T-square thermal resistivity and quasi-particle hydrodynamics

Kamran Behnia

Benoît Fauqué
Collège de France

Alexandre Jaoui
Now in Munich

Xiao Lin (2012-15)
Now in Hangzhou

Clément Collignon
(2014-18)
Now in Boston

Willem Rischau
(2015-17)
Now in Geneva

Outline

- T-square electrical resistivity of Fermi liquids and its thermal counterpart
- ^{3}He and a hydrodynamic view of T-square resistivity
- The origin of T-linear resistivity in cuprates

T-square resistivity

The electric resistivity of Fermi liquids follows:

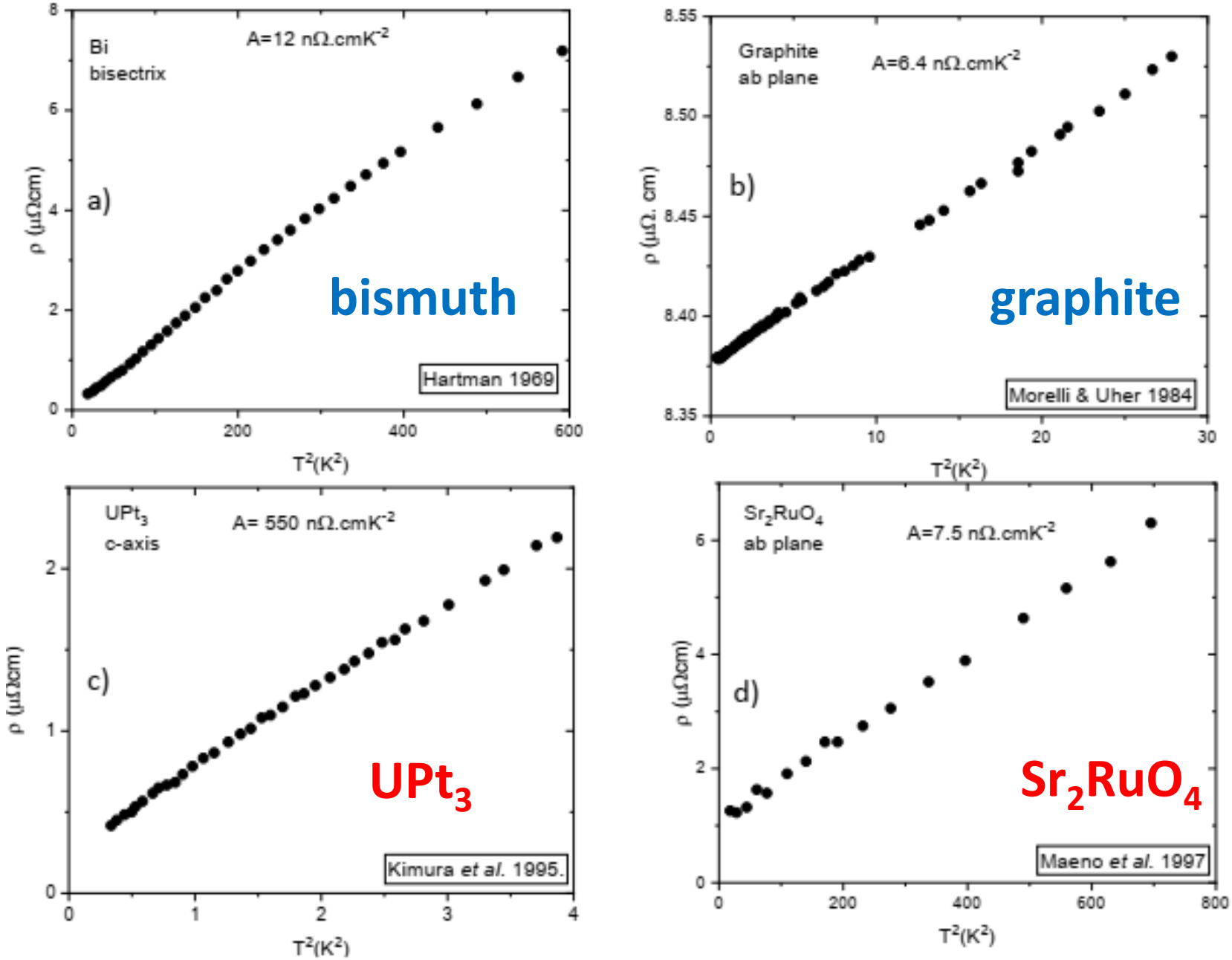
$$\rho = \rho_0 + AT^2$$

Scattering by impurities

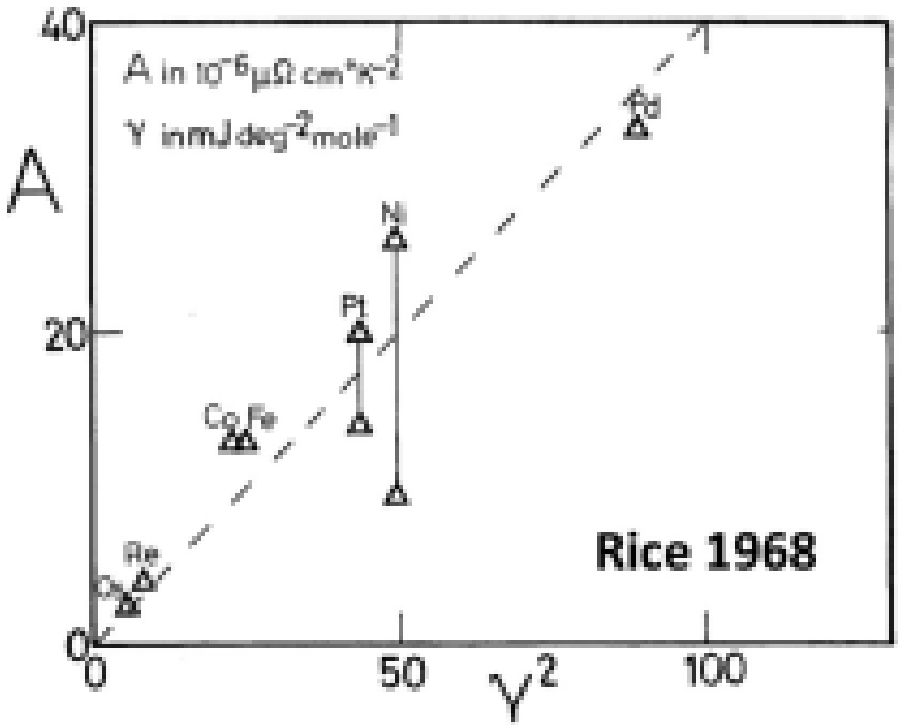
Electron-electron scattering

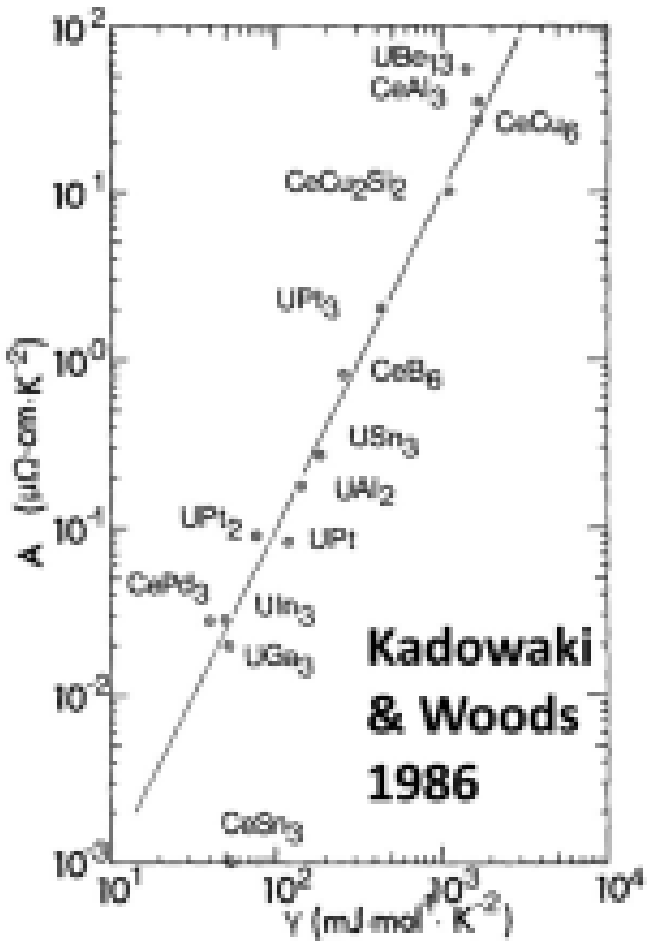
- Apply Pauli exclusion principle to each colliding electron. Then the phase space grows $\propto \left(\frac{k_B T}{E_F}\right)^2$
- Hard to see in common metals (overwhelmed by phonon scattering), but not in correlated or dilute metals.

T-square resistivity



What sets the amplitude of T-square resistivity?





- Kadowaki-Woods scaling postulates:

$A \propto \gamma^2$ **γ is the T-linear specific heat (Sommerfeld coefficient)**

The KW scaling works only for **dense** metals

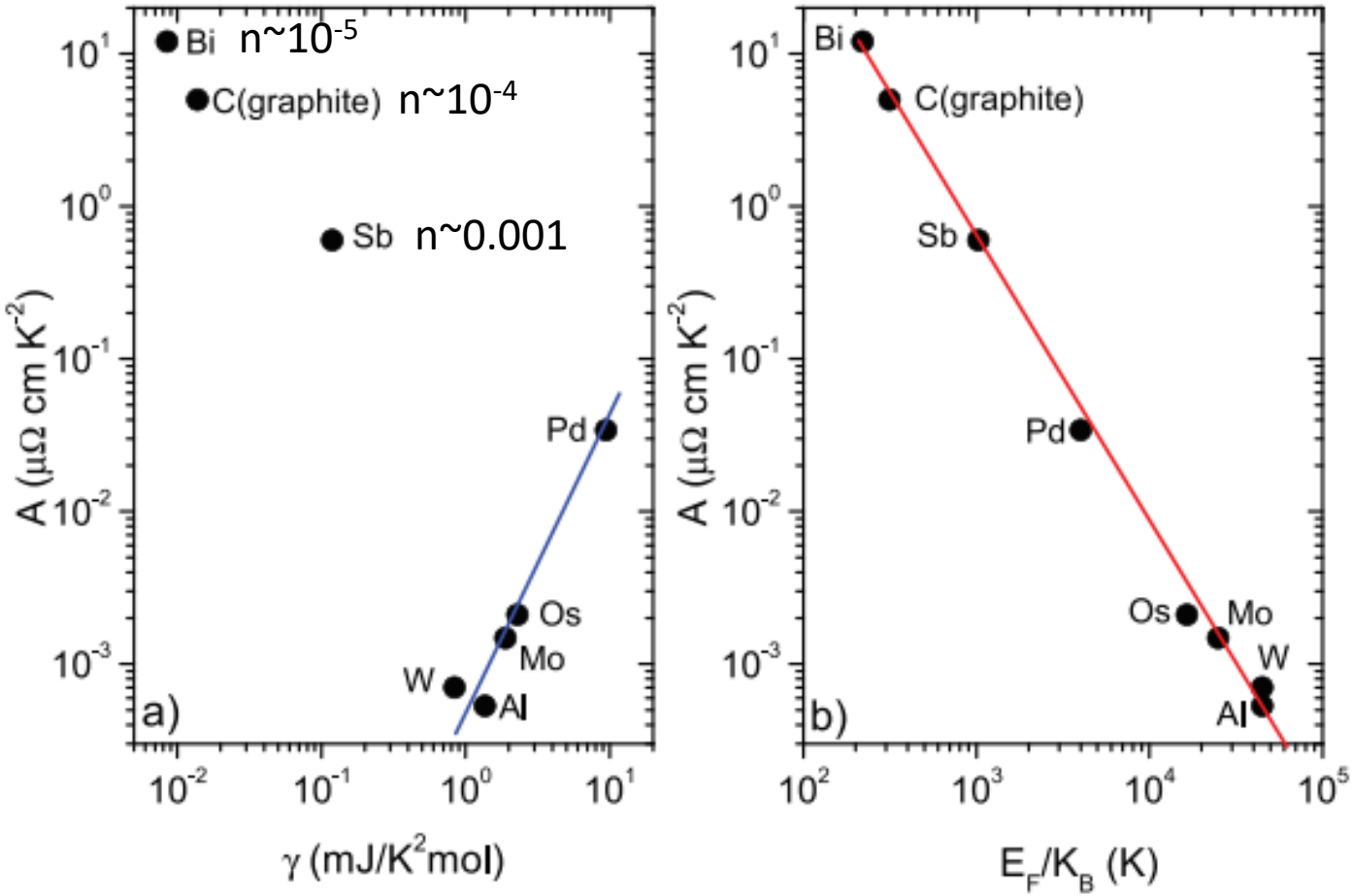
- Specific heat $\gamma = \frac{\pi^2}{2} k_B^2 \frac{n}{E_F}$ ← Number of électrons per unit cell
- T^2-resistivity $\rho = \rho_0 + AT^2$

$$A = \frac{\hbar}{e^2}\left(\frac{k_B}{E_F}\right)^2 \ell_{quad}$$

n~1 ⇒ $A \propto \gamma^2$

Whatever n ⇒ $A \propto \frac{1}{E_F^2}$

The “extended Kadowaki-Woods” scaling



Knowing the Fermi Energy, one can predict the magnitude of A.

REPORTS

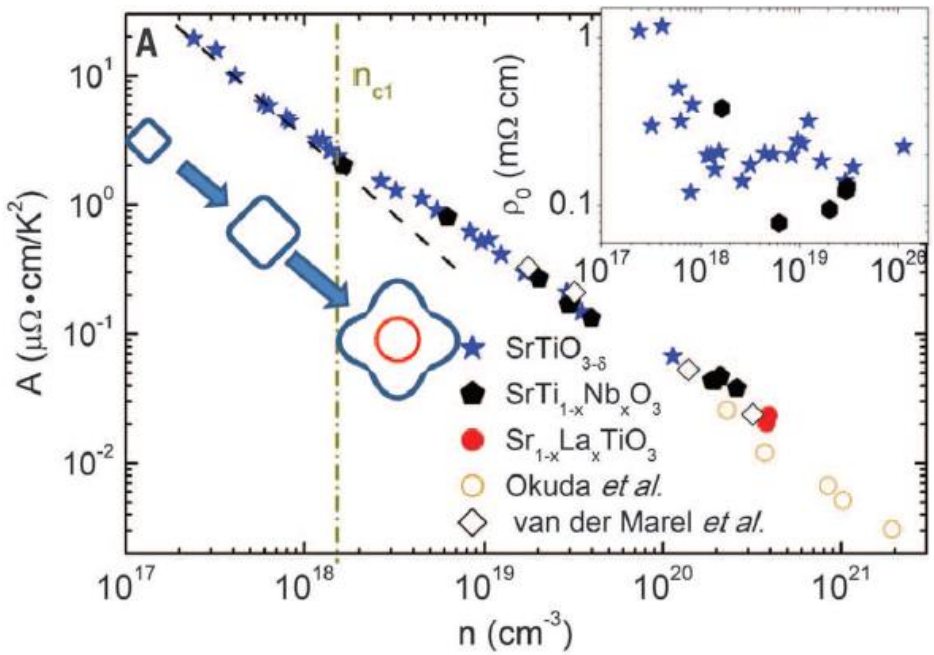
SOLID-STATE PHYSICS

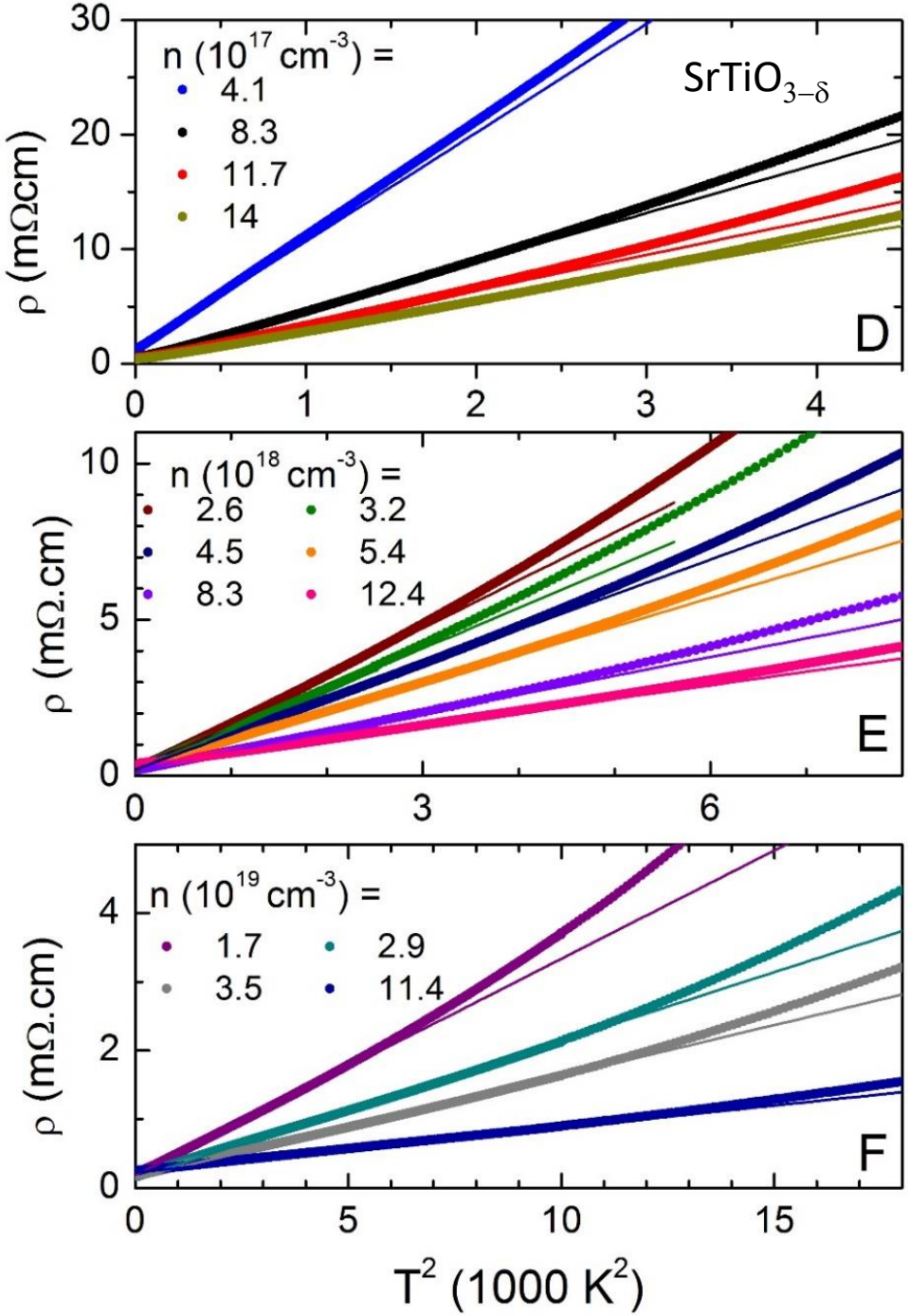
2015

Scalable T^2 resistivity in a small single-component Fermi surface

Xiao Lin, Benoît Fauqué, Kamran Behnia*

$$\rho = \rho_0 + AT^2$$





ARTICLE

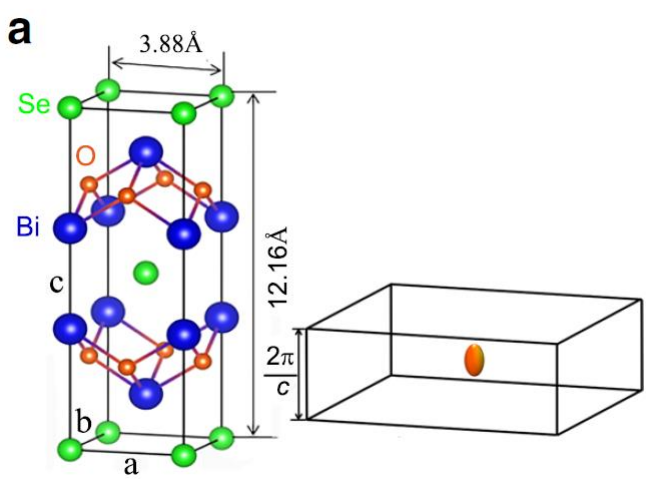
https://doi.org/10.1038/s41467-020-17692-6 OPEN

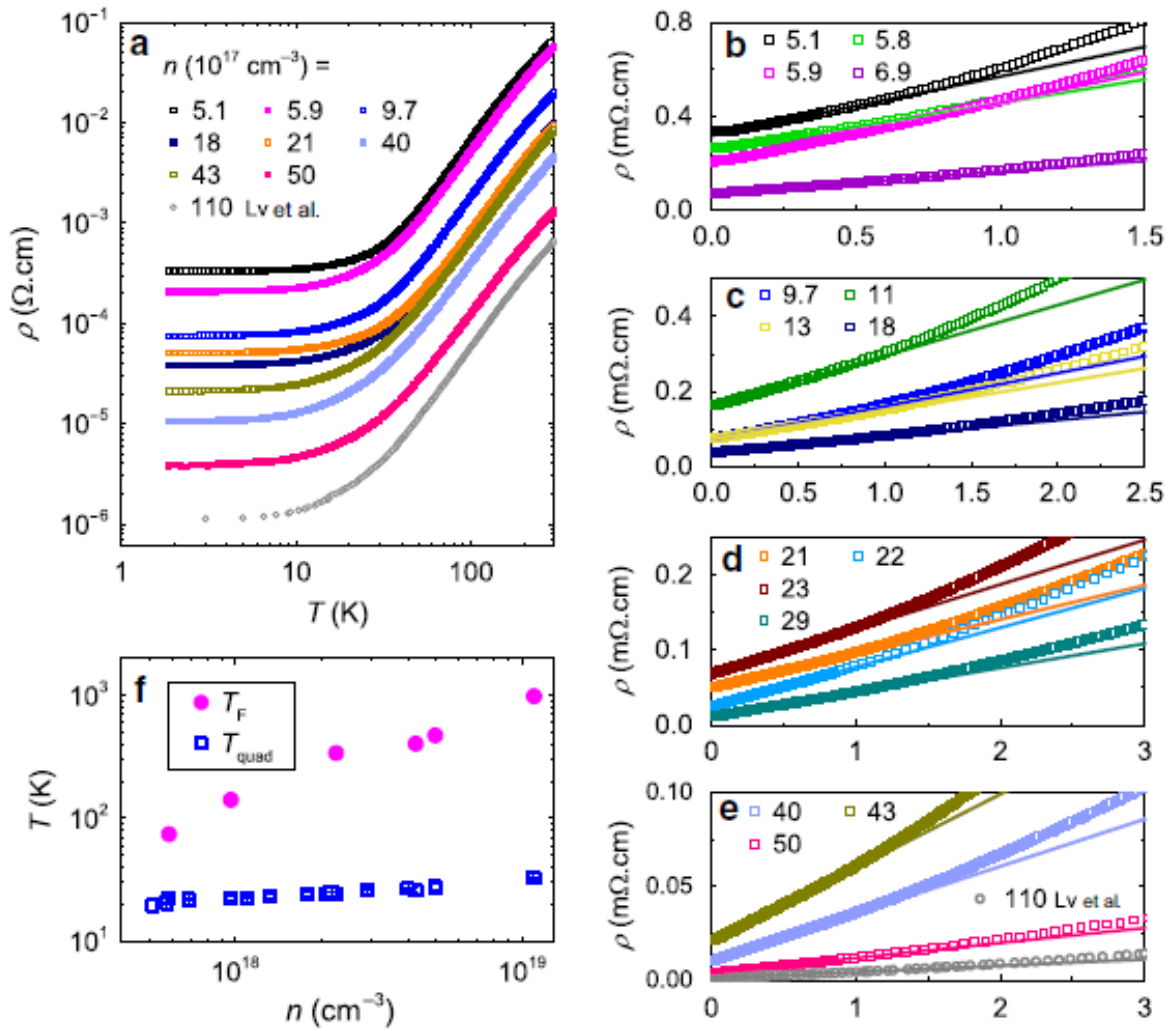
T-square resistivity without Umklapp scattering in dilute metallic Bi_2O_2Se

2020

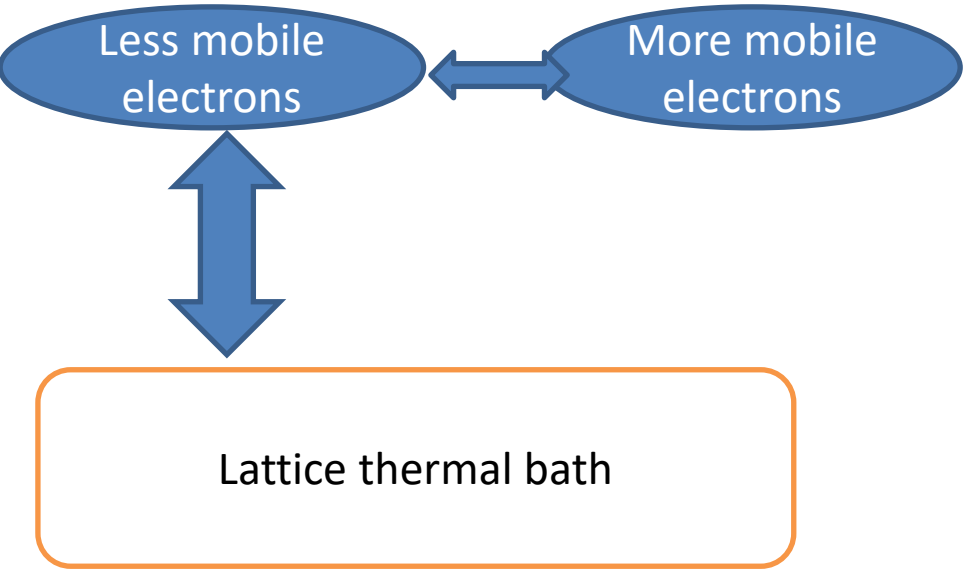
Jialu Wang[1,2], Jing Wu[1,2], Tao Wang[1,2], Zhuokai Xu[1,2], Jifeng Wu[1,2], Wanghua Hu[1,2], Zhi Ren[1,2], Shi Liu[1,2], Kamran Behnia[3] & Xiao Lin[1,2]✉

STO is not alone!

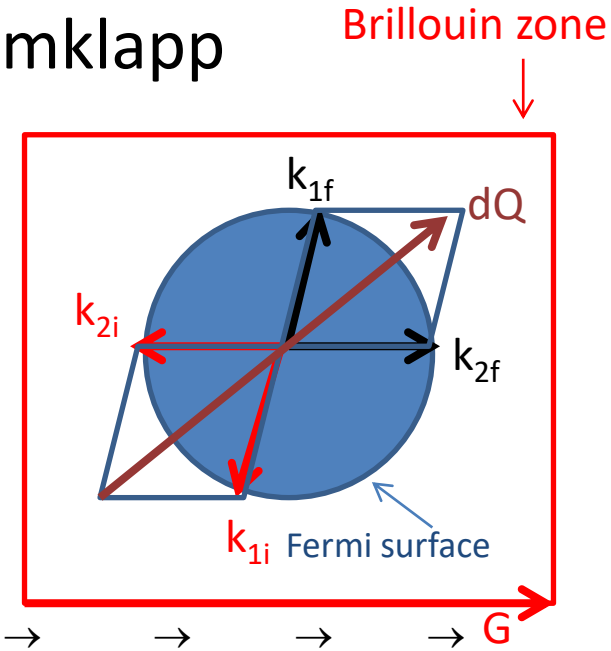




A) Baber



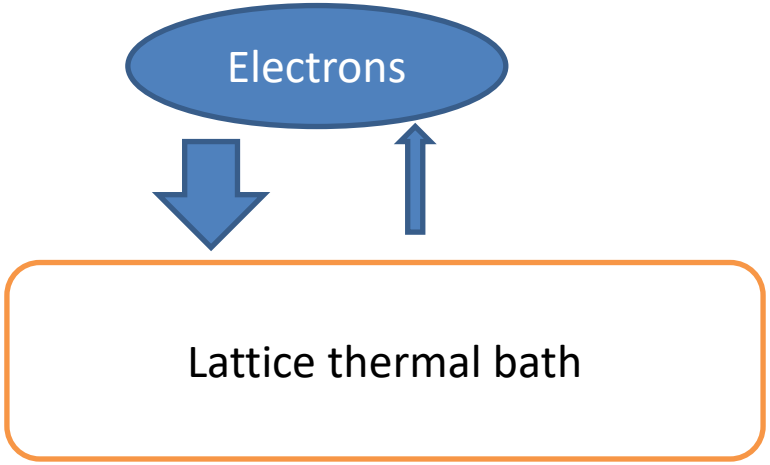
B) Umklapp

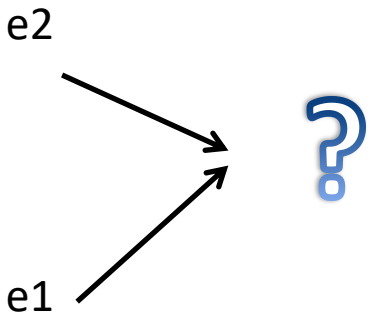


$$\vec{dQ} = \vec{k}_{2f} + \vec{k}_{1f} - \vec{k}_{2i} - \vec{k}_{1i}$$

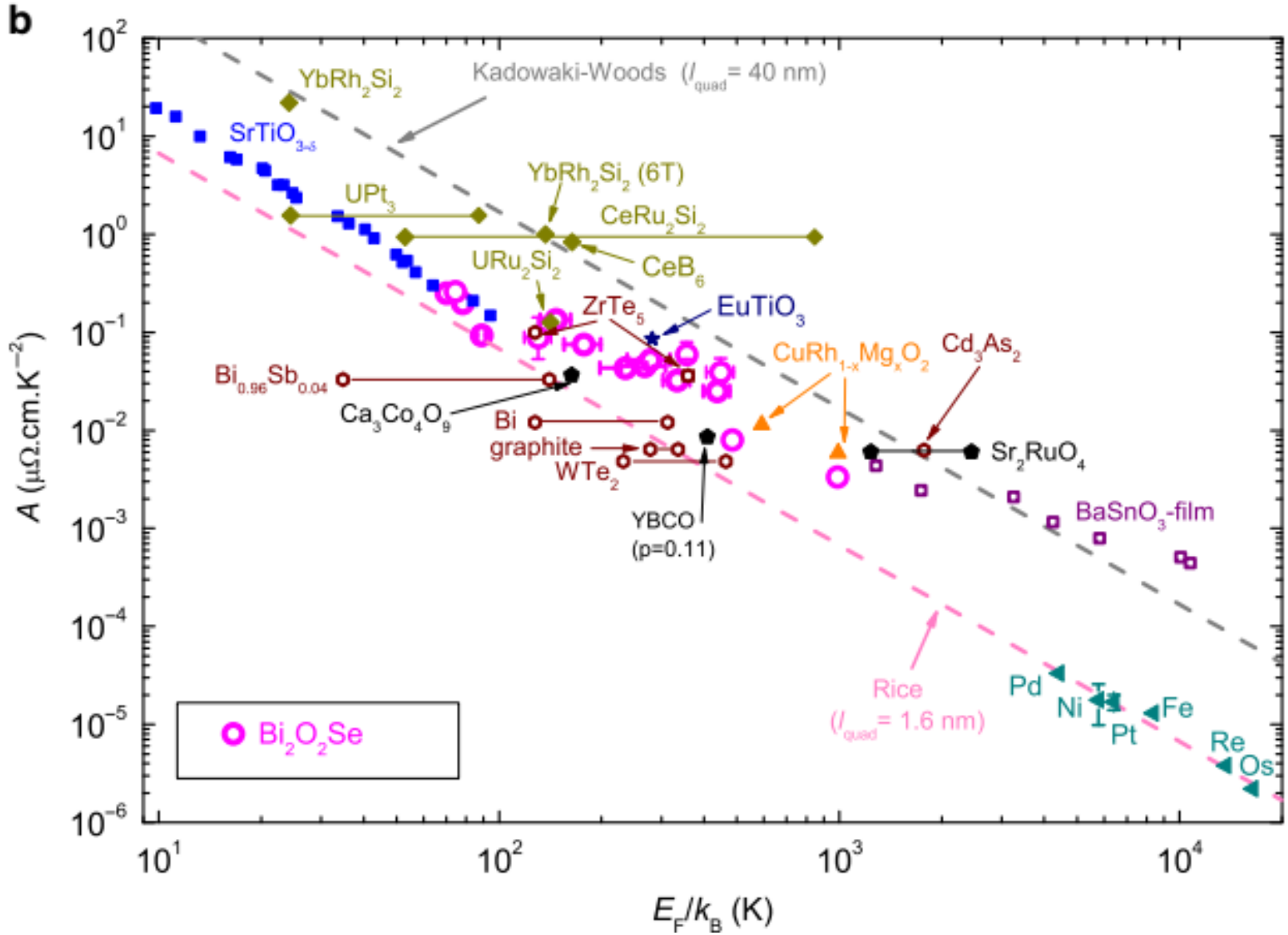
U process when: $\vec{dQ} = \vec{G}$

C)





The “extended Kadowaki-Woods” scaling

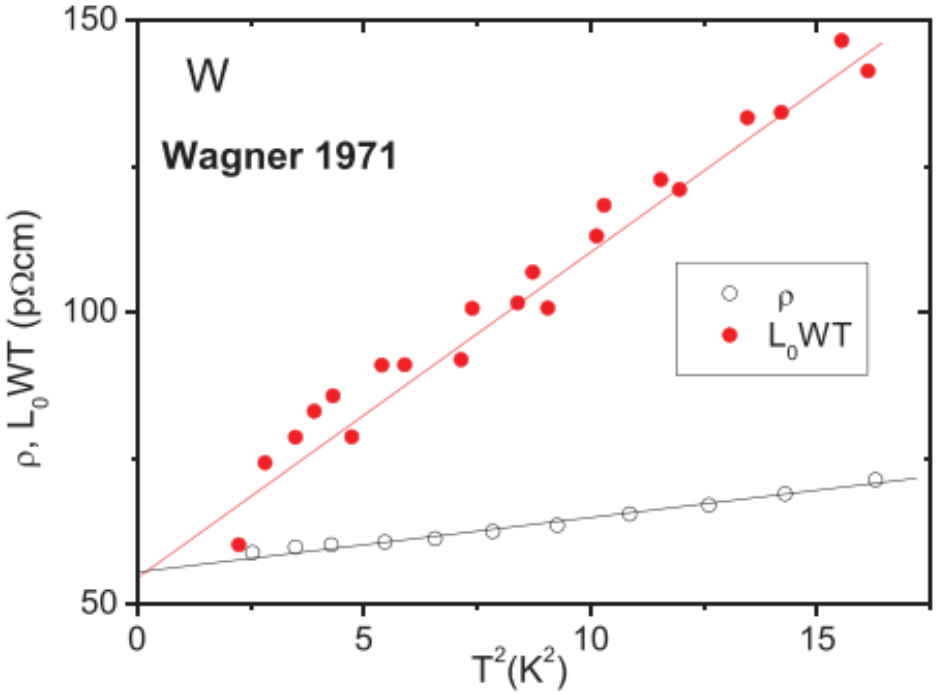


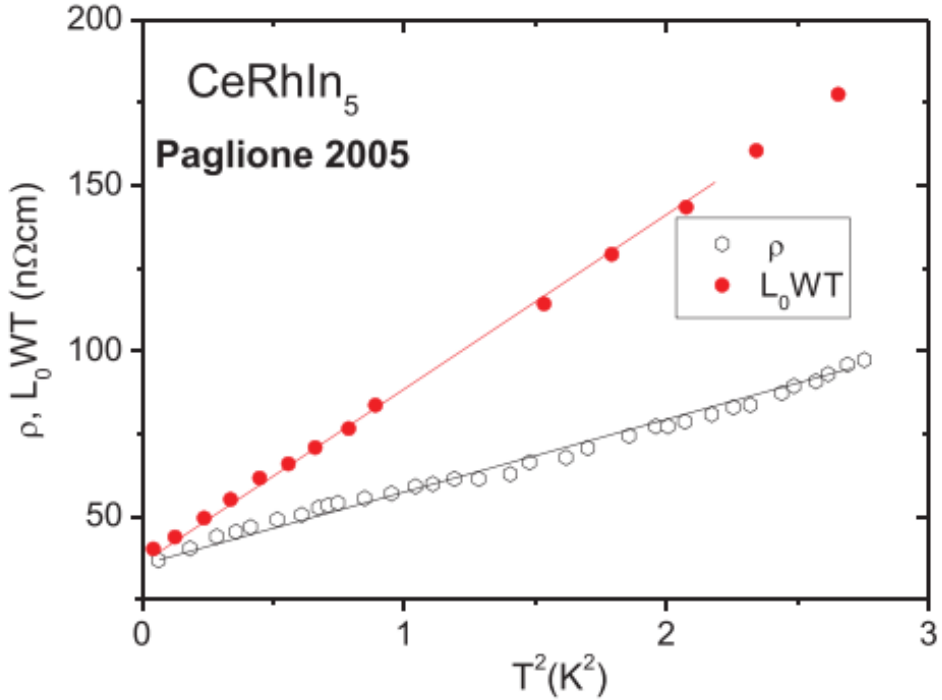
Knowing the Fermi Energy, one can predict the magnitude of A.

Two puzzles about T-square resistivity in Fermi liquids

- Why is it universally linked to the Fermi energy?
- Why does it persist without Umklapp?
- Let us turn our attention to thermal transport.

T-square thermal resistivity





$$L_0 = \frac{\pi^2}{3}\left(\frac{\kappa_B}{e}\right)^2$$

$$WT = (WT)_0 + BT^2$$

$$\rho_0 = L_0(WT)_0$$

$$L_0 B > A$$

$$WT = \left(\frac{\kappa}{T}\right)^{-1}$$

$$\rho = \rho_0 + AT^2$$

What happens to Liquid Helium 3 at very low Temperatures?

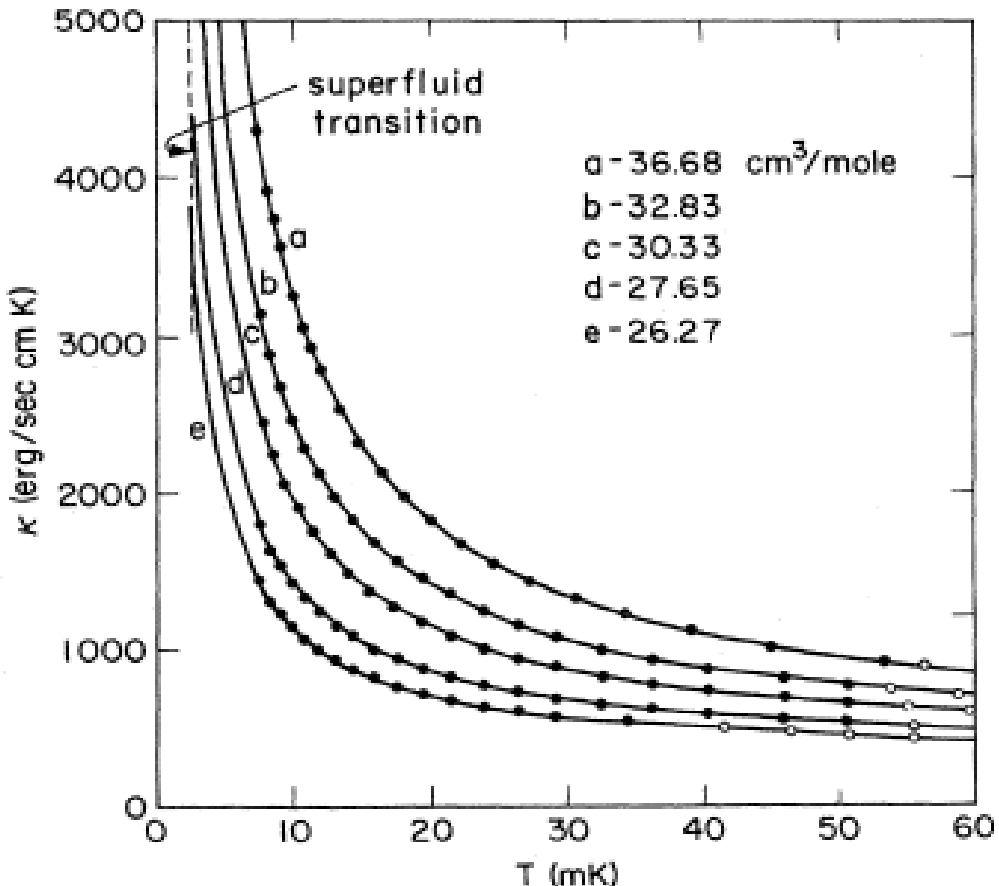
By E. R. Dobbs, London*)

The time between collisions is proportional to T^{-2}..., the viscosity of ^{3}He rises dramatically..., becoming at **3** mK, the same as olive oil at **40 °C!**

PHYSICAL REVIEW B VOLUME 29, NUMBER 9 1 MAY 1984

Thermal conductivity of normal liquid ^{3}He

Dennis S. Greywall
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
(Received 13 October 1983)

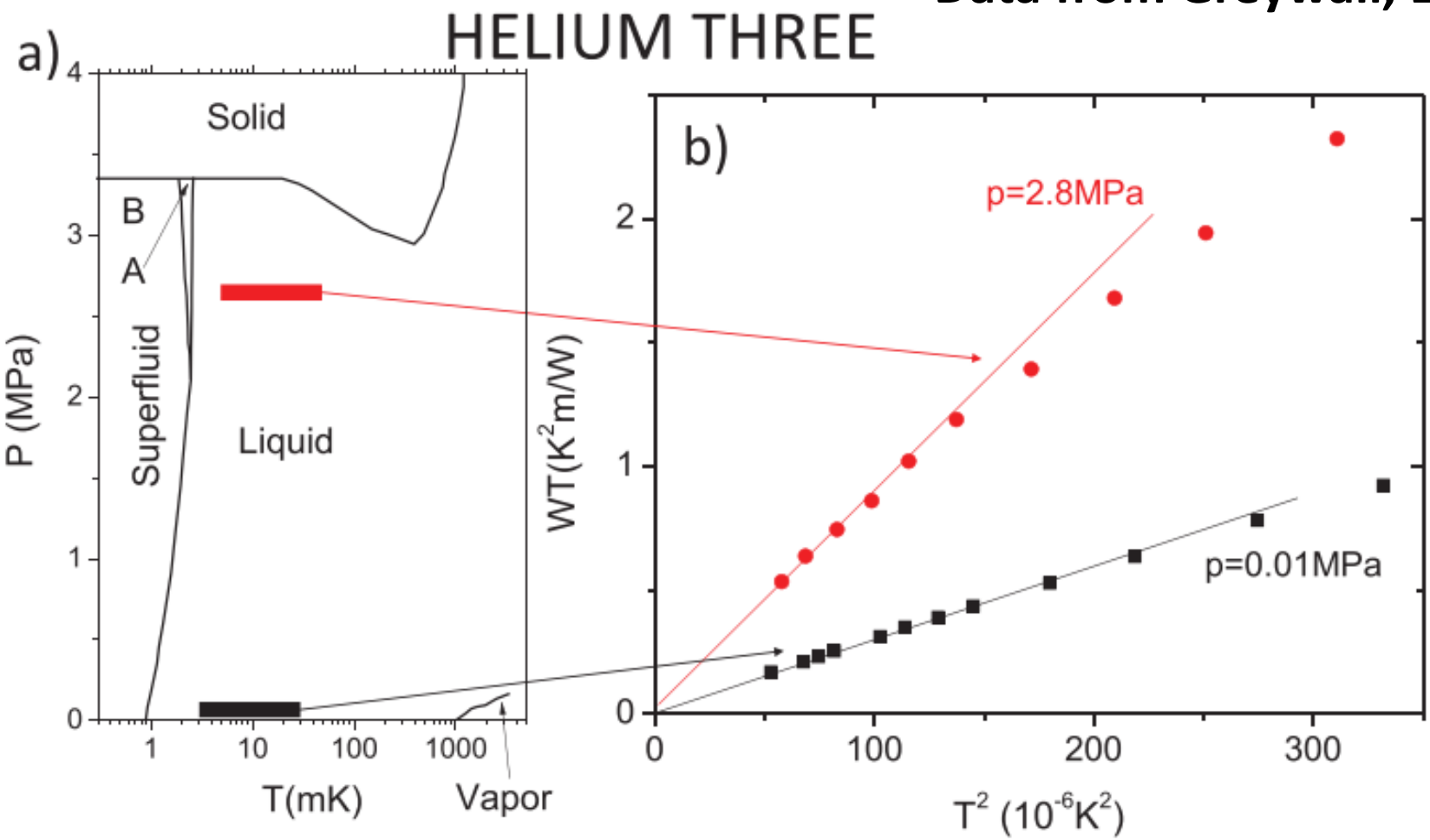


- In ^{3}He: $\kappa \propto T^{-1}$
- This is equivalent to $WT \propto T^2$

$$WT = \left(\frac{\kappa}{T}\right)^{-1}$$

T-square thermal resistivity in ^{3}He

Data from Greywall, 1984



No Umklapp here!

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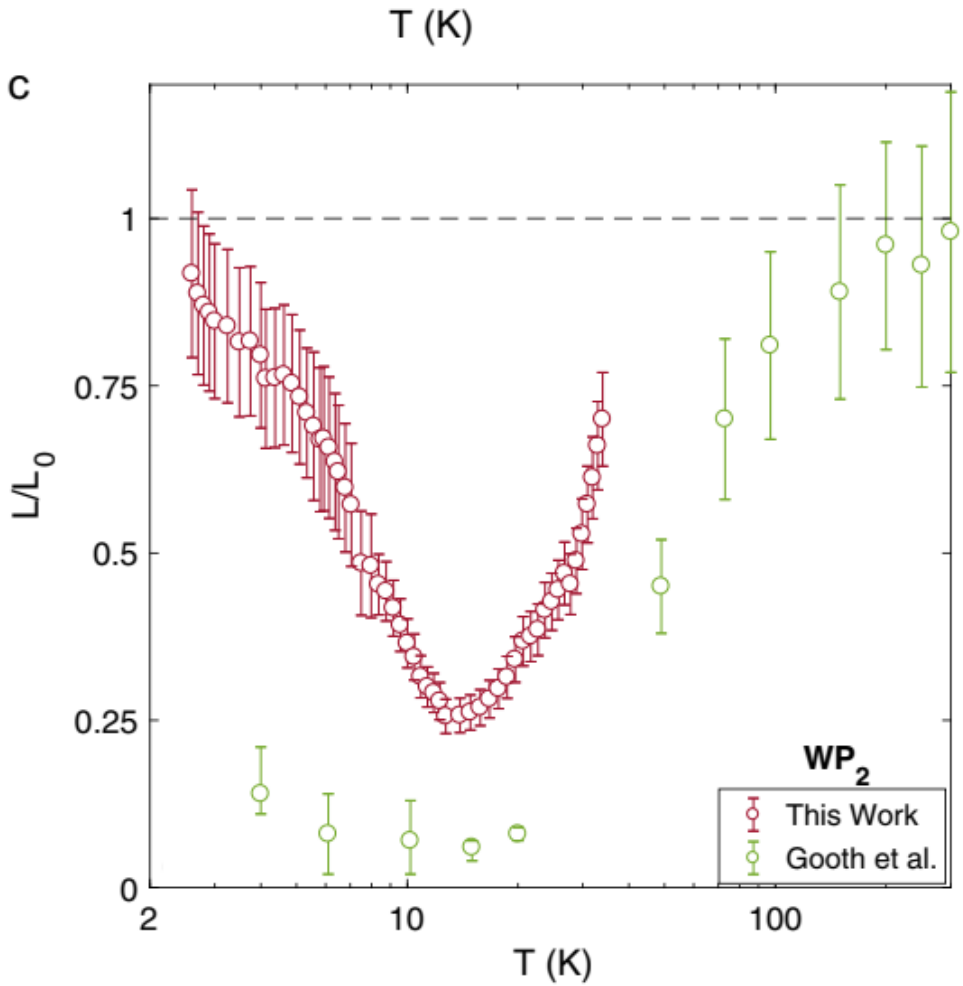
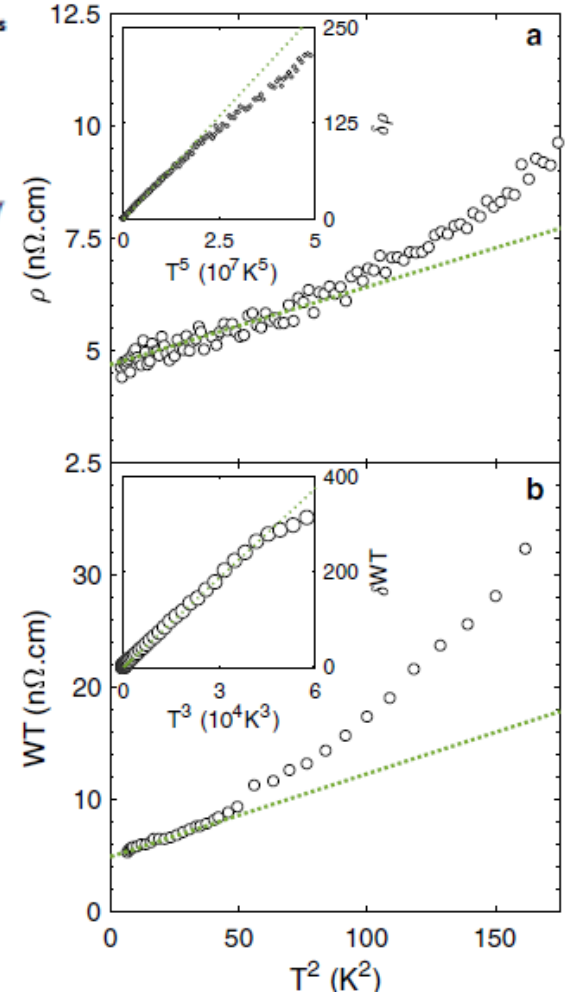
ARTICLE OPEN

Departure from the Wiedemann–Franz law in WP_2 driven by mismatch in T-square resistivity prefactors

Alexandre Jaoui[1,2], Benoit Fauqué[1,2], Carl Willem Rischau[2,3], Alaska Subedi[4,5], Chenguang Fu[6], Johannes Gooth[6], Nitesh Kumar[6], Vicky Süß[6], Dmitrii L. Maslov[7], Claudia Felser[6] and Kamran Behnia[2,8]





$$\rho = \rho_0 + A_2 T^2 + A_5 T^5$$

$$WT = W_0 T + B_2 T^2 + B_3 T^3$$

$$\boxed{B_2 / A_2 \sim 5}$$

What sets the mismatch between T-square prefactors in a given solid?

Material	ρ_0 (nΩcm)	A_2 (pΩcmK^{-2})	B_2(pΩcmK^{-2})	B_2/A_2
WP_2	4	17	76	5
W	0.06	0.9	6.2	6
Ni	3	25	61	2.5
UPt_3	200	$1.6\ 10^6$	$2.4\ 10^6$	1.5
$CeRhIn_5$	37	$2.1\ 10^4$	$5.7\ 10^4$	2.5

Theory:

- Herring (1967): The ratio is quasi-universal and ~ 2!
- Li & Maslov (2019) : No boundary! It can become arbitrarily large!

T-square thermal resistivity does not require Umklapp events!

Two possible explanations of the T-square mismatch

- The electrical T-square prefactor (A) is NOT affected by horizontal events.
- The thermal T-square prefactor (B) is affected by both horizontal and vertical events.
- B>A, because some collisions are horizontal!

- The electrical T-square prefactor (A) quantifies momentum-relaxing collisions.
- The thermal T-square prefactor (B) quantifies momentum-conserving collisions.
- B>A, because some e-e collisions conserve momentum!

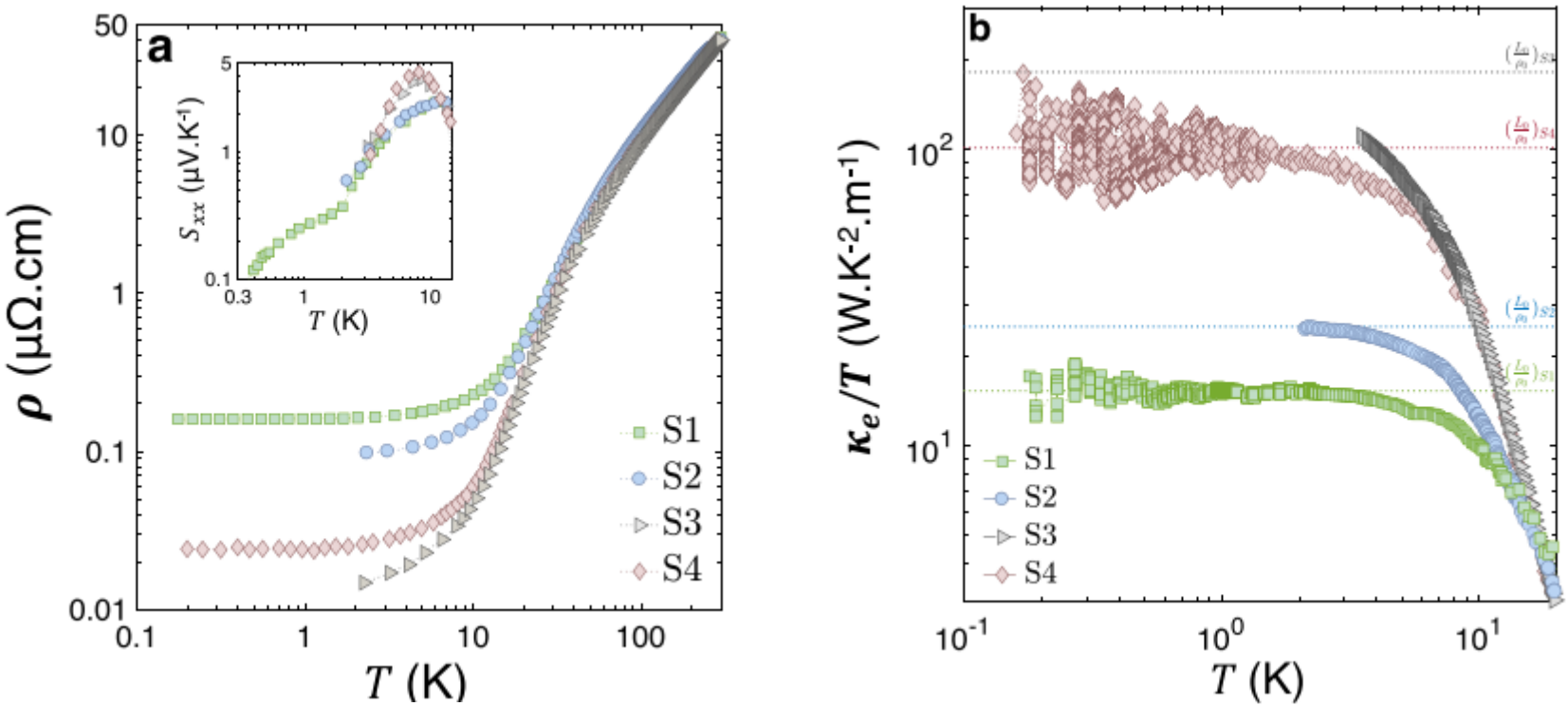
Look at the size dependence of B/A in a solid with ballistic electronic transport!

ARTICLE

https://doi.org/10.1038/s41467-020-20420-9 OPEN

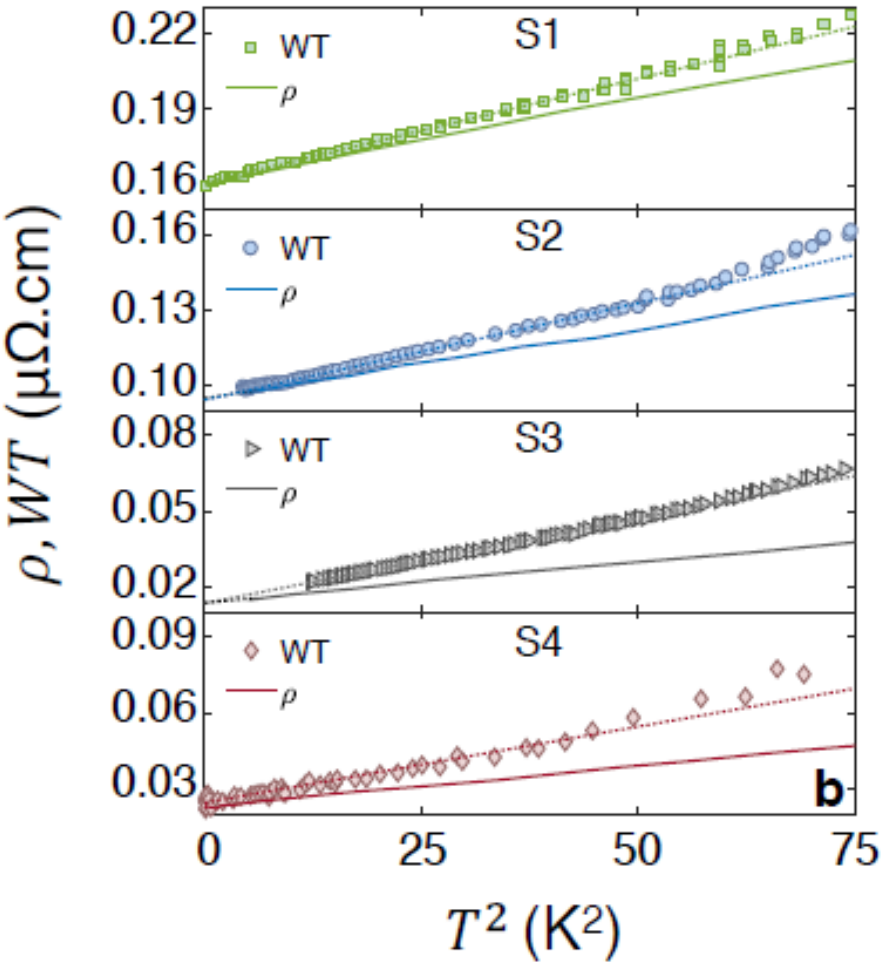
Thermal resistivity and hydrodynamics of the degenerate electron fluid in antimony

Alexandre Jaoui[1,2✉], Benoît Fauqué[1] & Kamran Behnia[2]

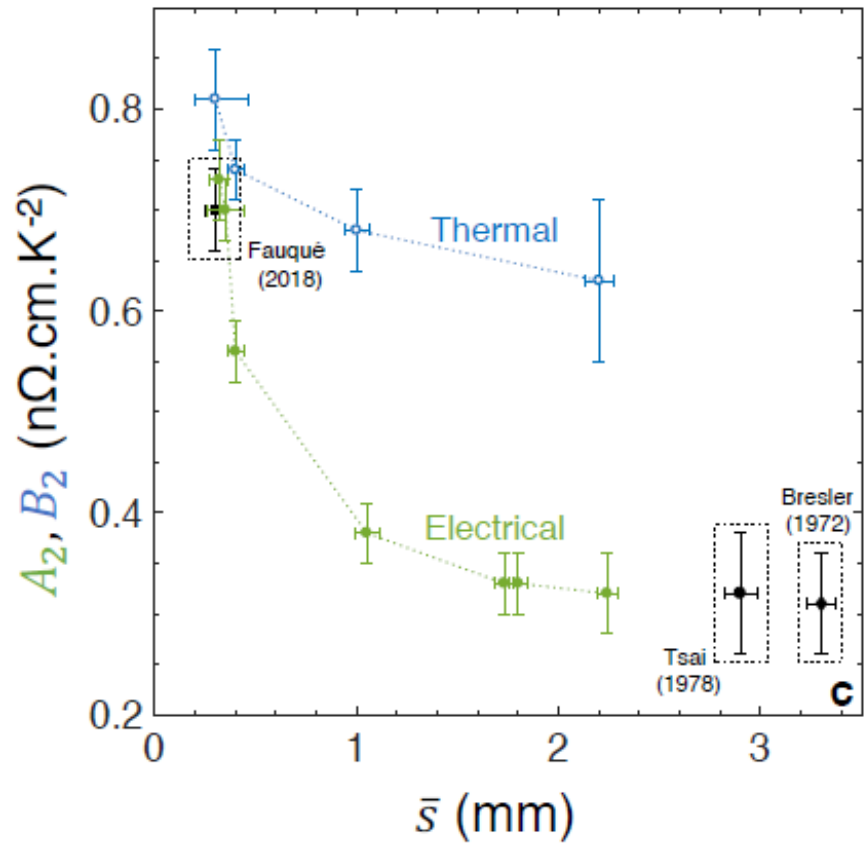


Electric conductivity and and electronic thermal conductivities are both size dependent.

Evolution of T-square resistivities



Jaoui et al. (2021)



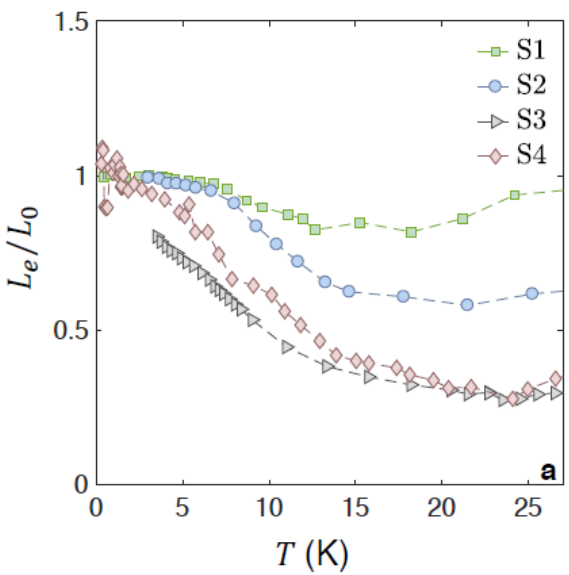
The larger the sample the higher the B/A ratio!

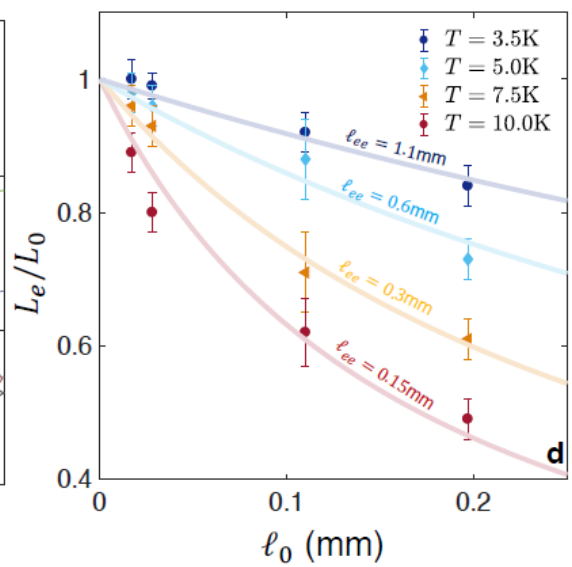
A substantial fraction of e-e scattering is momentum-conserving

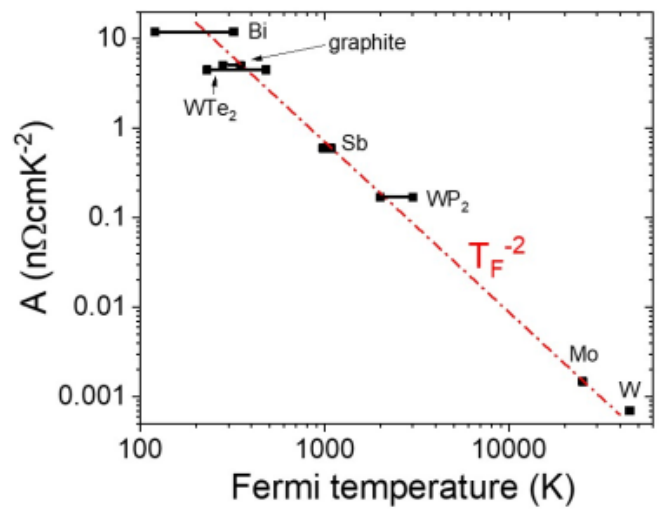
PRL **115,** 056603 (2015) PHYSICAL REVIEW LETTERS week ending 31 JULY 2015

Violation of the Wiedemann-Franz Law in Hydrodynamic Electron Liquids

Alessandro Principi* and Giovanni Vignale
Department of Physics and Astronomy, University of Missouri, Columbia, Missouri 65211, USA
(Received 16 June 2014; revised manuscript received 16 January 2015; published 31 July 2015)







Jaoui et al. (2021)

Origin of T-square thermal resistivity in ^{3}He

$$D = \frac{1}{3}\tau v_{\mathrm{m}}^2$$

Diffusivity

Scattering time

Mean velocity

$$\tau \propto T^{-2}$$

Energy diffusivity: $\mathrm{D} \propto T^{-2}$

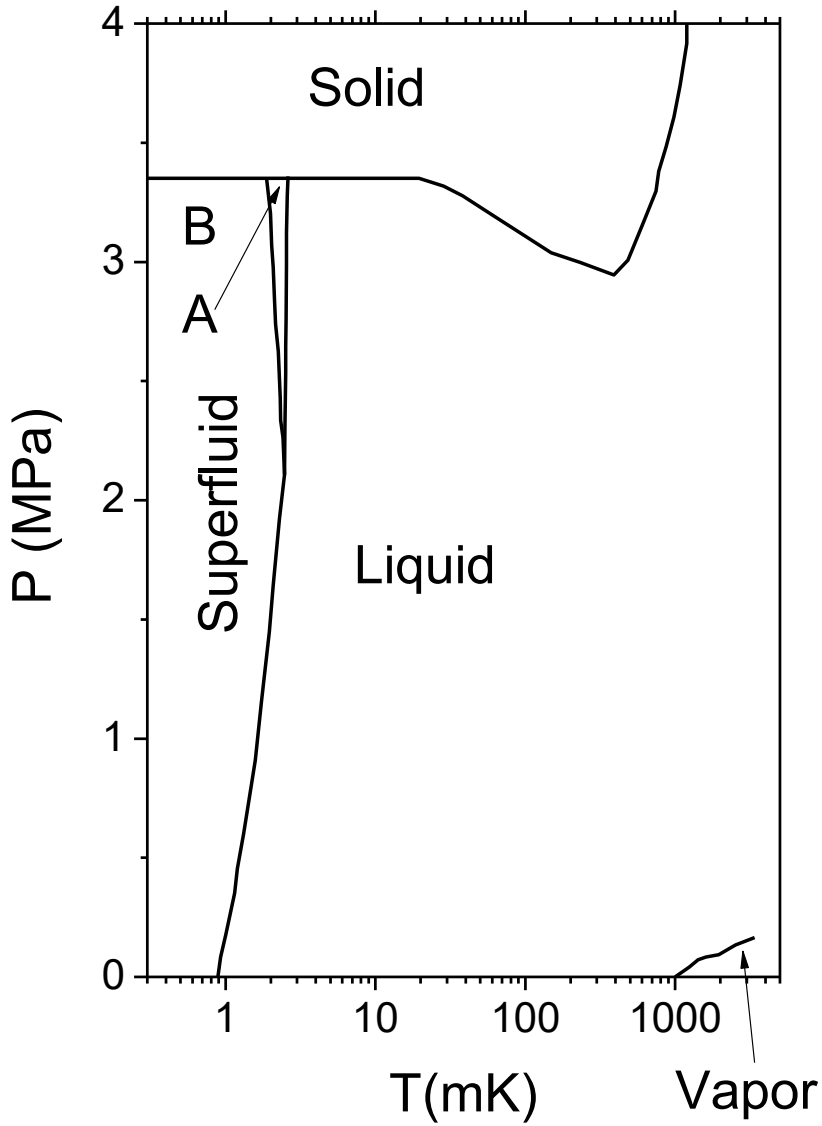
$\kappa = C \times D \propto T^{-1}$

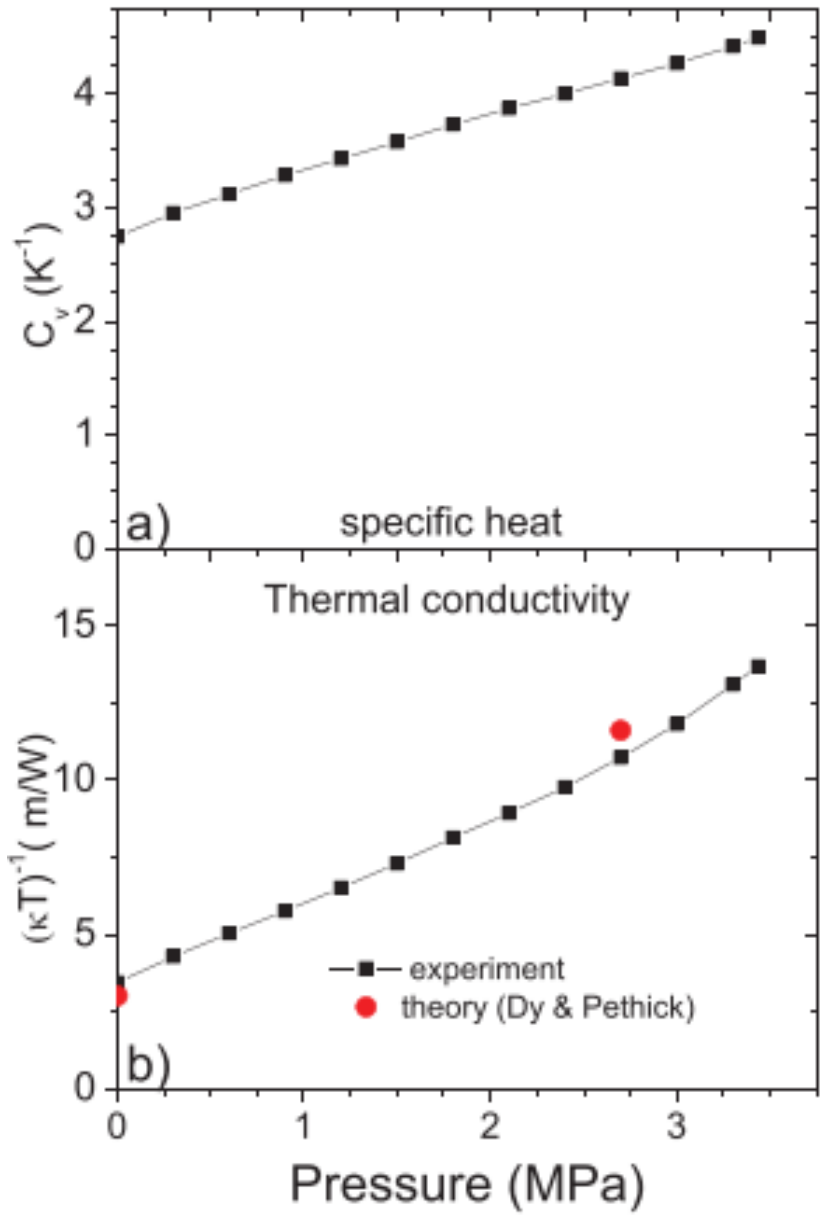
Specific heat: $\mathrm{C} \propto T$

Momentum diffusivity (Viscosity): $\eta \propto T^{-2}$

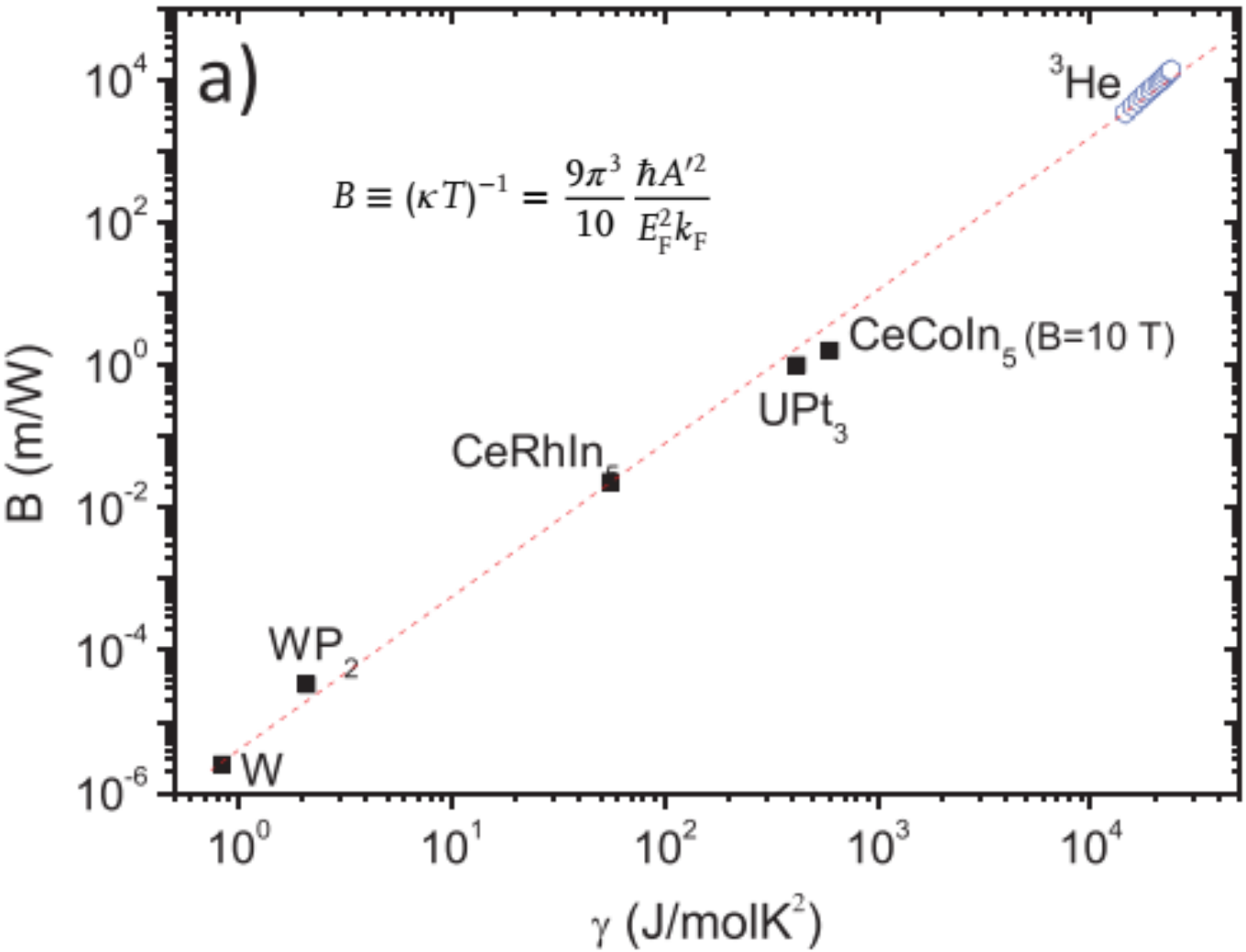
$$WT \propto T^2$$

^{3}He under pressure



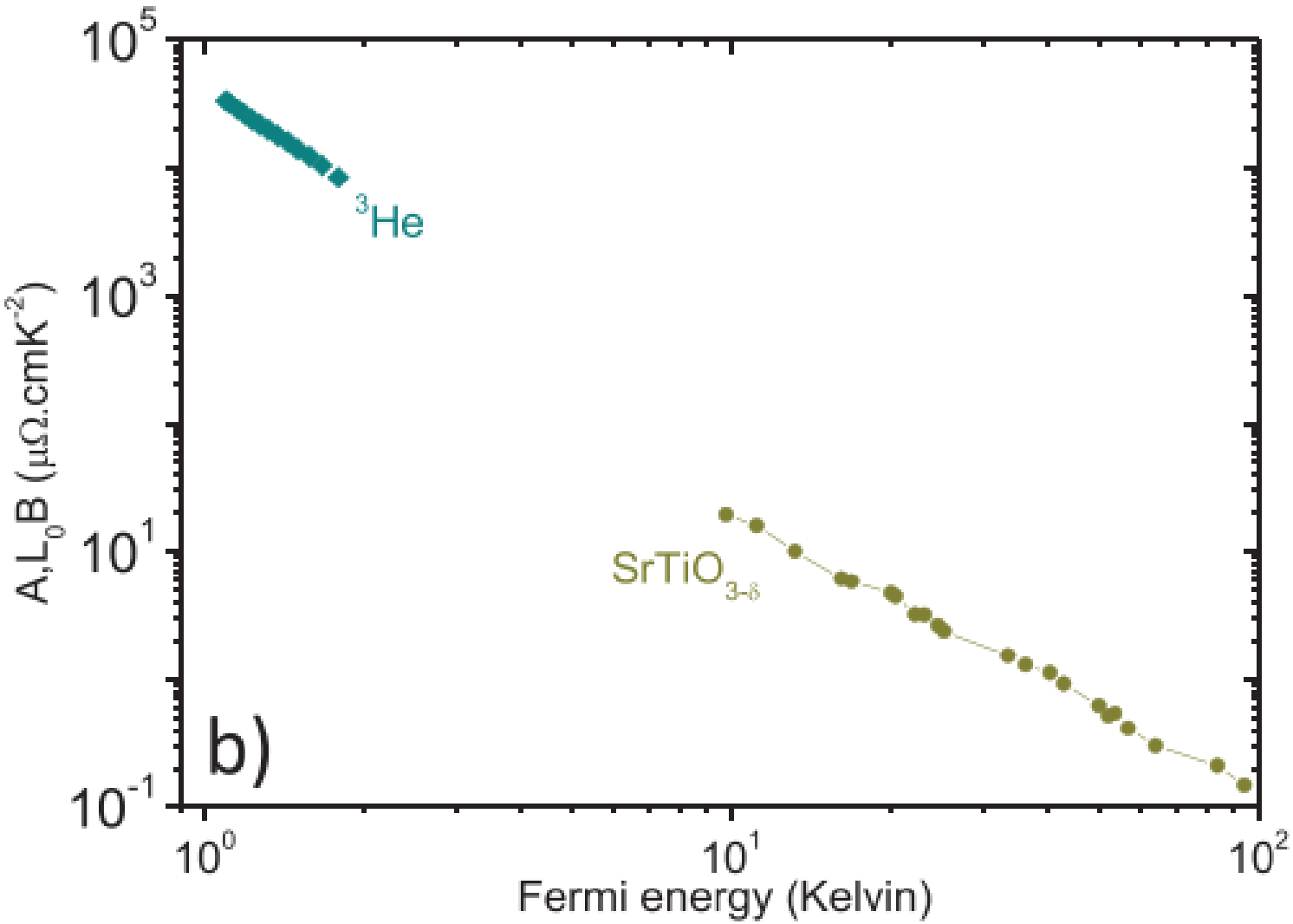


^{3}He and metals



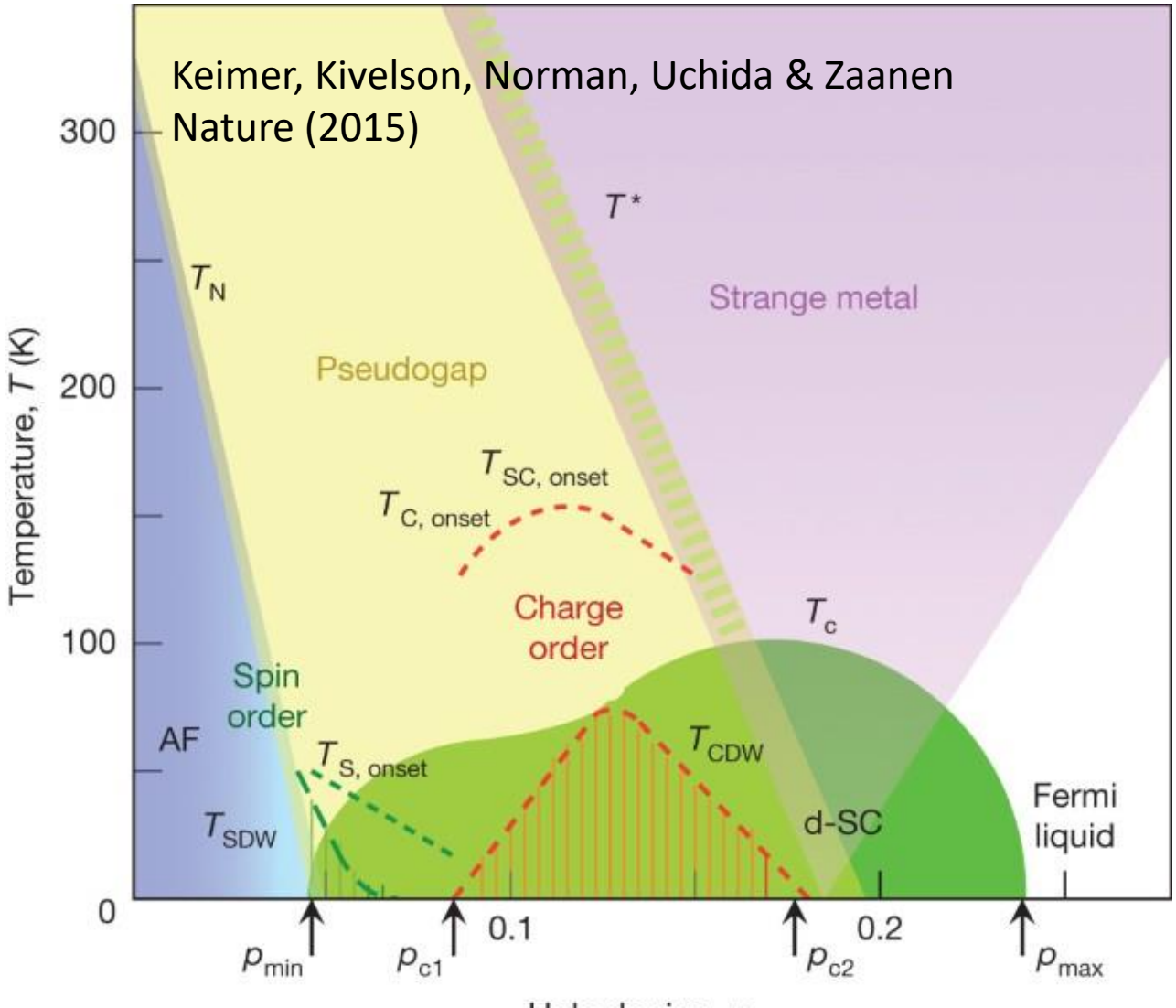
T-square electrical thermal resistivity

^{3}He and metals



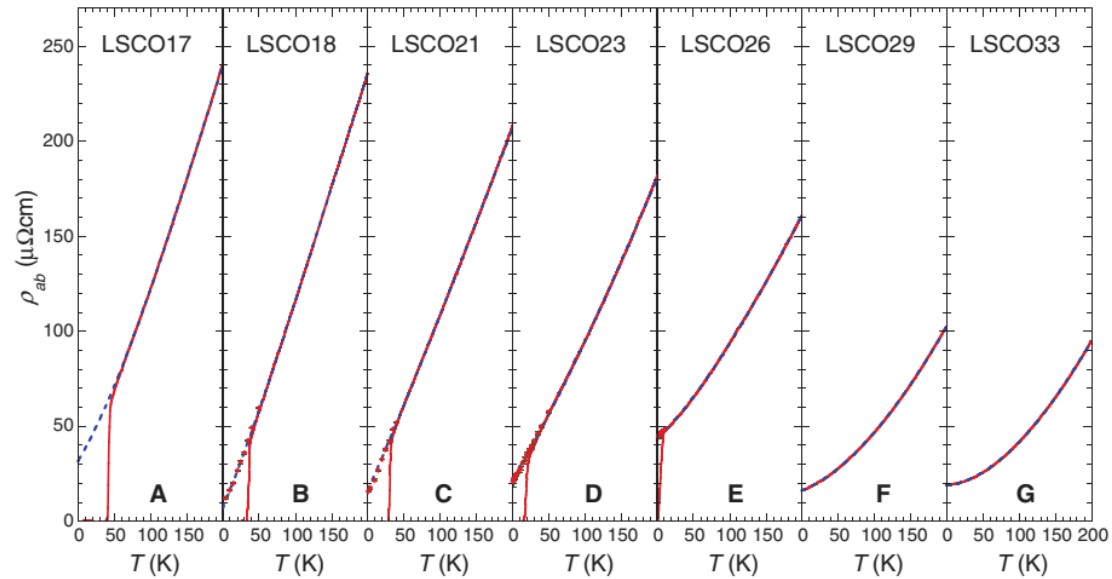
T-square electrical resistivity can occur without Umklapp

Fermion-fermion scattering in cuprates



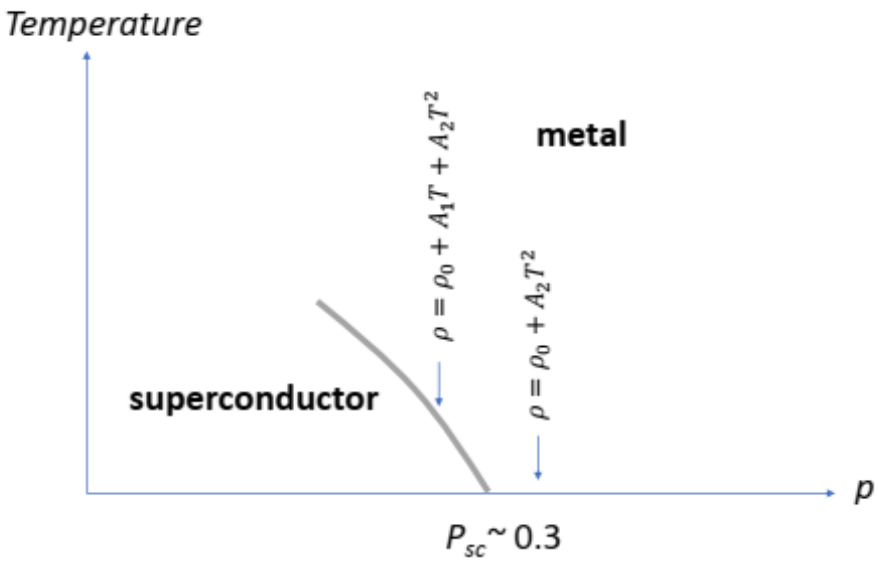
SCIENCE VOL 323 30 JANUARY 2009

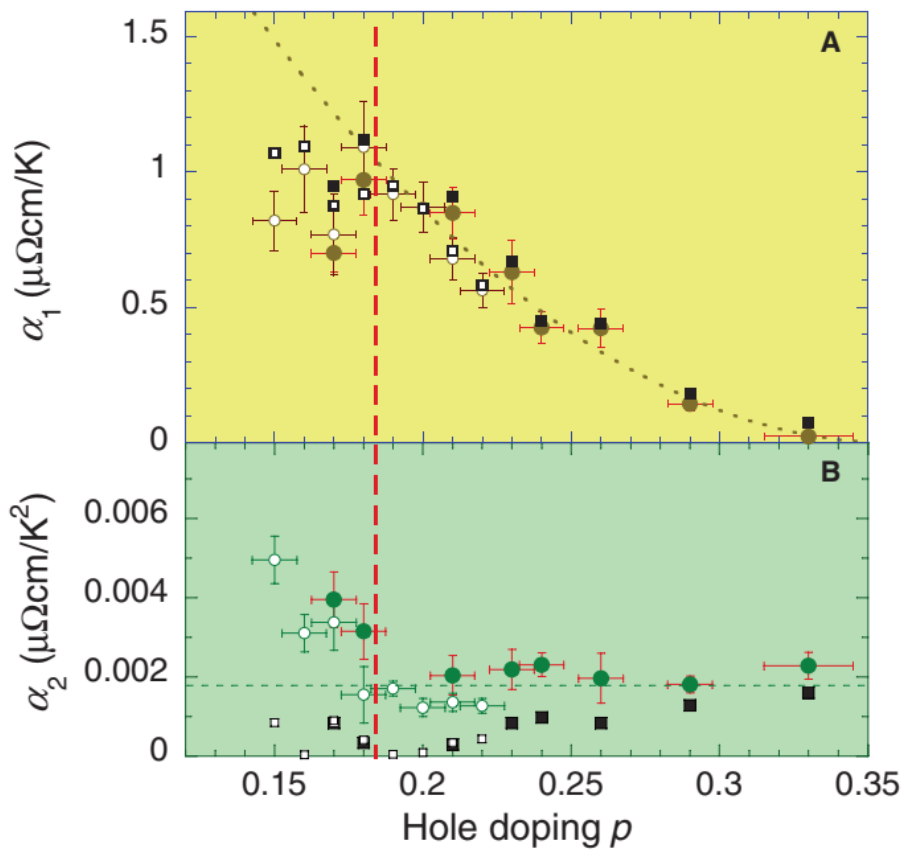
Why does resistivity ceases to be T-square inside the dome?



Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4]* H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1]†

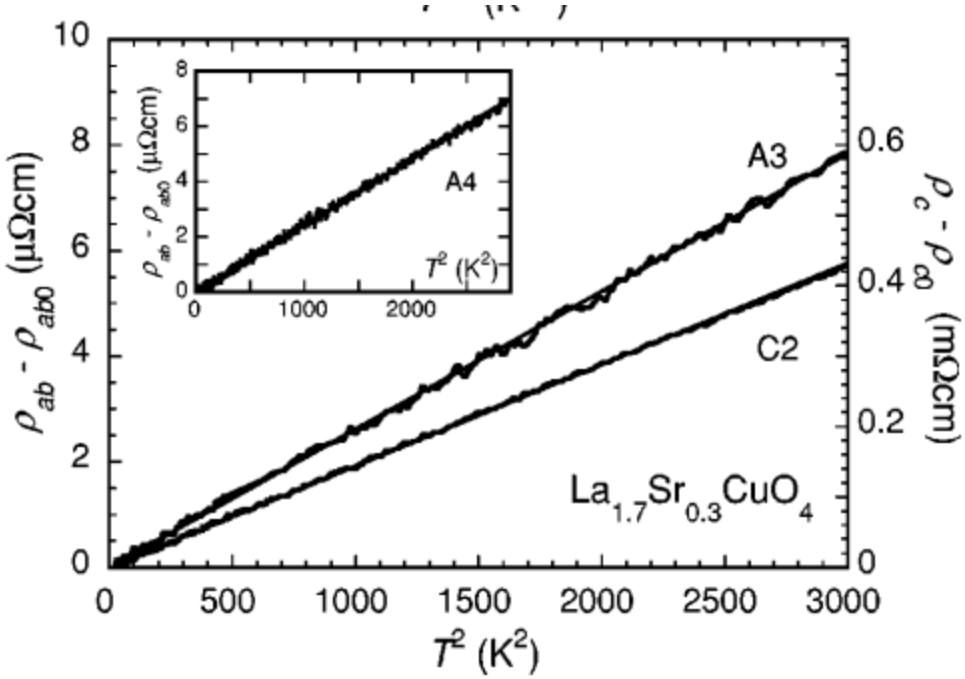


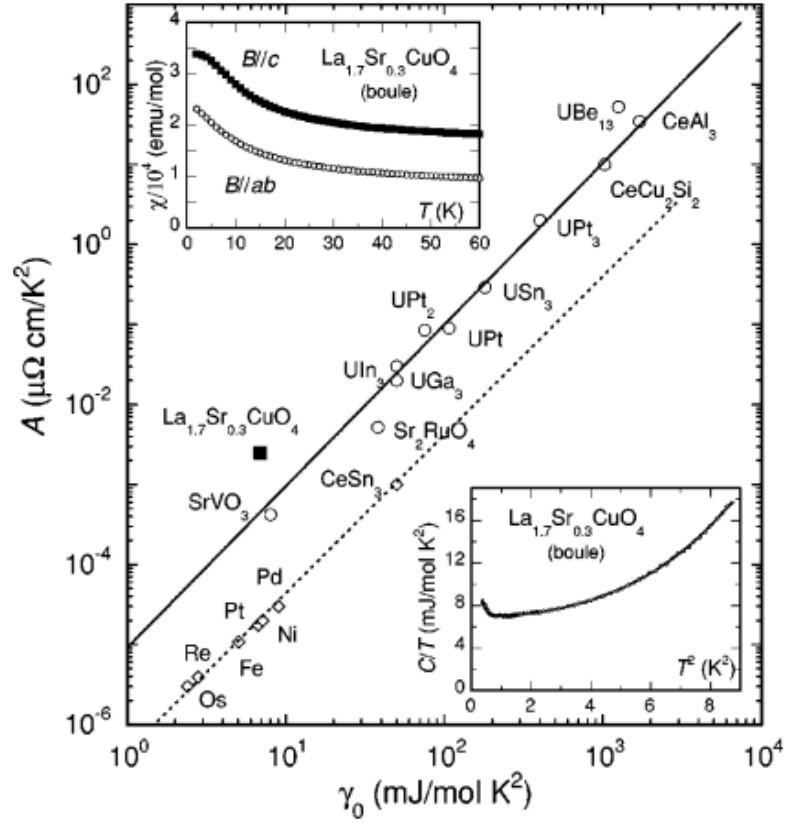


PHYSICAL REVIEW B **68**, 100502(R) (2003)

Electronic ground state of heavily overdoped nonsuperconducting $La_{2-x}Sr_xCuO_4$

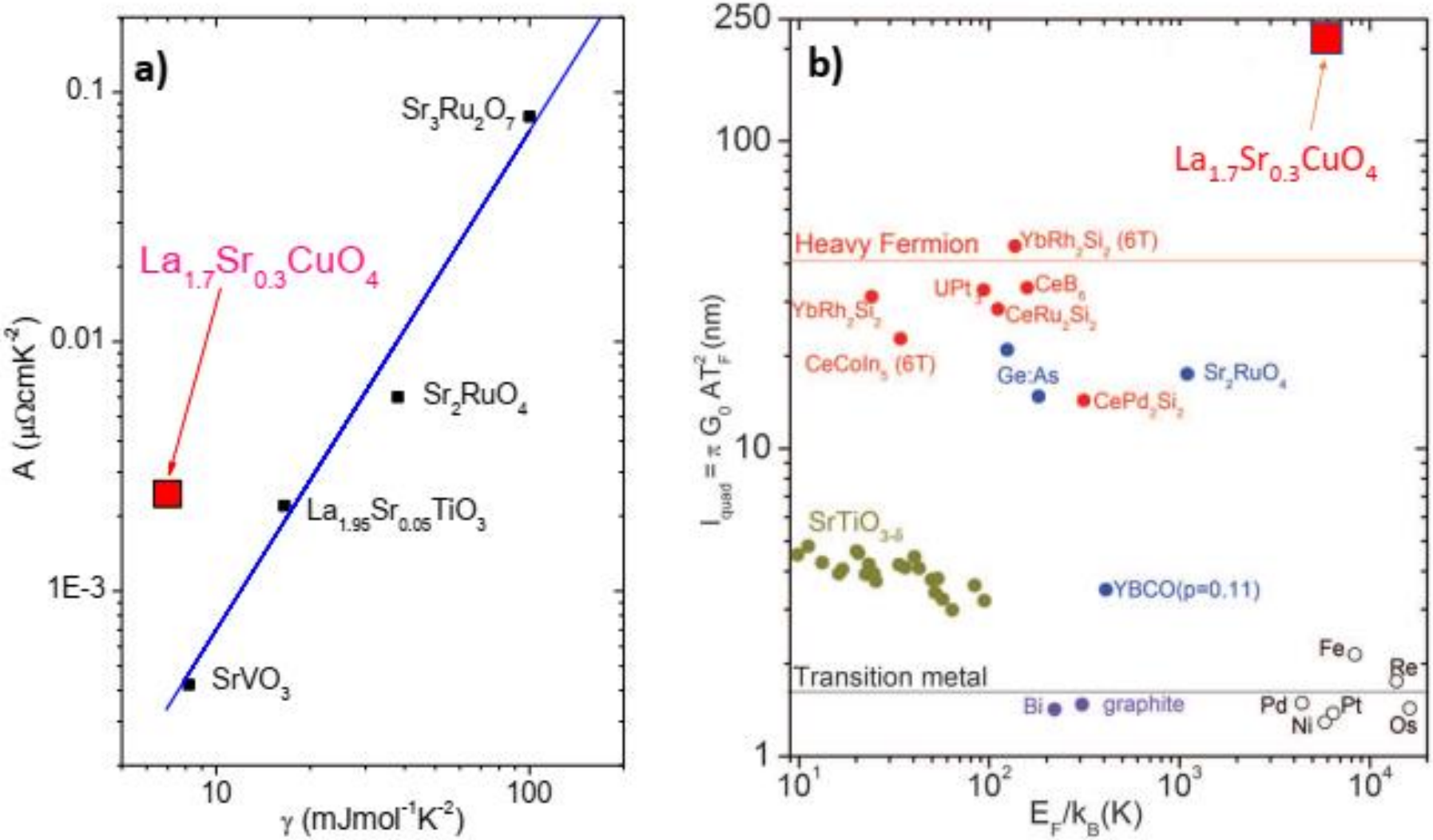
S. Nakamae,[1,*] K. Behnia,[1] N. Mangkorntong,[2] M. Nohara,[2] H. Takagi,[2,3,4] S. J. C. Yates,[5,†] and N. E. Hussey[5]





The prefactor of T-square resistivity in heavily overdoped LSCO is unusually large.

Electron-electron scattering in overdoped cuprates is comparatively larger than any other known Fermi liquid!



Two time scales

Time for fermions to scatter off each other: $\tau_{int.} = \tau_{ff} \left(\frac{k_B T}{\epsilon_F}\right)^2$

Time required for two fermions to commute: $\tau_{FD} = \frac{\hbar}{\epsilon_F}$

$$\zeta = \frac{\tau_{FD}}{\tau_{int.}}$$

$$\boxed{\zeta = (\tau_{ff} T^2)^{-1} \frac{\hbar E_F}{k_B^2}}$$

How large can ζ be?

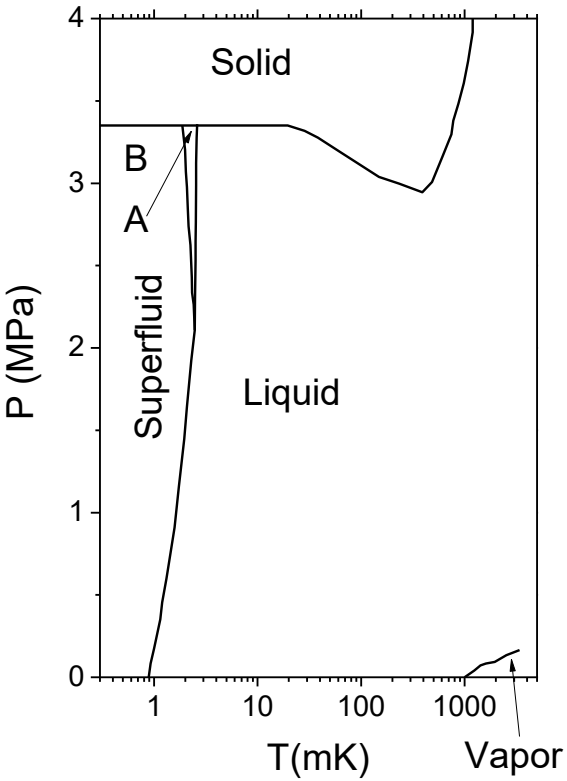
A comparison

System	$k_F(nm^{-1})$	$m^\star/m_0$	E_F (K)	ζ
^{3}He (p=0)	7.9	2.8	1.8	35
^{3}He (p=3.4MPa)	8.9	5.8	1.1	60
$La_{1.67}Sr_{0.33}CuO_4$	5.6	5	5900	24-61
UPt_3	5	16-130	90	≈ 10
Sr_2RuO_4	5	3.3-16	1800	≈ 16
Sb	0.8	0.07-1	1100	≈ 0.1
$SrTiO_{3-\delta}$ (n=$4\times10^{17}cm^{-3}$)	0.23	1.8	18	≈ 0.1

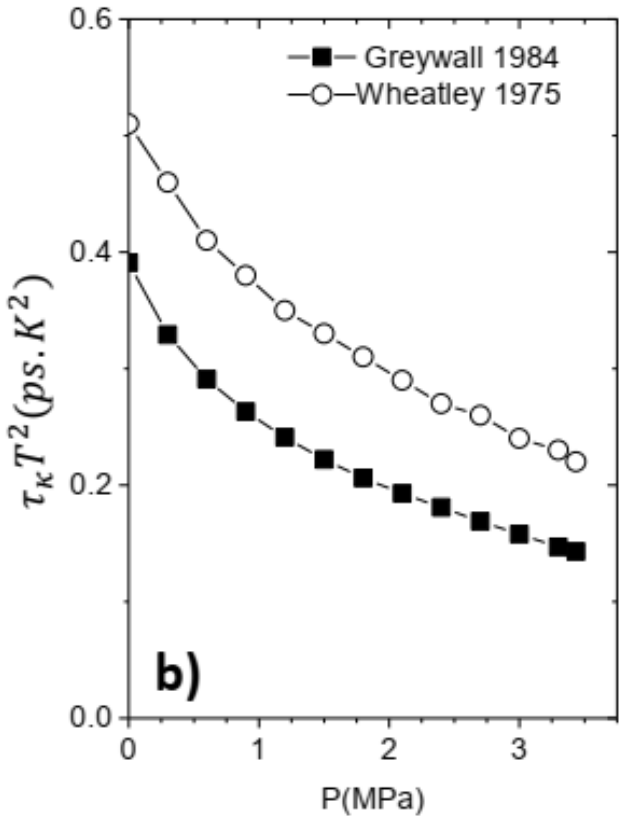
This magnitude for ζ is exceptionally large in cuprates.

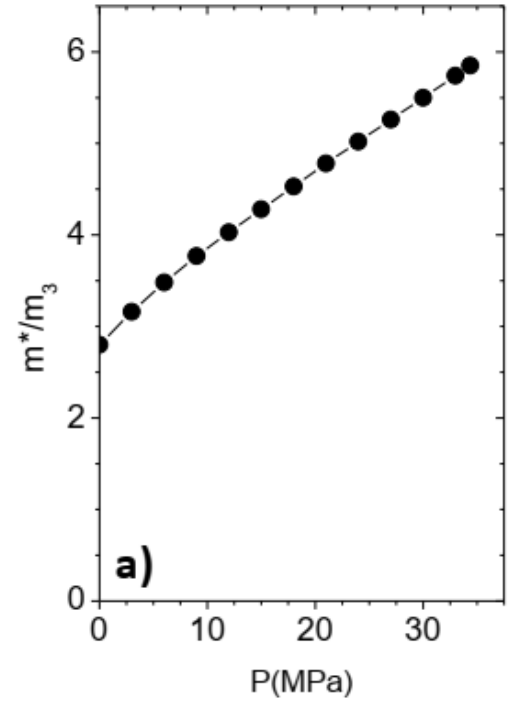
Attention: Only in a single-band FL, ζ can be determined without ambiguity.

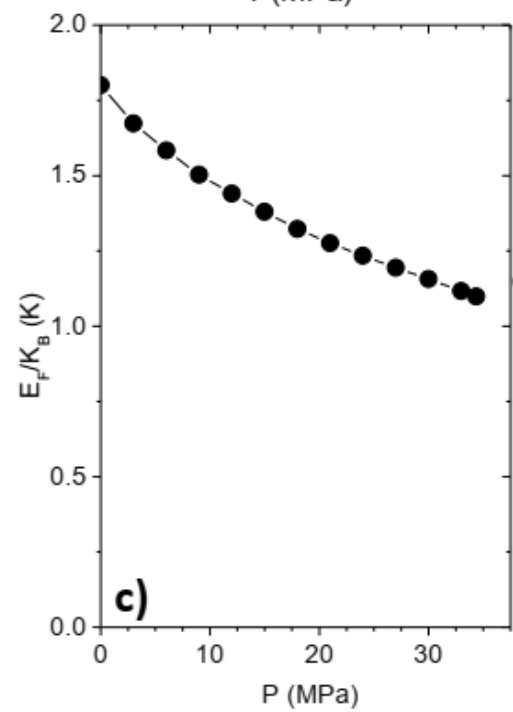
^{3}He



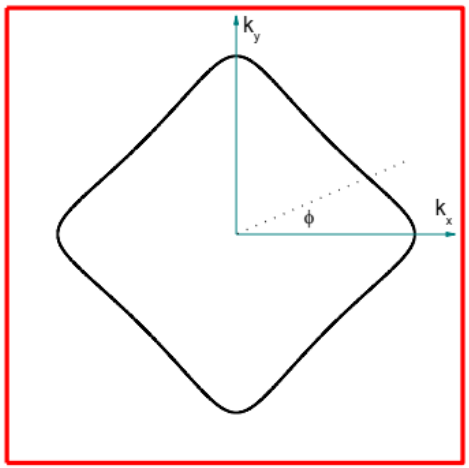
"The melting transition in 3He is our best example of a Mott transition." P. W. Anderson



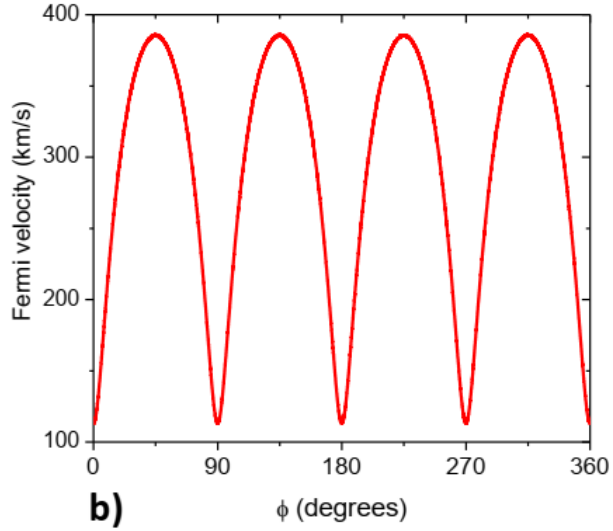




The Fermi surface of $La_{1.67}Sr_{0.33}CuO_4$



a)



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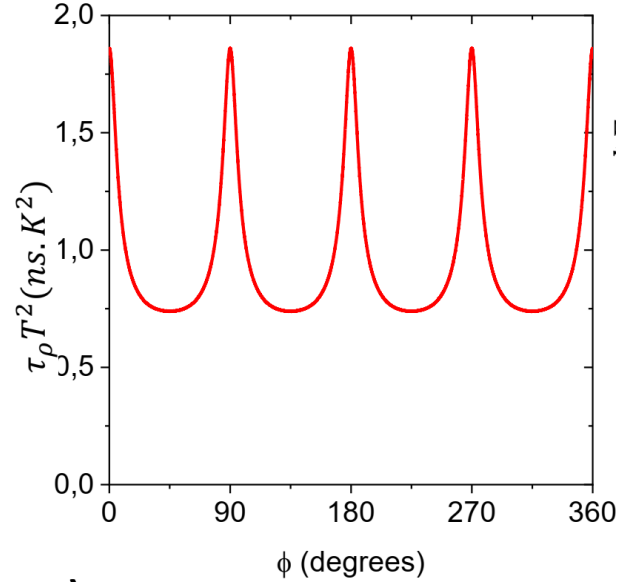
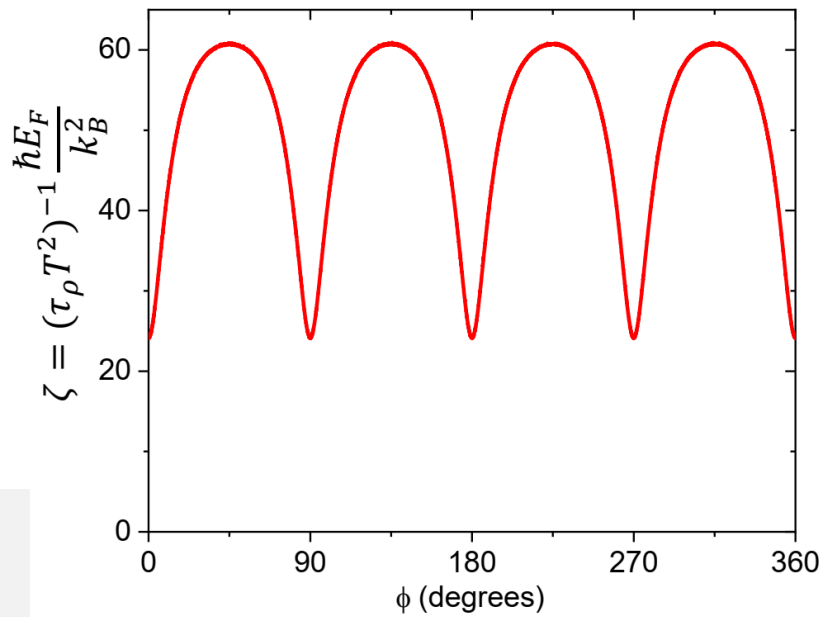


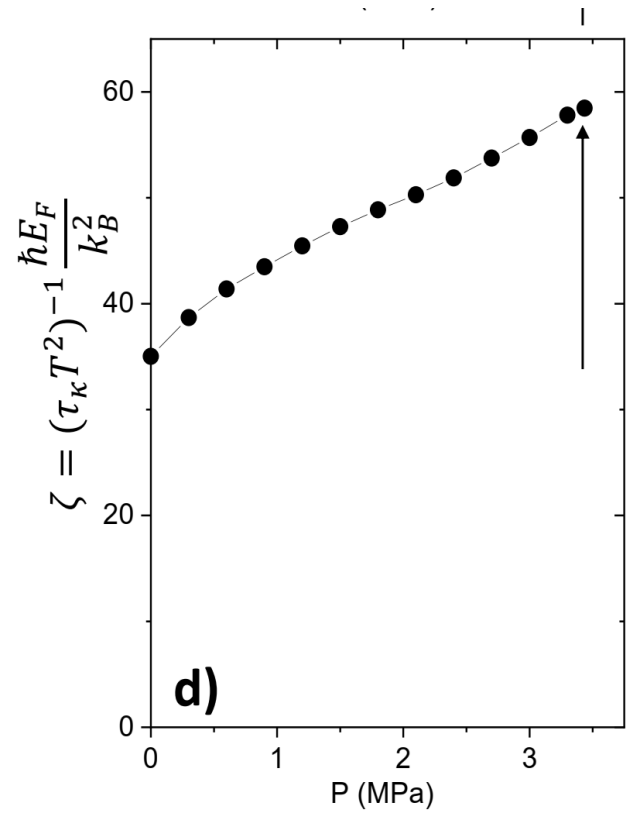
$$\tau_\rho T^2 = A^{-1} \frac{\hbar}{e^2} \frac{2\pi c}{k_F v_F}$$

Unbearable shortness of scattering time for nodal quasi-particles



Nodal quasi-particles

$\zeta \simeq 60$



^{3}He atoms the onset of solidification

$\zeta \simeq 60$

Shall Landau parameters diverge at the Mott transition?

Two-particle Fermi liquid parameters at the Mott transition: Vertex divergences, Landau parameters, and incoherent response in dynamical mean-field theory

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[2]*Radboud University, Institute for Molecules and Materials, NL-6525 AJ Nijmegen, The Netherlands*

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(Received 6 November 2018; published 13 June 2019)

We consider the interaction-driven Mott transition at zero temperature from the viewpoint of microscopic Fermi liquid theory. To this end, we derive an exact expression for the Landau parameters within the dynamical mean-field theory (DMFT) approximation to the single-band Hubbard model. At the Mott transition, the symmetric and the antisymmetric Landau parameters diverge. The vanishing compressibility at the Mott

But, is this realistic?

What happens if nodal quasi-particles freeze out of the Fermi surface?

- The phase space of scattering between one electron (inside the Fermi sea) and another (out of it) will be linear (and not quadratic) in T!

- With decrease in carrier concentration, the linear term will grow and the T-square term will hit a ceiling!

Theory of the strange metal $Sr_3Ru_2O_7$

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Edited by Zachary Fisk, University of California, Irvine, CA, and approved December 21, 2019 (received for review September 2, 2019)

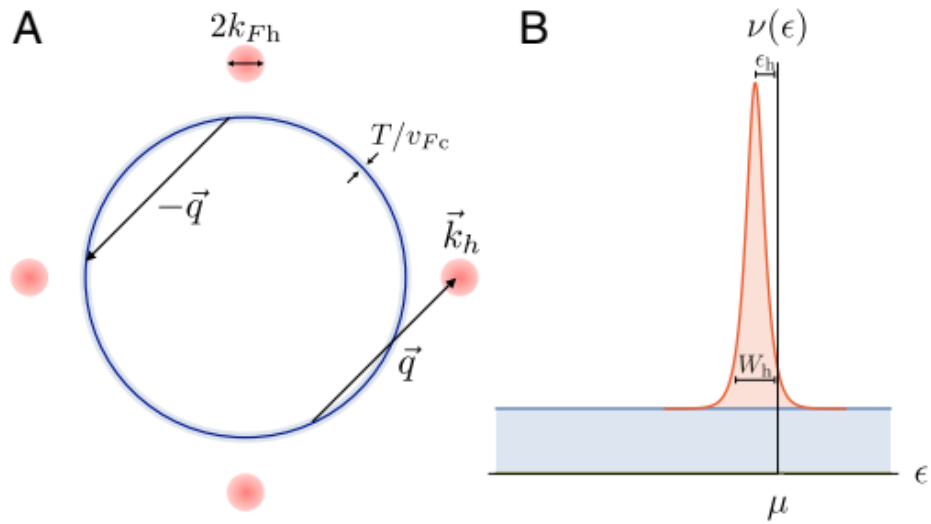


Fig. 3. (*A*) Illustration of cc → ch scattering, in which one of the fermions is scattered from the cold Fermi surface (blue) into the hot region (red). In general, the hot regions could also be on the same Fermi sheet as the cold fermions. (*B*) A sharp peak in the density of states $\nu(\epsilon)$ close to the chemical potential, due to the hot fermions.

A linear scattering rate of $\sim \frac{k_B T}{\hbar}$ is expected when a degnerate electron is scattered by a non-degenerate electron.

Summary

- There is a deep (hitherto unnoticed) connection between the amplitude of T-square resistivity in metals and in ^{3}He.

- Implications for two puzzles : i) why does T-square resistivity persist without Umklapp; and ii) why its prefactor can be guessed knowing the Fermi energy.

- The dimensionless amplitude of T-square resistivity in heavily-doped LSCO is significantly larger than in other Fermi liquids. If the latter has an upper boundary, then a subset of carriers will meet it first.

Quantum critical behaviour in magic-angle twisted bilayer graphene

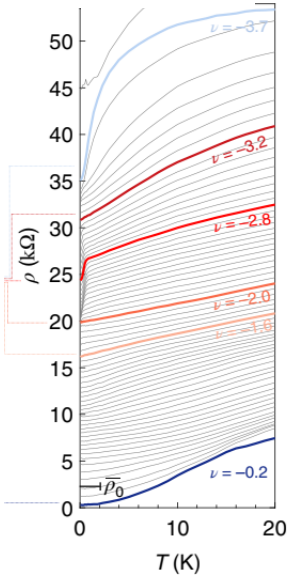
Alexandre Jaoui[1]✉, Ipsita Das[1], Giorgio Di Battista[1], Jaime Díez-Mérida[1], Xiaobo Lu[1], Kenji Watanabe[2], Takashi Taniguchi[2], Hiroaki Ishizuka[3,4], Leonid Levitov[4] and Dmitri K. Efetov[1]✉

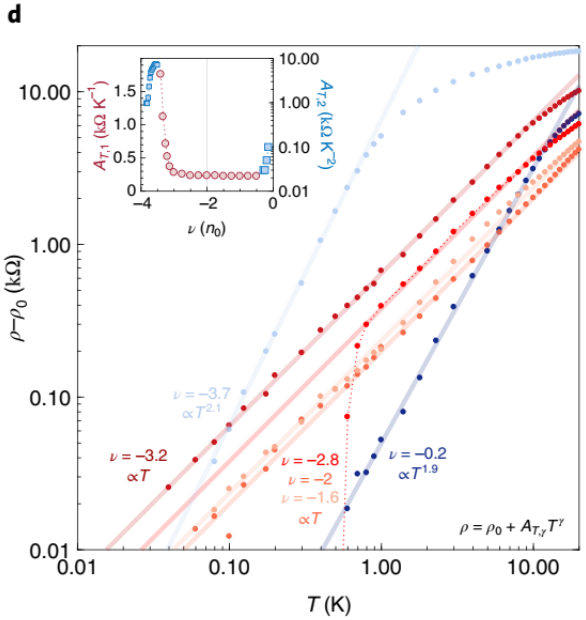
T_F=24 K

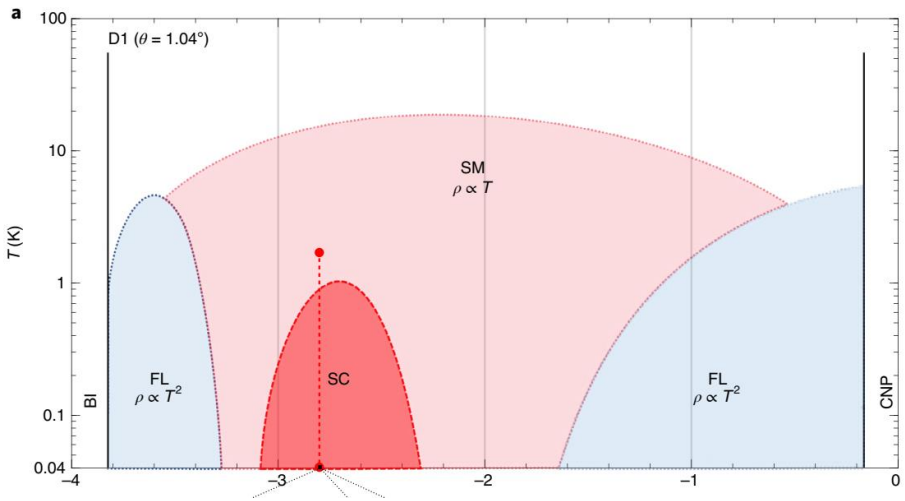
k_F = 6.2e+07 m^{-1}

A ~ 100 Ω/K^2

$$\zeta = (\tau_\kappa T^2)^{-1} \frac{\hbar E_F}{k_B^2}$$







Can Fermion-fermion scattering become arbitrarily large?

$$\zeta = (\tau_\kappa T^2)^{-1} \frac{\hbar E_F}{k_B^2}$$

- **No, t**he Landau parameters of a Fermi liquid are two-particle correlators. They can become large, but note infinite! There should be cut-off.

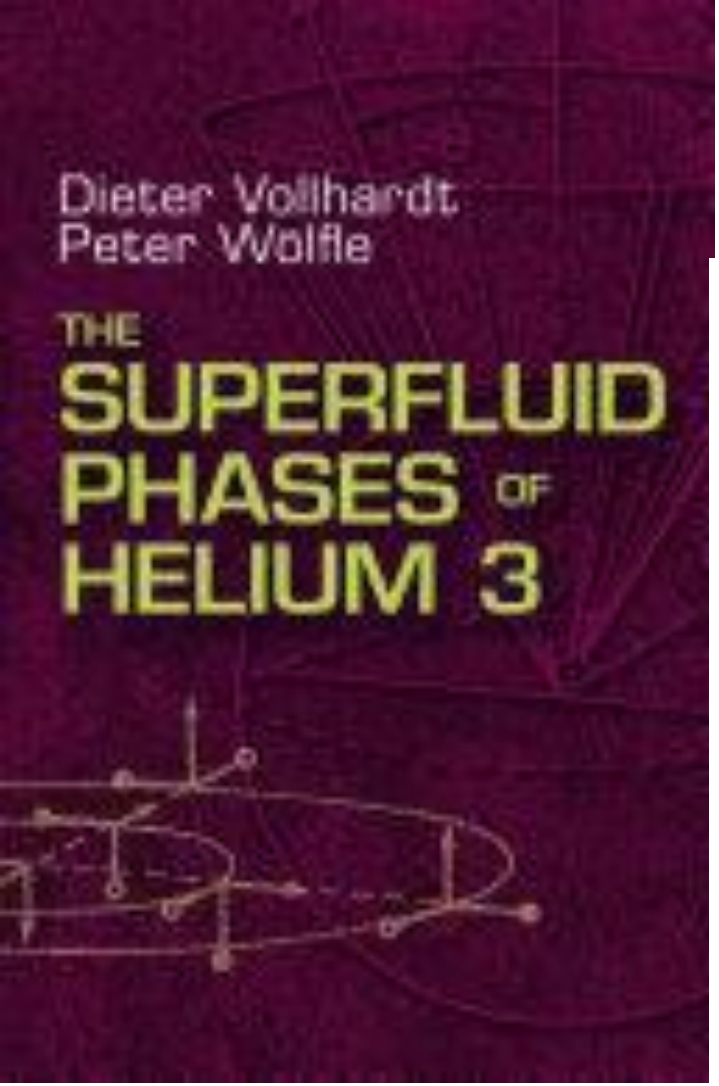
PHYSICAL REVIEW B VOLUME 29, NUMBER 9 1 MAY 1984

Thermal conductivity of normal liquid ^{3}He

Dennis S. Greywall
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
(Received 13 October 1983)

TABLE V. Smoothed zero-temperature parameters derived from the measured thermal conductivity. The quantities τ_κ and v_F are based on m_3^* values from Ref. 1. The quantity b is defined by Eq. (10).

P (bar)	V (cm^3/mol)	p_F (10^{-20} g cm/sec)	v_F (10^3 cm/sec)	κT (erg/sec cm)	$\tau_\kappa T^2$ (10^{-12} sec K^2)	b (cm sec/erg K)
0	36.84	8.28	6.00	29.08	0.391	−0.42
3	33.87	8.52	5.42	23.36	0.329	−0.60
6	32.07	8.67	5.04	19.89	0.291	−0.78
9	30.76	8.79	4.72	17.37	0.263	−0.97
12	29.71	8.89	4.47	15.35	0.241	−1.19
15	28.86	8.98	4.24	13.71	0.222	−1.42
18	28.13	9.06	4.03	12.30	0.206	−1.69
21	27.56	9.12	3.86	11.20	0.193	−1.96
24	27.06	9.18	3.71	10.24	0.181	−2.26
27	26.58	9.23	3.57	9.32	0.169	−2.63
30	26.14	9.28	3.44	8.47	0.158	−3.06
33	25.71	9.34	3.30	7.64	0.147	−3.62
34.36	25.54	9.36	3.24	7.31	0.143	−3.89



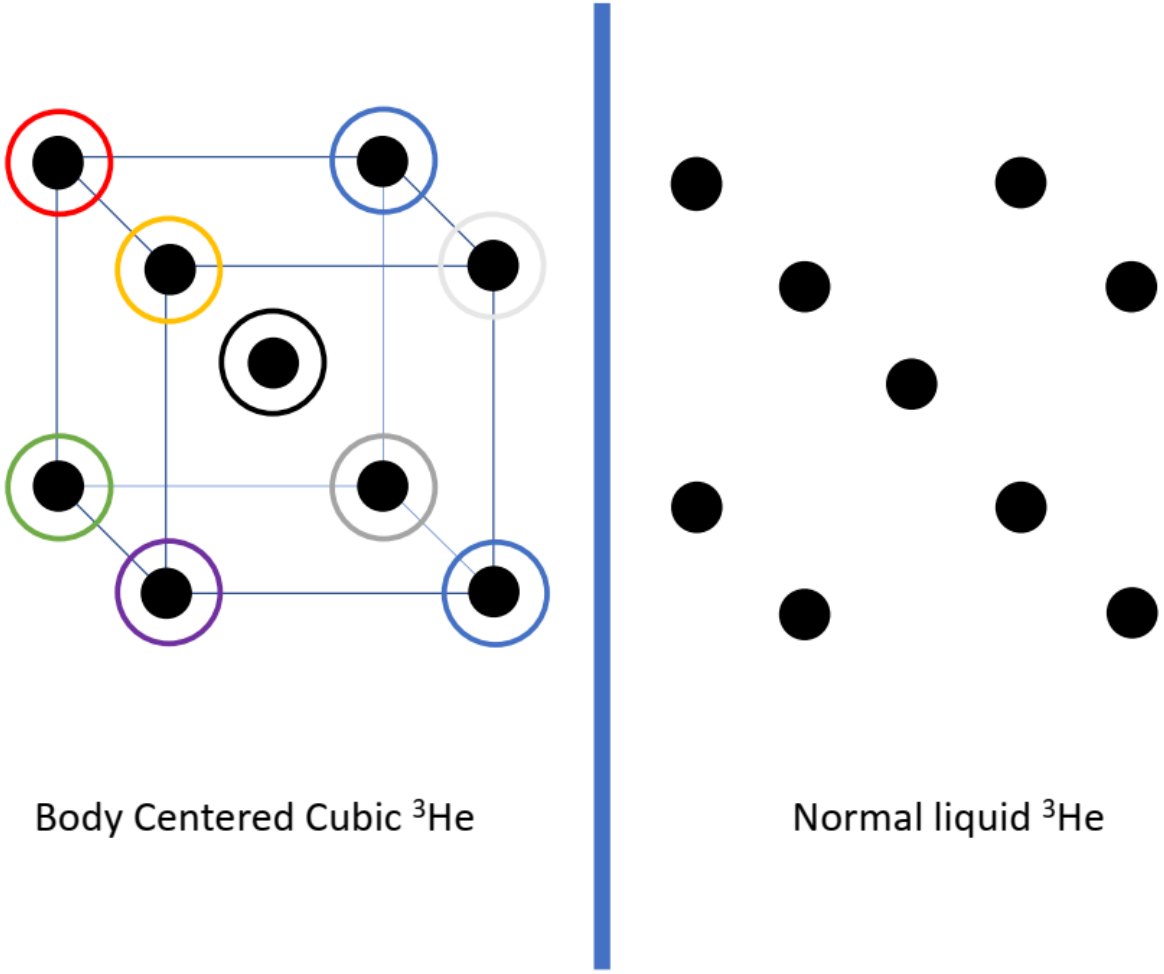
Landau parameters amplify with pressure!

Table 2.1 Values of the Landau parameters F_0^a, F_0^s and F_1^s in ^{3}He together with the molar volume and the effective mass ratio m^*/m for pressures between $P=0$ and melting pressure. The values of V, m^*/m and F_1^s are taken from Greywall (1986), whereas F_0^a, F_0^s are from Wheatley (1975) but corrected for the newly determined m^*/m. At the highest pressure ($P=34.39$ bar) this recalculation was done using Wheatley's values at $P=34.36$ bar.

P (bar)	V (cm^3)	m^*/m	F_1^s	F_0^s	F_0^a
0	36.84	2.80	5.39	9.30	−0.695
3	33.95	3.16	6.49	15.99	−0.723
6	32.03	3.48	7.45	22.49	−0.733
9	30.71	3.77	8.31	29.00	−0.742
12	29.71	4.03	9.09	35.42	−0.747
15	28.89	4.28	9.85	41.73	−0.753
18	28.18	4.53	10.60	48.46	−0.757
21	27.55	4.78	11.34	55.20	−0.755
24	27.01	5.02	12.07	62.16	−0.756
27	26.56	5.26	12.79	69.43	−0.755
30	26.17	5.50	13.50	77.02	−0.754
33	25.75	5.74	14.21	84.79	−0.755
34.39	25.50	5.85	14.56	88.47	−0.753

Large, yet finite, at the solidification pressure!

Distinguishability and solidification

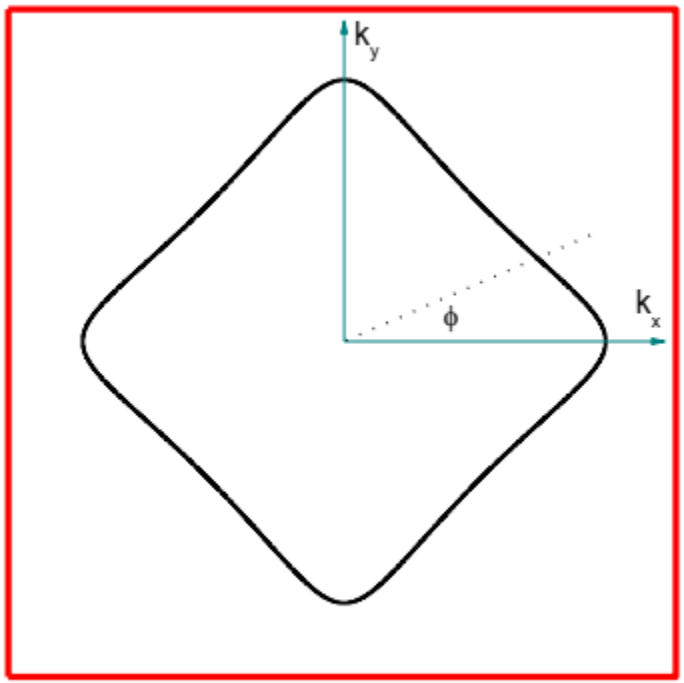


Dynamical indistinguishability and statistics in quantum fluids

Alessio Zaccone[1,2] and Kostya Trachenko[3]
[1] *Department of Physics "A. Pontremoli", University of Milan, via Celoria 16, 20133 Milan, Italy.*
[2] *Cavendish Laboratory, University of Cambridge, JJ Thomson Avenue, CB30HE Cambridge, U.K. and*
[3] *School of Physics and Astronomy, Queen Mary University of London, Mile End, London, U.K.*

For a system to qualify as a quantum fluid, quantum-statistical effects should operate in addition to quantum-mechanical ones. Here, we address the hitherto unexplored dynamical condition for the quantum-statistical effects to be manifested, and consider particle exchange events in the gaslike regime of fluid dynamics as a dynamical process with an intrinsic time scale. We subsequently

The Fermi surface of $La_{1.67}Sr_{0.33}CuO_4$



a)

The Fermi surface seen by ARPES can be described with a tight-binding model

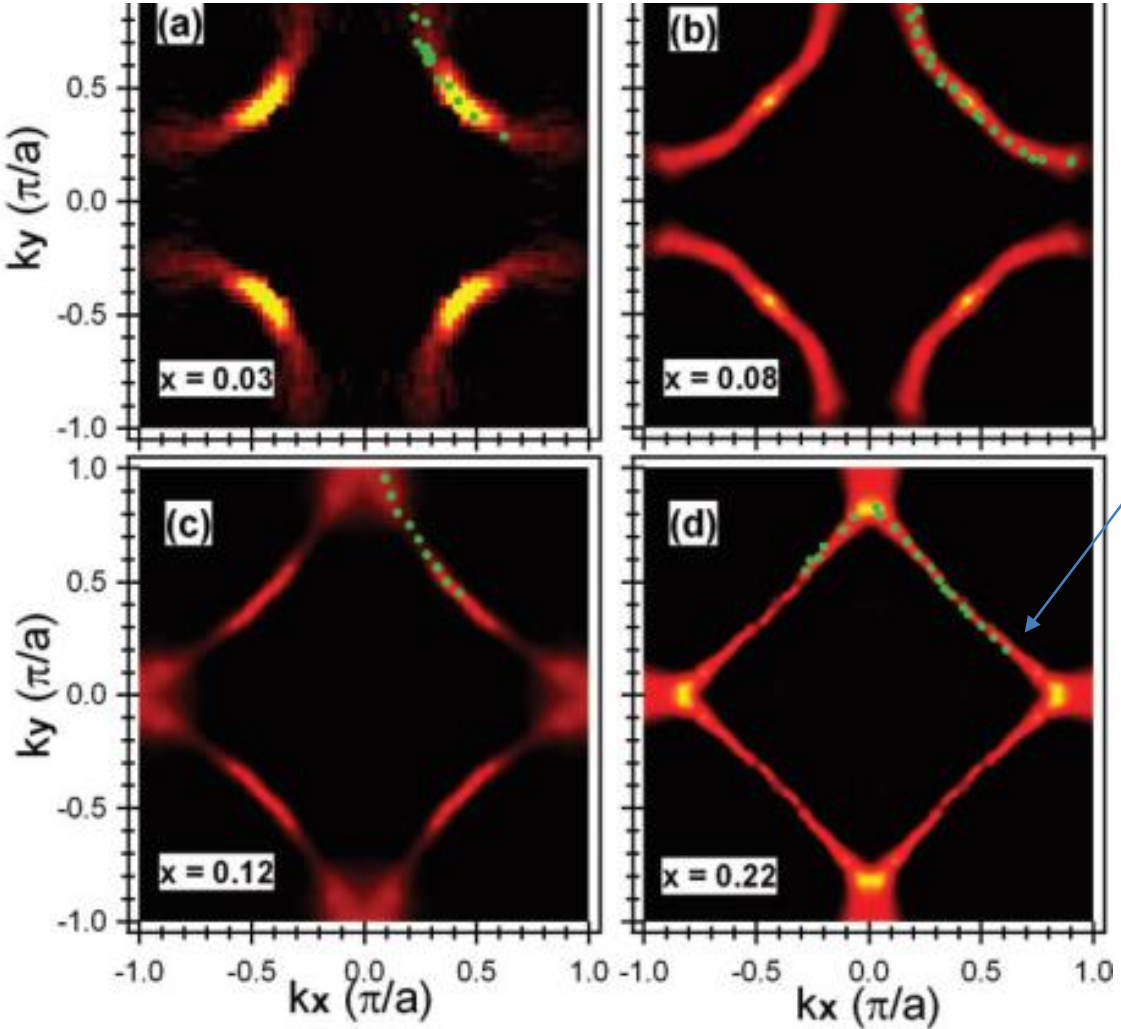
$$\begin{aligned}\varepsilon_{\mathbf{k}} + \mu = &-2t[\cos(k_x x) + \cos(k_y y)] \\ &- 4t_1 \cos(k_x x)\cos(k_y y) \\ &- 2t_2[\cos(2k_x x) + \cos(2k_y y)] \\ &- 4t_3(\cos(2k_x x)\cos(k_y y) + \cos(k_x x)\cos(2k_y y)) \\ &- 4t_4 \cos(2k_x x)\cos(2k_y y),\end{aligned}$$

$$t = 1.72eV$$

$$\frac{t_1}{t} = -0.136 \qquad \frac{t_2}{t} = 0.068 \qquad \frac{t_3}{t} = 0 \qquad \frac{t_4}{t} = -0.02$$

The Fermi surface and band folding in $La_{2-x}Sr_xCuO_4$, probed by angle-resolved photoemission

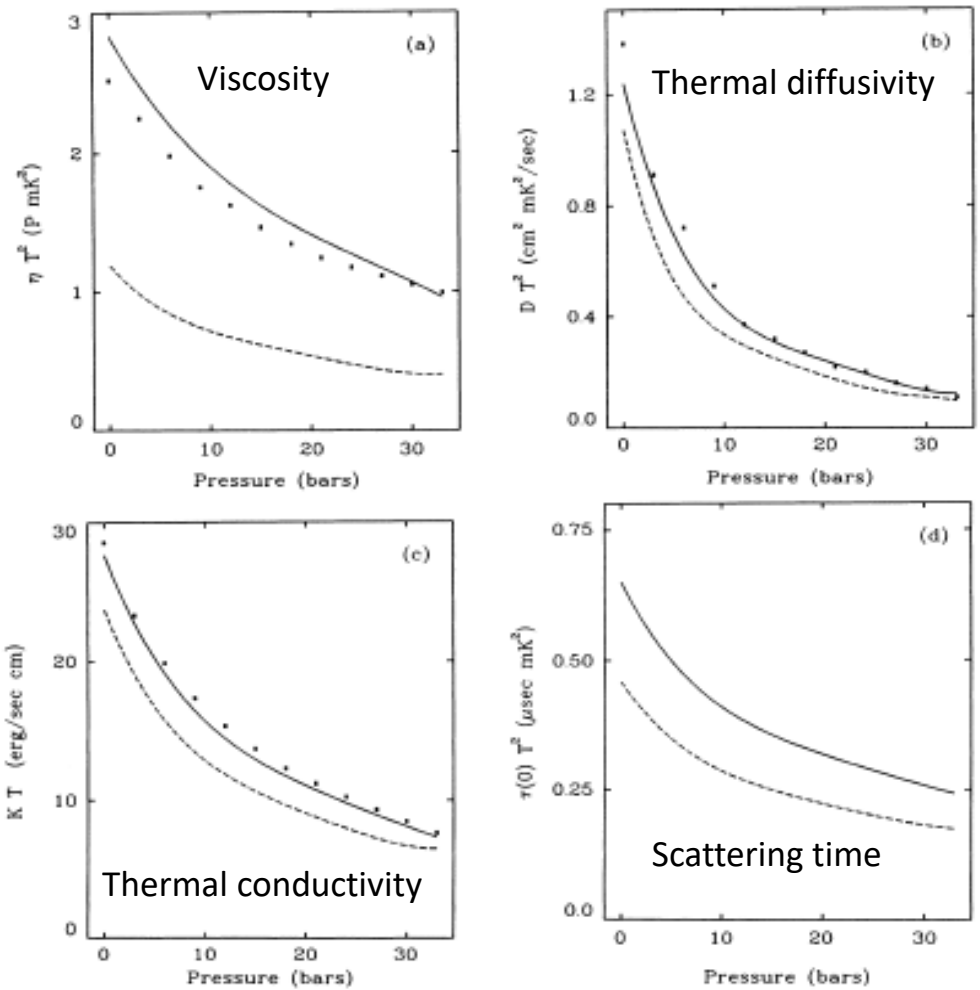
E Razzoli[1,2], Y Sassa[3], G Drachuck[4], M Månsson[2,3], A Keren[4], M Shay[4], M H Berntsen[5], O Tjernberg[5], M Radovic[1,2], J Chang[3], S Pailhès[6], N Momono[7], M Oda[8], M Ido[8], O J Lipscombe[9], S M Hayden[9], L Patthey[1], J Mesot[2,3] and M Shi[1,10]



Above 0.22, it is a simple Fermi surface

Pressure dependence

Ainsworth & Bedell, 1987



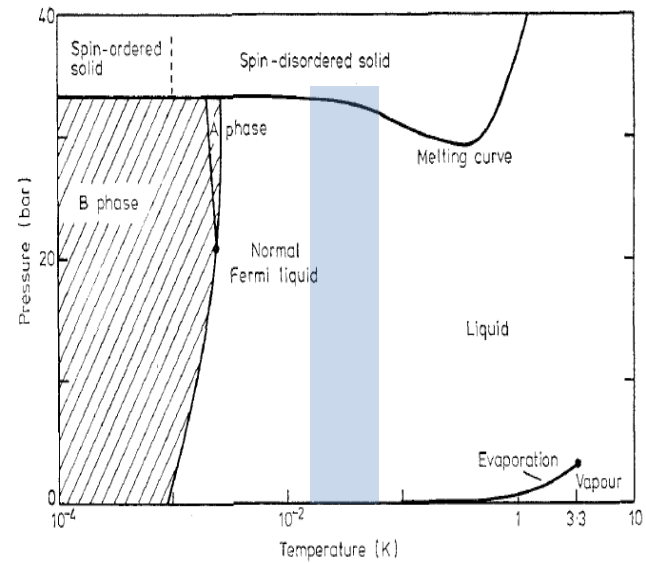
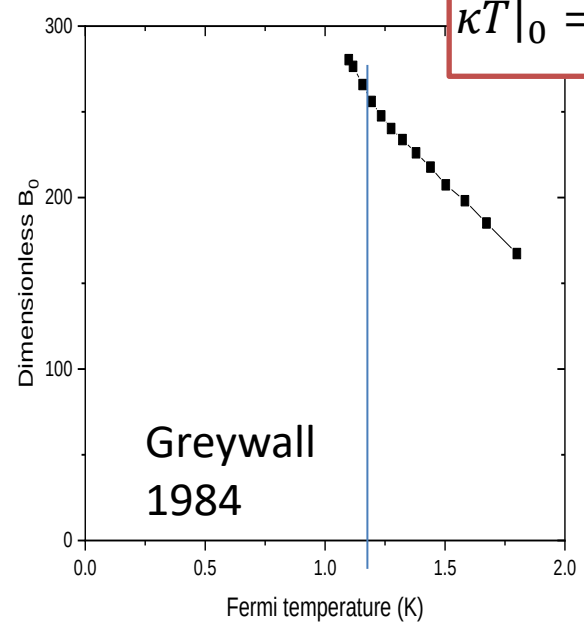


Figure 2. Phase diagram of ^{3}He.

$$\kappa T|_0 = \frac{E_F^2 k_F}{B_0\,\hbar}$$



Cross section B_0 and the length l_{quad}

$$\kappa T|_0 = \frac{1}{B_0}\frac{E_F^2 k_F}{\hbar}$$

$$B_0\left(\frac{\hbar}{k_F E_F^2}\right)\frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2 = B_2 = \ell_{quad}\left(\frac{\hbar}{e^2}\right)$$

$$B_0\frac{\pi^2}{3} = k_F \ell_{quad}$$

$$B_0 = \frac{\pi}{6}\frac{\ell_{quad}}{\lambda_F}$$

Physical meaning of phenomenological l_{quad}

Electron-electron cross-section

$$A = \frac{P_F \sigma_{CS}}{e^2}\left(\frac{k_B}{E_F}\right)^2$$

Mott (1990)

$$A = \frac{\hbar}{e^2}\left(\frac{k_B}{E_F}\right)^2 \ell_{quad}$$

$$\boxed{\ell_{quad} = \sigma_{CS} k_F}$$

It represents the collision cross section divided by the Fermi wavelength!
Expected to be larger in more correlated systems!

The law of Wiedemann and Franz

Lorenz number

$$\boxed{\mathrm{L} = \frac{\kappa}{\sigma T}} \quad \boxed{\mathrm{L} = L_0}$$

Sommerfeld ratio

$$\boxed{L_0 = \frac{\pi^2}{3}\left(\frac{\kappa_B}{e}\right)^2} = 2.45\ 10^{-8} \mathrm{W}\ \Omega\ /\ \mathrm{K}^2$$

- **The ratio of quanta of charge and entropy!**
- **Why power of two? Because the quanta are present in both the Onsager force AND the Onsager flux!**
- **Why $\pi^2/3$?** Ask Sommerfeld!