


Archimedean screw and time quasi-crystals in driven chiral magnets

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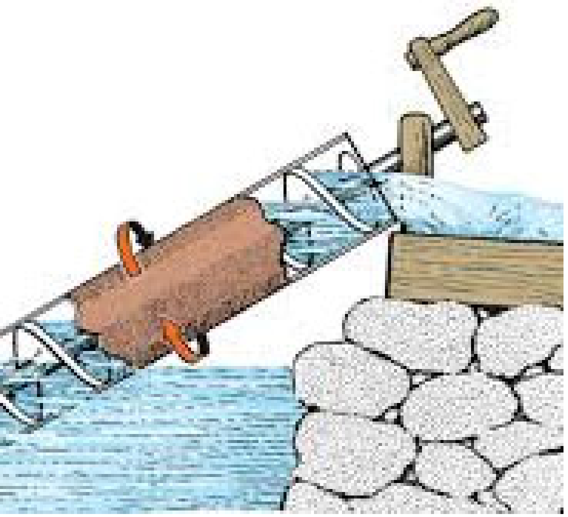
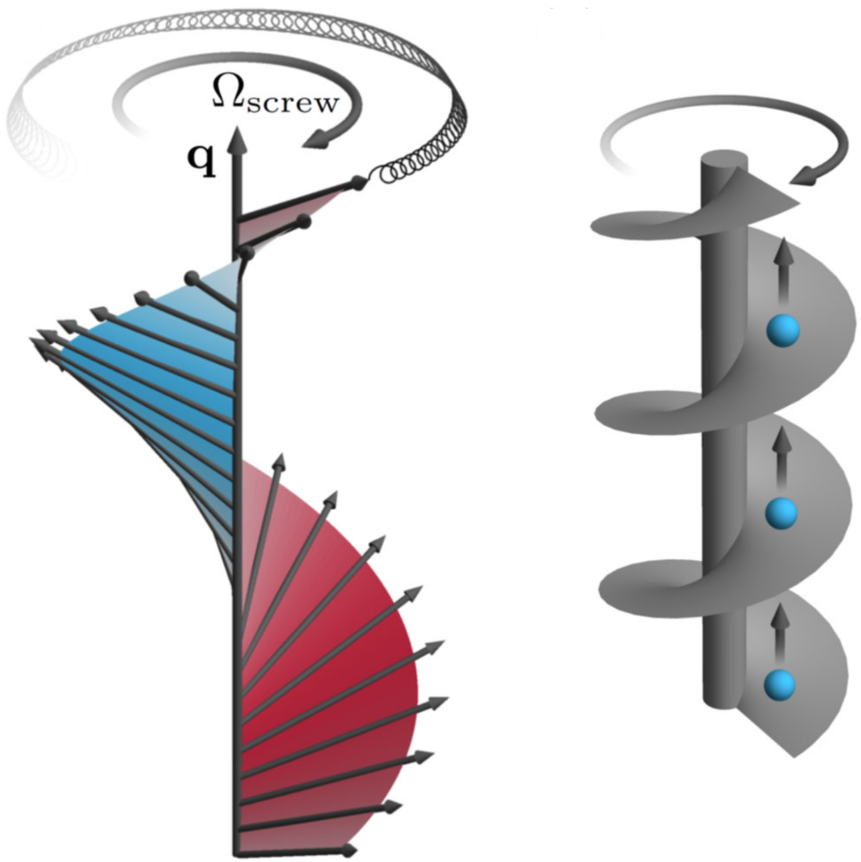
- driving helical magnets by GHz radiation
- Archimedean-screw like motion
- time-quasicrystals
- and some dancing skyrmions….

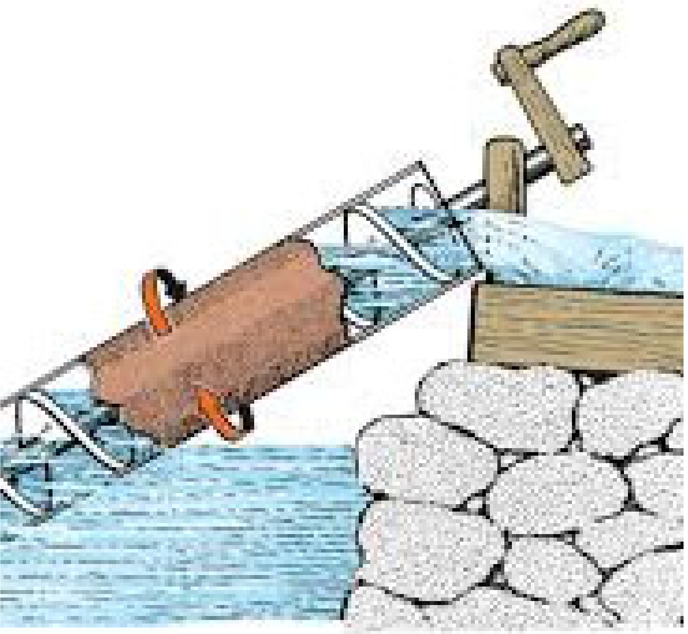


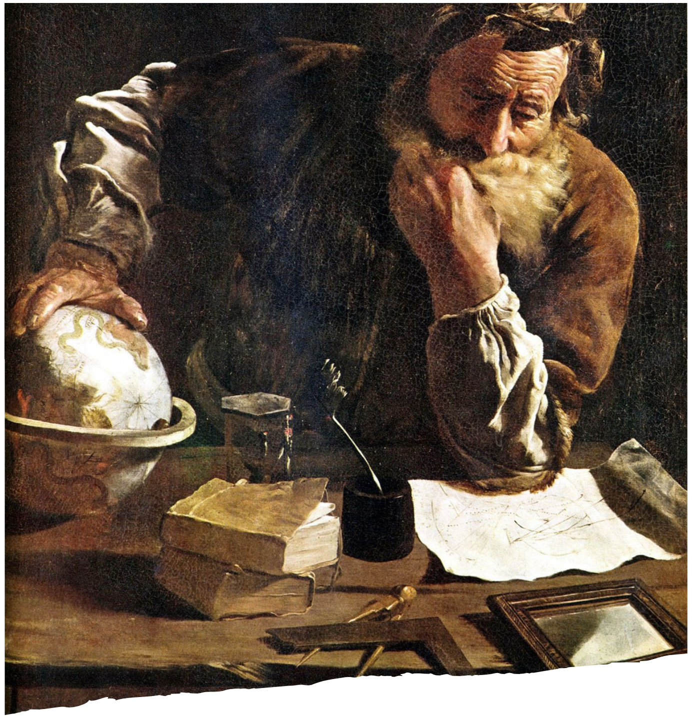


Can we build some small machines using driven quantum matter?

- to explore physical mechanisms & fundamental concepts
- to create something useful?

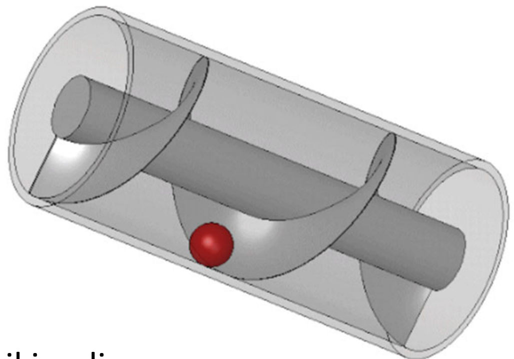
one of the oldest machines:
Archimedean screw





Archimedes of Syracuse *267BC
mathematician, inventor, engineer,
physicist

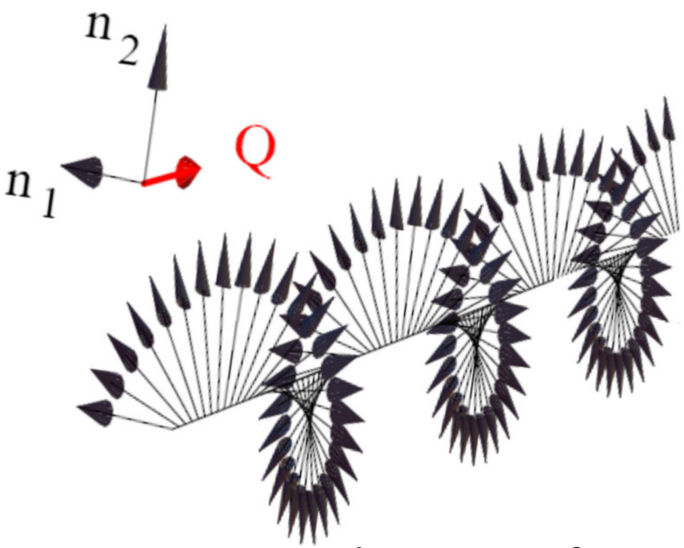
use screw to drain ships



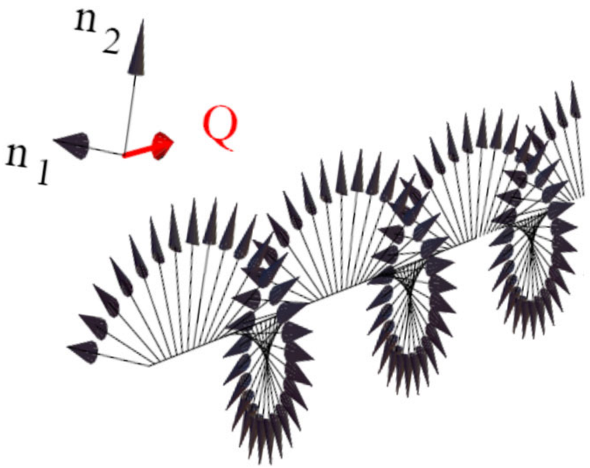
wikipedia

screw principle:
rotation implies translation

our screw:
helical (or conical) phase of chiral magnet

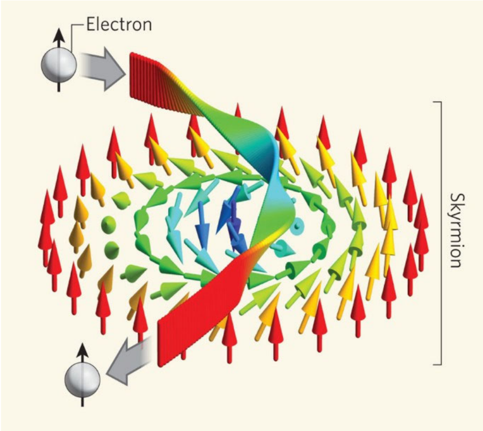


symmetry: combination of spin-rotation and translation
locked by chiral spin-orbit interaction



materials:

- **O(100) different systems** with pitch of helix ranging from nm to μm
- metals, insulators, semiconductors,…
  both at room temperatures and low temperatures
- often studied because of magnetic skyrmion phases
- simplest class: cubic magnets without inversion symmetry where
  DMI interactions twist ferromagnet into a helix
- most famous: MnSi (very clean metal), Cu_2OSeO_3 (insulator)



The model:

chiral magnet in its helical or conical phase (including dipolar interactions)
in a **small oscillating external magnetic field**

$$\boldsymbol{B}_{\text{ext}} = \begin{pmatrix} \boldsymbol{B}_{\perp}^{x} \cos(\boldsymbol{\omega t}) \\ 0 \\ B_0^z \end{pmatrix}$$

$$F = \int \mathrm{d}^3 r \left[-\frac{J}{2} \hat{\mathbf{M}} \cdot \nabla^2 \hat{\mathbf{M}} + D \hat{\mathbf{M}} \cdot (\boldsymbol{\nabla} \times \hat{\mathbf{M}}) - \mathbf{M} \cdot \mathbf{B}_{\text{ext}} \right] + F_{\text{demag}}[\mathbf{M}]$$

$$\dot{\mathbf{M}} = \gamma \mathbf{M} \times \mathbf{B}_{\text{eff}} - \frac{\gamma}{|\gamma|} \alpha \hat{\mathbf{M}} \times \dot{\mathbf{M}}$$

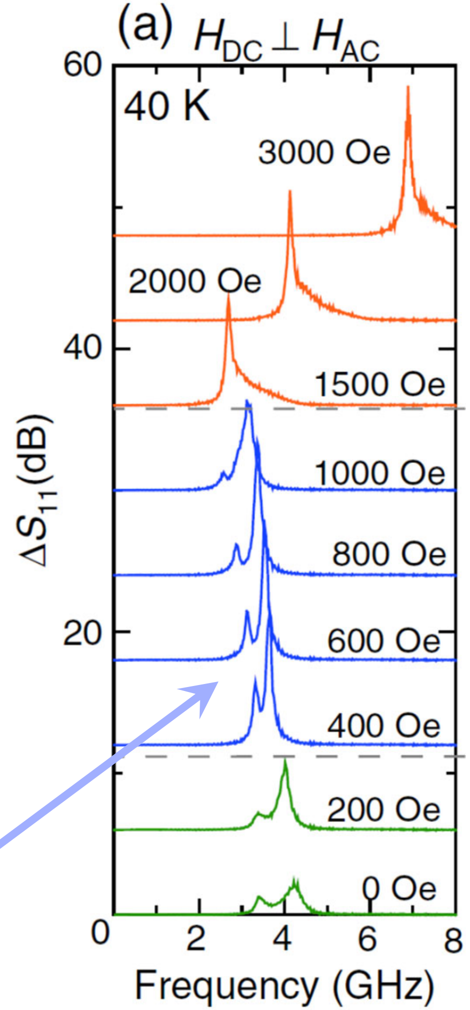
$$\boldsymbol{B}_{\mathrm{ext}} = \begin{pmatrix} \boldsymbol{B}_\perp^{\boldsymbol{x}} \cos(\boldsymbol{\omega t}) \\ 0 \\ B_0^z \end{pmatrix}$$

$O(B_\perp^0)$ no perturbation: conical state

$$\boldsymbol{M} = M_0 \begin{pmatrix} \sin\theta_0 \cos qz \\ \sin\theta_0 \sin qz \\ \cos\theta_0 \end{pmatrix}$$

$O(B_\perp^1)$ linear response: oscillation at frequency $\boldsymbol{\omega}$

resonantly enhanced at k=0 magnon frequencies
measured via microwave absorption:
Onose, Okamura, Seki, Ishiwata,Tokura, PRL (2012)
Schwarze et al., Nature Materials (2015)



two resonances in
conical phase split
by dipolar interactions

$$\boldsymbol{B}_{\rm ext} = \begin{pmatrix} \boldsymbol{B}_\perp^x \cos(\boldsymbol{\omega t}) \\ 0 \\ B_0^z \end{pmatrix}$$

$O(B_\perp^0)$ no perturbation: conical state $\boldsymbol{M} = M_0 \begin{pmatrix} \sin\theta_0 \cos qz \\ \sin\theta_0 \sin qz \\ \cos\theta_0 \end{pmatrix}$

$O(B_\perp^1)$ linear response: oscillation at frequency $\boldsymbol{\omega}$

$O(B_\perp^2)$ quadratic response at frequencies 2ω and 0

pumping into the Goldstone mode
allowed by symmetries for arbitrary "perpendicular" pumping

Analytics

expand direction of magnetization on powers of oscillating field
useful: use angles to parametrize magnetization

$$\theta = \theta_0 + \theta_1(z,t) + \theta_2(z,t) + O\left(B_{1,\perp}^3\right)$$

$$\phi = \phi_0 + \phi_1(z,t) + \phi_2(z,t) + O\left(B_{1,\perp}^3\right)$$

solve Landau-Lifshitz-Gilbert equation order by order

linear order full analytic solution possible

resonance frequencies:
(including dipolar interactions)

$$\omega_{\text{res}}^{\pm} = \frac{1}{2}\sqrt{\Big[c^2\left(\delta^2(2N_xN_y - N_x - N_y) - 4 - 4\delta\right) + (\delta+2)(\delta(N_x+N_y)+4) \pm \sqrt{\left(\left(c^2\left(\delta^2(2N_xN_y - N_x - N_y) - 4 - 4\delta\right) + (\delta+2)(\delta(N_x+N_y)+4)\right)^2 - 4\left(c^2\left(2\delta + \delta^2 N_x + 2\right) - (\delta+2)(\delta N_x + 2)\right)\left(c^2\left(2\delta + \delta^2 N_y + 2\right) - (\delta+2)(\delta N_y + 2)\right)\right)}\Big]}$$

Analytics

quadratic order shown below: formulas without dipolar interactions

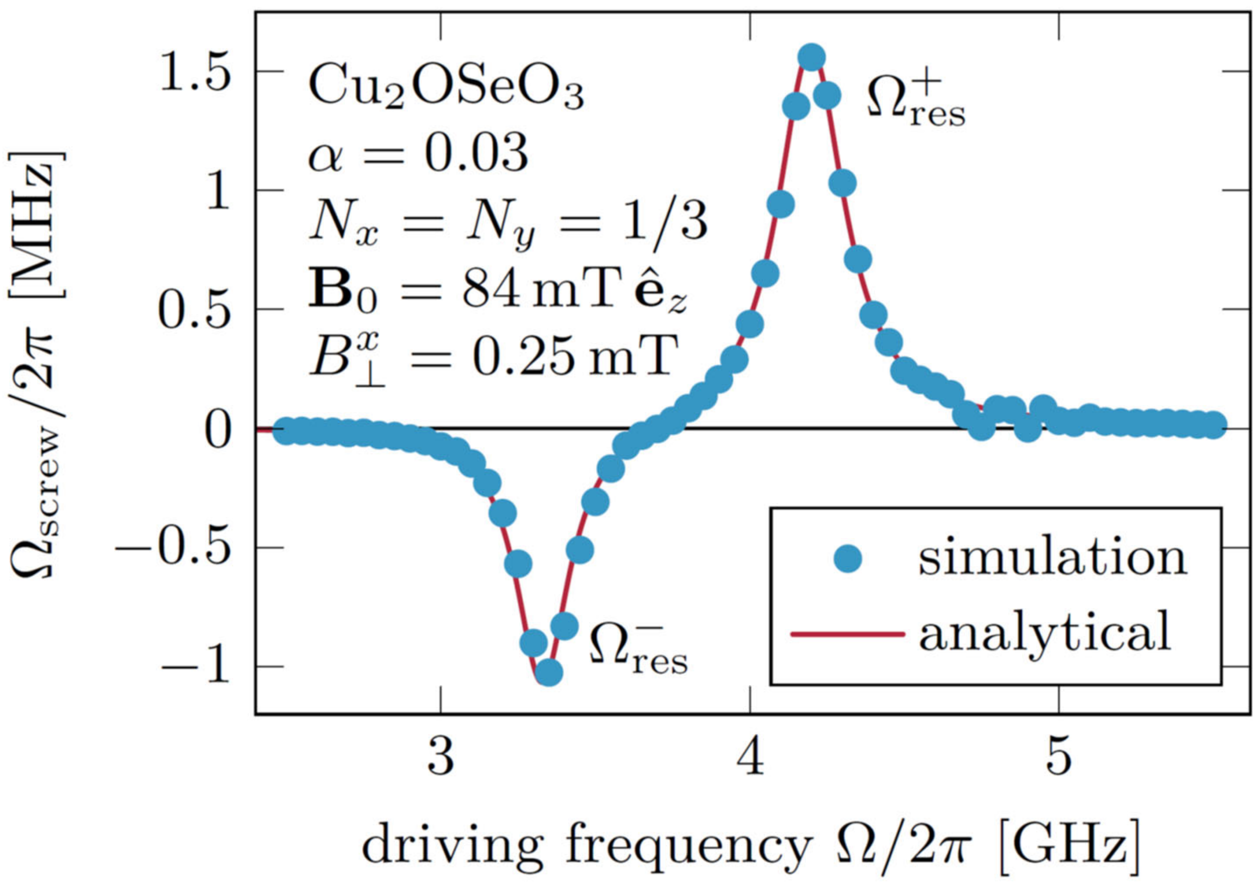
$$\begin{aligned}
\mathrm{sgn}(\gamma)\dot\theta_2 - \alpha(s\dot\phi_2 + c\theta_1\dot\phi_1) &= -2c\theta_1'\phi_1' - c\theta_1\phi_1'' - s\phi_2'' + \phi_1\left(b_x(t)\cos(z) + b_y(t)\sin(z)\right) \qquad (7)\\
\mathrm{sgn}(\gamma)s(2c\theta_1\dot\phi_1 + s\dot\phi_2) + \alpha(c\theta_1\dot\theta_1 + s\dot\theta_2) &= s\theta_2'' + c\theta_1\theta_1'' - s^2c\phi_1'^2 - \frac{5}{2}cs^2\theta_1^2 - s^3\theta_2\\
&\quad + (c^2 - s^2)\theta_1\left[b_x(t)\cos(z) + b_y(t)\sin(z)\right] + sc\phi_1\left[b_y(t)\cos(z) + b_x(t)\sin(z)\right]
\end{aligned}$$

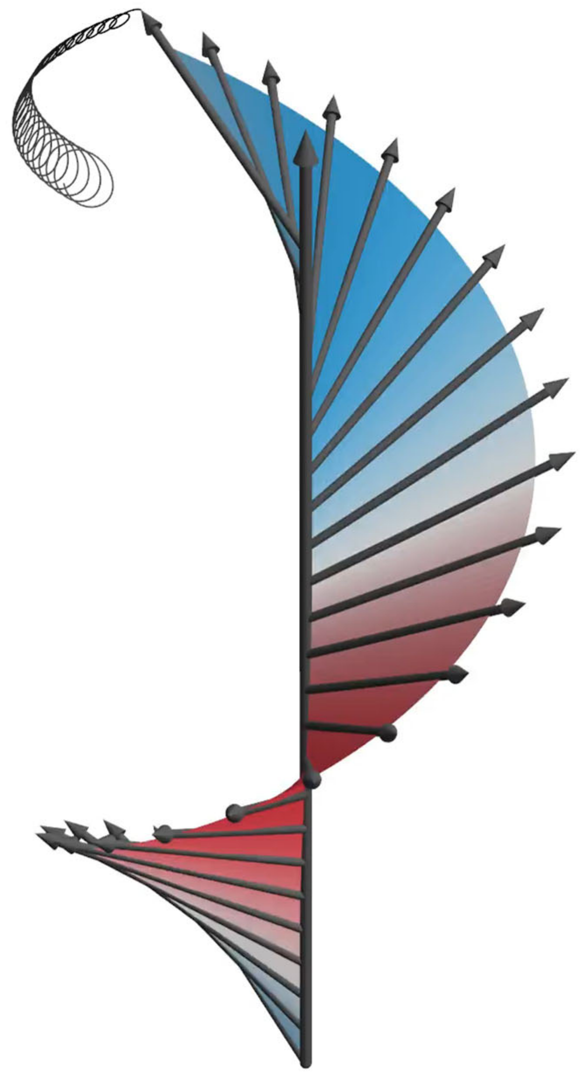
Fourier-transformation: equations do **not** have a solution at *ω = 0, k=0*

solution:

$$\phi_2(t) = \Omega_{\mathrm{screw}} t + \ldots \qquad \Omega_{\mathrm{screw}} \propto B_\perp^2$$

angle grows linear in time: **screw-like motion**
rotations & translation of helix with constant (angular) velocity

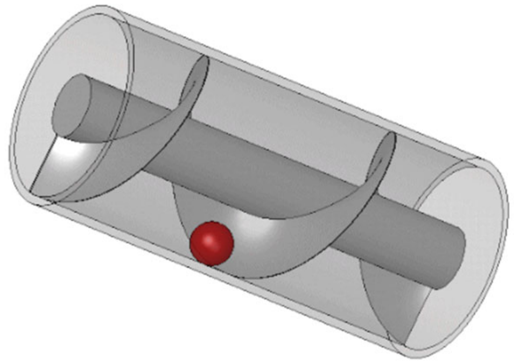




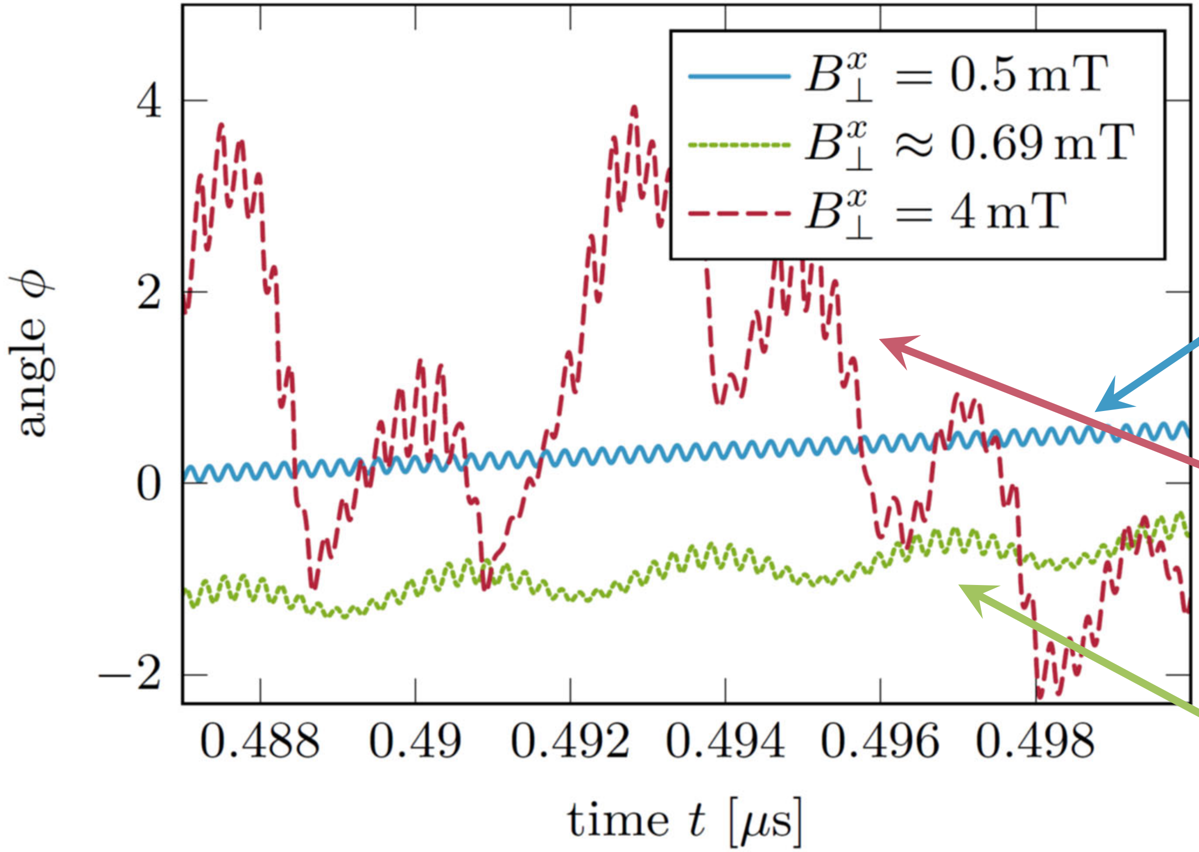
parameters: Cu_2OSeO_3 (α too large)

Archimedean screw solution found

stability?



numerics: track a single spin in a large unit cell

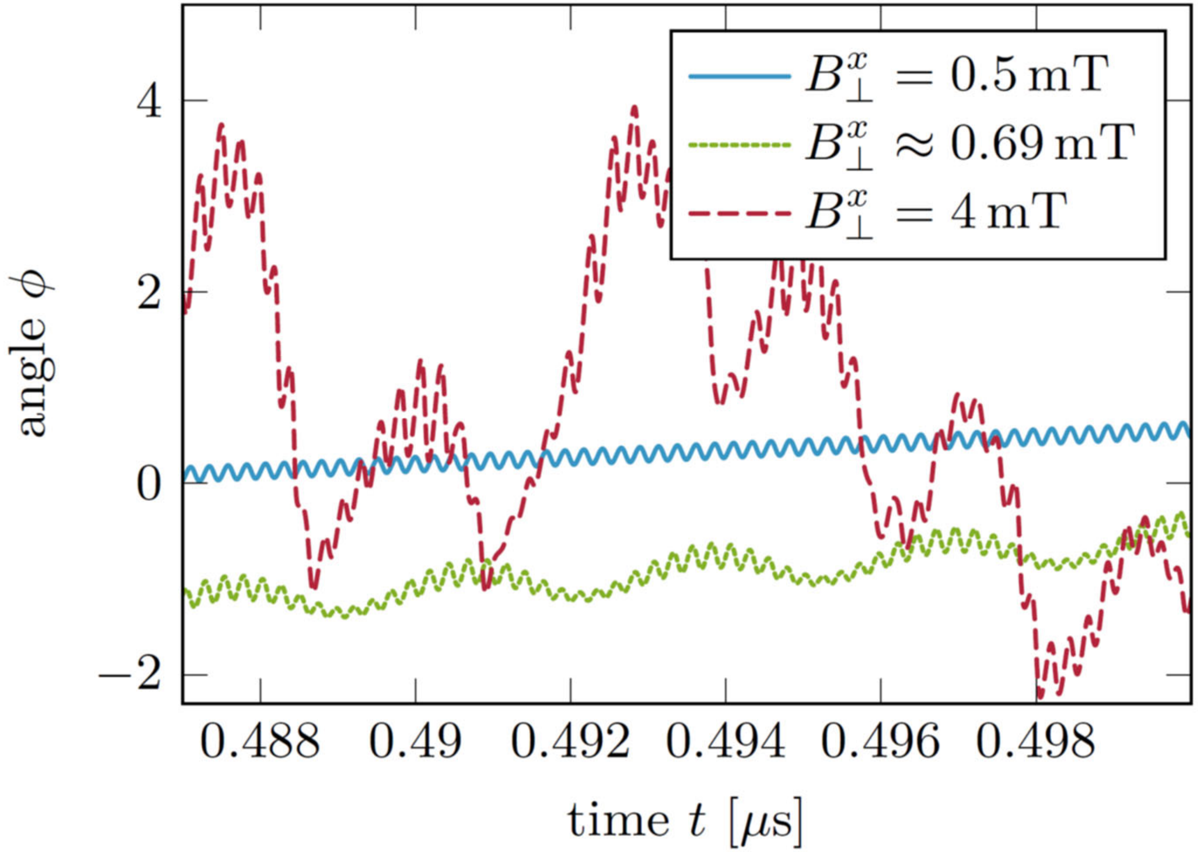


Archimedean screw solution

chaotic solution

time-quasi-crystal
spontaneos breaking of discrete time translation invariance: oscillating with incommensurate period (space & time)

numerics: track a single spin in a large unit cell



numerics alone: **not reliable**

depends strongly on size of simulated region

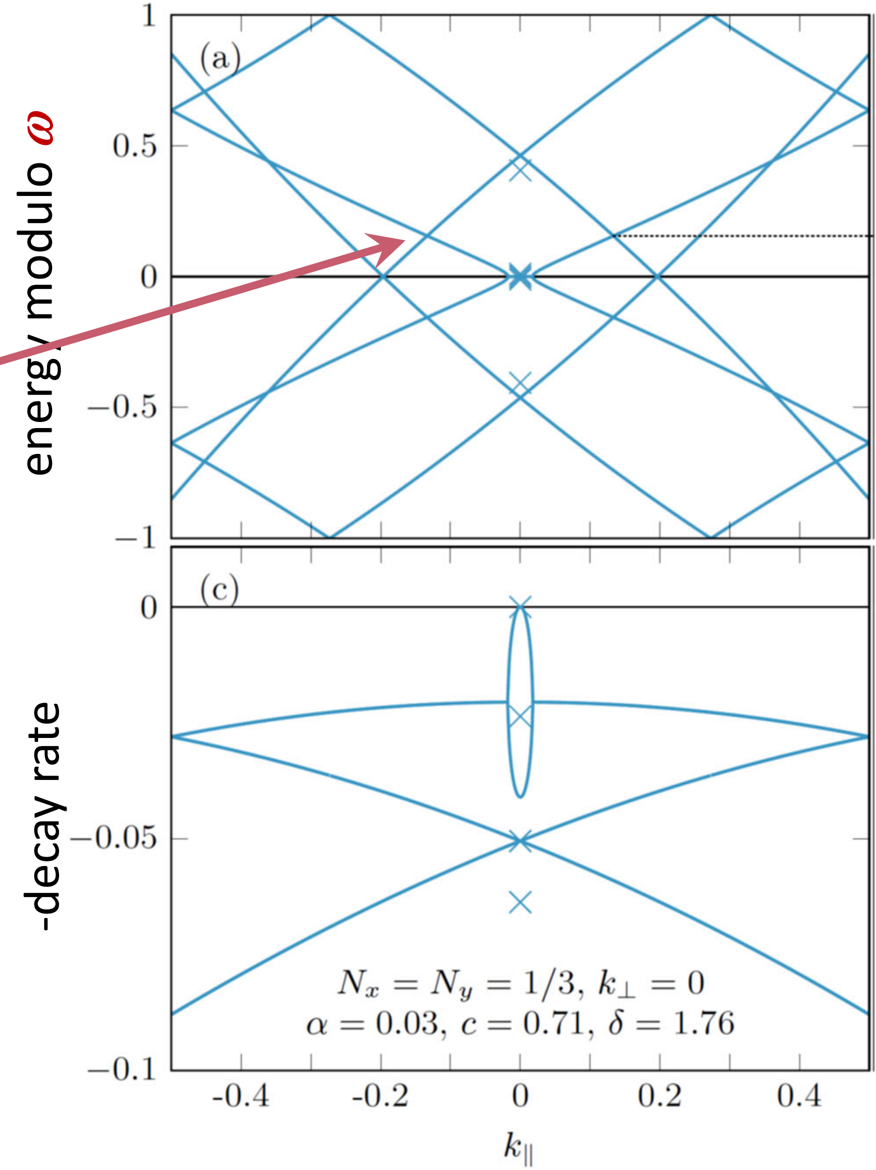
Stability of Archimedean screw: analytics

Bogolioubov-Floquet spin-wave theory
including damping terms

unavoidable:
crossing points describing **resonant creation of a magnon-pair**

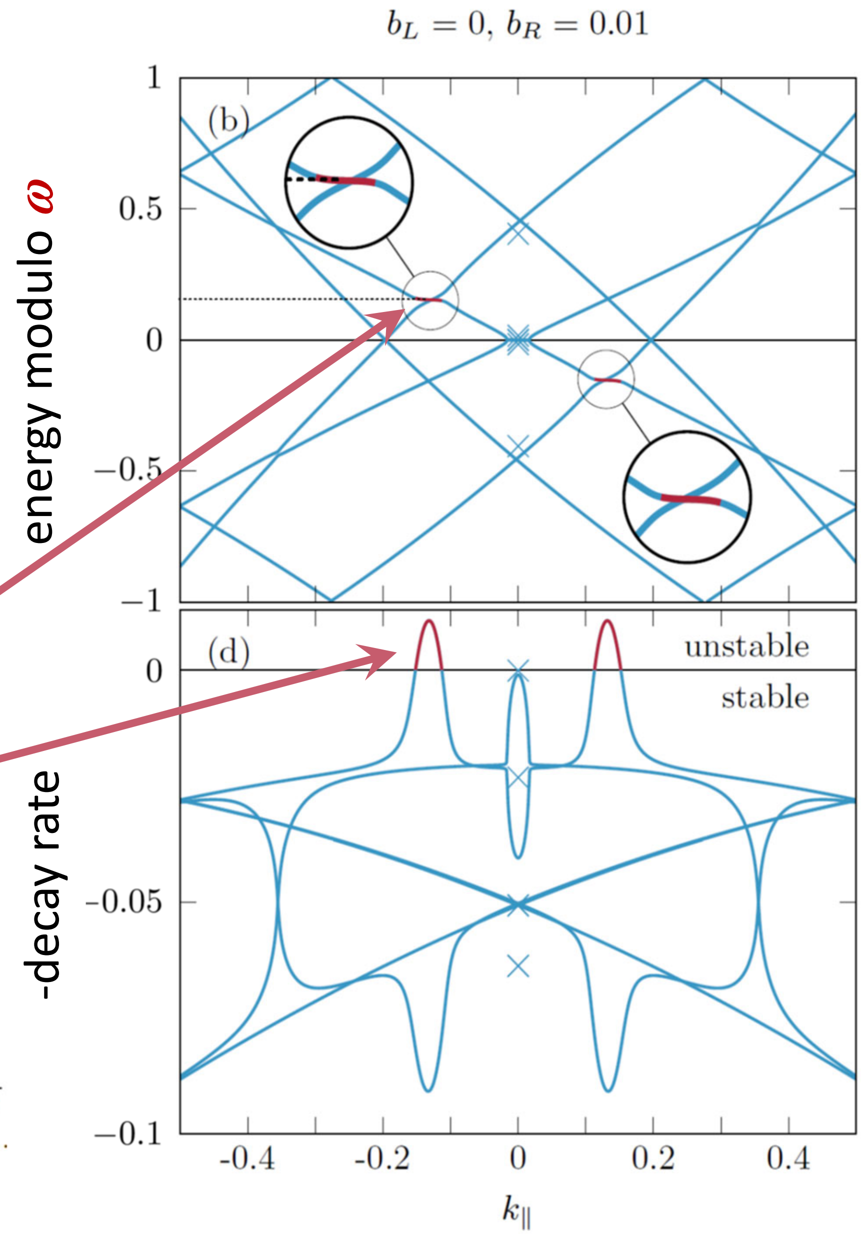
$$\epsilon_{i,\boldsymbol{k}} + \epsilon_{j,-\boldsymbol{k}} = \boldsymbol{\omega}$$

unperturbed system



Stability of Archimedean screw: analytics

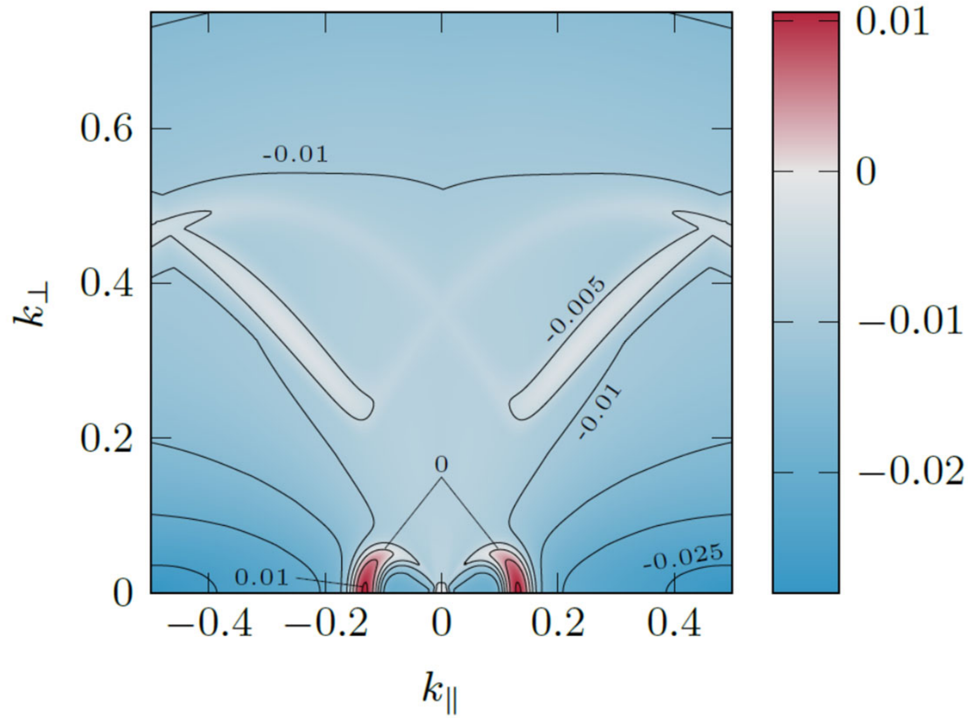
Bogolioubov-Floquet spin-wave theory
including damping terms



instability when driving sufficiently large
close to resonant condition

$$\epsilon_{i,\boldsymbol{k}} + \epsilon_{j,-\boldsymbol{k}} = \omega$$

$$\lambda^{\pm}_{\text{res}} = \epsilon^0_{i,\mathbf{k}} - i\alpha\frac{\Gamma_1+\Gamma_2}{2} \pm i\sqrt{\mu^{(1)}_{\omega}\mu^{(2)}_{\omega} + \alpha^2\left(\frac{\Gamma_1-\Gamma_2}{2}\right)^2}.$$

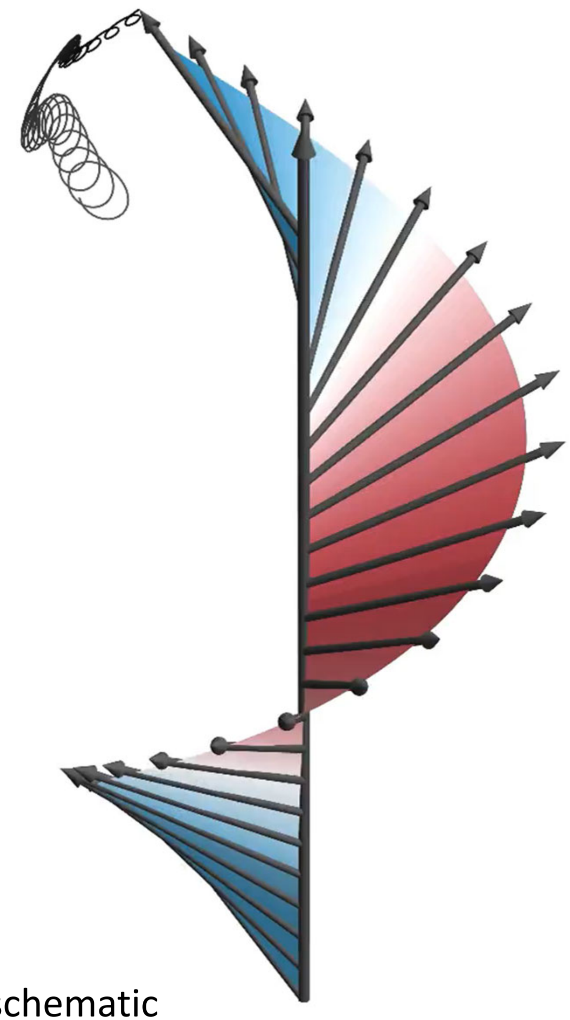


0.01
0
−0.01
−0.02
-0.01
-0.005
-0.01
0
0.01
-0.025
0.6
0.4
0.2
0
−0.4
−0.2
0
0.2
0.4
$k_\perp$
$k_\parallel$

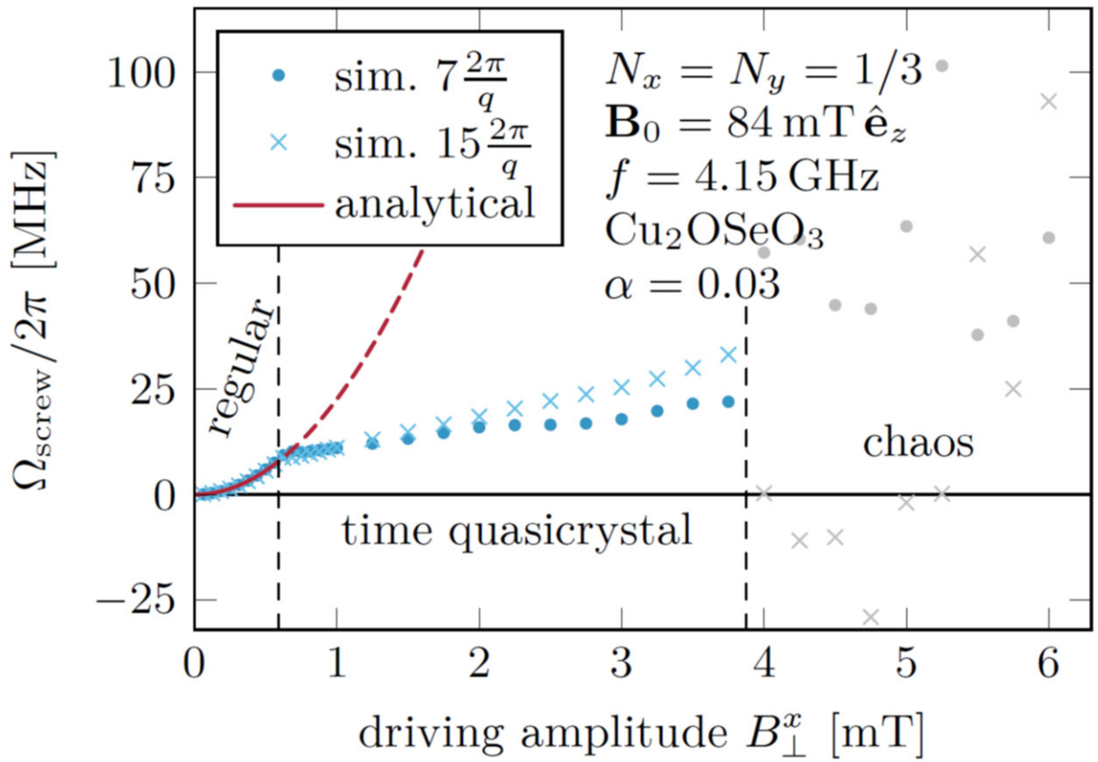
gain

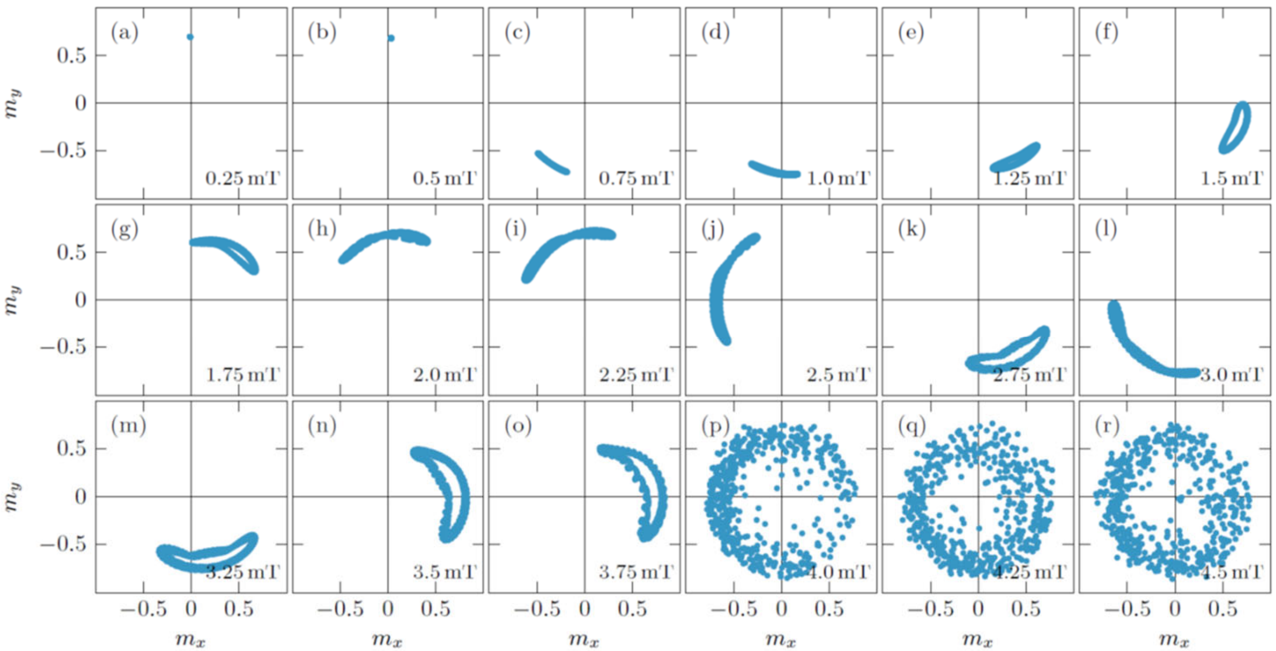
loss

magnon laser = time quasi crystal

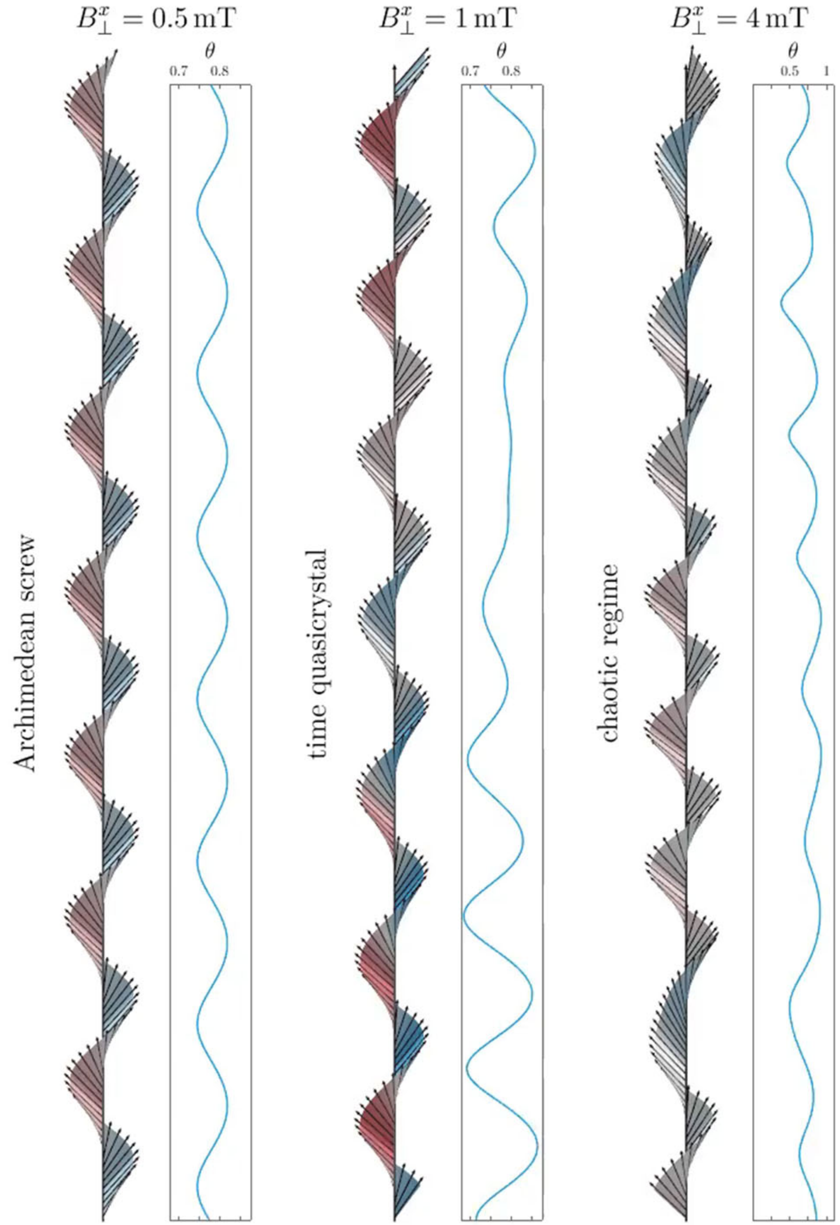


schematic





stroboscopic time evolution



generic properties of magnonic systems driven by GHz/THz B-fields

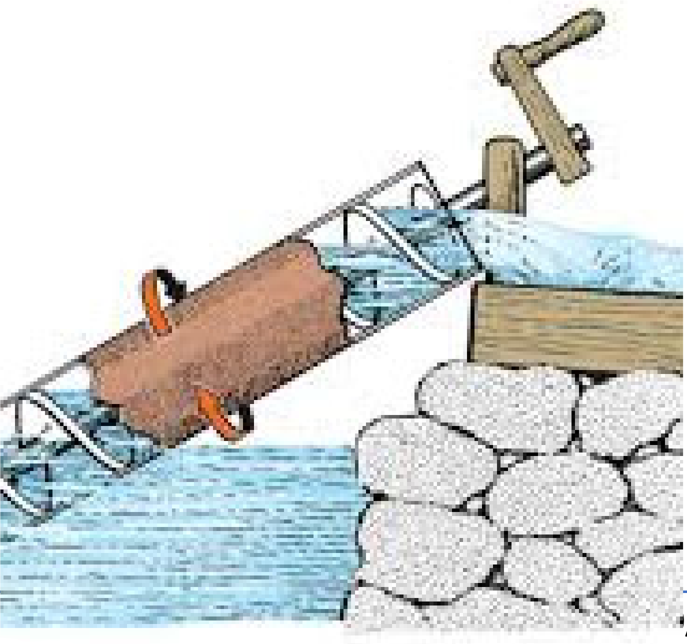
- for incommensurate magnetic order & if symmetry-allowed: translational **Goldstone mode activated** for arbitrariy weak driving (in absence of pinning) $\Omega_{\text{screw}} \propto B_\perp^2$
- next leading instability: **resonant creation of magnon pairs** $\epsilon_{i,\boldsymbol{k}_0} + \epsilon_{j,-\boldsymbol{k}_0} = \omega$
- **stabilization** of periodically driven phases only due **to magnon damping** (extrinsic or due to magnon-magnon interactions)
- consequence of secondary instability:

magnon laser = oscillating texture with momentum $\boldsymbol{k}_0$ and frequency $\epsilon_{i,\boldsymbol{k}_0}$
= **time quasi crystal**

back to Archimedes

Can we pump something ?
charge – heat – spin

now: charge pump



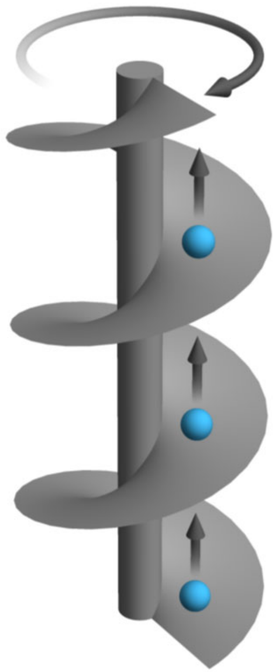
kinetic energy

$$H = \int \boldsymbol{c}^\dagger \left(\frac{p^2}{2m} + \lambda\, \boldsymbol{p} \cdot \boldsymbol{\sigma} + J_H \left(\boldsymbol{n}(\boldsymbol{r}, t) \cdot \sigma \right) + V_{\mathrm{dis}}(\boldsymbol{r}) \right) \boldsymbol{c} + H_{\mathrm{int}}$$

spin-orbit coupling
(needed !!)

disorder potential

Archimedean screw coupled by exchange field



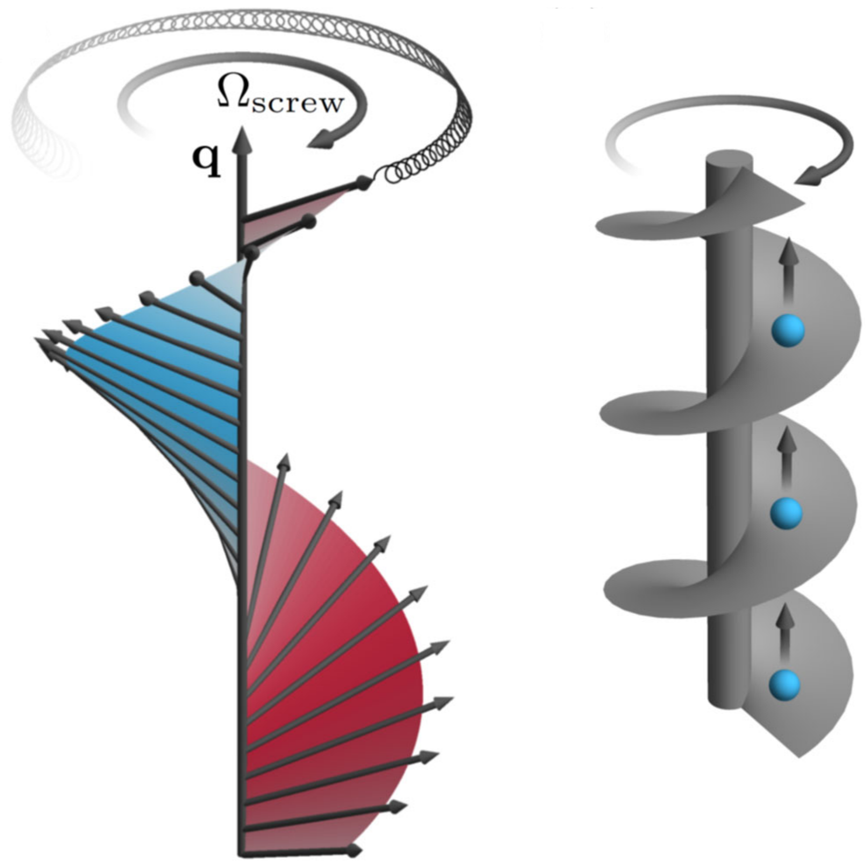
clean limit

no disorder, no Umklapp scattering from atomic lattice

transformation to comoving coordinate system

$$\boldsymbol{r} \rightarrow \boldsymbol{r} - \boldsymbol{v}_{\text{screw}} t, \qquad \boldsymbol{v}_{\text{screw}} = \Omega_{\text{screw}} \lambda_{\text{helix}}$$

➡ electric current density $j = e n_e v_{\text{screw}}$
highly efficient pump (later)



dirty limit

different transformation: spin-quantization axis parallel to local magnetization

$$\tilde{H} \approx \sum_{\sigma,\mathbf{k}} \epsilon_{\sigma,\mathbf{k}} d^\dagger_{\sigma,\mathbf{k}} d_{\sigma,\mathbf{k}} + H_1(t) + H_{\text{dis}}$$

$$H_1(t) = \sum_{\sigma,\mathbf{k}} \frac{\hbar s k_\perp \lambda}{2} (d^\dagger_{\sigma,\mathbf{k}} d_{\sigma,\mathbf{k}+\mathbf{q}} e^{-i\omega_{\text{screw}} t} + h.c.)$$

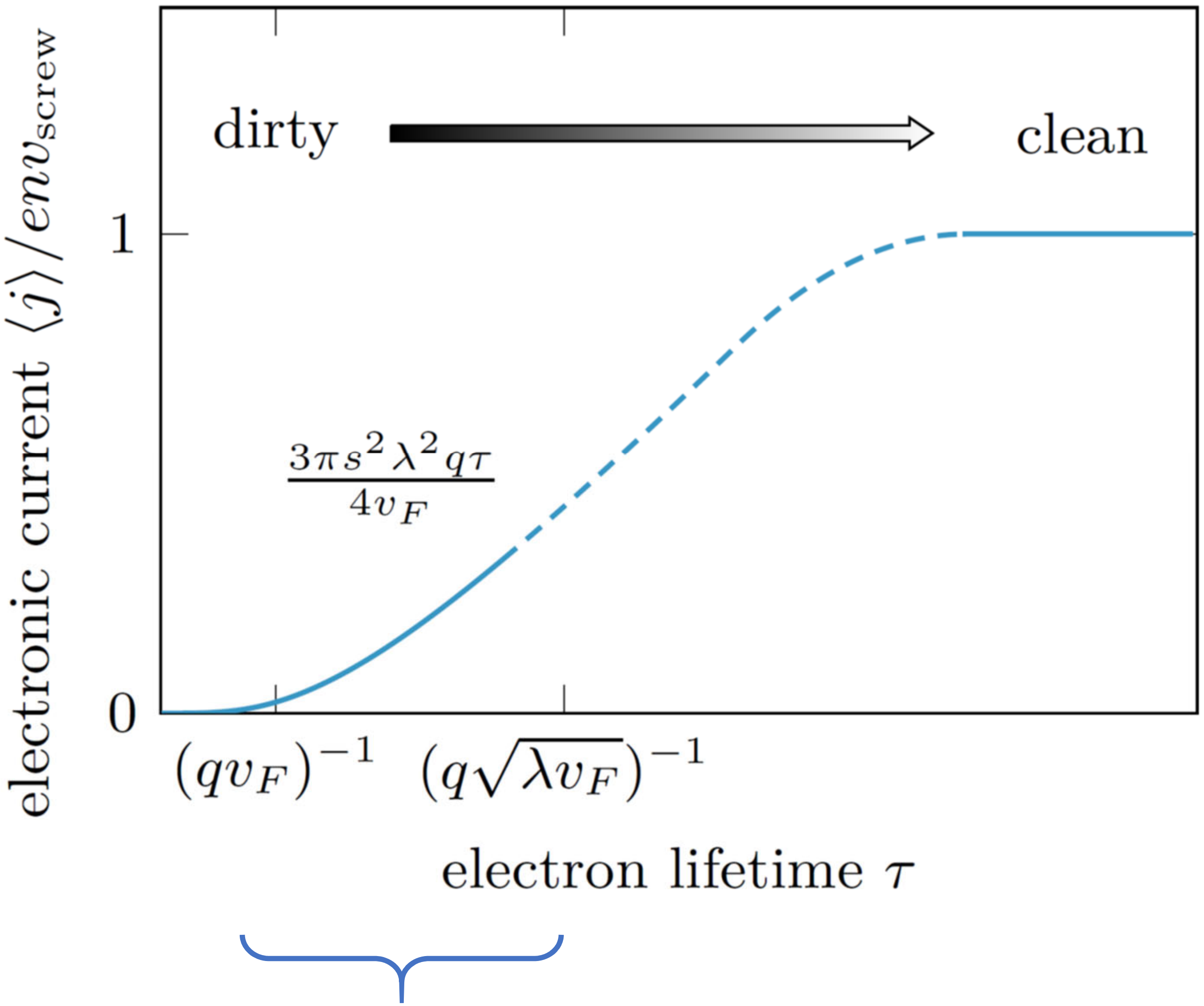
driving only due to spin-orbit interactions

$$\epsilon_{\uparrow/\downarrow,\mathbf{k}} \approx \frac{\hbar^2}{2m} \left((k_\| \mp k_0)^2 + k_\perp^2\right) \mp J_H$$

to do: 2nd order Keldysh-PT in oscillating term (ignoring disorder-induced vertex corrections)

$$\langle J_\| \rangle = J_0 \sum_{\sigma,\mathbf{k}} \frac{k_\perp^2 (k_\| - \sigma k_0)(n_{\sigma,\mathbf{k}} - n_{\sigma,\mathbf{k}+\mathbf{q}})(\epsilon_{\sigma,\mathbf{k}} - \epsilon_{\sigma,\mathbf{k}+\mathbf{q}})}{\left((\epsilon_{\sigma,\mathbf{k}+\mathbf{q}} - \epsilon_{\sigma,\mathbf{k}})^2 + (\hbar\tau^{-1})^2\right)^2}$$

$$J_0 = \frac{2\lambda^2 s^2 e \hbar^4 q v_{\text{screw}}}{m}$$



experimental regime
e.g. bulk MnSi, $\ell_s \sim 1000\,\text{Å}$ at T=0

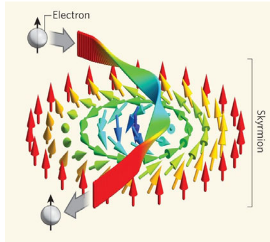
numbers

$$\Omega_{\rm screw} \sim 1\,{\rm MHz}, \quad \lambda_h \sim 200\text{Å} \qquad\qquad v_{\rm screw} \sim 20\,{\rm mm/s}$$

the biggest enemy: **pinning** of the helix by **disorder**

compare to depinning of skyrmions in MnSi
(similar pinning forces expected

$$v_{\rm skyrmion} \sim 0.2\,{\rm mm/s}$$



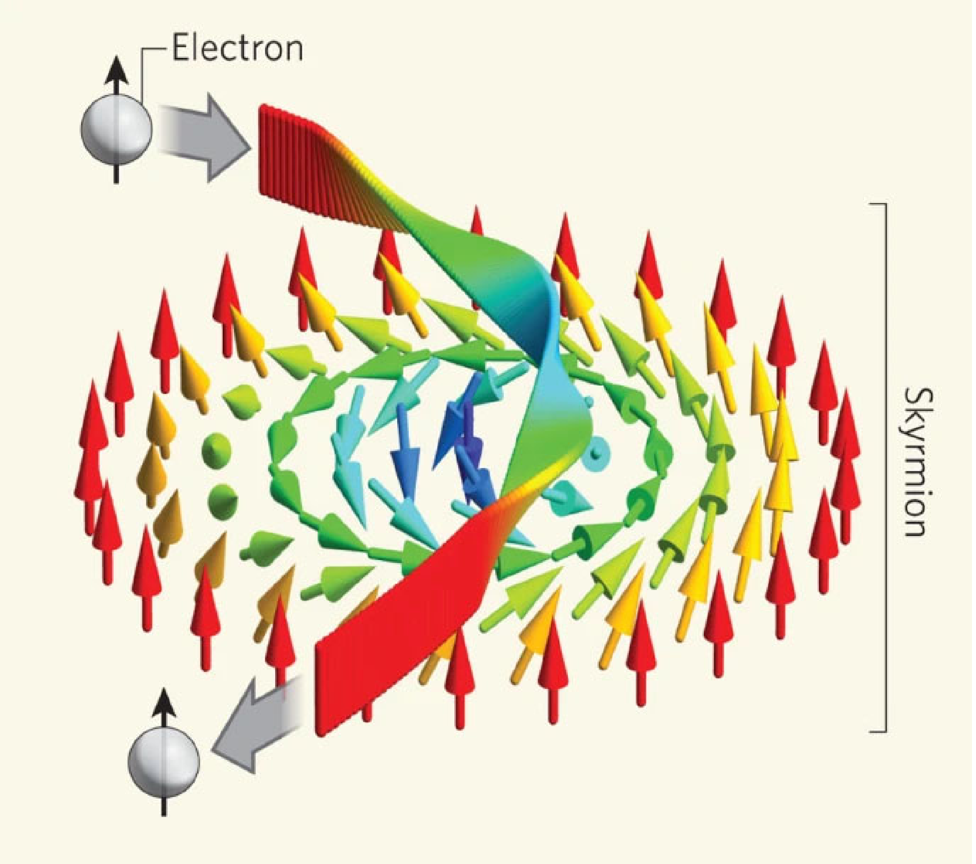
➡ pinning most likely **not a problem** in clean systems like bulk MnSi

order-of-magnitude estimate of
achievable current densities (MnSi type parameters)
voltage drop easily measurable

$$j \sim 10^{3\dots 5}\,{\rm A/m^2}$$

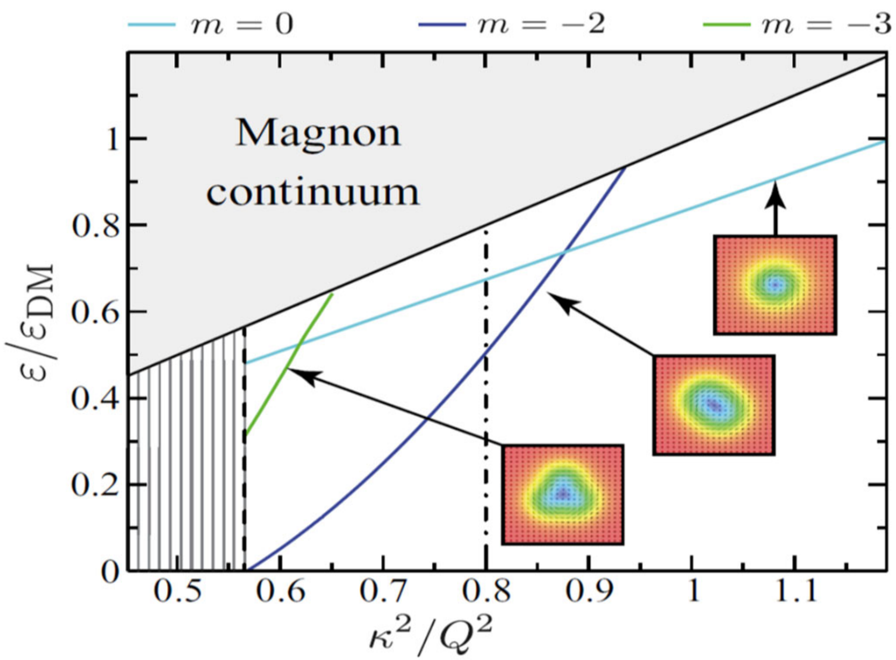
possible issue: sample thinner than GHz penetration depth (μm)
high surface quality

Can we move also other stuff?



magnetic skyrmions
in an oscillating magnetic field

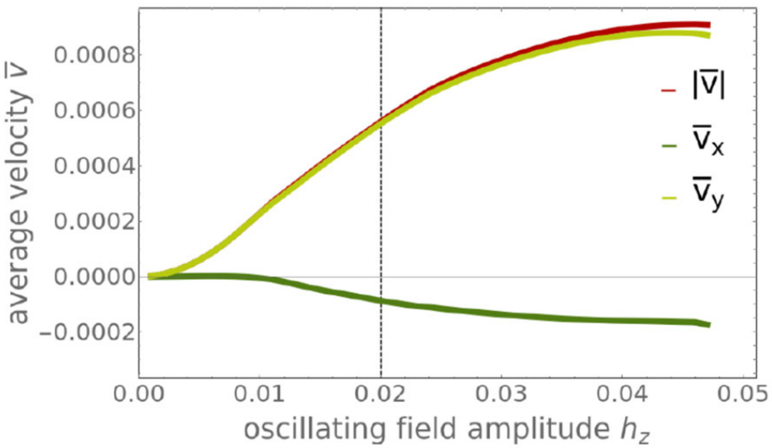
Weiwei Wang, Marijan Beg, Bin Zhang, Wolfgang Kuch, Hans Fangohr, Phys. Rev. B 92, 020403 (2015)
master thesis, Bernd Große Jüttermann (2017)



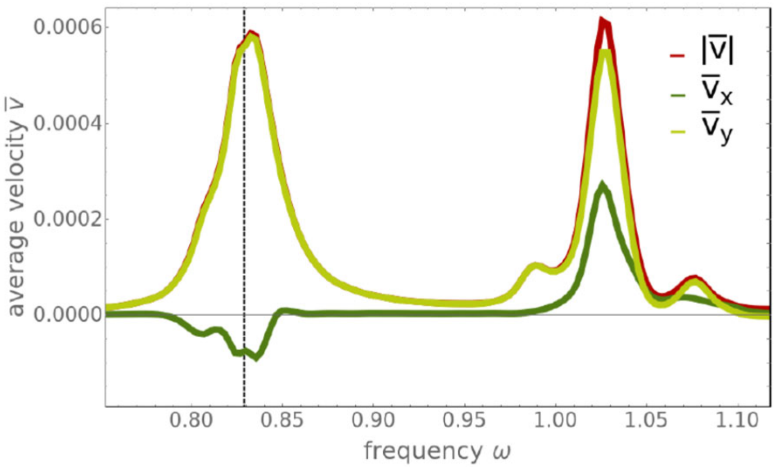
Schütte, Garst, 2014

to excite translational motion = “Goldstone mode“

- excite
  “breathing mode“ = oscillations of skyrmion size of skyrmion
  by GHz oscillating fields
- break rotation symmetry somehow:
  - by static tilted magnetic field
  - by oscillating field
  - by edge of sample
  - by other skyrmions
  - ....

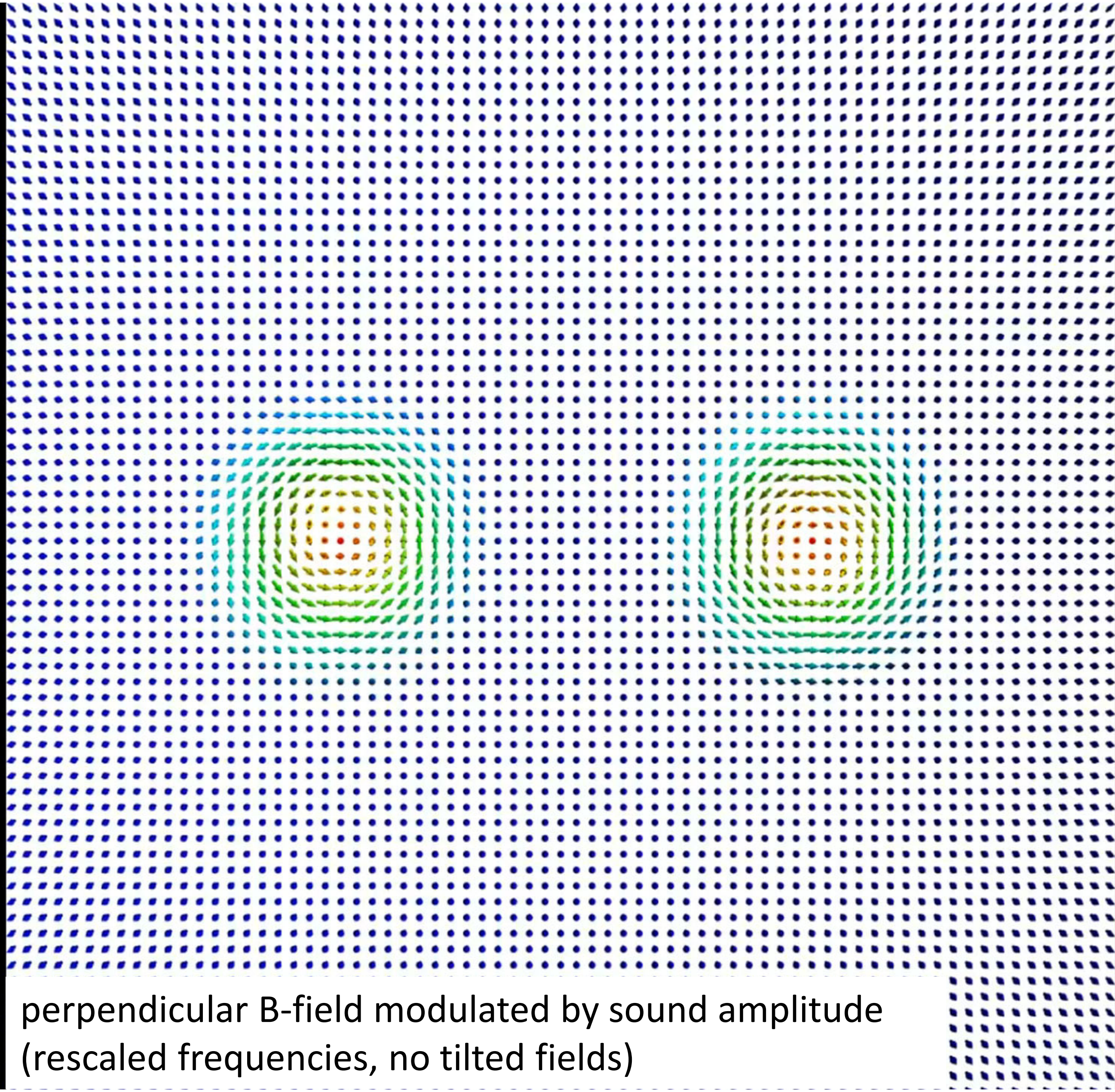


(b) Velocity vs. magnetic field h_z

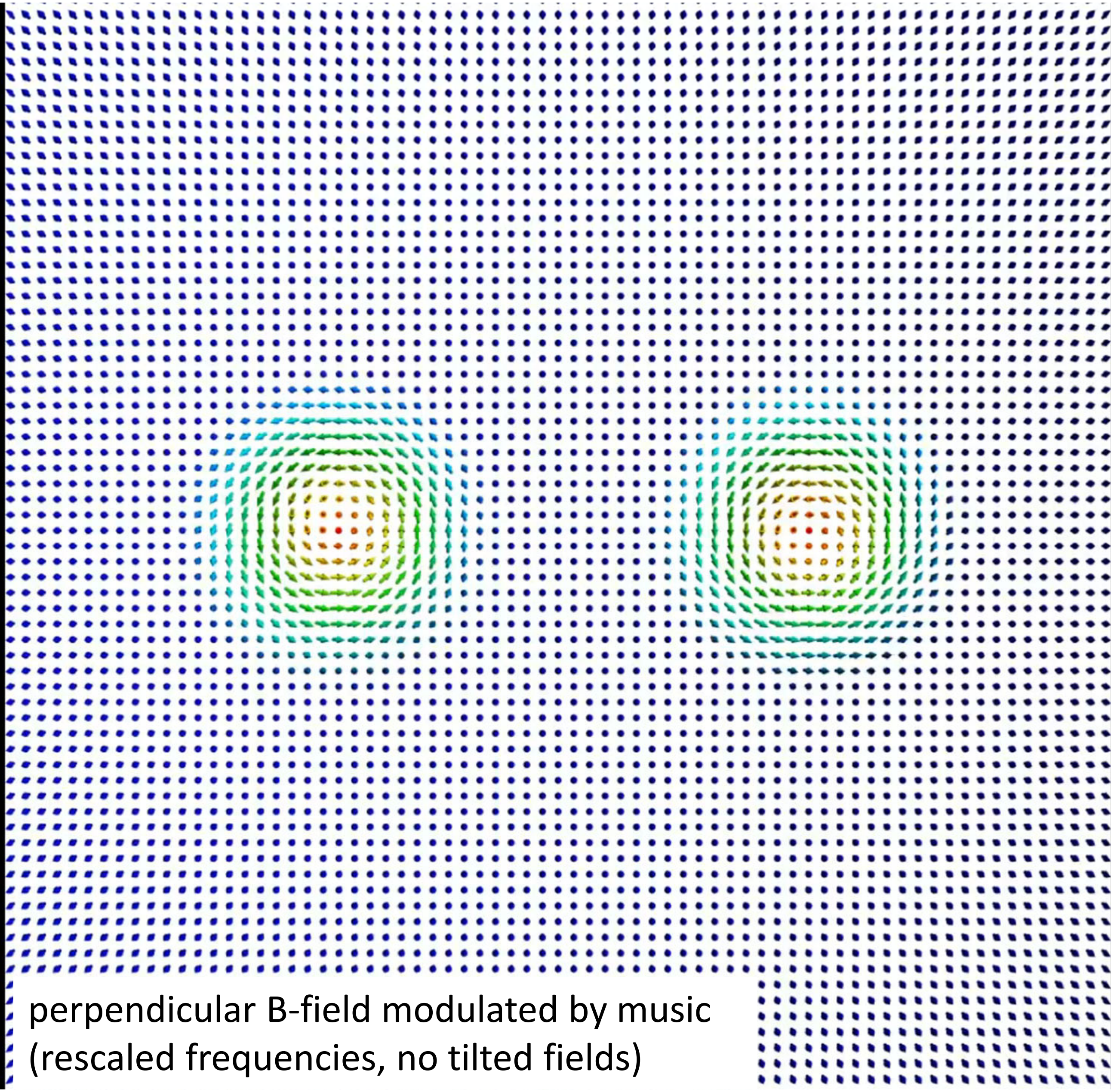


(c) Velocity vs. frequency ω

- motion maximal when in resonance with either the breathing mode or the Kittel mode of the background magnet
- velocities similar to screw reachable (10 mm/s)



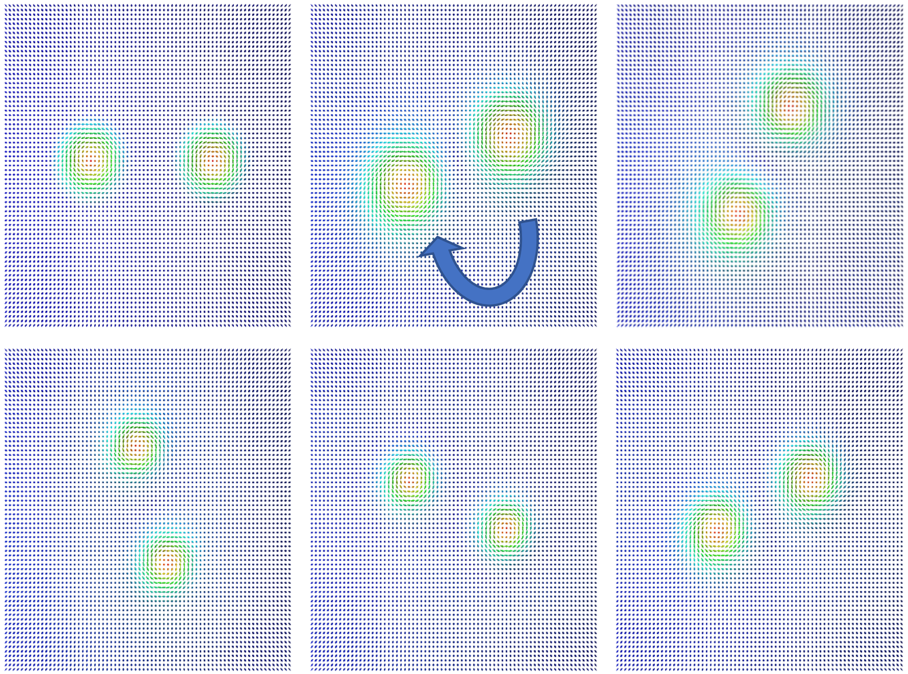
perpendicular B-field modulated by sound amplitude
(rescaled frequencies, no tilted fields)



perpendicular B-field modulated by music
(rescaled frequencies, no tilted fields)

some lessons

- advice to backpipe players for the skyrmion dance:
  tune one of your pipes to the skyrmion-breathing mode
- combination of oscillating field & deformation of skyrmions creates attractive force & circular motion
- pumping of Goldstone modes works when allowed by symmetry

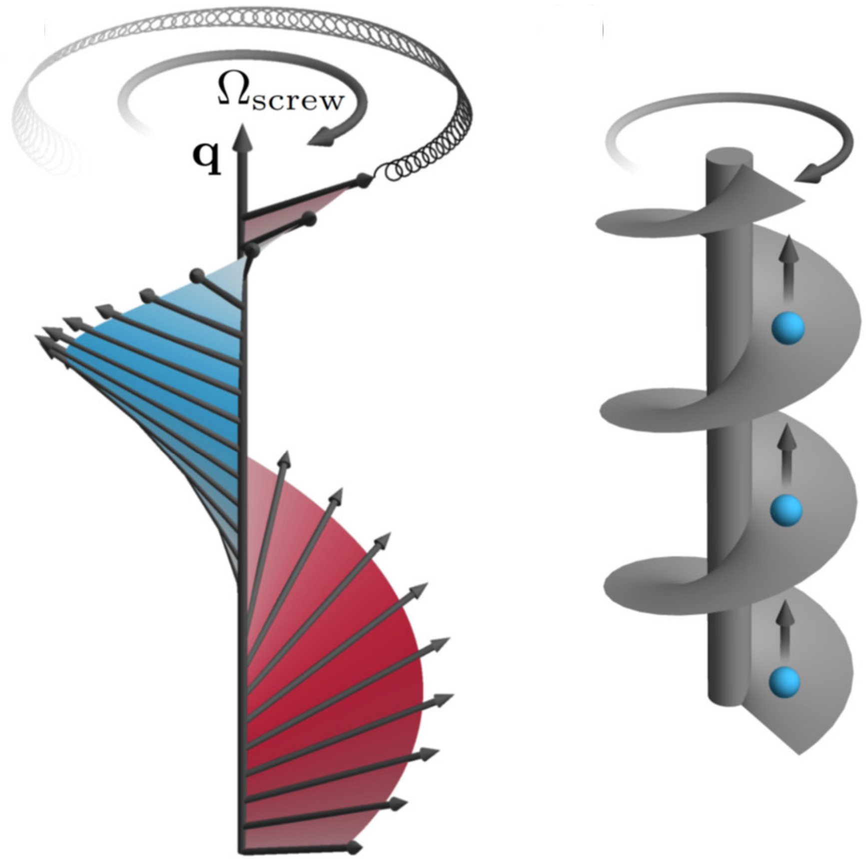


wikipedia

conclusions

- activation of Goldstone mode as a general principle
- drive domain walls, skyrmions, …
- screw like motion in helical magnets realizes Archimedean screw
- efficient pumping of charge, spin, heat,…
- additional generic instability of driven system:
  resonant magnon emission
  magnon laser may form (time quasi crystal)

- promising for experimental observations



Nina del Ser, Lukas Heinen, Achim Rosch, SciPost Phys. 11, 009 (2021)