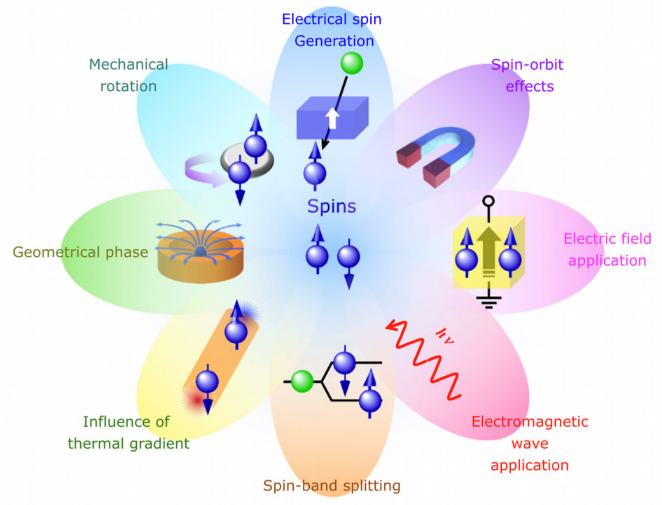
Alexander Mook

University of Basel

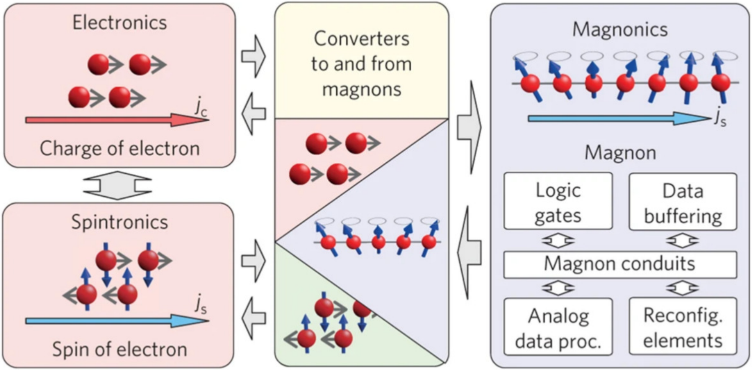
Interacting and higher-order topological spin excitations

Spintronics



☞ Hirohata *et al.*, J. Mag. Mag. Mat., 509, 166711 (2020)

Magnon Spintronics



☞ Chumak *et al.*, Nature Physics **11**, 453–461 (2015)

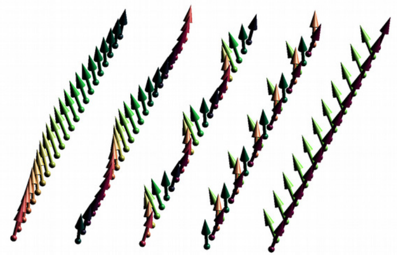
Magnons

Interacting localized spin moments

$$\hat{H} = \sum_{i,j} \hat{\boldsymbol{S}}_i \cdot \mathbb{I}_{i,j} \cdot \hat{\boldsymbol{S}}_j - \sum_i \boldsymbol{B} \cdot \hat{\boldsymbol{S}}_i$$

Classical: Landau-Lifshitz-Gilbert equation

$$\dot{\boldsymbol{S}}_i = -\frac{\gamma}{\mu\,(1+\alpha^2)}\left[\boldsymbol{S}_i \times \boldsymbol{B}_i^{\mathrm{eff}} + \alpha \boldsymbol{S}_i \times \left(\boldsymbol{S}_i \times \boldsymbol{B}_i^{\mathrm{eff}}\right)\right]$$

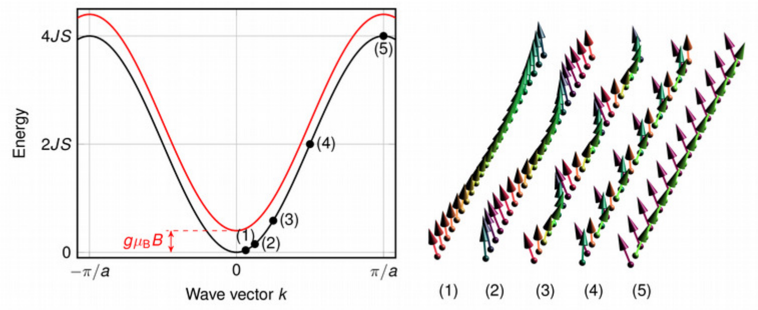


Quantum: Holstein-Primakoff transformation from spins $\hat{\boldsymbol{S}}_{\boldsymbol{r}}$ to bosons $\hat{a}_{\boldsymbol{r}}^{(\dagger)}$

$$\hat{\boldsymbol{S}}_{\boldsymbol{r}} = \sqrt{\frac{S}{2}}\left[\left(\hat{f}_{\boldsymbol{r}}\hat{a}_{\boldsymbol{r}} + \hat{a}_{\boldsymbol{r}}^{\dagger}\hat{f}_{\boldsymbol{r}}\right)\hat{\boldsymbol{x}}_{\boldsymbol{r}} - \mathrm{i}\left(\hat{f}_{\boldsymbol{r}}\hat{a}_{\boldsymbol{r}} - \hat{a}_{\boldsymbol{r}}^{\dagger}\hat{f}_{\boldsymbol{r}}\right)\hat{\boldsymbol{y}}_{\boldsymbol{r}}\right] + \left(S - \hat{a}_{\boldsymbol{r}}^{\dagger}\hat{a}_{\boldsymbol{r}}\right)\hat{\boldsymbol{z}}_{\boldsymbol{r}}, \quad \hat{f}_{\boldsymbol{r}} = \sqrt{1 - \frac{\hat{a}_{\boldsymbol{r}}^{\dagger}\hat{a}_{\boldsymbol{r}}}{2S}}$$

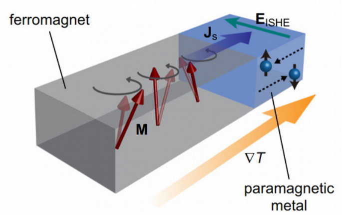
$$\hat{H} = \hat{H}_0 + \hat{H}_1 + \hat{H}_2 + \hat{H}_3 + \ldots$$

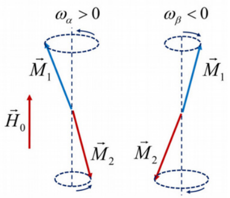
Magnons



Spintronics: Spin Seebeck effect

Nonequilibrium spin current caused by temperature gradient: $\langle \boldsymbol{J}_\gamma \rangle = \chi_\gamma \boldsymbol{\nabla} T$

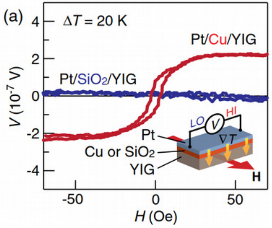




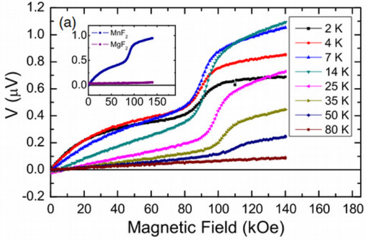
☞ Uchida *et al.*, J. Appl. Phys. 111, 103903 (2012);

Uchida *et al.*, Nat. Mat. 9, 894-897 (2010)

☞ Rezende *et al.*, J. Appl. Phys. 126, 151101 (2019)



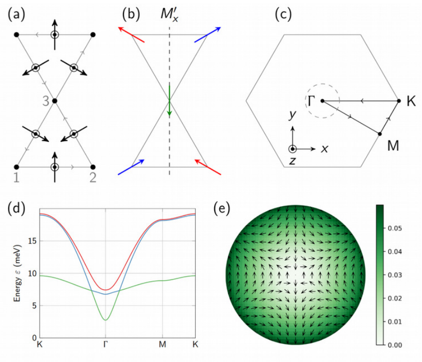
☞ Kikkawa *et al.*, PRL 110, 067207 (2013)

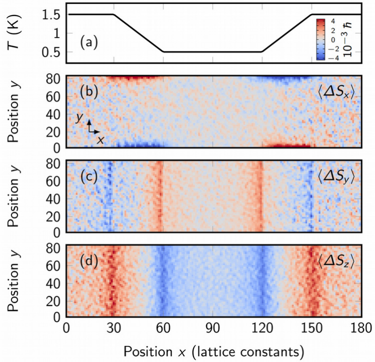


☞ Wu *et al.*, PRL 116, 097204 (2016)

Magnon spin transport in noncollinear magnets

Nonequilibrium spin current caused by temperature gradient: $\langle \boldsymbol{J}_\gamma \rangle = \chi_\gamma \boldsymbol{\nabla} T$





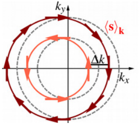
☞ **AM**, Neumann, Henk, Mertig, PRB 100, 100401(R) (2019); **AM**, Neumann, Johansson, Henk, Mertig, PRRes 2, 023065 (2020); Neumann, **AM**, Henk, Mertig, PRL 125, 117209 (2020)

☞ See also: Okuma, PRL 119, 107205 (2017); Flebus *et al.*, PRB 99, 224410 (2019); Li *et al.*, PRRes 2, 013079 (2020)

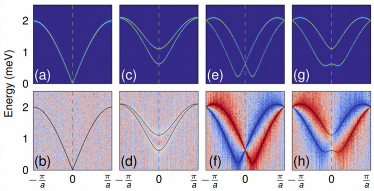
Magnonic thermal Edelstein effect

Nonequilibrium spin-polarization caused by temperature gradient: $\langle \boldsymbol{S} \rangle = \chi \boldsymbol{\nabla} T$

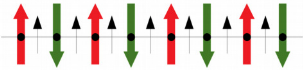
Reminder: electronic EE $\langle \boldsymbol{S} \rangle = \chi \boldsymbol{E}$



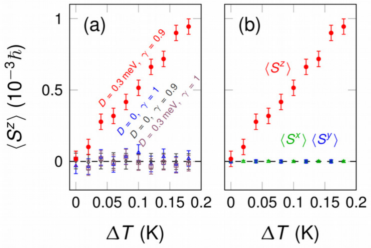
- Nonconserved spin
- Broken inversion symmetry



Toy model: AFM spin chain



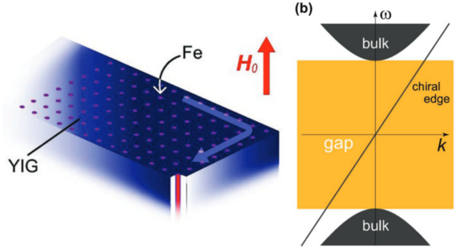
$$H = J \sum_i \left(\gamma S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \lambda S_i^z S_{i+1}^z \right) + D \sum_i \hat{\boldsymbol{z}} \cdot \boldsymbol{S}_i \times \boldsymbol{S}_{i+1}$$



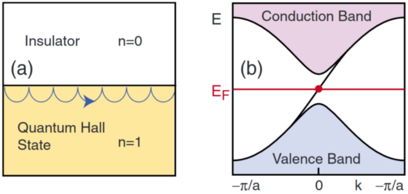
☞ Li, **AM**, Raeliarijaona, Kovalev, PRB 101, 024427 (2020)

☞ See also: Shitade, Yanase, PRB 100, 224416 (2019); Zhang, Cheng, Appl. Phys. Lett. 117, 222402 (2020)

Topological magnonics



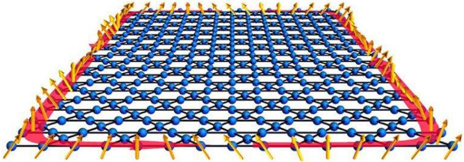
☞ Shindou *et al.*, PRB 87, 174427 (2013)



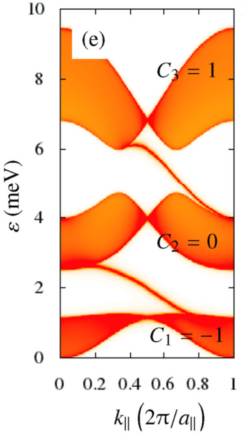
☞ Hasan, Kane, Rev. Mod. Phys. 82, 3045 (2010)

Topological magnon insulators

Cu(1,3-benzenedicarboxylate)



☞ Chisnell *et al.*, PRL 115, 147201 (2015) & press release; Katsura *et al.*, PRL 104, 066403 (2010); Zhang *et al.*, PRB 87, 144101 (2013)



Chern number

$$C_n = \frac{1}{2\pi} \int_{BZ} \Omega_n(\boldsymbol{k}) \mathrm{d}\boldsymbol{k}$$

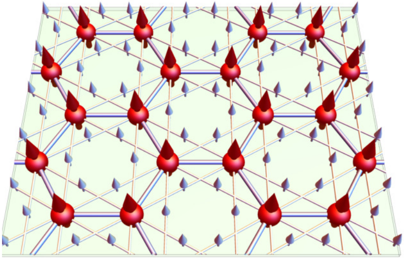
$$\Omega_n(\boldsymbol{k}) = \mathrm{i}\langle \nabla_{\boldsymbol{k}} u_n(\boldsymbol{k})| \times |\nabla_{\boldsymbol{k}} u_n(\boldsymbol{k})\rangle$$

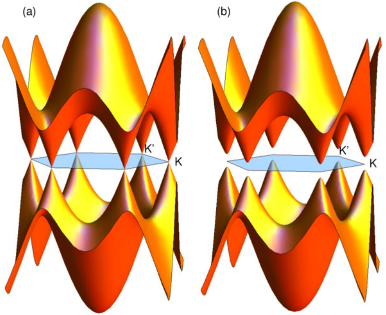
Bulk-boundary correspondence

☞ **AM**, Henk, Mertig, PRB **90**, 024412 (2014)

Honeycomb lattice ferromagnet

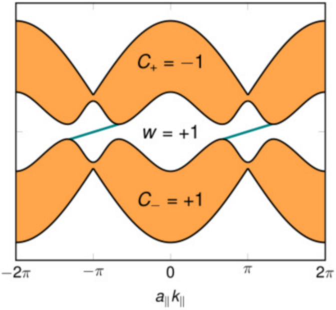
$$\hat{H} = \frac{1}{2}\sum_{\langle \boldsymbol{r},\boldsymbol{r}'\rangle} -J\hat{\boldsymbol{S}}_{\boldsymbol{r}} \cdot \hat{\boldsymbol{S}}_{\boldsymbol{r}'} + \frac{1}{2}\sum_{\langle\langle \boldsymbol{r},\boldsymbol{r}'\rangle\rangle} \boldsymbol{D}_{\boldsymbol{r},\boldsymbol{r}'} \cdot \left(\hat{\boldsymbol{S}}_{\boldsymbol{r}} \times \hat{\boldsymbol{S}}_{\boldsymbol{r}'}\right) = \hat{H}_0 + \hat{H}_2 + \hat{H}_3 + \ldots$$

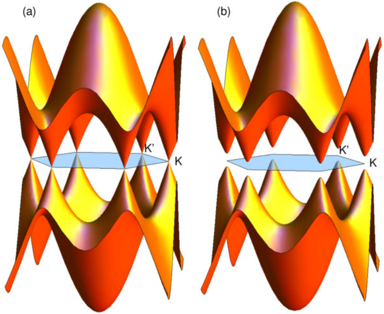




Honeycomb lattice ferromagnet

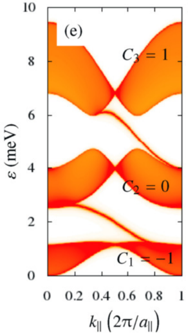
$$\hat{H} = \frac{1}{2} \sum_{\langle \boldsymbol{r},\boldsymbol{r}' \rangle} -J \hat{\boldsymbol{S}}_{\boldsymbol{r}} \cdot \hat{\boldsymbol{S}}_{\boldsymbol{r}'} + \frac{1}{2} \sum_{\langle\langle \boldsymbol{r},\boldsymbol{r}' \rangle\rangle} \boldsymbol{D}_{\boldsymbol{r},\boldsymbol{r}'} \cdot \left(\hat{\boldsymbol{S}}_{\boldsymbol{r}} \times \hat{\boldsymbol{S}}_{\boldsymbol{r}'} \right) = \hat{H}_0 + \hat{H}_2 + \hat{H}_3 + \ldots$$





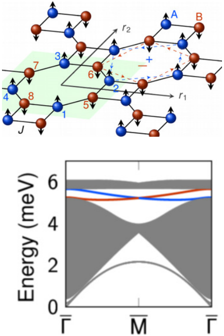
Topological phases of magnons (incomplete)

Chern insulator



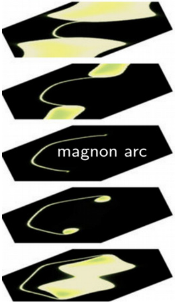
☞ Shindou *et al.*, PRB 87, 174427 (2013); Zhang *et al.*, PRB 87, 144101 (2013); **AM**, Henk, Mertig, PRB 90, 024412 (2014)

$\mathbb{Z}_2$ topological insulator



☞ **AM**, Göbel, Henk, Mertig, PRB 97, 140401(R) (2018); Nakata *et al.*, PRB 96, 224414 (2017)

Dirac/Weyl magnons



☞ **AM**, Henk, Mertig, PRL 117, 157204 (2016); Li *et al.*, Nat. Comm. 7, 12691 (2016); Fransson *et al.*, PRB 94, 075401 (2016)

Topological magnon materials (incomplete)

Compound	Magnon topology	Theory	Experiment
$Lu_2V_2O_7$	Weyl magnons	[1]	[2]
Cu(1,3-bdc)	Chern insulating	[3]	[4]
CrI_3	Chern insulating	[5]	[6]
$CrBr_3$	Dirac magnons	[7]	[8]
	Chern insulating	[9]	[9]
Cu_2OSeO_3	Weyl magnons	[10]	[10]
YMn_6Sn_6	Dirac magnons	[11]	[11]
FeSn	Dirac magnons	[12]	[12]
Cu_3TeO_6	Dirac magnons	[13]	[14]
$CoTiO_3$	Nodal lines	[15]	[15]
$CrSiTe_3/CrGeTe_3$	Chern insulating	[16]	[16]
Skyrmion crystals	Chern insulating	[17]	[18]
Elemental Gd	Nodal lines	[19]	[19]
Antiskyrmion crystals	Second-order	[20]	[21]
van der Waals magnet stacks	Second-order	[22]	–
Magnonic metamaterials	various	...	...
⋮	⋮	⋮	⋮

Topological magnon materials (incomplete); references

☞ [1] **AM**, Henk, Mertig, PRL 117, 157204 (2016)

☞ [2] Onose *et al.*, Science 329, 5989 (2010); Mena *et al.*, PRL 113, 047202 (2014)

☞ [3] Katsura *et al.*, PRL 104, 066403 (2010); Zhang *et al.*, PRB 87, 144101 (2013)

☞ [4] Chisnell *et al.*, PRL 115, 147201 (2015)

☞ [5] Owerre, J. Phys.: Condens. Matter 28, 386001 (2016); Aguilera *et al.*, Phys. Rev. B 102, 024409 **(2020)**; Costa *et al.*, 2D Mater. 7 045031 **(2020)**

☞ [6] Chen *et al.*, PRX 8, 041028 (2018)

☞ [7] Pershoguba *et al.*, PRX 8, 011010 (2018)

☞ [8] Samuelsen *et al.*, PRB 3, 157 (1971); Yelon and Silberglitt, PRB 4, 2280 (1971)

☞ [9] Cai *et al.*, PRB 104, L020402 **(2021)**

☞ [10] Zhang *et al.*, PRRes 2, 013063 **(2020)**

☞ [11] Zhang *et al.*, PRB 101, 100405(R) **(2020)**

☞ [12] Do *et al.*, arXiv:2107.08915 **(2021)**

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☞ [14] Yao *et al.*, Nature Physics 14, 1011-1015 (2018); Bao *et al.*, Nat. Comm. 9, 2591 (2018)

☞ [15] Yuan *et al.*, PRX 10, 011062 **(2020)**; Elliot *et al.*, Nat. Comm. 12, 3936 **(2021)**

☞ [16] Zhu *et al.*, arXiv:2107.03835 **(2021)**

☞ [17] van Hoogdalem *et al.*, PRB 87, 024402 (2013); Roldán-Molina *et al.*, New J. Phys. 18, 045015 (2016); Díaz *et al.*, PRL 122, 187203 (2019)

☞ [18] Mochizuki *et al.*, Nat. Mat. 13, 241-246 (2014)

☞ [19] Schele *et al.*, arXiv:2107.11372 **(2021)**

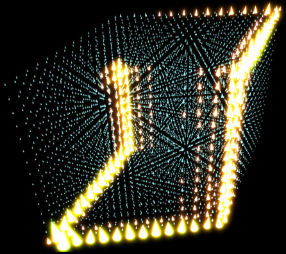
☞ [20] Hirosawa *et al.*, PRL 125, 207204 **(2020)**

☞ [21] Jena *et al.*, Research Square **(2021)**

☞ [22] **AM**, Díaz, Klinovaja, Loss, PRB 104, 024406 **(2021)**

Central questions

How to go 3D?



AM, Díaz, Klinovaja, Loss, PRB 104, 024406 (2021)

What do interactions do?

Topology in quantum matter without particle conservation

$$\hat{H} = \hat{H}_2 + \hat{H}_3 + \hat{H}_4 + \ldots$$

Single-particle sector

$\updownarrow$

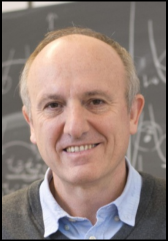
Two-particle sector

AM, Klinovaja, Loss, PRRes 2, 033491 (2020)

AM, Plekhanov, Klinovaja, Loss, PRX 11, 021061 (2021)

Collaborators

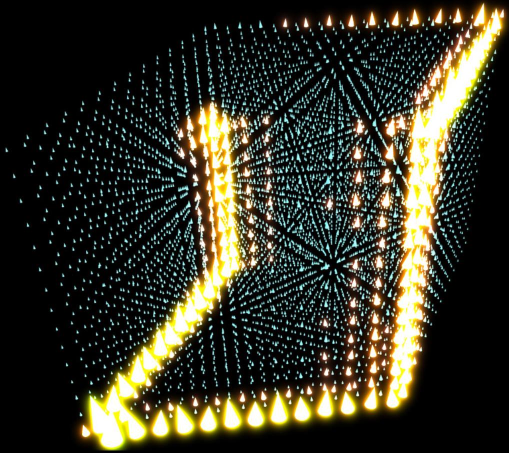


Kirill Plekhanov, Sebastián Díaz, Jelena Klinovaja, Daniel Loss

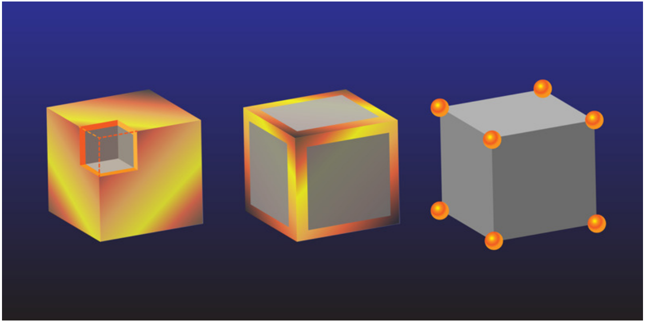
Second-order topological chiral hinge magnons

—Topological magnonics goes 3D—



AM, Díaz, Klinovaja, Loss, PRB 104, 024406 (2021)

Second-and-higher-order topology

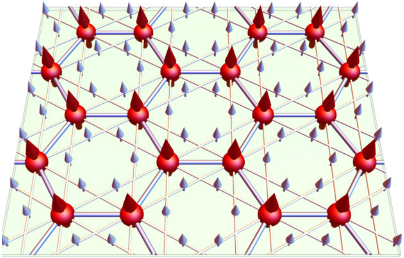


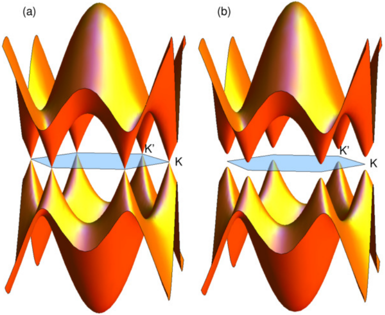
☞ Figure: APS, Alan Stonebraker

☞ Benalcazar *et al.*, Science 357, 6346, 61–66 (2017); Schindler *et al.*, Science Advances 4, 6, (2018); Schindler *et al.*, Nature Physics 14, 918–924 (2018)

Honeycomb lattice ferromagnet

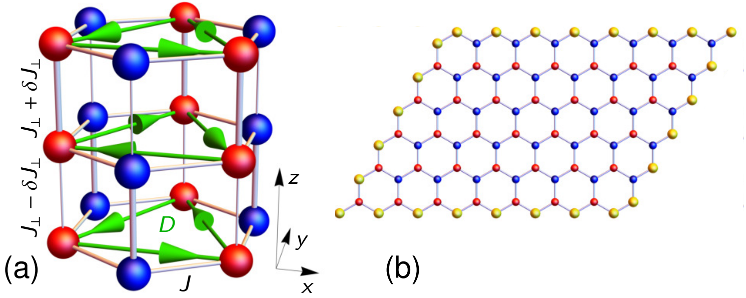
$$\hat{H} = \frac{1}{2}\sum_{\langle \boldsymbol{r},\boldsymbol{r}'\rangle} -J\hat{\boldsymbol{S}}_{\boldsymbol{r}} \cdot \hat{\boldsymbol{S}}_{\boldsymbol{r}'} + \frac{1}{2}\sum_{\langle\langle \boldsymbol{r},\boldsymbol{r}'\rangle\rangle} \boldsymbol{D}_{\boldsymbol{r},\boldsymbol{r}'} \cdot \left(\hat{\boldsymbol{S}}_{\boldsymbol{r}} \times \hat{\boldsymbol{S}}_{\boldsymbol{r}'}\right) = \hat{H}_0 + \hat{H}_2 + \hat{H}_3 + \ldots$$





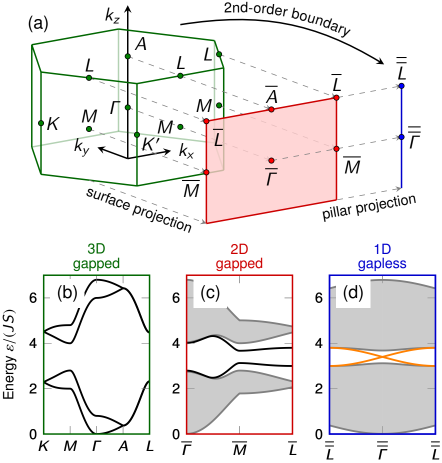
Stack of honeycomb-lattice ferromagnets

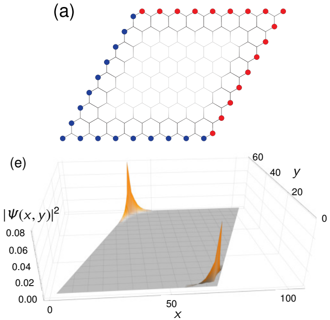
$$\hat{H} = \sum_{\ell} \left[\hat{H}^{(\ell)}_{\text{honeycomb}}(J, D^{(\ell)}) + \hat{H}^{(\ell \to \ell+1)}_{\text{coupling}}(J_\perp, \delta J_\perp) \right], \qquad D^{(\ell)} = (-1)^{\ell} D$$



☞ **AM**, Díaz, Klinovaja, Loss, PRB 104, 024406 (2021)

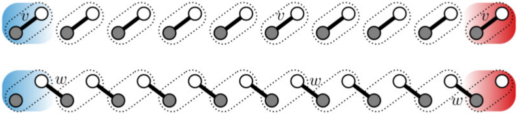
Second-order topology and chiral hinge magnons





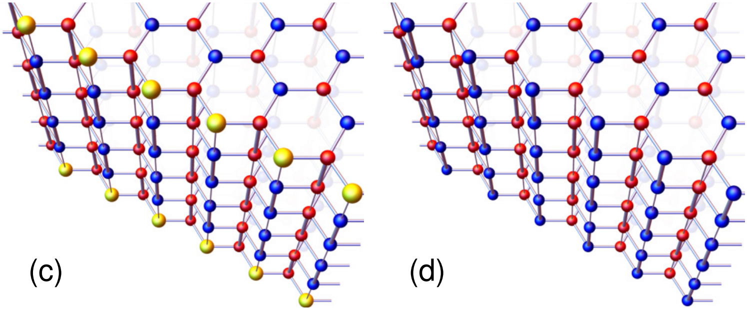
☞ **AM**, Díaz, Klinovaja, Loss, PRB 104, 024406 (2021)

SSH chains along stacking direction



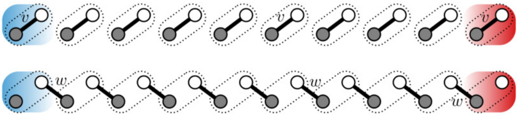
☞ Asbóth *et al.*, "A Short Course on Topological Insulators," Springer International Publishing

$\delta J_{\perp} > 0$ $\delta J_{\perp} < 0$



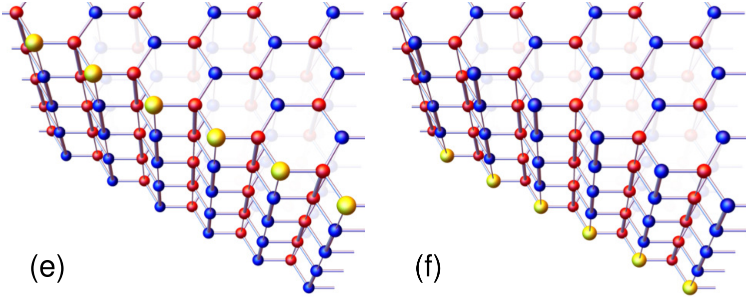
☞ **AM**, Díaz, Klinovaja, Loss, PRB 104, 024406 (2021)

SSH chains along stacking direction



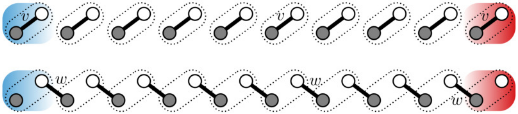
☞ Asbóth *et al.*, "A Short Course on Topological Insulators," Springer International Publishing

$\delta J_\perp > 0$ $\delta J_\perp < 0$

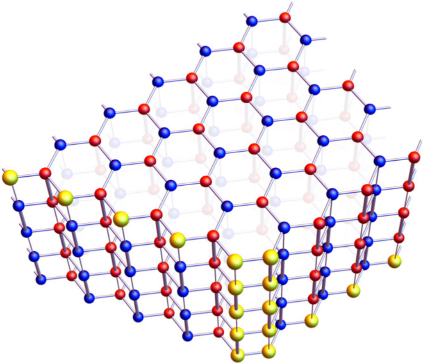


☞ **AM**, Díaz, Klinovaja, Loss, PRB 104, 024406 (2021)

SSH chains along stacking direction: Domain walls

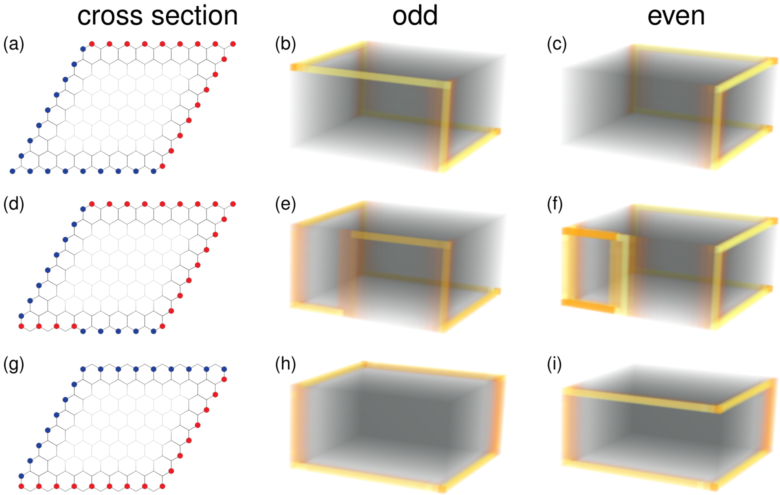


☞ Asbóth *et al.*, “A Short Course on Topological Insulators,” Springer International Publishing



☞ **AM**, Díaz, Klinovaja, Loss, PRB 104, 024406 (2021)

Second-order topology and chiral hinge magnons



☞ **AM**, Díaz, Klinovaja, Loss, PRB 104, 024406 (2021)

Spin dynamics simulations

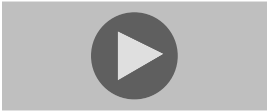
Landau-Lifshitz-Gilbert equation

$$\dot{\boldsymbol{S}}_i = -\frac{\gamma}{\mu\left(1+\alpha^2\right)}\left[\boldsymbol{S}_i \times \boldsymbol{B}_i^{\mathrm{eff}} + \alpha \boldsymbol{S}_i \times \left(\boldsymbol{S}_i \times \boldsymbol{B}_i^{\mathrm{eff}}\right)\right]$$

☞ **AM**, Díaz, Klinovaja, Loss, PRB 104, 024406 (2021)

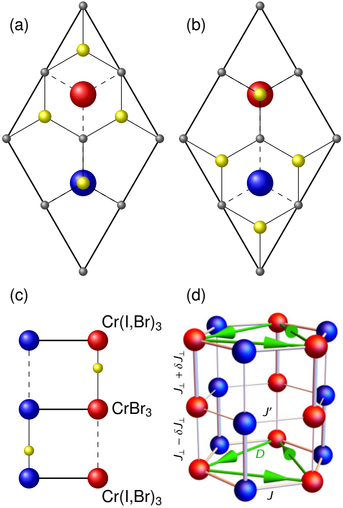
Spin dynamics simulations with disorder

$$R = \frac{\text{Disorder strength}}{\text{Band gap}}$$

$R = 0.21$	$R = 0.64$	$R = 1.07$
		

☞ **AM**, Díaz, Klinovaja, Loss, PRB 104, 024406 (2021)

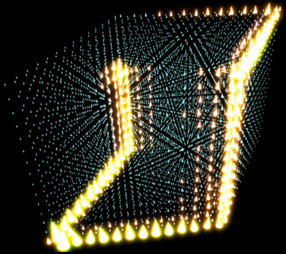
Proposal for experimental realization



☞ **AM**, Díaz, Klinovaja, Loss, PRB 104, 024406 (2021)

Central questions

How to go 3D?



AM, Díaz, Klinovaja, Loss, PRB 104, 024406 (2021)

What do interactions do?

Topology in quantum matter without particle conservation

$$\hat{H} = \hat{H}_2 + \hat{H}_3 + \hat{H}_4 + \ldots$$

Single-particle sector

$\updownarrow$

Two-particle sector

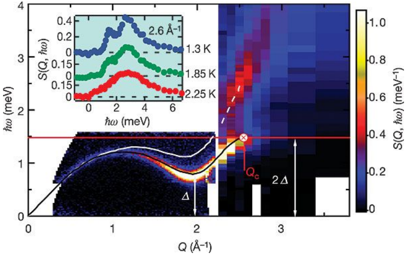
AM, Klinovaja, Loss, PRRes 2, 033491 (2020)

AM, Plekhanov, Klinovaja, Loss, PRX 11, 021061 (2021)

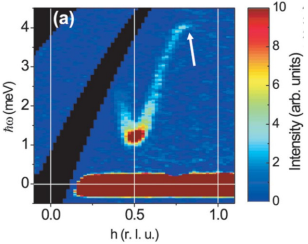
Spontaneous quasiparticle decay

$$\hat{H} = \hat{H}_0 + \hat{H}_2 + \hat{H}_3 + \hat{H}_4 + \ldots$$

$$\hat{\alpha}^\dagger_{\boldsymbol{k},\lambda}\hat{\alpha}^\dagger_{\boldsymbol{q},\mu}\hat{\alpha}_{\boldsymbol{p},\nu}$$



☞ Stone *et al.*, Nature 440, 187 (2006)

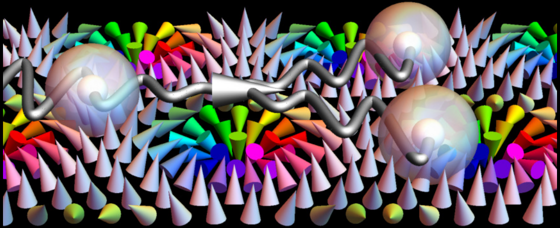


☞ Masuda *et al.*, PRL 96, 047210 (2006)

1) Does single-particle topology survive many-body interactions?
2) Can many-body interactions bring about nontrivial topology?

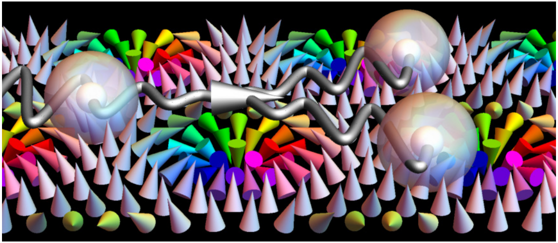
Spontaneous magnon decay in skyrmion crystals

—Topological magnons **in spite of** interactions—

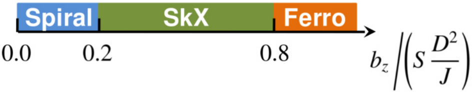


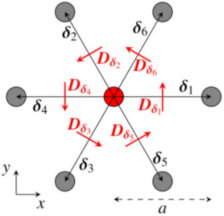
AM, Klinovaja, Loss, PRRes 2, 033491 (2020)

Spontaneous magnon decay in skyrmion crystals

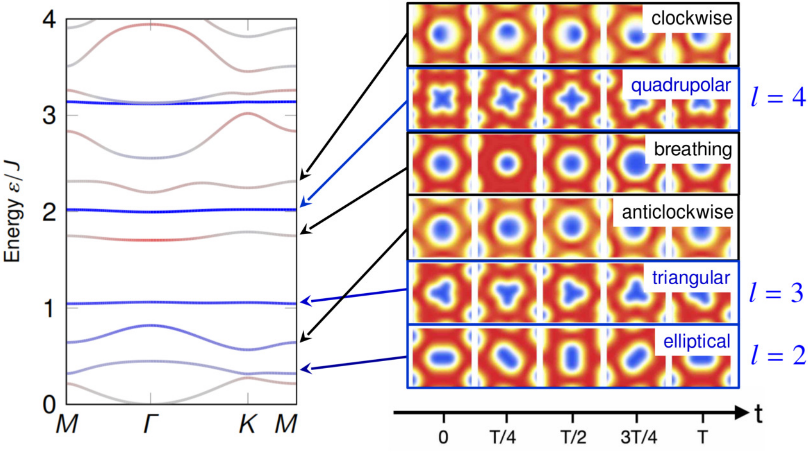


$$\hat{H} = \frac{1}{2}\sum_{\boldsymbol{r},\boldsymbol{r}'}\left(-J\hat{\boldsymbol{S}}_{\boldsymbol{r}}\cdot\hat{\boldsymbol{S}}_{\boldsymbol{r}'} + \boldsymbol{D}_{\boldsymbol{r},\boldsymbol{r}'}\cdot\hat{\boldsymbol{S}}_{\boldsymbol{r}}\times\hat{\boldsymbol{S}}_{\boldsymbol{r}'}\right) - b_z\sum_{\boldsymbol{r}}\hat{S}^z_{\boldsymbol{r}}$$



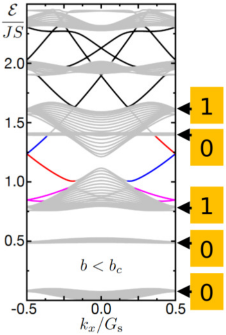


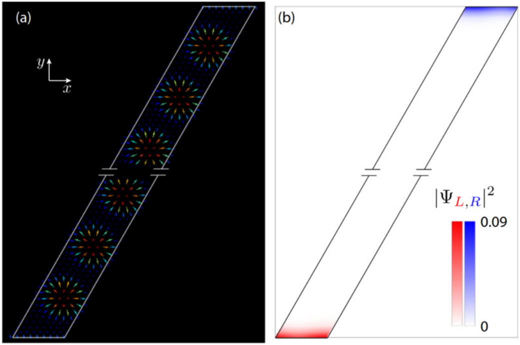
Skyrmion crystal eigenmodes



☞ Back *et al.*, J. Phys. D: Appl. Phys. 53, 363001 (2020), Garst's chapter

Topology of skyrmion crystal eigenmodes





☞ Díaz *et al.*, PRRes 2, 013231 (2020)

☞ Roldán-Molina *et al.*, New J. Phys. 18, 045015 (2016)

Noncollinear texture causes topological magnon bands.
BUT: Noncollinear texture also causes spontaneous magnon decay!

Any hope to detect topological magnons?

Many-body theory of spontaneous magnon decay

Perturbation theory

$$\hat{H} = \hat{H}_2 + \hat{V}, \qquad \hat{V} = \hat{H}_3 + \hat{H}_4$$

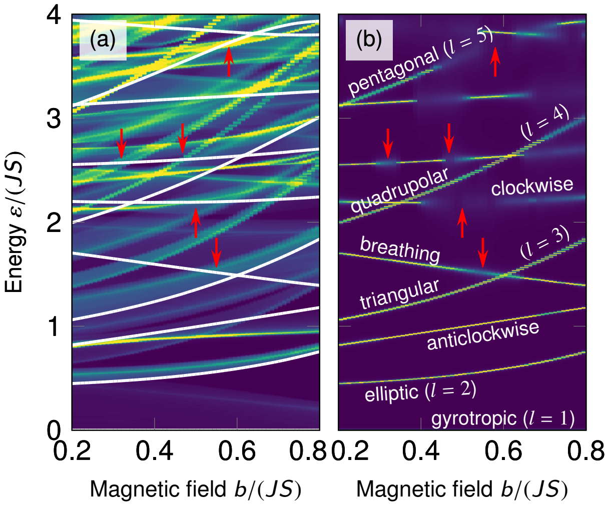
Spontaneous damping (inverse lifetime)

$$\Gamma_{\boldsymbol{k}}^{\nu} = \frac{\pi}{2N} \sum_{\boldsymbol{q}\in\mathrm{BZ}} \sum_{\lambda,\mu} \left| \mathcal{V}_{\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q}\leftarrow\boldsymbol{k}}^{\lambda\mu\leftarrow\nu} \right|^2 \delta\left(\varepsilon_{\nu,\boldsymbol{k}} - \varepsilon_{\lambda,\boldsymbol{q}} - \varepsilon_{\mu,\boldsymbol{k}-\boldsymbol{q}}\right)$$

Two-magnon density of states

$$D_{\boldsymbol{k}}(\varepsilon) = \frac{\pi}{2N} \sum_{\boldsymbol{q}\in\mathrm{BZ}} \sum_{\lambda,\mu} \delta\left(\varepsilon - \varepsilon_{\lambda,\boldsymbol{q}} - \varepsilon_{\mu,\boldsymbol{k}-\boldsymbol{q}}\right)$$

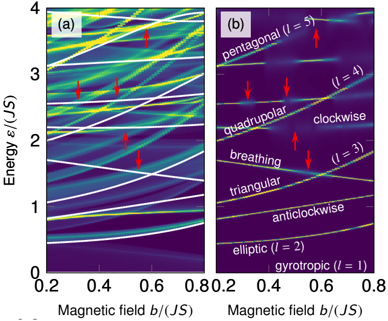
Magnon spectral function



☞ **AM**, Klinovaja, Loss, PRRes 2, 033491 (2020)

Implications

- Great prospects for topological magnonics
- Field-dependent quantum damping of C mode

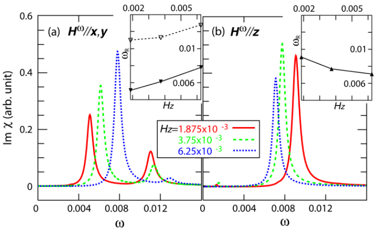


☞ **AM**, Klinovaja, Loss, PRRes 2, 033491 (2020)

Quantum vs. classical damping

$$\frac{\Gamma_{\mathrm{C}}^{\mathrm{qm.}}}{\Gamma_{\mathrm{C}}^{\mathrm{Gilbert}}} \approx \frac{0.1}{\alpha S}\left(\frac{D}{J}\right)^2 \approx 3.6$$

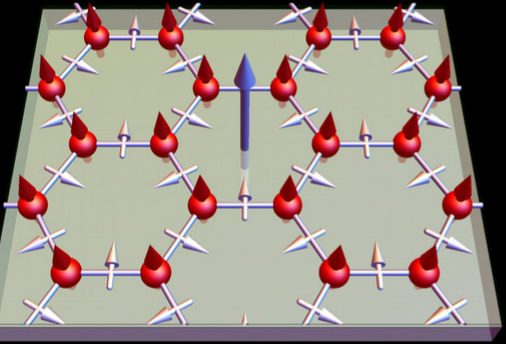
(CuO_2SeO_3)



☞ Mochizuki, PRL 108, 017601 (2012)

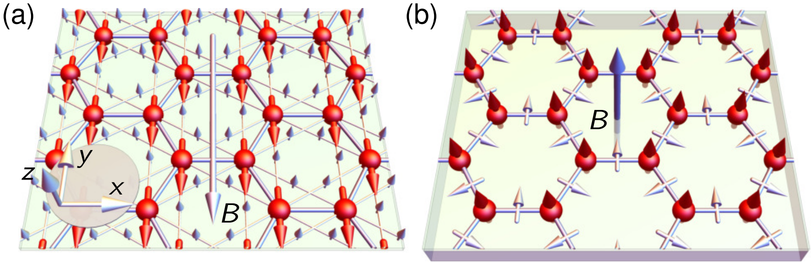
Quantum mass of Dirac magnons

—Topological magnons **because of** interactions—



AM, Plekhanov, Klinovaja, Loss, PRX 11, 021061 (2021)

Models



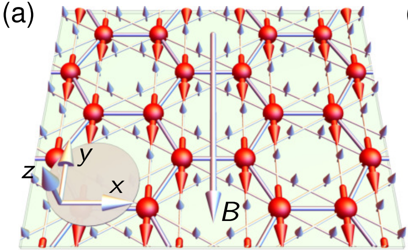
Achiral

Chiral

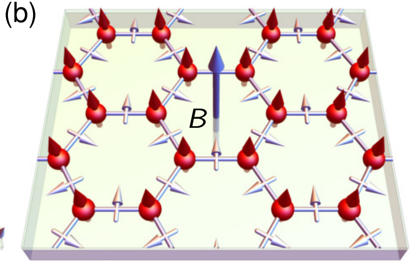
$$\hat{H}^{\mathrm{A}} = -\frac{J}{2}\sum_{\langle \boldsymbol{r},\boldsymbol{r}'\rangle} \hat{\boldsymbol{S}}_{\boldsymbol{r}} \cdot \hat{\boldsymbol{S}}_{\boldsymbol{r}'} - \sum_{\boldsymbol{r}} \boldsymbol{B} \cdot \hat{\boldsymbol{S}}_{\boldsymbol{r}} + \frac{D_z}{2} \sum_{\langle\langle \boldsymbol{r},\boldsymbol{r}'\rangle\rangle} \nu_{\boldsymbol{r},\boldsymbol{r}'} \boldsymbol{z} \cdot \left(\hat{\boldsymbol{S}}_{\boldsymbol{r}} \times \hat{\boldsymbol{S}}_{\boldsymbol{r}'}\right)$$

$$\hat{H}^{\mathrm{C}} = -\frac{J}{2}\sum_{\langle \boldsymbol{r},\boldsymbol{r}'\rangle} \hat{\boldsymbol{S}}_{\boldsymbol{r}} \cdot \hat{\boldsymbol{S}}_{\boldsymbol{r}'} - \sum_{\boldsymbol{r}} \boldsymbol{B} \cdot \hat{\boldsymbol{S}}_{\boldsymbol{r}} + \frac{D}{2} \sum_{\langle \boldsymbol{r},\boldsymbol{r}'\rangle} \boldsymbol{d}_{\boldsymbol{r},\boldsymbol{r}'} \cdot \left(\hat{\boldsymbol{S}}_{\boldsymbol{r}} \times \hat{\boldsymbol{S}}_{\boldsymbol{r}'}\right)$$

Models



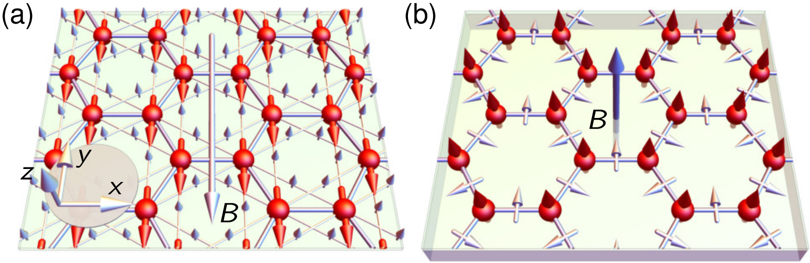
Achiral



Chiral

	Achiral	Chiral
Time-reversal symmetry	$\mathcal{R}(\pi, \boldsymbol{z})\mathcal{T}$	–
Parity symmetry	$\mathcal{I}$	$\mathcal{R}(\pi, \boldsymbol{z})\mathcal{I}$

Models



Achiral Chiral

$$\hat{H}^{\mathrm{A/C}} - \hat{H}_0 \approx \hat{H}_2(J, B) + \hat{H}_3^{\mathrm{A/C}}(D) + \hat{H}_4(J)$$

Models differ only in particle-number nonconserving interactions.

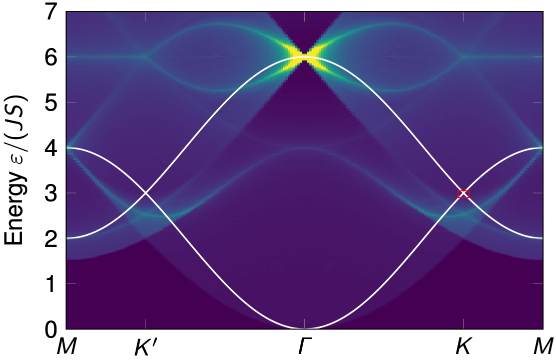
Harmonic theory

Free-magnon dispersion

$$\varepsilon_{\boldsymbol{k},\pm} = JS\,(3 \pm |\gamma_{\boldsymbol{k}}|) + B$$

Dirac cones at $\boldsymbol{k} = \boldsymbol{K}, \boldsymbol{K}'$ $(\gamma_{\boldsymbol{k}} = 0)$

$$\varepsilon_{\mathrm{D}} = 3JS + B$$



Anharmonic theory

Three-magnon interaction:

$$\hat{H}_3 = \frac{1}{2\sqrt{N}} \sum_{\lambda,\mu,\nu=1}^{2} \sum_{\boldsymbol{k},\boldsymbol{q},\boldsymbol{p}}^{\boldsymbol{p}=\boldsymbol{k}+\boldsymbol{q}} \left(\mathcal{V}_{\boldsymbol{k},\boldsymbol{q}\leftarrow\boldsymbol{p}}^{\lambda\mu\leftarrow\nu} \hat{b}_{\boldsymbol{k},\lambda}^{\dagger} \hat{b}_{\boldsymbol{q},\mu}^{\dagger} \hat{b}_{\boldsymbol{p},\nu} + \mathrm{H.c.} \right)$$

Interacting Green's function:

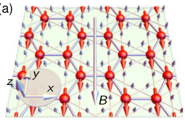
$$\mathcal{G}_{\boldsymbol{k}}^{-1}(\varepsilon, T) = (\varepsilon + \mathrm{i}0^{+}) - \mathcal{E}_{\boldsymbol{k}} - \Sigma_{\boldsymbol{k}}(\varepsilon, T)$$

Self-energy:

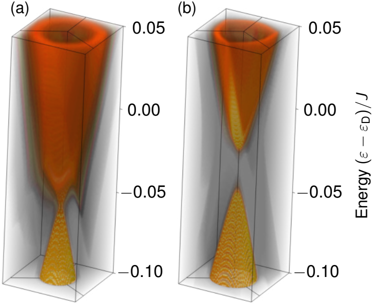
$$\Sigma_{\boldsymbol{k}}^{\alpha\beta}(\varepsilon, T) = \frac{1}{2N} \sum_{\boldsymbol{q}\in\mathrm{BZ}} \sum_{j,j'} \frac{\mathcal{V}_{\boldsymbol{k}\leftarrow\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q}}^{\alpha\leftarrow jj'} \mathcal{V}_{\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q}\leftarrow\boldsymbol{k}}^{jj'\leftarrow\beta}}{\varepsilon + \mathrm{i}0^{+} - \varepsilon_{j,\boldsymbol{q}} - \varepsilon_{j',\boldsymbol{k}-\boldsymbol{q}}}$$

Interaction-renormalized Dirac cones

Spectral function $A_{\boldsymbol{k}}(\varepsilon) = -\frac{1}{\pi}\mathrm{Im}\mathrm{Tr}\mathcal{G}_{\boldsymbol{k}}(\varepsilon)$

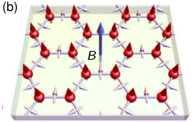


Achiral



Achiral

Chiral

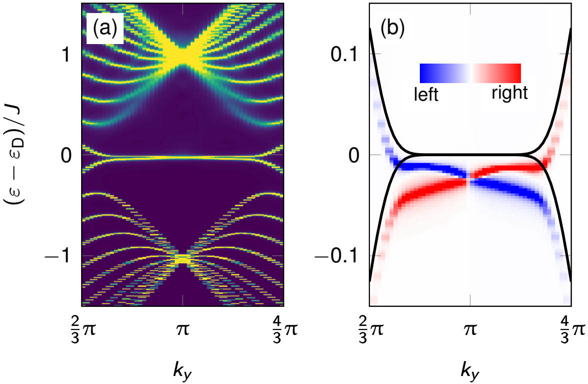


Chiral

Chiral magnet exhibits a spontaneous Dirac mass gap.

☞ **AM**, Plekhanov, Klinovaja, Loss, PRX 11, 021061 (2021)

Slab calculation



Interaction-induced topological band gap with chiral edge magnons.

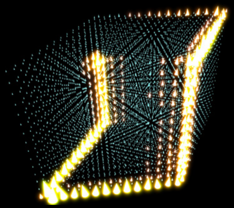
☞ **AM**, Plekhanov, Klinovaja, Loss, PRX 11, 021061 (2021)

Summary

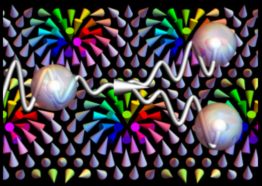
Second-order topological magnonics goes 3D!

Great prospects for topological magnonics in skyrmion crystals!

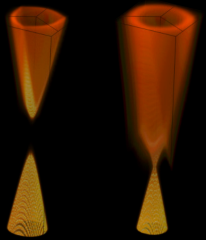
Magnon-magnon interactions cause topology!



AM, Díaz, Klinovaja, Loss, PRB 104, 024406 (2021)

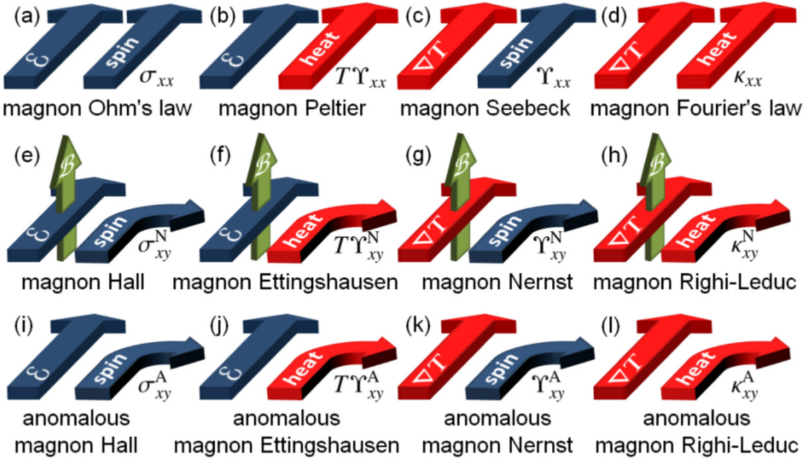


AM, Klinovaja, Loss, PRes 2, 033491 (2020)



AM, Plekhanov, Klinovaja, Loss, PRX 11, 021061 (2021)

Anomalous transverse transport



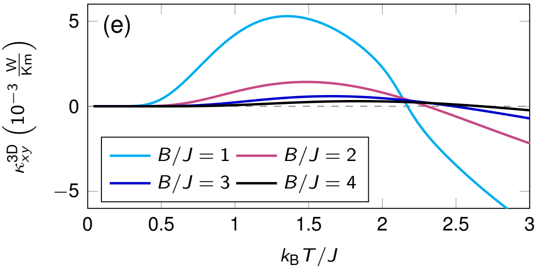
☞ **AM**, Göbel, Henk, Mertig, PRB 97, 140401(R) (2018)

Anomalous transverse transport

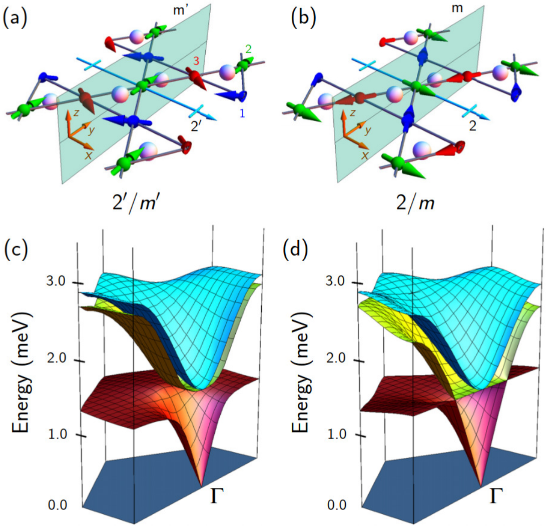
$$\boldsymbol{j} = L^{(0)}\nabla B - L^{(1)} T^{-1} \nabla T,$$
$$\boldsymbol{q} = L^{(1)}\nabla B - L^{(2)} T^{-1} \nabla T,$$

$$L_{xy}^{(i)}(T) \propto \sum_{j=\pm} \int_{\mathrm{BZ}} c_i[\rho(\varepsilon_{\boldsymbol{k},j}, T)]\Omega_{\boldsymbol{k},j} \, \mathrm{d}^2 k$$

☞ Matsumoto, Murakami, PRL 106, 197202 (2011)

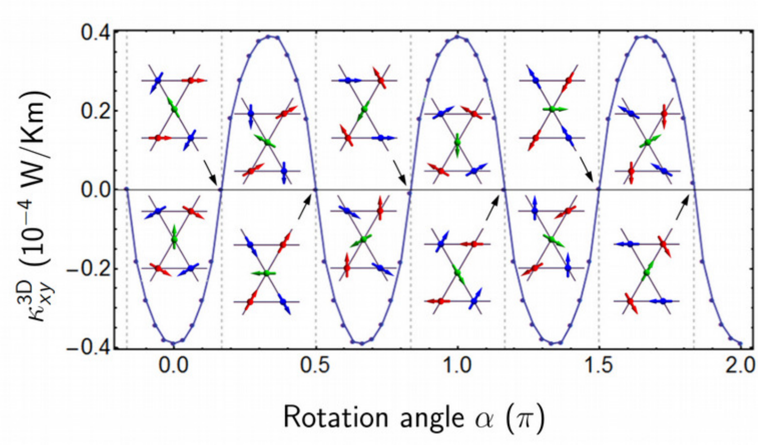


Magnon Chern insulator in antiferromagnets



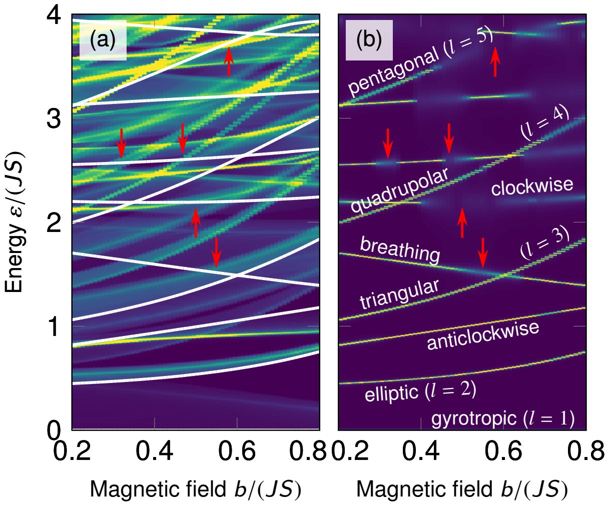
☞ **AM**, Henk, Mertig, PRB 99, 014427 (2019)

Magnon Chern insulator in antiferromagnets



☞ **AM**, Henk, Mertig, PRB 99, 014427 (2019)

Magnon spectral function

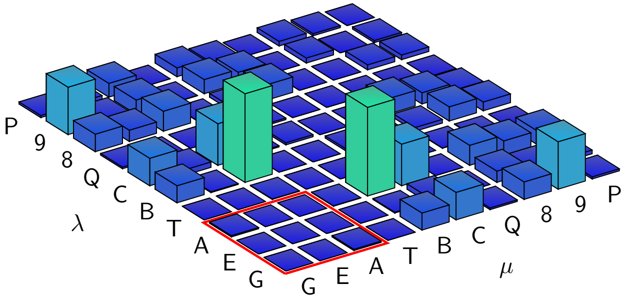


☞ **AM**, Klinovaja, Loss, PRRes 2, 033491 (2020)

Anticlockwise mode is very stable

$$\Gamma^{\alpha}_{\boldsymbol{k}} = \frac{\pi}{2N} \sum_{\boldsymbol{q}\in \mathrm{BZ}} \sum_{\lambda,\mu} \left| \mathcal{V}^{\lambda\mu\leftarrow\alpha}_{\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q}\leftarrow\boldsymbol{k}} \right|^2 \delta\left(\varepsilon - \varepsilon_{\lambda,\boldsymbol{q}} - \varepsilon_{\mu,\boldsymbol{k}-\boldsymbol{q}}\right)$$

(a) Anticlockwise mode

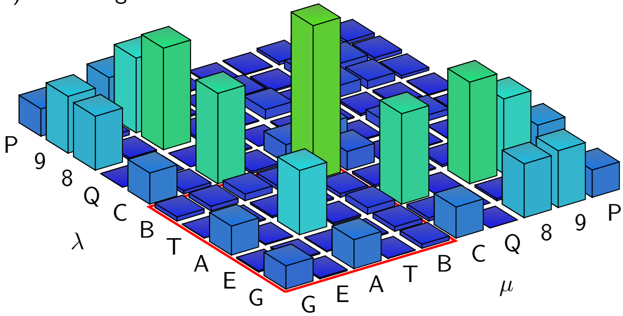


☞ **AM**, Klinovaja, Loss, PRRes 2, 033491 (2020)

Breathing mode is also stable

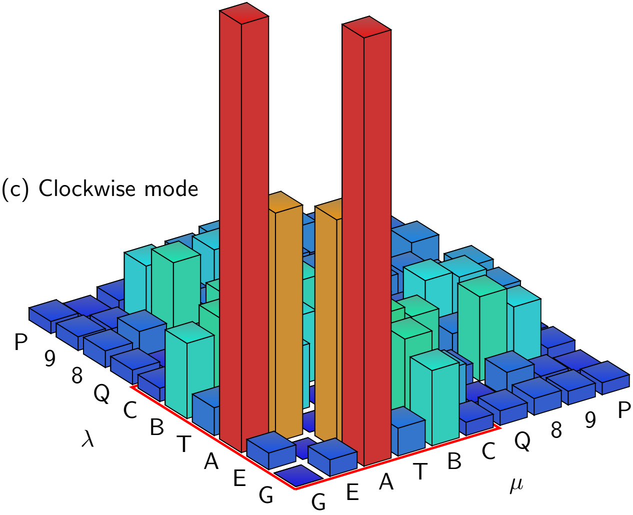
$$\Gamma_{\boldsymbol{k}}^{\alpha} = \frac{\pi}{2N} \sum_{\boldsymbol{q} \in \mathrm{BZ}} \sum_{\lambda,\mu} \left| \mathcal{V}_{\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q} \leftarrow \boldsymbol{k}}^{\lambda\mu \leftarrow \alpha} \right|^2 \delta\left(\varepsilon - \varepsilon_{\lambda,\boldsymbol{q}} - \varepsilon_{\mu,\boldsymbol{k}-\boldsymbol{q}}\right)$$

(b) Breathing mode



☞ **AM**, Klinovaja, Loss, PRRes 2, 033491 (2020)

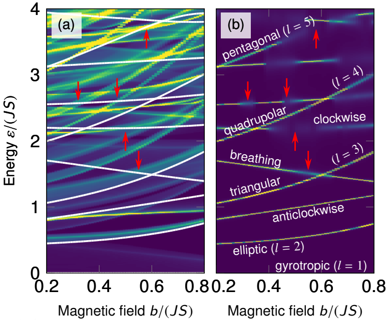
Clockwise mode is very unstable



☞ **AM**, Klinovaja, Loss, PRRes. 2, 033491 (2020)

Implications

Zero-temperature magnon lifetimes are strongly momentum and field-dependent.

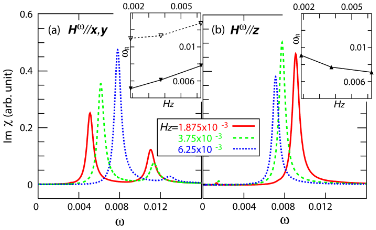


☞ **AM**, Klinovaja, Loss, PRRes. 2, 033491 (2020)

Quantum vs. classical damping

$$\frac{\Gamma_{\mathrm{C}}^{\mathrm{qm.}}}{\Gamma_{\mathrm{C}}^{\mathrm{Gilbert}}} \approx \frac{0.1}{\alpha S}\left(\frac{D}{J}\right)^2 \approx 3.6$$

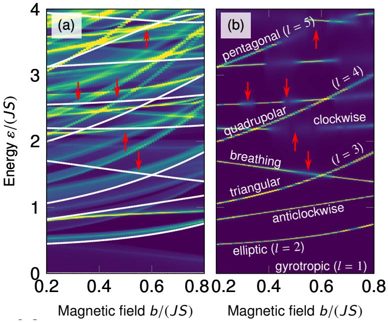
(CuO_2SeO_3)

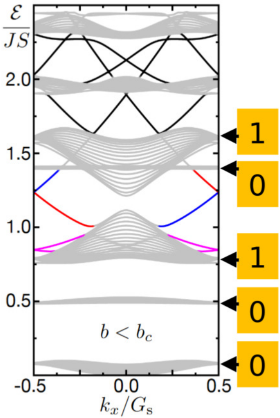


☞ Mochizuki, PRL 108, 017601 (2012)

Implications

Excellent prospects for "topological magnonics" in skyrmion crystals!





☞ Díaz *et al.*, PRes 2, 013231 (2020)

☞ **AM**, Klinovaja, Loss, PRRes 2, 033491 (2020)