

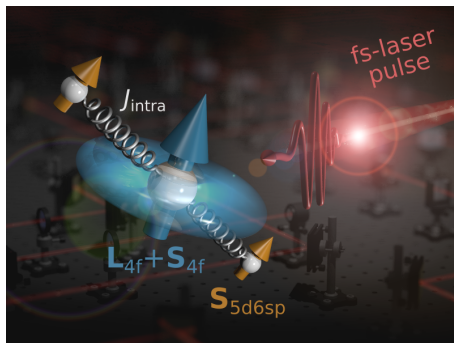
Spin dynamics: the Landau-Lifshitz equation and beyond

Uli Nowak

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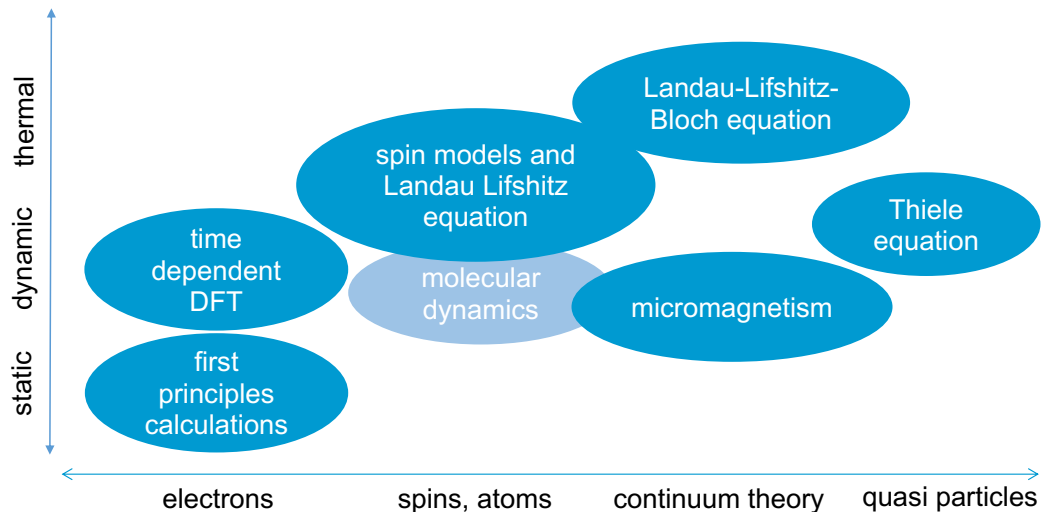
Topics:

- introduction: modelling spin dynamics
- orbital-resolved spin models and inverse Faraday effect in AFMs
- beyond LLG equation: field-derivative torques and nutation
- ultrafast transfer of spin angular momentum into the lattice.



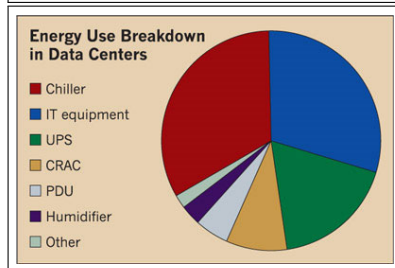
Multi-scale modelling in magnetism

- **goals:** understanding spin structures and dynamics, involving
 - time scales from femtoseconds to years
 - length scales from electronic to sample size
 - temperatures from zero up to the Curie temperature
 - opto-magnetic effects, charge currents, rapid heating



Energy consumption of data centers

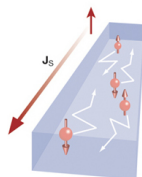
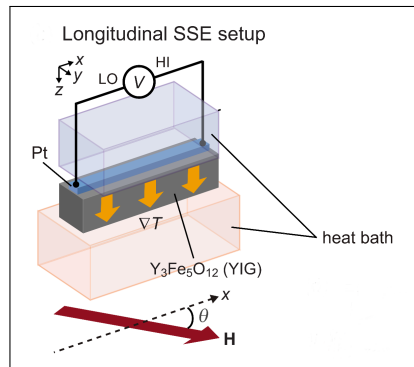
- In 2013, data centers in the U.S. consumed an estimated **91 billion kWh of electricity**.
<http://ecmweb.com/energy-efficiency/data-center-efficiency-trends>
- per the U.S. Energy Information Administration, that is about **7% of total commercial electric energy consumption**
- this number is only going to go up: by 2020, it is estimated consumption will increase to 140 billion kWh, costing about 13 billion \$ in power bills.



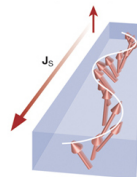
New transport properties: spin caloritronics etc.

Combined transport of spin (with/without charge) and heat

- **Spin Seebeck effect:** generation a spin voltage through a temperature gradient in a conducting ferromagnet (K. Uchida et al., *Nature* **455**, 778 (2008)) or even an insulator (K. Uchida et al., *Nature* **9**, 894 (2010)); Xiao et al. *Phys. Rev. B* **81**, 214418 (2010))
- **Spin current:** two possible types of carriers (Y. Kajiwara et al., *Nature* **464**, 262 (2010))



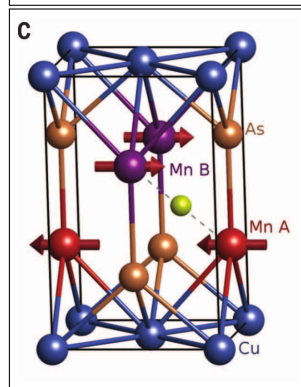
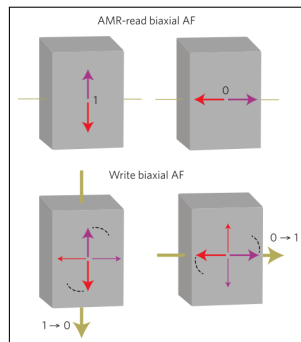
- spin current via polarized electrons



- chargeless spin current via magnons

Spintronics with antiferromagnets

- **read antiferromagnetic order parameter** in biaxial AFMs via anisotropic magneto-resistance (*Jungwirth et al. Nat. Nanotech.* **11**, 91 (2017))
- **write with current-induced staggered fields** (*Wadley et al., Science* **351**, 6273 (2016))
- **advantages of AFM:**
 - no coupling via stray field
 - not susceptible to external fields
 - faster dynamics
 - new class of materials (low damping, insulators)



Ultrafast: all-optical magnetization switching

- uses **circularly polarised light** pulses of ≈ 100 fs duration
- writing bits **without applying an external magnetic field**
- magnetisation direction **defined by helicity of light**
- time scale some hundred femtoseconds
- many open questions regarding fundamental **interactions between laser, electron spins, lattice**

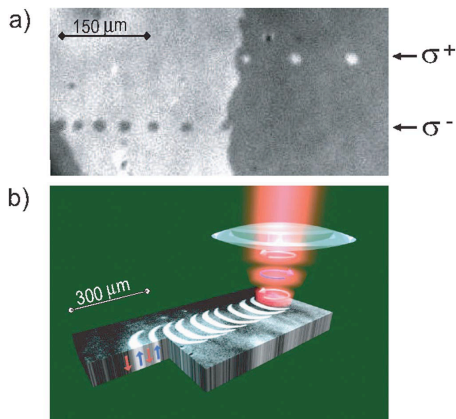
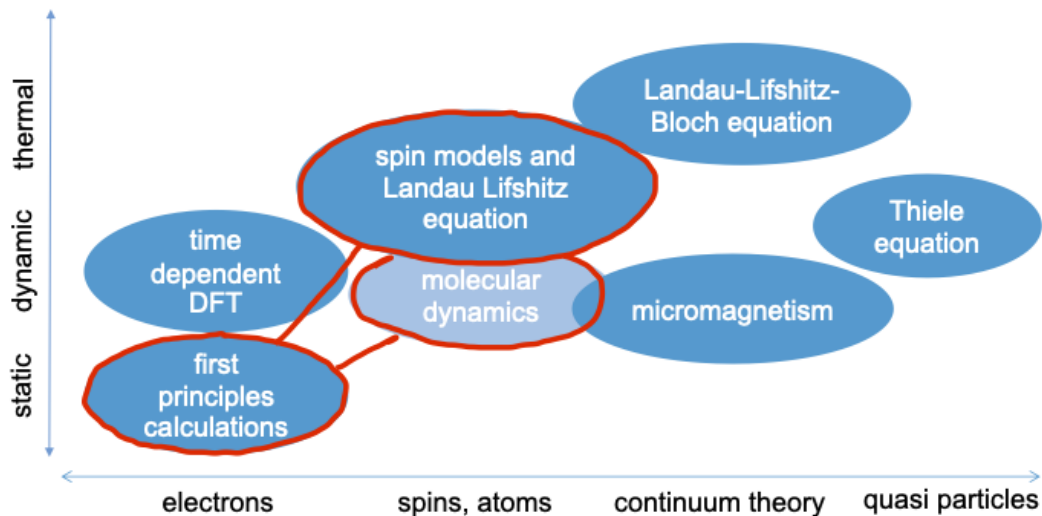


FIG. 4 (color). All-optical magnetic recording by femtosecond laser pulses. (a) The effect of single 40-fs circular polarized laser pulses on the magnetic domains in $\text{Gd}_{22}\text{Fe}_{74,6}\text{Co}_{3,4}$. The domain pattern was obtained by sweeping at high-speed (~ 50 mm/s) circularly polarized beams across the surface so that every single laser pulse landed at a different spot. The laser fluence was about 2.9 mJ/cm². The small size variation of the written domains is caused by the pulse-to-pulse fluctuation of the laser intensity.

Stanciu et al. PRL **99**, 047601 (2007)

Multi-scale modelling in magnetism

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Micromagnetic continuum theory

- on a length scale \gg lattice constant a the **magnetisation** is assumed as field $\mathbf{m}(\mathbf{r})$ with constant length and **energy**:

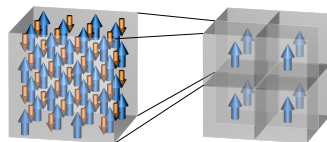
$$E = \underbrace{\frac{J}{2a} \int_V (\nabla \mathbf{m})^2 dV}_{\text{exchange}} - \underbrace{\frac{d_z}{a^3} \int_V S_z^2 dV}_{\text{anisotropy}} - \underbrace{M_s \int_V \mathbf{m} \cdot \mathbf{B} dV}_{\text{external field}} - \underbrace{\frac{\mu_0}{2} \int_V \mathbf{m} \cdot \mathbf{H}_s dV}_{\text{magneto-static}}$$

- equation of motion:

Landau-Lifshitz-Gilbert equation

$$\dot{\mathbf{m}} = -\frac{\gamma}{(1 + \alpha^2)M_s} \mathbf{m}_i \times \mathbf{H}_{\text{eff}} - \frac{\alpha\gamma}{(1 + \alpha^2)M_s} \mathbf{m} \times (\mathbf{m}_i \times \mathbf{H}_{\text{eff}})$$

- with effective field
- describes domain structures and magnetisation dynamics
- **at finite temperatures:**
 - Langevin dynamics: add stochastic field
(*Brown, Phys. Rev.* **130**, 1677 (1963))
 - ☺ at elevated temperatures approach will fail since spin wave spectrum is limited by cell size
(*Grinstein and Koch, PRL* **90**, 207201 (2003))



Atomistic spin model

- Hamiltonian for spins $\underline{S}_i = \underline{\mu}_i / \mu_s$ on a given lattice:

spin model including relativistic interactions

$$\mathcal{H} = -\frac{1}{2} \sum_{i,j} \mathbf{S}_i J_{ij} \mathbf{S}_j - \sum_i d_i^z (S_i^z)^2 - \mathbf{B} \cdot \sum_i \mu_i \mathbf{S}_i - \sum_{i,j} \frac{\mu_0 \mu_i \mu_j}{8\pi} \frac{3(\mathbf{S}_i \cdot \mathbf{e}_{ij})(\mathbf{e}_{ij} \cdot \mathbf{S}_j) - \mathbf{S}_i \cdot \mathbf{S}_j}{r_{ij}^3}$$

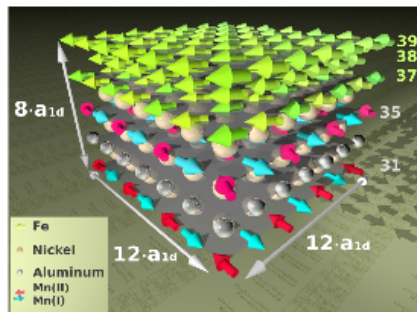
exchange anisotropy external field dipole-dipole

- **tensorial exchange interactions** J_{ij} can be decomposed in:

$$\mathcal{H}_{\text{ex}} = -\frac{1}{2} \sum_{i,j} J_{ij}^{\text{iso}} \mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{2} \sum_{i,j} \mathbf{S}_i J_{ij}^{\text{S}} \mathbf{S}_j - \frac{1}{2} \sum_{i,j} \mathbf{D}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j)$$

isotropic exchange two-site anisotropy Dzyaloshynskii-Moriya

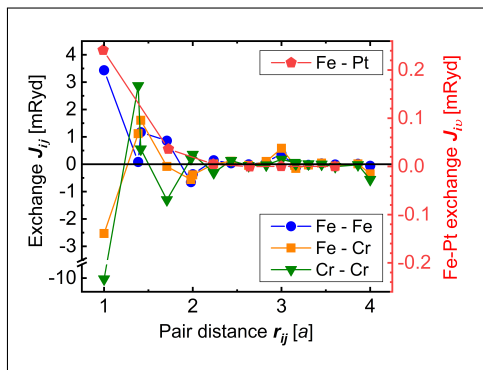
- different types of **anisotropies** . . .
- **dipole-dipole interaction** leads to:
 - shape anisotropy
 - domain structures
 - large numerical effort



Yanes et al., PRB 96, 064435 (2017)

Spin model parameters from first principles

- exchange integrals, isotropic but **orbital resolved**
(*Lichtenstein et al., JMMM 67, 65 (1987)*)
- fully **relativistic** screened Korringa-Kohn-Rostoker (SKKR) method plus **spin cluster expansion** for layered systems and clusters
(*Szunyogh et al., PRB 83, 024401(2011)*)
- calculations of **opto-magnetic effects** such as IFE
(*Berritta et al., PRL 117, 137203 (2016)*;
John et al., Scientific Reports 7, 4114 (2017))



Schmidt et al., PRB 102, 214436 (2020)

Gd	orbital-resolved dynamics	<i>Frietsch et al., Nature Com. 6 8262 (2015)</i>
Tb	orbital-resolved dynamics	<i>Frietsch et al., Science Advances, 6, eabb1601 (2020)</i>
CrPt	switching with IFE in an AFM	<i>Dannegger et al., submitted (2021)</i>

Equation of motion

stochastic Landau-Lifshitz-Gilbert equation

$$\dot{\mathbf{S}}_i = - \frac{\gamma}{(1+\alpha^2)\mu_s} \mathbf{S}_i \times \mathbf{H}_i(t) - \frac{\alpha\gamma}{(1+\alpha^2)\mu_s} \mathbf{S}_i \times (\mathbf{S}_i \times \mathbf{H}_i(t))$$

precession

dissipation

with $\mathbf{H}_i(t) = -\frac{\partial \mathcal{H}}{\partial \mathbf{S}_i} + \zeta_i(t)$

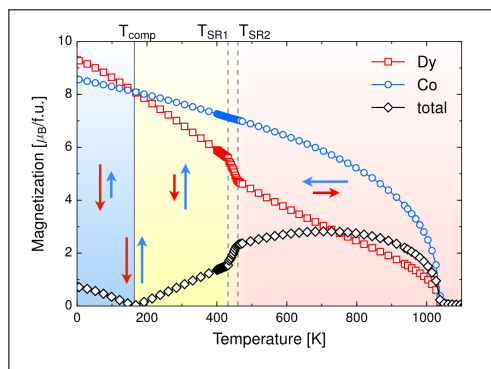
fluctuations

and $\langle \zeta_i(t) \rangle = 0$, $\langle \zeta_{i\eta}(0) \zeta_{j\vartheta}(t) \rangle = \delta_{ij} \delta_{\eta\vartheta} \delta(t) 2\alpha k_B T \mu_s / \gamma$.

α : quantifies coupling to the electronic/phononic heat bath

spin dynamics

- **numerical integration** of the stochastic LLG equation
(Lyberatos et al., *J. Phys. C* **5**, 8911 (1993))
- simulation of 10^8 spins possible
- statistical average in the **canonical ensemble**
- ☺ realistic dispersion relations; non-linear processes; critical behavior methods
- ☹ classical approximation; large numerical effort



Donges et al., *PRB* **96**, 024412 (2017)

Spin dynamics: the Landau-Lifshitz equation and beyond

Tobias Dannegger, Andreas Donges, Severin Selzer, Uli Nowak
University of Konstanz, Germany

In collaboration with:

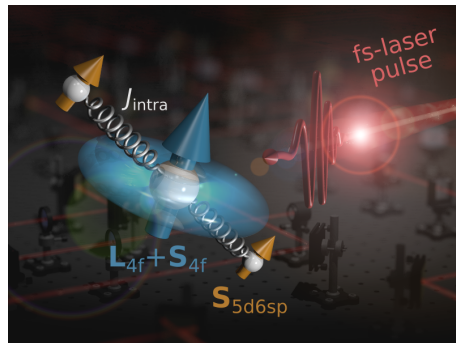
M. Berritta, P. Oppeneer, University of Uppsala

C. Karva, Charles University, Prague

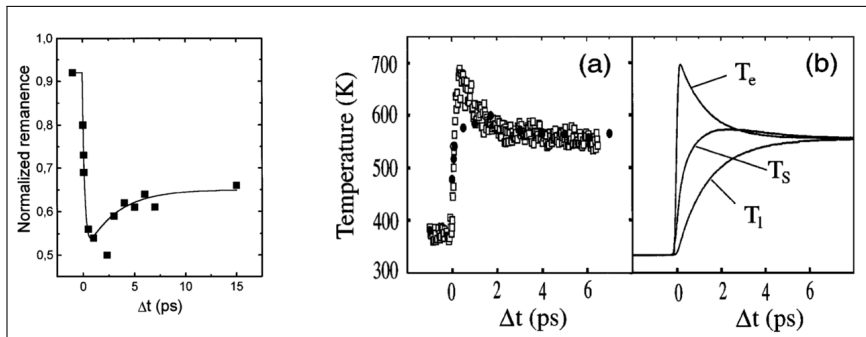
B. Frietsch, U. Ritzmann, J. Bowlan, R. Carley, M. Teichmann, M. Weinelt, FU Berlin

Topics:

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- **orbital-resolved spin models and inverse Faraday effect in AFMs**
- beyond LLG equation:
field-derivative torques and nutation
- ultrafast transfer of spin angular momentum into the lattice.

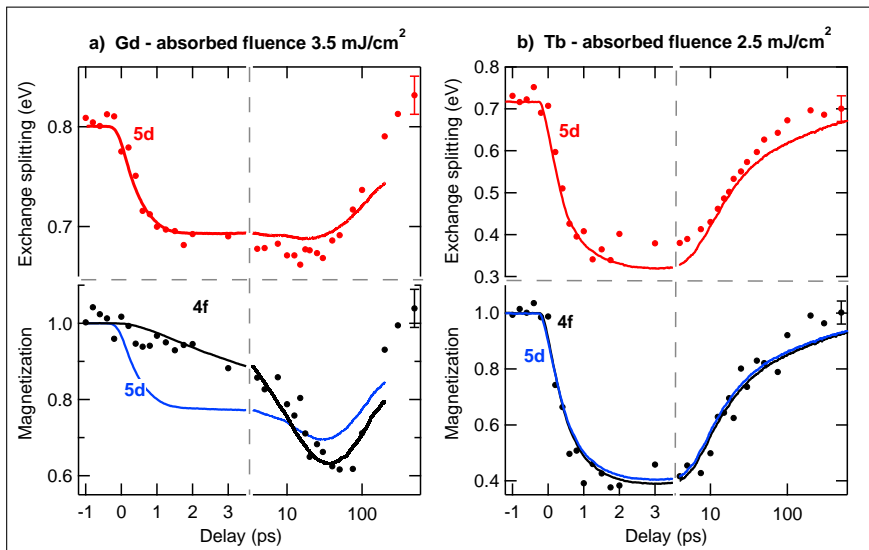


Laser-induced ultrafast spin dynamics



- following a fs laser pulse **magnetisation can break down on a time scale of some hundred femtoseconds and recover on a ps time scale**
(*Beaurepaire et al., PRL 76, 4250 (1996)*)
- the phenomenological **three-temperature model** gives an idea of the energy flow
- flow of **angular momentum** remains an open question
- coupling between **microscopic degrees of freedom** (spin, electrons, lattice) hardly understood

Demagnetization dynamics of Gd and Tb



(Frietsch et al., *Science Adv.* **6**, eabb1601 (2020))

- photoemission data: concurrent measurement of 5d-6s-exchange splitting and 4f magnetic linear dichroism (B. Frietsch et al., *Nature Com.* **6**, 8262 (2015))
- Gd shows **distinct demagnetization dynamics** of d and f electrons
- Tb does not
- excellent agreement between theory (lines) and measurement (points)

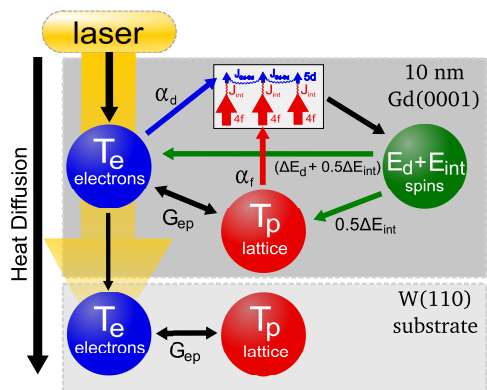
Modelling the magnetic response to the laser pulse: Gd and Tb

- **two temperature model** for electron and phonon temperature (*Kaganov et al. JETP 4, 173 (1957)*)

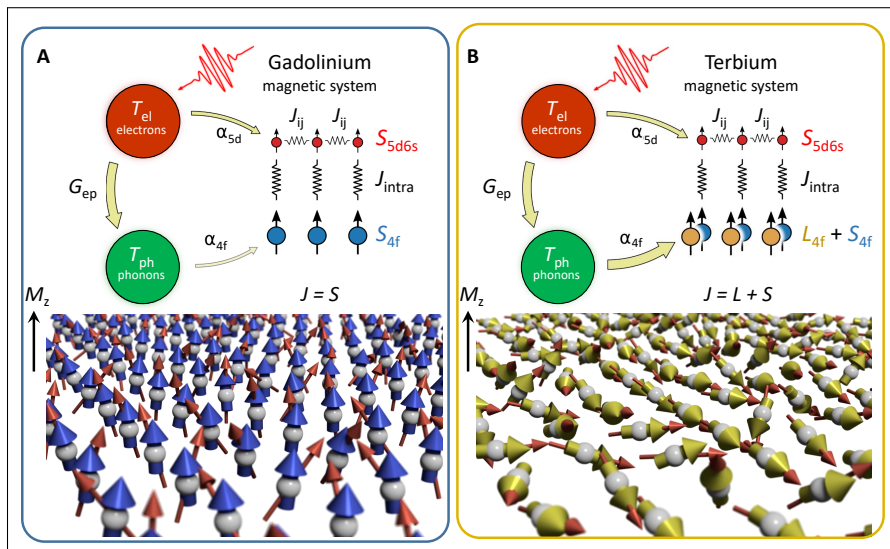
$$C_e \frac{dT_e}{dt} = -G_{el}(T_e - T_p) + P(t)$$

$$C_p \frac{dT_p}{dt} = G_{el}(T_e - T_p)$$

- add heat diffusion and substrate
- **orbital-resolved spin model** for spin dynamics simulations with parameters from ab-initio calculations (*Carva, Oppeneer*)
- cooling effect of spin system taken into account



Demagnetization dynamics of Gd and Tb

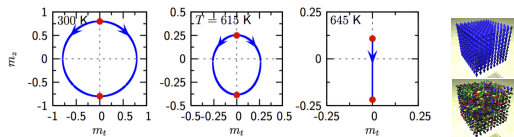


(Frietsch et al., Science Adv. **6**, eabb1601 (2020))

- Gd shows distinct demagnetization dynamics of d and f electrons
- Tb does not because of the **larger coupling to the phonons**

Switching with inverse Faraday effect

- **helicity-dependent all-optical switching** demonstrated in many materials
- strong influence of heating, **switching is linear**:

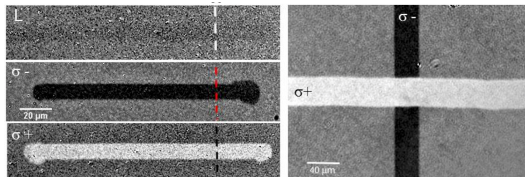


(Kazantseva et al., *Europhys. Lett.* **86**, 27006 (2009))

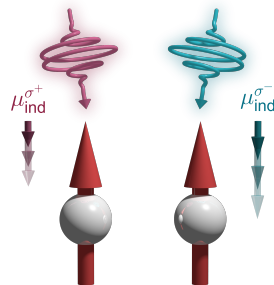
- due to **inverse Faraday effect**: circularly polarized light induces magnetization in a sample
(Battiato et al. *PRB* **89**, 0144413 (2014))

- what about antiferromagnets?

all-optical writing of an FePt nanoparticle recording medium

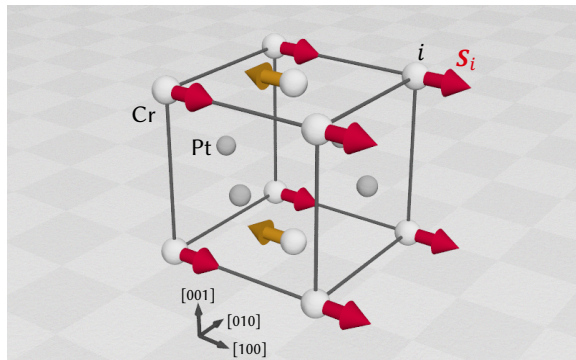


John et al., *Scientific Reports* **7**, 4114 (2017)

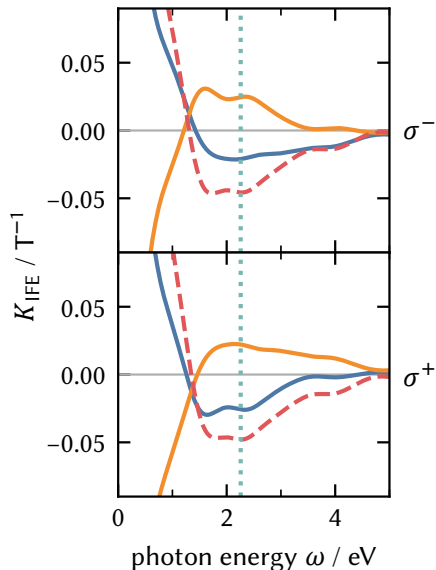


Inverse Faraday effect in CrPt

- CrPt: **antiferromagnet** with two sublattices
- **IFE quantified** by first principles calculations
(Berritta *et al.*, *Phys. Rev. Lett.* **117**, 137203 (2016))



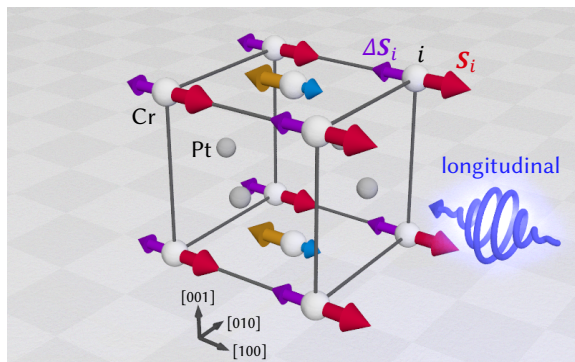
- lattice and spin moments



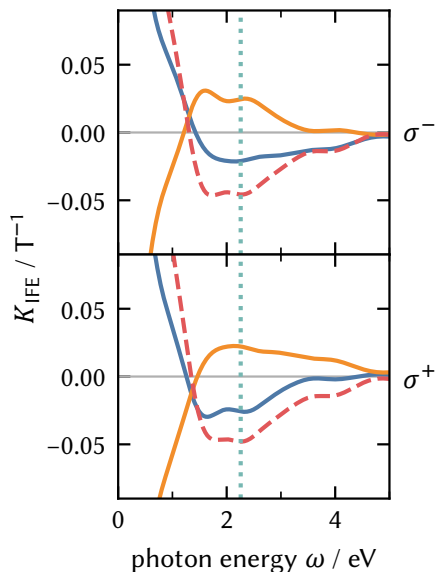
IFE constants Cr moment direction parallel and antiparallel to k -vector of the light. Red line indicates difference.

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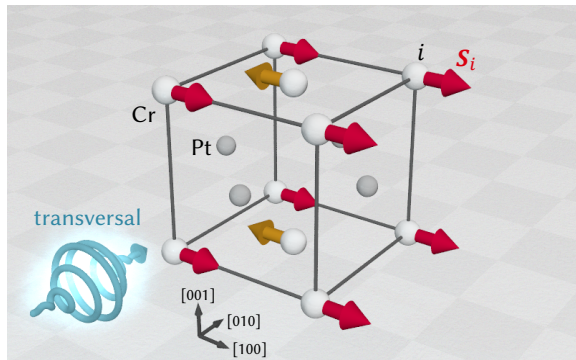
- IFE leads to **staggered induced moments**



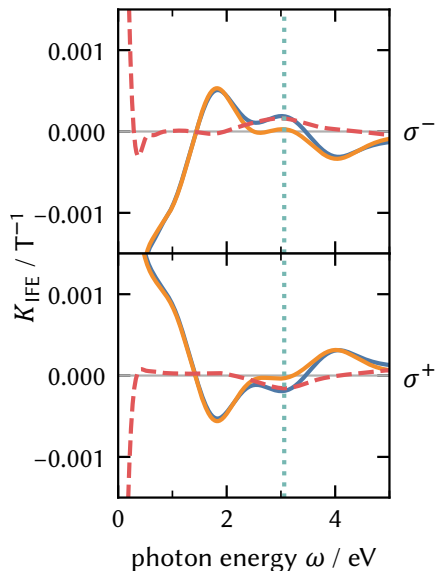
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(Berritta et al., Phys. Rev. Lett. **117**, 137203 (2016))



- perpendicular IFE very small



IFE constants Cr moment direction parallel and antiparallel to k -vector of the light. Red line indicates difference.

Atomistic spin dynamics

- **induced magnetic moments** from DFT calculations via:

$$\mu_{\text{ind}} = \frac{K_{\text{IFE}}^{\sigma}(\omega)V}{c} I(t)$$

- add to the existing moments:

$$\boldsymbol{\mu}_i \rightarrow \boldsymbol{\mu}_i + \boldsymbol{\mu}_{\text{ind}}, \quad \mathbf{S}_i \rightarrow \mathbf{S}_i + \Delta \mathbf{S}_i$$

spin model

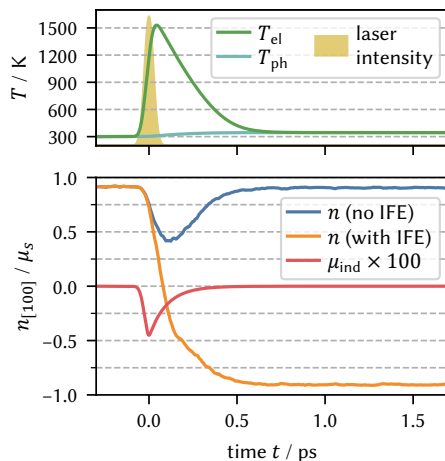
$$\mathcal{H} = - \sum_{\langle ij \rangle} J_{ij} (\mathbf{S}_i + \Delta \mathbf{S}_i) \cdot (\mathbf{S}_j + \Delta \mathbf{S}_j) - d_z \sum_i S_{iz}^2$$

- induced moments **couple via exchange interaction** with neighbouring spins
- dynamics via stochastic LLG equation

two-temperature model:

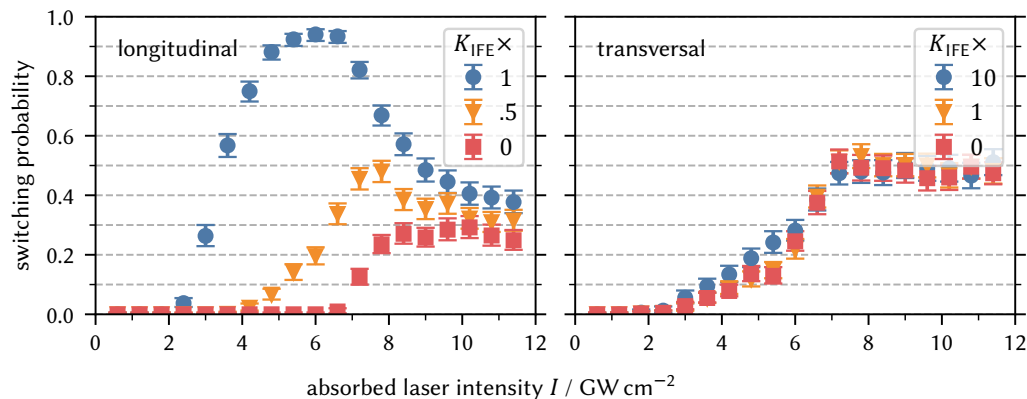
$$C_{\text{el}} \dot{T}_{\text{el}} = G(T_{\text{ph}} - T_{\text{el}}) + P(t),$$

$$C_{\text{ph}} \dot{T}_{\text{ph}} = G(T_{\text{el}} - T_{\text{ph}})$$



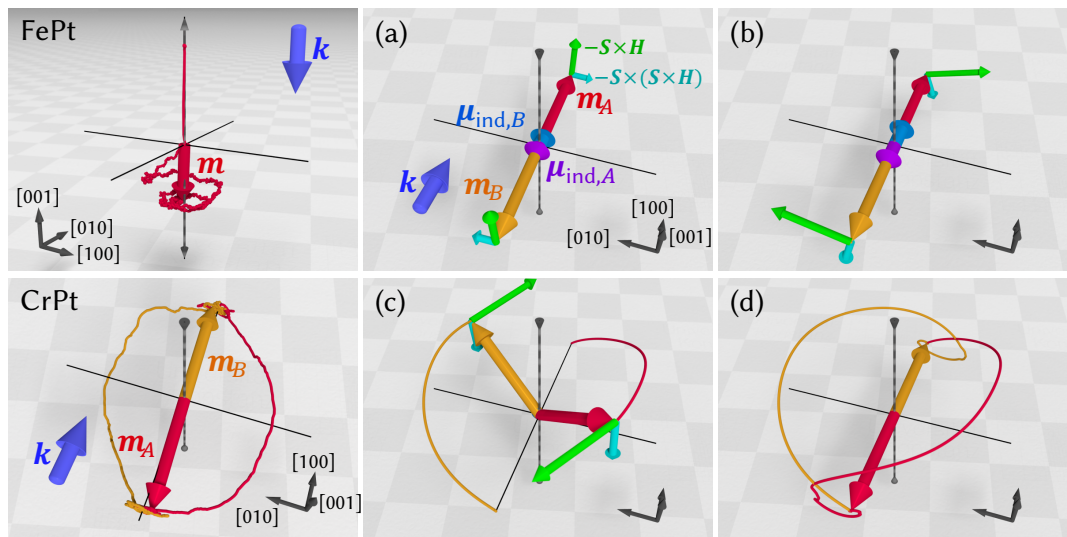
time-dependence: $\mu_{\text{ind}} \sim e^{-t/\tau}$

Statistics of the switching process



- switching process can be stochastic due to heating
- **deterministic switching** observed e.g. for $I \approx 6 \text{GW}/\text{cm}^2$, $\tau \gtrsim 200\text{fs}$
- perpendicular switching also possible for lower intensities

Exchange-enhanced switching dynamics



- in the ferromagnet FePt switching is linear
- in CrPt we find an **elliptical switching path**
- dynamics **exchange-enhanced** and driven by precessional torque
- very fast, time scale $< 500\text{fs}$

Spin dynamics: the Landau-Lifshitz equation and beyond

Ritwik Mondal, Sebastian Großenbach, Levente Rózsa, Andreas Donges, and Uli Nowak
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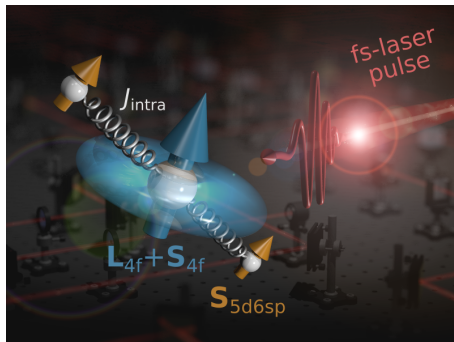
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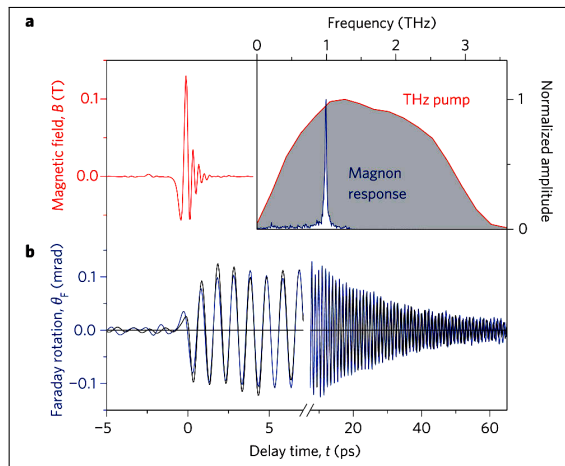
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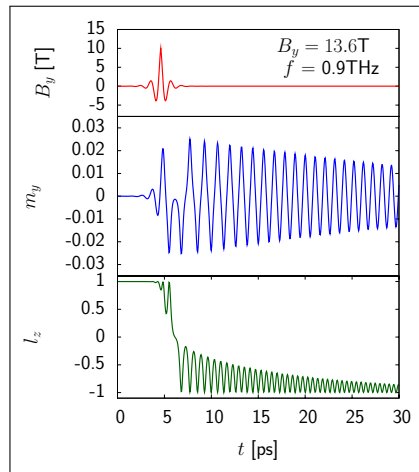


Excitation of NiO with single cycle THz pulses



Kampfrath et al., *Nature Photonics* **5**, 31(2010)

- **resonant excitation** with B field component of an intense ultrashort THz laser pulse



Wienholdt et al., *PRL* **111**, 217202 (2013)

- simulations show: even **switching** possible at 13.6T with frequency around 0.9THz

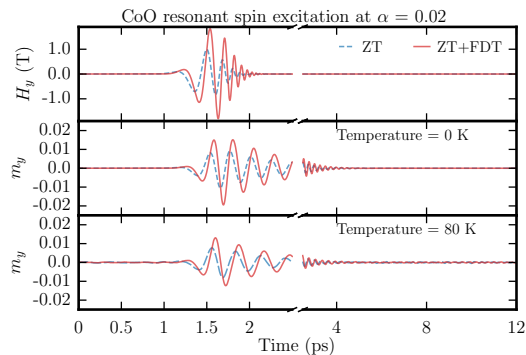
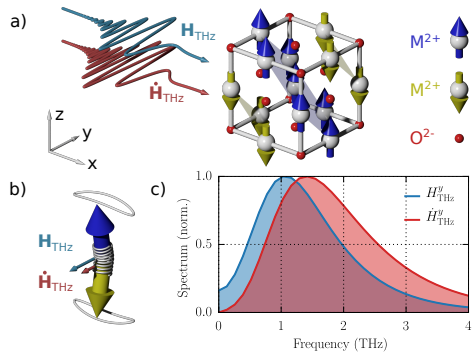
Beyond LLG: field derivative torque

- equation of motion for spins derived starting from relativistic Dirac-Kohn-Sham equation
- several new extensions to LLG equation: one is a **field-derivative torque** (Mondal et al., PRB 94, 144419 (2016))

extended LLG equation

$$\dot{\mathbf{m}}_i = -\frac{\gamma}{(1+\alpha^2)\mu_s} \mathbf{m}_i \times \left(\mathbf{H}_i^{\text{eff}} - \frac{\alpha a^3}{\gamma} \frac{\partial \mathbf{H}}{\partial t} \right) - \frac{\gamma \alpha}{(1+\alpha^2)\mu_s} \mathbf{m}_i \times \left(\mathbf{m}_i \times \left(\mathbf{H}_i^{\text{eff}} - \frac{\alpha a^3}{\gamma} \frac{\partial \mathbf{H}}{\partial t} \right) \right)$$

- simulations of the stochastic LLG equation with field-derivative torque for CoO show:
- field-derivative torque leads to **phase shift and higher amplitude**



Mondal et al., PRB 100, 060409R (2019)

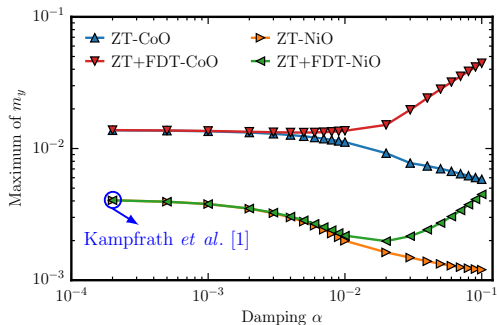
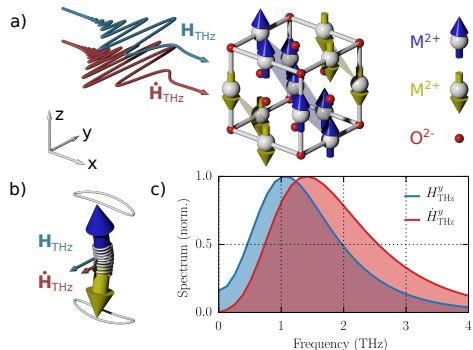
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- simulations of the stochastic LLG equation with field-derivative torque for CoO show:
- effect **enhanced for high resonance frequency and spin-orbit coupling**



Mondal *et al.*, PRB **100**, 060409R (2019)

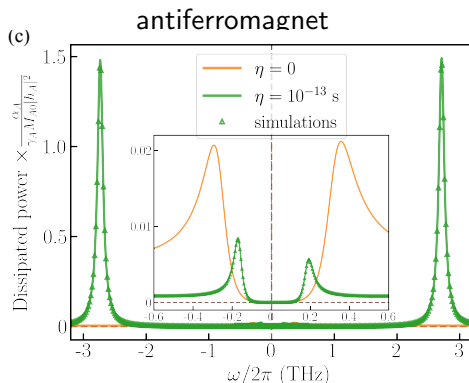
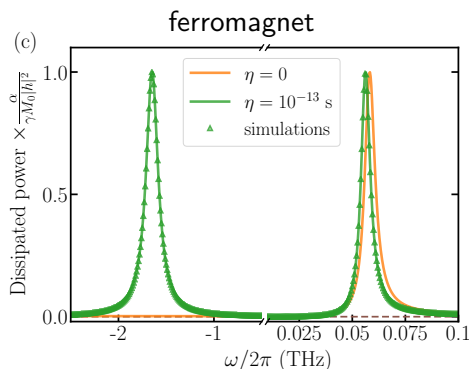
Beyond LLG: nutation

- another extension to LLG equation is a second order time derivative (Mondal et al., PRB **94**, 144419 (2016))
- new effects: **inertia and nutation** (Ciornei et al. PRB **83**, 020410R (2011); Thonig et al., Sci. Rep. **7**, 931 (2017); Neeraj et al., Nat. Phys. **17**, 245 (2021))

inertial LLG equation

$$\dot{\mathbf{M}}_i = -\gamma_i \mathbf{M}_i \times \mathbf{H}_i + \frac{\alpha_i}{M_{i0}} \mathbf{M}_i \times \dot{\mathbf{M}}_i + \frac{\eta}{M_{i0}} \mathbf{M}_i \times \ddot{\mathbf{M}}_i$$

- calculations of the (A)FMR resonance spectra show:
 - precession frequencies decrease
 - inertia reduces the effective damping
 - additional high-frequency nutation peaks
 - effects larger in AFMs



Mondal et al., PRB **103**, 104404 (2021)

Spin dynamics: the Landau-Lifshitz equation and beyond

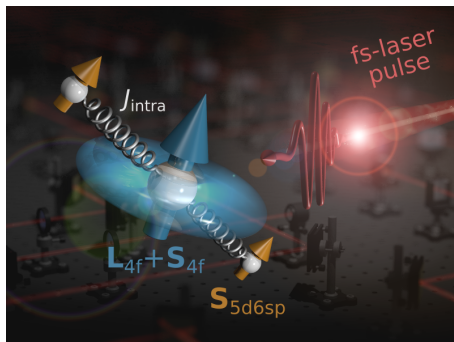
Martin Evers, Hannah Lange, Andreas Donges, and Uli Nowak
University of Konstanz, Germany

In collaboration with:

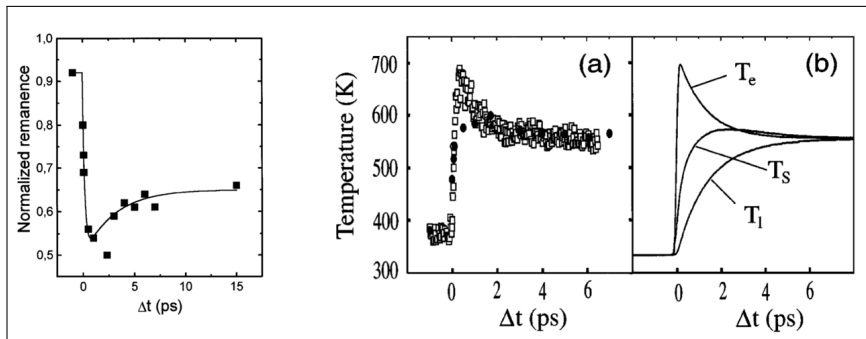
S. Tauchert, M. Volkov, D. Ehberger, D. Kazenwadel, P. Baum, University of Konstanz
A. Book, W. Kreuzpaintner, LMU München

Topics:

- introduction: modelling spin dynamics
- orbital-resolved spin models and inverse Faraday effect in AFMs
- beyond LLG equation: field-derivative torques and nutation
- **ultrafast transfer of spin angular momentum into the lattice**

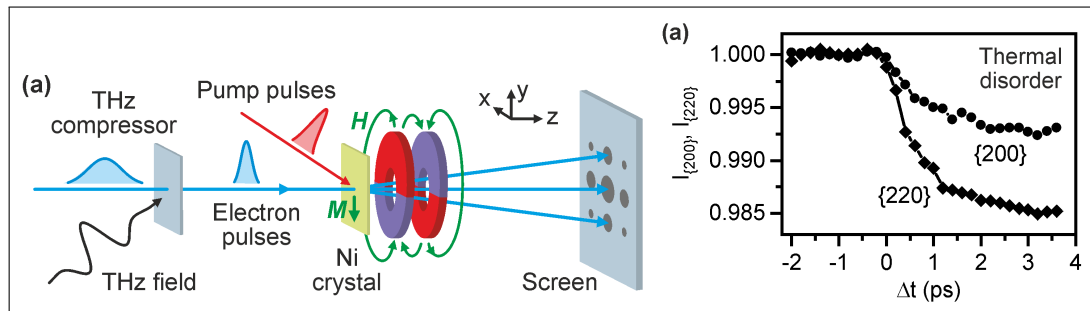


Laser-induced ultrafast spin dynamics



- following a fs laser pulse **magnetisation can break down on a time scale of some hundred femtoseconds and recover on a ps time scale**
(*Beaurepaire et al., PRL 76, 4250 (1996)*)
- the phenomenological **three-temperature model** gives an idea of the energy flow
- flow of **angular momentum** remains an open question
- coupling between **microscopic degrees of freedom** (spin, electrons, lattice) hardly understood

Ultrafast generation of chiral phonons

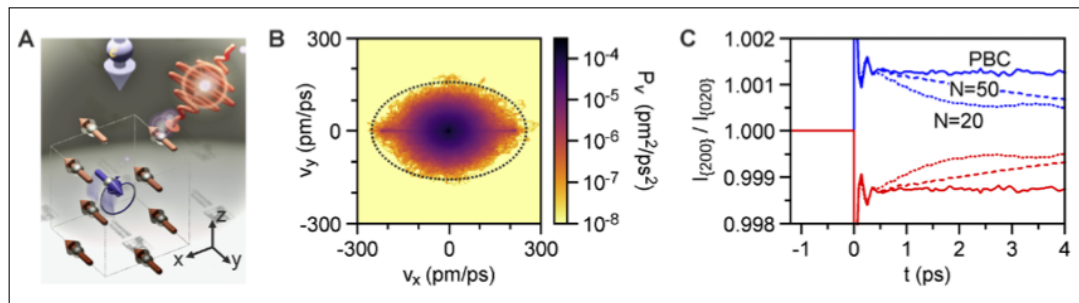


experiment by *S. Tauchert et al.*

- **time-resolved electron diffraction** delivers structural dynamics and atomic motions with picometer and femtosecond resolutions in space and time
- THz fields compress electron pulses to < 100 fs duration
- electron diffraction of a single-crystalline Ni layer on Si membrane reveals the epitaxy and distinct **Bragg spots**
- when laser pulse hits the sample (2 2 0) spot decays by ≈ 1.5 % with a < 500 fs time constant indicating the ultrafast population of phonons

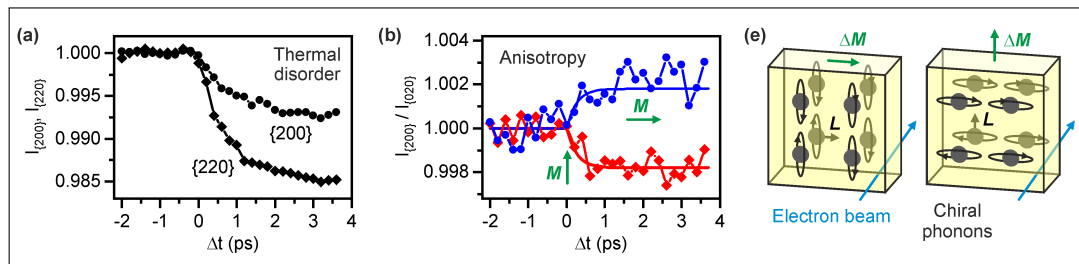
Ultrafast generation of chiral phonons: theory

How can we detect the **phonon angular momentum**?



- idea: develop a model for local phonon excitations of the lattice with finite angular momentum $L = \hbar \mathbf{e}_M = m \Delta \mathbf{r} \times \mathbf{p}$
- number of local excitation corresponding to a demagnetization of 50 %
- **molecular dynamics simulations** using LAMMPS: free Ni crystal with 500000 atoms
- calculate and analyze diffraction patterns
- broadening of (200) and (020) peaks different \Rightarrow **anisotropy indicates chiral character of phonons**

Ultrafast generation of chiral phonons



experiment by *S. Tauchert et al.*

- after laser excitation time-resolved electron diffraction shows an **anisotropy of the (normally equivalent) Bragg peaks within less than a ps**
- this indicates a **transfer of angular momentum from the spin system into the lattice** within less than a ps
- this is not an ultrafast Einstein-de Haas effect (*Dornes et al., Nature 565, 209 (2019)*)

Spin lattice coupling

spin-lattice Hamiltonian

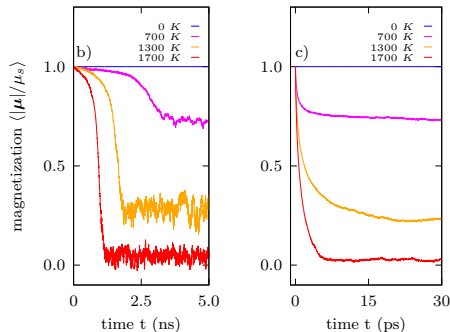
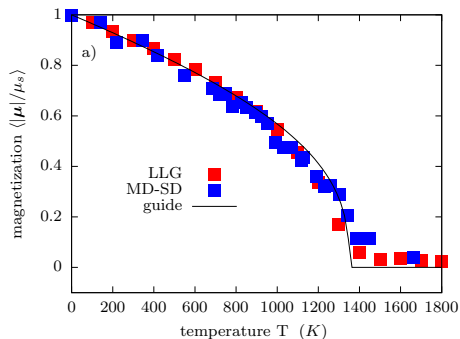
$$\mathcal{H} = - \underbrace{\sum_{i,j} J(r_{ij}) (\mathbf{S}_i \cdot \mathbf{S}_j)}_{\text{exchange}} + \lambda \underbrace{\sum_{i,j} \frac{(\mathbf{S}_i \cdot \mathbf{r}_{ij})(\mathbf{r}_{ij} \cdot \mathbf{S}_j)}{r_{ij}^6}}_{\text{spin-lattice}}$$

$$- \underbrace{\mu_s \mathbf{B} \cdot \sum_i \mathbf{S}_i}_{\text{Zeeman}} + \underbrace{\sum_i \frac{p_i^2}{2m_i}}_{\text{kinetic}} + \underbrace{\sum_{i,j} V(r_{ij})}_{\text{lattice}}$$

equations of motion

$$\dot{\mathbf{r}}_i = \frac{\partial \mathcal{H}}{\partial \mathbf{p}_i} \quad \dot{\mathbf{p}}_i = - \frac{\partial \mathcal{H}}{\partial \mathbf{r}_i} \quad \dot{\mathbf{S}}_i = \frac{\gamma}{\mu_s} \mathbf{S}_i \times \frac{\partial \mathcal{H}}{\partial \mathbf{S}_i}$$

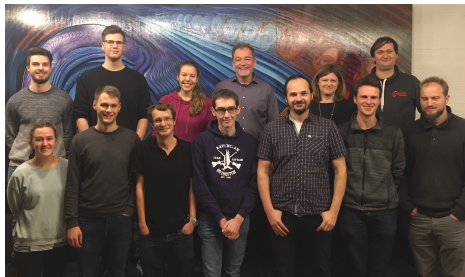
- **no heat bath** necessary
- same equilibrium magnetisation as with stochastic LLG equation but different dynamics
- see also: *Strungaru et al., PRB 103, 024429 (2021)*



Abmann et al, JMMM 469, 217 (2019)

Thanks to ...

My group in Konstanz:



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Controlling Nanostructures



Schwerpunktprogramm
Skymionics

SFB 1214



Anisotropic nanoparticles

SFB 1432



Fluctuations

Summary: Landau-Lifshitz equation and beyond

- **stochastic LLG equation:**
 - ultrafast demagnetization with orbital-resolved spin models for Gd and Tb
 - switching with the inverse Faraday effect in the antiferromagnet CrPt
- **beyond LLG equation:**
 - THz dynamics with field-derivative torques in NiO and CoO
 - (A)FMR with inertial dynamics and nutation
- **no LLG equation** (but should be connected):
 - ultrafast transfer of spin angular momentum into the phonon systems
 - outlook: a framework for spin-lattice dynamics