Challenging energy-speed limits in antiferromagnets

Johan H. Mentink

Institute for Molecules and Materials

Radboud University

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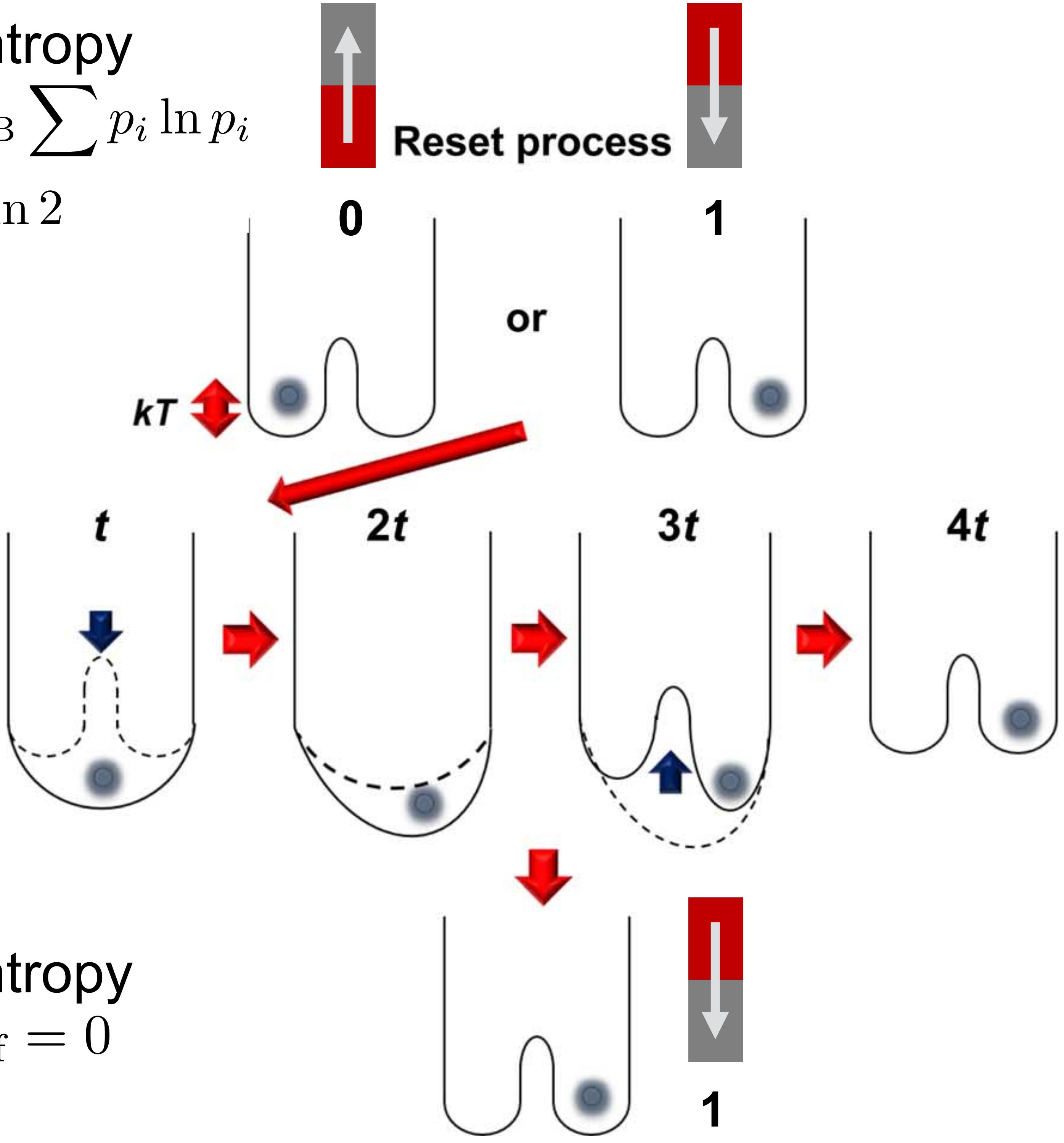
Thermodynamics of computation

Entropy
$$S_{\mathrm{i}} = -k_{\mathrm{B}} \sum p_i \ln p_i$$
$$= k_{\mathrm{B}} \ln 2$$



Entropy
$$S_{\mathrm{f}} = 0$$

$$\Delta E_{\mathrm{int}} = Q + W = 0$$

$$W_{\mathrm{min}} = -Q = -T\Delta S$$

Landauer limit

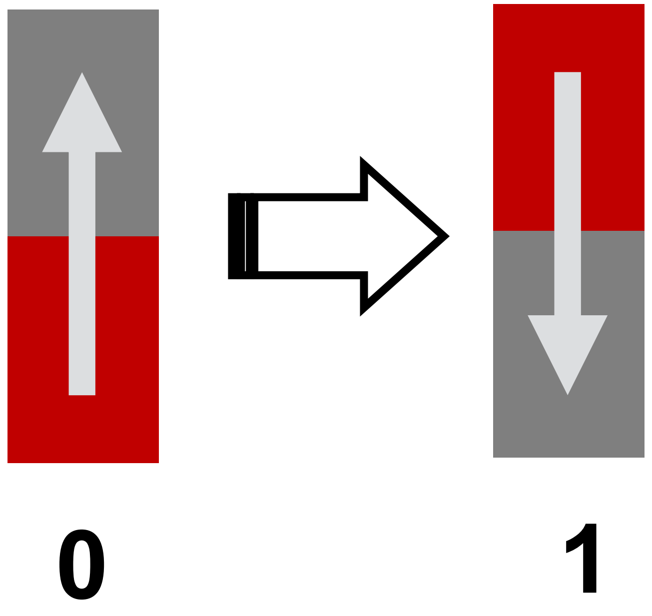
$$W_{\mathrm{min}} = k_{\mathrm{B}} T \ln 2$$

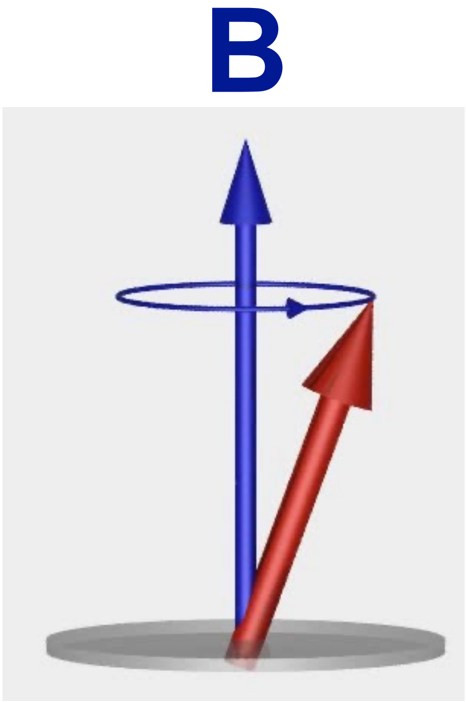
~ zJ = 10^{-21} Joule

Hong *et al.*, Sci. Adv. (2016)

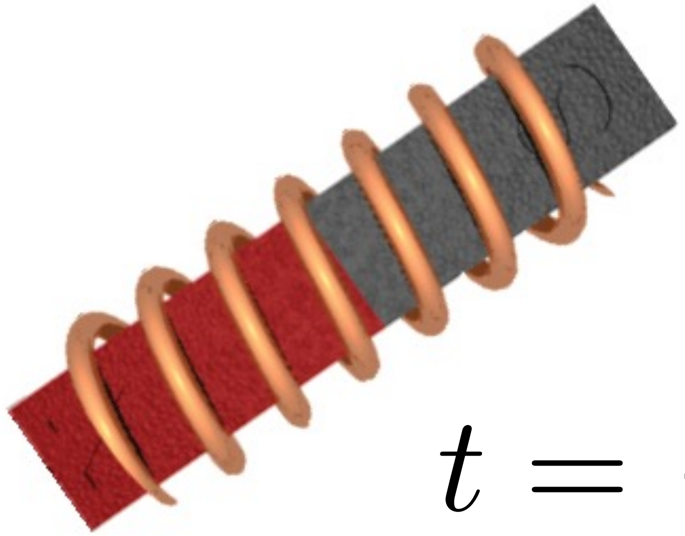
Landauer, IBM J Res Dev (1961)

Energy–time dilemma





$\omega = \gamma B$



$$t = \frac{\pi}{\gamma B}$$

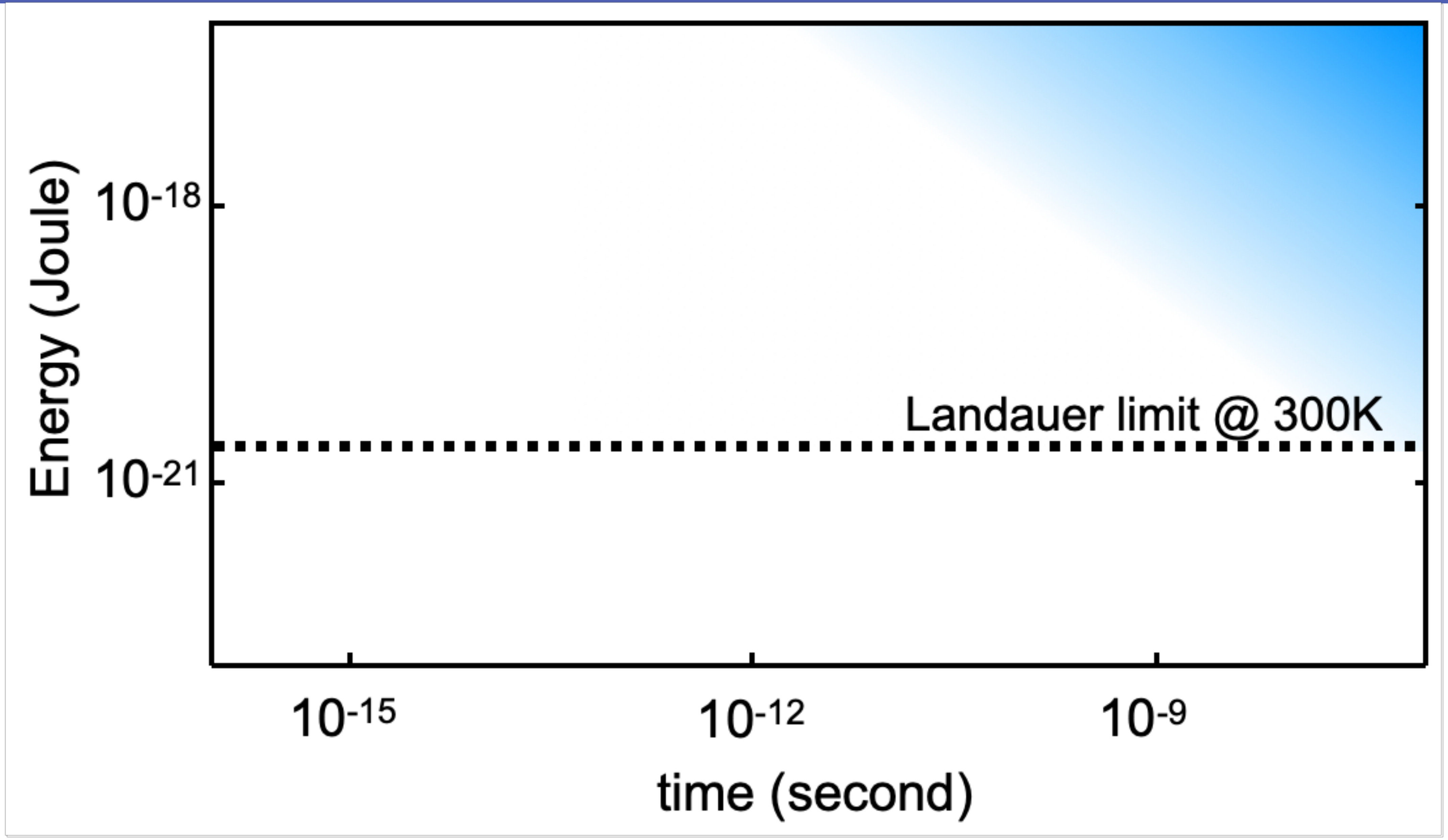
Quantum speed limit

$$t \geq h/4E$$

$$\gamma = 2\frac{\mu_B}{\hbar} \qquad E = \mu_B B$$

S. Rijmer, B.Sc. thesis (2019)
Margolus and Levitin, Phys D. 120, 188 (1998)

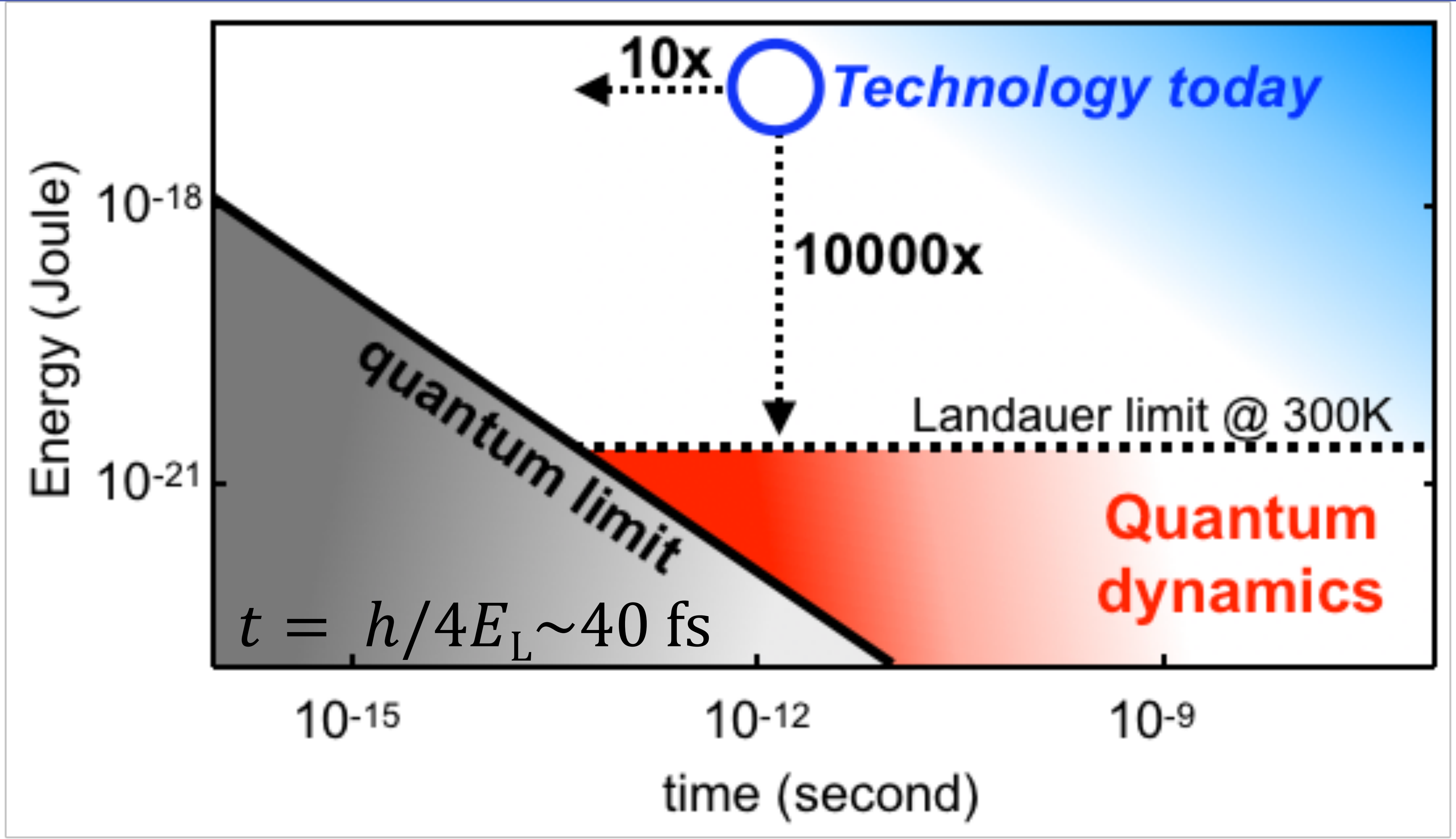
Physical laws



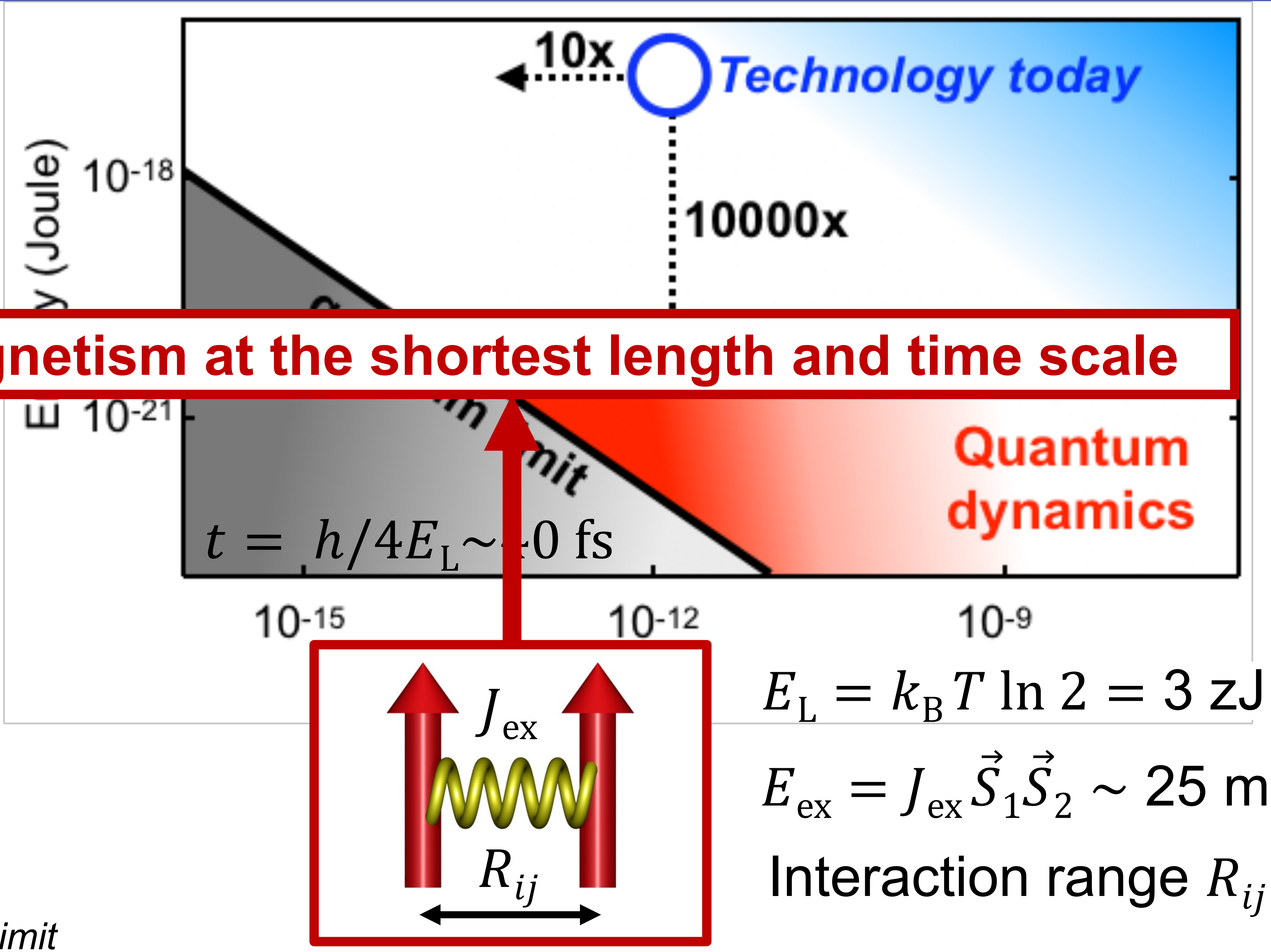
Quantum speed limit
Margolus and Levitin, Phys D. 120, 188 (1998)

Landauer, IBM J Res Dev (1961)

Physical laws



Physical laws



Magnetism at the shortest length and time scale

$E_L = k_B T \ln 2 = 3$ zJ = 25 meV

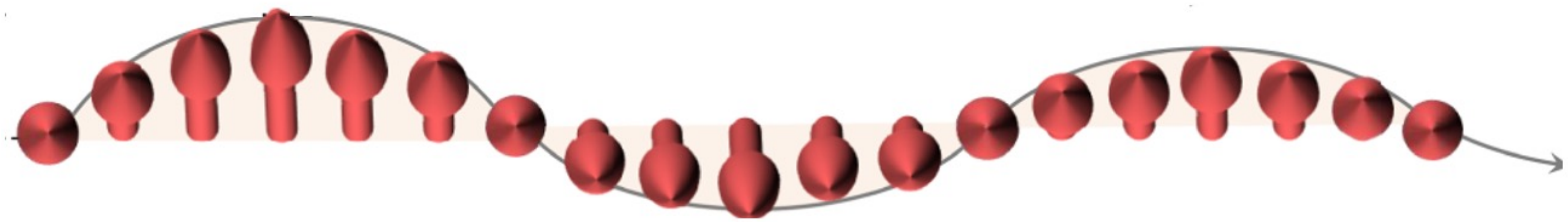
$E_{ex} = J_{ex} \vec{S}_1 \vec{S}_2 \sim 25$ meV

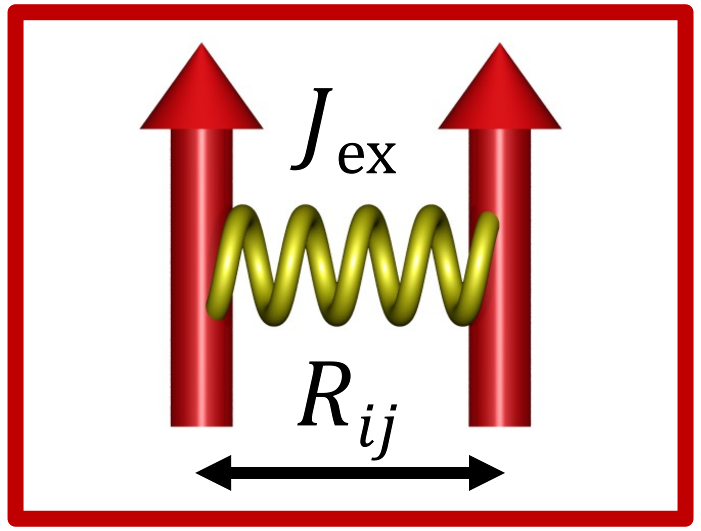
Interaction range $R_{ij} \sim$ nm

Quantum speed limit
Margolus and Levitin, Phys D. 120, 188 (1998)

Landauer, IBM J Res Dev (1961)

The smallest and fastest magnetic waves





Wavelength $\lambda \rightarrow Ri_j \sim 5\text{Å}$

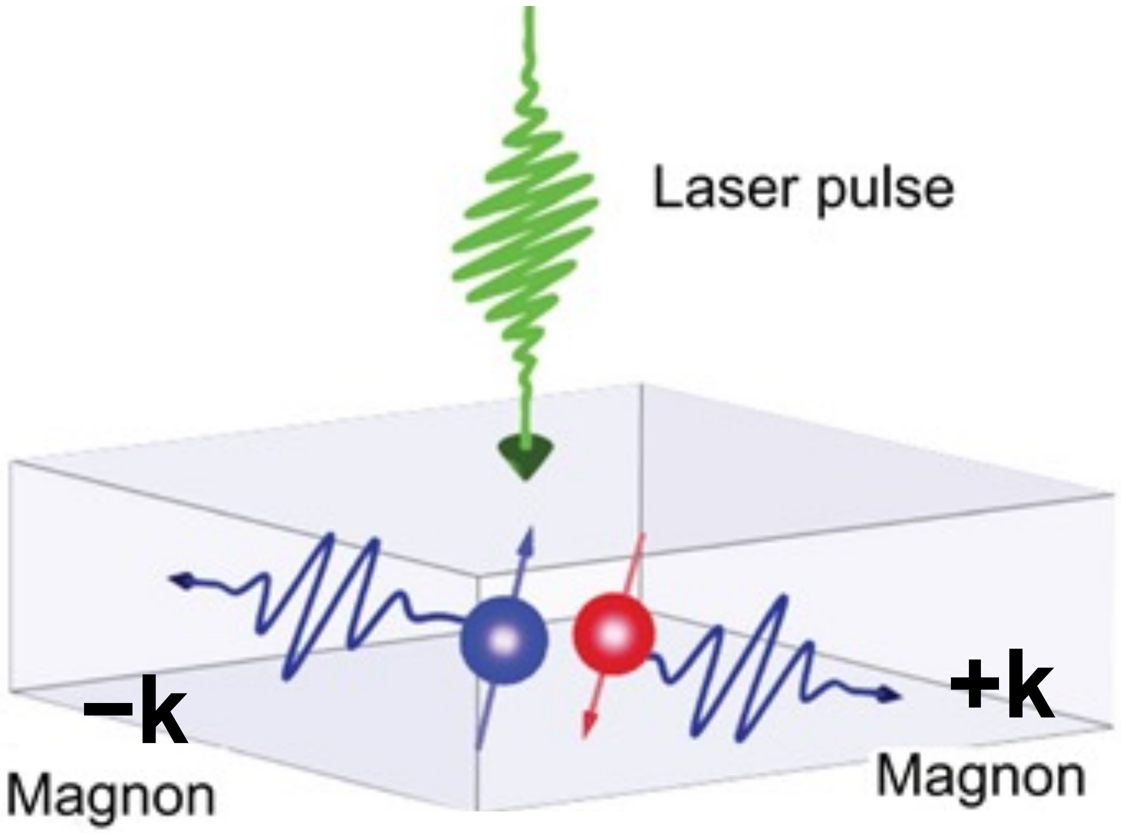
$\hbar\omega \rightarrow E_{\text{ex}} = J_{\text{ex}}\vec{S}_1\vec{S}_2 \sim$ 25 meV

Coherent: almost no dissipation

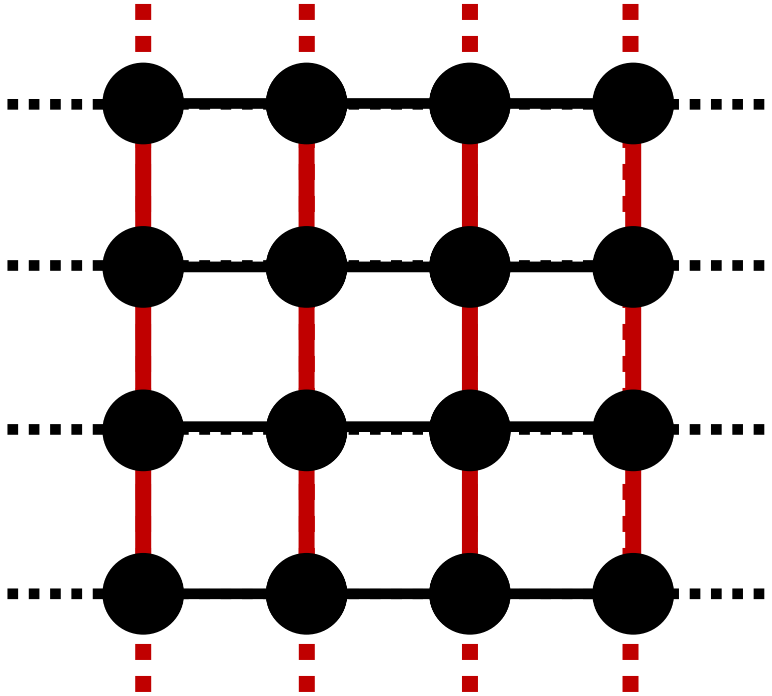
Excitation of the smallest and fastest magnons

$$\hat{H} = J_{\text{ex}} \sum_{i,\delta} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_{i+\delta}$$

$$\Delta\hat{H}(t) = \Delta J_{\text{ex}}\, f(t) \sum_{i,\delta} (\hat{e}\cdot\vec{\delta})^2 \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_{i+\delta}$$



$\Delta\mathbf{k}_{\text{tot}}$=0, ΔS_{tot}=0



$$\hat{H} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \left(\hat{\alpha}_{\mathbf{k}}^\dagger \hat{\alpha}_{\mathbf{k}} + \hat{\alpha}_{-\mathbf{k}}^\dagger \hat{\alpha}_{-\mathbf{k}} + 1\right)$$

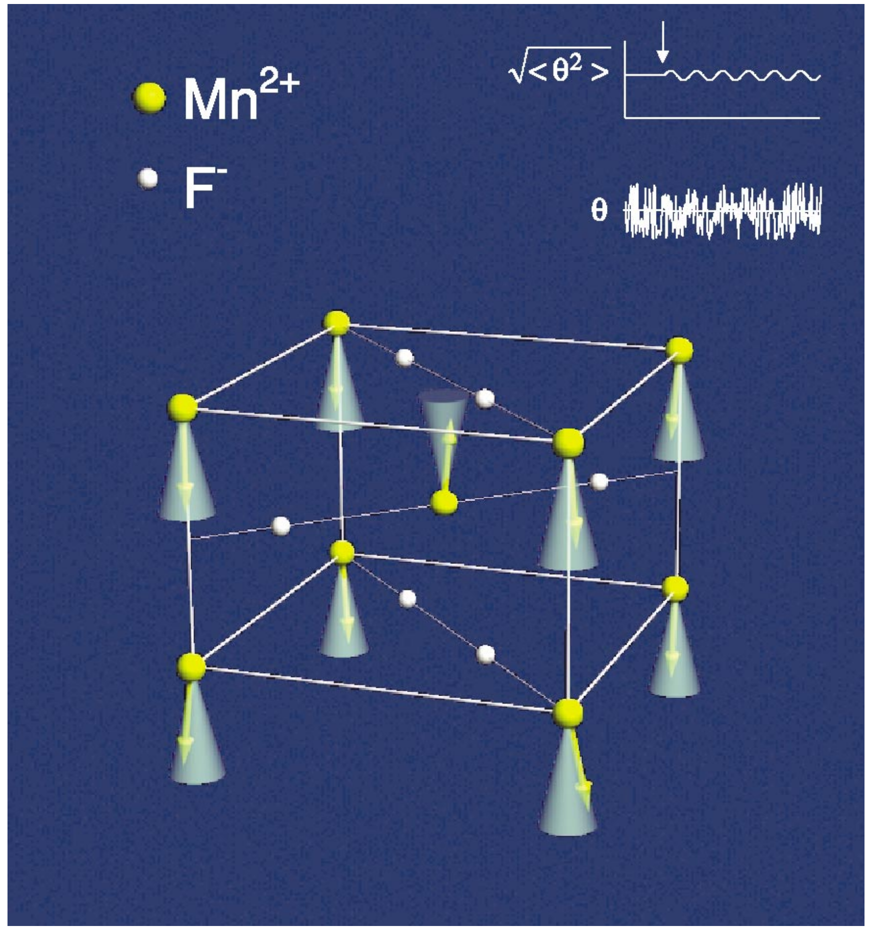
$$\Delta\hat{H}(t) = f(t) \sum_{\mathbf{k}} V_{\mathbf{k}} \left(\hat{\alpha}_{\mathbf{k}}^\dagger \hat{\alpha}_{-\mathbf{k}}^\dagger + \hat{\alpha}_{\mathbf{k}} \hat{\alpha}_{-\mathbf{k}}\right)$$

$V_{\mathbf{k}}$ dominant at $\mathbf{k} = (0, \pm\pi)$

Spontaneous RS: Fleury and Loudon, Phys Rev. 1968
Ultrafast: Mentink et al., Nat. Commun. 2015

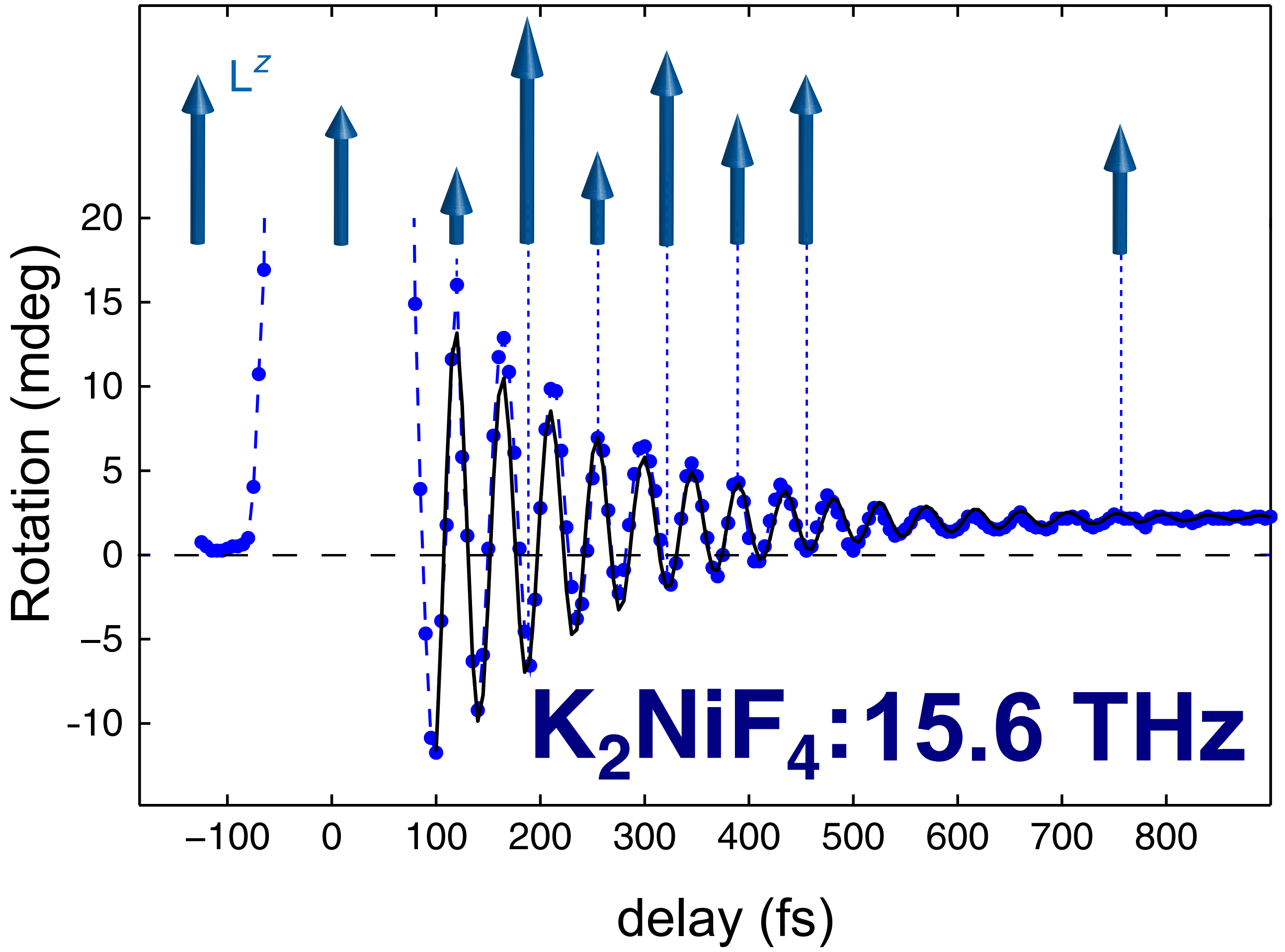
J. Zhao et al., PRL 93, 107203 (2004)
Bossini et al., Nat. Commun. 7, 10645 (2016)
D. Bossini, .. ,O. Gomonay, …, J.H. Mentink, .. et al., PRB 100, 024428 (2019)

Time-resolved dynamics of the smallest and fastest magnons



Magnon squeezing

Zhao, Merlin et al.,
PRL 2004*, PRB* 2006



Coherent *longitudinal* dynamics
Magnon entanglement

D. Bossini et al.,
Nat. Commun. (2016)*, PRB* (2019)

Outline

1. Magnon-pair physics
   J. Zhao et al., PRL **93,** 107203 (2004);
   D. Bossini et al, Nat. Commun. **7**, 10645 (2016);
   D. Bossini, .. , O. Gomonay, .., J.H. Mentink et al., PRB **100**, 024428 (2019)

2. Supermagnonic propagation of magnon pairs
   G. Fabiani, M.D. Bouman, J.H. Mentink
   Phys. Rev. Lett. 127, 097202 (2021)

3. Semi-classical approach to nonlinear magnon-pair dynamics
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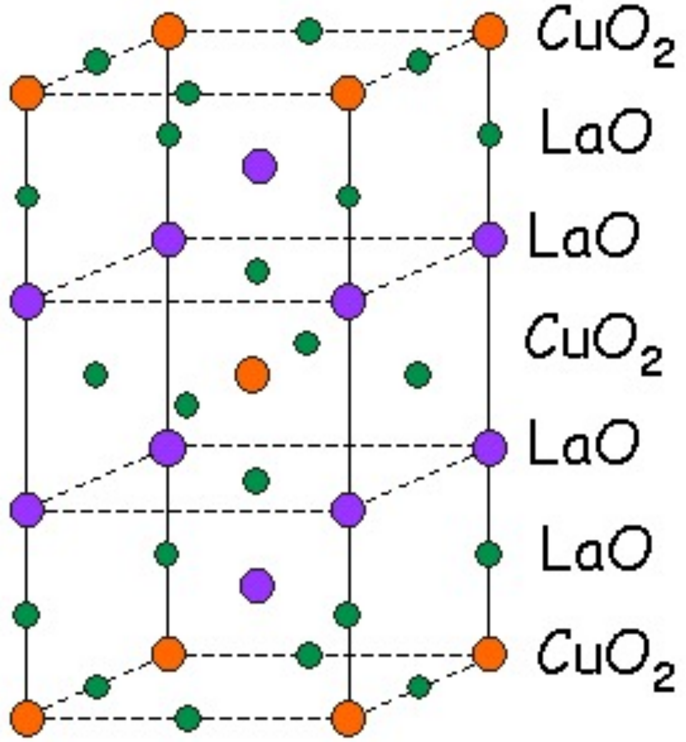
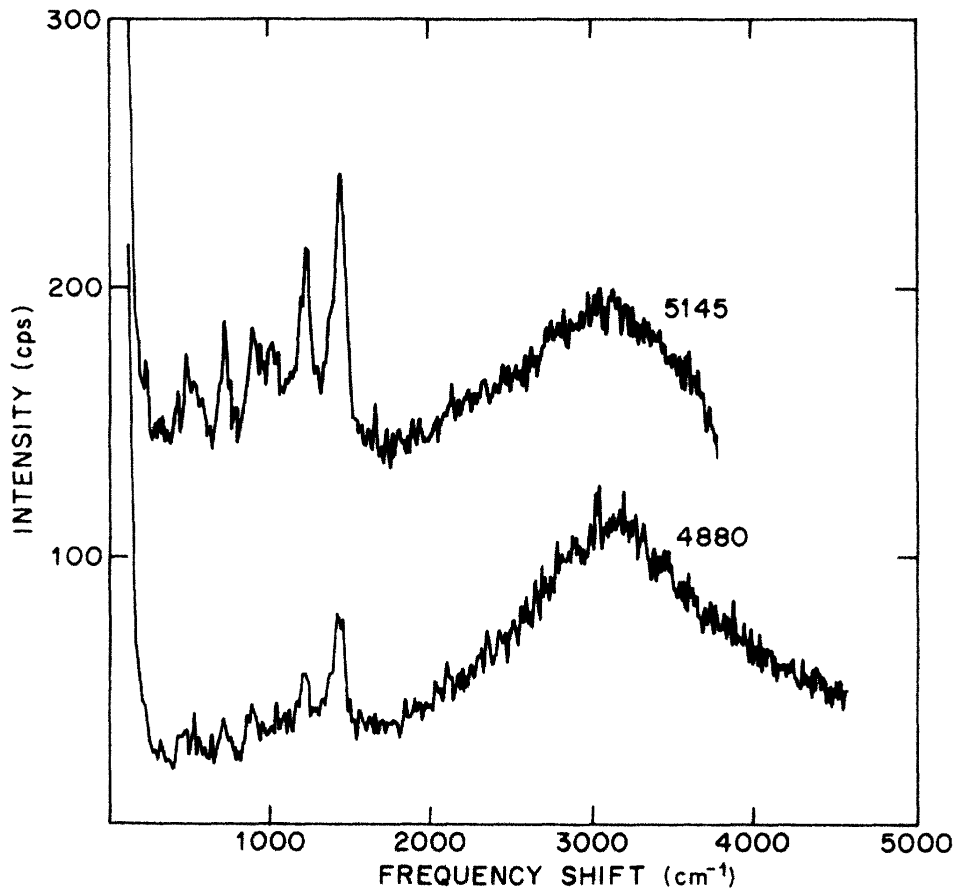
   Appl. Phys. Lett. 120, 152402 (2022)

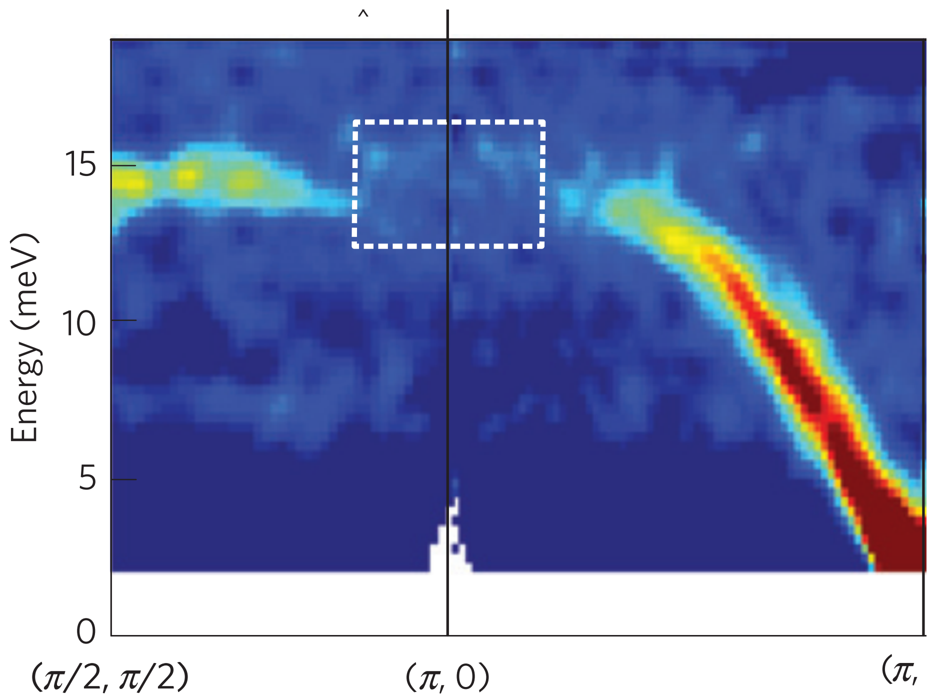
Challenges at the Edge of the Brillouin Zone

parent compounds
cuprates
(high-Tc, Nobelprize 1987)
La_2CuO_4



metal-organic compound
$Cu(DCOO)_2 \cdot 4D_2O$ (CFTD)





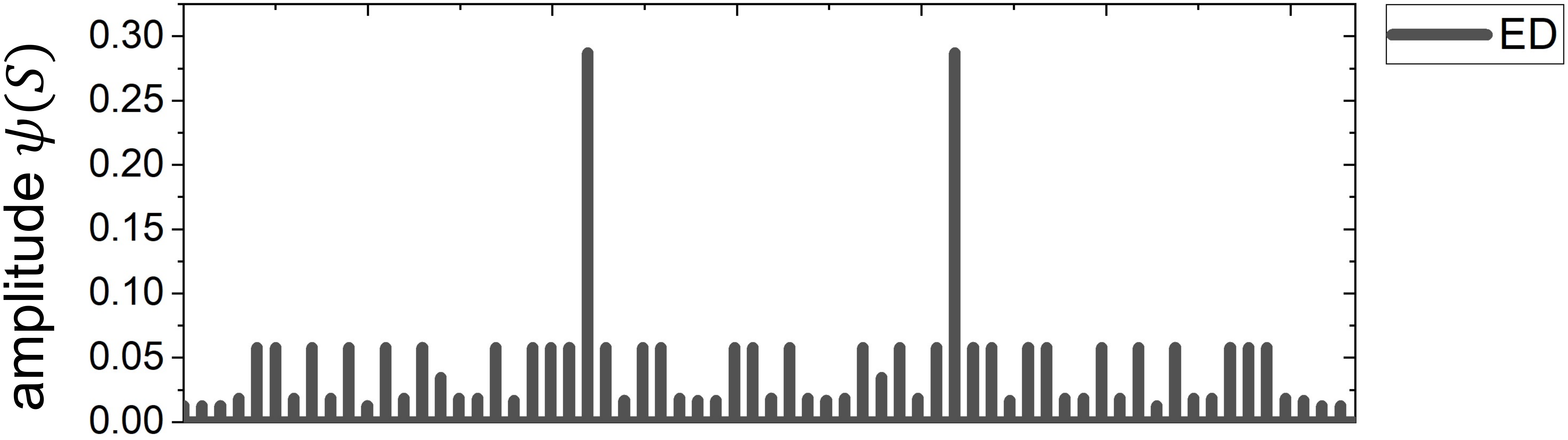
S=1/2 Heisenberg antiferromagnet in 2D

$$\hat{H} = J_{\mathrm{ex}} \sum_{i,\delta} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_{i+\delta}$$

Lyons et al., PRB **37**, 2353 (1988)

B. Dalla Piazza et al., Nature Physics **11**, 62 (2015)
H. Shao, et al., PRX **7** 041072 (2017)

The many-body wavefunction



All possible quantum states $|S_1\rangle \ldots |S_{2^N}\rangle$

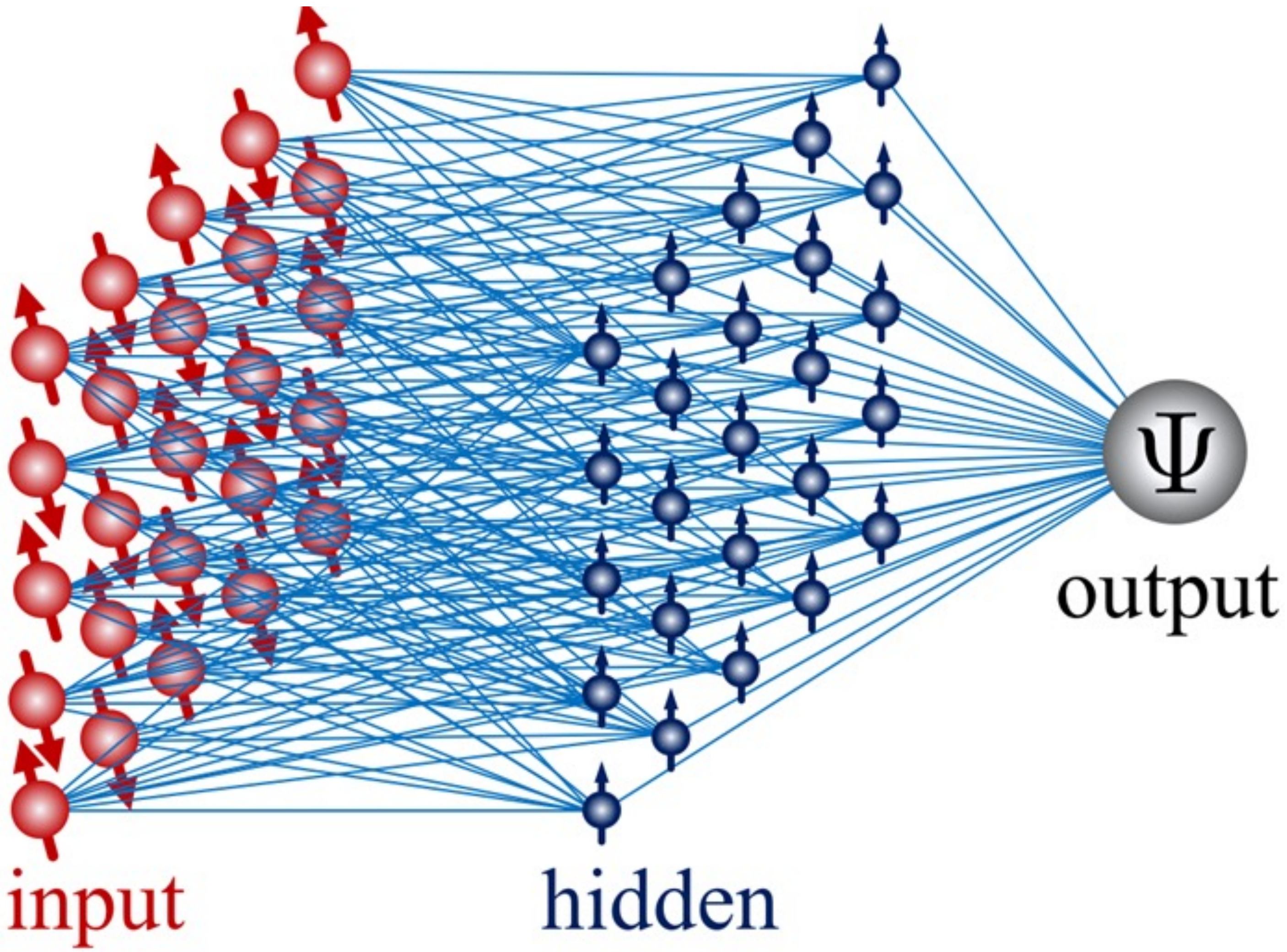
2^N states

$|S\rangle = |\uparrow\downarrow\uparrow \cdots \uparrow\rangle$

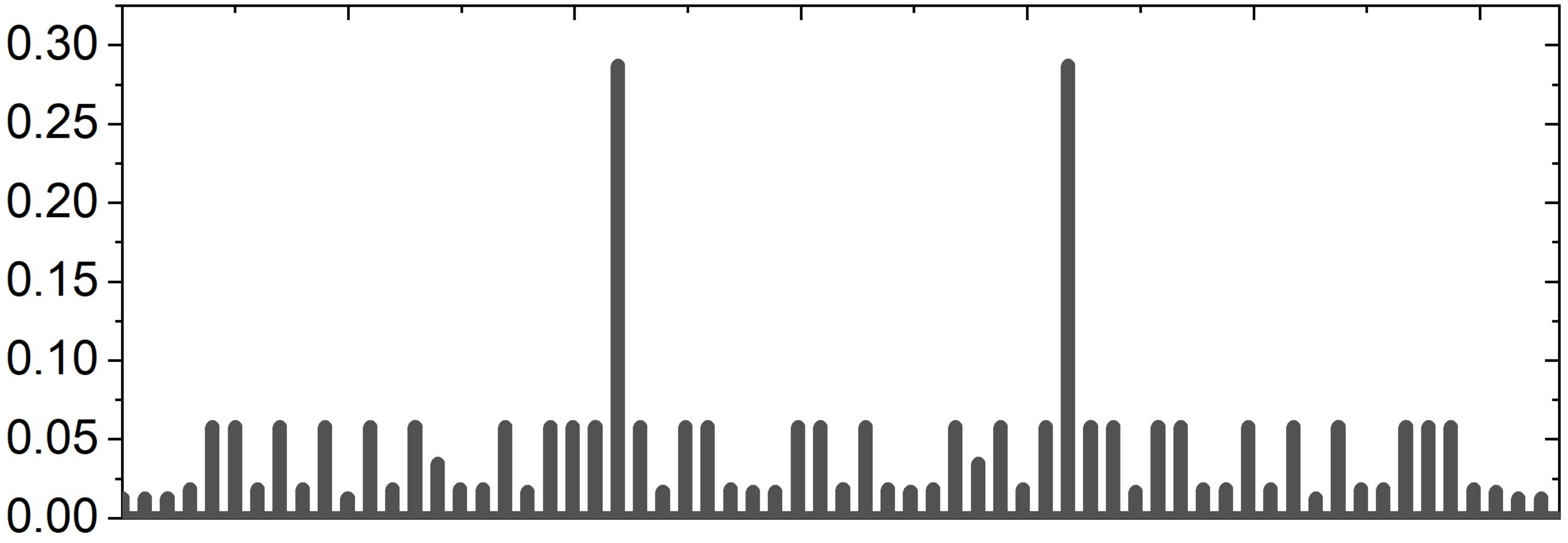
N spins

4x4 Heisenberg model

Wave function as artificial neural network



$|S\rangle = |\uparrow\downarrow\uparrow \cdots \uparrow\rangle$

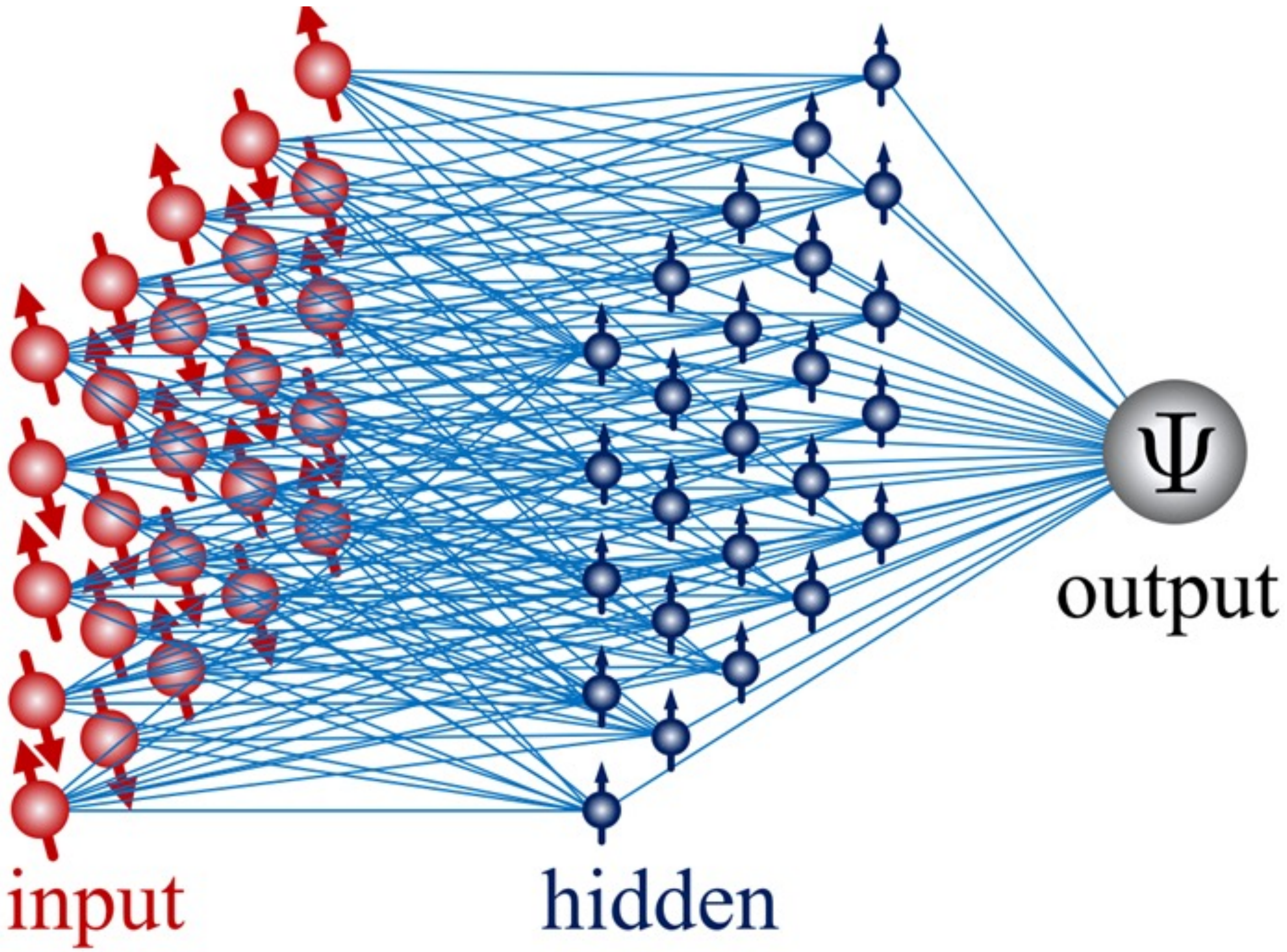


All possible quantum states $|S_1\rangle \dots |S_{2^N}\rangle$

- Universal function approximation theorem
- Reduction from 2^N to αN parameters
- Much reduced limits on simulation time / system size

G. Carleo, M. Troyer
Science 355, 602 (2017)

Wave function as artificial neural network



$|S\rangle = |\uparrow\downarrow\uparrow \cdots \uparrow\rangle$

G. Carleo, M. Troyer
Science 355, 602 (2017)

Neural-network quantum states (NQS)

$$\psi_{\mathcal{W}}(S) = \sum_{\{h_i\}} e^{\sum_j a_j s_j^z + \sum_i b_i h_i + \sum_{ij} w_{ij} s_i^z h_j}$$

Restricted Boltzmann Machine

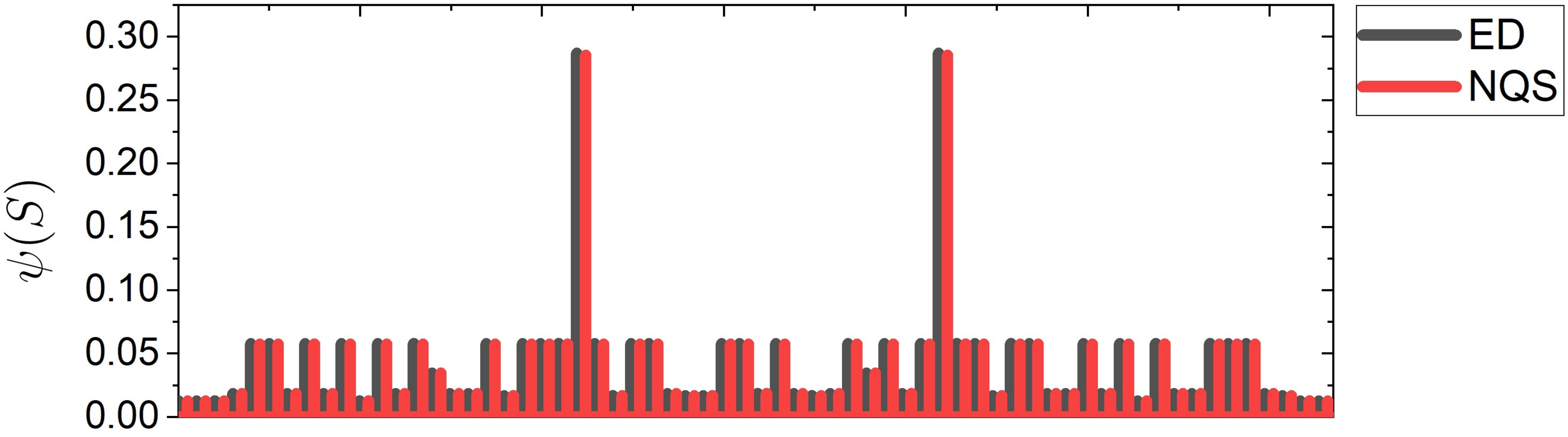
Optimization

Ground state: minimize $\left\| (\hat{\mathcal{H}} - E)\psi_{\mathcal{W}} \right\|$

Dynamics: minimize $\left\| \mathrm{i}\partial_t \psi_{\mathcal{W}}(t) - \hat{\mathcal{H}}(t)\, \psi_{\mathcal{W}}(t) \right\|$

"unsupervised" learning from samples

Ground state NQS vs Exact Diagonalization (ED)



Network with $M = \alpha N = 64$ parameters already gives accurate results

Ground state N=4x4 Heisenberg model

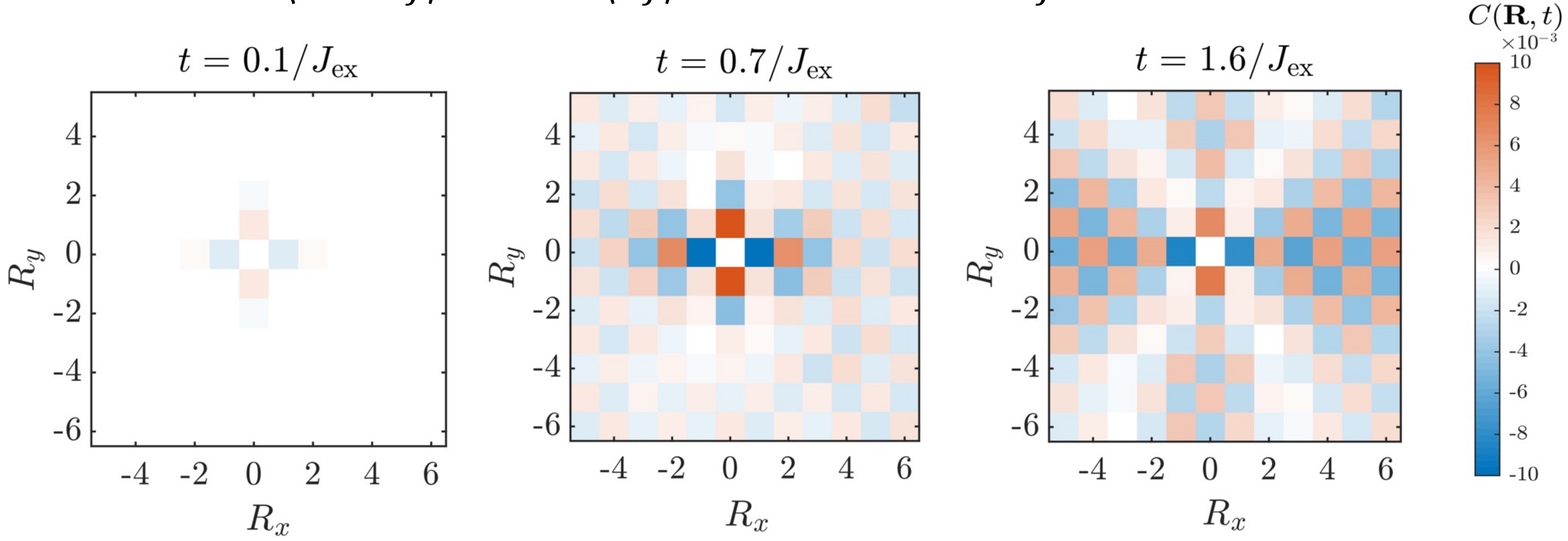
G. Fabiani, JHM, SciPost. Phys. **7**, 004 (2019)

Propagation of fastest magnetic waves

$\Delta J_{\mathrm{ex}} f(t)$

$$\Delta \hat{H}(t) = \Delta J_{\mathrm{ex}}\, f(t) \sum_{i,\delta} (\hat{e} \cdot \vec{\delta})^2\, \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_{i+\delta}$$

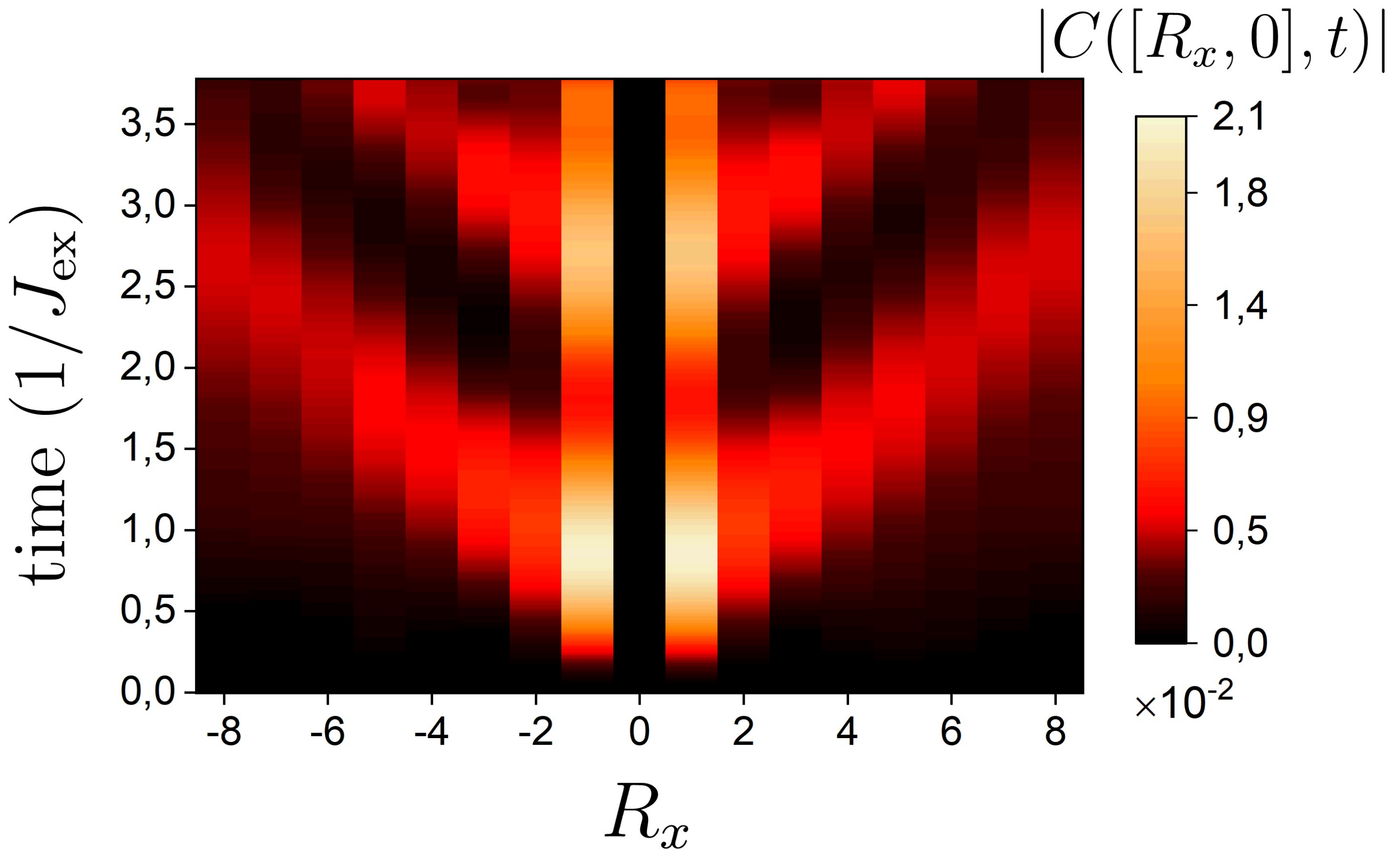
$$C(\boldsymbol{R}, t) = \langle \boldsymbol{S}_i \cdot \boldsymbol{S}_j \rangle - \langle \boldsymbol{S}_i \rangle \langle \boldsymbol{S}_j \rangle, \qquad \boldsymbol{R} = \boldsymbol{r}_i - \boldsymbol{r}_j$$

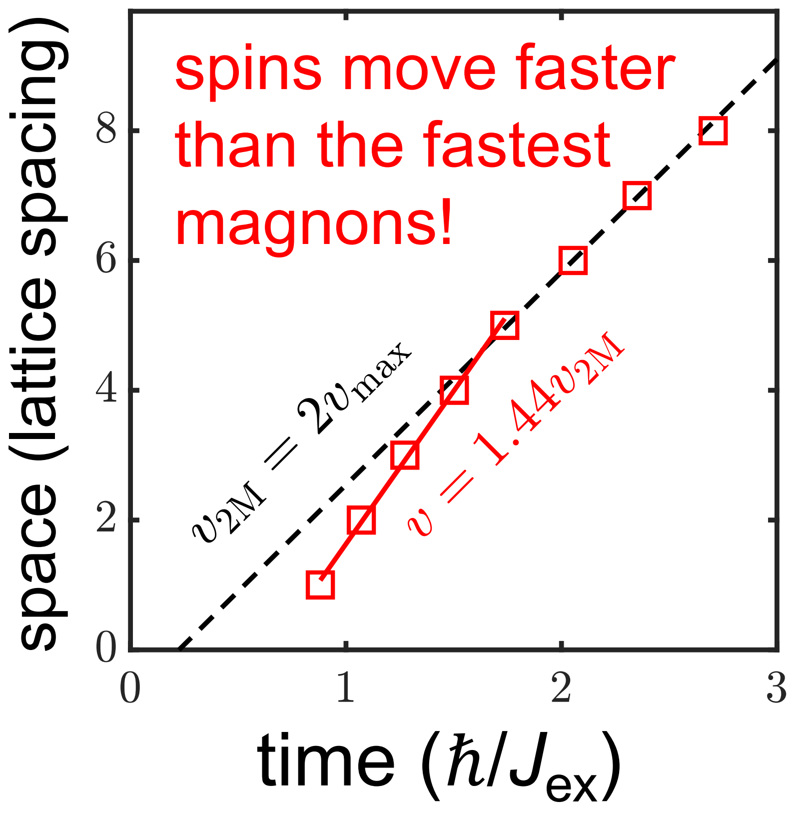


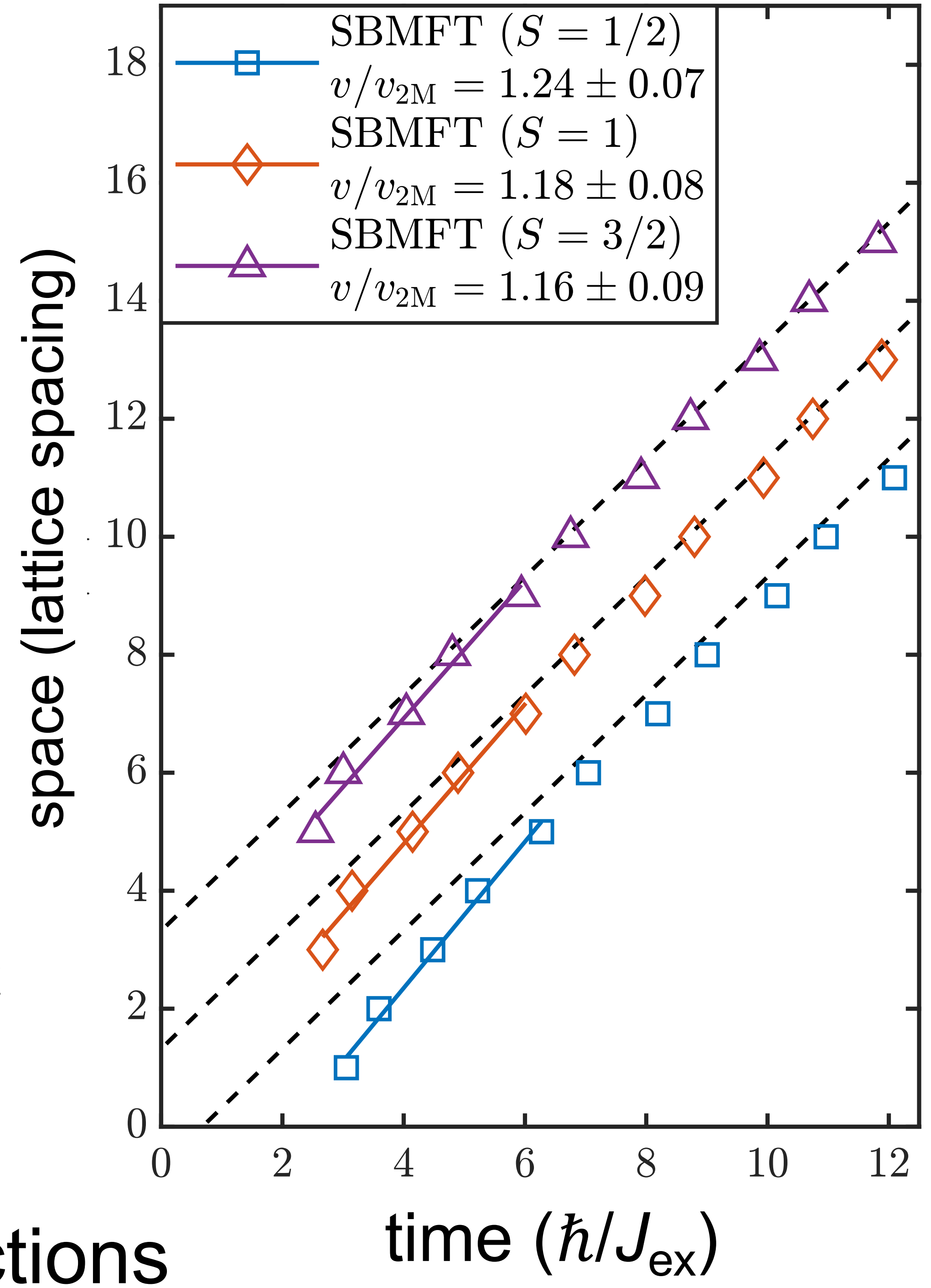
Anisotropy of propagation determined by symmetry of light-matter interaction

G. Fabiani, M.D. Bouman, JHM, Phys. Rev. Lett. 127, 097202 (2021)

Supermagnonic propagation







For small R_x *faster*

$v(\mathrm{NQS}) \approx 4.71\, aJ_{\mathrm{ex}}$ **40% higher** than $v(\mathrm{LSWT}) \approx 3.28\, aJ_{\mathrm{ex}}$

$\approx 20\ \mathrm{km/s}$ for $J_{\mathrm{ex}} \approx 6$ meV, $a \approx 5$Å

Consequence of exceptionally strong magnon-magnon interactions

G. Fabiani, M.D. Bouman, JHM, Phys. Rev. Lett. 127, 097202 (2021)

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   Appl. Phys. Lett. 120, 152402 (2022)

Magnon-pair operator algebra

$$\hat{\mathcal{H}} = \sum_{\boldsymbol{k}} \omega_{\boldsymbol{k}} \underbrace{\left(\hat{\alpha}^\dagger_{\boldsymbol{k}}\hat{\alpha}_{\boldsymbol{k}} + \hat{\beta}^\dagger_{-\boldsymbol{k}}\hat{\beta}_{-\boldsymbol{k}} + 1\right)}_{2\hat{K}^z_{\boldsymbol{k}}} + f(t)\sum_{k} V_{\boldsymbol{k}}\left(\underbrace{\hat{\alpha}^\dagger_{\boldsymbol{k}}\hat{\beta}^\dagger_{-\boldsymbol{k}}}_{\hat{K}^+_{\boldsymbol{k}}} + \underbrace{\hat{\alpha}_{\boldsymbol{k}}\hat{\beta}_{-\boldsymbol{k}}}_{\hat{K}^-_{\boldsymbol{k}}}\right)$$

SU(2)

$$[\hat{S}^z_i, \hat{S}^\pm_j] = \pm\hat{S}^\pm_i\,\delta_{i,j}$$

$$[\hat{S}^-_i, \hat{S}^+_j] = -2\hat{S}^z_i\,\delta_{i,j}$$

$$\hat{S}^2_i = \left(\hat{S}^z_i\right)^2 + \frac{1}{2}\left(\hat{S}^-_i\hat{S}^+_{\boldsymbol{i}} + \hat{S}^+_i\hat{S}^-_i\right) = \text{const}$$

$$\left(\hat{S}^z_i\right)^2 + \left(\hat{S}^x_i\right)^2 + \left(\hat{S}^y_i\right)^2$$

SU(1,1)

$$[\hat{K}^z_{\boldsymbol{k}}, \hat{K}^\pm_{\boldsymbol{k'}}] = \pm\hat{K}^\pm_{\boldsymbol{k}}\,\delta_{\boldsymbol{k},\boldsymbol{k'}}$$

$$[\hat{K}^-_{\boldsymbol{k}}, \hat{K}^+_{\boldsymbol{k'}}] = 2\hat{K}^z_{\boldsymbol{k}}\,\delta_{\boldsymbol{k},\boldsymbol{k'}}$$

$$\hat{K}^2_{\boldsymbol{k}} = -\left(\hat{K}^z_{\boldsymbol{k}}\right)^2 + \frac{1}{2}\left(\hat{K}^-_{\boldsymbol{k}}\hat{K}^+_{\boldsymbol{k}} + \hat{K}^+_{\boldsymbol{k}}\hat{K}^-_{\boldsymbol{k}}\right) = \text{const}$$

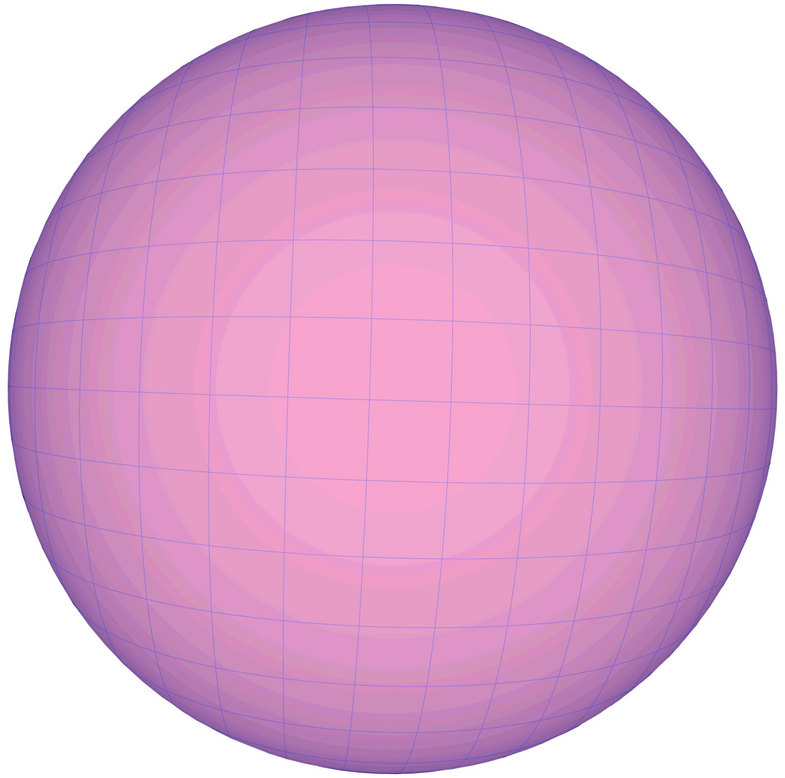
$$-\left(\hat{K}^z_{\boldsymbol{k}}\right)^2 + \left(\hat{K}^x_{\boldsymbol{k}}\right)^2 + \left(\hat{K}^y_{\boldsymbol{k}}\right)^2$$

D. Bossini, .. ,O. Gomonay, …, J.H. Mentink, .. et al., PRB 100, 024428 (2019)

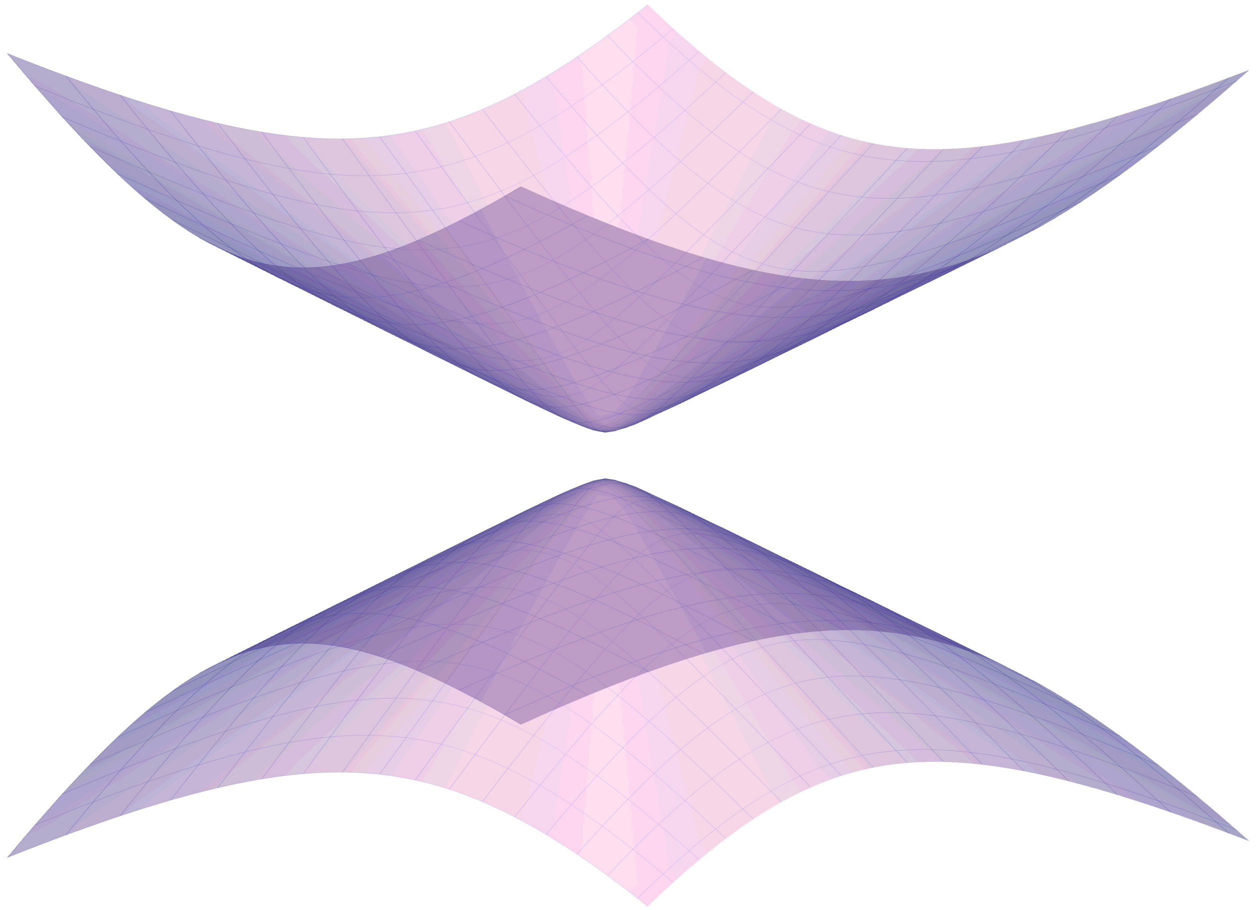
Magnon-pair operator algebra

$$\hat{\mathcal{H}} = \sum_{\boldsymbol{k}} \omega_{\boldsymbol{k}} \underbrace{\left(\hat{\alpha}^\dagger_{\boldsymbol{k}}\hat{\alpha}_{\boldsymbol{k}} + \hat{\beta}^\dagger_{-\boldsymbol{k}}\hat{\beta}_{-\boldsymbol{k}} + 1\right)}_{2\hat{K}^z_{\boldsymbol{k}}} + f(t)\sum_{k} V_{\boldsymbol{k}}\left(\underbrace{\hat{\alpha}^\dagger_{\boldsymbol{k}}\hat{\beta}^\dagger_{-\boldsymbol{k}}}_{\hat{K}^+_{\boldsymbol{k}}} + \underbrace{\hat{\alpha}_{\boldsymbol{k}}\hat{\beta}_{-\boldsymbol{k}}}_{\hat{K}^-_{\boldsymbol{k}}}\right)$$

SU(2)



SU(1,1)



D. Bossini, .. ,O. Gomonay, …, J.H. Mentink, .. et al., PRB 100, 024428 (2019)

Quantum to semi-classical

$$\hat{\mathcal{H}} = \sum_{\boldsymbol{k}} 2\omega_{\boldsymbol{k}} \hat{K}_{\boldsymbol{k}}^z + f(t) \sum_k V_{\boldsymbol{k}} \left(\hat{K}_{\boldsymbol{k}}^+ + \hat{K}_{\boldsymbol{k}}^- \right)$$

$$H_{\text{cl}} = \langle \hat{\mathcal{H}} \rangle = 2 \sum_{\boldsymbol{k}} [\omega_{\boldsymbol{k}} \mathcal{J}_{\boldsymbol{k}}^z + f(t) V_{\boldsymbol{k}}\, \mathcal{J}_{\boldsymbol{k}}^x]$$

Heisenberg equations

$$\frac{d\langle \hat{K}_{\boldsymbol{k}}^z \rangle}{dt} = i f(t) V_{\boldsymbol{k}} \left(\langle \hat{K}_{\boldsymbol{k}}^- \rangle - \langle \hat{K}_{\boldsymbol{k}}^+ \rangle \right)$$

$$\frac{d\langle \hat{K}_{\boldsymbol{k}}^\pm \rangle}{dt} = 2i\omega_{\boldsymbol{k}} \langle \hat{K}_{\boldsymbol{k}}^\pm \rangle \pm 2i f(t) V_{\boldsymbol{k}} \langle \hat{K}_{\boldsymbol{k}}^z \rangle$$

Dynamics on pseudo-sphere

$$\frac{d\boldsymbol{\mathcal{J}}_{\boldsymbol{k}}}{dt} = -\boldsymbol{\mathcal{J}}_{\boldsymbol{k}} \times \boldsymbol{B}_{\boldsymbol{k}}$$

$$\boldsymbol{B}_{\boldsymbol{k}} = -\frac{dH_{\text{cl}}}{d\boldsymbol{\mathcal{J}}_{\boldsymbol{k}}} = -2\left(f(t) V_{\boldsymbol{k}}, 0, -\omega_{\boldsymbol{k}} \right)$$

Pseudo-spin vector

$$\boldsymbol{\mathcal{J}}_{\boldsymbol{k}} = \left(\langle \hat{K}_{\boldsymbol{k}}^x \rangle, \langle \hat{K}_{\boldsymbol{k}}^y \rangle, \langle \hat{K}_{\boldsymbol{k}}^z \rangle \right)$$

$$(\boldsymbol{a} \times \boldsymbol{b})^x = a^y b^z - a^z b^y$$

$$(\boldsymbol{a} \times \boldsymbol{b})^y = a^z b^x - a^x b^z$$

$$(\boldsymbol{a} \times \boldsymbol{b})^z = -(a^x b^y - a^y b^x)$$

G. Fabian and JHM Appl. Phys. Lett. 120, 152402 (2022)

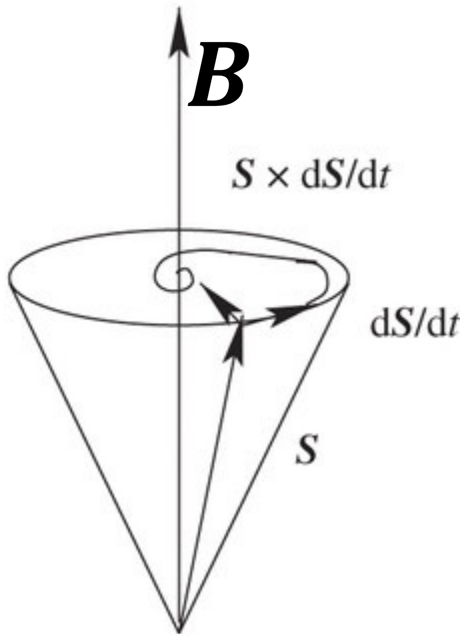
Semi-classical magnon pair dynamics

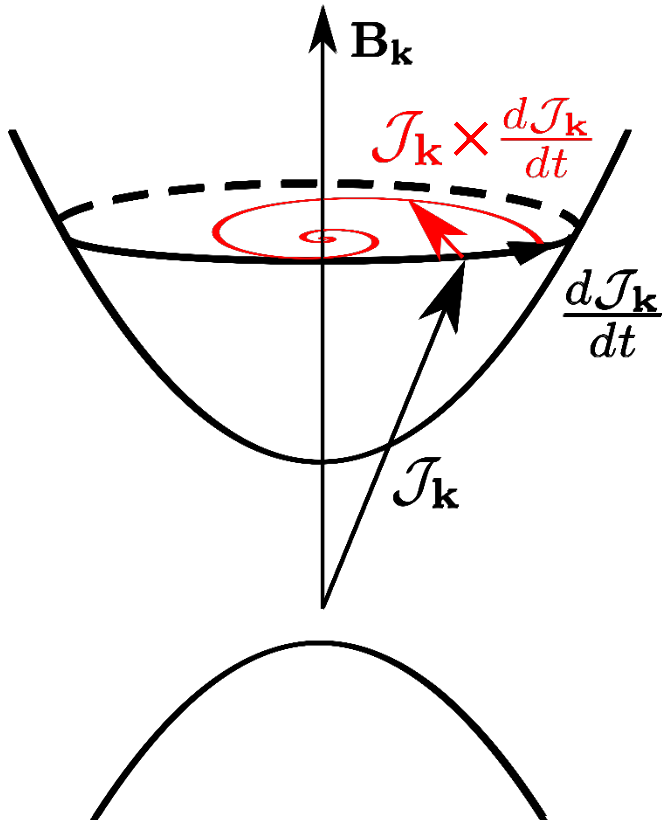
LLG equation on sphere

$$\frac{d\boldsymbol{S}}{dt} = -\gamma \boldsymbol{S}\times\boldsymbol{B} + \frac{\eta}{S}\boldsymbol{S}\times\frac{d\boldsymbol{S}}{dt}$$

"LLG" equation on pseudo-sphere

$$\frac{d\boldsymbol{\mathcal{J}}_k}{dt} = -\boldsymbol{\mathcal{J}}_k\times\boldsymbol{B}_k + \frac{\eta}{\mathcal{J}_k}\boldsymbol{\mathcal{J}}_k\times\frac{d\boldsymbol{\mathcal{J}}_k}{dt}$$



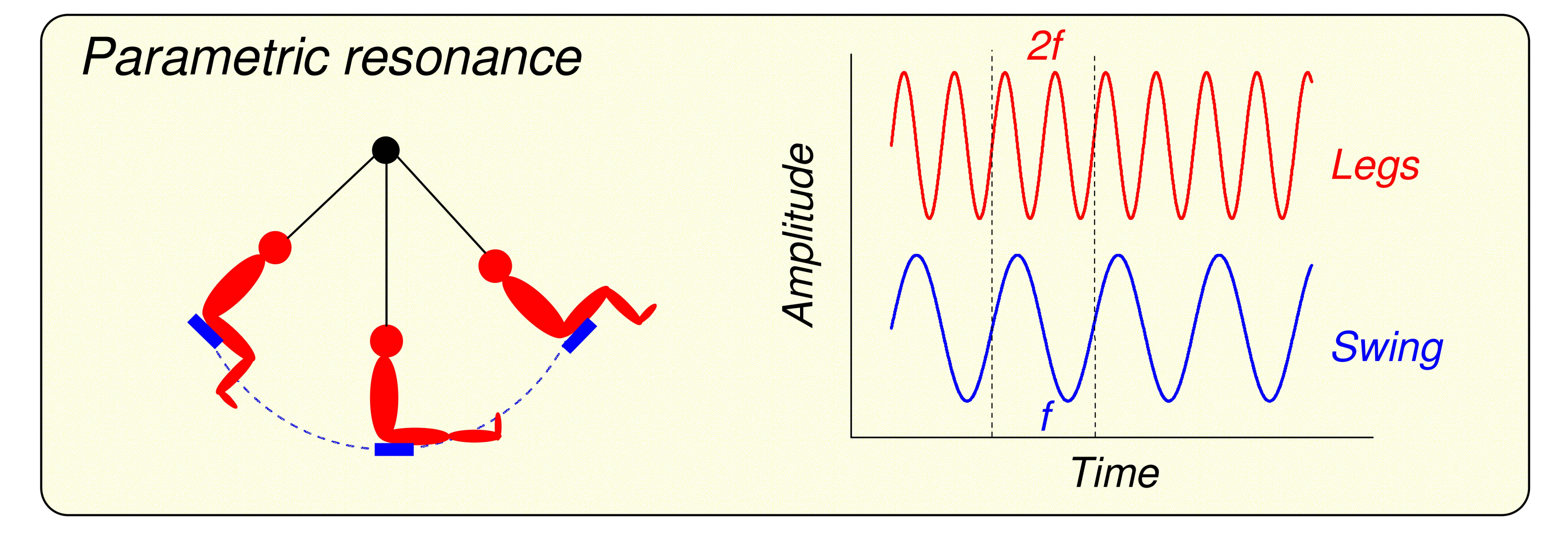


$$(\mathbf{a}\times\mathbf{b})^x = a^y b^z - a^z b^y$$
$$(\mathbf{a}\times\mathbf{b})^y = a^z b^x - a^x b^z$$
$$(\mathbf{a}\times\mathbf{b})^z = a^x b^y - a^y b^x$$

G. Fabiani and JHM, Appl. Phys. Lett. 120, 152402 (2022)

$$(\boldsymbol{a}\times\boldsymbol{b})^x = a^y b^z - a^z b^y$$
$$(\boldsymbol{a}\times\boldsymbol{b})^y = a^z b^x - a^x b^z$$
$$(\boldsymbol{a}\times\boldsymbol{b})^z = -(a^x b^y - a^y b^x)$$

Parametric oscillations

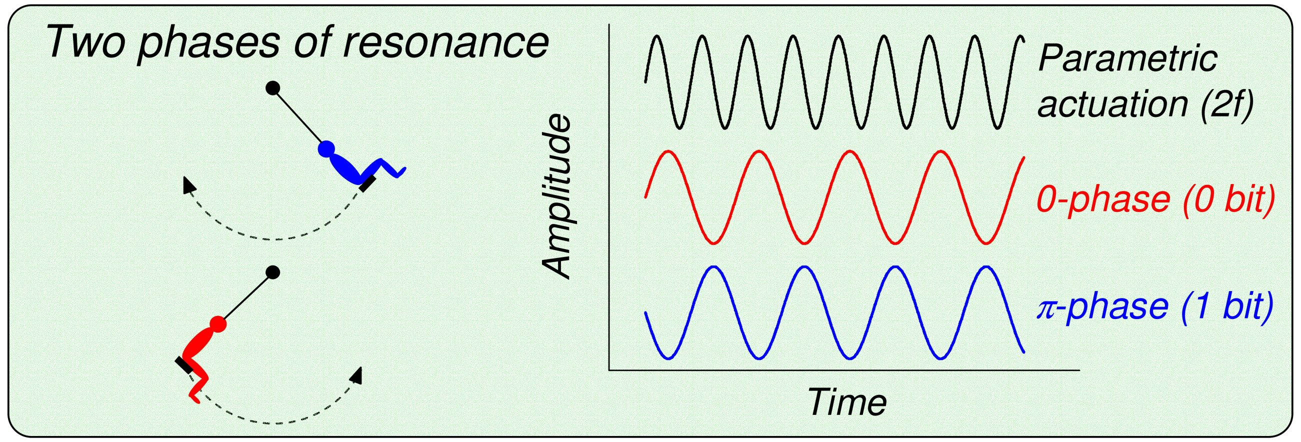


$$\left[\frac{d^2}{dt^2} + \eta\omega_0 \frac{d}{dt} + \omega_0^2(1 + \alpha + \beta x^2 - 2\Gamma \sin 2\omega_0 t)\right] x(t) = 0$$

Ansatz: $(\alpha = \beta = 0)$

$$x(t) = x_c(t)\cos\omega_0 t \; + \; x_s(t)\sin\omega_0 t$$

$$x_s = x_s(0)e^{\omega_0(\Gamma-\eta)t}$$
$$x_c = x_c(0)e^{-\omega_0(\Gamma+\eta)t}$$



Pictures adapted from I. Mahboob and H. Yamaguchi, Nature Nanotech **3,** 275 (2008)

Parametric excitation of magnon pairs

Excitation of parametric resonance by periodic modulation $f(t) = \cos(2\omega_M t)$

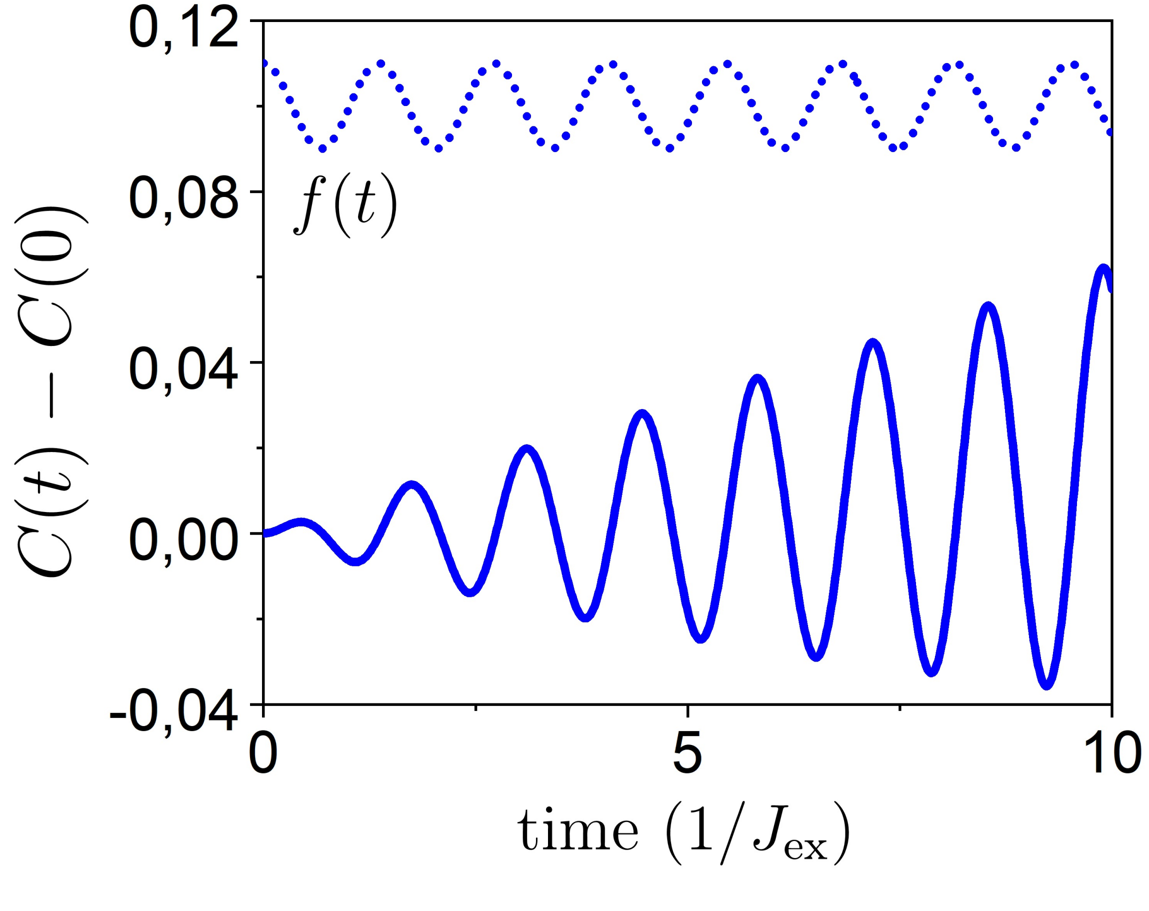
$$\frac{d\boldsymbol{\mathcal{J}}_k}{dt} = -\boldsymbol{\mathcal{J}}_k \times \boldsymbol{B}_k + \frac{\eta}{\mathcal{J}_k}\boldsymbol{\mathcal{J}}_k \times \frac{d\boldsymbol{\mathcal{J}}_k}{dt}$$

$$\boldsymbol{B}_k = -2(V_k \cos(2\omega_M t), 0, -\omega_k)$$

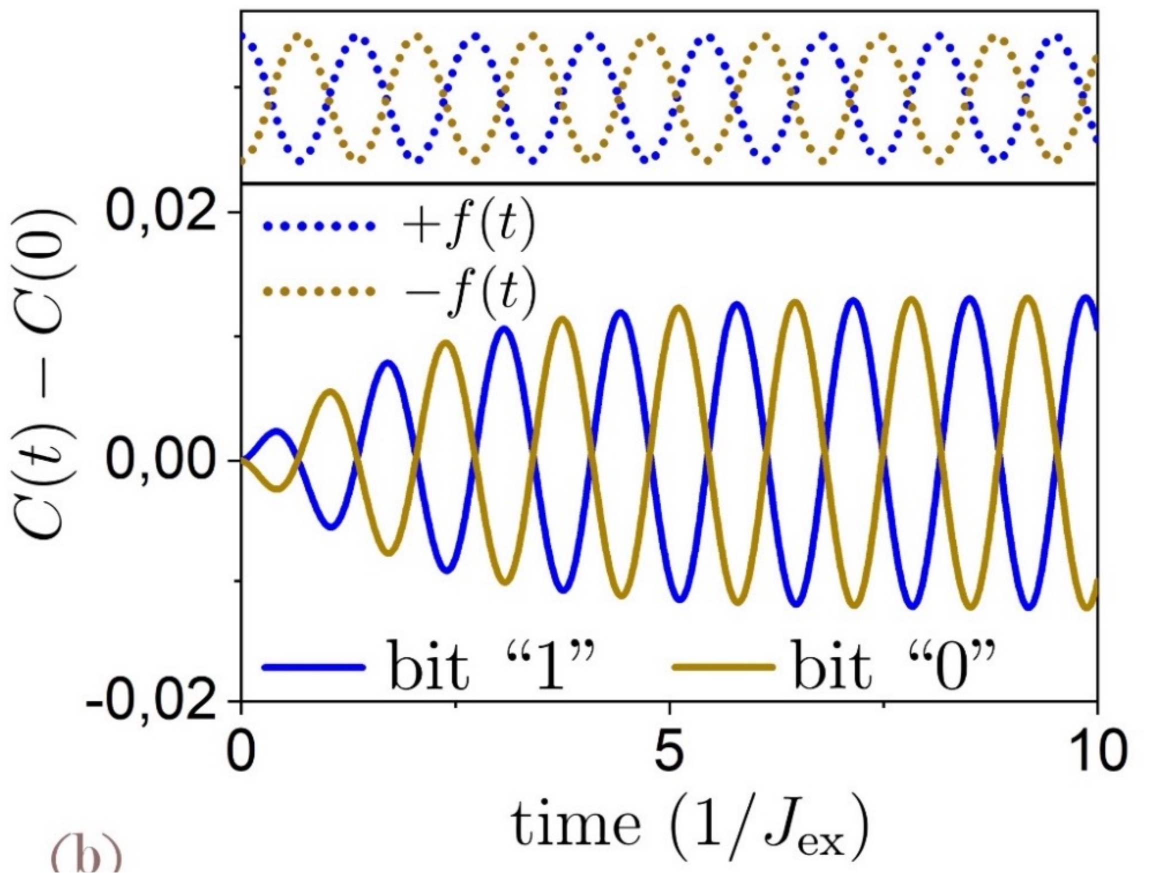
$$C(t) = \langle \hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_{i+\delta x} \rangle = \sum_k \boldsymbol{\Theta}_k \cdot \boldsymbol{\mathcal{J}}_k$$

Bits as oscillation states

$\eta = 0$



$\eta = 0.1$



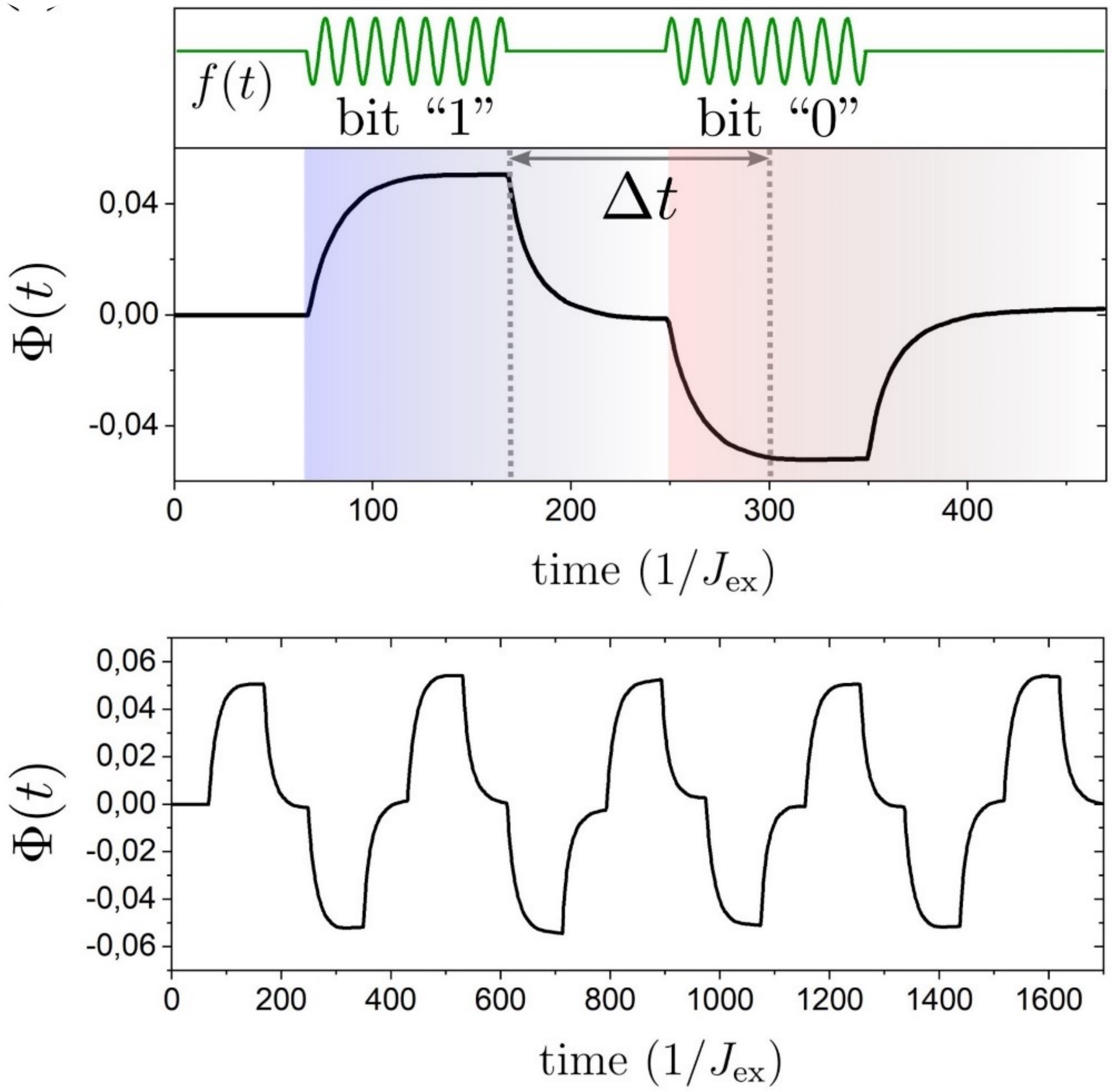
E. Goto, The Parametron, a Digital Computing Element Which Utilizes Parametric Oscillation *Proc. IRE*, **47**, 1304 (1959)

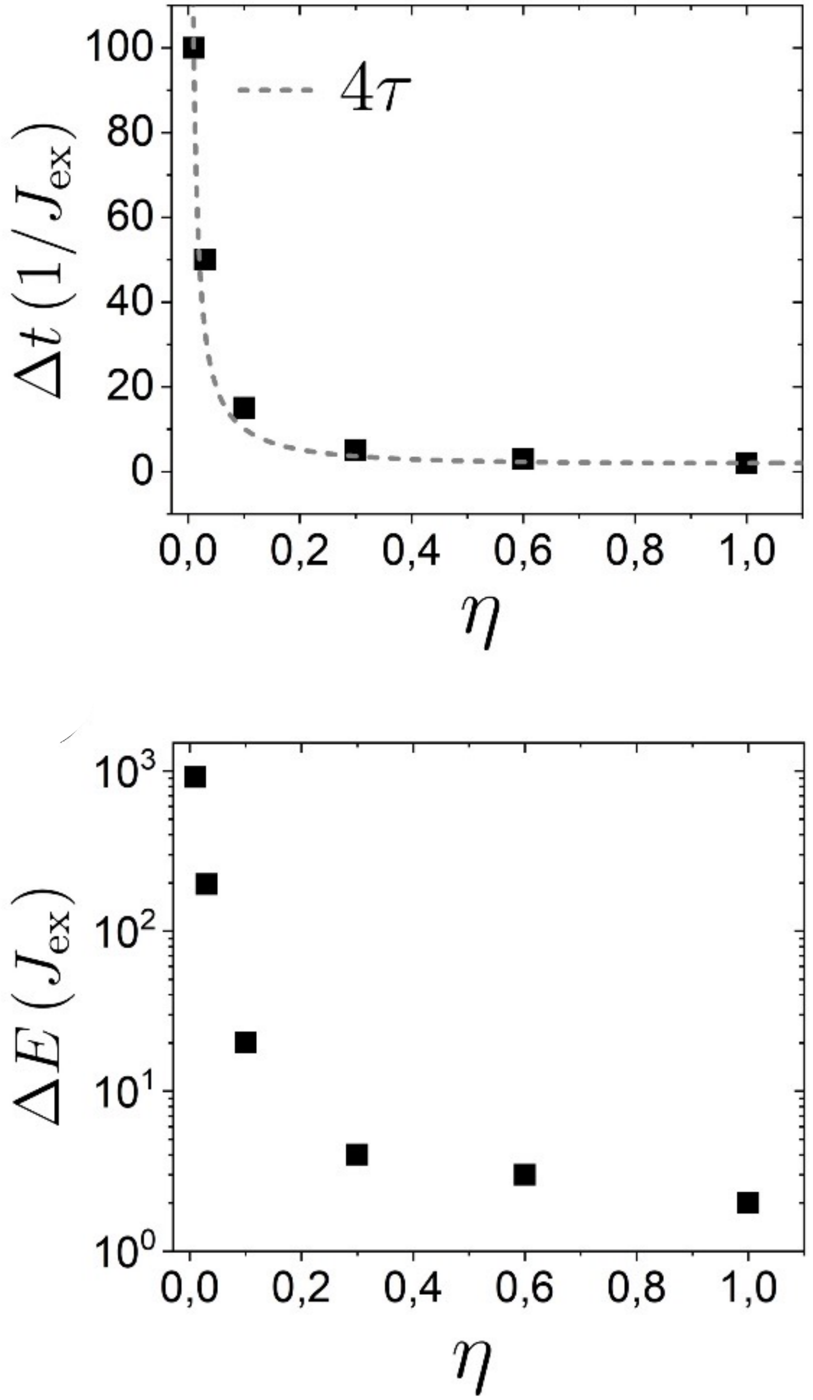
Bits as phases of magnon pair oscillations

$$\Phi(t) = \int_{t}^{t+T} dt' C(t') \cdot \cos 2\omega_{\boldsymbol{k}} t'$$

$J_{ex} = 10^{-2} eV \implies t = 100/J_{ex} \sim 6$ ps

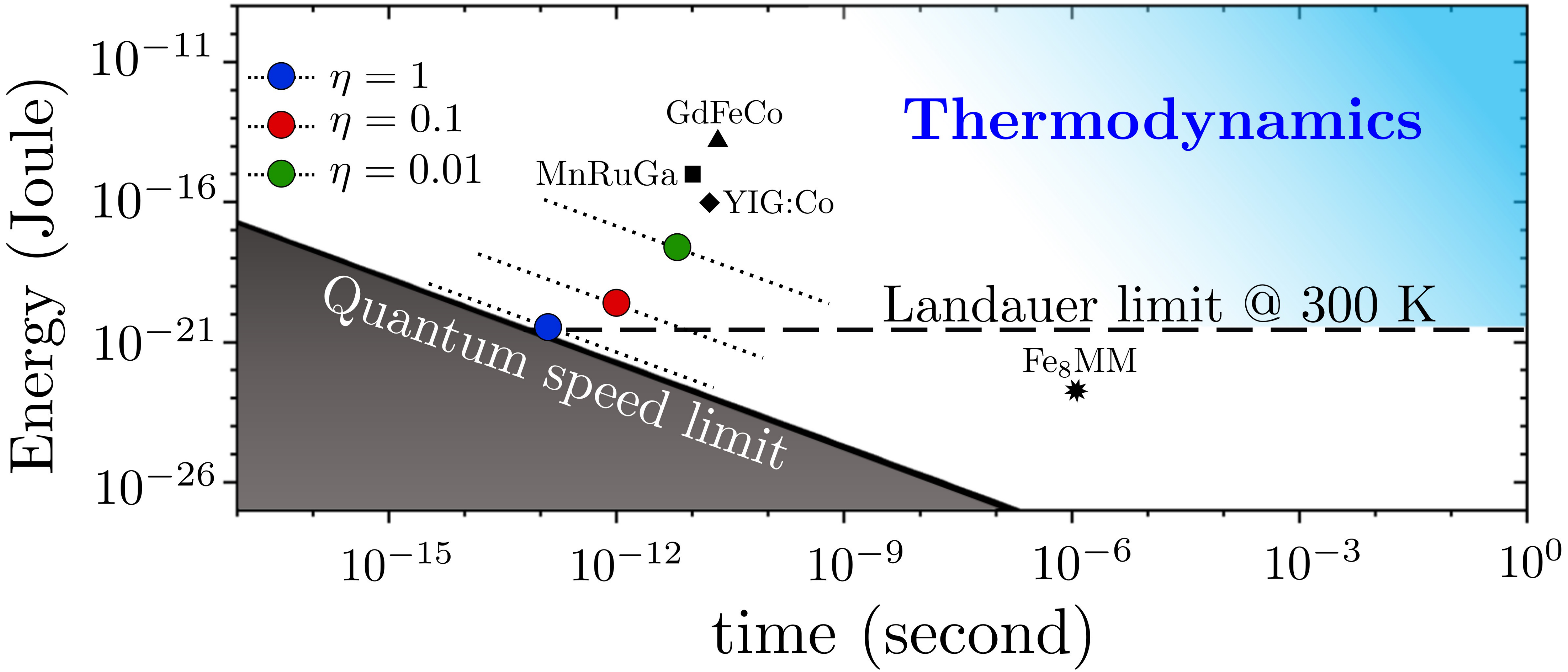
$\eta = 0.01$





G. Fabiani and JHM, Appl. Phys. Lett. 120, 152402 (2022)

Approaching the fundamental limits



- Strong damping $\eta \sim 1$ approaches "ultimate" physical limits
- Up to 5 orders of magnitude better than best magnets so far

GdFeCo: K. Vahaplar Phys. Rev. Lett. 103, 117201(2009)

YIG:Co: A. Stupakiewicz, Nature 542, 71–74 (2017).

MnRuGa: C. Banerjee, et al., Nature communications 11, 4444 (2020)

Fe_8MM: R. Gaudenzi, et al., Nature Phys. 14,565–568 (2018)

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Institute for Molecules and Materials
Radboud University

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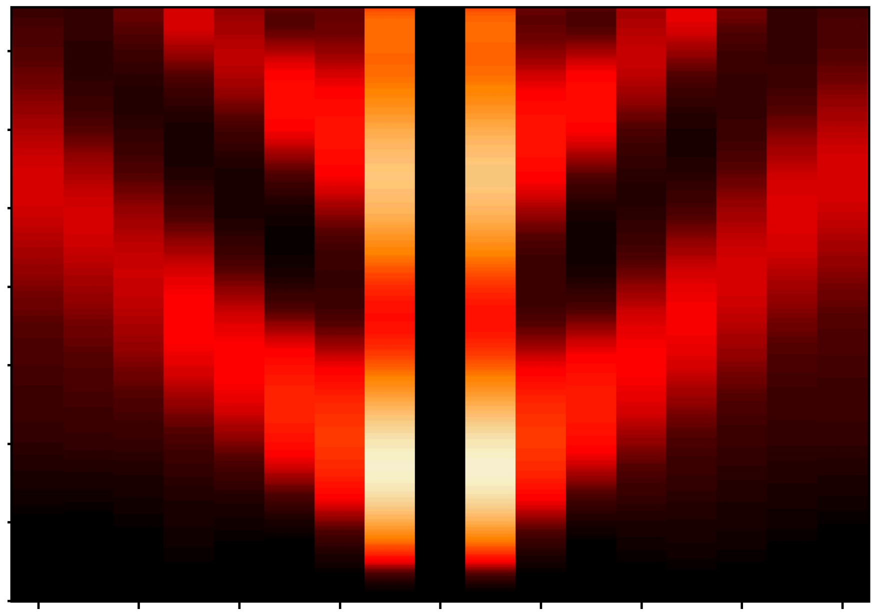
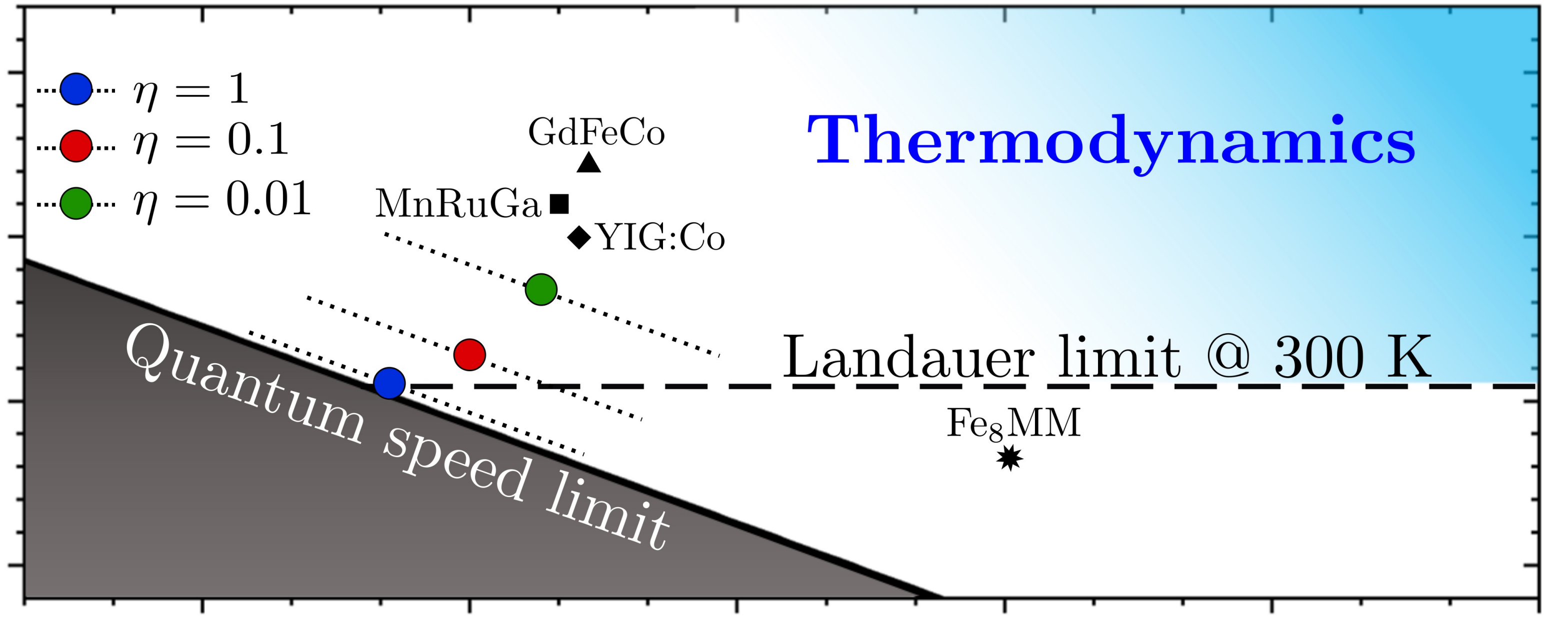
j.mentink@science.ru.nl

Summary

- **Supermagnonic propagation**
  magnons at the edge can propagate up to 40% faster
  extraordinary strong magnon-magnon interactions in 2D

- **Parametric excitation of magnon-pairs**
  Semi-classical dynamics reveals switching near fundamental limits





SciPost. Phys. 7, 004 (2019) Phys. Rev. Lett. 127, 097202 (2021) Appl. Phys. Lett. 120, 152402 (2022)