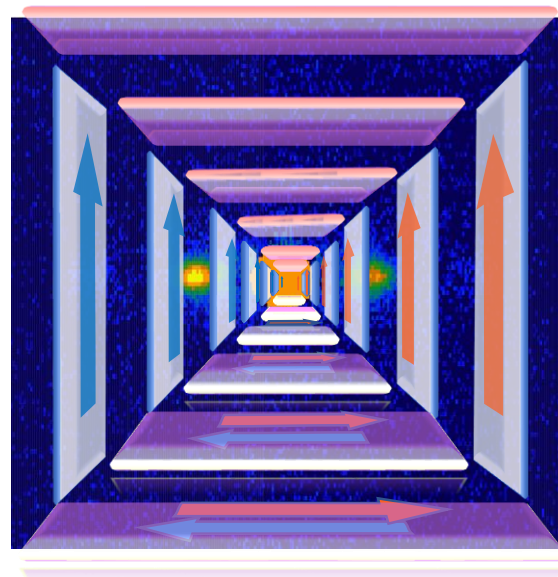


Improper Dzyaloshinskii spirals & metamagnetic textures – and where to look for them



Leibniz-Institut
für Festkörper- und
Werkstoffforschung
Dresden

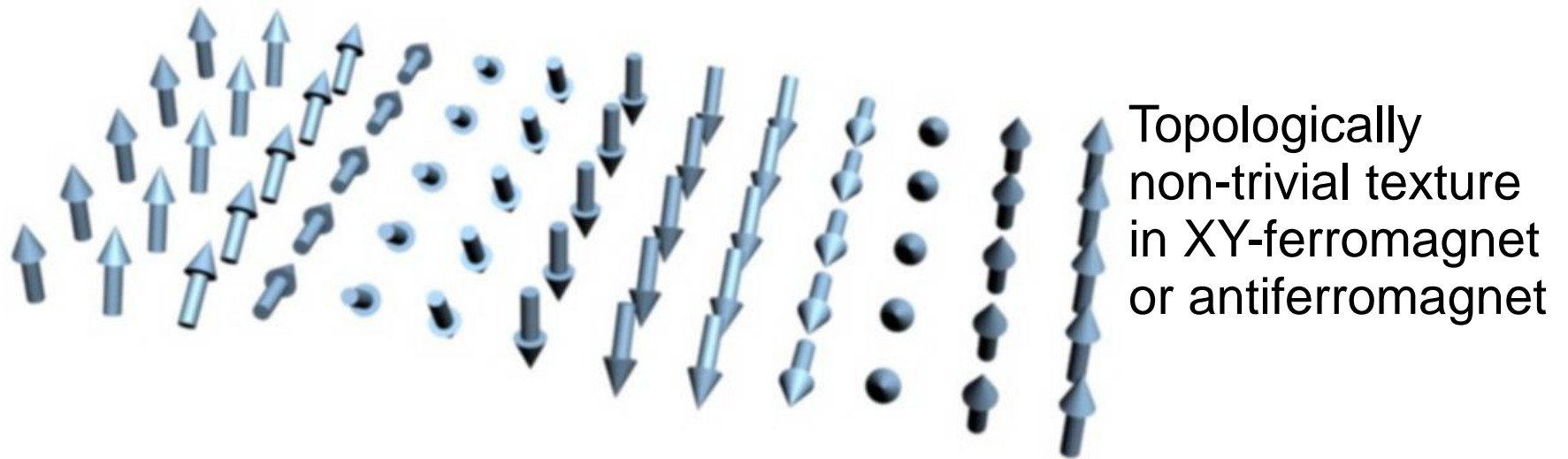
Ulrich K. Rößler
IFW Dresden



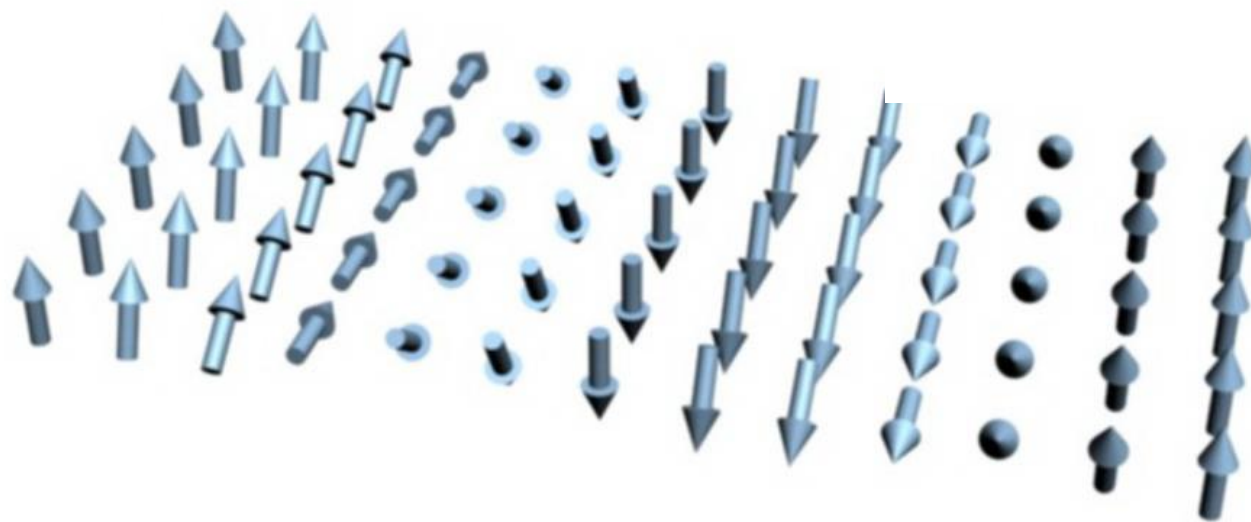
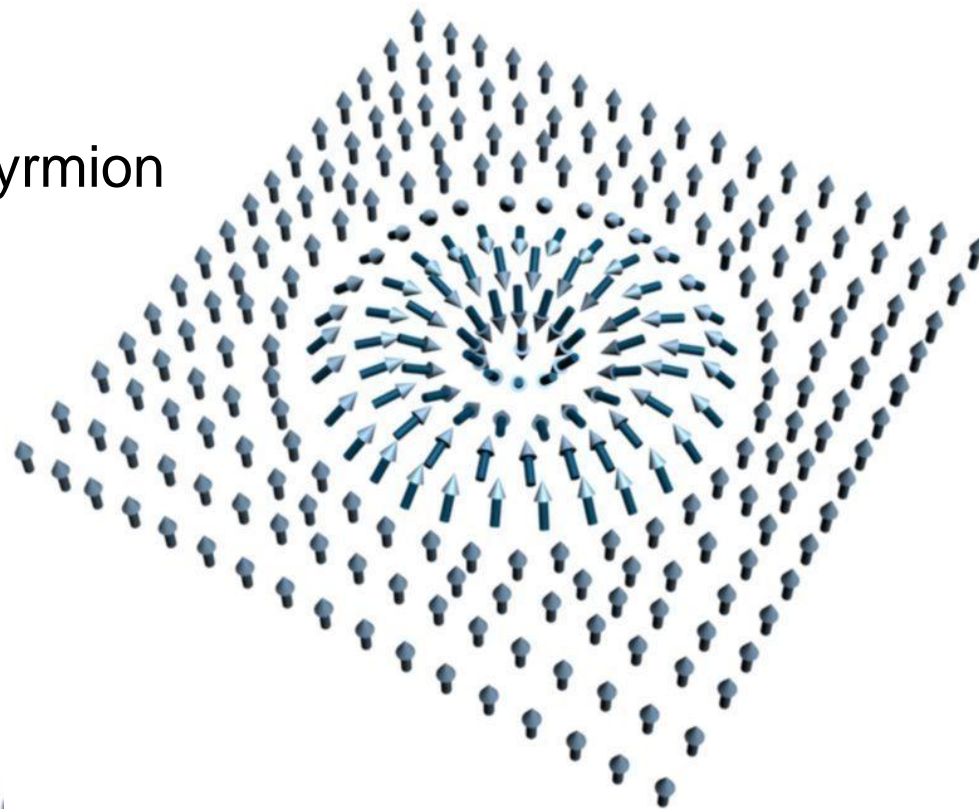
Contents

- Motivation – textures in chiral magnetism beyond spirals, skyrmions
- Phenomenology as guide
- Lifshitz-type invariants : broken inversion symmetry
- Multisublattice magnetic systems & multicriticality
- Conjectures on amorphous glass-like ground-states

Some topological textures in magnetism

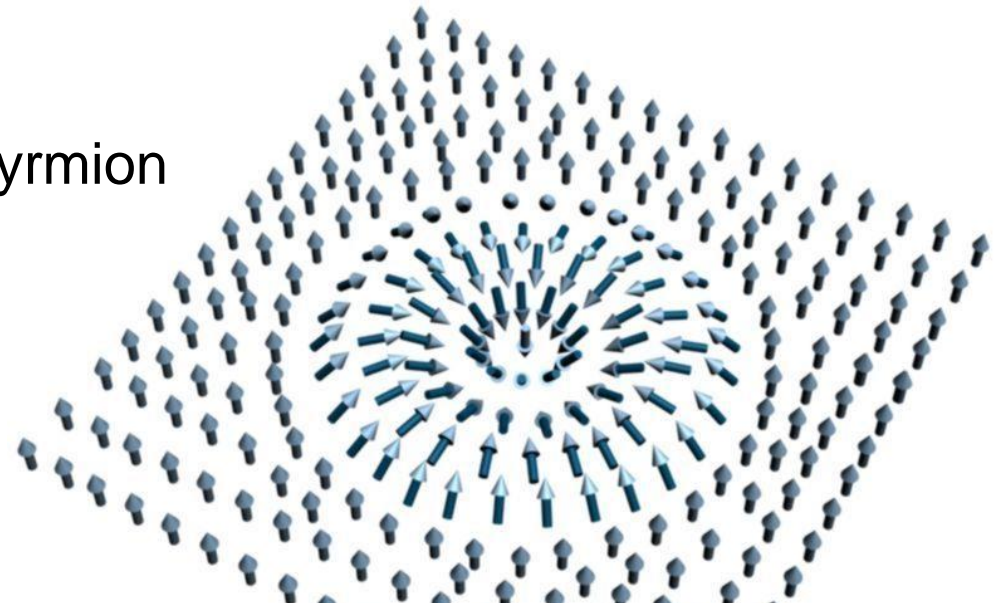


Magnetic 2D skyrmion



Topologically non-trivial texture in XY-ferromagnet or antiferromagnet

Magnetic 2D skyrmion



Thermodynamically stable “vortices” in magnetically ordered crystals. The mixed state of magnets

A. N. Bogdanov and D. A. Yablonskiĭ

Physicotechnical Institute, Donetsk, Academy of Sciences of the Ukrainian SSR

(Submitted 20 April 1988)

Zh. Eksp. Teor. Fiz. **95**, 178–182 (January 1989)

It is shown that in magnetically ordered crystals belonging to the crystallographic classes C_n , C_{nv} , D_n , D_{2d} , and S_4 ($n = 3, 4, 6$), in a certain range of fields, a thermodynamically stable system of magnetic vortices, analogous to the mixed state of superconductors, can be realized.

Dzyaloshinskii models in magnetism

Landau -Ginzburg-functional for some order parameter \mathbf{l}

$$\begin{aligned} f = f_0(\mathbf{l}) + \sum_{\mathbf{x}=\mathbf{a},\mathbf{b},\mathbf{c}} A_{\mathbf{x}} (\partial_{\mathbf{x}} \mathbf{l} \cdot \partial_{\mathbf{x}} \mathbf{l}) \\ + \sum B_{ijkl} (\partial_i l_k \partial_j l_l) \\ + \sum_{\mathbf{x}=\mathbf{a},\mathbf{b},\mathbf{c}} d_{kl}^{(\mathbf{x})} (l_l \partial_{\mathbf{x}} l_k - l_k \partial_{\mathbf{x}} l_l) \end{aligned}$$

Lifshitz invariants

OP \mathbf{l} must be a pure symmetry mode (transforming like an irrep of a (little) space group)

Dzyaloshinskii textures are described by free energies with (several) Lifshitz invariants

Standard Landau theory of 2nd order phase transitions is not applicable!



Lifshitz type invariants:

$$\left(\frac{dM_{AFM}}{dz}\right) F(M_{FM})$$

Couple various magnetic modes!

Where do they come from? Mostly SOC in non-centrosymmetric magnets!

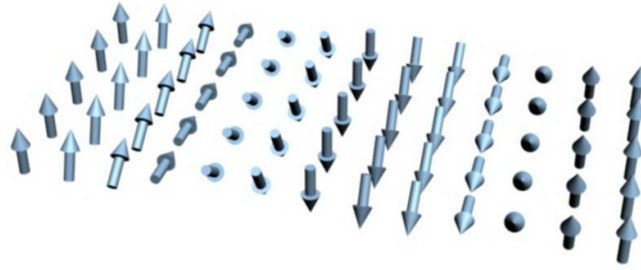
Beyond standard phenomenology of magnetism!

Scaling argument

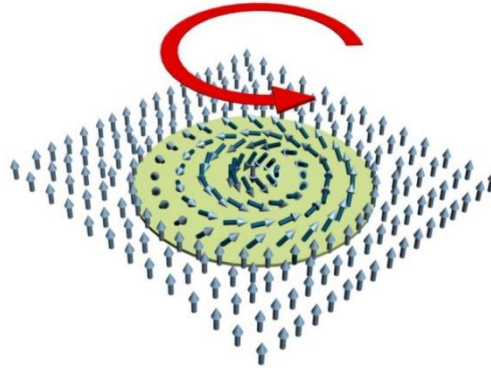
Stability of static
solitonic units

(Derrick, 1958)

$D = 1$ kinks, walls

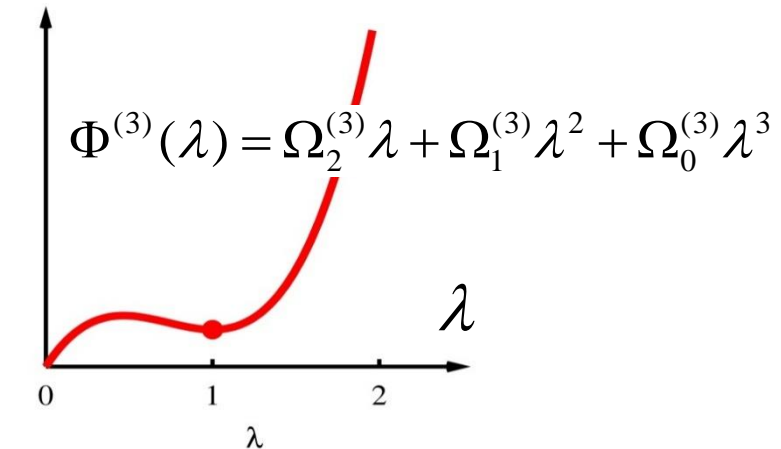
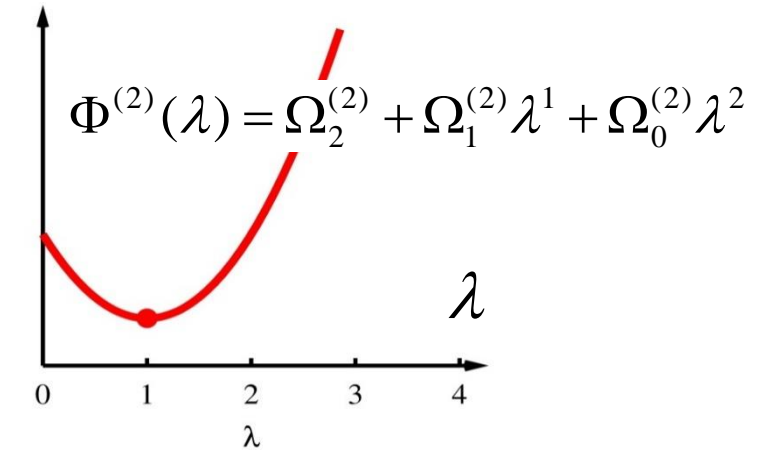
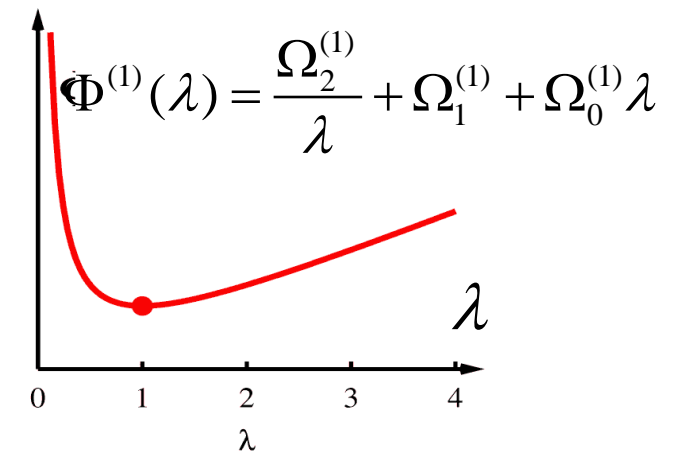
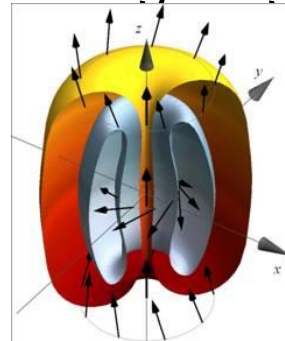


$D = 2$ vortices, baby-Skyrmions



A.N. Bogdanov, JETP Lett. 62 (1995) 249

$D = 3$ hedgehogs, spherulites



Scaling argument

$D = 1$ kinks, walls



$$\Phi^{(1)}(\lambda) = \frac{\Omega_2^{(1)}}{\lambda} + \Omega_1^{(1)} + \Omega_0^{(1)}\lambda$$

Stability of static

sol New localized solutions of the nonlinear field equations

(De

A. Bogdanov

Physicotechnical Institute, Ukrainian National Academy of Sciences, 340114 Donetsk, Ukraine

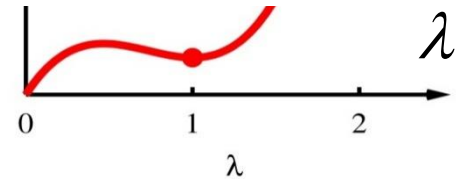
(Submitted 29 May 1995)

Pis'ma Zh. Éksp. Teor. Fiz. **62**, No. 3, 231–235 (10 August 1995)

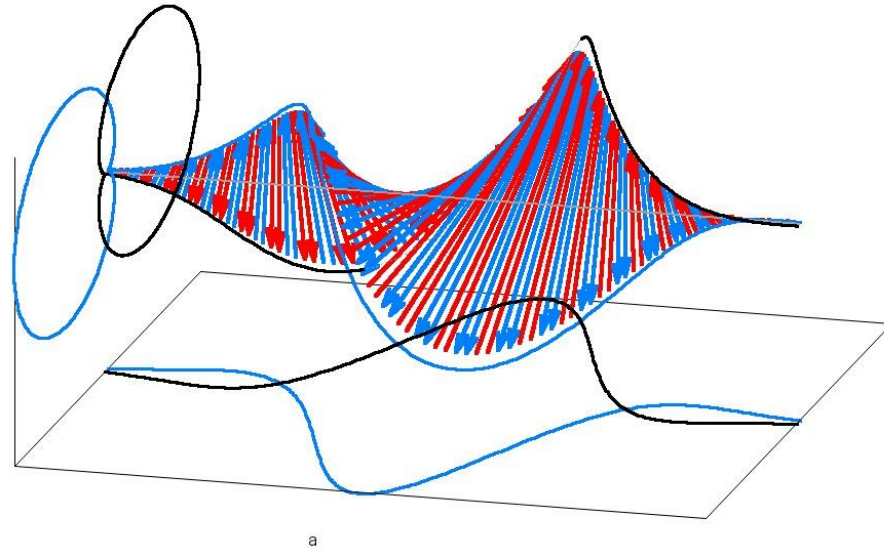
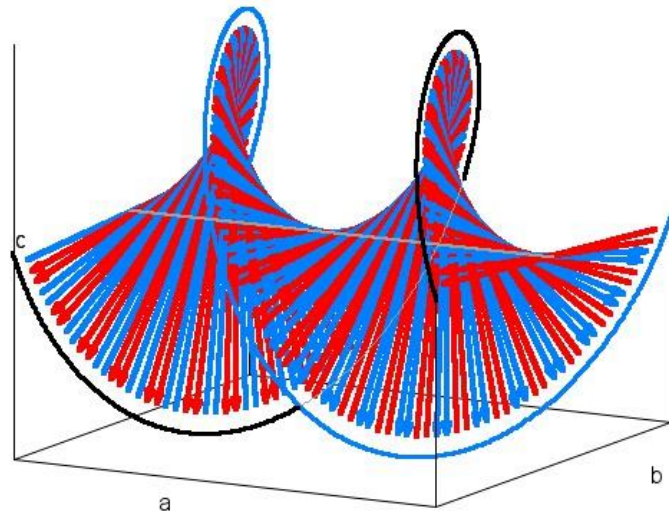
The interactions described by invariants which are linear in the first spatial derivatives (Lifshitz invariants) stabilize two- and three-dimensional localized states. The interaction force at large distances is determined for two-dimensional localized states (vortices). © 1995 American Institute of Physics.

$$\Omega_2^{(2)}\lambda^2$$

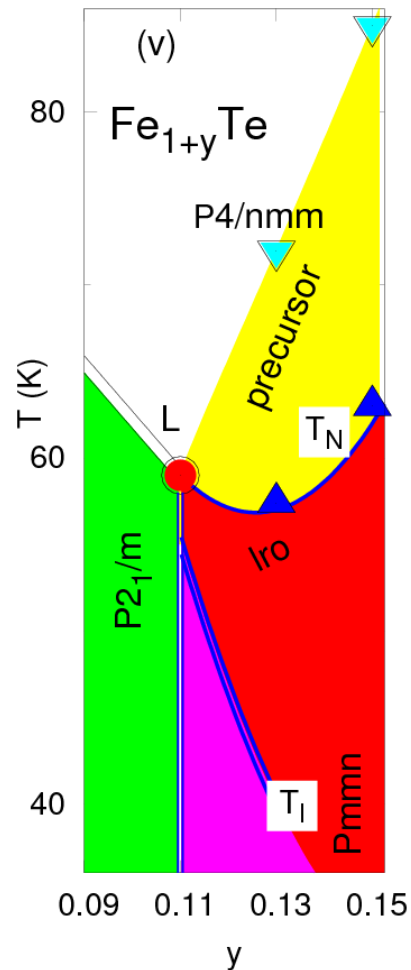
$$\Omega_0^{(3)}\lambda^3$$



Incommensurate (magnetic) phases

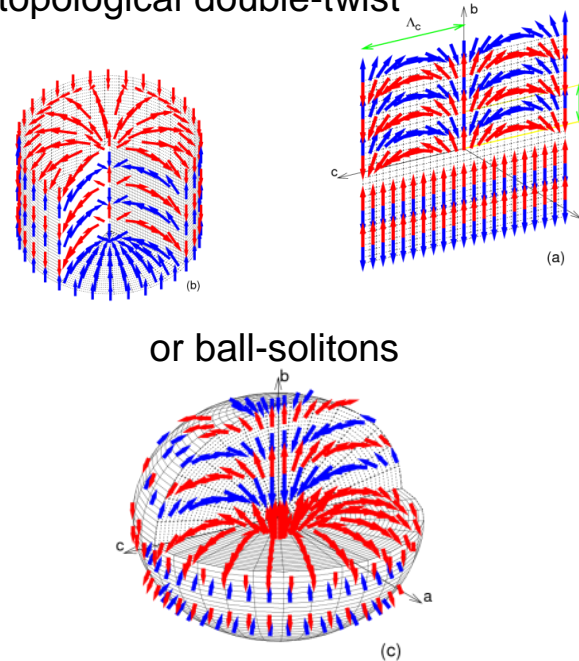


Magnetic precursor state predicted and found near AFM-spin-density-wave state in Fe_{1+y}Te

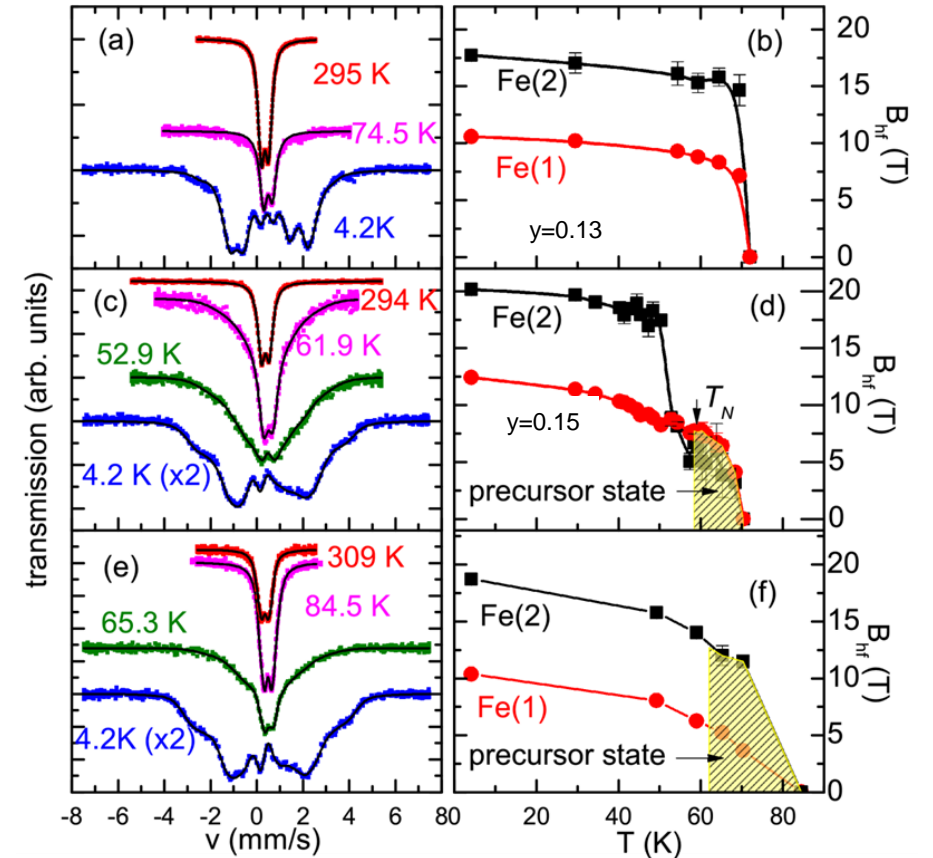


Prediction based on elementary phenomenological theory for incommensurate helical SDW-state

Precursor state: non-topological double-twist



Mössbauer experiment



Frustration model 4 component OP

Glasses –

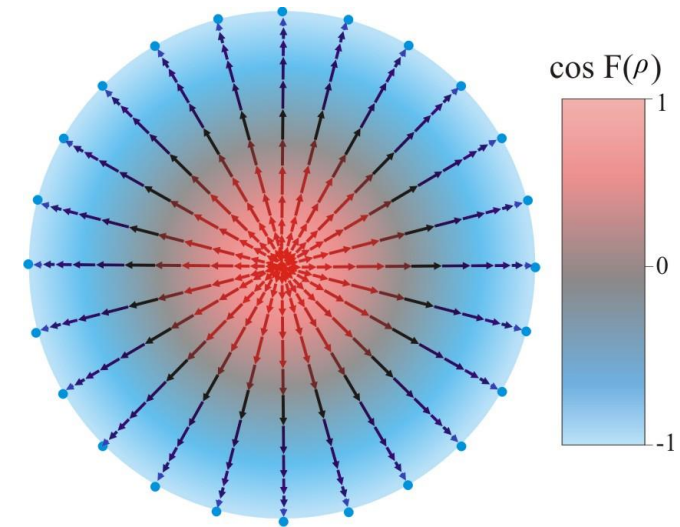
$$F_{so4} = (\partial_\alpha \phi_i)^2 + \eta (\partial_\mu q)^2 + \kappa q^2 [\phi_\mu \partial_\mu \phi_0 - \phi_0 \partial_\mu \phi_\mu - \varepsilon_{\alpha\beta\gamma} \phi_\alpha \partial_\beta \phi_\gamma] + f(q),$$

Chiral magnets

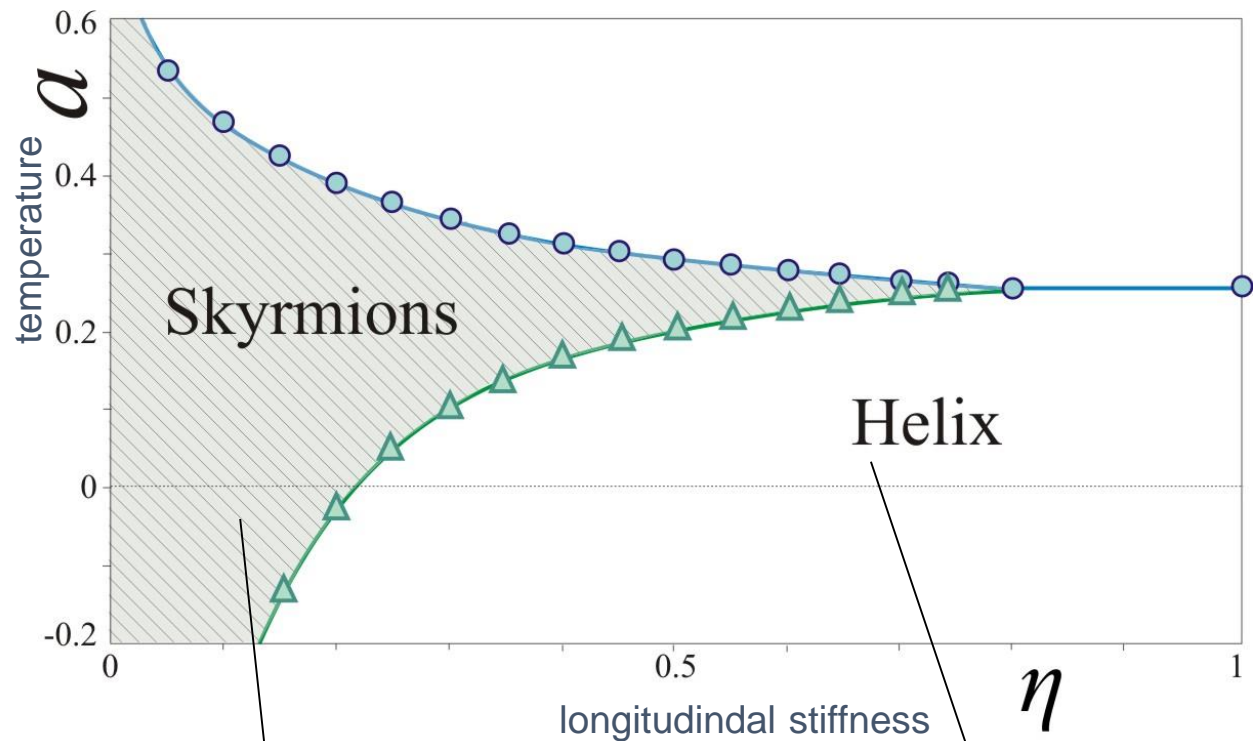
$$F_{so3} = A m^2 (\partial_\nu n_\mu)^2 + A \eta (\partial_\nu m)^2 + D \varepsilon_{\alpha\beta\gamma} m_\alpha \partial_\beta m_\gamma + f_m(m),$$

Skyrme's B=1 hedgehog ansatz

$$(\vec{\phi}, \phi_0) = \begin{pmatrix} \sin F(\rho) \sin \theta \cos \varphi \\ \sin F(\rho) \sin \theta \sin \varphi \\ \sin F(\rho) \cos \theta \\ \cos F(\rho) \end{pmatrix}$$



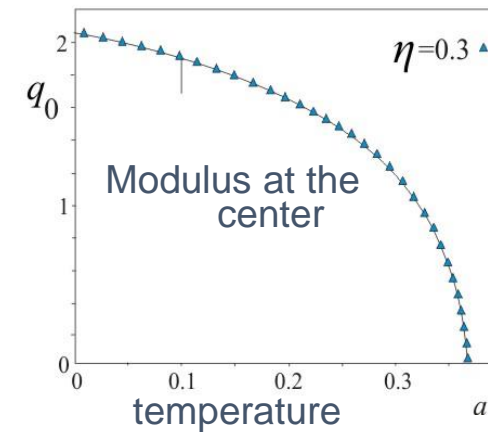
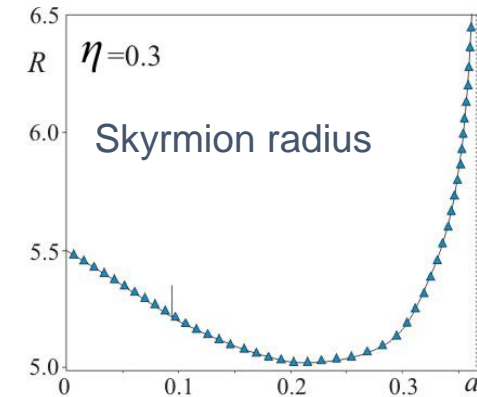
Phase diagram for the frustration model



amorphous and defect-ordered phases

1D modulation

“ideal” crystalline order



Multidimensional solitons – field theoretical

S. Coleman, Erice Lectures 1976 **Classical lumps and their quantum descendants**

Two mechanisms for multidimensional solitons

(1) Gauge and matter fields (Faddeev)

(2) Higher-order-gradient terms (Skyrme)

SOME COMMENTS ON THE MANY-DIMENSIONAL SOLITONS

L.D. FADDEEV

CERN – Geneva and Steklov Mathematical Institute, Leningrad, U.S.S.R.

ABSTRACT. The possibilities for the existence of truly localized soliton solutions in the realistic three-dimensional case are discussed. The gauge invariant theory of a non-linear chiral field is shown to be a good candidate for a model with solitons.

Lett. Math. Phys 1, 289 (1976)

Nuclear Physics B228 (1983) 552–566
© North-Holland Publishing Company

STATIC PROPERTIES OF NUCLEONS IN THE SKYRME MODEL

Gregory S. ADKINS¹

Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544, USA

Chiara R. NAPPI

The Institute for Advanced Study, Princeton, New Jersey 08540, USA

Edward WITTEN¹

Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544, USA

Received 20 June 1983

We compute static properties of baryons in an $SU(2) \times SU(2)$ chiral theory (the Skyrme model) whose solitons can be interpreted as the baryons of QCD. Our results are generally within about 30% of experimental values. We also derive some relations that hold generally in soliton models of baryons, and therefore, serve as tests of the $1/N$ expansion.

Multidimensional solitons – in magnets

(1) Chiral helimagnets

Dzyaloshinskii-Moriya coupling

Lifshitz invariants

$$W = \int \left\{ \frac{1}{2} \alpha \left(\frac{\partial \mathbf{M}}{\partial x_i} \right)^2 - \frac{1}{2} \beta M_x^2 - H M_x - \frac{1}{2} \mathbf{M} \mathbf{H}_M + w' \right\} dV,$$

$$\omega_1 = M_x \frac{\partial M_x}{\partial x} - M_x \frac{\partial M_x}{\partial x} + M_x \frac{\partial M_y}{\partial y} - M_y \frac{\partial M_x}{\partial y},$$

$$\omega_2 = M_x \frac{\partial M_x}{\partial y} - M_x \frac{\partial M_x}{\partial y} - M_x \frac{\partial M_y}{\partial x} + M_y \frac{\partial M_x}{\partial x},$$

(2) Higher-order-gradient terms

Exchange frustration

Higher-order gradient terms

$$H_0 = \frac{1}{2} \int J(\nabla \mathbf{S})^2 d^2x \quad (1)$$

$$H_1 = \frac{1}{2} \int [\kappa(\Delta \mathbf{S})^2 + \lambda(1 - S_z^2)] d^2x, \quad (5)$$

Thermodynamically stable “vortices” in magnetically ordered crystals. The mixed state of magnets

A. N. Bogdanov and D. A. Yablonskii

Physicotechnical Institute, Donetsk, Academy of Sciences of the Ukrainian SSR

(Submitted 20 April 1988)

Zh. Eksp. Teor. Fiz. **95**, 178–182 (January 1989)

It is shown that in magnetically ordered crystals belonging to the crystallographic classes C_n , C_{nv} , D_n , D_{2d} , and S_4 ($n = 3, 4, 6$), in a certain range of fields, a thermodynamically stable system of magnetic vortices, analogous to the mixed state of superconductors, can be realized.

RAPID COMMUNICATIONS

PHYSICAL REVIEW B

VOLUME 58, NUMBER 14

1 OCTOBER 1998-II

Skyrmion in a real magnetic film

Ar. Abanov

Department of Physics, Texas A&M University, College Station, Texas 77843-4242

V. L. Pokrovsky

Department of Physics, Texas A&M University, College Station, Texas 77843-4242
and Landau Institute of Theoretical Physics, Moscow, Russia

(Received 10 July 1998)

Skyrmions are magnetic defects in ultrathin magnetic films, similar to the bubble domains in the thicker films. Even weak uniaxial anisotropy determines their radii unambiguously. We derive equations of slow dynamics for Skyrmions. We show that the discreteness of the lattice in an isotropic two-dimensional magnet leads to a slow rotation of the local magnetization in the Skyrmion and even a small dissipation leads to decay of the Skyrmion. The radius of such a Skyrmion as a function of time is calculated. We prove that uniaxial anisotropy stabilizes the Skyrmion and study the relaxation process. [S0163-1829(98)50438-9]

Multidimensional solitons –

S. Coleman, Erice Lectures 1976

Classical lumps and their quantum descendants

Two mechanism for multidimensional solitons

(1) Gauge and matter fields (Faddeev)

SOME COMMENTS ON THE MANY-DIMENSIONAL SOLITONS

L.D. FADDEEV

CERN – Geneva and Steklov Mathematical Institute, Leningrad, U.S.S.R.

ABSTRACT. The possibilities for the existence of truly localized soliton solutions in the realistic three-dimensional case are discussed. The gauge invariant theory of a non-linear chiral field is shown to be a good candidate for a model with solitons.

Lett. Math. Phys 1, 289 (1976)

(2) Higher-order-gradient terms (Skyrme)

Nuclear Physics B228 (1983) 552–566
© North-Holland Publishing Company

STATIC PROPERTIES OF NUCLEONS IN THE SKYRME MODEL

Gregory S. ADKINS¹

Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544, USA

Chiara R. NAPPI

The Institute for Advanced Study, Princeton, New Jersey 08540, USA

Edward WITTEN¹

Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544, USA

Received 20 June 1983

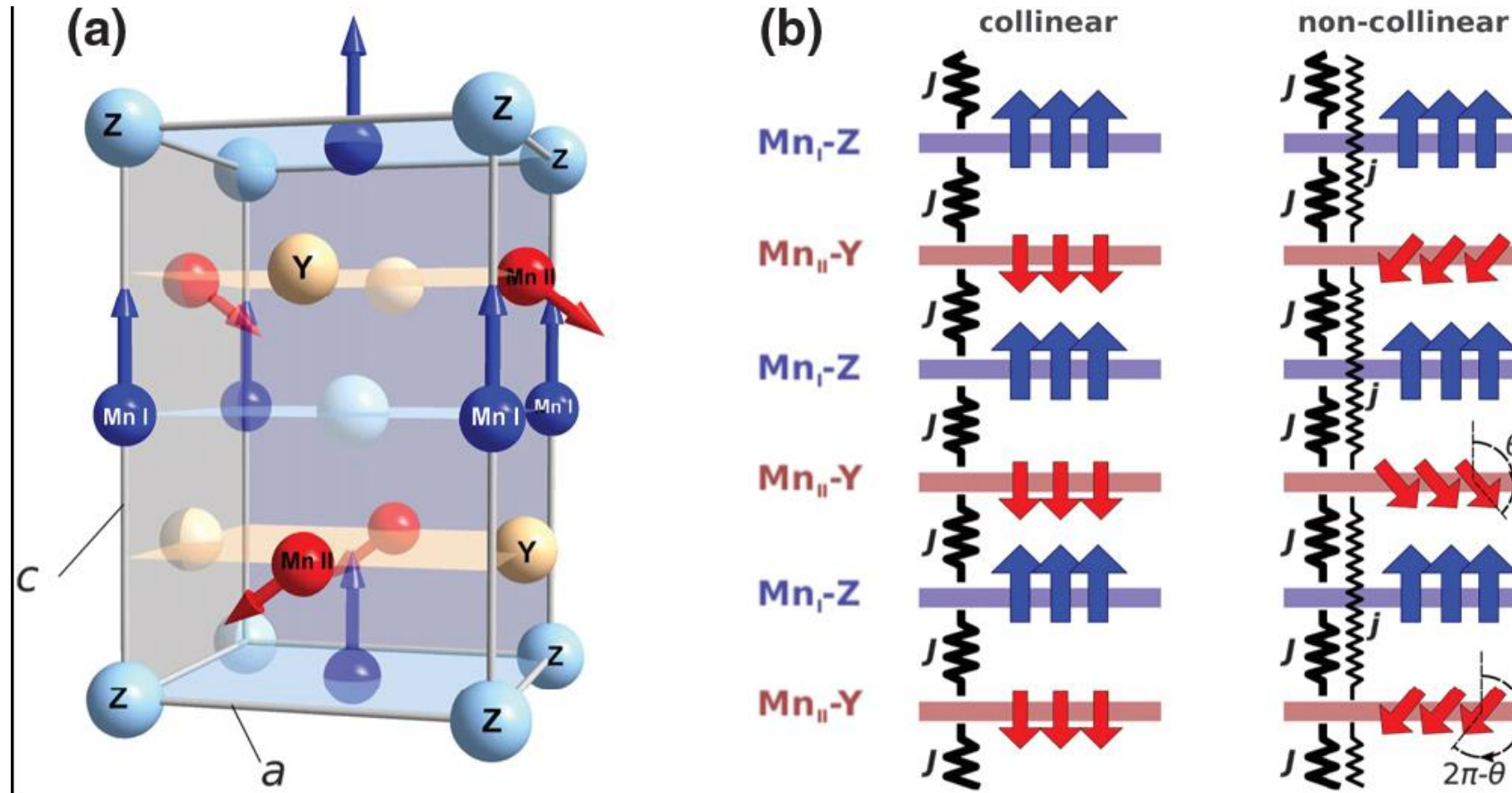
We compute static properties of baryons in an $SU(2) \times SU(2)$ chiral theory (the Skyrme model) whose solitons can be interpreted as the baryons of QCD. Our results are generally within about 30% of experimental values. We also derive some relations that hold generally in soliton models of baryons, and therefore, serve as tests of the $1/N$ expansion.

Synthesis of the two mechanisms ? I.L.Bogolubsky, A.A. Bogolubskaya!

Can we find more types of textures -
- in magnetism ?



More complex underlying magnetic order



As realized in
acentric invervse
tetragonal
Heusler alloys

Synthesis + Experiments +
Electronic structure
calculations

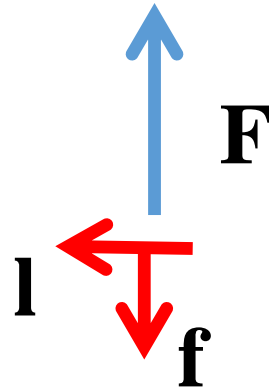
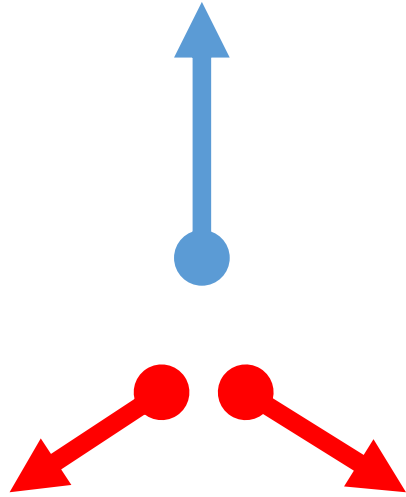
MPI-CPFS A.J. Nayak,
O.Meshcheriakova,
S.Chadov, C. Felser and
many co-workers



MPI for the Chemical
Physics of Solids
Chemistry Department

Phenomenological Landau-Ginzburg model

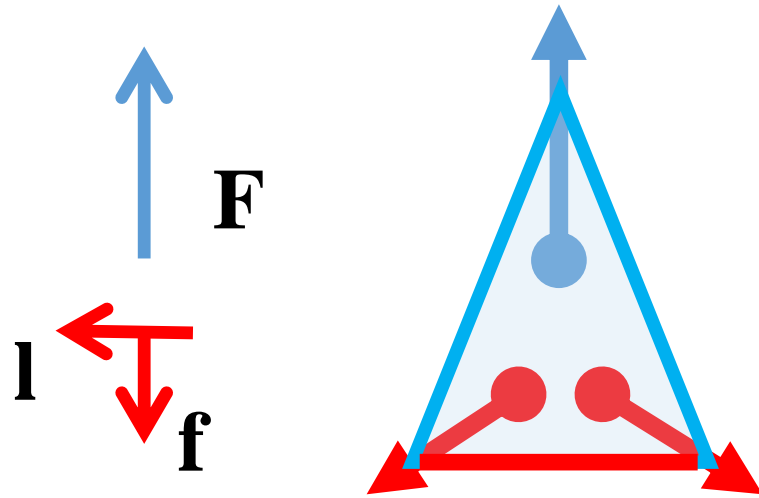
Intermetallic ternary compounds – inverse tetragonal Heusler structure



$$\begin{aligned}w_0 = & a_F \mathbf{F} \cdot \mathbf{F} + b_F (\mathbf{F} \cdot \mathbf{F})^2 \\ & + a_l \mathbf{l} \cdot \mathbf{l} + b_f (\mathbf{l} \cdot \mathbf{l})^2 \\ & + a_f \mathbf{f} \cdot \mathbf{f} \\ & + c_f \mathbf{F} \cdot \mathbf{f} + c' \mathbf{F} \cdot \mathbf{l} \\ & + b_{Ff} |\mathbf{F}|^2 |\mathbf{f}|^2 + b_{Fl} |\mathbf{F}|^2 |\mathbf{l}|^2 + b_{fl} |\mathbf{f}|^2 |\mathbf{l}|^2 \\ & + b_c (\mathbf{F} \cdot \mathbf{f})^2 + b' (\mathbf{F} \cdot \mathbf{l})^2 \\ & + \text{h.o.t.} \\ & - 2(\mathbf{F} + \mathbf{f}) \cdot \mathbf{H}, \\ & + \text{many Lifshitz-type invariants} \\ & + \text{anisotropies}\end{aligned}$$

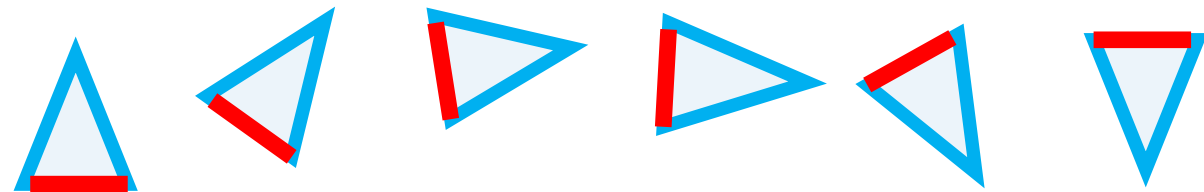
Phenomenological Landau-Ginzburg model

Intermetallic ternary compounds – inverse tetragonal Heusler structure



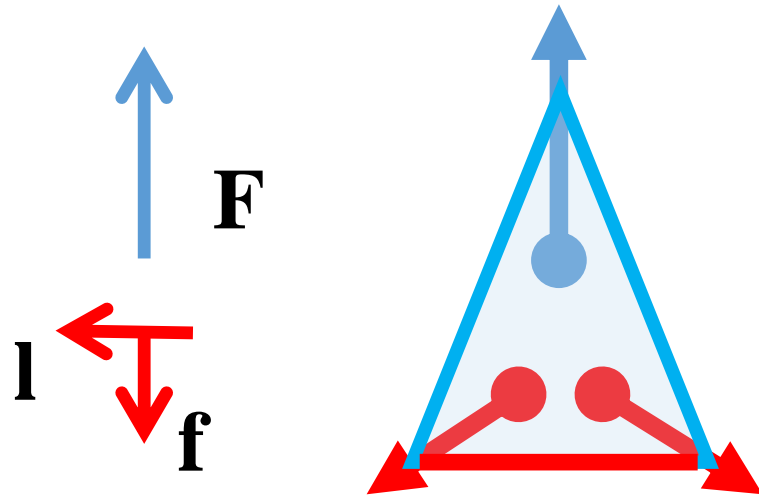
Textures

Dzyaloshinskii spiral – handed rigid body rotation



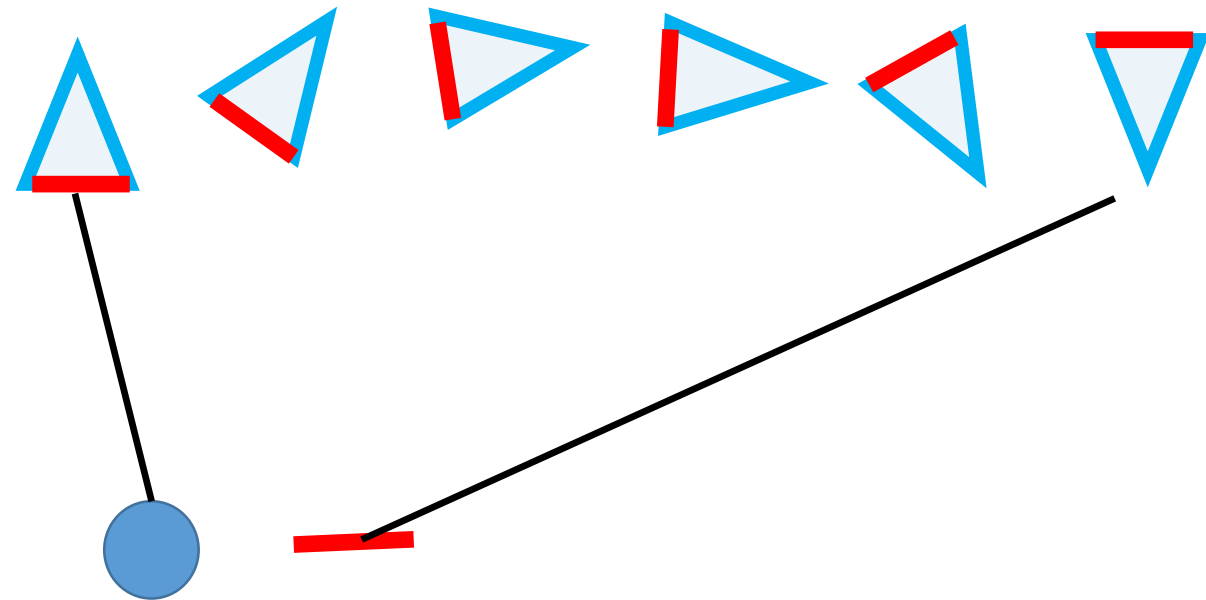
Phenomenological Landau-Ginzburg model

Intermetallic ternary compounds – inverse tetragonal Heusler structure



Textures

Dzyaloshinskii spiral – handed rigid body rotation

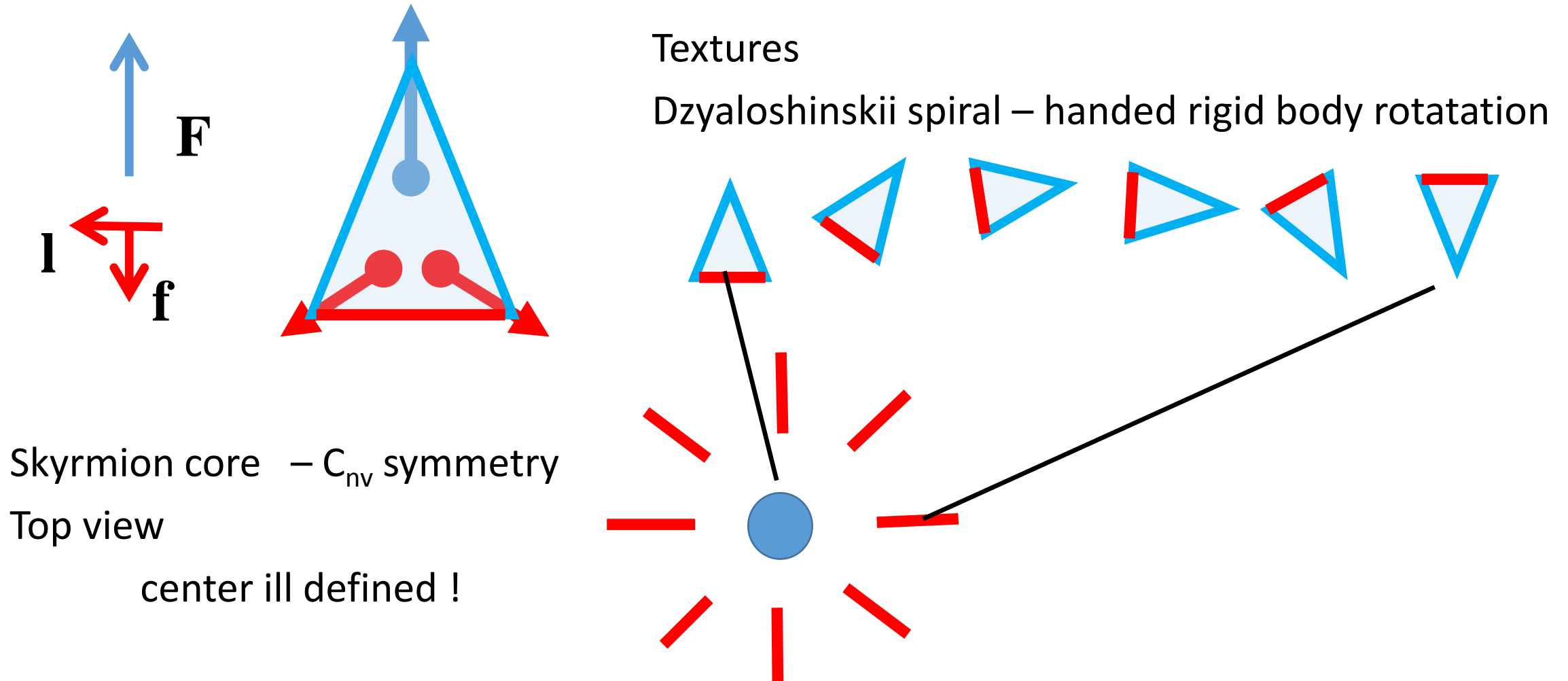


Building up a skyrmion core

Top view

Phenomenological Landau-Ginzburg model

Intermetallic ternary compounds – inverse tetragonal Heusler structure



Improper Dzyaloshinskii textures

Lifshitz-type invariants!

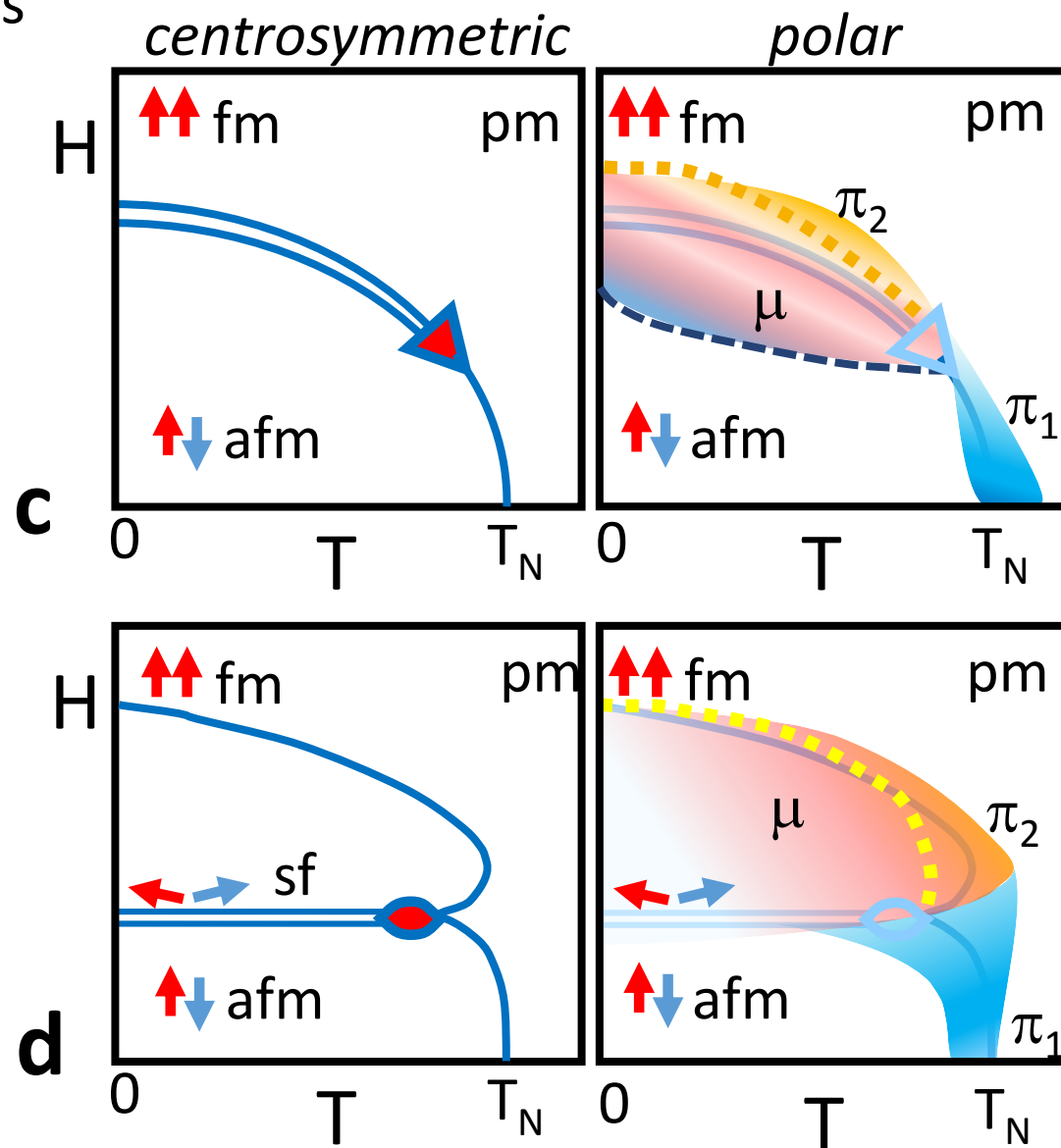
System with co-existing ordering modes : e.g. **l** and **f**

$$\mathbf{g}_{ij}^{(\mathbf{x})} (l_i \partial_{\mathbf{x}} f_j - f_j \partial_{\mathbf{x}} l_i)$$

BUT these terms play a role only near multicritical point

Metamagnetic textures

Uniaxial antiferromagnets
in a field



Experiments

D.A Sokolov
Experiments



R. Cubitt,
SANS, D33



J. White, SANS
SANS-II



E. Ressouche



M. Bleuel



N. Kikugawa,
crystal growth



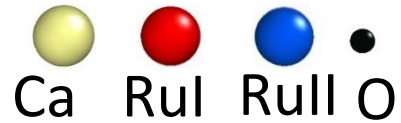
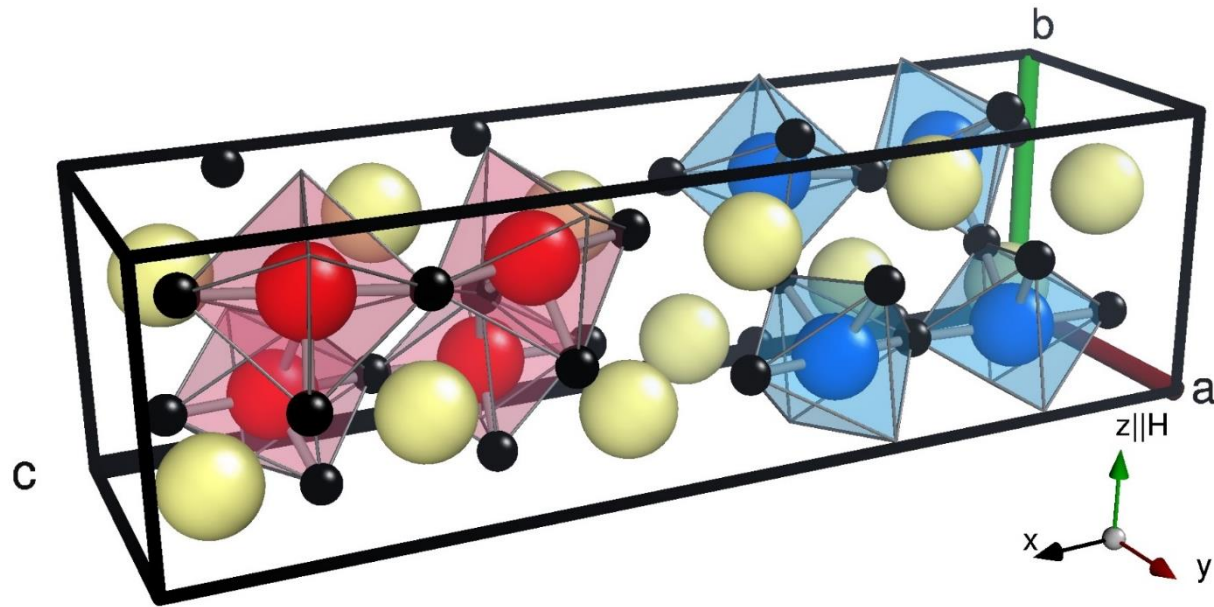
K. Kummer, XMCD,
ID32



A. P. Mackenzie
C. W. Hicks
H. Borrmann
U. Burkhardt



Experiment : $\text{Ca}_3\text{Ru}_2\text{O}_7$



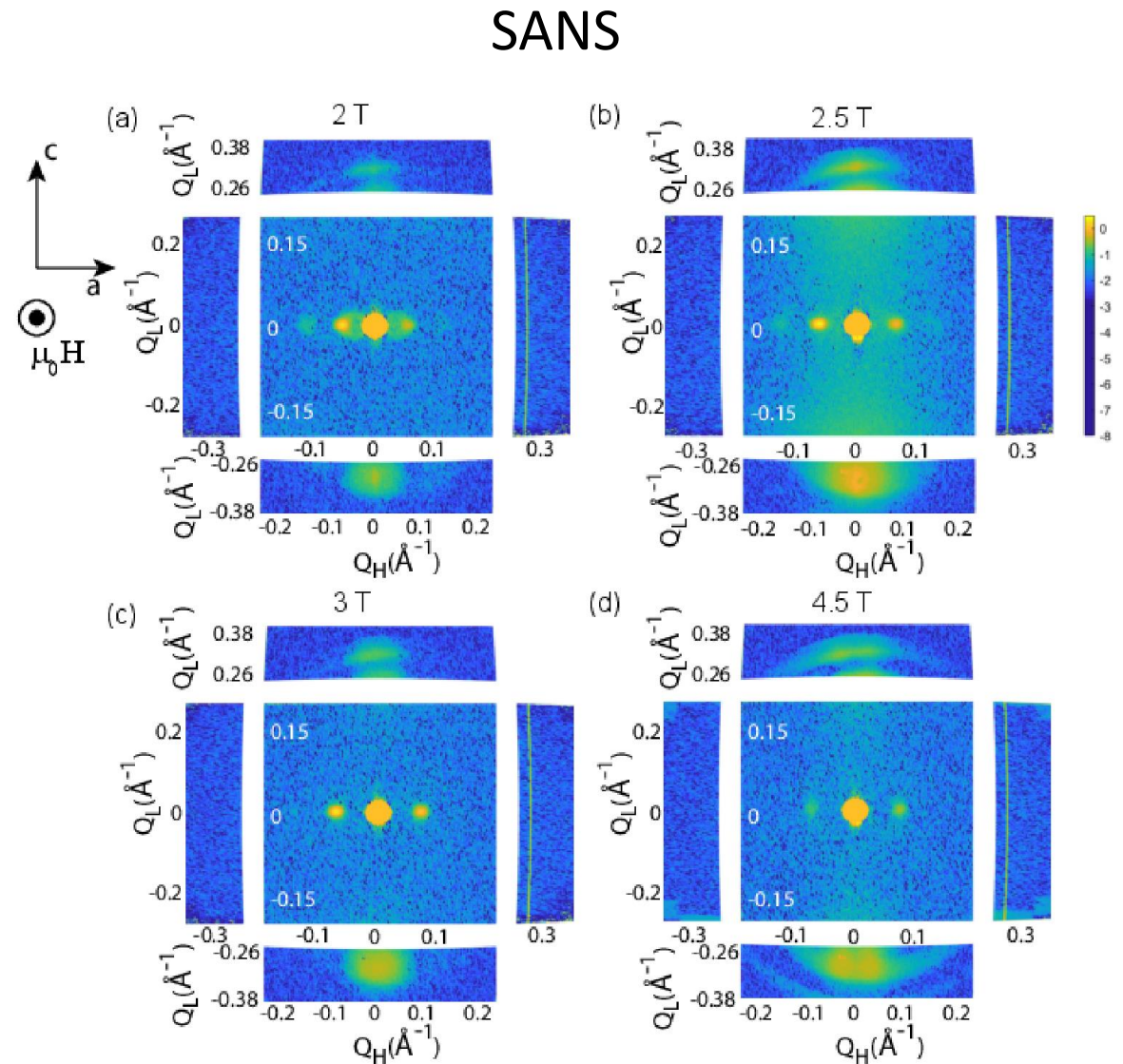
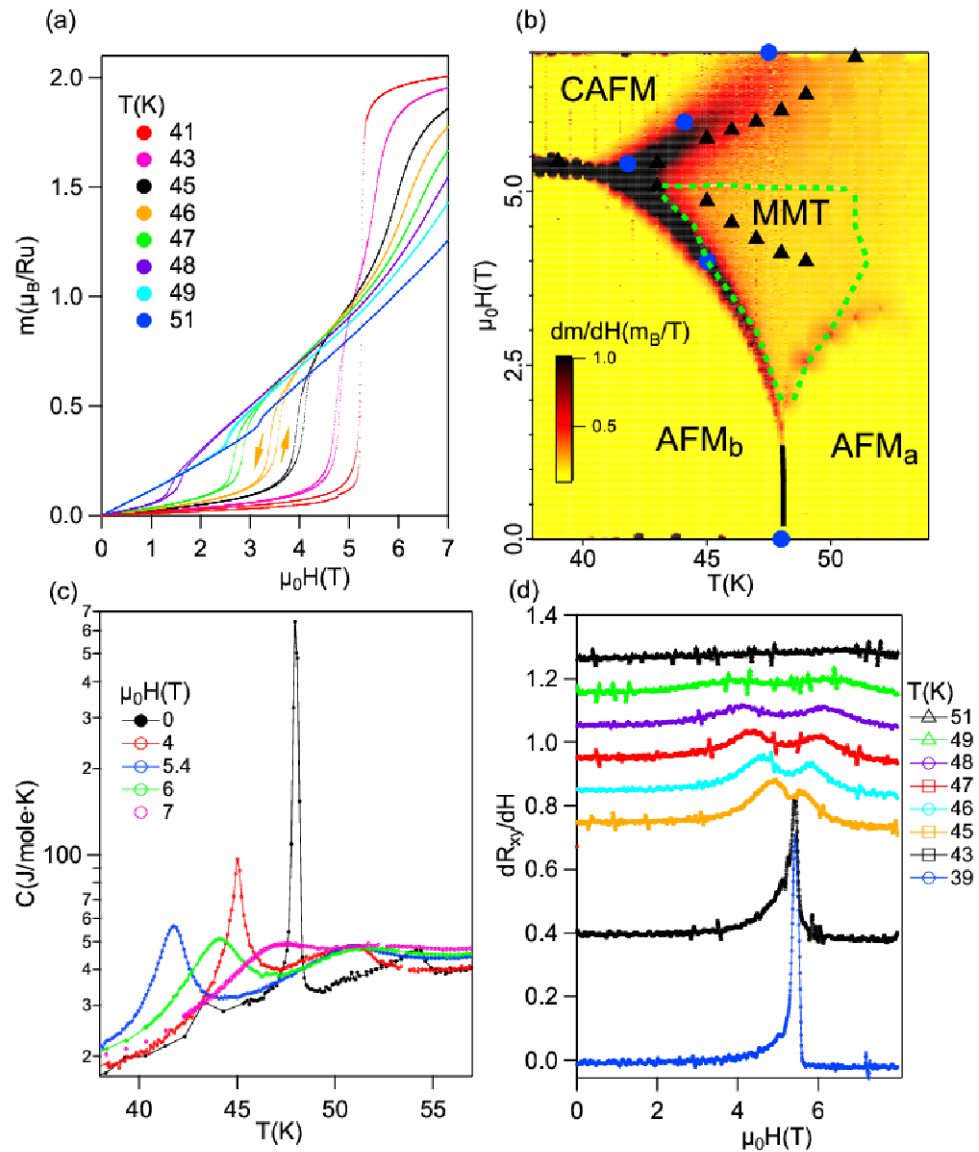
Ruddlesden Popper bilayer phase

polar / non-centrosymmetric

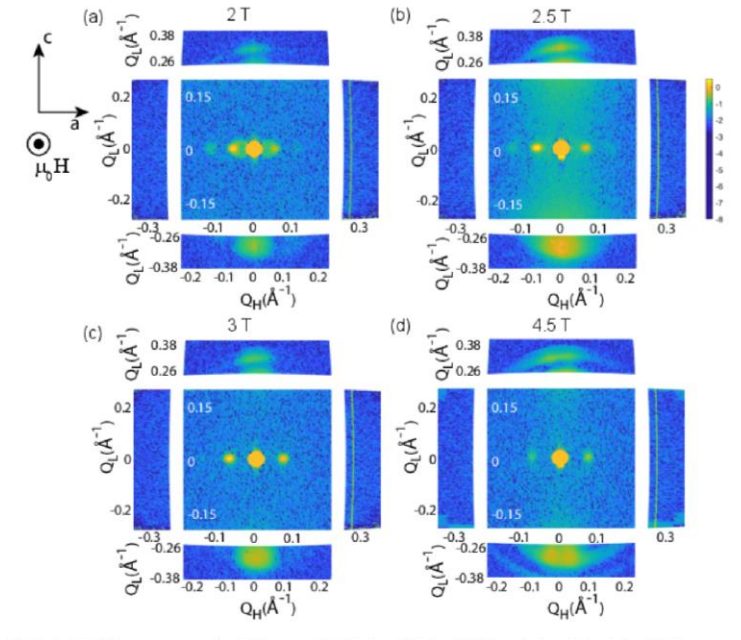
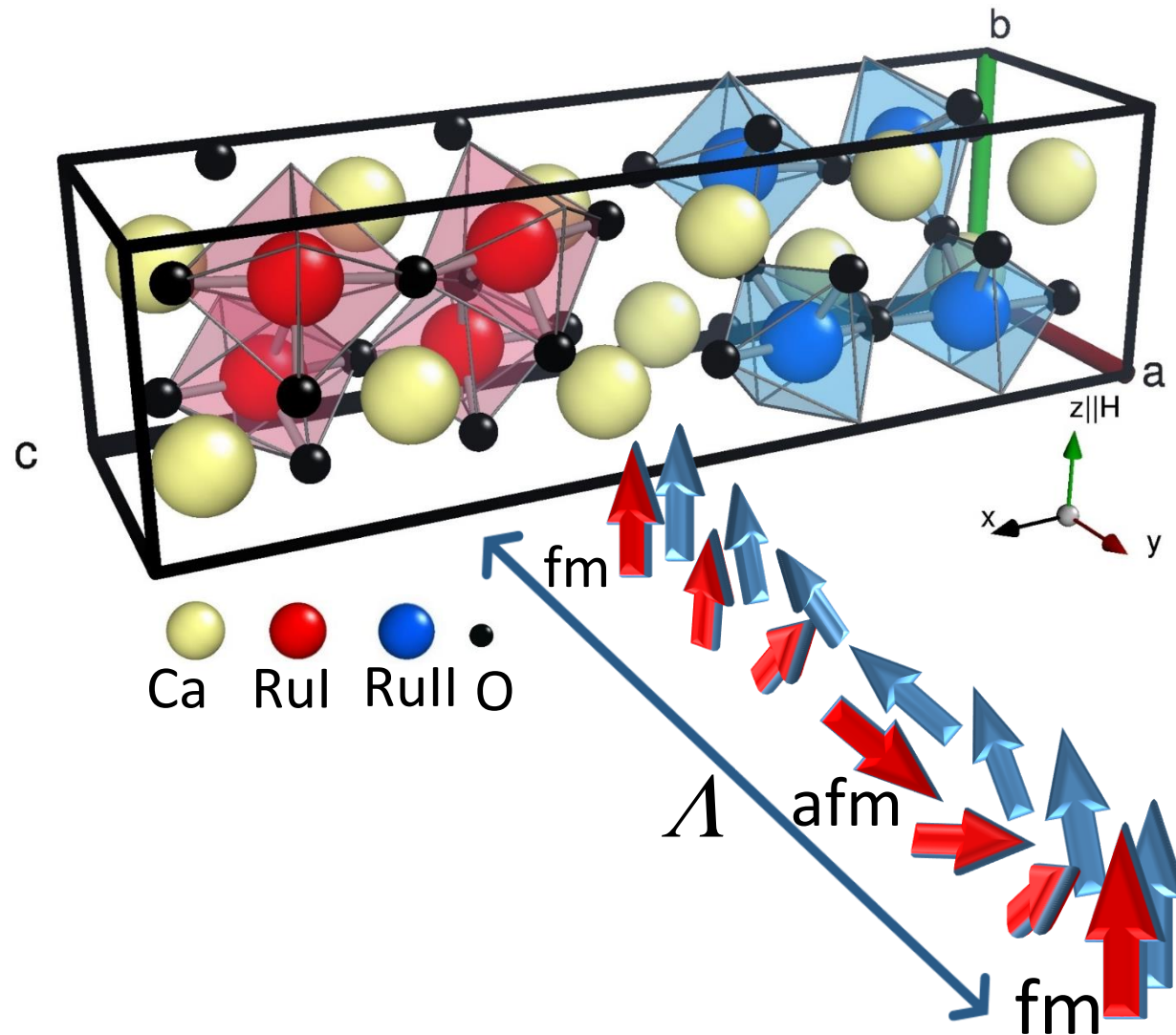
orthorhombic crystal structure $Bb2_1m$ (C_{2v})

simple layered antiferromagnetic order

Experiment : $\text{Ca}_3\text{Ru}_2\text{O}_7$



Metamagnetic texture in $\text{Ca}_3\text{Ru}_2\text{O}_7$



Theory for $\text{Ca}_3\text{Ru}_2\text{O}_7$

Free energy near tricritical point

$$w = w_E + w_0 + w_D + w_F + w_\alpha + w_\mu + w_\Delta = w_4$$

Exchange

$$w_E = A_l (\nabla \mathbf{l})^2 + A_f (\nabla \mathbf{f})^2 + \dots,$$

Landau part

$$\begin{aligned} w_0 &= a_l |\mathbf{l}|^2 + a_f |\mathbf{f}|^2 \\ &+ b_l |\mathbf{l}|^4 + b_f |\mathbf{f}|^4 \\ &+ c_1 |\mathbf{l}|^2 |\mathbf{f}|^2 \\ &+ c_l |\mathbf{l}|^6 + c_f |\mathbf{f}|^6 \\ &+ c_2 |\mathbf{l}|^4 |\mathbf{f}|^2 + c_3 |\mathbf{l}|^2 |\mathbf{f}|^4. \end{aligned}$$

+ Anisotropies

$$\begin{aligned} w_\alpha &= K_z l_z^2 + k_z f_z^2 \\ &+ \kappa_x l_x^2 + \kappa_{xy} l_x l_y + \kappa_y l_y^2 \\ &+ \nu_x f_x^2 + \nu_{xy} l_x l_y + \nu_y l_y^2, \end{aligned}$$

Theory for $\text{Ca}_3\text{Ru}_2\text{O}_7$

Lifshitz invariants $\Gamma_{ij}^{(\gamma)}(\mathbf{x}) \equiv (x_i \partial_\gamma x_j - x_j \partial_\gamma x_i)$

$$w_D = D_x \Gamma_{zx}^{(x)}(\mathbf{l}) + D_y \Gamma_{yy}^{(y)}(\mathbf{l})$$

$$w_F = F_x \Gamma_{zx}^{(x)}(\mathbf{f}) + F_y \Gamma_{yz}^{(y)}(\mathbf{f}),$$

$$w_\mu = \sum_{\alpha=x,y,z} \sum_{\beta=x,y} \left(a_\alpha f_\alpha^2 \Gamma_{\beta z}^{(\beta)}(\mathbf{l}) + b_\alpha l_\alpha^2 \Gamma_{\beta z}^{(\beta)}(\mathbf{m}) \right)$$

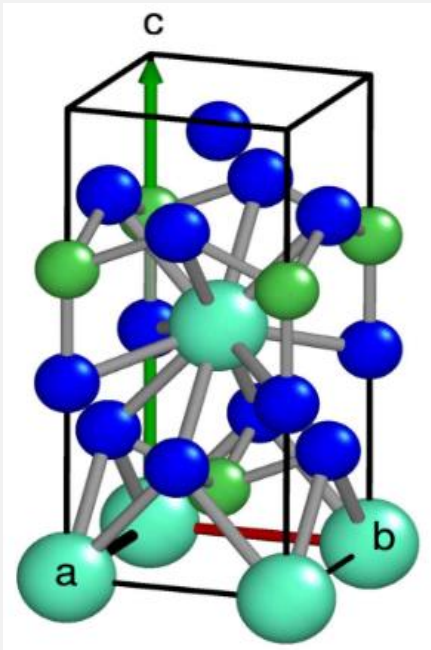
Lifshitz-type invariants

$$\begin{aligned} w_4 = & \sum_{\alpha=x,y,z} \sum_{\beta=x,y} \left(\eta_\alpha f_\alpha^2 \Gamma_{\beta z}^{(\beta)}(\mathbf{f}) + \tau_\alpha l_\alpha^2 \Gamma_{\beta z}^{(\beta)}(\mathbf{l}) \right) \\ & + \sigma_1 f_x f_y \Gamma_{yz}^{(x)}(\mathbf{f}) \\ & + \sigma_2 f_x f_y \Gamma_{zx}^{(y)}(\mathbf{f}) \\ & + \sigma_3 f_y f_z \Gamma_{zx}^{(y)}(\mathbf{f}) \\ & + \sigma_4 f_z f_x \Gamma_{yz}^{(x)}(\mathbf{f}) \\ & + \sigma_5 l_x l_y \Gamma_{yz}^{(x)}(\mathbf{l}) \\ & + \sigma_6 l_x l_y \Gamma_{zx}^{(y)}(\mathbf{l}) \\ & + \sigma_7 l_y l_z \Gamma_{zx}^{(y)}(\mathbf{l}) \\ & + \sigma_8 l_z l_x \Gamma_{yz}^{(x)}(\mathbf{l}). \end{aligned}$$

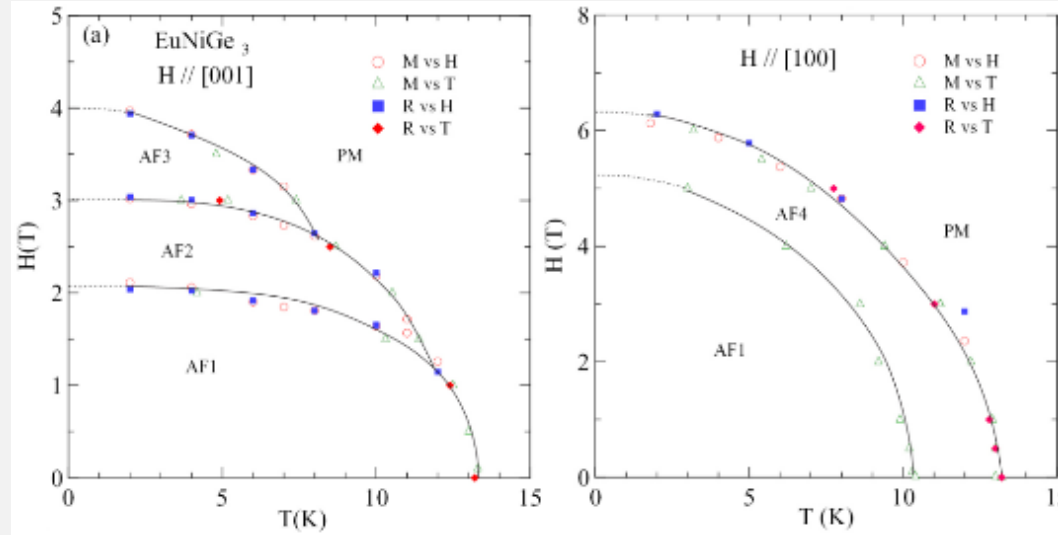
$$\begin{aligned} w_\Delta = & \Delta_1 f_x f_y \Gamma_{xy}^{(z)}(\mathbf{l}) \\ & + \Delta_2 f_x f_y \Gamma_{zx}^{(z)}(\mathbf{l}) \\ & + \Delta_3 f_x f_y \Gamma_{yz}^{(x)}(\mathbf{l}) \\ & + \Delta_4 f_x f_y \Gamma_{yz}^{(z)}(\mathbf{l}) \\ & + \Delta_5 f_x f_y \Gamma_{zx}^{(y)}(\mathbf{l}) \\ & + \Delta_6 f_z f_x \Gamma_{xy}^{(y)}(\mathbf{l}) \\ & + \Delta_7 f_z f_x \Gamma_{zx}^{(z)}(\mathbf{l}) \\ & + \Delta_8 f_y f_z \Gamma_{xy}^{(x)}(\mathbf{l}) \\ & + \Delta_9 f_y f_z \Gamma_{yz}^{(z)}(\mathbf{l}) \\ & + \Xi_1 l_x l_y \Gamma_{xy}^{(z)}(\mathbf{f}) \\ & + \Xi_2 l_x l_y \Gamma_{zx}^{(z)}(\mathbf{f}) \\ & + \Xi_3 l_x l_y \Gamma_{yz}^{(x)}(\mathbf{f}) \\ & + \Xi_4 l_x l_y \Gamma_{yz}^{(z)}(\mathbf{f}) \\ & + \Xi_5 l_x l_y \Gamma_{zx}^{(y)}(\mathbf{f}) \\ & + \Xi_6 l_z l_x \Gamma_{xy}^{(y)}(\mathbf{f}) \\ & + \Xi_7 l_z l_x \Gamma_{zx}^{(z)}(\mathbf{f}) \\ & + \Xi_8 l_y l_z \Gamma_{xy}^{(x)}(\mathbf{f}) \\ & + \Xi_9 l_y l_z \Gamma_{yz}^{(z)}(\mathbf{l}), \end{aligned}$$

Candidate systems

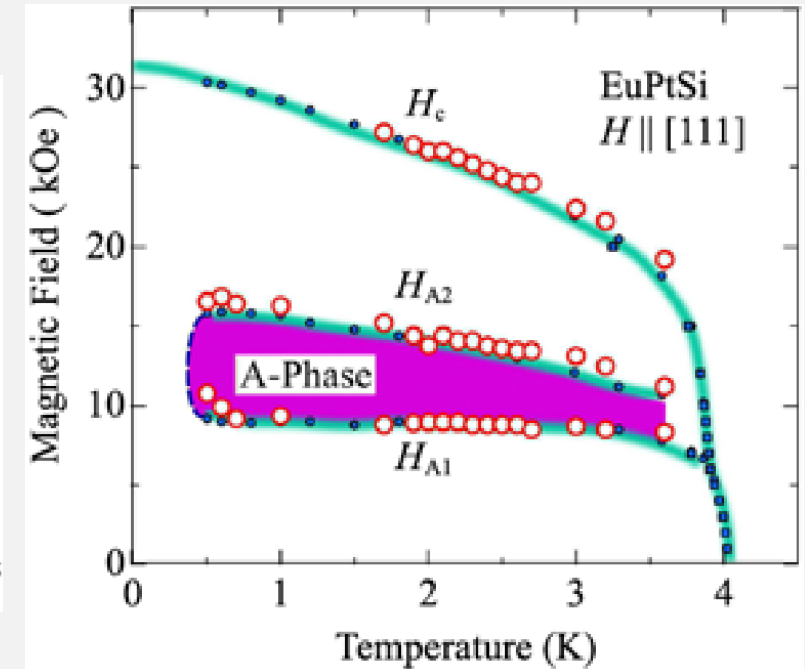
EuNiGe₃
acentric (I4mm)
BaNiSn₃-type
antiferromagnet



Complex metamagnetic phases



Maurya et al. JPCM 2014



Unique Helical Magnetic Order and Field-Induced Phase in Trillium Lattice Antiferromagnet EuPtSi
Koji Kaneko^{1,2+}, Matthias D. Frontzek³, Masaaki Matsuda³, Akiko Nakao⁴,
Koji Munakata⁴, Takashi Ohhara², Masashi Kakihana⁵, Yoshinori Haga⁶,
Masato Hedo⁷, Takao Nakama⁷, and Yoshichika Ōnuki

Journal of the Physical Society of Japan 88,
013702 (2019)
<https://doi.org/10.7566/JPSJ.88.013702>

α -FeOOH goethite

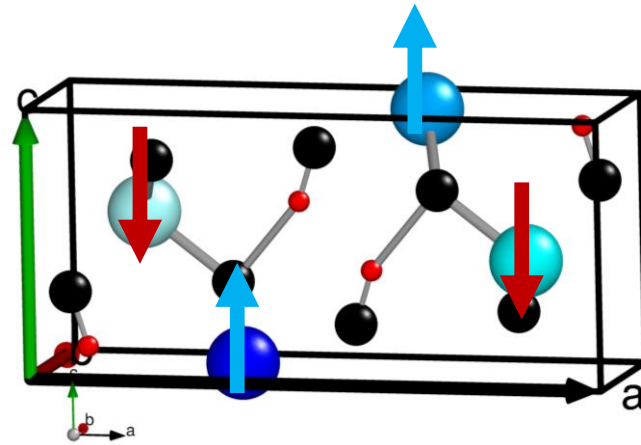
Antiferromagnet T_{Neel} 325 – 405 K

Unclear high-field spin-flop

Centrosymmetric space group (Pnma)

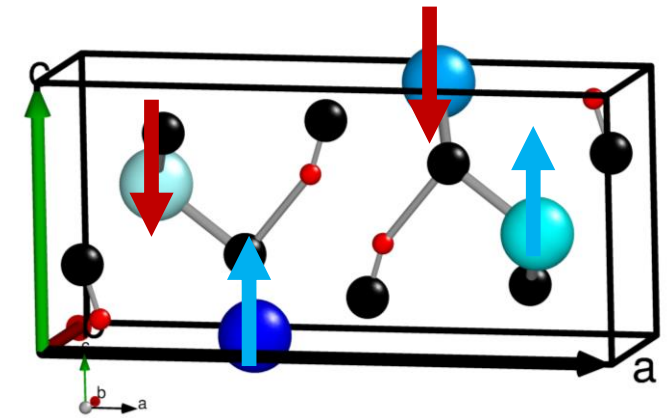
Nearly degenerate magnetic configurations AFM 1 & 2

Energy difference 28 meV / f.u.
from preliminary
DFT results (fplo) GGA



ground state AFM 1

Spatial parity **even**



AFM 2

odd

Can we make it more complicated ?

Aiming at glassy groundstates in magnetic systems
without quenched disorder

Spin liquids ...

Chiral Spin Liquid Ground State in $\text{YBaCo}_3\text{FeO}_7$

Experiments W. Schweika, J. Reim (FZ Jülich, Lund), M. Valldor (IFW now U Oslo)

$\text{YBaCo}_3\text{FeO}_7$

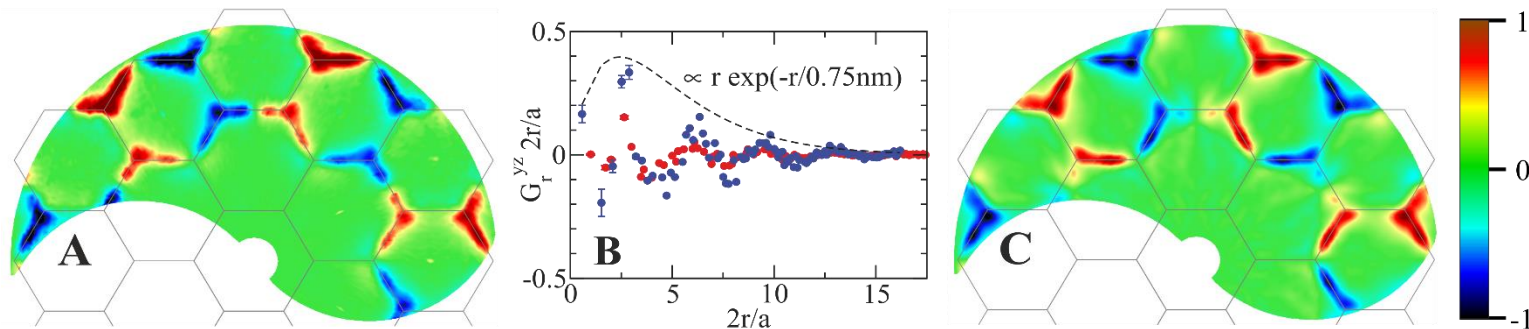
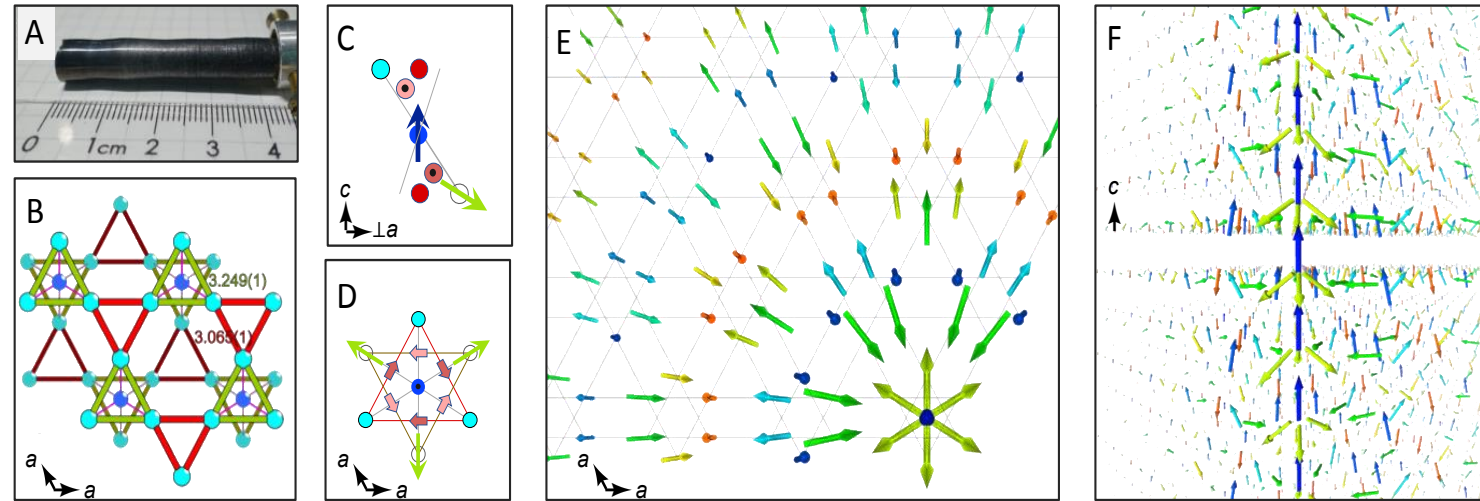
Swedenborgite type

3D geometric frustrated

Polar crystal structure P6₃mc

Polarized neutron scattering

Chirality of magnetic structure factor



Primitive model for swedenborgite spin-structure

Lifshitz invariants

$$D (m_x \partial_x m_z - m_z \partial_x m_x + m_y \partial_y m_z - m_y \partial_x m_z) \quad (T7)$$

Lifshitz type invariants

$$g_{lm} (l_x \partial_x m_z - m_z \partial_x l_x + l_y \partial_y m_z - m_z \partial_x l_y) \quad (T8)$$

$$f_{lm} (l_x \partial_z m_x - m_x \partial_z l_x).$$

Continuum theory with „many“ ordering modes

Lifshitz invariants

$$\mathbf{D}_{\alpha\beta}^{(\gamma)} (\varphi_{1\alpha} \partial_\gamma \varphi_{1\beta} - \varphi_{1\beta} \partial_\gamma \varphi_{1\alpha}) \cdot \quad (\text{T1})$$

Landau-Ginzburg free energy functional

$$w_i = \mathbf{A}_i (\nabla \varphi_i)^2 + \mathbf{D}_{i\alpha\beta}^{(\gamma)} (\varphi_{i\alpha} \partial_\gamma \varphi_{i\beta} - \varphi_{i\beta} \partial_\gamma \varphi_{i\alpha}) + a_i (T - T_{ci}) (\varphi_i)^2 + b_i ((\varphi_i)^2)^2, \quad i=1,2 \quad (\text{T2})$$

Lifshitz type invariants:

$$\mathbf{G}_{\alpha\beta}^{(\gamma)} (\varphi_{1\alpha} \partial_\gamma \varphi_{2\beta} - \varphi_{2\beta} \partial_\gamma \varphi_{1\alpha}). \quad (\text{T3})$$

Combined (large) order parameter

$$\phi = (\varphi_1, \varphi_2)$$

$$\mathbf{W} = \mathbf{A} (\nabla \phi)^2 + \Delta_{\alpha\beta}^{(\gamma)} (\phi_\alpha \partial_\gamma \phi_\beta - \phi_\beta \partial_\gamma \phi_\alpha) \quad (\text{T4})$$

Rewritten – Lifshitz- and Lifshitz-type invariants act as gauge-vector potential:

$$\mathbf{W} = [(\partial_\gamma + \mathbf{d}_{\beta\alpha}^{(\gamma)}) \phi_\alpha]^2 + \text{anisotropic terms}, \quad (\text{T5})$$

Bogomolnyi-type equations

$$(\partial_\gamma + \mathbf{d}_{\beta\alpha}^{(\gamma)}) \phi_\alpha = 0. \quad (\text{T6})$$

Gauge freedom

$$W = [(\partial_\gamma + d_{\beta\alpha}^{(\gamma)}) \phi_\alpha]^2 + \text{anisotropic terms}, \quad (\text{T5})$$

$$\phi' = \mathbf{R}(\mathbf{x}) \phi \quad (\text{T10})$$

$$d'_{\beta\alpha} = \mathbf{g}(\mathbf{R}(\mathbf{x})) d_{\beta\alpha} \quad (\text{T11})$$

Creation of local non-collinear lumps of mixed mode-character
gauge-fixing locally impossible – case of Elitzur theorem

Lumps are no skyrmions !

No topological stabilization

geometry of spin-textures Maps ($S^3 \rightarrow M$).

(Infinite) crystal, $E^3 \cup \{\infty\} = S^3$ onto order parameter manifold M

But topology & sphalerons Klinkhamer, Manton 1984, Manton 2019

k^{th} homotopy group : $\Pi_k(\text{Maps}(S^k \rightarrow M)) = \Pi_{k+1}(M)$ not always trivial

1. J.A. Hertz, Gauge models for spin-glasses, *Phys. Rev. B* **18** (1978) 4875
2. F.R. Klinkhamer, N.S. Manton, A saddle-point solution in the Weinberg-Salam theory, *Phys. Rev. D* **30** (1984) 2212
3. N.S. Manton, The inevitability of Sphalerons in Field Theory, *arXiv:1903.11573*
4. C.H. Taubes, The existence of a non-minimal solution to the SU(2) Yang-Mills-Higgs equations on R^3 ; Part I. *Commun. Math. Phys.* **86** (1982) 257, Part II. *ibid.* 299.

Glassiness and frozen gauge fields

PHYSICAL REVIEW B **69**, 014208 (2004)

Avoided phase transitions and glassy dynamics in geometrically frustrated systems and non-Abelian theories

Zohar Nussinov*

Institute Lorentz for Theoretical Physics, Leiden University, P.O. Box 9506, 2300 RA Leiden, The Netherlands

(Received 16 March 2003; revised manuscript received 16 July 2003; published 28 January 2004)

We demonstrate that the application of any external uniform non-Abelian gauge background, no matter how small, leads to a greatly enhanced degeneracy. This degeneracy is so large that even a non-Abelian background field of infinitesimal strength leads to a shocking change in the thermodynamics. The critical temperature might be discontinuously depressed and an “avoided critical point” will emerge. We focus on how this arises in models previously employed to describe the microscopics of metallic glasses and correctly predicted the structure factor peaks. Some of the best fits, to date, to the dynamics of supercooled liquids were inspired by such notions for which we now provide a suggestive microscopic basis. We generalize the Mermin-Wagner inequality to high dimensions and discuss how extensive configurational entropy may be computed, by replica calculations, for a multitude of glass models (including non-Abelian gauge backgrounds). This extensive configurational entropy then allows a possible derivation of Vogel-Fulcher dynamics. We fortify earlier ideas suggesting avoided critical dynamics.

Where to look : The ingredients

- Magnetic system with multicriticalities
- Many sublattices with competing primary modes (irreps)
- Geometric frustration can help
- **Lifshitz-type invariants** : broken inversion symmetry mandatory

Gradient part of free energy : frozen gauge field background

System	Crystal symmetry
$\text{PbCuTe}_2\text{O}_6$	$P4_132$
$\text{Cu}_2\text{Te}_2\text{O}_5\text{X}_2$ (X=Cl, Br)	P-4
FeCrAs	P-62m
$\text{Bi}_3\text{Mn}_4\text{O}_{12}(\text{NO}_3)$	P3
$\text{A}(\text{TiO})\text{Cu}_4(\text{PO}_4)_4$ (A=Ba, Pb, Sr)	$P42_12$
$\text{Ca}_{10}\text{Cr}_7\text{O}_{28}$ Whitlockite-type	R3c
$\text{Na}_4\text{Ir}_3\text{O}_8$	$P4_132$
$\text{La}_4\text{Ru}_6\text{O}_{19}$	I23
LalrSi-type intermetallic compounds EuPtSi, CeIrSi	$P2_13$
Swedenborgite type magnets $\text{Y}_{0.5}\text{Ca}_{0.5}\text{BaCo}_4\text{O}_7$ YBaCo_4O_7 $\text{CaBaCo}_3\text{FeO}_7$ $\text{YBaCo}_3\text{FeO}_7$	$P6_3mc$ $Pbn2_1$ $Pbn2_1$ $P6_3mc$

Improper Dzyaloshinskii textures in magnetic materials

What

No true 3D long-range order in LGW description (magnetic space groups or representation analysis)

Why

Lifshitz invariants couple & twist many magnetic modes

Where

Materials hosting lumps of magnetic order, but condense without long-range order: *magnetic skyrmions, textures, spin liquids*

Beyond $\text{Ca}_3\text{Ru}_2\text{O}_7$: **EuPtSi**

cubic $P2_13$, AFM, field induced helical A-phase, K. Kaneko et al., JPSJ, 2019

EuNiGe₃, tetra $I4mm$, AFM, transition in $\sim 2\text{T}$

Classical!

Coupled AF modes:

α -FeO(OH): AF mode breaks inv. symm.

Fe₂P: hex, multicritical point under pressure. J. Staunton et al., PRB 2013

Classical!

Chiral Spin liquids due to Lifshitz invariants: frustration in 3D, role of *quantum fluctuations*?

YBaCo₃FeO₇: hex, $P6_3mc$, no order, strong frustration. M. Valldor et al., PRB 2011

LaIrSi: cubic $P2_13$, no order, SC. B. Chevalier et al., SSC, 1982

La₄Ru₆O₁₉: cubic $I2_13$, no order, nFI, QCP? P. Khalifa et al., Nature, 2001, PRB 2009

Na₄Ir₃O₈: cubic $P4_132$, Y. Okamoto et al., PRL 2007

Curtsey, Dmitry Sokolov

Thank you for listening !

