Topological phases with Rydberg atoms

Hans Peter Büchler

Institute for theoretical physics III,
and Center for Integrated Quantum Science and Technology

Universität Stuttgart, Germany

Experimental collaboration:

S. de Léséleuc, V. Lienhard, P. Scholl,
D. Barredo, T. Lahaye, and A. Browaeys

Research group:

Felix Roser, **Johannes Mögerle, Nastasia Makki**,
Katharina Brechtelsbauer, Jan Kumlin, Tobias Ilg,
Kevin Kleinbeck, **Sebastian Weber, Rukmani Bai**,
Nicolai Lang

DFG Deutsche Forschungsgemeinschaft

ANR-DFG: Systems of Rydberg atoms to realize Bosonic Topological Insulators

QRyd Demo

Federal Ministry of Education and Research

Rydberg Quantum Computer Demonstrator

erc

Strongly interacting Rydberg slow light polaritons

Baden-Württemberg Stiftung

FOR 2247

iRyd

How to characterize states of matter?

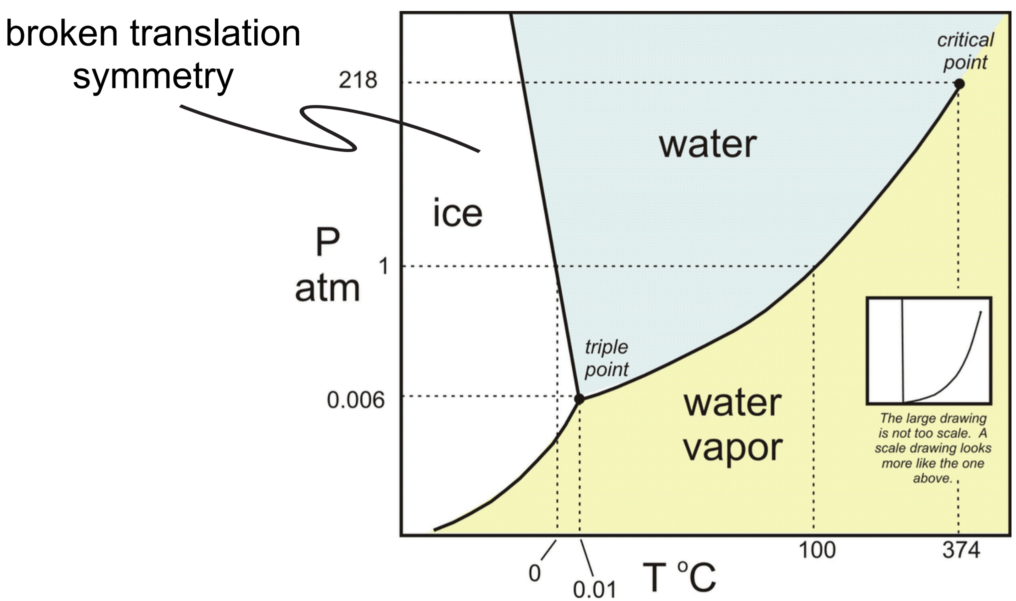
Characterize phases of matter

- based on Landau paradigm
- symmetry breaking
- order parameter and long range order
- thermal and quantum phase transitions

Extremely successful

- band insulator/Fermi liquids
- crystals
- superfluids
  - Bose-Einstein condensate
  - superfluid Helium
- superconductors
- ferromagnets and anti-ferromagnets

phase diagram of water



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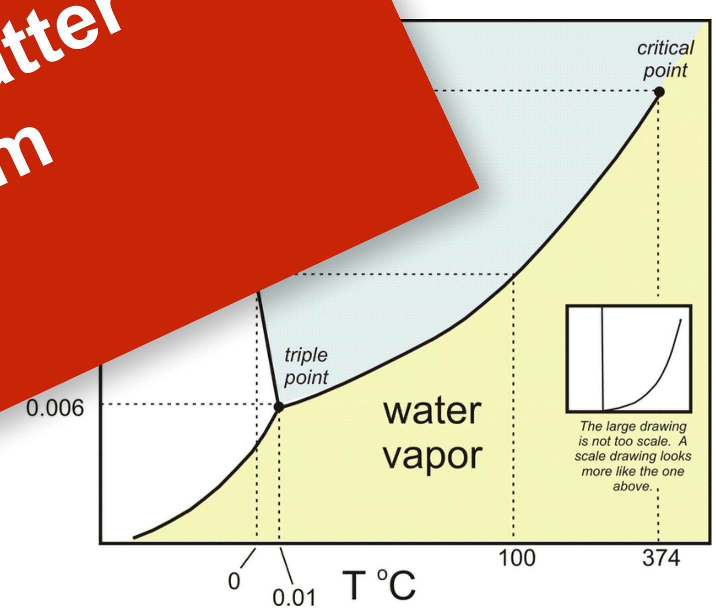


But, not all states of matter follow this paradigm

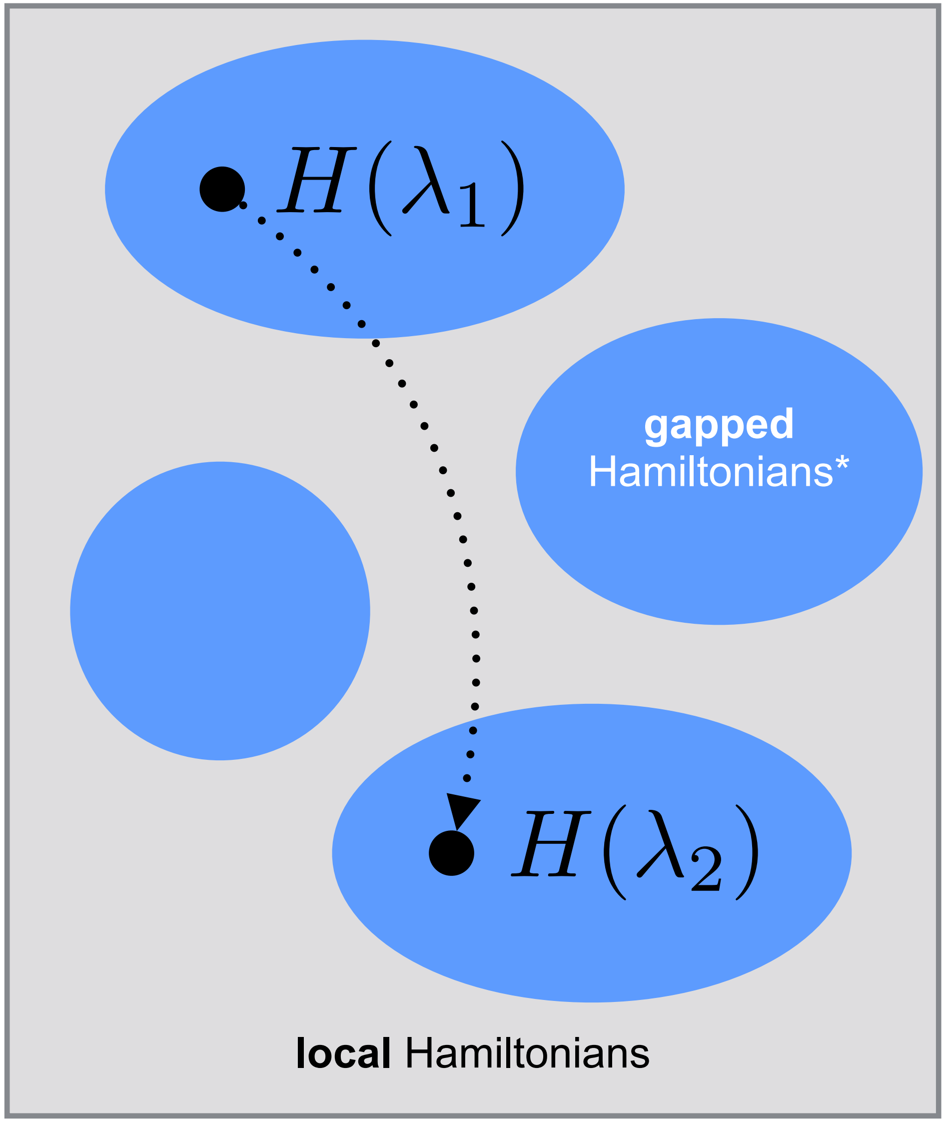
Topological phases

Characterizing ground states of quantum many-body systems at T=0

- absence of symmetry breaking

- gapped phases

Two dimensions



Definition:

two states are in the same topological phase if they can be smoothly transformed into each other without closing the gap

Properties of topological phases in 2D

Topological character

- **ground state degeneracy** scales with genus of the manifold

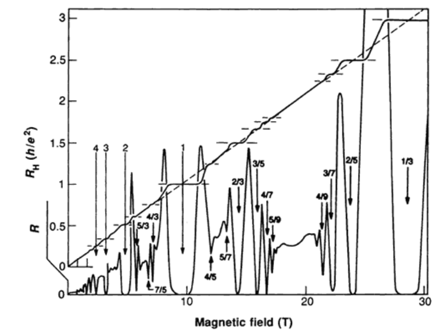
- ground states are indistinguishable to any local probe

- **anyonic excitations**: the statistics is neither bosons nor fermions

- **long-range entanglement** with topological entanglement entropy

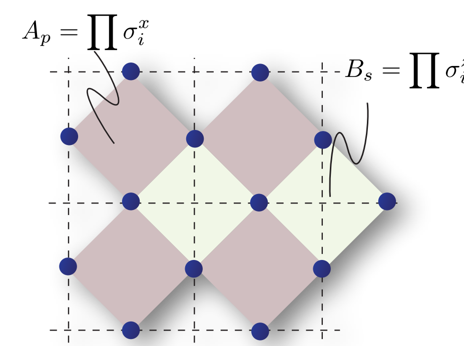
Examples

- fractional quantum Hall states



- fractional Chern insulators

- bosonic models: toric code, Fibonacci anyons, bosonic fractional Chern insulators



$$\mathcal{S}_L = \alpha L - \gamma$$

area law

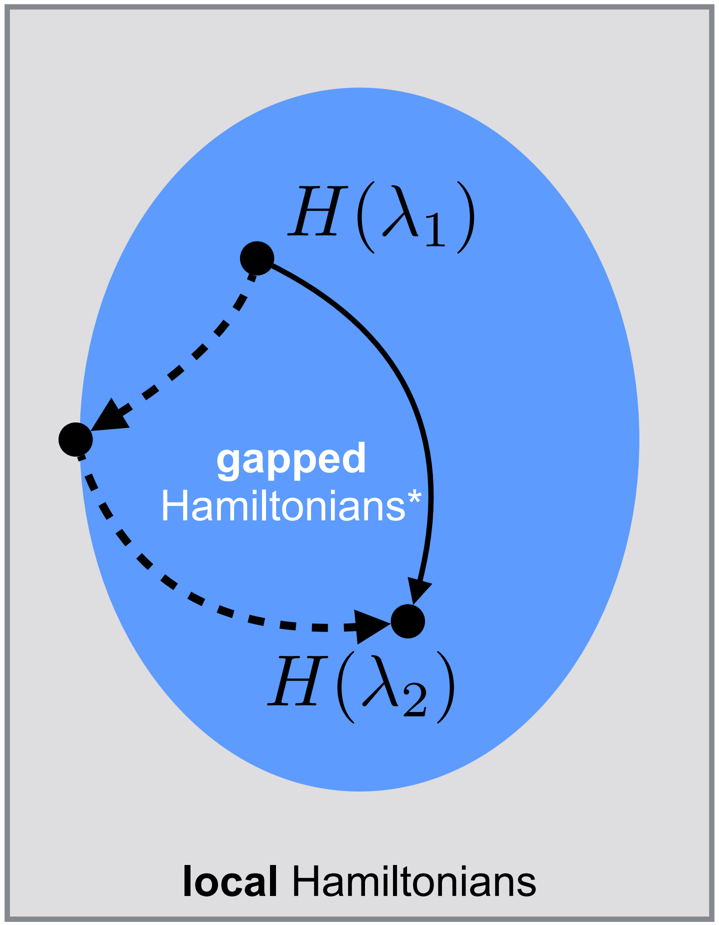
$$\gamma = \log \sqrt{\sum_a d_a^2}$$

: topological entanglement entropy

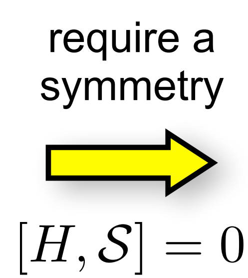
Are there other topological phases also in 1D?

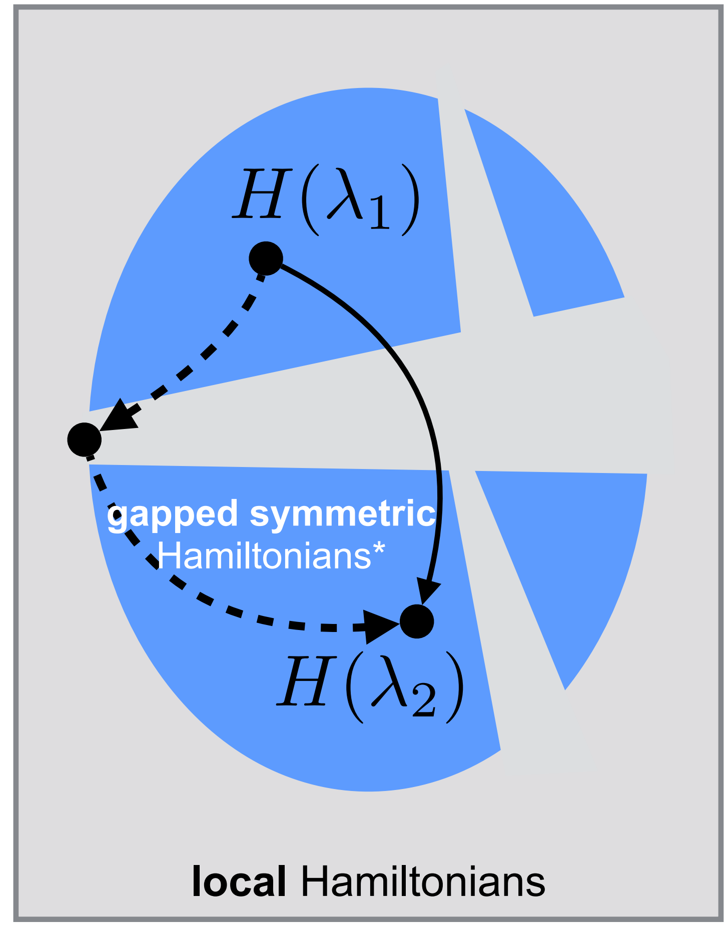
Symmetry protected topological phases

- restrict to systems, which satisfy a certain symmetry
- symmetry group $\mathcal{S}$ with $[H, \mathcal{S}] = 0$



One dimension





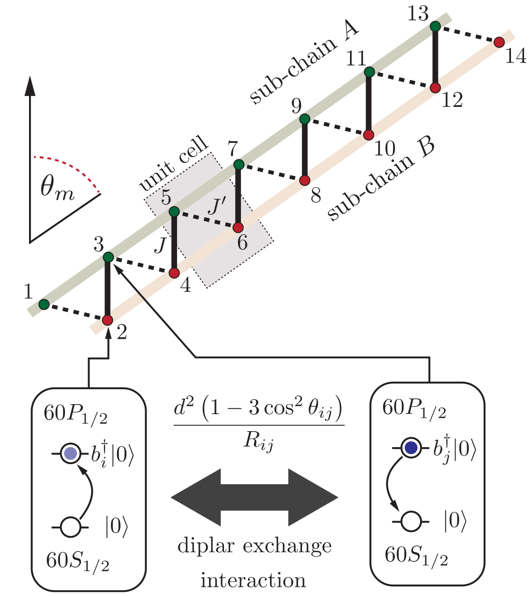
Outline

Symmetry protected Topological phases with Rydberg atoms

- experimental setup of Rydberg atoms in optical tweezers

- Symmetry protected topological phase
  S. de Léséleuc, *et al*, Science 365, 775 (2019)|

- Haldane Spin-1 phase

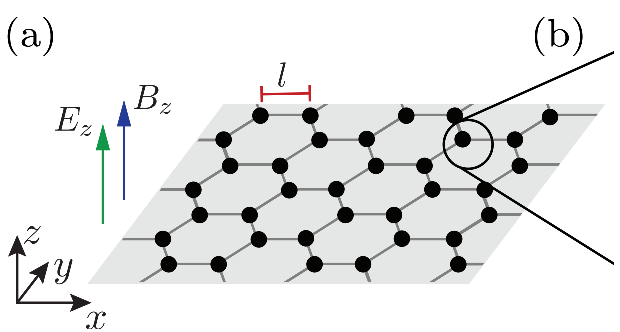


Bosonic fractional Chern insulator with Rydberg atoms

- topological band structures

- proof of principle experiments on a triangle
  V. Lienhard, *et al.*, Phys. Rev. X 10, 021031 (2020)

- blueprint for realization of bosonic fractional Chern insulator
  S. Weber *et al.*, arxiv:2202.00699

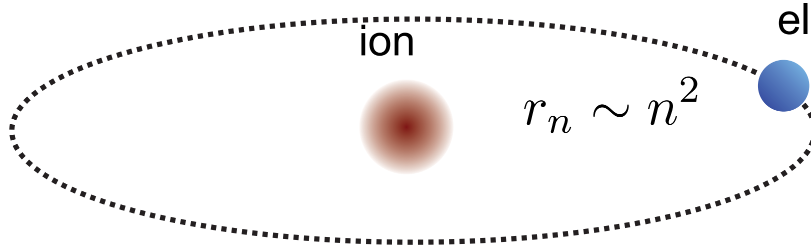


Experimental setup with Rydberg atoms

Rydberg atoms

- one electron excited into a state with high principal quantum number *n*

- here, Rubidium atoms *n*~40 -100, excited into s-states and p-states

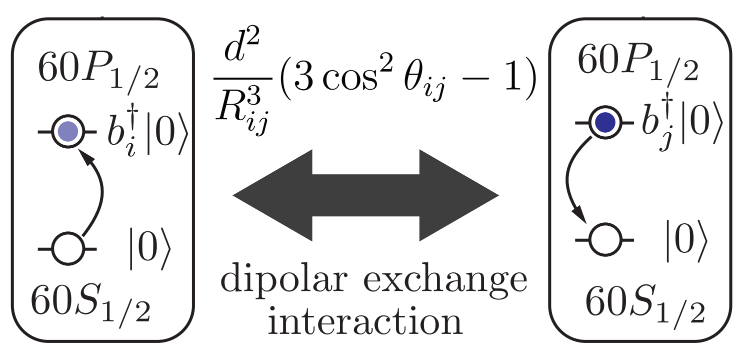


n^{th} : principal quantum number

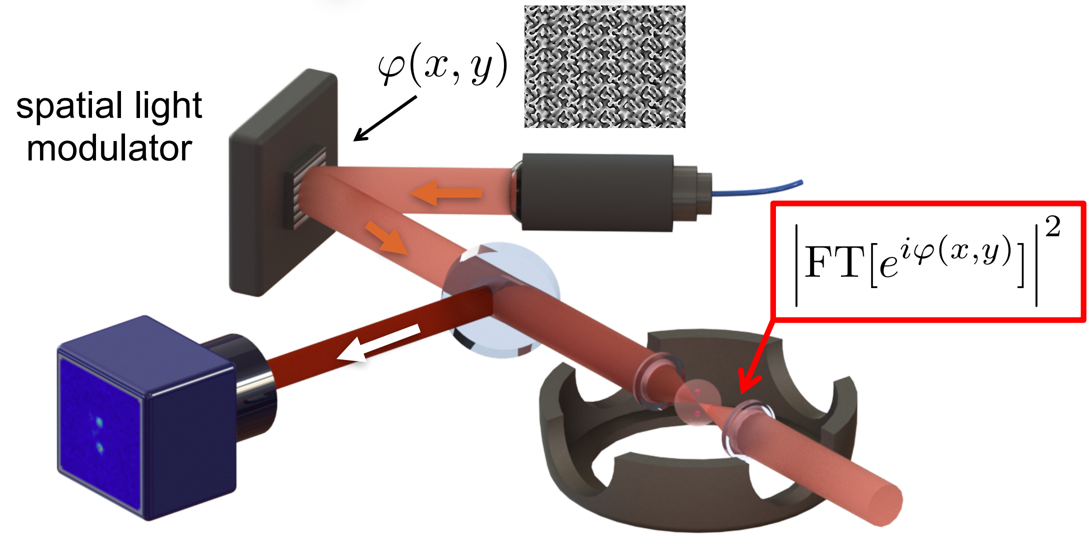
Rydberg-Rydberg interaction

- strong van der Waals interactions between Rydberg states
  - attractive or repulsive
  - $C_6 \sim n^{11}$

- dipolar exchange interactions
  - exchange of excitation between two different Rydberg states
  - $d \sim n^2$



Experimental setup with Rydberg atoms



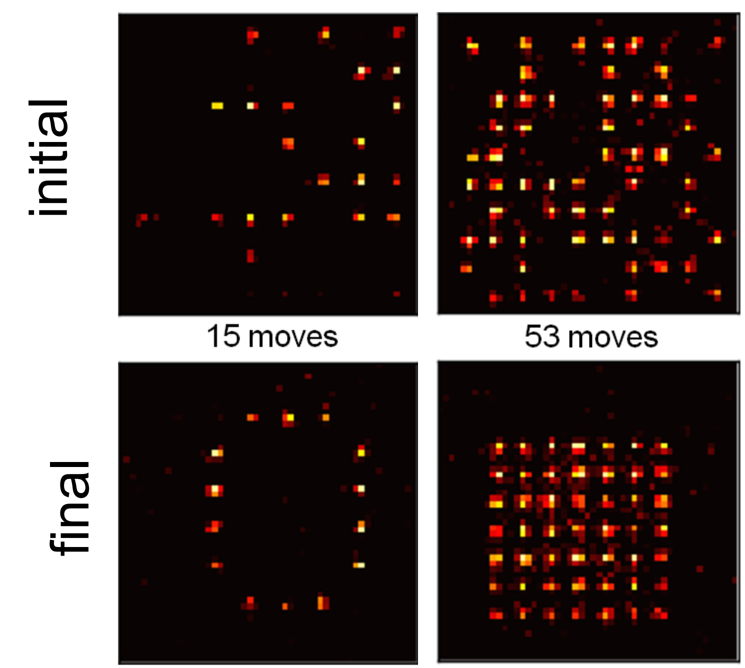
Single atoms trapped in optical tweezers

- individual traps for a single atom

- not in ground state of the trapping potential

- single site resolution

Barredo, *et al.*, Science **354**, 1021 (2016)



Deterministic assembly in arbitrary structures and lattices

- loading from a cold thermal cloud
    - stochastic loading

- prepare lattice structure by moving the filled traps

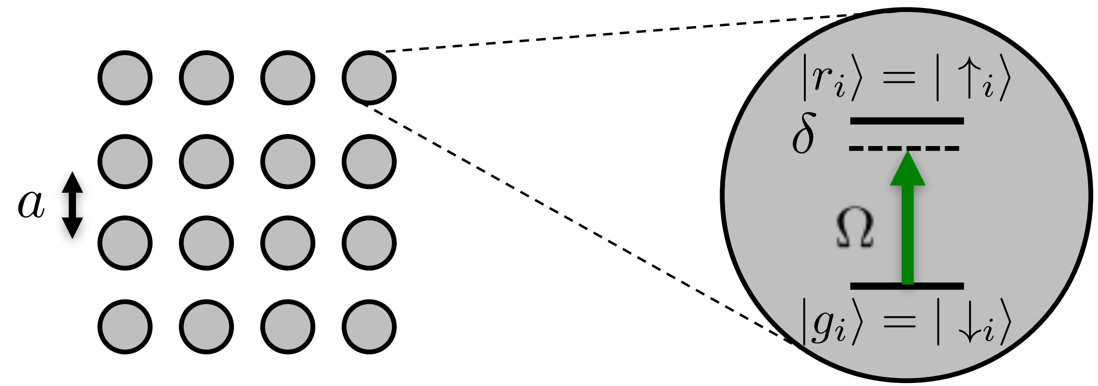
- prepare arbitrary 2D as well as 3D structures

- achieved by different groups:
  Paris, Science **354**, 1021 (2016)
  Harvard, Science **354**,1024 (2016)
  Korea, Nat. Comm. **7**, 13317 (2016)

Experimental setup with Rydberg atoms

Quantum Ising like models

- all atoms coupled to a Rydberg S-state
- van der Waals interaction between Rydberg states



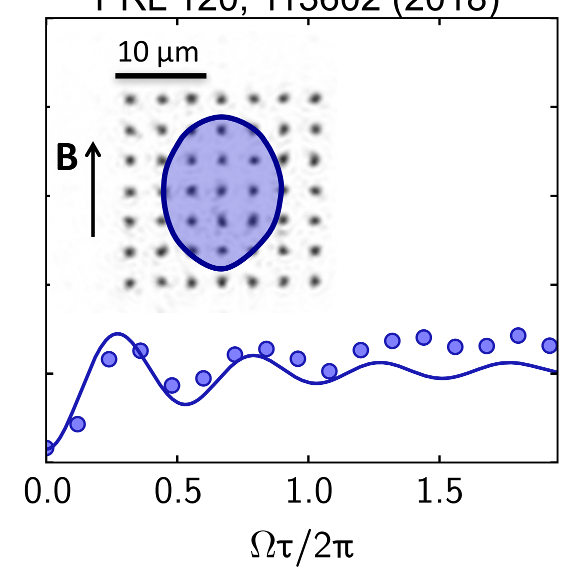
$$H = \underbrace{\Omega \sum_i \sigma_i^x}_{\text{transverse field}} + \underbrace{\sum_i \Delta_i \sigma_i^z}_{\text{longitudinal field}} + \underbrace{\sum_{i \neq j} \frac{C_6}{|\mathbf{r}_i - \mathbf{r}_j|^6} n_i n_j}_{\text{Ising type interaction}}$$

$$n_j = \frac{1 + \sigma_j^z}{2}$$

Quantum simulation of spin models

- non-equilibrium quench dynamics
- time dependent driven and disordered systems
- up to several hundreds of atoms
- Z_2 spin liquid in analogy to toric code

PRL 120, 113602 (2018)



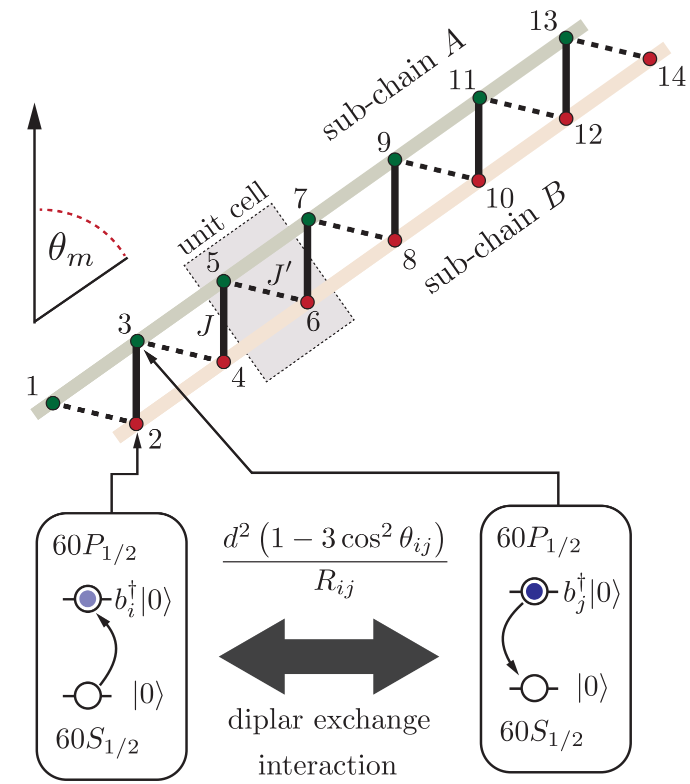
- Labuhn, *et. al*., Nature 534, 667 (2016)
- Bernien, *et al*., Nature 551, 579 (2017)
- Ebadi, et. al. Nature 595, 227 (2021)
- Scholl, et al., Nature 595, 233 (2021)
- Semeghini, et al., Science 374, 1242 (2021)

Experimental setup with Rydberg atoms

Dipolar exchange interaction

- vacuum state: all atoms in a Rydberg S-state
- bosonic particles: excitations into a Rydberg P-state
- hopping by dipolar exchange
- hard-core bosons: strong interactions

$$H = \sum_{ij} b_i^\dagger \hat{H}_{ij} b_j = \sum_{i\in A, j\in B} J_{ij} \left[b_i^\dagger b_j + b_j^\dagger b_i \right]$$



Alternative view in spin language: **hard core bosons** ⟺ **spin 1/2 system**

Rydberg S-state: spin down $|\downarrow\rangle$

Rydberg p-state: spin up $|\uparrow\rangle$

⇒ XY model for spin 1/2

$$H = \sum_{ij} J_{ij} \left[S_i^+ S_j^- + S_i^- S_j^+ \right]$$
$$= \sum_{ij} 2J_{ij} \left[S_i^x S_j^x + S_i^y S_j^y \right]$$

Experimental setup with Rydberg atoms

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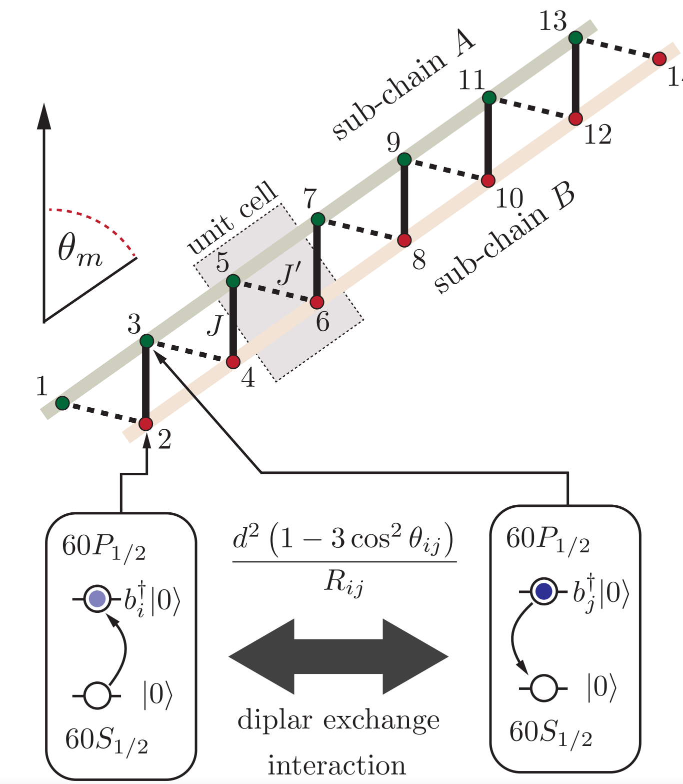


Reminder: Fermionic SSH chain — topological insulator

- non-interacting Fermions
- sub-lattice (chiral) symmetry (anti-unitary operator)

$$\mathcal{S}_S = \prod_i \left[c_i + (-1)^i c_i^\dagger \right] K$$

complex conjugation

- on single particle Hamiltonian

$$U_S \hat{H} U_S^\dagger = -\hat{H} \qquad (U_S)_{ij} = (-1)^j \delta_{ij}$$

- Jordan Wigner transformation

$$b_j = e^{i\pi \sum_{k<j} c_k^\dagger c_k} c_j$$

: non-local transformation

non-interacting fermions ⟺ hard-core bosons

Symmetry protected topological phase

Protecting symmetries

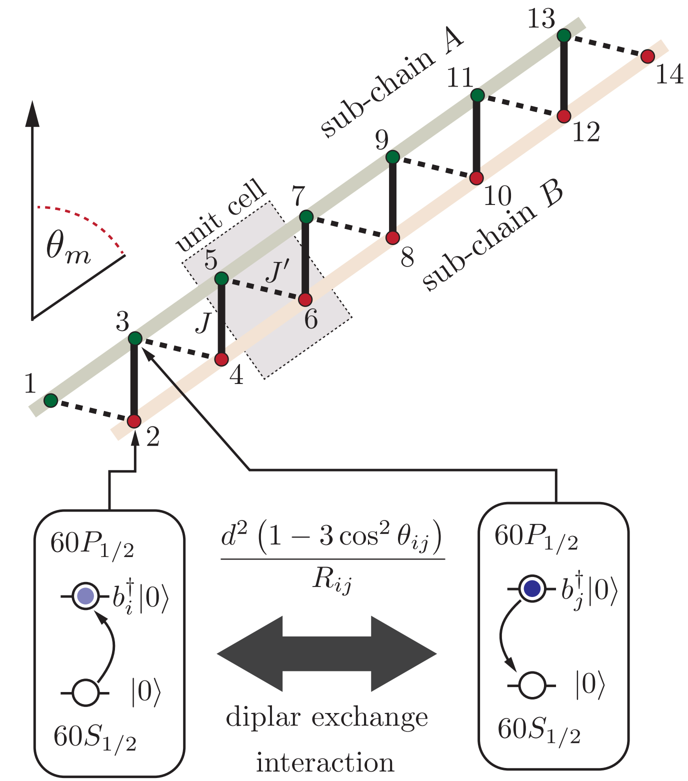
- particle conservation

- discrete operation $\mathcal{S}_B = \prod_i \left[b_i^\dagger + b_i \right] K$

- symmetry group $U(1) \times Z_2^T$

- Hamiltonian $[H, \mathcal{S}_B] = 0$

- allows for 4 different SPT phases
  X.-G. Wen, *et al*, Science (2012)
  F. Pollman, *et al*, PRB (2010)



Special point: $J' = 0$

$$|m, m'\rangle = \left(b_1^\dagger\right)^m \left(b_L^\dagger\right)^{m'} \prod_{i \in \text{even}} \frac{1}{\sqrt{2}} \left(b_i^\dagger + b_{i+1}^\dagger\right)|0\rangle$$

SPT phase

- gapped ground state at half-filling

- four-fold ground state degeneracy

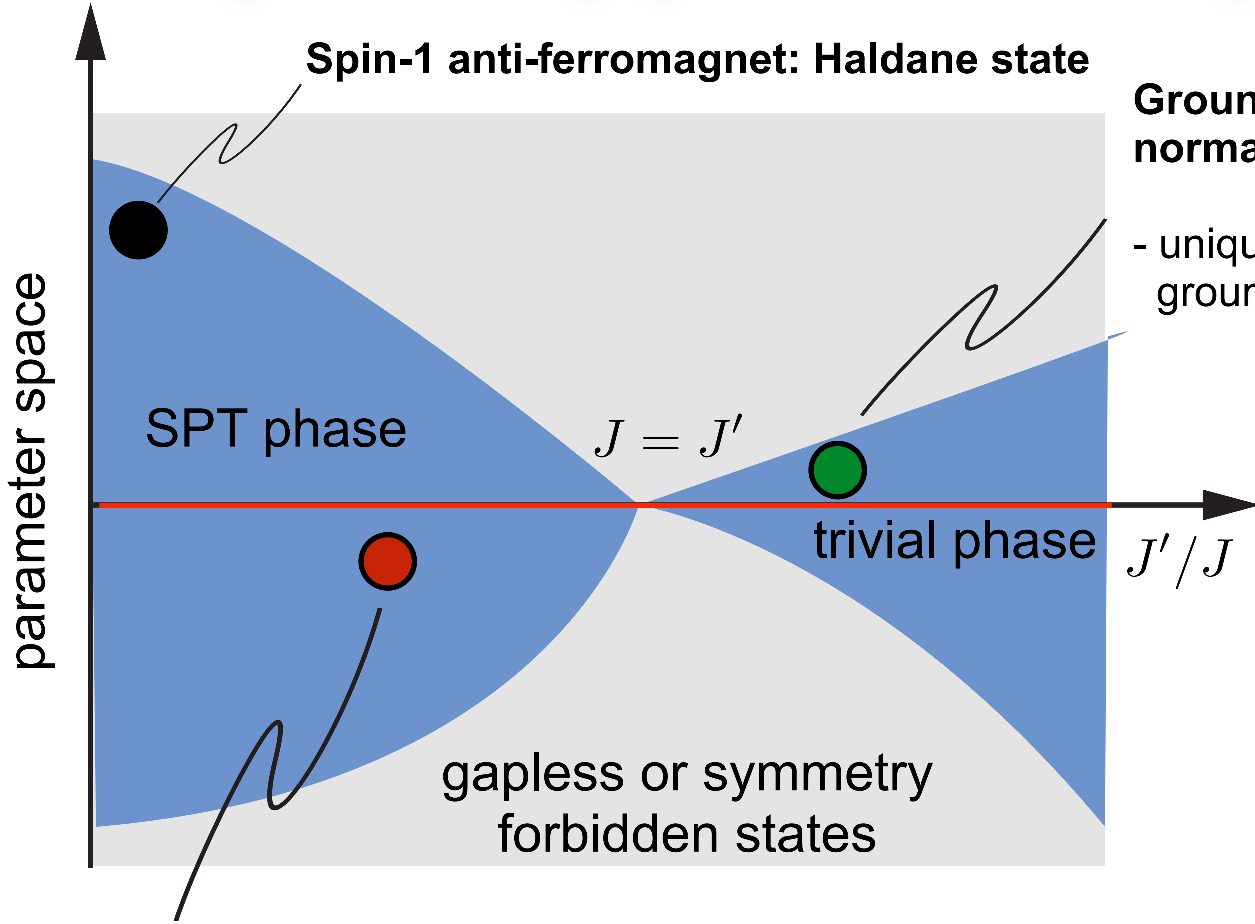
- zero energy edge states

Perturbations respecting the symmetry:

- arbitrary hoppings (also complex) $b_i^\dagger b_j + b_j^\dagger b_i$

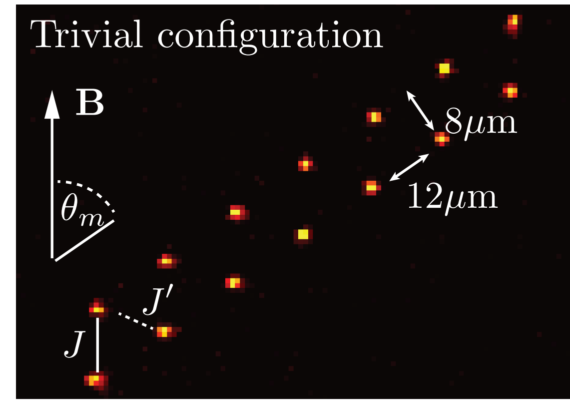
- interactions $\left(b_i^\dagger b_i - 1/2\right)\left(b_j^\dagger b_j - 1/2\right)$

Symmetry protected topological phase



Ground state in normal phase

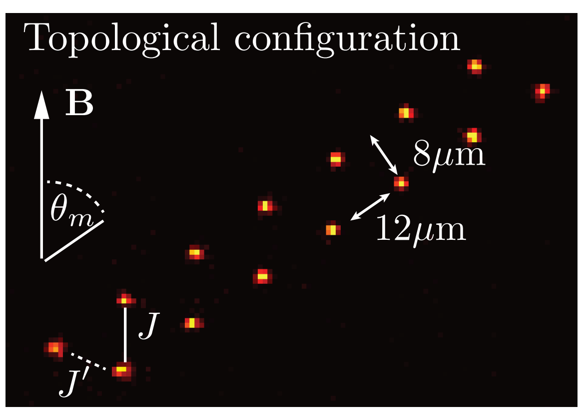
- unique gapped ground state

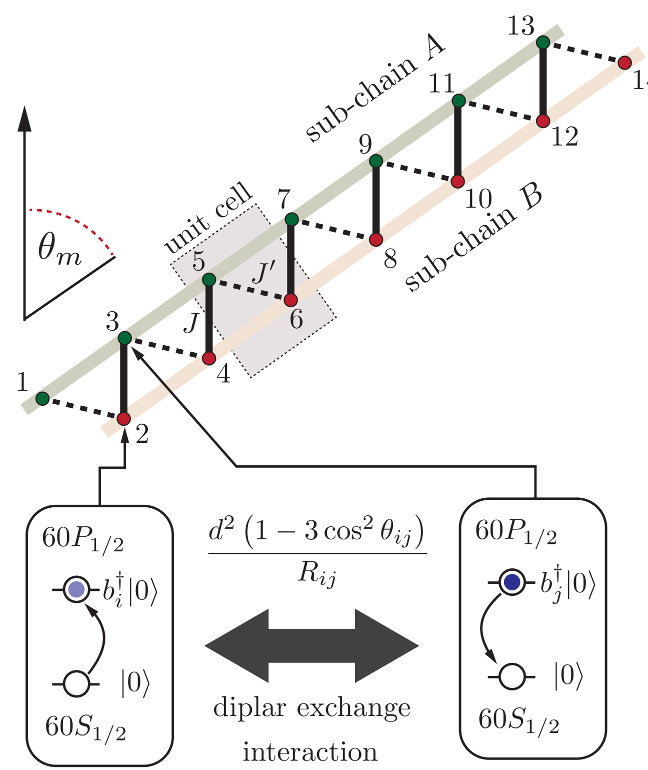


Ground state in SPT phase

- ground state degeneracy

- edge states





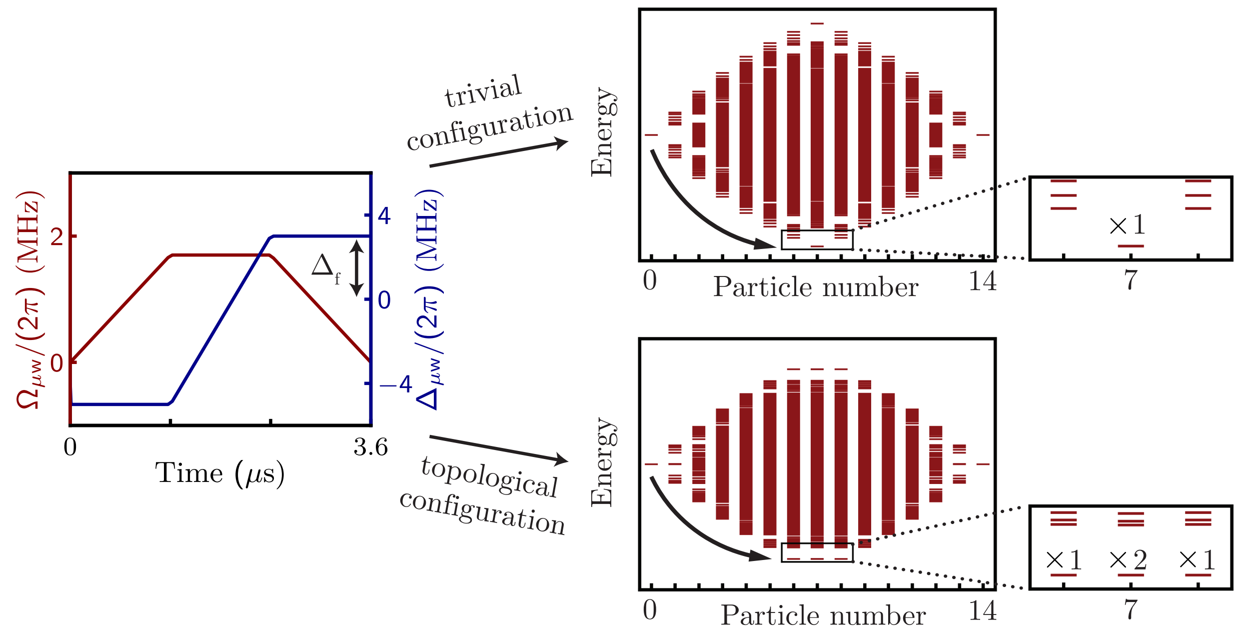
Preparation of ground state at half-filling

Adiabatic preparation of quantum many-body ground state

- prepare the ground state at half-filling

- highly efficient due to large gap

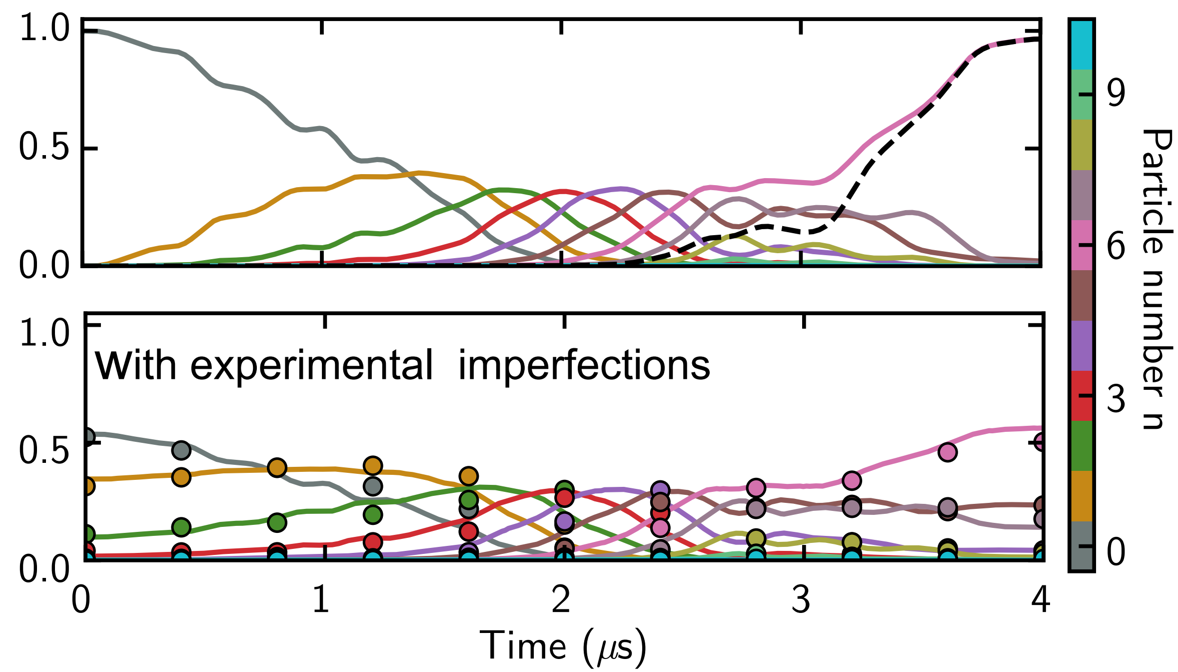
- ramping scheme motivated for prefect dimerization



Full numerical simulation

- efficient preparation of ground state

- high fidelity to prepare exact ground state with N/2 excitations



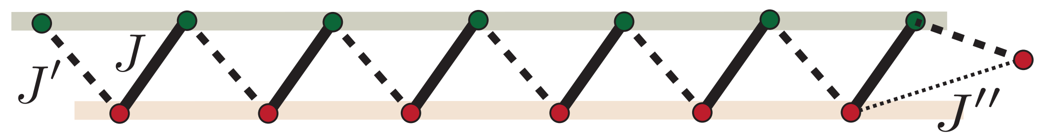
Symmetry protected topological phase

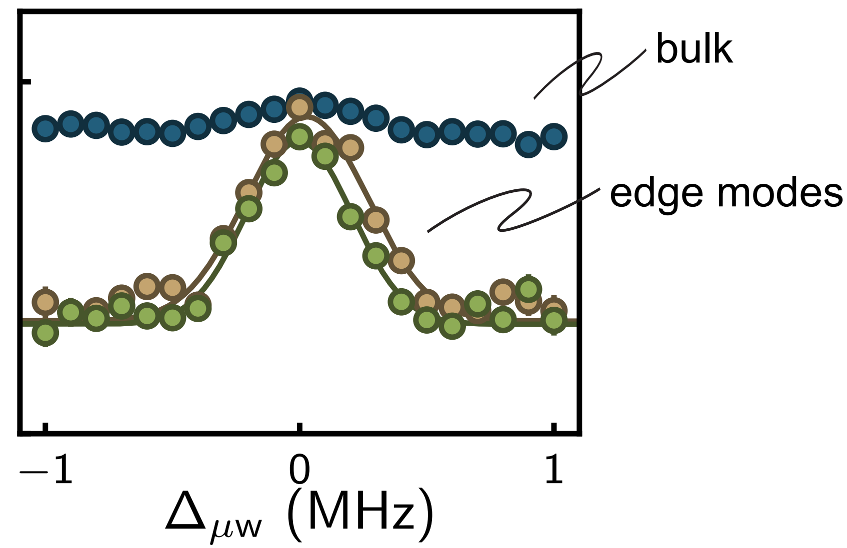
Characterization of topological phase

- half-filling in the bulk
- occupation of edge states depending on ramping procedure

→ ground state degeneracy

- zero-energy modes in spectroscopy
- robust to perturbation
- absence of any spontaneous symmetry breaking
- experimental detection of string order parameter





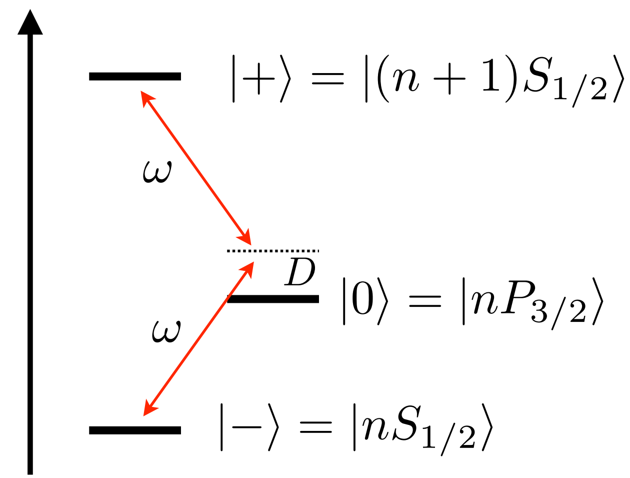
- edge modes excitation is at zero detuning
- in perturbed and unperturbed setup
- line-width well accounted for by finite pulse shape

What about spin fractionalization?

Spin-1 Haldane phase/AKLT model

- spin-1 at each lattice site
- edges carry spin-1/2 degree of freedom

⇨ spin fractionalization



Setup with Rydberg atoms

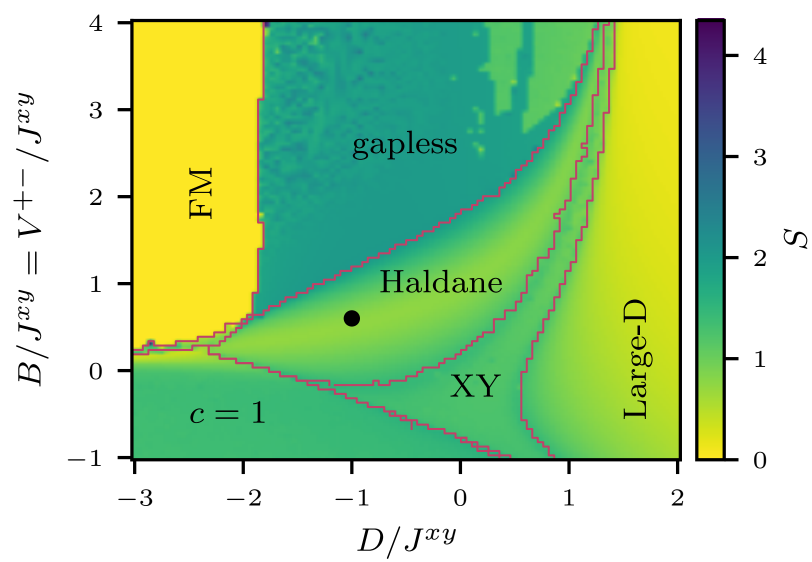
- three Rydberg levels to model spin-1
- dipolar exchange hopping
- suppress symmetry breaking terms

$$H = J^{xy} \sum_{i>j} \frac{1}{|i-j|^3} \left[S_i^+ S_j^- + S_i^- S_j^+\right] + D \sum_i \left(S_i^z\right)^2$$

$$+ \frac{B}{4} \sum_{i>j} \frac{1}{|i-j|^6} \left[A_{ij} - A_{ij}^\dagger A_{ij} + h.c.\right]$$

enhanced by Förster resonance

$$A_{ij} = \left(S_i^+\right)^2 \left(S_j^-\right)^2$$



parameters optimized with pair-interaction software: S. Weber,et al, J. Phys. B (2017)

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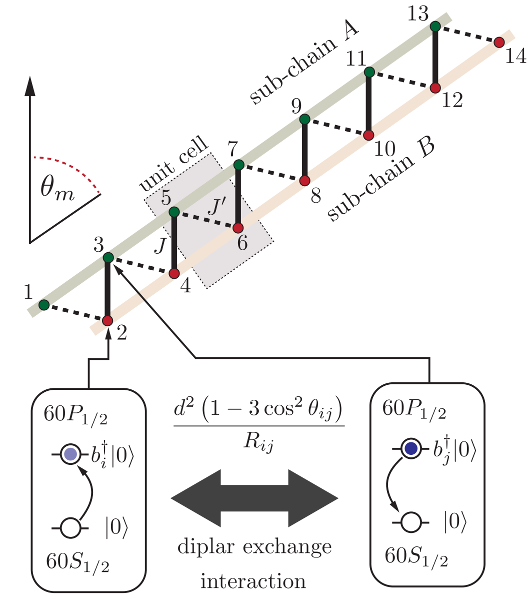
  S. de Léséleuc, *et al*, Science 365, 775 (2019)|

- Haldane Spin-1 phase



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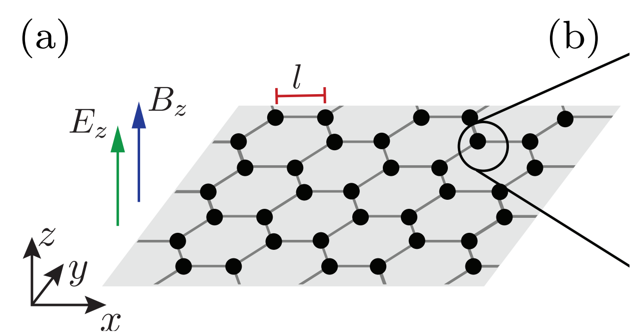
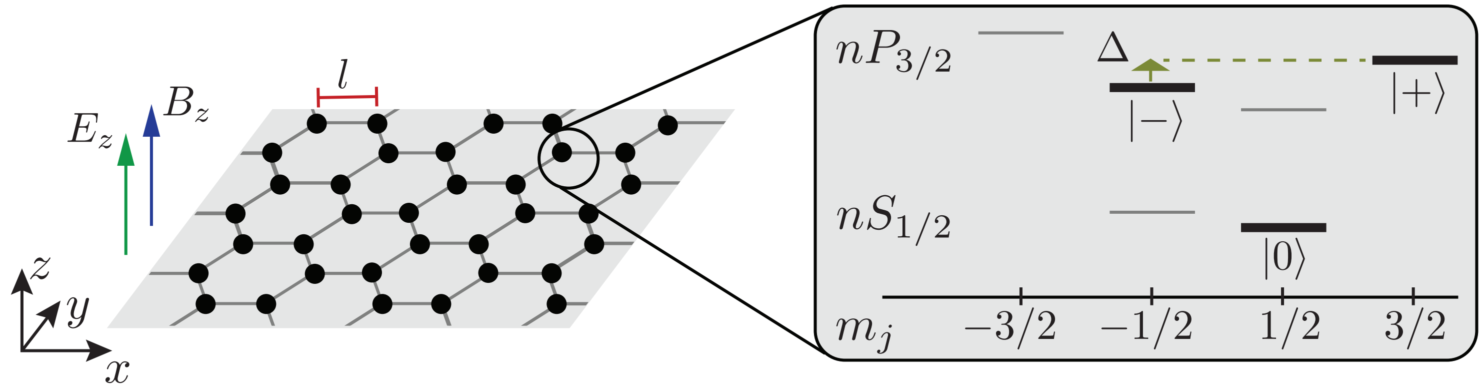


Array of Rydberg atoms in two-dimensions



$$|+\rangle = |(n+1)S_{1/2}\rangle$$

Setup

- one atom per lattice site with quenched tunneling

- static external electric field and magnetic field

broken time reversal symmetry

- select three internal states

$|-\rangle_i \quad |+\rangle_i$: two excited states

$|0\rangle_i$: ground state

Mapping onto two hard-core bosons, e.g., $(a_i)^2 = 0$

Bosonic operators:

$$a_i^\dagger |0\rangle = |+\rangle_i$$

$$b_i^\dagger |0\rangle = |-\rangle_i$$

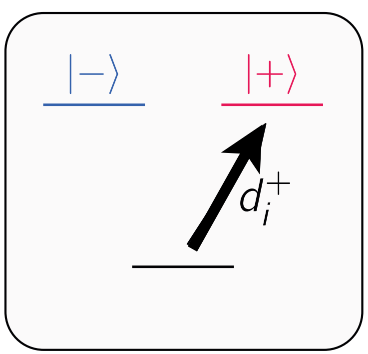
Dipolar exchange interactions

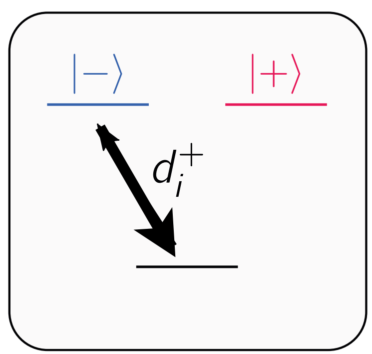
Hamiltonian

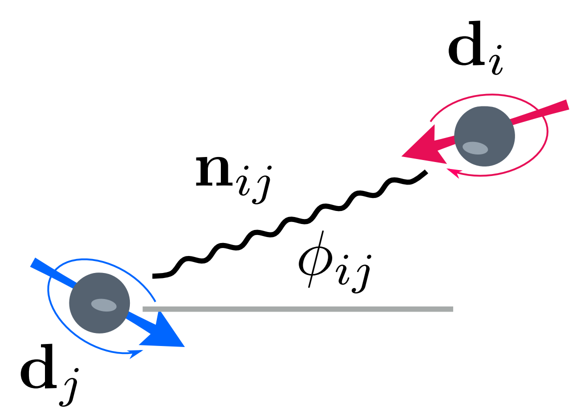
- dipolar interaction restricted to the three internal levels

$$d^0 \equiv d^z \qquad d^+ \equiv d^x + i d^y \qquad d^- \equiv d^x - i d^y$$

$$H_{ij} = \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|^3} \left[\underbrace{d_i^0 d_j^0}_{\text{dipole interaction}} + \frac{1}{2} \underbrace{\left(d_i^+ d_j^- + d_i^- d_j^+ \right)}_{\text{exchange interaction}} - \frac{3}{2} \underbrace{\left(d_i^+ d_j^+ e^{-i2\phi_{ij}} + d_i^- d_j^- e^{i2\phi_{ij}} \right)}_{\text{spin-orbit coupling}} \right]$$





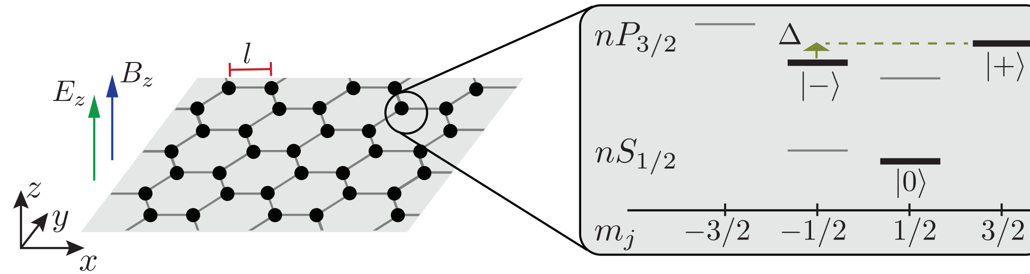


hopping with spin flip: intrinsic spin-orbit coupling

Quantum many-body Hamiltonian

Hard-core bosons

- honeycomb lattice



$$t_{ij}^a \sim \frac{t^a}{|\mathbf{R}_i - \mathbf{R}_j|^3}$$

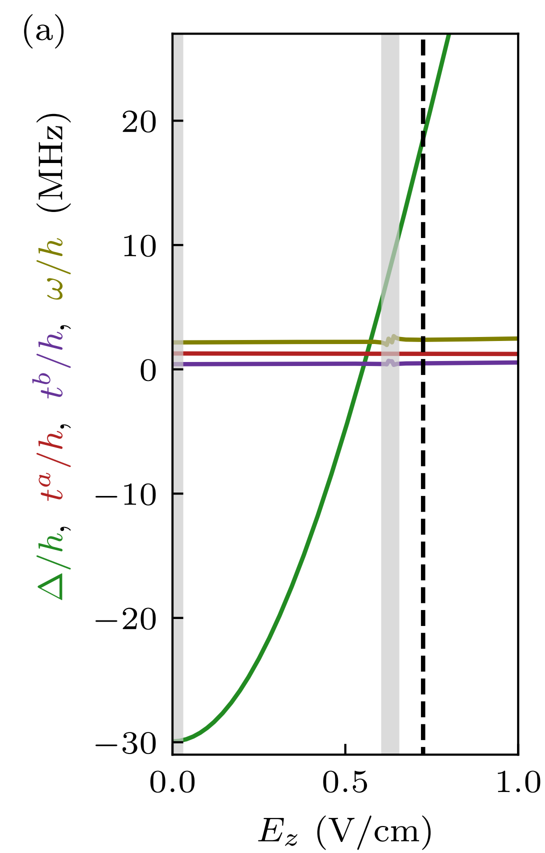
$$w_{ij} \sim \frac{e^{2i\phi_{ij}} w}{|\mathbf{R}_i - \mathbf{R}_j|^3}$$

$$n_i^a = a_i^\dagger a_i$$

$$H_0 = \sum_{i \neq j} \begin{pmatrix} a_i \\ b_i \end{pmatrix}^\dagger \begin{pmatrix} -t_{ij}^a & \omega_{ij} \\ \omega_{ij}^* & -t_{ij}^b \end{pmatrix} \begin{pmatrix} a_j \\ b_j \end{pmatrix} + \Delta \sum_i n_i^a$$

hopping Hamiltonian

energy difference



Parameters

- Rubidium atoms $^{87}\mathrm{Rb}$ in Rydberg state n=60
- lattice spacing $R = 12\mu\mathrm{m}$
- magentic field strength B=8G
- additional weak density-density interactions

microscopic derivation of parameters using

pair-interaction software

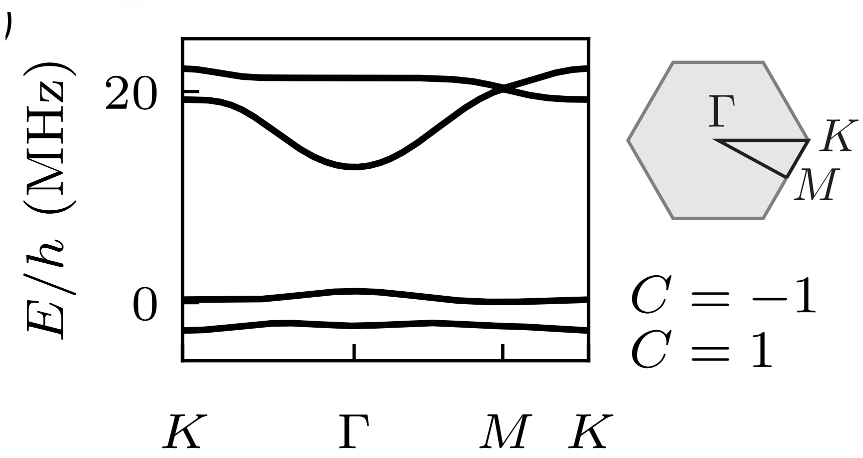
S. Weber,et al, J. Phys. B (2017)

Single particle properties

Topological band structure

- characterized by a topological invariant

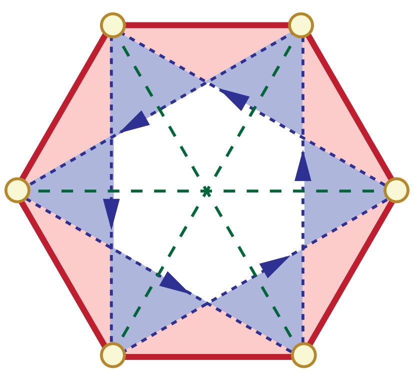
- relative high flatness and homogeneous Berry curvature



Interpretation

- adiabatic elimination of the $|+\rangle$ for large energy difference $\Delta \gg t, w$

- Haldane model on the honeycomb lattice for single particles



$$\lambda = -\frac{w^2}{\Delta}$$

$$H = -t \sum_{\langle i,j \rangle} b_i^\dagger b_j - t \sum_{\langle\langle i,j \rangle\rangle} \left(\frac{1}{3^{3/2}} + \lambda e^{i \frac{2\pi}{3} p_{ij}} (1 - n_{\bar{i}\bar{j}}) \right) b_i^\dagger b_j - t\nu \sum_{\langle\langle\langle i,j \rangle\rangle\rangle} b_i^\dagger b_j + 2t\lambda \sum_{\langle i,j \rangle} n_i n_j$$

n.n. hopping — n.n.n. hopping with correlated phases — n.n.n.n. hopping — n.n. interaction

(see also Ohler et al, arxiv:2202.03860)

Experimental proof of principle

V. Lienhard, *et al.*, Phys. Rev. X 10, 021031 (2020)

Minimal setup with three Rydberg atoms on a regular triangle

- hopping by dipolar exchange interactions

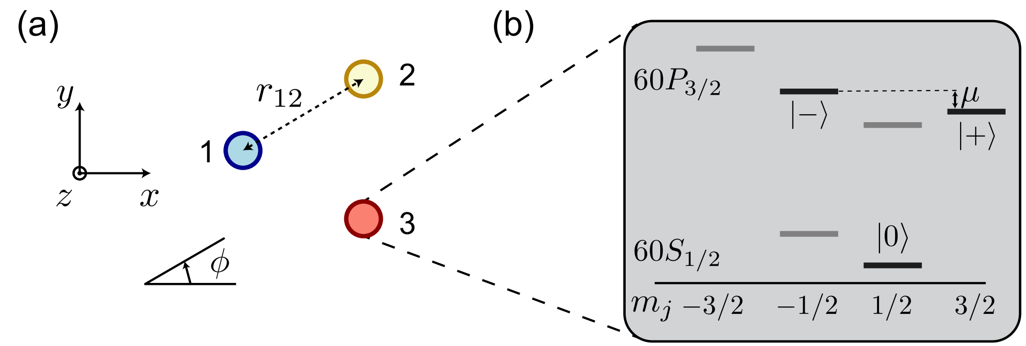
- intrinsic spin-orbit coupling

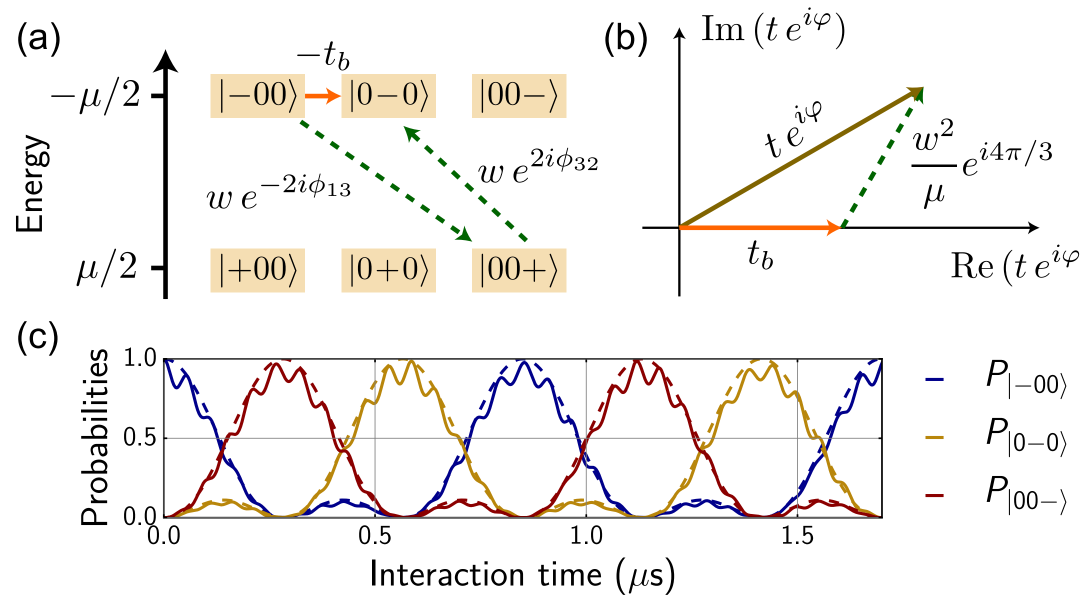
- adiabatic elimination of $|+\rangle$

$$te^{i\varphi} = t^a + e^{i4\pi/3}\frac{w^2}{\Delta}$$

- artificial gauge flux $\Phi = 3\varphi$ through the triangle

- perfect chiral motion of a single excitation for $\Phi = \pm\pi/2$





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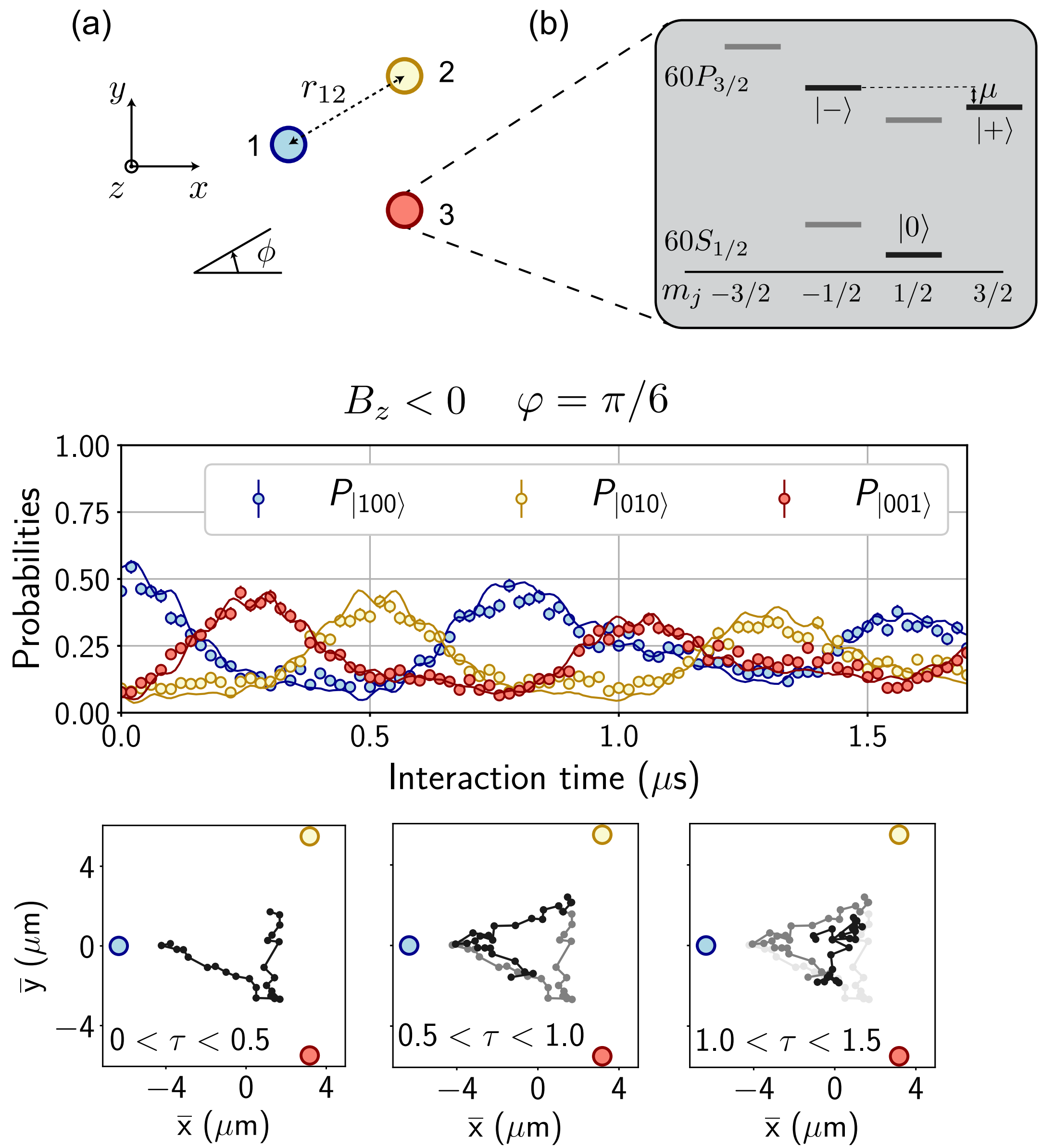
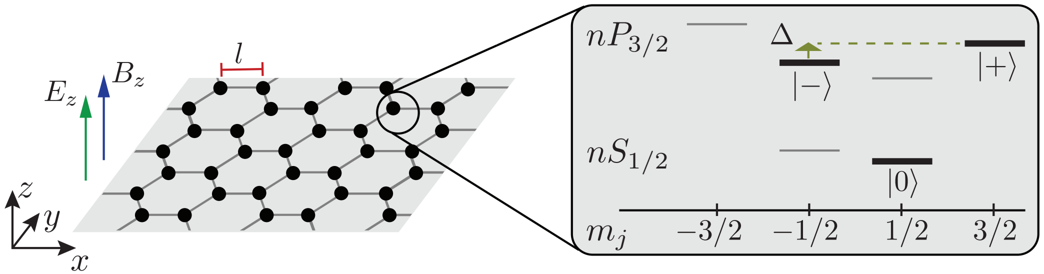


Quantum many-body Hamiltonian

Hard-core bosons

- honeycomb lattice



$$t_{ij}^a \sim \frac{t^a}{|\mathbf{R}_i - \mathbf{R}_j|^3}$$

$$w_{ij} \sim \frac{e^{2i\phi_{ij}} w}{|\mathbf{R}_i - \mathbf{R}_j|^3}$$

$$n_i^a = a_i^\dagger a_i$$

$$H_0 = \sum_{i \neq j} \begin{pmatrix} a_i \\ b_i \end{pmatrix}^\dagger \begin{pmatrix} -t_{ij}^a & \omega_{ij} \\ \omega_{ij}^* & -t_{ij}^b \end{pmatrix} \begin{pmatrix} a_j \\ b_j \end{pmatrix} + \Delta \sum_i n_i^a$$

hopping Hamiltonian

energy difference

- single particle band structure with Chern number C=1

- relative high flatness f= 2.7

- homogeneous Berry curvature

Does a fractional bosonic Chern insulator naturally appear?

Fractional bosonic Chern insulator

Quantum many-body ground state

- numerical methods:
    - exact diagonalization: three states per lattice site, up to 28 sites
    - infinite DMRG on torus

Signature of bosonic fractional Chern insulator

(i) incompressible plateau at filling $n = 1/4$

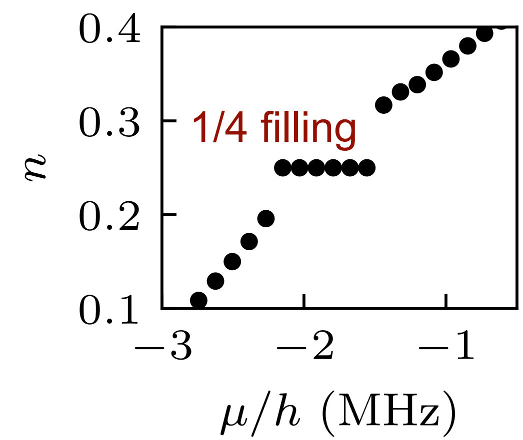
(ii) near two-fold ground state degeneracy and large excitation gap $\Delta E \sim t^b \approx t^a/3$

(iii) absence of long-range order, and exponential decay of correlations

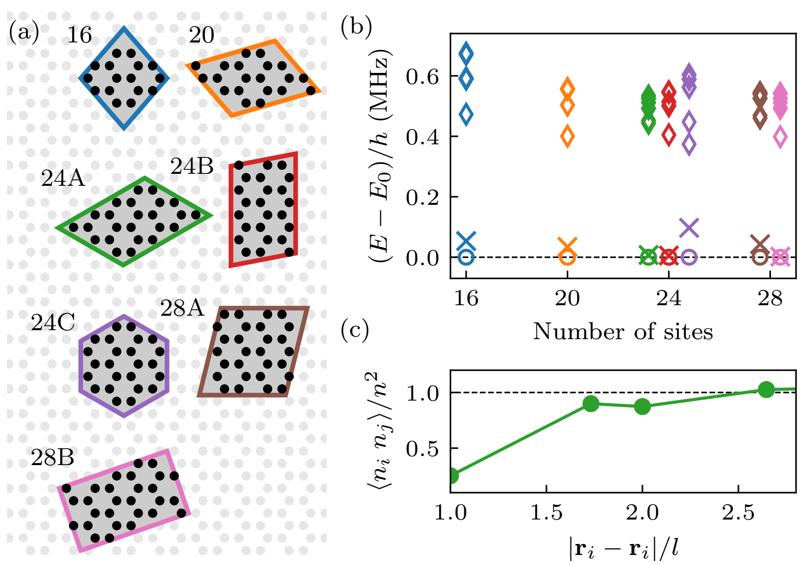
(iv) many-body Chern number C=1

(v) topological entanglement entropy

incompressible phase



excitation energies



Characteristic property of a $\nu = 1/2$ bosonic Laughlin state

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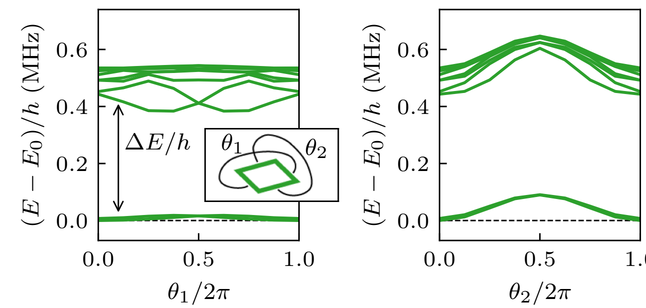
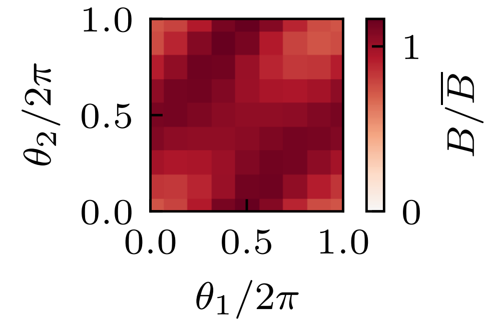
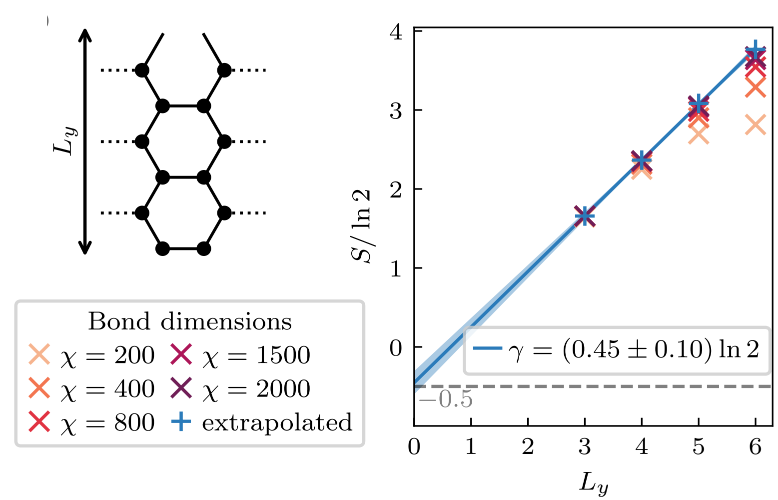
twisted boundary conditions



Berry curvature



topological entanglement entropy



Fractional bosonic Chern insulator

Adiabatic preparation

- all atoms in ground state $|0\rangle$

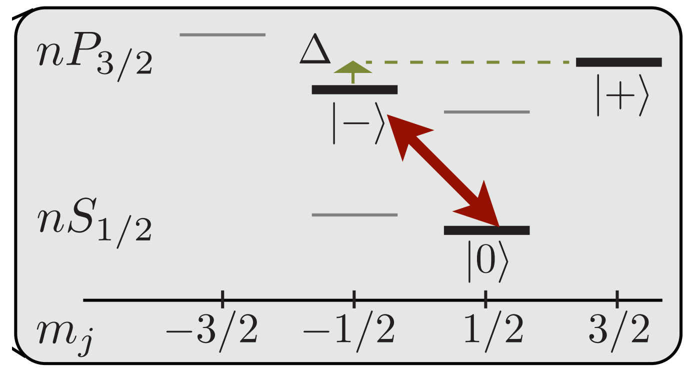
- generate excitations by microwave field

$$H_c = \sum_i \Omega(\tau)\left[b_i^\dagger + b_i\right] - \sum_i \delta(\tau)\left[n_i^a + n_i^b\right]$$

- final filling is controlled by detuning

⇨ gap closing for critical Rabi frequency

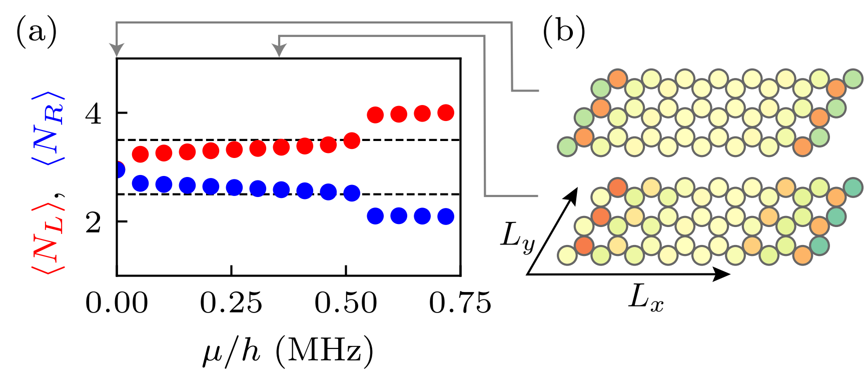
- increase finite size gap by inhomogeneous driving



Signatures of fractional excitations

- local chemical potential traps excitations

⇨ excitations appear in step with fractional charge



Raciunas, et al., Phys. Rev. A (2018)

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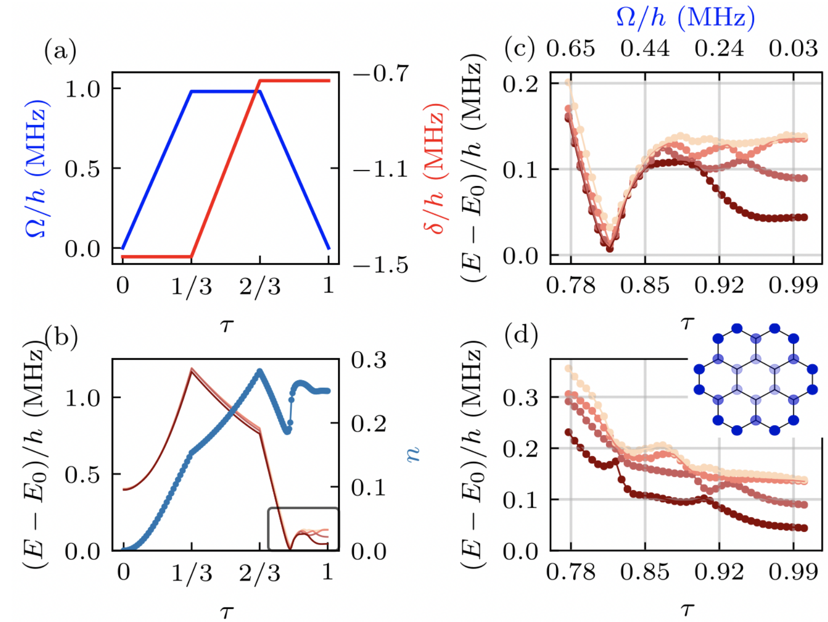

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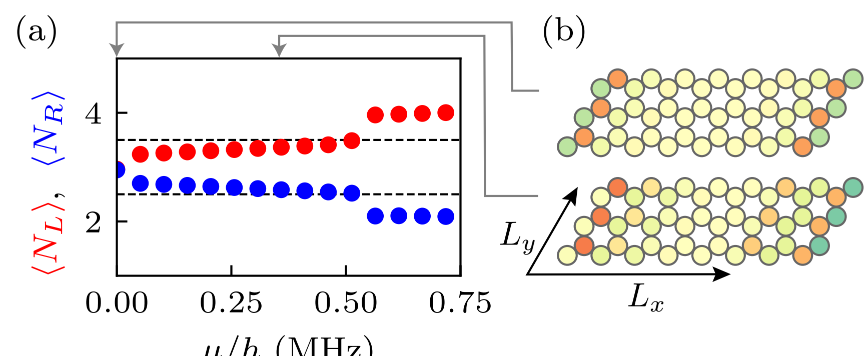
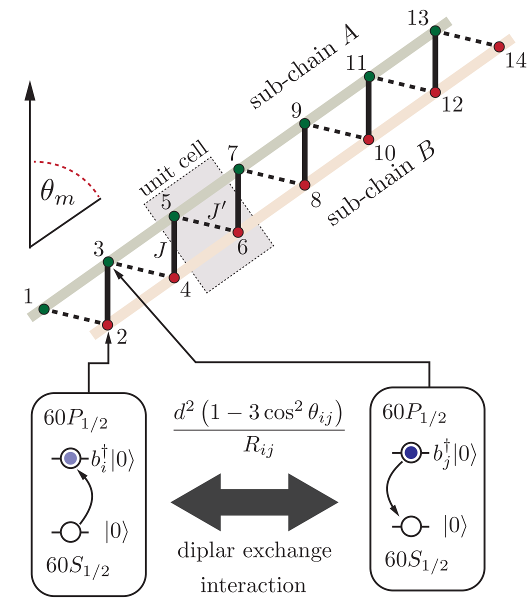


Raciunas, et al., Phys. Rev. A (2018)

Conclusion

Symmetry protected Topological phases with Rydberg atoms

- experimental setup of Rydberg atoms in optical tweezers
- Symmetry protected topological phase
  S. de Léséleuc, *et al*, Science 365, 775 (2019)|
- Haldane Spin-1 phase



Bosonic fractional Chern insulator with Rydberg atoms

- topological band structures
- proof of principle experiments on a triangle
  V. Lienhard, *et al.*, Phys. Rev. X 10, 021031 (2020)
- blueprint for realization of bosonic fractional Chern insulator
  S. Weber *et al.*, arxiv:2202.00699