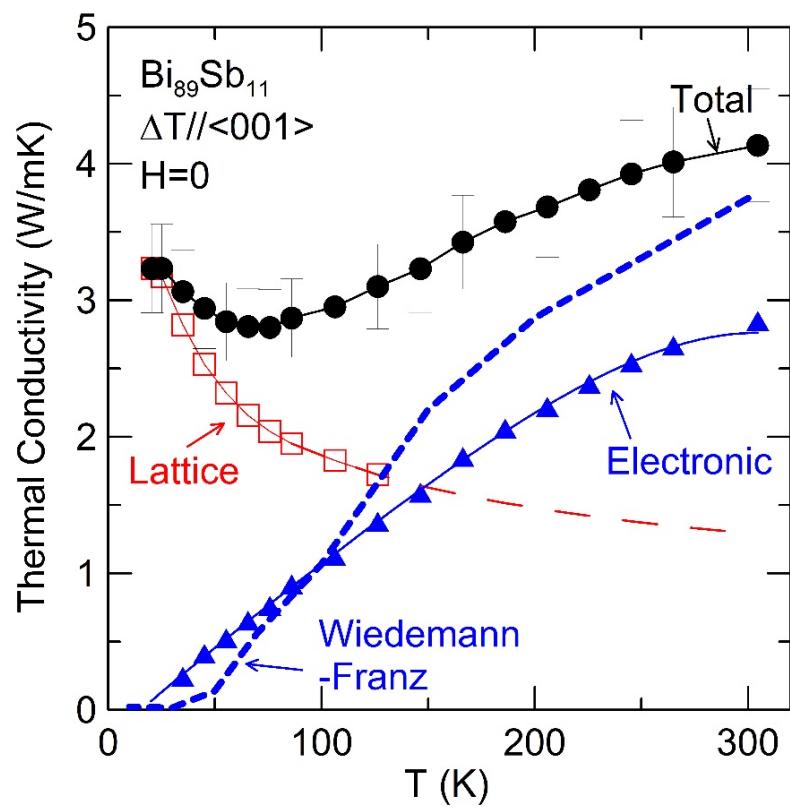
- Very low-field Hall measurements, B // bisectrix (no band movement)
- Freeze-out achieved: carrier concentration reduced to $2 \times 10^{15} \text{ cm}^{-3}$ ($T < 20-40\text{K}$)
- Chemical potential at energy of minimum DOS, i.e. midgap at zero field, at Weyl point in field.
- Mobility
 - at 10K: 2,000,000 $\text{cm}^2 / \text{V s}$
 - At 100K: 200,000 $\text{cm}^2 / \text{V s}$



Thermal conductivity κ_{33} trigonal direction, zero field (TI)



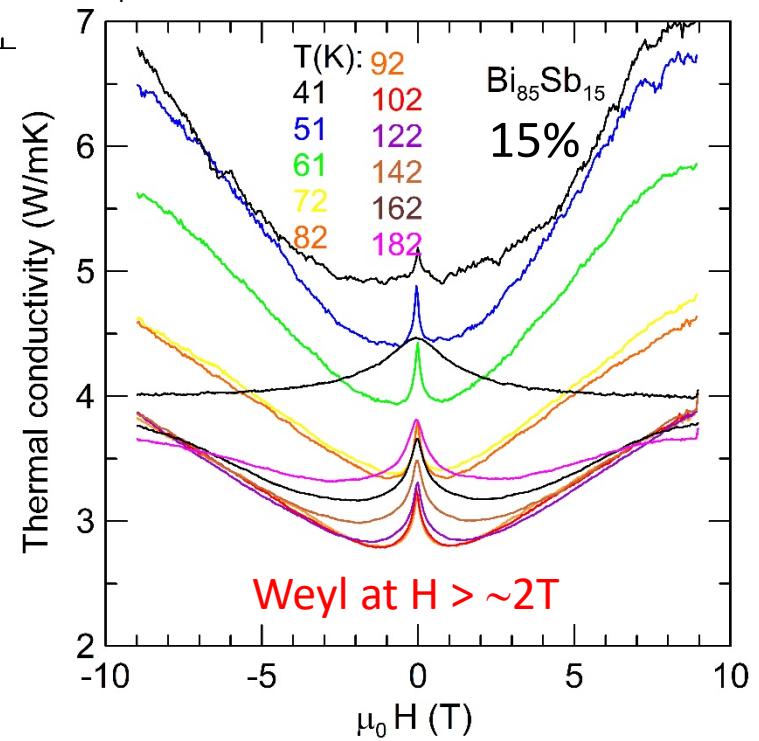
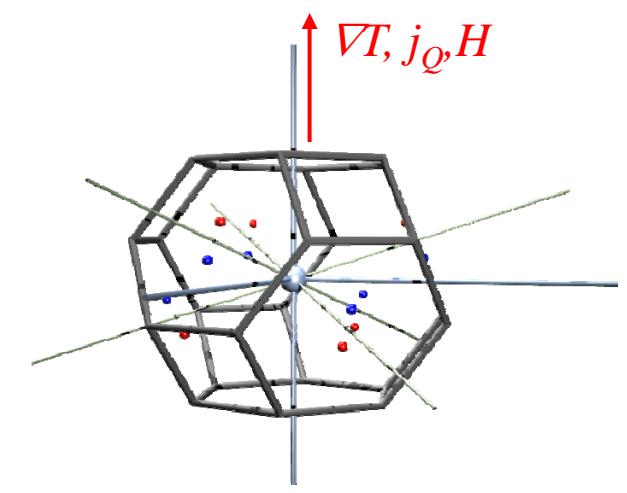
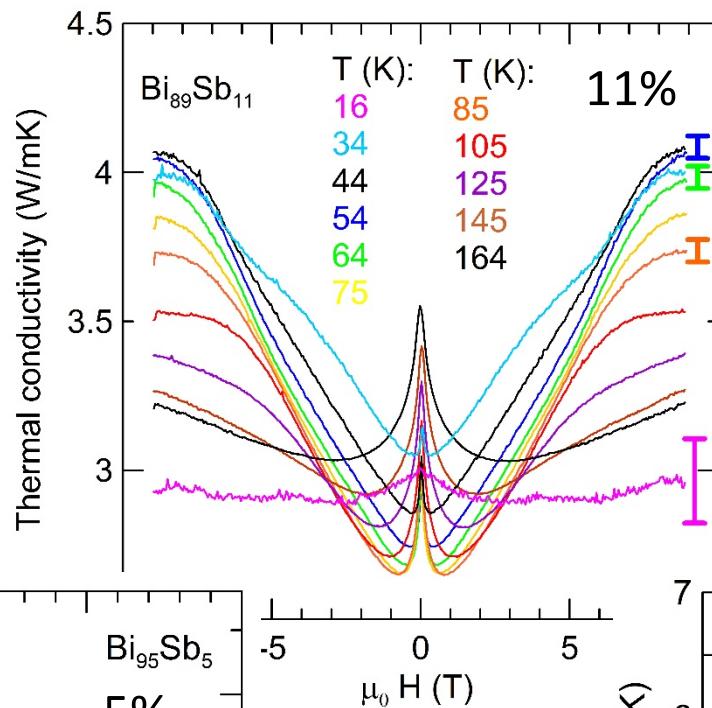
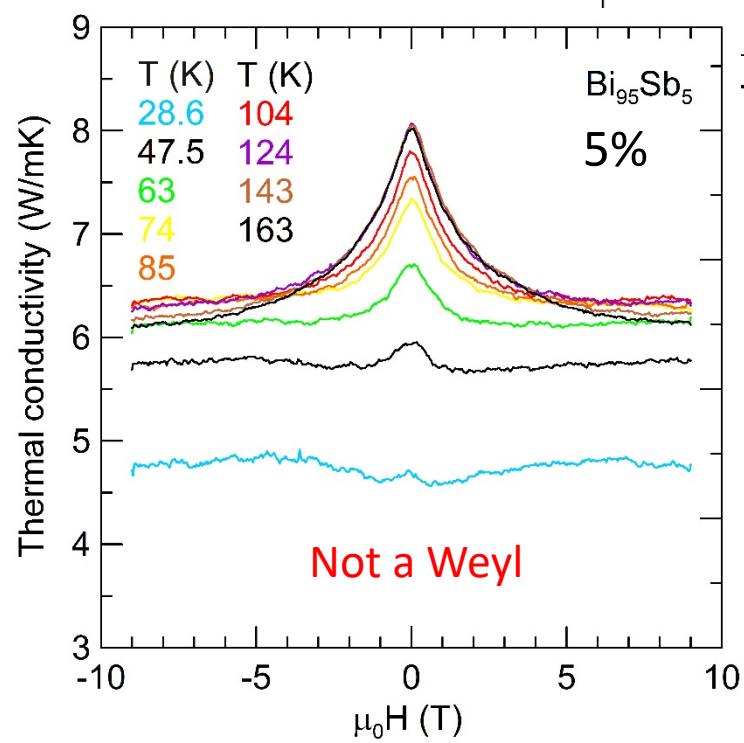
Thermal conductivity κ_{33} trigonal direction, zero field (TI)



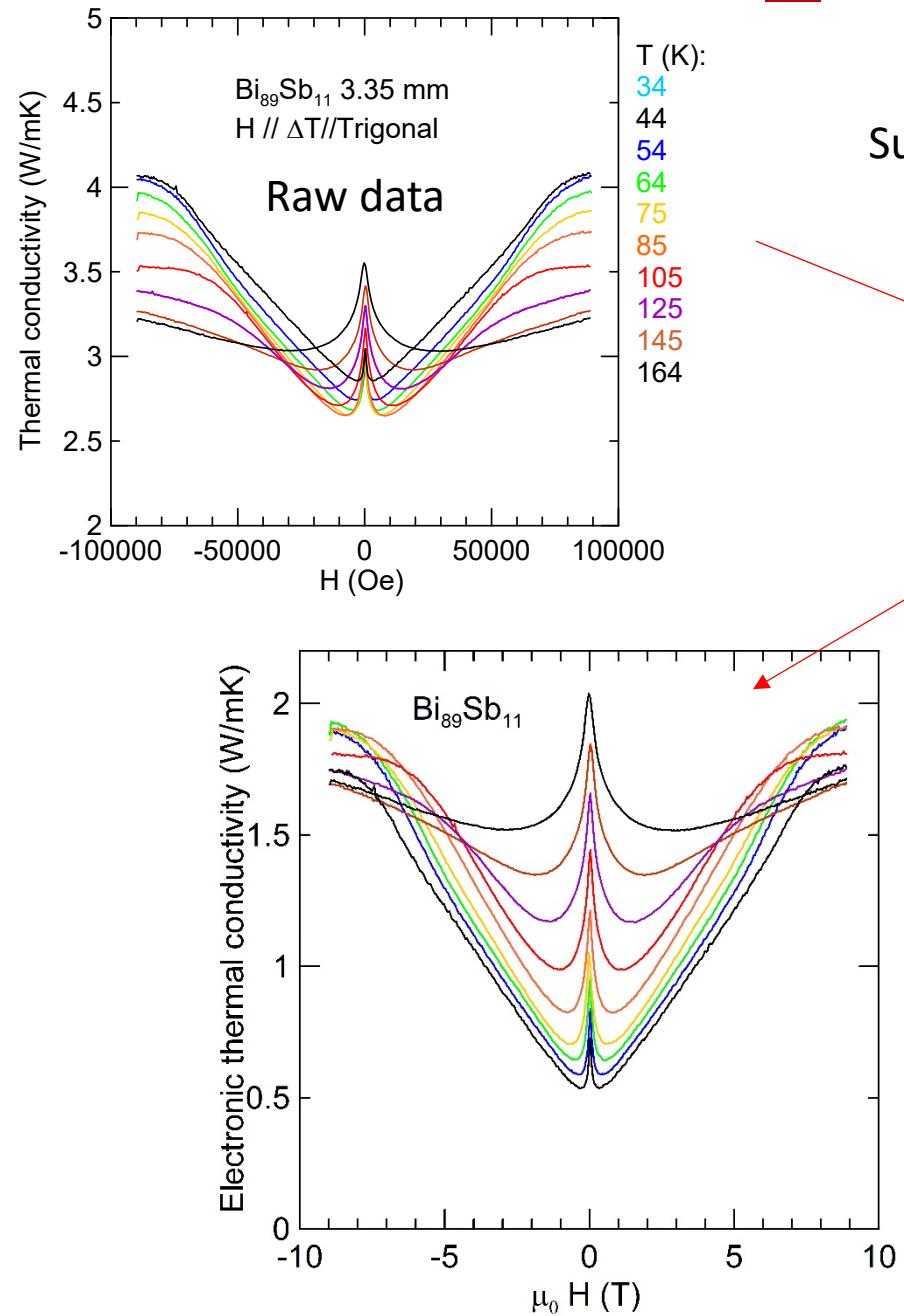
Dashed line: Wiedemann-Franz law, for later

Data: κ_{33} (B_3)

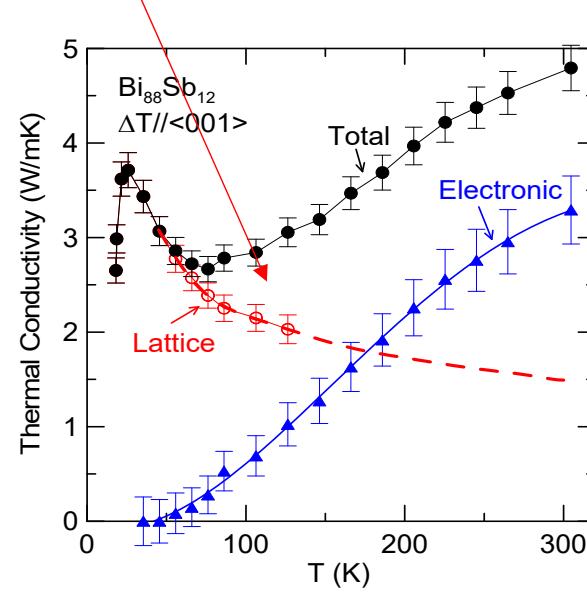
Weyl
at $H > \sim 1$ T



Electronic contribution to $\kappa_{33}(B_3)$



Subtract the lattice part

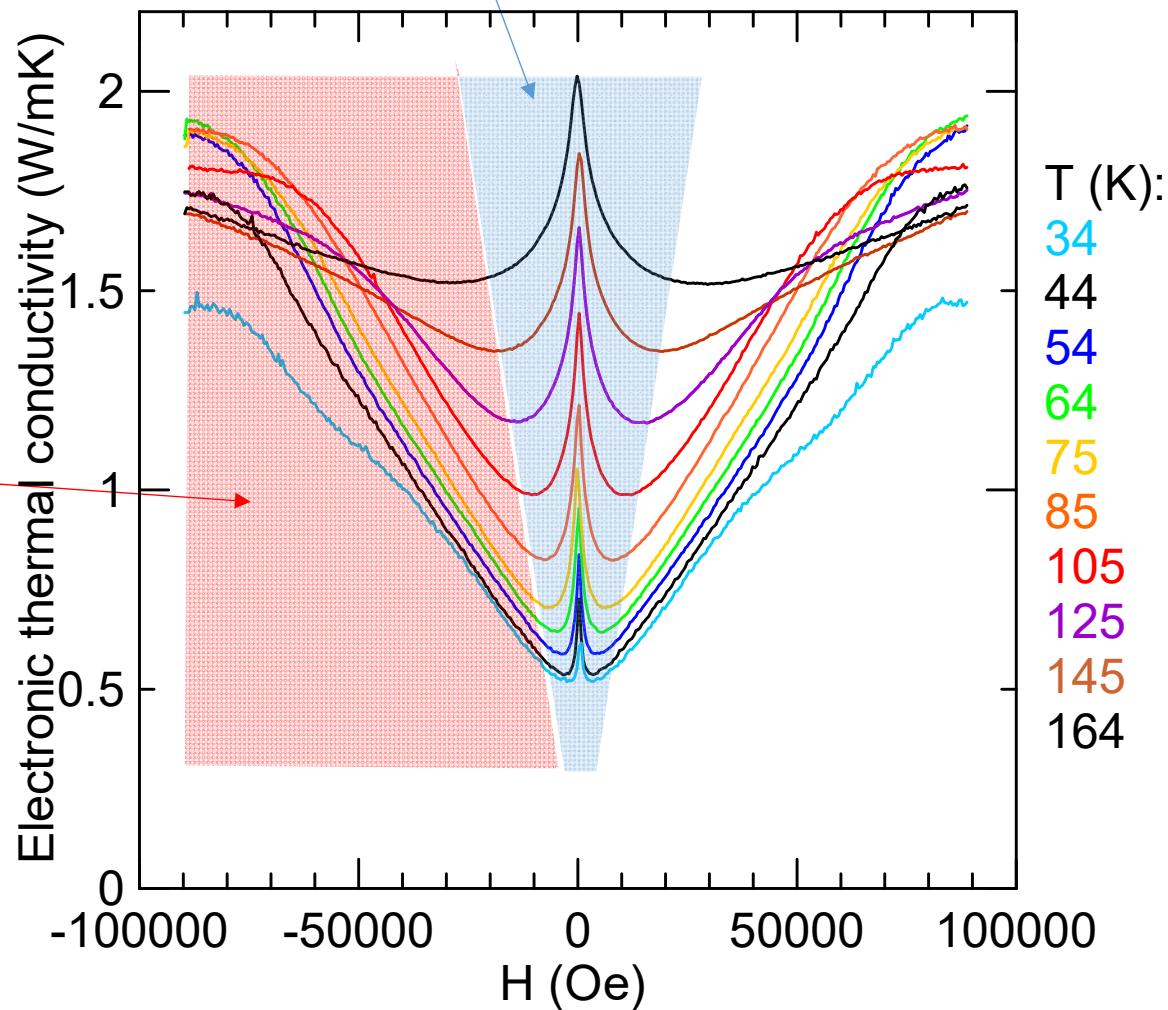


Electronic thermal conductivity

Low-field: conventional TI

Thermal conductivity decreases with field: normal magnetoresistance
decreases electronic contribution to thermal conductivity

Thermal conductivity increases:
Claim: the Chiral energy unbalance term
Analyzed next



Sample 1

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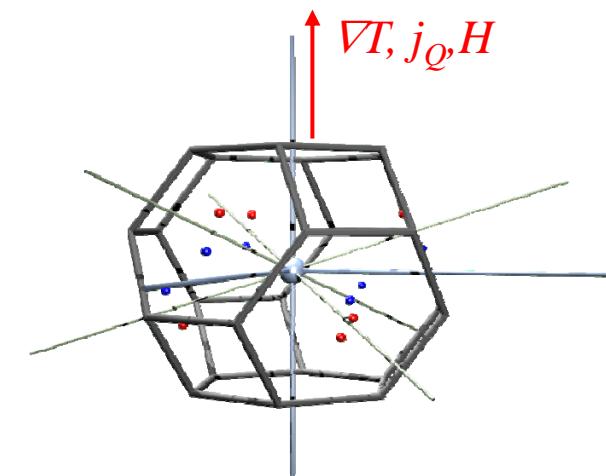
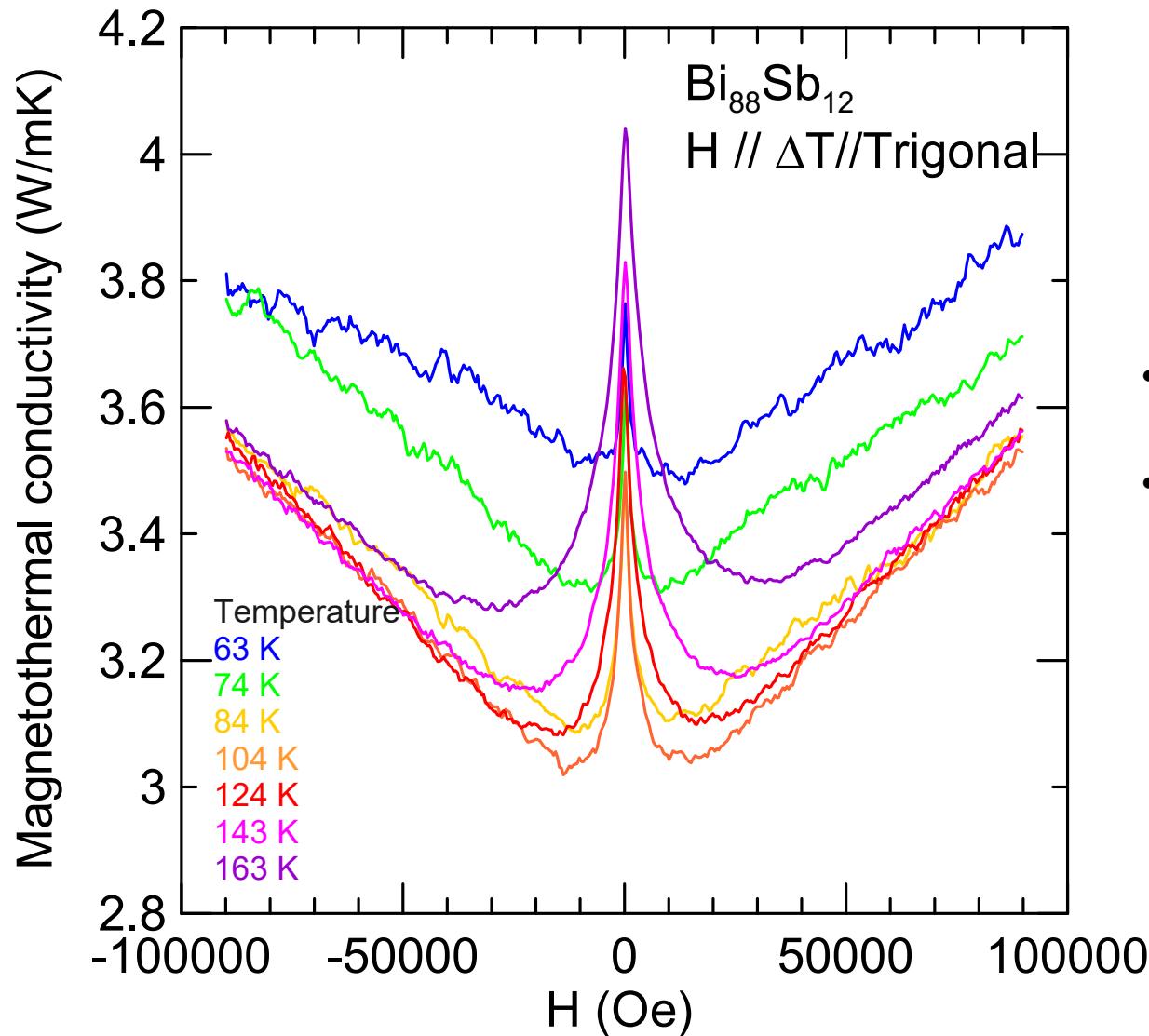
Rules for experimentalists

If you think you see an effect, you must also be able to:

1. Reproduce it on multiple samples.
2. Deliberately make it go away.

✓ Reproduce on Sample 2: $\text{Bi}_{88}\text{Sb}_{12}$

And sample 3 and sample 4



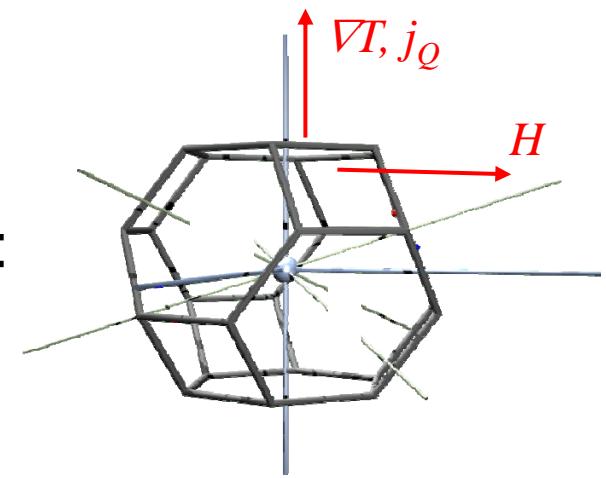
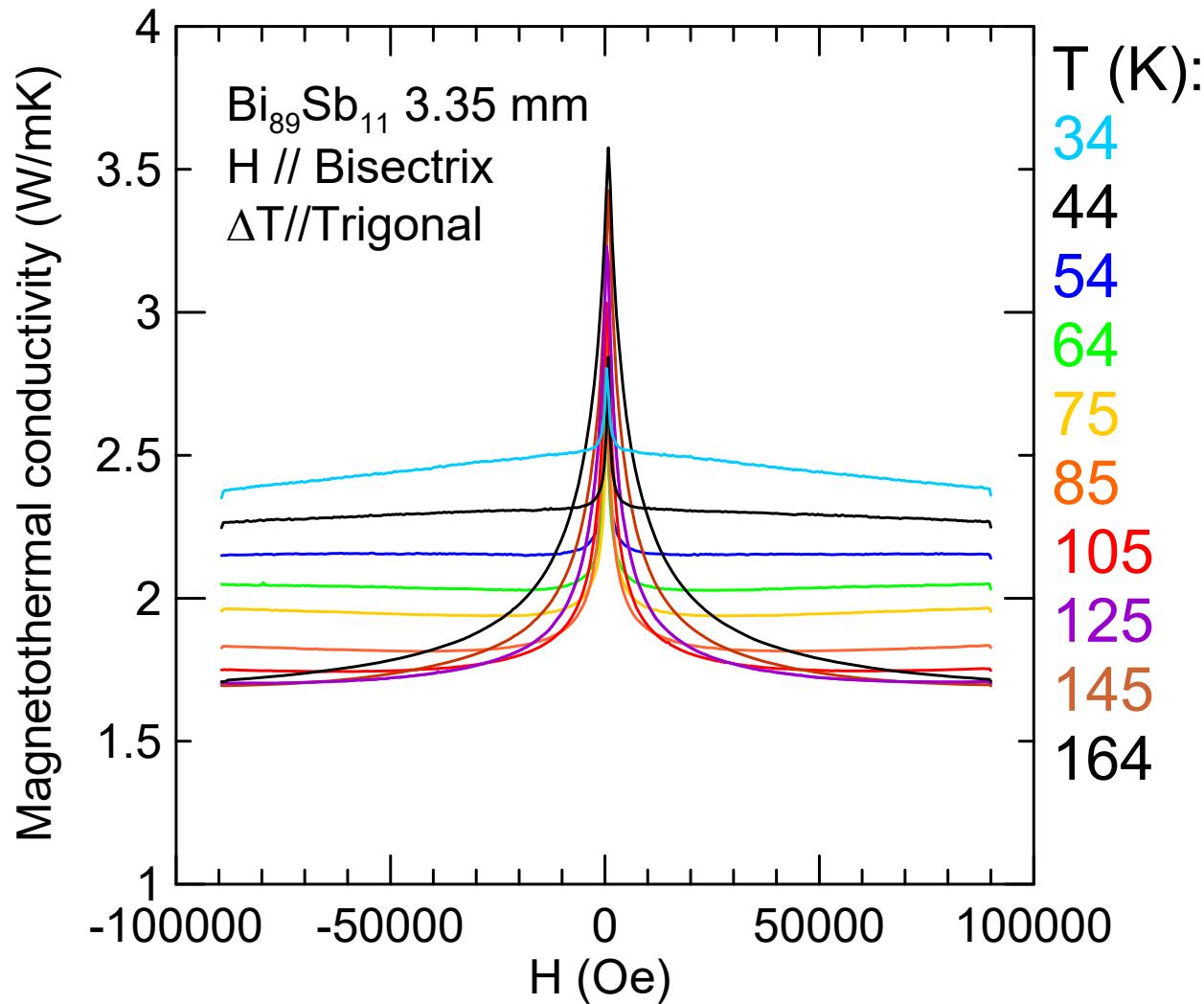
- Lower mobility $2 \times 10^4 \text{ cm}^{-3}$ at 10K
- Residual doping n-type $1 \times 10^{16} \text{ cm}^{-3}$

Effect persists in samples with 100 times lower mobility
=> Effect is robust to defect scattering

✓ Can make the effect go away:

Transverse field

Trigonal thermal conductivity sample 1



Field in wrong direction =>

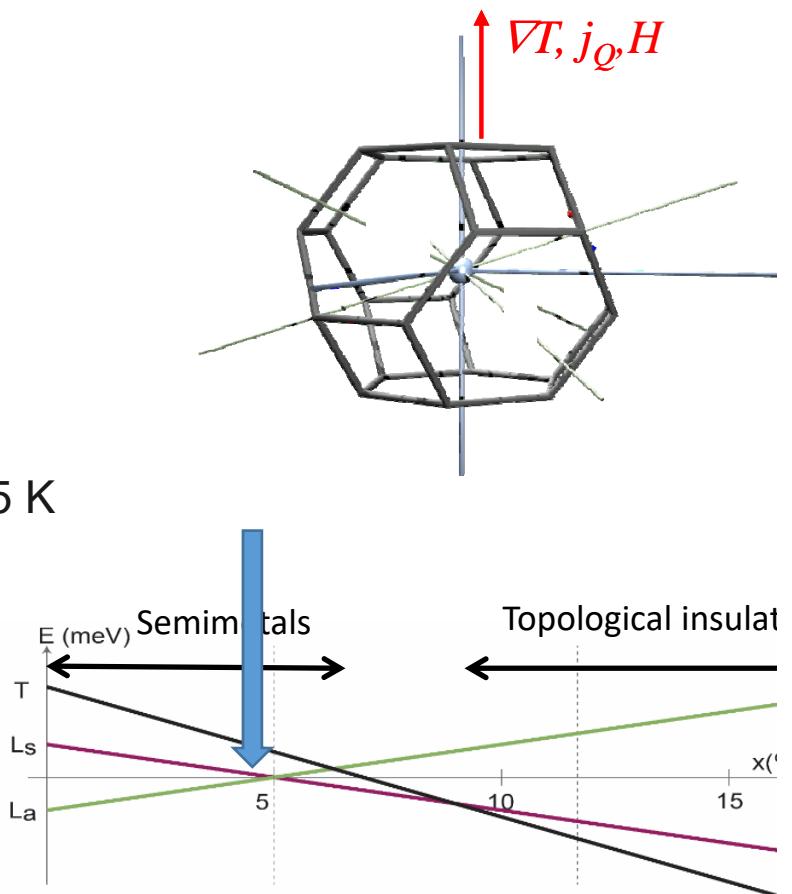
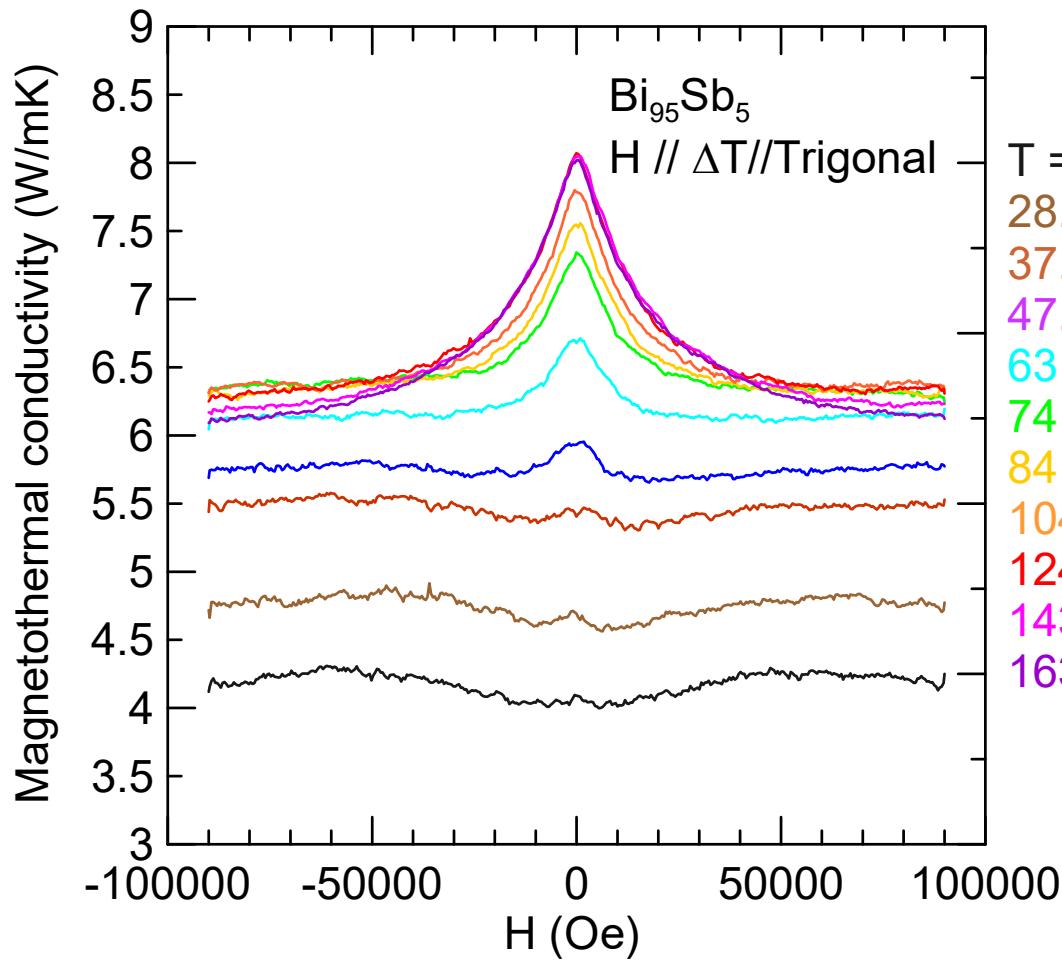
No chiral anomaly

Should **NOT** show the effect

✓ Can make the effect go away:

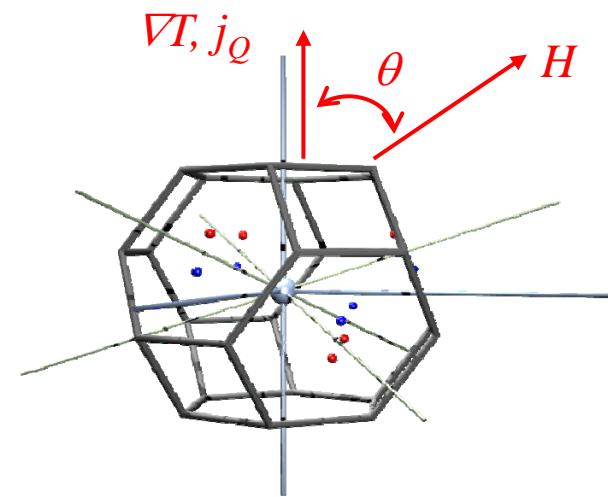
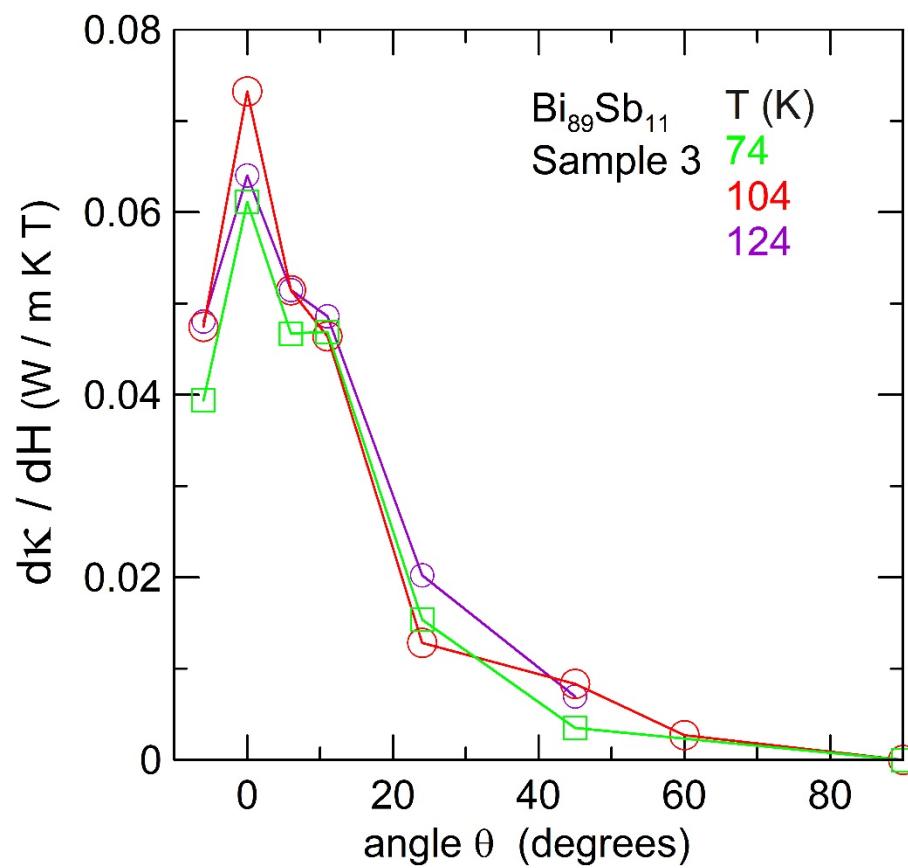
Composition at the Dirac point

Trigonal thermal conductivity $\text{Bi}_{95}\text{Sb}_5$



- Geometry is correct
- Bands are not inverted
- \Rightarrow no Weyl points
- Should **NOT** show the effect

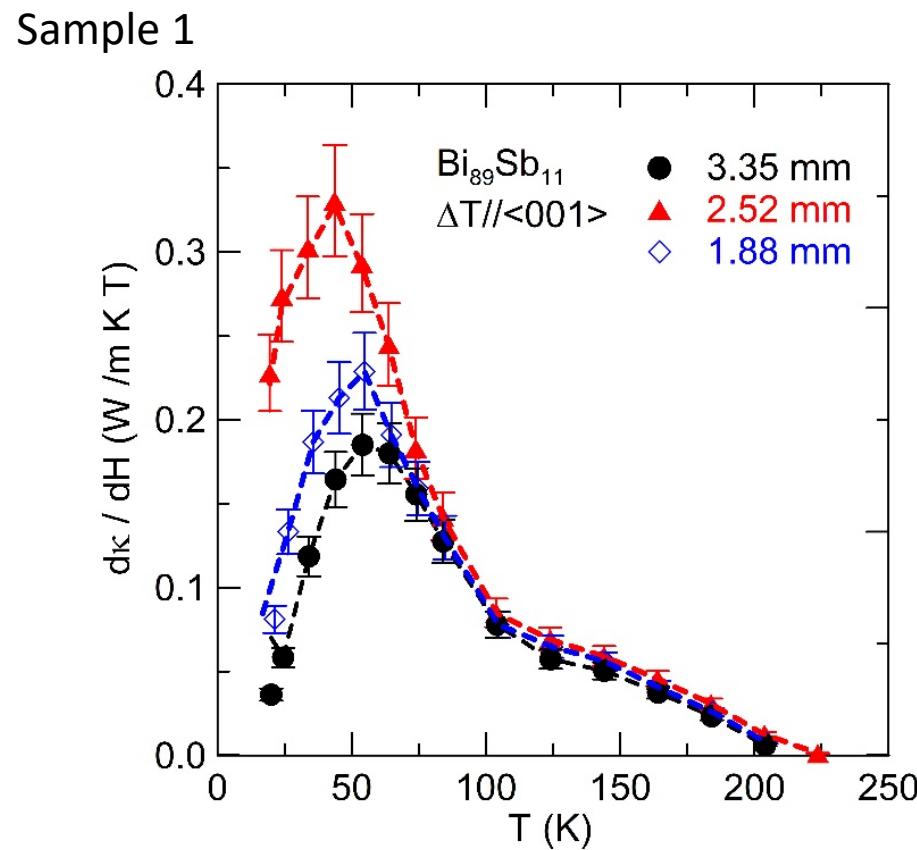
Angular dependence: angle from 3-axis in 3-2 plane



- Much sharper dependence than $\cos(\theta)$ or $\cos^2(\theta)$, possibly $\cos^N(\theta)$ with $N > 6$
- Signature feature of electrical chiral anomaly

Temperature and sample length dependence

- Slope of thermal conductivity at 7 Tesla
- No length dependence
- Temperature dependence analyzed next



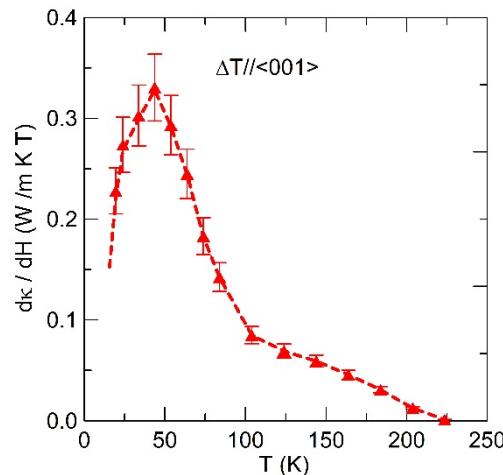
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Analysis of temperature-dependence, $k_B T > \mu$



1. Take these raw data for experimental temperature dependence at 7 T of $d\kappa_{zz}/dB_z$

2. Compare to the formula for the anomalous conductivity $d\kappa_{zz}/dB_z = \frac{\pi e v_F k_B^2}{\hbar^2} T \tau$

3. Solve for τ and plot versus $1/T$

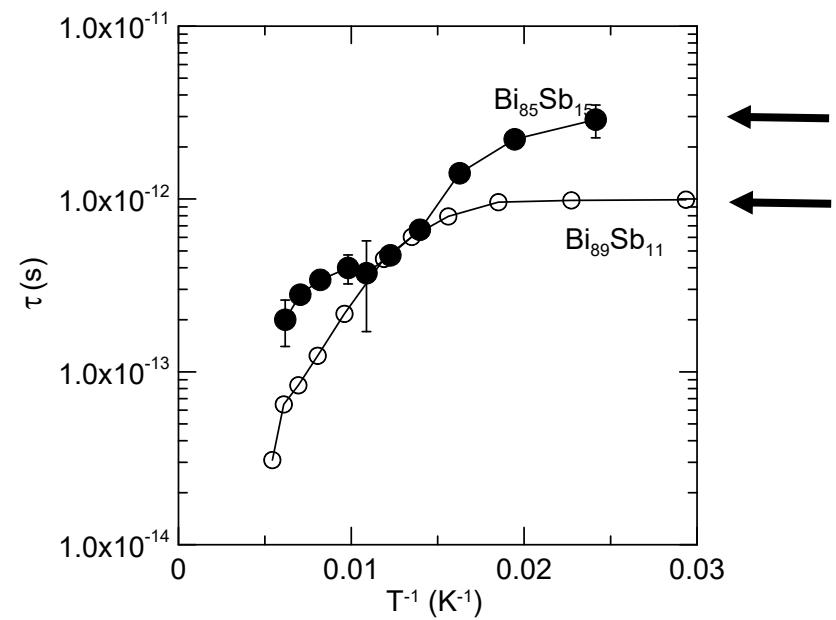
4. For $T < 50$ K,

$$\text{Bi}_{89}\text{Sb}_{11} \quad \tau \sim 1 \times 10^{-12} \text{ s}$$

$$\text{Bi}_{85}\text{Sb}_{15} \quad \tau \sim 3 \times 10^{-12} \text{ s}$$

τ is T-independent

τ is 10x larger than the resistivity scattering time

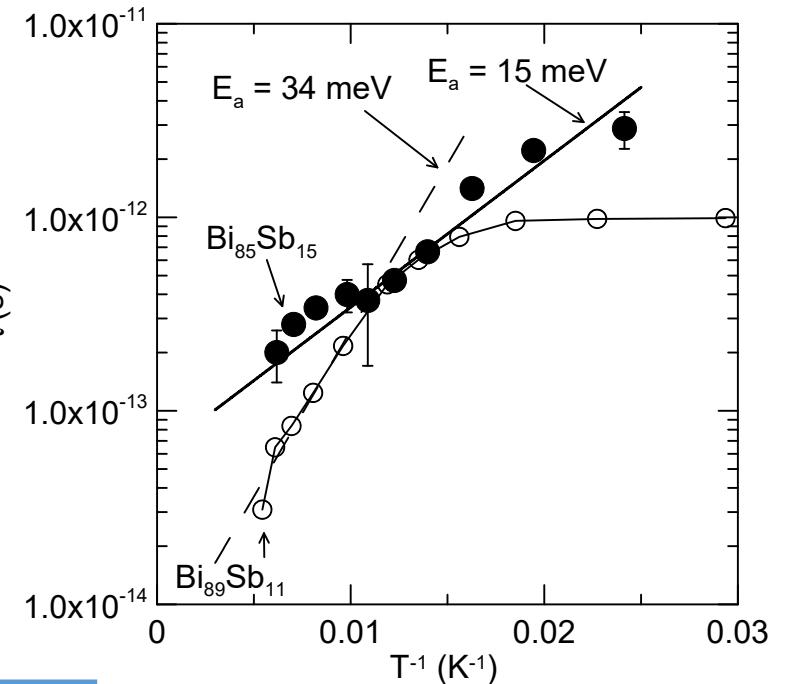
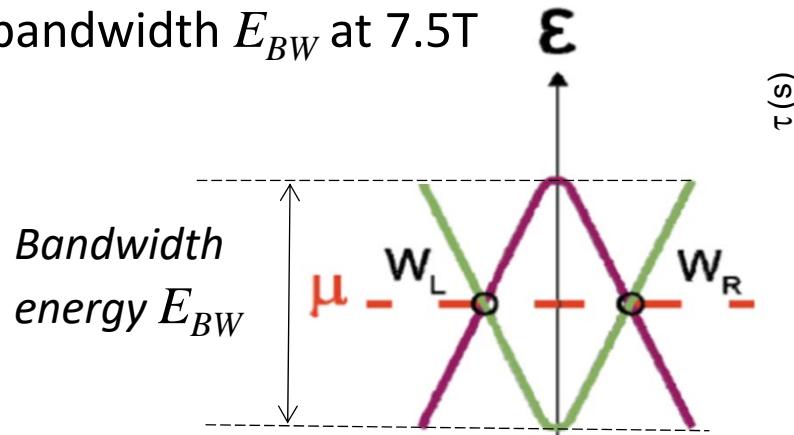


Analysis of temperature-dependence, high- T

6. $T > 100$ K: fit an activation-energy to $\tau(T)$:

$$\tau(T) = \tau_0 e^{E_A/k_B T}$$

7. Calculated bandwidth E_{BW} at 7.5T



Composition	Experimental E_A	Calculated E_{BW}
	meV	meV@7.5T
$\text{Bi}_{89}\text{Sb}_{11}$	34 ± 2	35
$\text{Bi}_{85}\text{Sb}_{15}$	15 ± 2	20

- ⇒ τ is the inter-Weyl point scattering time, thermal activation
- ⇒ The only energy scale in the observation is the width of the Weyl bands
- ⇒ The Wiedemann-Franz law is expected to hold



1. Introduction:

- Weyl semimetals: chiral anomaly and thermal conductivity
- Experimental difficulties
- Bi-Sb semiconductors alloys and TI's
- In the ultra-quantum limit: field-induced Weyls

2. Thermal conductivity

- Experimental evidence
- Robustness to phonons and defects
- Decay only via inter-Weyl point scattering

- The Wiedemann-Franz law:

$$K = K_E + K_L$$

Measure Isolate Subtract

A diagram showing the Wiedemann-Franz law equation $K = K_E + K_L$. The total thermal conductivity K is shown as a sum of two terms: K_E and K_L . The term K_E is highlighted with a blue circle and labeled 'Isolate' with an arrow pointing to it. The term K_L is highlighted with a red circle and labeled 'Subtract'.

- Can we prove experimentally

- That the electronic contribution $K_E = LT\sigma$

- with $L = L_o = \pi^3/3 \left(k_B/e \right)^2 = 2.45 \cdot 10^{-8} \left(V/K \right)^2$

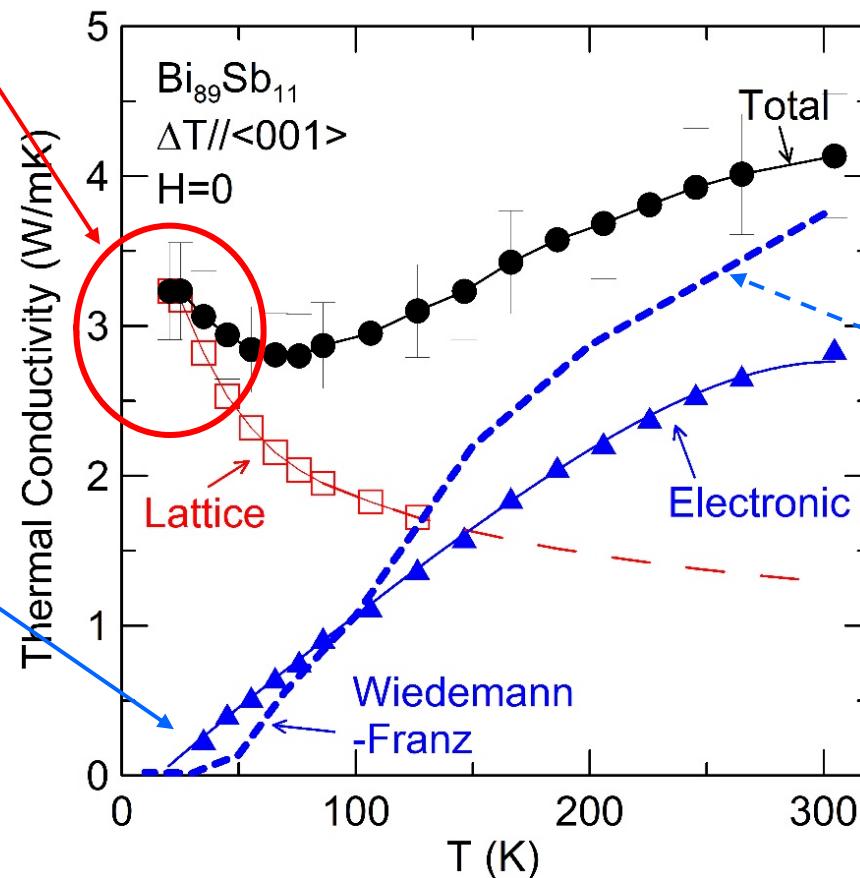
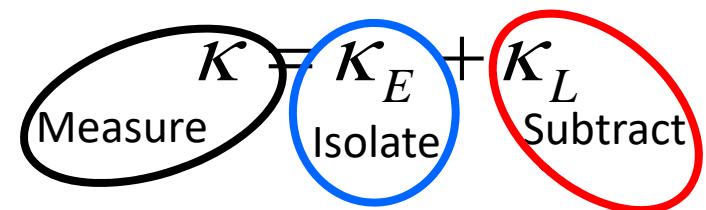
Wiedemann-Franz law (WFL) $\kappa_E = LT\sigma$

Predictions for regime of chiral anomaly:

- If inelastic relaxation rate (τ_{eff}^{-1}) is dominated by the helicity (τ_h^{-1}): $L = L_o$
Spivak, N. Z. and Andreev, A. V., Phys. Rev. B 93, 085107 (2016)
- If there are other inelastic relaxation mechanisms, $L = \frac{\tau_h^{-1}}{\tau_{eff}^{-1}} L_o < L_o$
- If there is ambipolar conduction: $\kappa_E = L_o T \sigma (1 + ZT); \quad L > L_o$
- Chiral zero sound: $L \gg L_o$

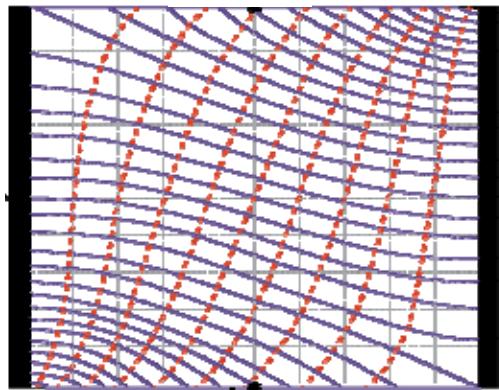
At zero field Wiedemann-Franz holds

Below 50 K
 $\kappa \sim \kappa_L$
⇒ extreme error
bars on κ_E
⇒ Procedure impossible
⇒ Example: at zero field one would get unphysical result $L \gg L_0$



- Take experimental values of electrical resistivity
- Calculate $\kappa_{WF} = L_0 T \sigma$
- Reproduces κ_E
- Wiedemann-Franz holds

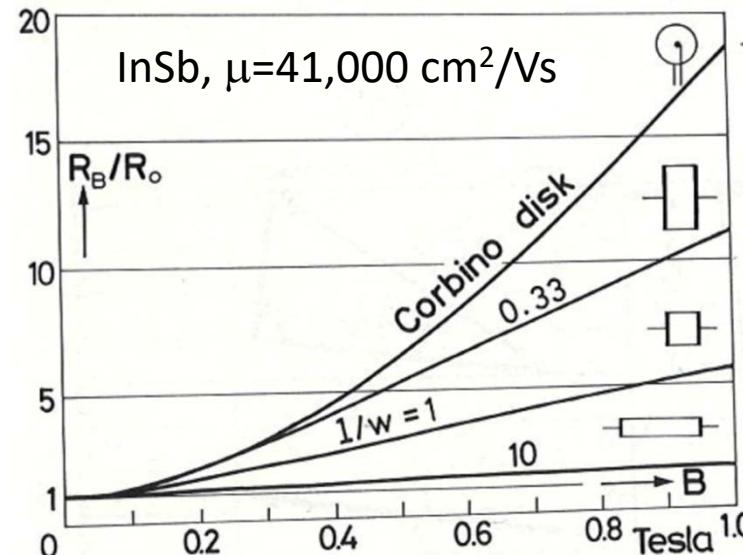
Error due to geometrical magnetoresistance in $\sigma_{33}(B_3)$



$$R(B) / R_0 = 1 + g \mu^2 B^2$$

Geometrical factor

Distortions of current lines by Lorenz force =>
Geometrical magnetoresistance in transverse field



Length /Width	<i>g</i>
0	1
0.33	0.6
1	0.28
10	0.05
∞	0

In our samples, $\mu \sim 0.7 \cdot 10^6 \text{ cm}^2/\text{Vs}$ @ 60 K =>
1T transverse gives, for square sample $R(B)/R(0)=1400$

Baker, D. R. and Heremans, J. P., Phys. Rev. B 59, 13927 (1999), InSb
data from Weiss, Zeitschrift für Physik, 13d. t38, S. 322--329 (t954).

Problem: geometrical magnetoresistance in $\sigma_{33}(B_3)$

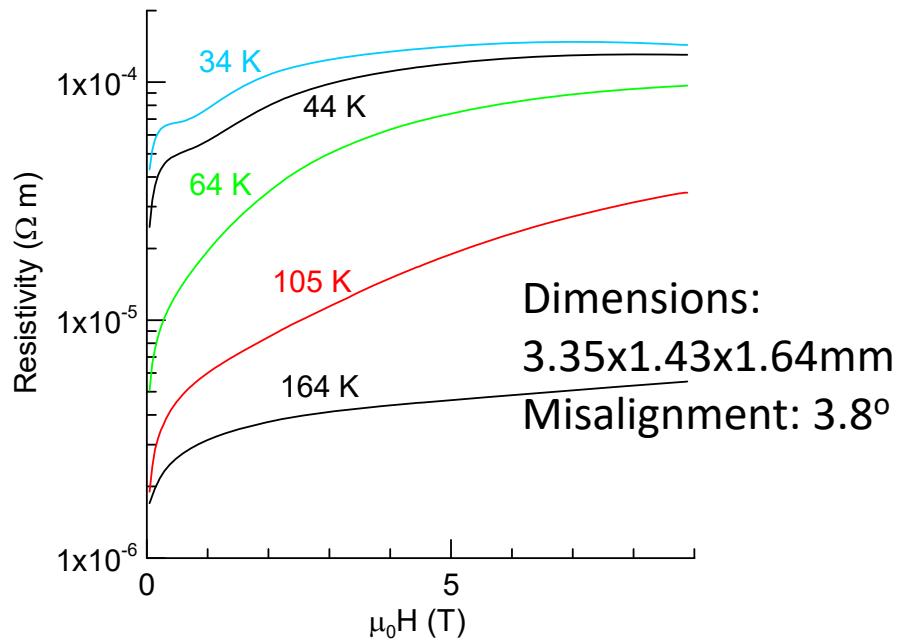
Source of error: misalignment of the longitudinal field, angle θ

$$\Delta R / R \Big|_{\text{ERROR}} = g \mu^2 B^2 \sin^2 \theta$$

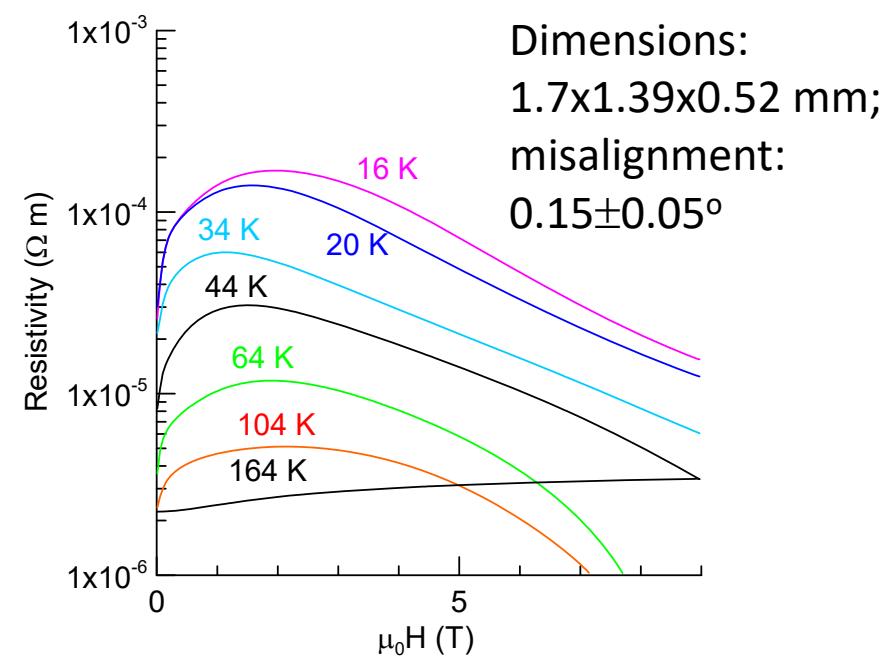
In our samples, $\mu \sim 0.7 \cdot 10^6 \text{ cm}^2/\text{Vs}$ @ 60 K
 \Rightarrow 1 degree misalignment 1 Tesla gives
 200% relative error

In Bi, $\mu \sim T^{-2}$ so error $\sim T^{-4}$ 5% error at 60 K gives 45% error at 35 K

Example: same $\text{Bi}_{89}\text{Sb}_{11}$ sample as used for thermal conductivity



Geometrical MR error at 60 K & 9T: factor 400
 Chiral anomaly masked



Geometrical MR error at 60 K & 9T: 25%
 Chiral anomaly visible

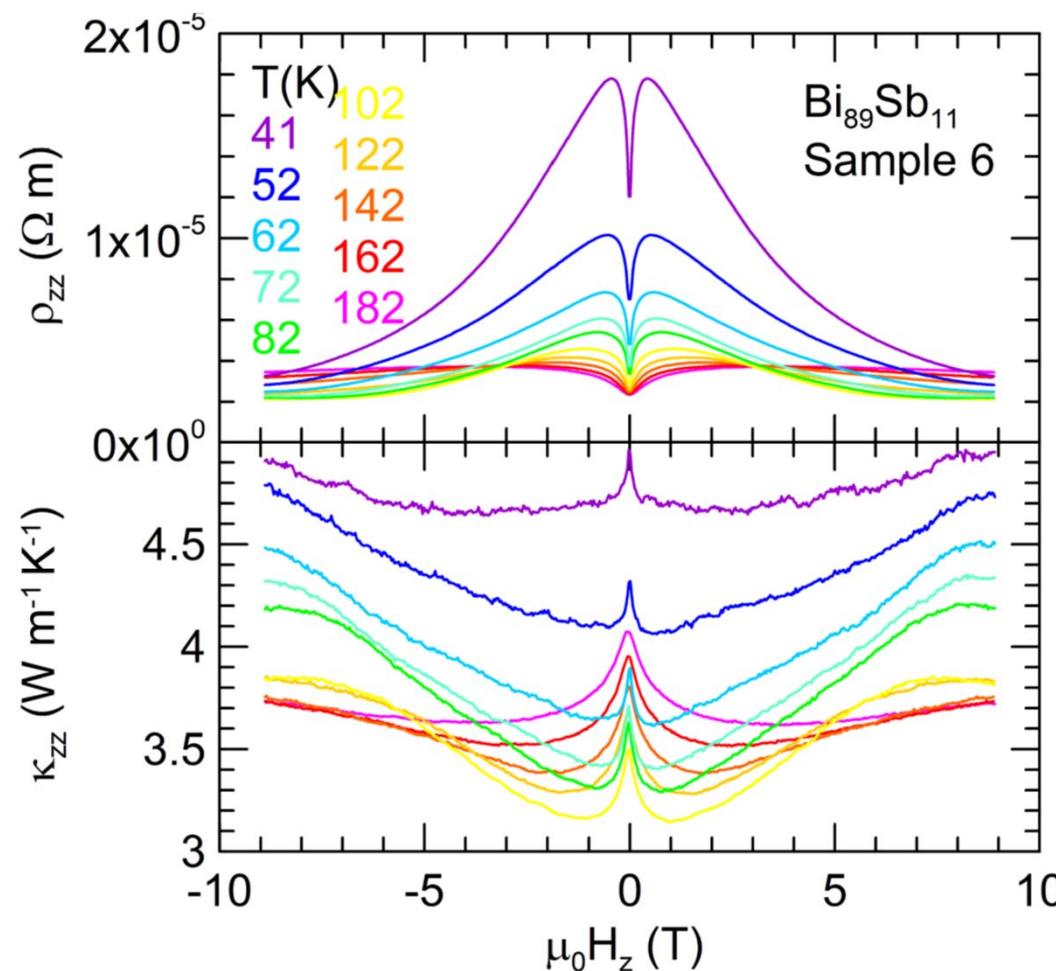
Wiedemann-Franz law (WFL) $\kappa_E = LT\sigma$

Predictions for the effect of measurement errors:

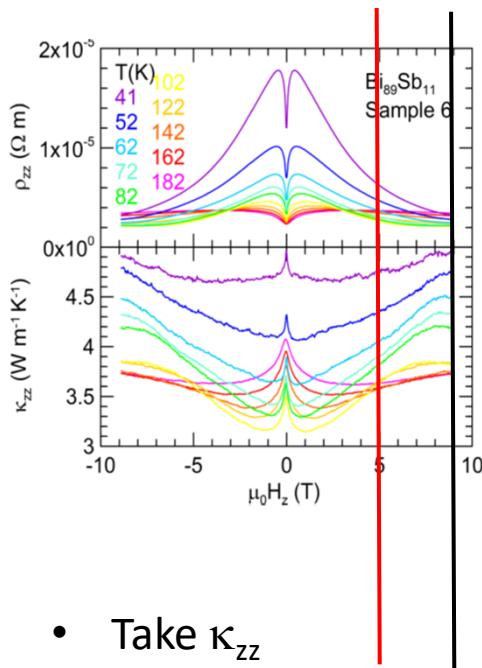
- If we underestimate the lattice contribution: $L \gg L_o$
- If current jetting effect contaminates measurement: $L < L_o$
- Small misalignment and geometrical MR contamination: $L \gg L_o$

$Bi_{89}Sb_{11}$ sample 6 specially cut & etched

1. Cut to $3 \times 0.6 \times 0.4$ mm L/W=7.5/1 ($g=0.06$)
2. Sides etched (surface roughness also gives geometrical MR)
3. Mounted with misalignment angle < 0.1 degree (goniometer + guides)
4. MR error at T=60 K and B=9T: < 7%.

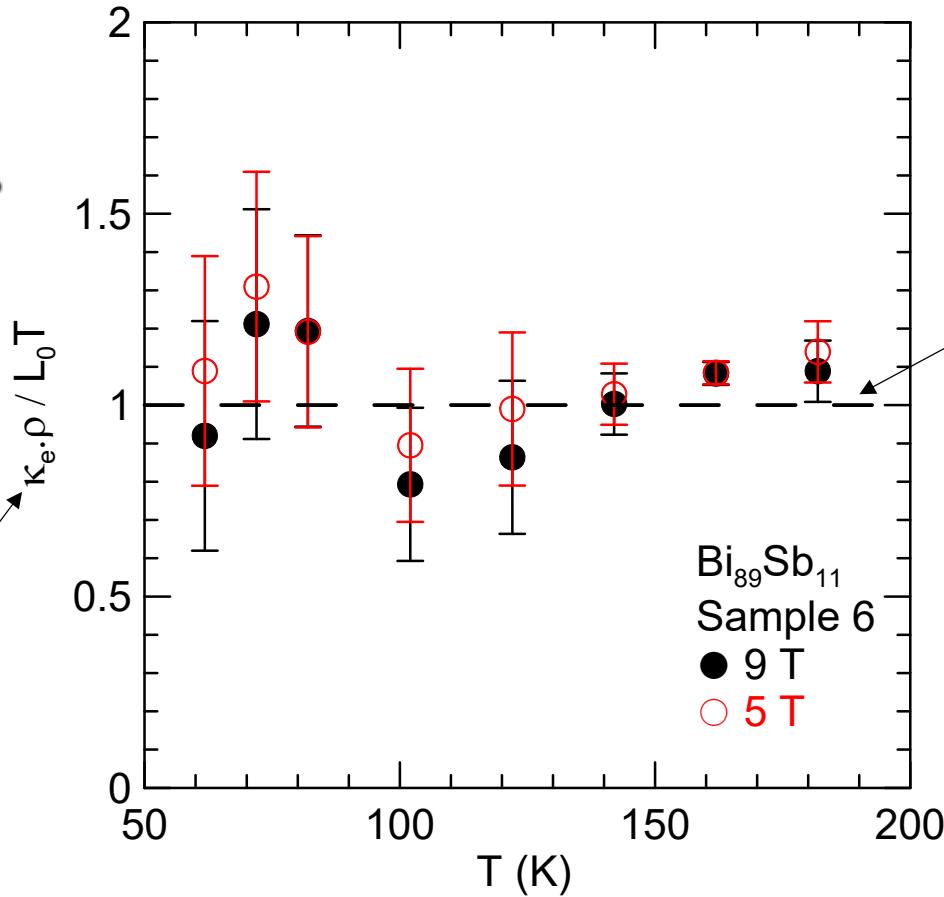


Wiedemann-Franz is obeyed at $T > 50$ K in the chiral anomaly regime



- Take κ_{zz}
- Subtract $\kappa_{zz,L}$
- Take ρ_{zz}
- Calculate

$$\frac{(\kappa_{zz} - \kappa_{zz,L})\rho_{zz}}{L_0 T}$$



$$L = L_0 = \pi^2 / 3 \left(k_B / e \right)^2$$

Below 50 K, error on MR and error on $(\kappa_{zz} - \kappa_{zz,L})$ are prohibitive => no data

Conclusions

1. “Chiral anomaly” in Weyl semimetals is a charge carrier density unbalance between Weyl points when $\vec{E} // \vec{B}$
2. There exists an equivalent energy unbalance when $\nabla T // \vec{B}$
3. Both effects are related by the Wiedemann-Franz law
4. $\text{Bi}_{1-x}\text{Sb}_x$ alloys ($x > 10\%$) are ideal Weyl semimetals in magnetic field
5. In them, the anomalies are:
 - 300 % at 9T
 - Robust against phonon scattering
 - Robust against defect scattering
 - Governed only by one energy scale: the width of the Weyl bands