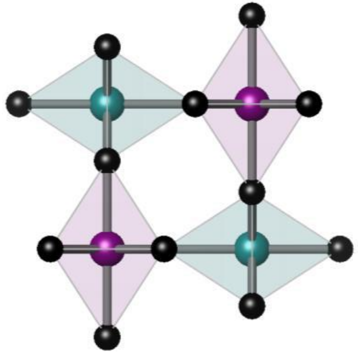
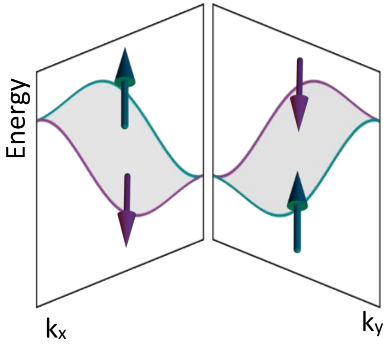
Altermagnetism



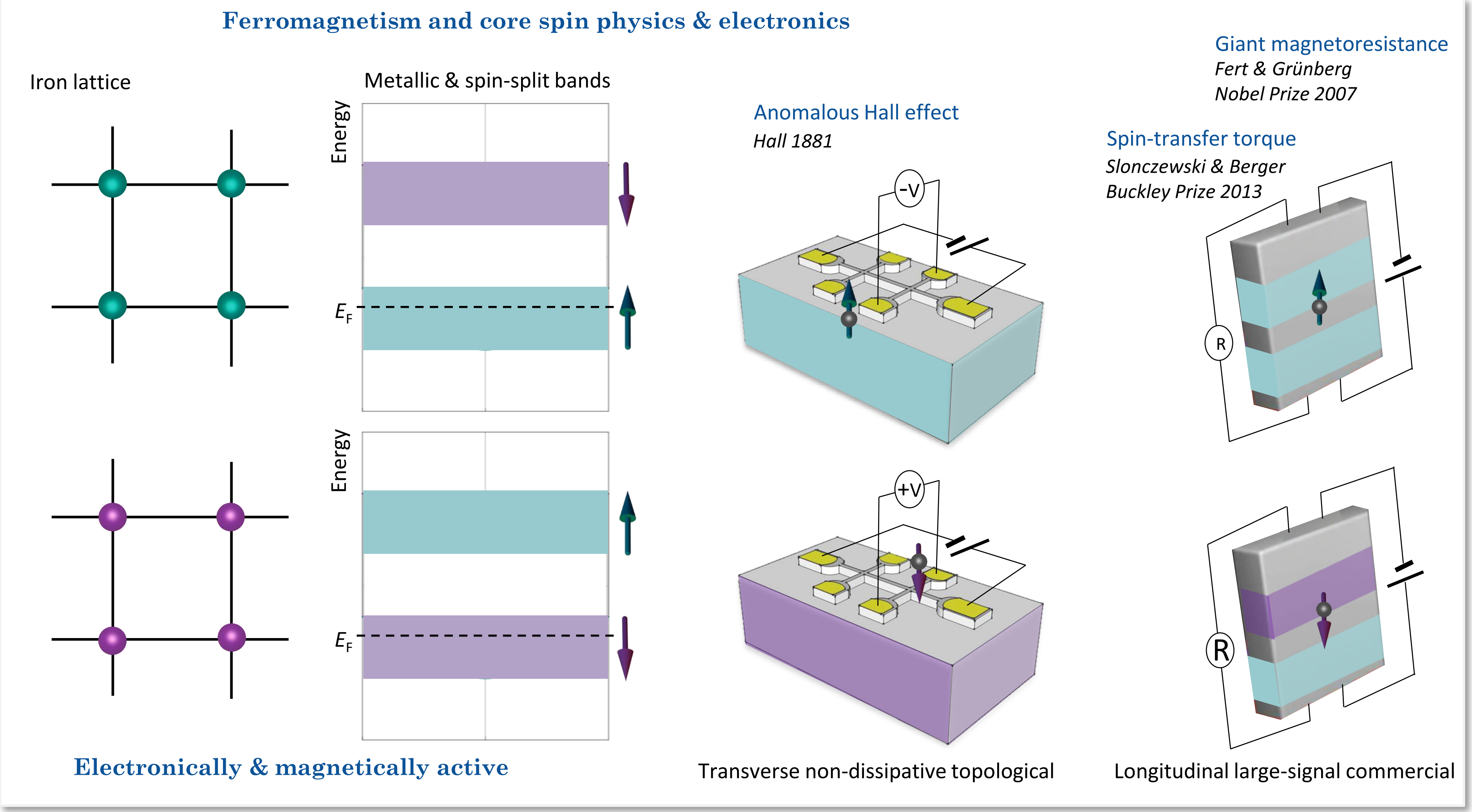
Libor Šmejkal[1,2], Jairo Sinova[1,2], Tomas Jungwirth[2,3]
[1]*University of Mainz, Germany*
[2]*Institute of Physics, Czech Academy of Sciences*
[3]*University of Nottingham, United Kingdom*

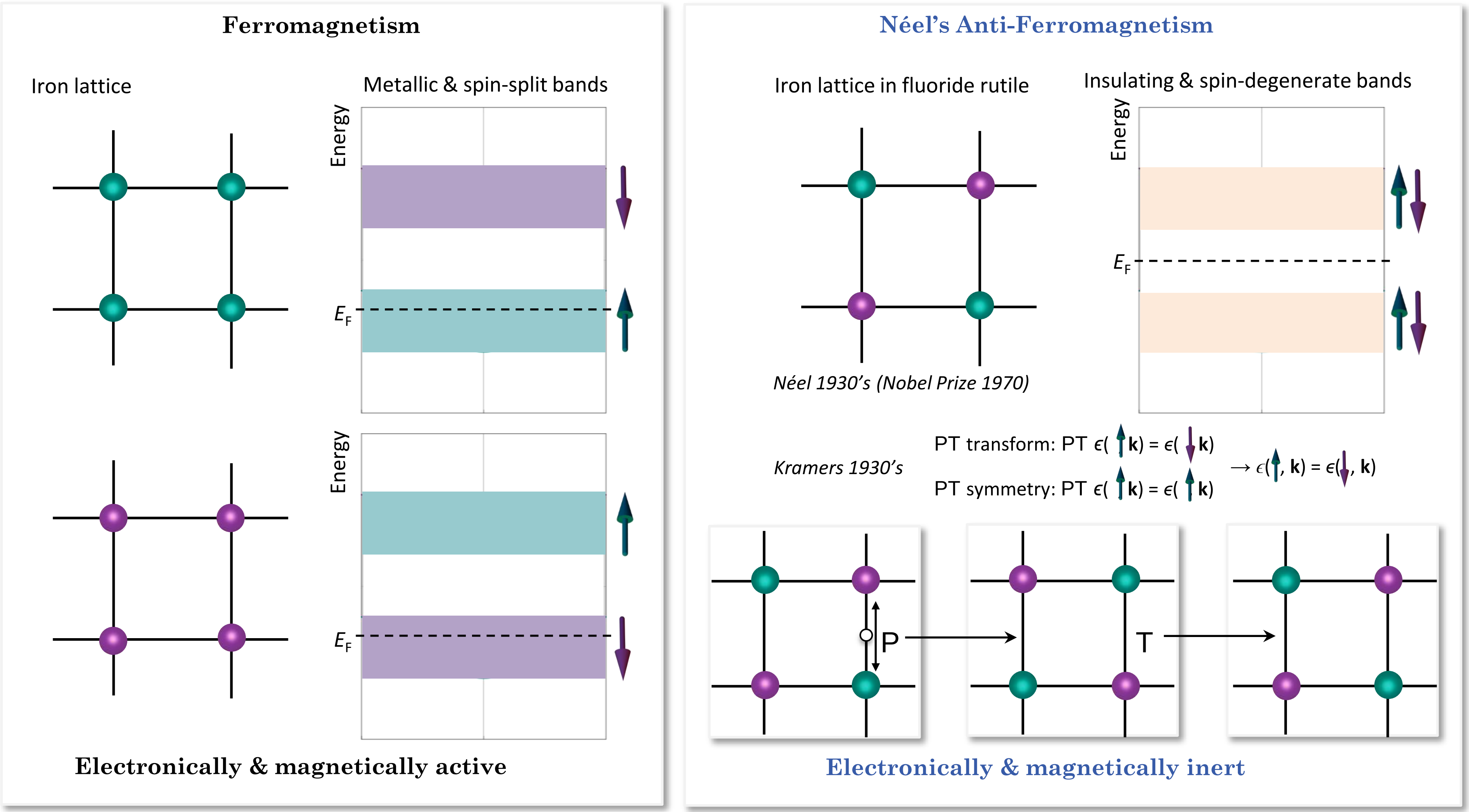


An emerging basic magnetic phase

- Compensated non-frustrated collinear magnetic structures
- Spin-split non-relativistic uncorrelated band structures
- Alternative phenomenology of core spin physics & electronics
- Separate spin-conserving symmetry class
- Abundant among magnetic materials
- Relevant in many fields

Šmejkal et al. Science Adv. 6, eaaz8809 (2020)
González-Hernández et al. PRL 126, 127701 (2021)
Šmejkal et al. Nat. Rev. Mater in press (arxiv:2107.03321)
Feng et al. arxiv: 2002.08712
Šmejkal et al. arxiv: 2103.12664
Šmejkal, Sinova & TJ arxiv:2105.05820

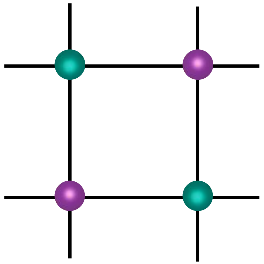

Ferromagnetism and core spin physics & electronics
Iron lattice
Metallic & spin-split bands
Energy
E_F
Energy
E_F
Anomalous Hall effect
Hall 1881
-V
+V
Giant magnetoresistance
Fert & Grünberg
Nobel Prize 2007
Spin-transfer torque
Slonczewski & Berger
Buckley Prize 2013
R
R
Electronically & magnetically active
Transverse non-dissipative topological
Longitudinal large-signal commercial



Ferromagnetism
Iron lattice
Metallic & spin-split bands
Energy
E_F
Energy
E_F
Electronically & magnetically active
Néel's Anti-Ferromagnetism
Iron lattice in fluoride rutile
Insulating & spin-degenerate bands
Energy
E_F
Néel 1930's (Nobel Prize 1970)
Kramers 1930's
PT transform: PT ϵ(↑ k) = ϵ(↓ k)
PT symmetry: PT ϵ(↑ k) = ϵ(↑ k)
→ ϵ(↑, k) = ϵ(↓, k)
P
T
Electronically & magnetically inert

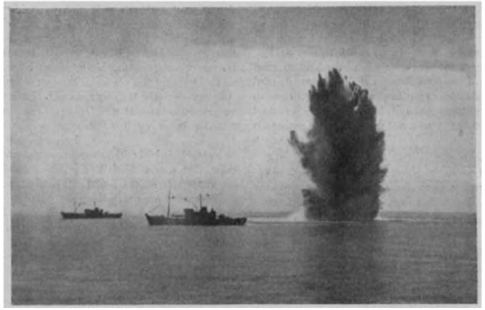
"Néel's paradox"

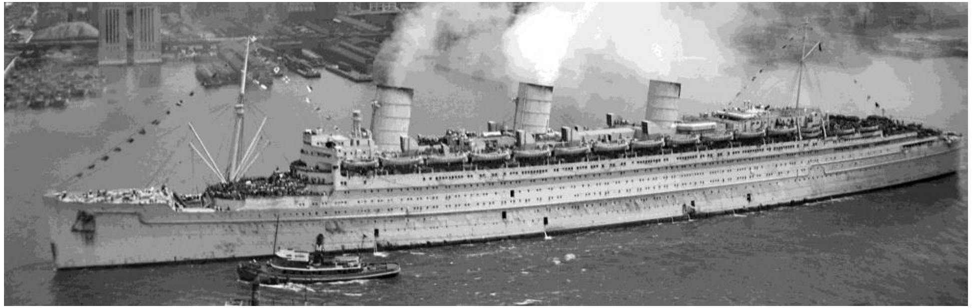
Nobel lecture on discovery of **antiferromagnetism**



"… interesting but does not appear to have any practical applications…"

Defense against magnetic mines by **demagnetizing** entire ship hulls during 2nd World War

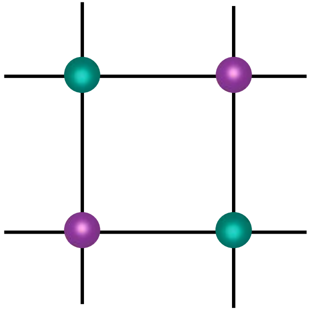




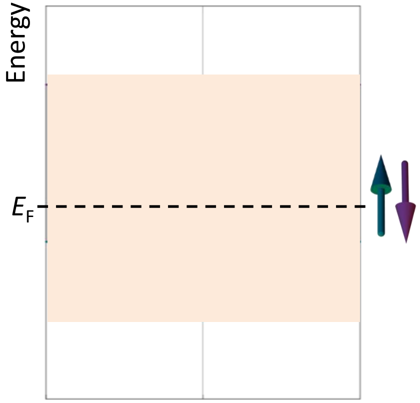
Why magnets that microscopically, precisely, and for free "demagnetize" themselves cannot be useful?

Antiferromagnetism

CuMnAs, Mn_2Au

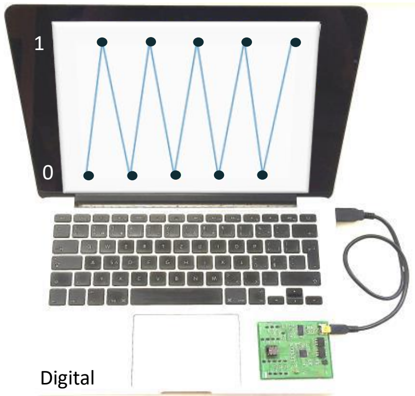


Metallic & spin-degenerate bands

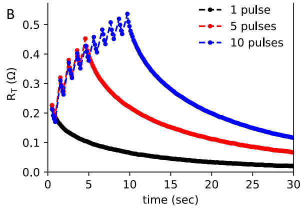


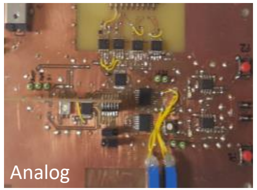
Spintronics without magnetization

but also without anomalous Hall effect, giant magnetoresistance and spin-transfer torque

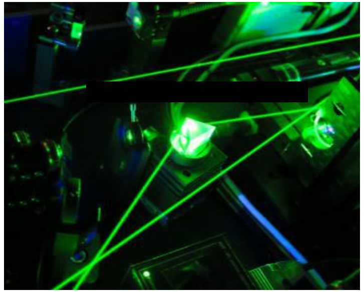


Proof-of-concept microelectronic memory bits





Stray-field-free & field-insensitive neuromorphic logic-in-memory

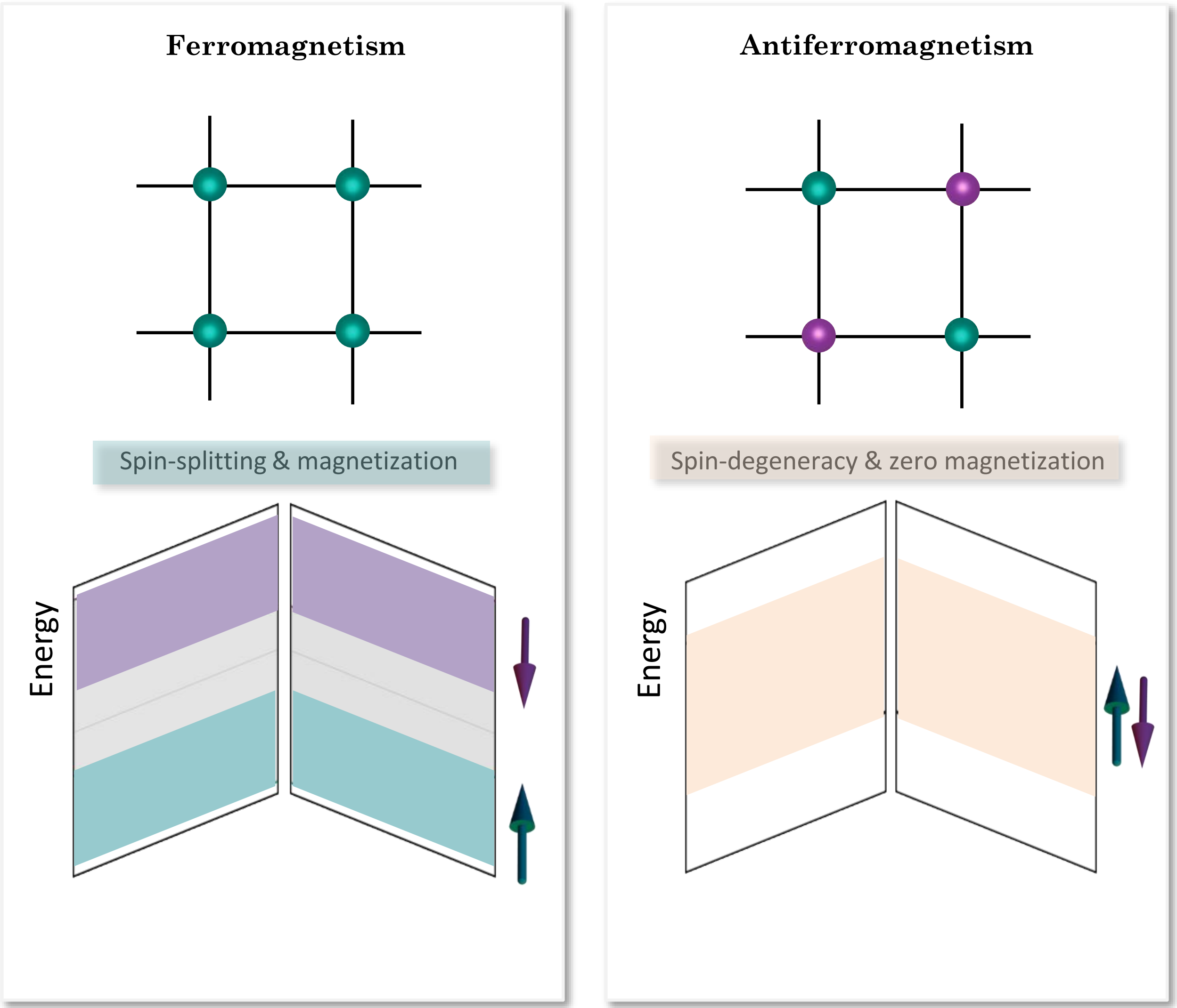


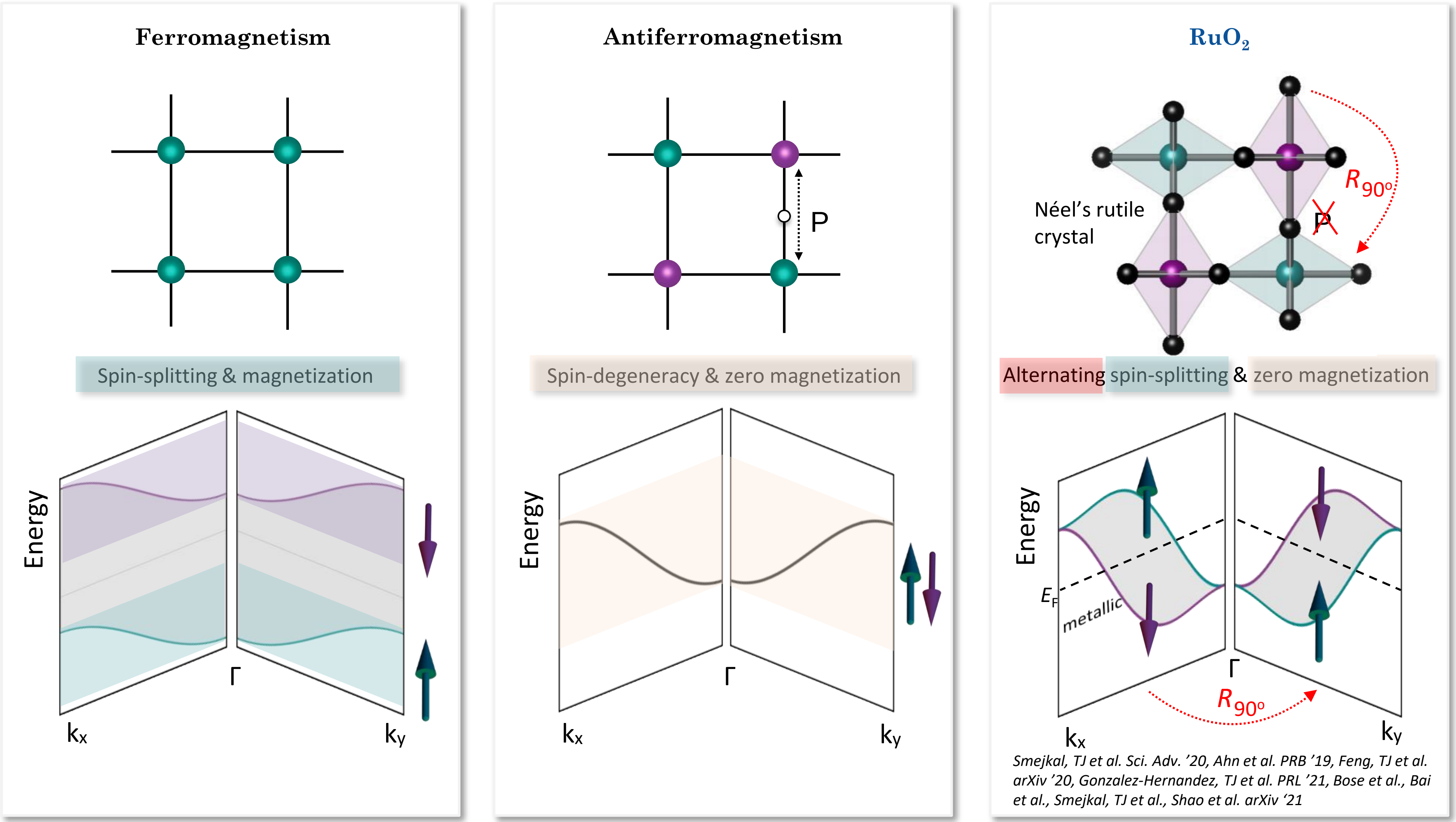
Electronic & THz & fs-laser pulse switching

Wadley, TJ et al., Science '16, Olejnik, TJ et al., Nature Commun. '17, Wadley, TJ et al., Nature Nanotech. '18

Olejnik, TJ et al. Sciene Adv. '18, Kaspar, TJ et al., Nature Electron. '21

Reviews: TJ et al. Nature Nanotech. '16, Nature Phys. Focus '18, Baltz et al. RMP '18, Song et al. Nanotech. '18, Siddiqui et al. J. Appl. Phys. '20


Ferromagnetism
Spin-splitting & magnetization
Energy
Antiferromagnetism
Spin-degeneracy & zero magnetization
Energy


Ferromagnetism
Spin-splitting & magnetization
Energy
Γ
kx
ky
Antiferromagnetism
P
Spin-degeneracy & zero magnetization
Energy
Γ
kx
ky
RuO₂
Néel's rutile crystal
R90°
P
Alternating spin-splitting & zero magnetization
Energy
EF
metallic
Γ
R90°
kx
ky
Smejkal, TJ et al. Sci. Adv. '20, Ahn et al. PRB '19, Feng, TJ et al. arXiv '20, Gonzalez-Hernandez, TJ et al. PRL '21, Bose et al., Bai et al., Smejkal, TJ et al., Shao et al. arXiv '21

Altermagnetism

RuO_2

Instead of a ferromagnetic or antiferromagnetic anomaly, an emerging 3rd basic magnetic phase

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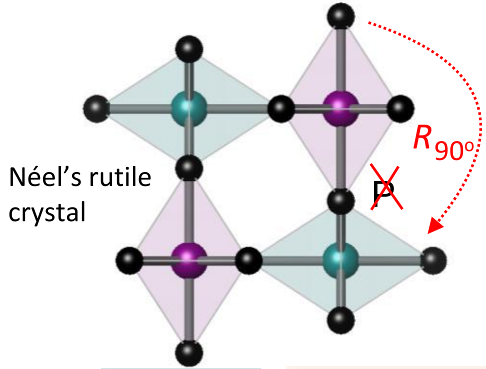
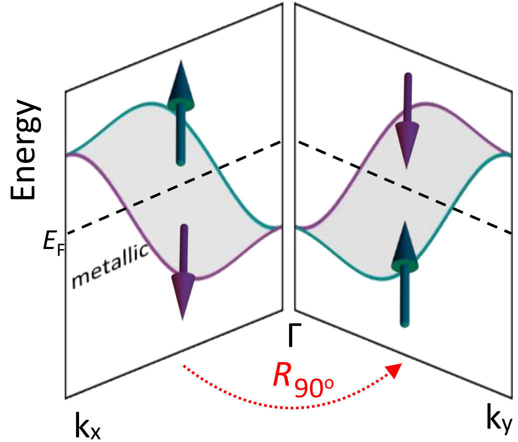


Alternating spin-splitting & zero magnetization



Šmejkal, Sinova & TJ arxiv:2105.05820

Smejkal, TJ et al. Sci. Adv. '20, Ahn et al. PRB '19, Feng, TJ et al. arXiv '20, Gonzalez-Hernandez, TJ et al. PRL '21, Bose et al., Bai et al., Smejkal, TJ et al., Shao et al. arXiv '21

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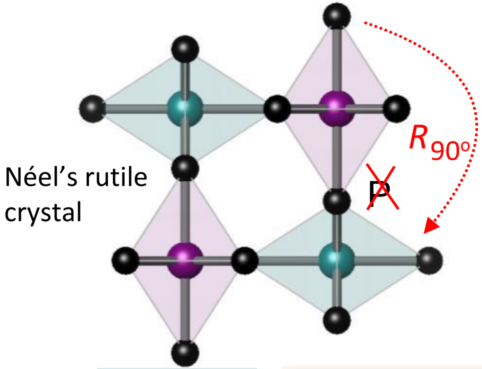
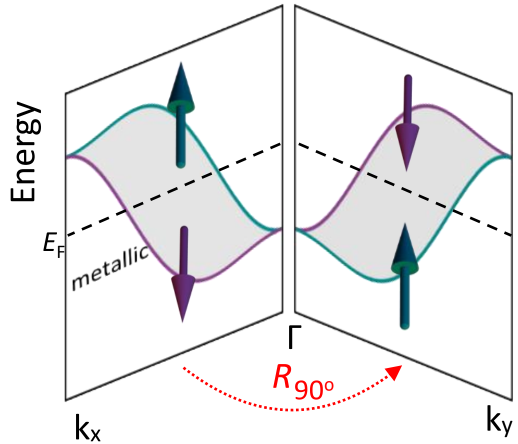
RuO_2



Alternating spin-splitting & zero magnetization



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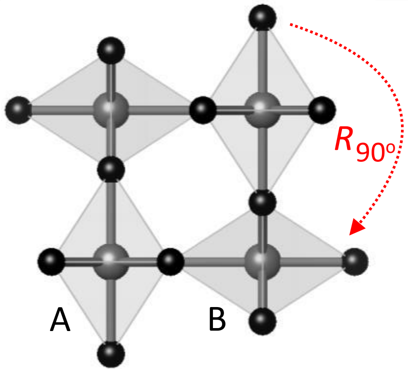
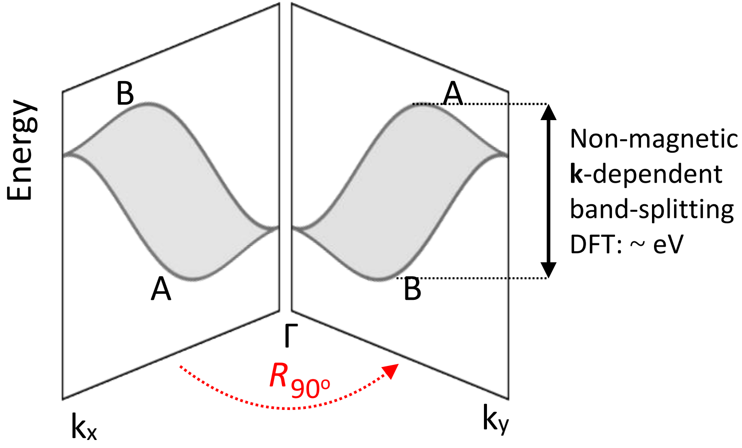
Size and k-dependence of altermagnetic spin splitting determined by electric crystal-field of non-magnetic phase

Non-magnetic
RuO_2



1) Band-anisotropy due to anisotropic electric crystal potential

2) Rotated between sublattice A and B bands due to R_{90^o}



Šmejkal, Sinova & TJ arxiv:2105.05820

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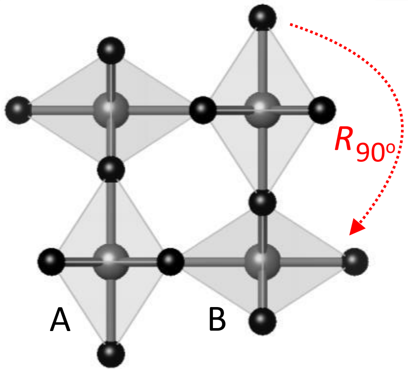
RuO_2



1) Band-anisotropy due to anisotropic electric crystal potential

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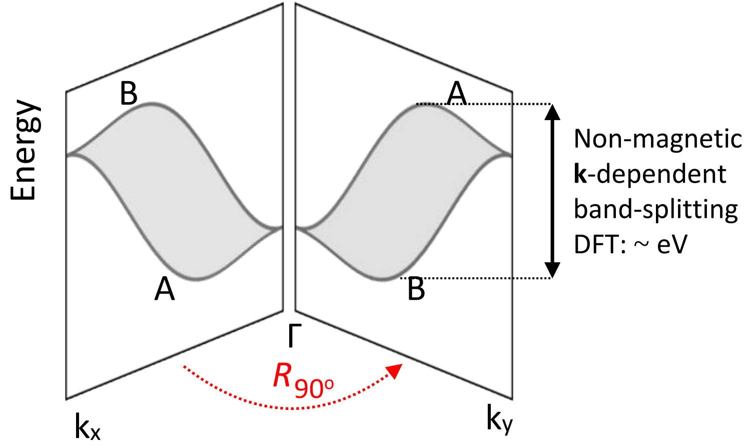
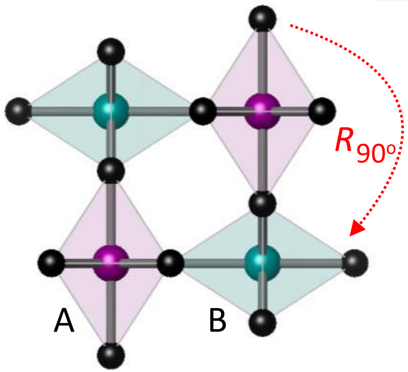
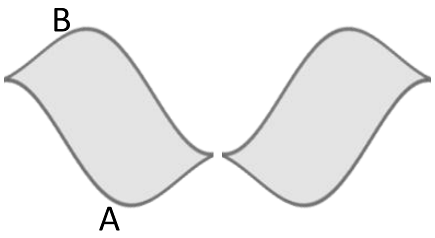


Altermagnetic
RuO_2

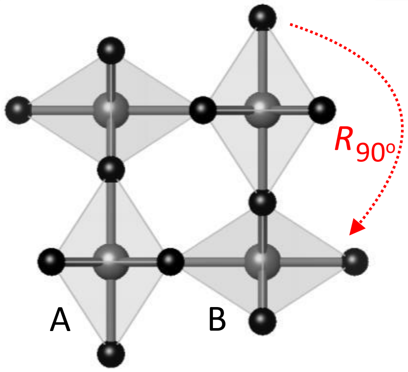




Šmejkal, Sinova & TJ arxiv:2105.05820

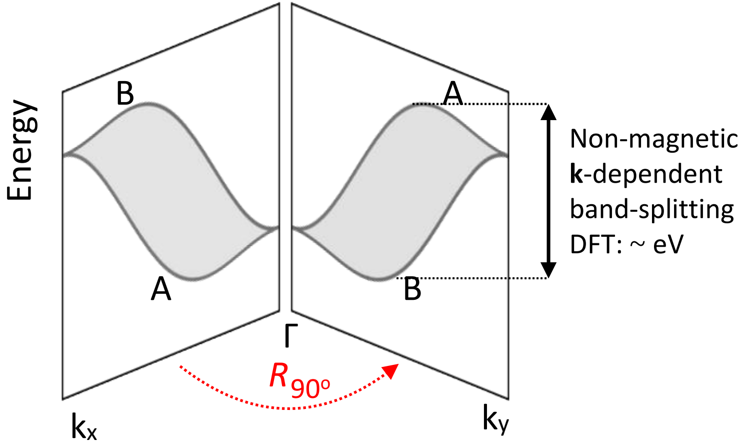
Size and k-dependence of altermagnetic spin splitting determined by electric crystal-field of non-magnetic phase

Non-magnetic RuO_2



1) Band-anisotropy due to anisotropic electric crystal potential

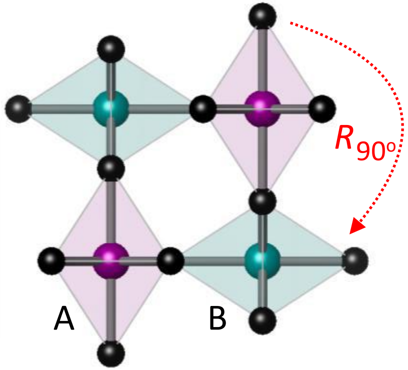
2) Rotated between sublattice A and B bands due to $R_{90°}$

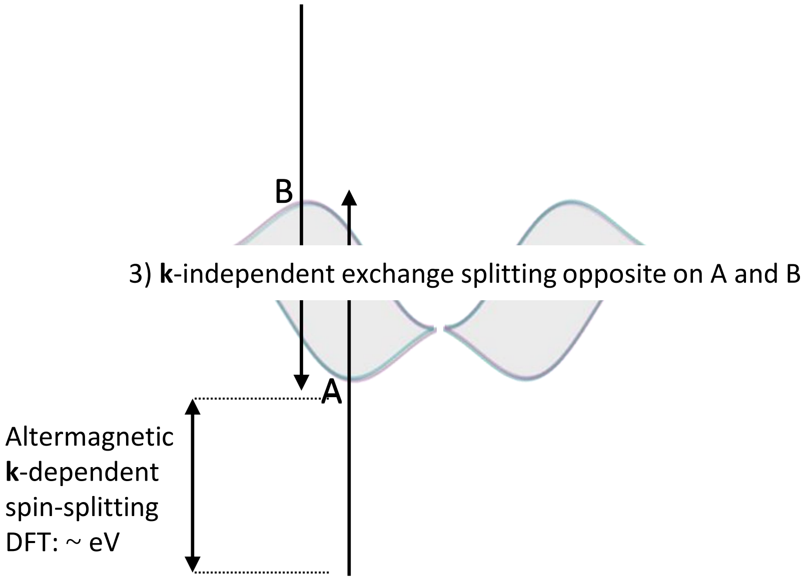


Altermagnetic RuO_2

cf.

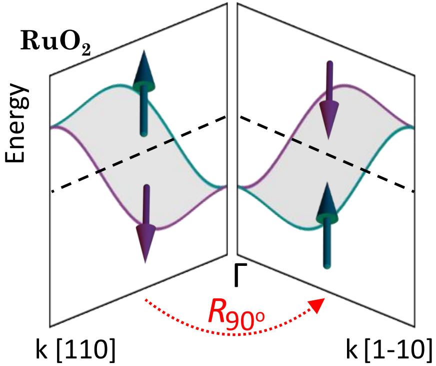
- *Magnetic-exchange global ferromagnetic order*
- *Electric & weak relativistic global inversion-asymmetry*

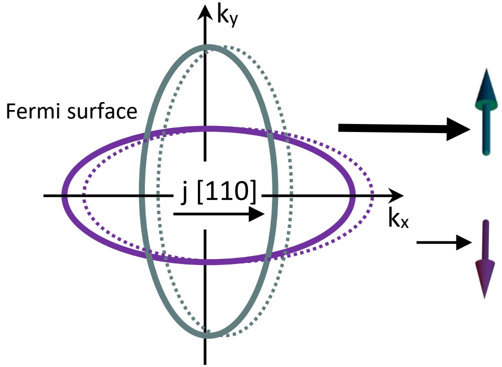




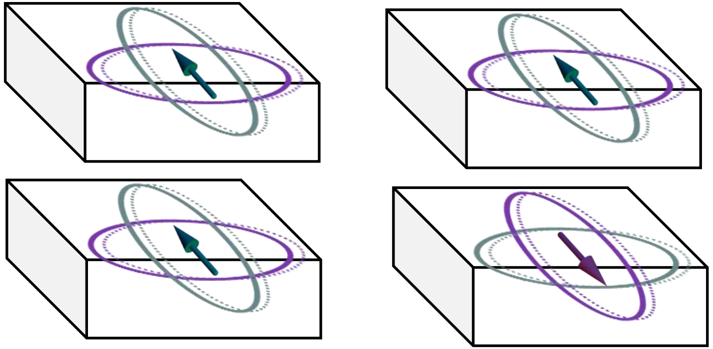
Šmejkal, Sinova & TJ arxiv:2105.05820

Longitudinal spin current & giant magnetoresistance





R DFT: GMR ~ 100 % R

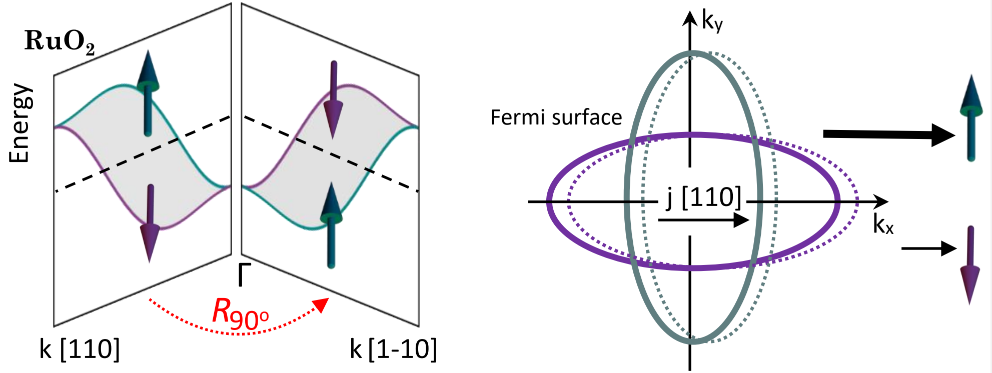


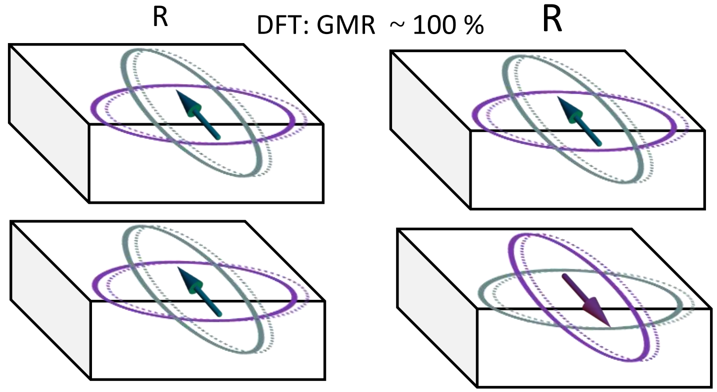
cf. stray-fields in ferromagnets

Also TMR ~ 100 %

Smejkal, TJ et al. arXiv '21, Shao et al. arXiv '21

Longitudinal spin current & giant magnetoresistance





cf. stray-fields in ferromagnets

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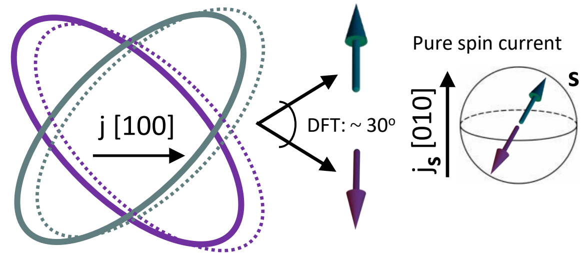
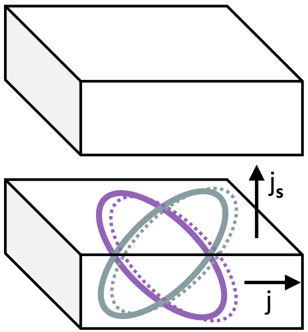
Smejkal, TJ et al. arXiv '21, Shao et al. arXiv '21

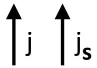
Transverse spin current & spin-splitter torque



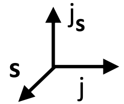


cf.

- *Ferromagnetic spin-transfer torque*



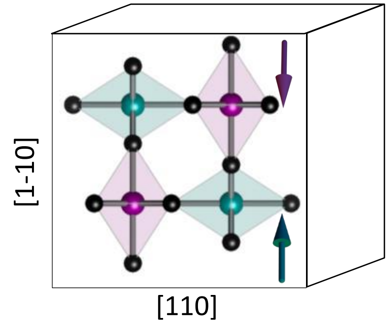
- *Weak relativistic spin-orbit torque*

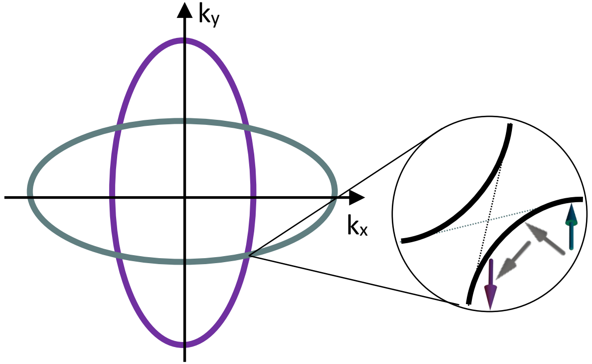


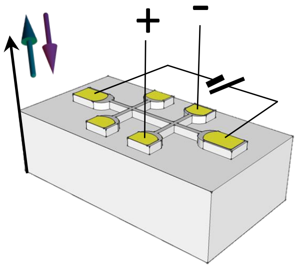
DFT: *Gonzalez-Hernandez, TJ et al. PRL '21*

Experiment: *Bose et al., Bai et al., Karube et al. arXiv '21*

Relativistic Berry curvature & anomalous Hall effect



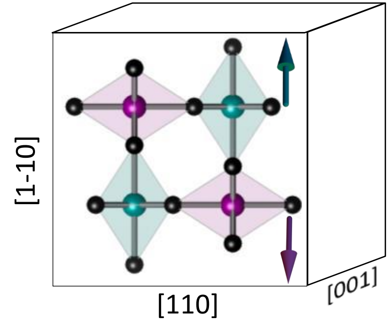


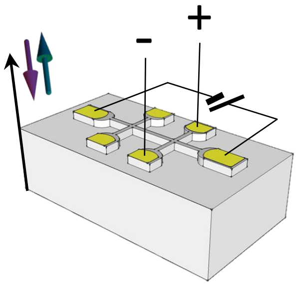


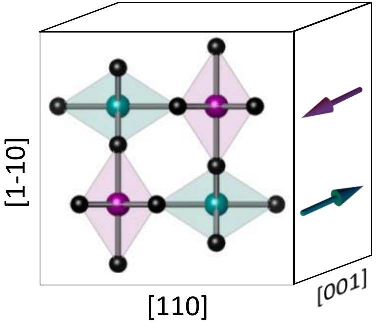
cf. accidental in ferromagnets

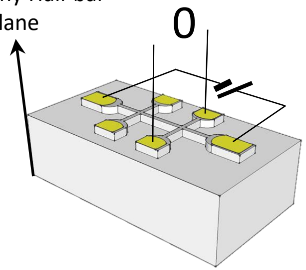
DFT: *Smejkal, TJ et al. Sci. Adv. '20*

Experiment: *Feng, TJ et al. arXiv '20*









cf. ~ **M** *in ferromagnets*

Review: Šmejkal, TJ et al. Nat. Rev. Mater. in press, arXiv'21

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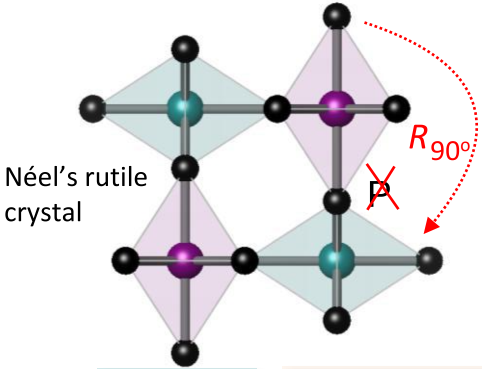
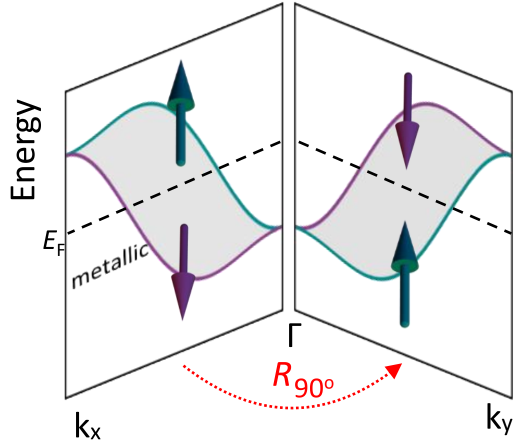


Alternating spin-splitting & zero magnetization



Šmejkal, Sinova & TJ arxiv:2105.05820

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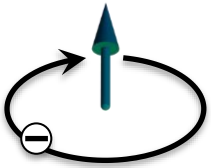
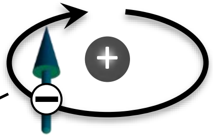
Coupled or uncoupled spin and real space?

Classical
Orbital magnetic moment

Relativistic QM
Spin-orbit coupling



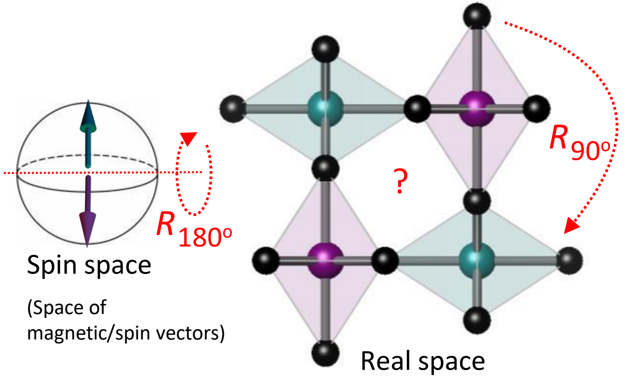


Magnetic symmetry transformations in coupled spin and real space:
Landau & Lifshitz, vol. 8 (1960)

$[R_{90^o}||R_{90^o}]$

~~$[R_{180^o}||R_{90^o}]$~~

RuO_2



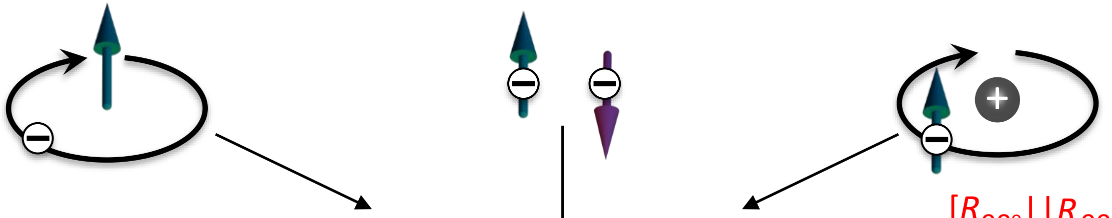
Šmejkal, Sinova & TJ arxiv:2105.05820

Coupled or uncoupled spin and real space?

Classical
Orbital magnetic moment

Non-relativistic QM (exchange)
Magnetic ordering of spins

Relativistic QM
Spin-orbit coupling



Magnetic symmetry transformations in coupled spin and real space:
Landau & Lifshitz, vol. 8 (1960)

$[R_{90^o} || R_{90^o}]$

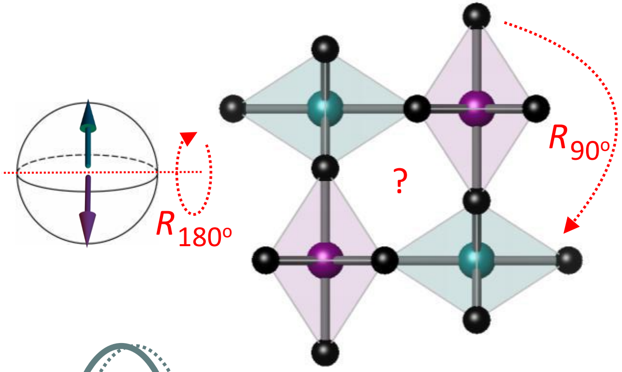
~~$[R_{180^o} || R_{90^o}]$~~

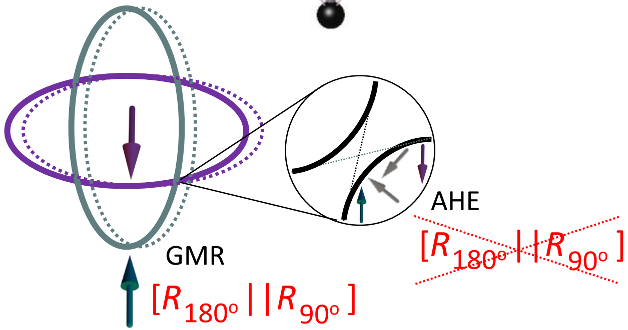
Spin symmetry transformations in uncoupled spin and real space:
Litvin & Opechowski, Physica (1974)

$[R_{90^o} || R_{90^o}]$

$[R_{180^o} || R_{90^o}]$

RuO_2





Only spin symmetry transformations describe strong non-relativistic spin physics & electronics

A huge generalization compared to magnetic symmetry transformations: The tool to describe new magnetic phases

Šmejkal, Sinova & TJ arxiv:2105.05820

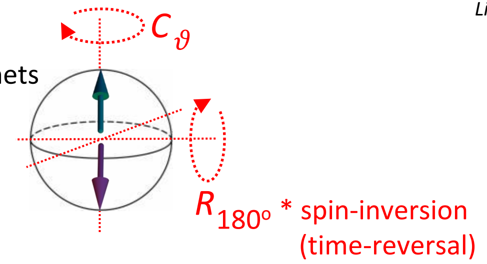
Spin symmetry transformations in uncoupled spin and real space

Spin space alone

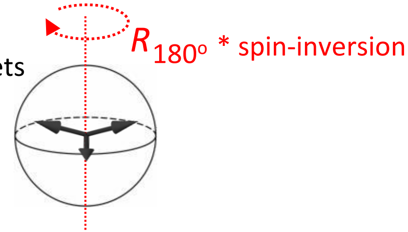
Litvin & Opechowski, Physica (1974)

Spin space & real space

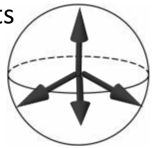
Collinear magnets

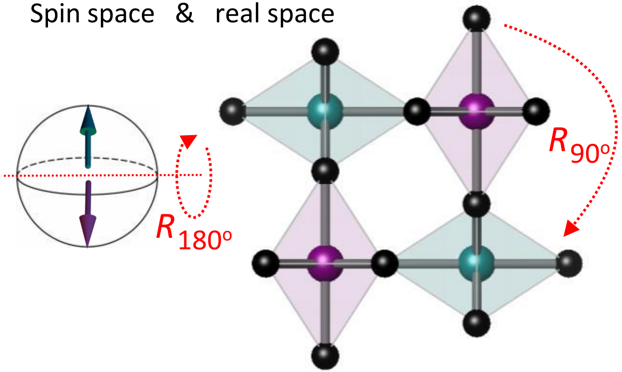


Coplanar magnets



Non-coplanar magnets





Only spin symmetry transformations delimit collinear non-frustrated magnets

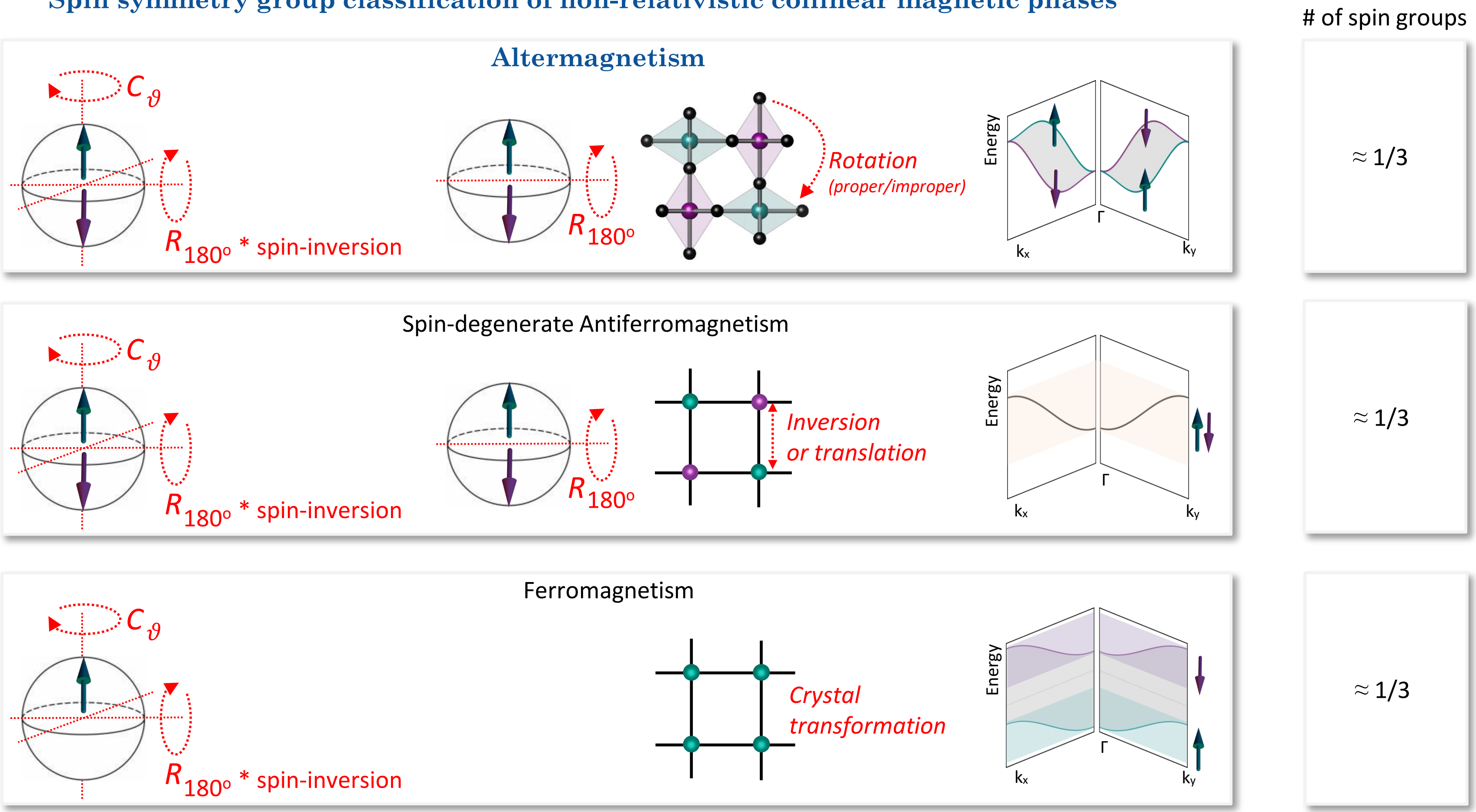
Spin-conserving altermagnetism

Only spin symmetry transformations describe strong non-relativistic spin physics & electronics

A huge generalization compared to magnetic symmetry transformations: The tool to describe new magnetic phases

Šmejkal, Sinova & TJ arxiv:2105.05820

Spin symmetry group classification of non-relativistic collinear magnetic phases



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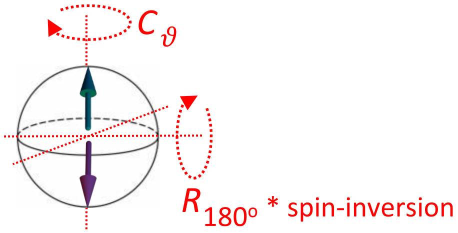
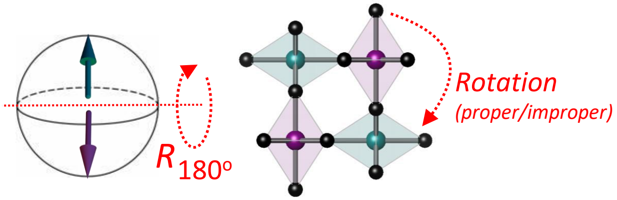
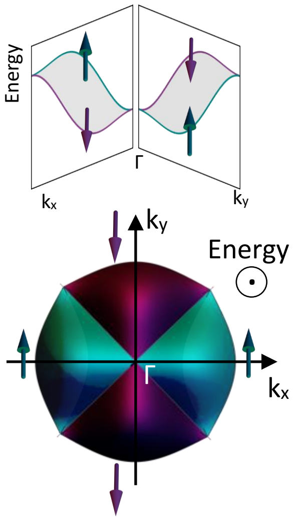
Altermagnetism

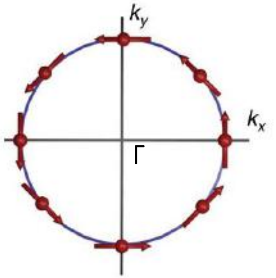






- Inversion symmetric or asymmetric magnetic crystals
- Bands always invariant under inversion of **k**
- **Γ**-point always spin-degenerate, other TRIMs can be spin-split
- **k**-independent spin axis
- Even spin winding number

cf. relativistic (Rashba)



- *Inversion-asymmetric non-magnetic crystals*
- *Bands inversion-asymmetric*
- *All TRIMs spin-degenerate*
- ***k**-dependent spin-texture*
- *Odd spin winding number*

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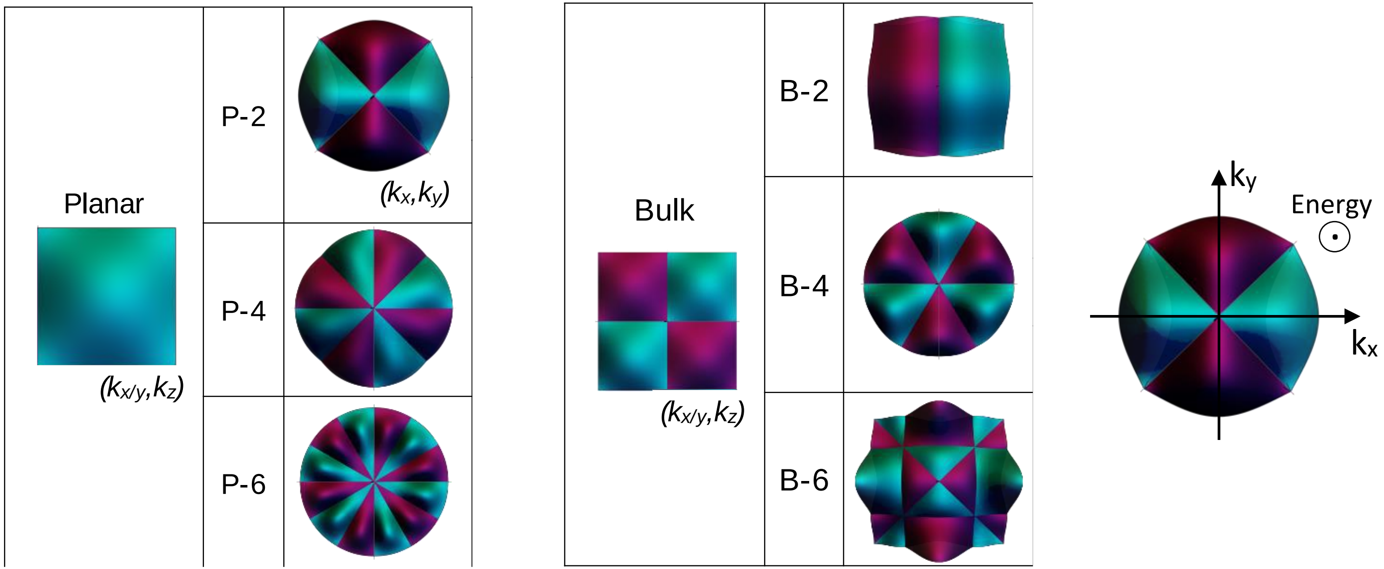
Altermagnetism

Planar & bulk spin winding number 2, 4, and 6



cf. relativistic planar Rashba, Dresselhaus, and bulk Weyl

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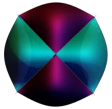
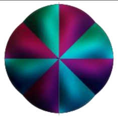
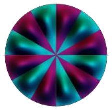
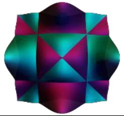
Altermagnetism

Recall: Non-relativistic spin groups = relativistic magnetic groups + **much more**

None of the altermagnetic spin groups has a corresponding magnetic group

AM spin group	AM spin winding number		
$2_m2_m1_m$ (8)	Planar  $(k_{x/y},k_z)$	P-2	 (k_x,k_y)
$2_{4/}1_m$ (8)			
$2_{4/}1_m1_m1_m$ (16)			
$1_{4/}1_m2_m2_m$ (16)		P-4	
$1_{6/}1_m2_m2_m$ (24)		P-6	
$2_{2/}2_m$ (4)	Bulk 	B-2	
$1_{\bar{3}}2_m$ (12)		B-4	
$2_{6/}2_m$ (12)			
$2_{6/}2_m2_m1_m$ (24)			
$1_m1_32_m$ (48)		B-6	

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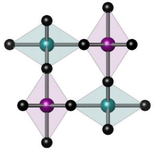
Altermagnetism

- Systematic altermagnetic symmetry description of earlier identified ”antiferromagnetic anomalies”



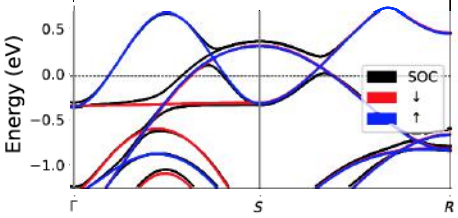
Lopez-Moreno et al. PRB ‘12
Noda et al. Phys. Chem Chem Phys. ‘16
Šmejkal, TJ et al. Sci. Adv. ’20
Ahn at al. PRB ‘19
Hayami et al. J. Phys.Soc. Jap. ‘19
Naka et al. Nat. Commun. ’19
Hayami et al. PRB ’20
Yuan et al. PRB ’20
Feng et al. arXiv ’20
Reichlova et al. arXiv ‘20
Yuan et al. PRM ‘21
Egorov et al. J. Phys, Chem Lett. ‘21
Gonzalez-Hernandez, TJ et al. PRL ‘21
Ma et al Nat. Commun. ‘21
Naka et al. PRB ‘21
Šmejkal, TJ et al. Nat. Rev. Mater. in press
Mazin et al. arXiv ‘21
Bose et al. arXiv ‘21
Bai et al. arXiv ‘21
Šmejkal, et al. arXiv ‘21
Shao et al. arXiv ‘21

Rutile RuO_2: metallic & T_N > 300K

Berlijn et al. PRL ‘17, Zhu et al. PRL ‘19

Spin-splitting & AHE anomaly

Šmejkal, TJ et al. Sci. Adv. ‘20



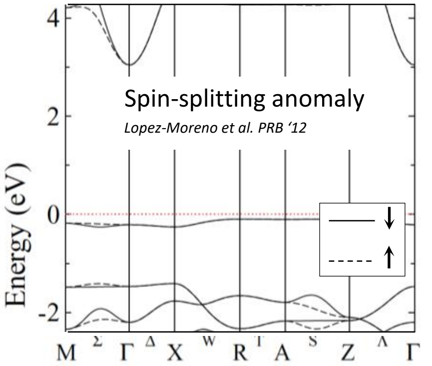
Other metallic:
Mn_5Si_3

Rutile FeF_2: insulating & $T_N \approx$ 80K

Néel RMP 1953

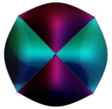
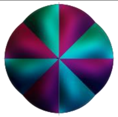
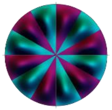
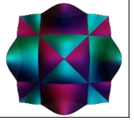
Spin-splitting anomaly

Lopez-Moreno et al. PRB ‘12



Other insulating/semiconducting:
MnO_2, MnF_2, CoF_2,
organic κ-Cl, MnTe, V_2Se_2O, $FeSb_2$,…

$[R_{180^o}||R_{90^o}]$ $2_{4/}1_m1_m1_m$

AM spin group		AM spin winding number	
$2_m2_m1_m$ (8)	Planar  $(k_{x/y},k_z)$	P-2	 (k_x,k_y)
$2_{4/}1_m$ (8)			
$1_{4/}1_m2_m2_m$ (16)		P-4	
$1_{6/}1_m2_m2_m$ (24)		P-6	
$2_{2/}2_m$ (4)	Bulk 	B-2	
$1_{\bar{3}}2_m$ (12)		B-4	
$2_{6/}2_m$ (12)			
$2_{6/}2_m2_m1_m$ (24)			
$1_m1_{\bar{3}}2_m$ (48)		B-6	

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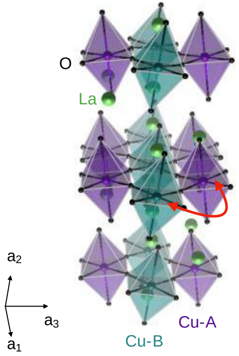
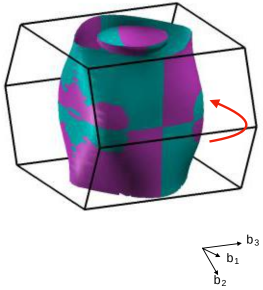
Altermagnetism

- Identification of altermagnetic materials

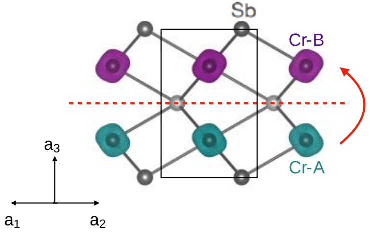
La_2CuO_4: Parent cuprate of high T_c *d*-wave superconductor & T_N > 300 K



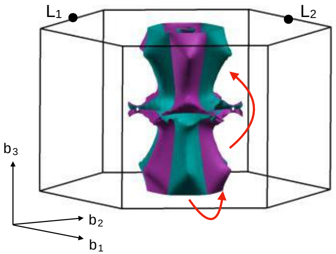
Fermi surface



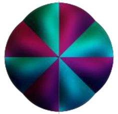
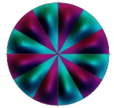
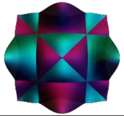
CrSb: Metallic & T_N > 700 K



Fermi surface



AM crystallographic group

AM material	AM spin group	AM crystallographic group	AM spin winding number			AM orbital harmonic
La_2CuO_4, $FeSb_2$	*${}^{2}m{}^{2}m{}^{1}m$ (8)*	*2/m*	Planar  $(k_{x/y},k_z)$	P-2	 (k_x,k_y)	*d*-wave
KRu_4O_8	*${}^{2}4/{}^{1}m$ (8)*					
RuO_2, MnO_2, MnF_2	*${}^{2}4/{}^{1}m{}^{1}m{}^{1}m$ (16)*	*mmm*				
$KMnF_3$	*${}^{1}4/{}^{1}m{}^{2}m{}^{2}m$ (16)*	*4/m*		P-4		*g*-wave
	${}^{1}6/{}^{1}m{}^{2}m{}^{2}m$ (24)	*6/m*		P-6		*i*-wave
CuF_2	*${}^{2}2/{}^{2}m$ (4)*	*$\bar{1}$*	Bulk 	B-2		*d*-wave
CoF_3, FeF_3, Fe_2O_3	*${}^{1}\bar{3}{}^{2}m$ (12)*	*$\bar{3}$*		B-4		*g*-wave
	${}^{2}6/{}^{2}m$ (12)					
CrSb, MnTe, VNb_3S_6	*${}^{2}6/{}^{2}m{}^{2}m{}^{1}m$ (24)*	*$\bar{3}m$*				
	${}^{1}m{}^{1}\bar{3}{}^{2}m$ (48)	*m3*		B-6		*i*-wave

Šmejkal, Sinova & TJ arxiv:2105.05820

Range of materials:

- 3D, 2D (no crystal rotation in 1D chains)

- insulating, semiconducting, metallic, superconducting

- rutiles, ruthenates, perovskites, cuprates, ferrites, silicides, pnictides, chalcogenides,…

Range of fields:

- Spintronics without magnetization and relativity
  Naka et al. Nat. Commun. '19 , Gonzalez-Hernandez, TJ et al. PRL '21, Naka et al. PRB '21, Bose et al. arXiv '21, Bai et al. arXiv '21, Smejkal, et al. arXiv '21, Shao et al. arXiv '21

- Spin-polarized quasi-particles near altermagnetic band-degeneracies
  Šmejkal, TJ arxiv '21, Liu et al. arXiv '21

- Valleytronics at time-reversal invariant momenta
  Reichlova et al. arXiv '20, Ma et al. Nat. Commun. '21, Smejkal, et al. arXiv '21

- Electro-magnetic multipoles in zero-dipole toroidal magnets
  Hayami et al. J. Phys.Soc. Jap. '19, Hayami et al. PRB '20

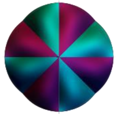
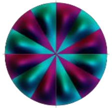
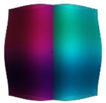
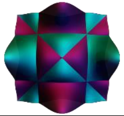
- Magnetic topological insulators (QAHE) with vanishing magnetization
  Šmejkal, TJ et al. Sci. Adv. '20, Nat. Rev. Mater. in press (arXiv '21)

- Fermi-liquid anisotropic (*d*-wave) instabilities without correlations
  Šmejkal, Sinova & TJ arXiv ,21

- Superconductivity and altermagnetism
  Šmejkal, Sinova & TJ arXiv ,21

Altermagnetism

AM crystallographic group ↓

AM material	AM spin group	AM crystallographic group	AM spin winding number			AM orbital harmonic
La_2CuO_4, $FeSb_2$	*$2_m2_m1_m$ (8)*	*2/m*	Planar  ($k_{x/y}$,k_z)	P-2	 (k_x,k_y)	*d*-wave
KRu_4O_8	*$2_{4/}1_m$ (8)*					
RuO_2, MnO_2, MnF_2	*$2_{4/}1_m1_m1_m$ (16)*	*mmm*				
$KMnF_3$	*$1_{4/}1_m2_m2_m$ (16)*	*4/m*		P-4		*g*-wave
	$1_{6/}1_m2_m2_m$ (24)	*6/m*		P-6		*i*-wave
CuF_2	*$2_{2/}2_m$ (4)*	*$\bar{1}$*	Bulk 	B-2		*d*-wave
CoF_3, FeF_3, Fe_2O_3	*$1_{\bar{3}}2_m$ (12)*	*$\bar{3}$*		B-4		*g*-wave
	$2_{6/}2_m$ (12)					
CrSb, MnTe, VNb_3S_6	*$2_{6/}2_m2_m1_m$ (24)*	*$\bar{3}m$*				
	$1_m1_32_m$ (48)	*m3*		B-6		*i*-wave

Šmejkal, Sinova & TJ arxiv:2105.05820

Altermagnetism:

The 1930s quantum mechanics of uncorrelated non-relativistic band theory of solids and magnetism is not a closed chapter in physics but, still today, can guide us to new discoveries, alongside the correlated and relativistic quantum mechanics.

Šmejkal, Sinova & TJ arxiv:2105.05820

Acknowledgment

JOHANNES GUTENBERG UNIVERSITÄT MAINZ
JG|U

FZU Institute of Physics of the Czech Academy of Sciences

University of Nottingham
UK | CHINA | MALAYSIA

erc
European Research Council
Established by the European Commission

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DFG
Deutsche Forschungsgemeinschaft

GAČR
CZECH SCIENCE FOUNDATION

MŠMT
MINISTRY OF EDUCATION, YOUTH AND SPORTS

Neuron
FOND NA PODPORU VĚDY

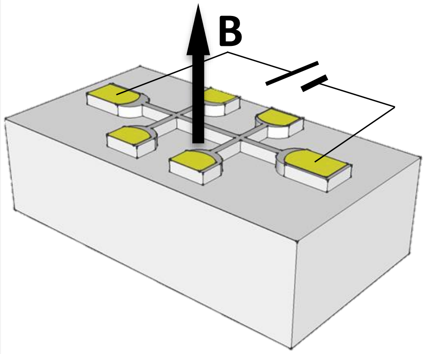
Non-dissipative Hall effect

Longitudinal current $\mathbf{j} = \sigma \mathbf{E}$ Dissipative (Joule heating) $\mathbf{j} \cdot \mathbf{E} \neq 0$

Hall effect $\mathbf{j}_{\mathrm{Hall}} = \mathbf{h} \times \mathbf{E}$ Non-dissipative (no Joule heating) $\mathbf{j}_{\mathrm{Hall}} \cdot \mathbf{E} = 0$

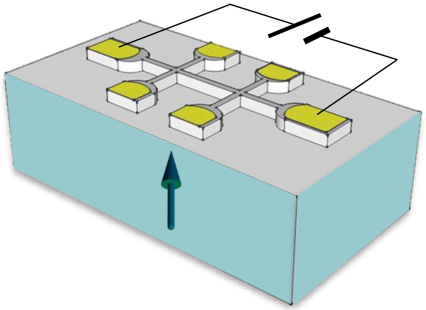
Quantum Hall effect (2D topological insulator) $\mathbf{j} = 0$ & $\mathbf{j}_{\mathrm{Hall}} = \mathbf{h} \times \mathbf{E}$

1879: Au
1980: 2D-Si



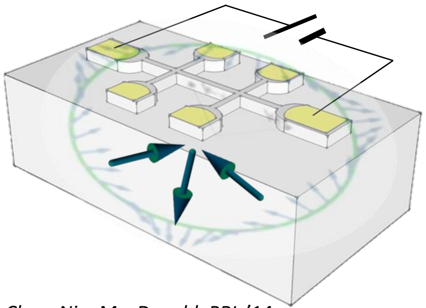
Hall
Klitzing et al.

1881: Ni



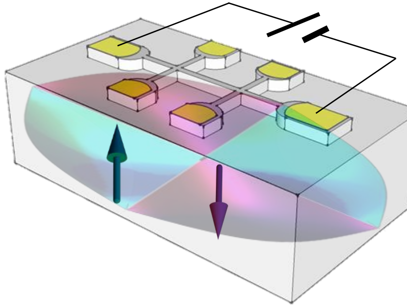
Hall

2014: Mn_3Ir



Chen, Niu, MacDonald, PRL '14
Kubler, Felser, EPL '14
Nakatsuji, Kiyohara, Higo, Nature '15
Nayak et al. Science Adv. '16

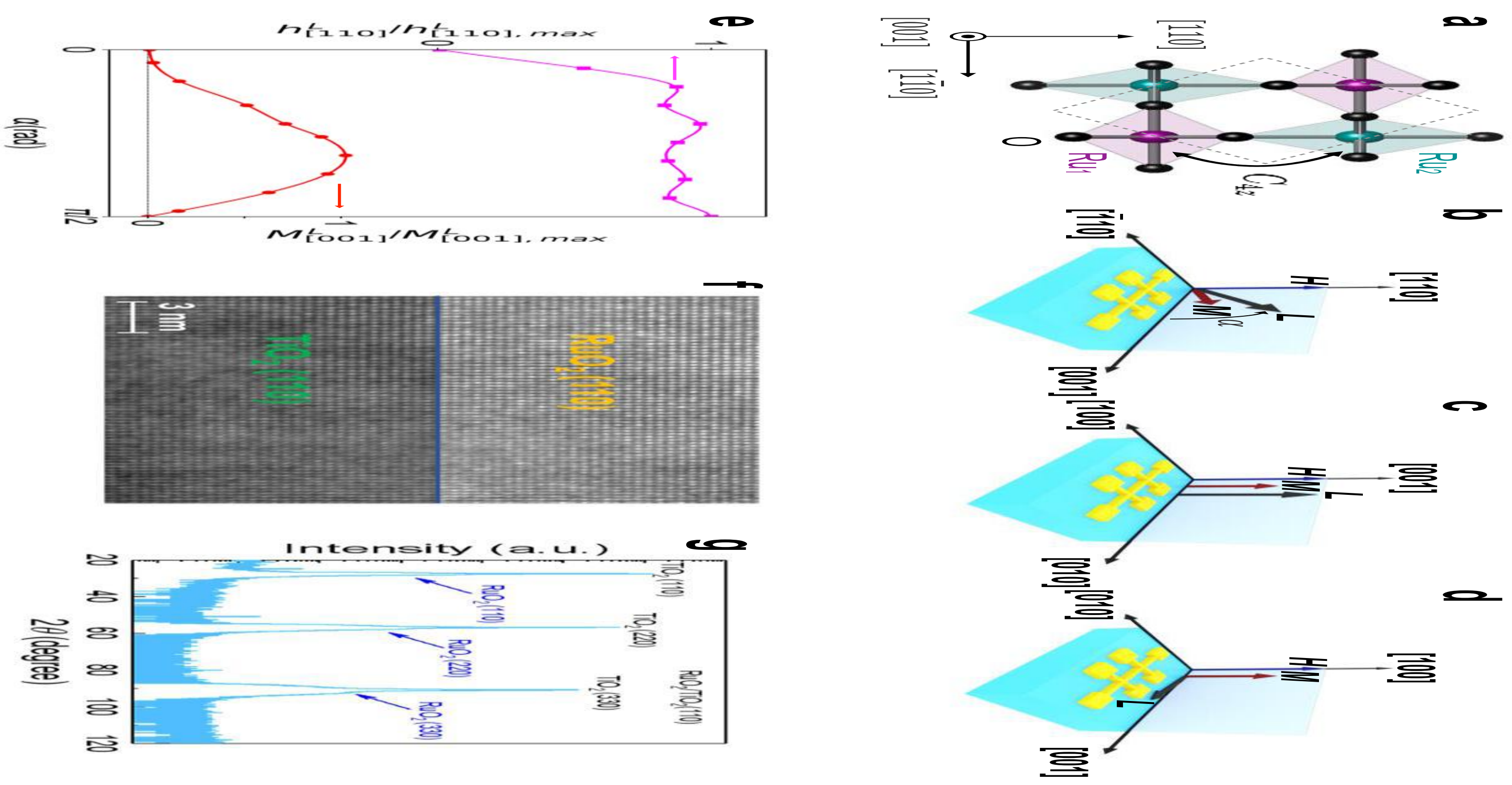
2020: RuO_2



Smejkal, et al. Science Adv. '20
Feng, et al. arXiv '20
Smejkal, et al. arXiv '21

Experimental observation of anomalous Hall effect in RuO_2

Feng, et al. arXiv2002.08712



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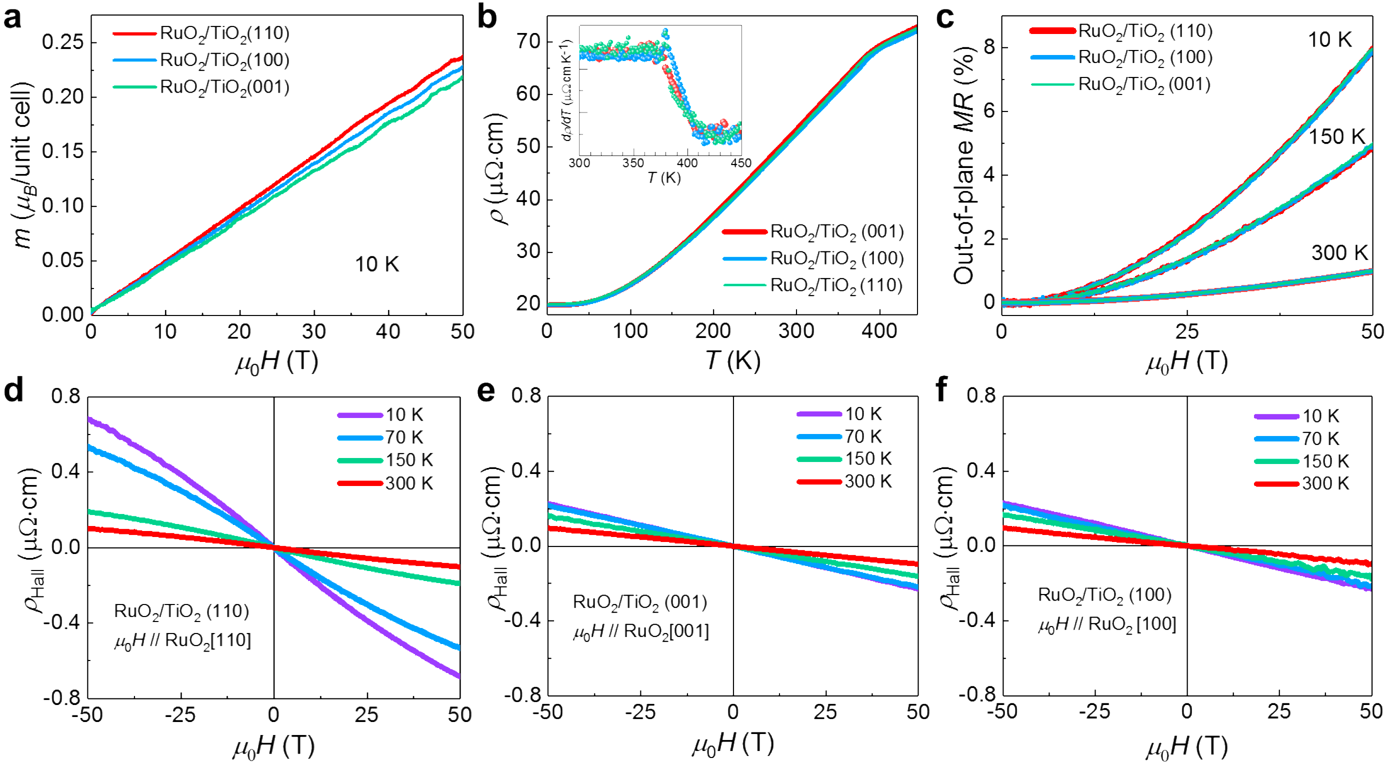

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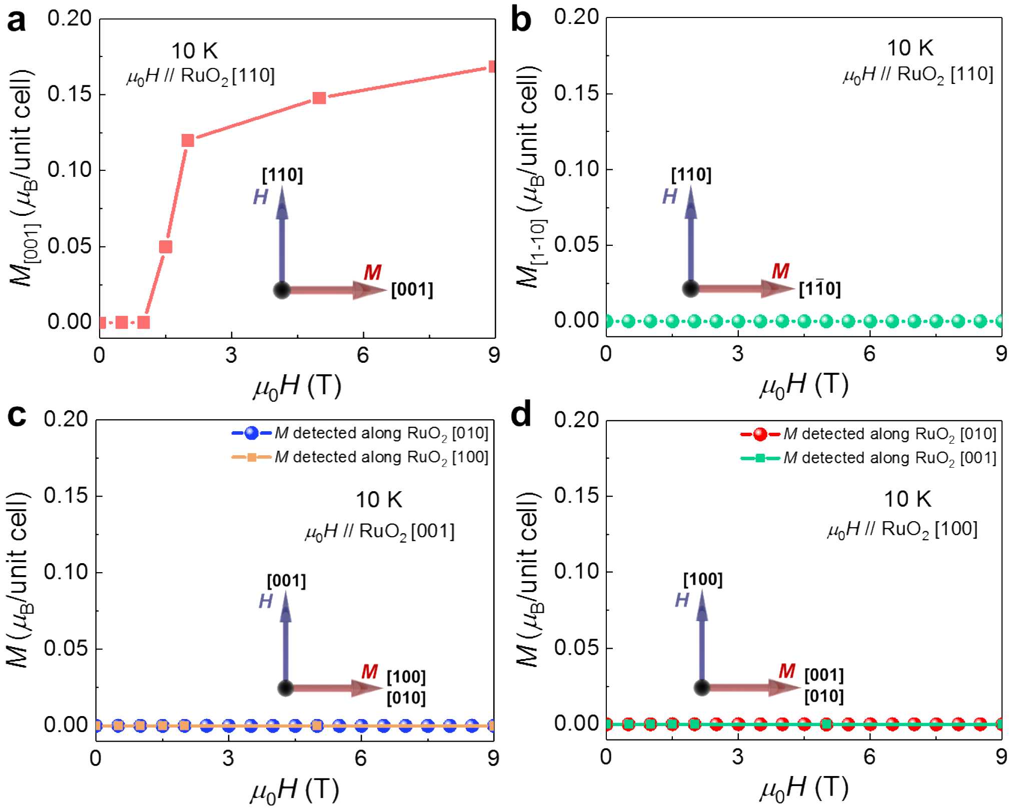

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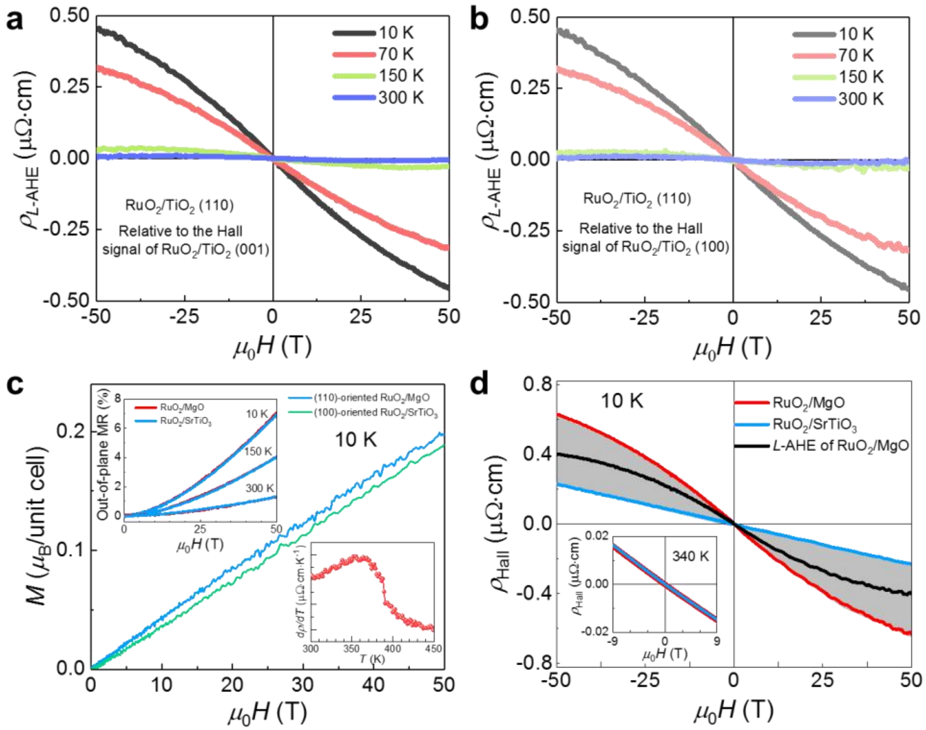
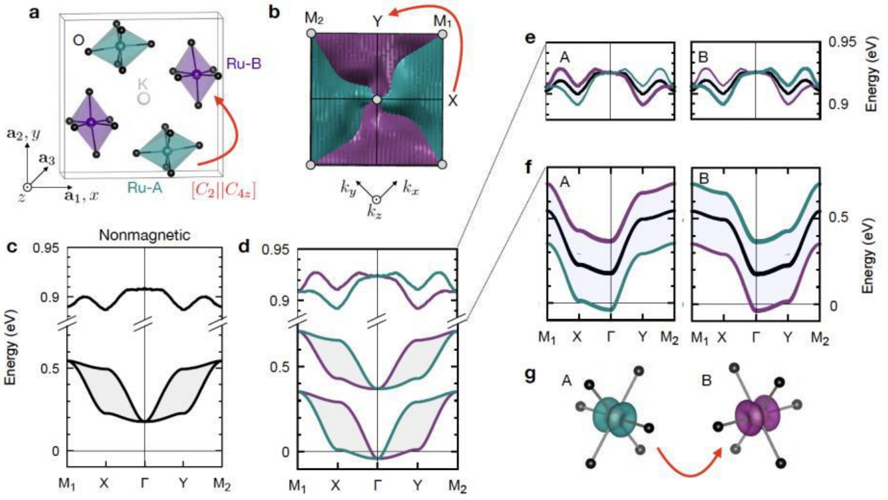
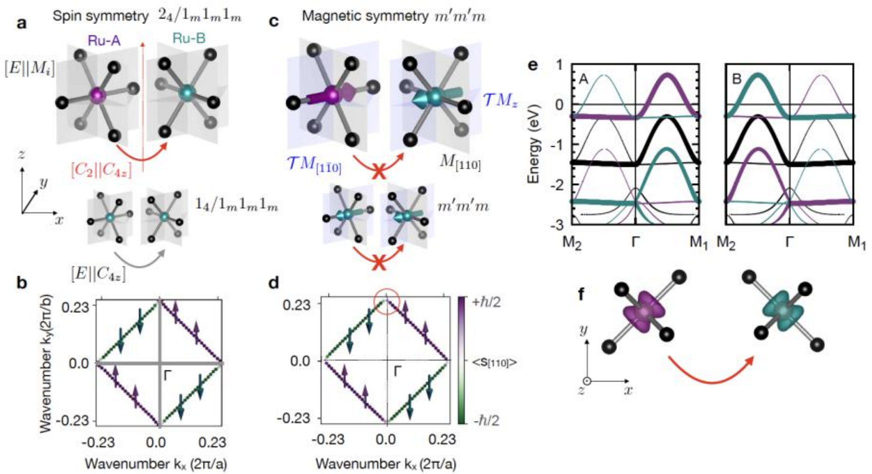


KRu_4O_8


a
O
K
Ru-B
Ru-A
$[C_2||C_{4z}]$
$\mathbf{a}_2, y$
$\mathbf{a}_3$
$\mathbf{a}_1, x$
z
b
M2
Y
M1
X
k_y
k_x
k_z
c
Nonmagnetic
Energy (eV)
0.95
0.9
0.5
0
M1 X Γ Y M2
d
0.95
0.9
0.5
0
M1 X Γ Y M2
e
A
B
0.95
0.9
Energy (eV)
f
A
B
0.5
0
Energy (eV)
M1 X Γ Y M2 M1 X Γ Y M2
g
A
B

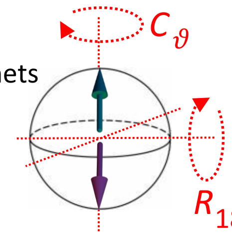
RuO_2


a
Spin symmetry $2_4/1_m1_m1_m$
Ru-A
Ru-B
$[E||M_i]$
$[C_2||C_{4z}]$
$1_4/1_m1_m1_m$
$[E||C_{4z}]$
z
y
x
c
Magnetic symmetry $m'm'm$
$\mathcal{T}M_z$
$\mathcal{T}M_{[1\bar{1}0]}$
$M_{[110]}$
$m'm'm$
b
0.23
0.0
-0.23
Wavenumber k_y (2π/b)
Γ
-0.23 0.0 0.23
Wavenumber k_x (2π/a)
d
0.23
0.0
-0.23
Γ
-0.23 0.0 0.23
Wavenumber k_x (2π/a)
+ħ/2
<S[110]>
-ħ/2
e
A
B
1
0
-1
-2
-3
Energy (eV)
M2 Γ M1 M2 Γ M1
f
y
z
x

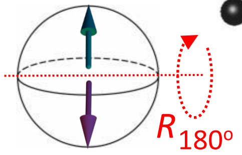
Altermagnetic spin-group derivation

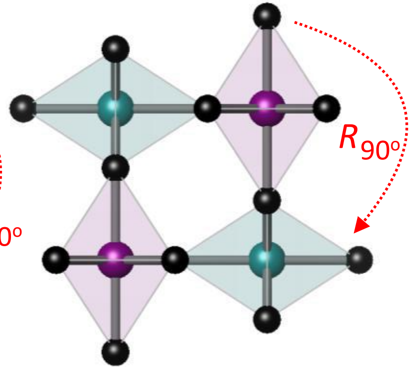
Šmejkal, Sinova & TJ arxiv:2105.05820

Collinear magnets



×





$[R_\theta||E]$ × Groups of spin-space transformations $[R_\alpha||R_\beta]$ Groups of real-space (crystal) transformations

Isomorphism theorem: $\mathbf{X} = \mathbf{x} + X_1\mathbf{x} + X_2\mathbf{x} + \dots$ $\quad \mathbf{Y} = \mathbf{y} + Y_1\mathbf{y} + Y_2\mathbf{y} + \dots$

$[\mathbf{x}||\mathbf{y}] + [X_1||Y_1]\,[\mathbf{x}||\mathbf{y}] + [X_2||Y_2]\,[\mathbf{x}||\mathbf{y}] + \dots$

Ferromagnetism: $\mathbf{S_1} = \{E\}$ $\quad \mathbf{G}$

$[E||\mathbf{G}]$

Spin-degenerate
Antiferromagnetism
Paramagnetism: $\mathbf{S}_2 = \{E, R_{180^o}\}$ $\quad \mathbf{G}$

$[E||\mathbf{G}] + [R_{180^o}||\mathbf{G}]$

?: $\mathbf{S}_2 = \{E\} + R_{180^o}\{E\}$ $\quad \mathbf{G} = \mathbf{H} + A\mathbf{H}$

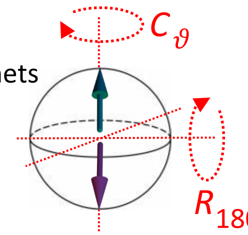
$\{E\}$: halving subgroup of $\mathbf{S}_2$ $\quad$ $\mathbf{H}$: halving subgroup of $\mathbf{G}$

$[E||\mathbf{H}] + [R_{180^o}||A]\,[E||\mathbf{H}]$

Altermagnetic spin-group derivation

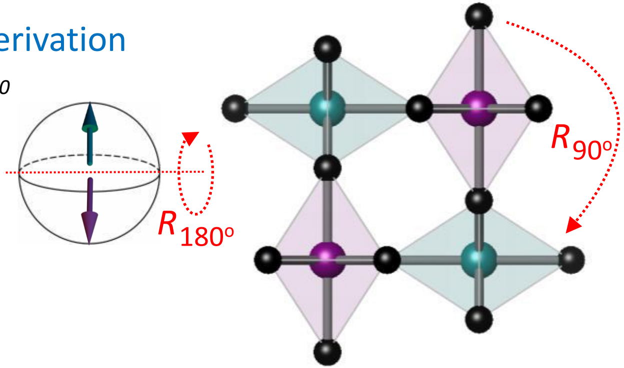
Šmejkal, Sinova & TJ arxiv:2105.05820

Collinear magnets



R_{180^o} * spin-inversion (time-reversal)

×



$[R_\theta||E]$ × Groups of spin-space transformations $[R_\alpha||R_\beta]$ Groups of real-space (crystal) transformations

$\mathbf{S}_2 = \{E\} + R_{180^o}\{E\}$

$\{E\}$: halving subgroup of $\mathbf{S}_2$

$\mathbf{G} = \mathbf{H} + A\mathbf{H}$

$\mathbf{H}$: halving subgroup of $\mathbf{G}$

$[E||\mathbf{H}] + [R_{180^o}||A]\,[E||\mathbf{H}]$

1. P is not in **H**, i.e., in *A***H**

$[E||\mathrm{P}]\,[E||\mathbf{H}] = [E||\mathbf{G}]$

Spin-degenerate
Antiferromagnetism
Paramagnetism

$[E||\mathbf{G}] + [R_{180^o}||\mathbf{G}]$

2. P is in **H**, i.e., not in *A***H**

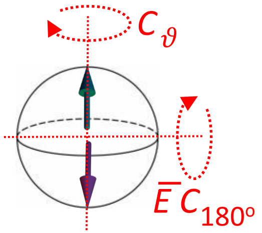
$[E||\mathrm{P}]\,[E||\mathbf{H}] = [E||\mathbf{H}]$

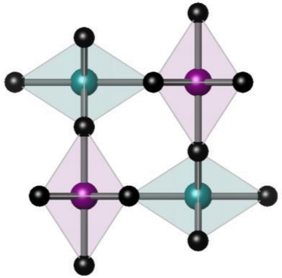
Altermagnetism

$[E||\mathbf{H}] + [R_{180^o}||A]\,[E||\mathbf{H}]$

Invariance of bands under inversion of **k**

Šmejkal, Sinova & TJ arxiv:2105.05820





No correspondence in magnetic groups ← $[R_\theta||E]$ E is real-space identity

Spin-only groups

↓

- Collinear magnets: $[\bar{E}\,C_{180^o}||E]$, $[C_\vartheta||E]$

Spin-inversion $\bar{E}$ enters via time-reversal T

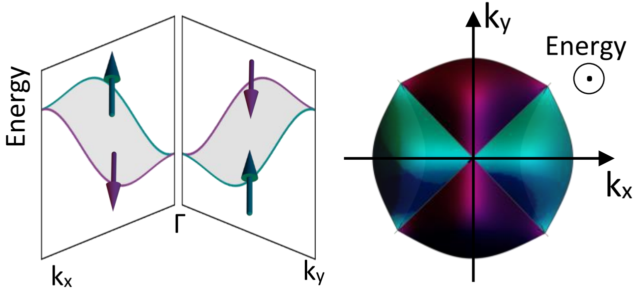
$[\bar{E}\,C_{180^o}||\mathrm{T}]$

Time in spin and real space is not decoupled

Real-space transformation: T crystal = E crystal

Only spin symmetries can delimit the non-frustrated collinear magnets

Litvin & Opechowski, Physica (1974)



Non-relativistic bands of collinear-magnetic crystals (including inversion-asymmetric) are invariant under inversion of **k**

Transformation:

$[\bar{E}\,C_{180^o}||\mathrm{T}]\ \epsilon(\uparrow, \mathbf{k}) = \epsilon(\uparrow, -\mathbf{k})$

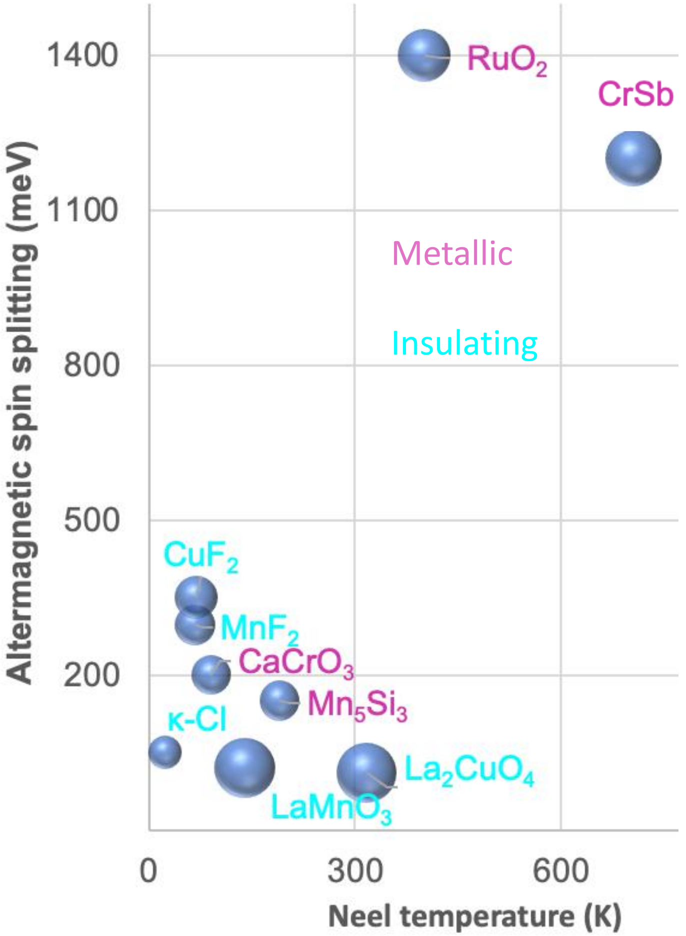
$\longrightarrow \epsilon(\uparrow, \mathbf{k}) = \epsilon(\uparrow, -\mathbf{k})$

Symmetry:

$[\bar{E}\,C_{180^o}||\mathrm{T}]\ \epsilon(\uparrow, \mathbf{k}) = \epsilon(\uparrow, \mathbf{k})$

Šmejkal, Sinova & TJ arxiv:2105.05820

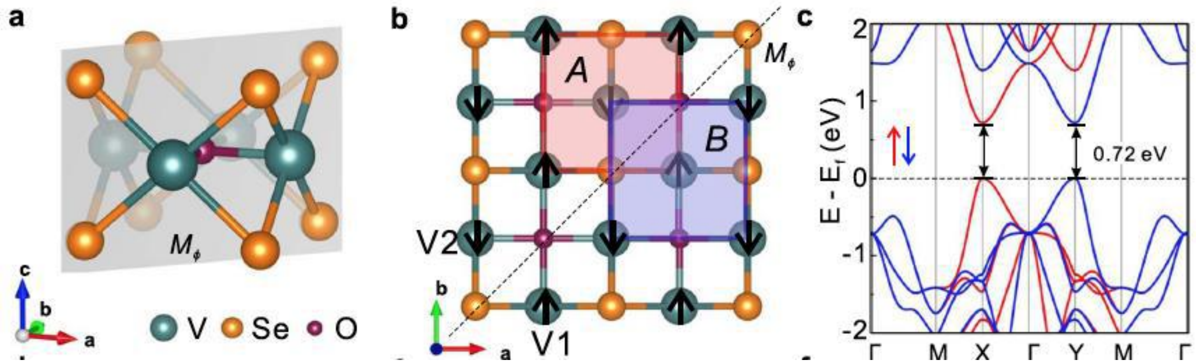
Altermagnetic spin-splitting vs. Néel temperature



2D Altermagnetism

Ma et al. Nat. Commun 12, 2846 (2021)

V_2Se_2O



Pekar-Rahba model is in coupled spin and real space

SOVIET PHYSICS JETP VOLUME 20, NUMBER 5 MAY, 1965

COMBINED RESONANCE IN CRYSTALS IN INHOMOGENEOUS MAGNETIC FIELDS

S. I. PEKAR and É. I. RASHBA

Institute of Semiconductors, Academy of Sciences, Ukrainian S.S.R.

Submitted to JETP editor May 21, 1964

J. Exptl. Theoret. Phys. (U.S.S.R.) **47**, 1927-1932 (November, 1964)

$$H = -\frac{\hbar^2}{2}\sum_{ij}(m^{-1})_{ij}\hat{k}_i\hat{k}_j + \beta_0\boldsymbol{\sigma}\,[\mathcal{B}(\mathbf{k}_0) + (\hat{\mathbf{k}}\nabla_{\mathbf{k}_0})\,\mathcal{B}(\mathbf{k}_0)], \quad (4)$$

where m^{-1} is the tensor of the reciprocal effective mass, $\hat{\mathbf{k}} = -i\nabla + e\mathbf{A}/c\hbar$, and $\mathbf{A}$—vector potential of the macroscopic induction $\mathbf{B} = \overline{\mathbf{h}}$;

$$\mathcal{B}(\mathbf{k}_0 + \mathbf{k}) = \overline{|u_{\mathbf{k}_0+\mathbf{k}}(\mathbf{r})|^2\,\mathbf{h}(\mathbf{r})},$$

$$\nabla_{\mathbf{k}_0}\mathcal{B}_i = [\nabla_{\mathbf{k}}\mathcal{B}_i(\mathbf{k}_0 + \mathbf{k})]_{\mathbf{k}=0}. \quad (5)$$

- No symmetries discussed, but spin and real space are coupled
- Some model weak dipolar-field mechanism of magnetic ordering; not the strong QM exchange mechanism
- Toy k.p model, not first principles calculation

No relevant symmetry or microscopic-physics guidance

$R_{180^o}\mathbf{t}$ - transformation argument on Type IV magnetic groups is invalid

Type IV magnetic groups have T **t** symmetry where T is time-reversal and **t** is traslation

$$\mathrm{T}\,\mathbf{t}\ \text{transform: } \mathrm{T}\,\mathbf{t}\,\epsilon(\uparrow\mathbf{k}) = \epsilon(\downarrow\text{-}\mathbf{k}) \qquad \rightarrow\ \epsilon(\uparrow, \mathbf{k}) = \epsilon(\downarrow, \text{-}\mathbf{k})$$

$$\mathrm{T}\,\mathbf{t}\ \text{symmetry: } \mathrm{T}\,\mathbf{t}\,\epsilon(\uparrow\mathbf{k}) = \epsilon(\uparrow\mathbf{k})$$

Only collinear/coplanar non-relativistic magnets have R_{180^o} * spin-inversion, i.e., $R_{180^o}\mathrm{T}$

For these T **t** = $R_{180^o}\mathbf{t}$

$$R_{180^o}\,\mathbf{t}\ \text{transform: } R_{180^o}\,\mathbf{t}\,\epsilon(\uparrow, \mathbf{k}) = \epsilon(\downarrow, \mathbf{k}) \qquad \rightarrow\ \epsilon(\uparrow, \mathbf{k}) = \epsilon(\downarrow, \mathbf{k})$$

$$R_{180^o}\,\mathbf{t}\ \text{symmetry: } R_{180^o}\,\mathbf{t}\,\epsilon(\uparrow, \mathbf{k}) = \epsilon(\uparrow, \mathbf{k})$$

But Type IV, or any magnetic groups in general, do not distinguish collinear/coplanar and non-coplanar magnets.

Adding $R_{180^o}\mathrm{T}$ to Type IV magnetic group symmetries, therefore, does not provide an argument that Type IV magnetic groups must result in spin-degenerate bands in the absence of spin-orbit coupling.