Measurement induced phase transitions in ground states

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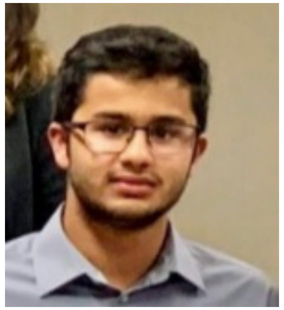
Sam Garratt

Zack Weinstein



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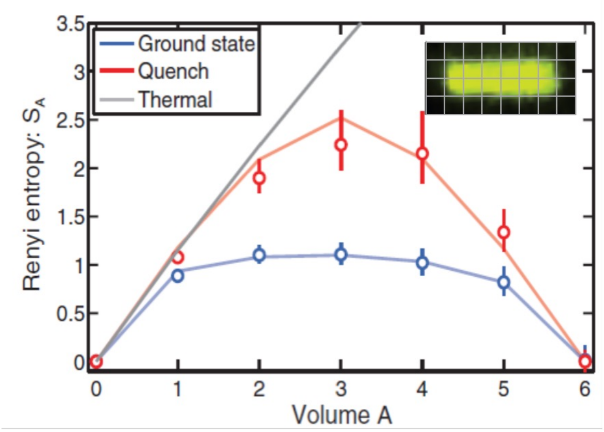
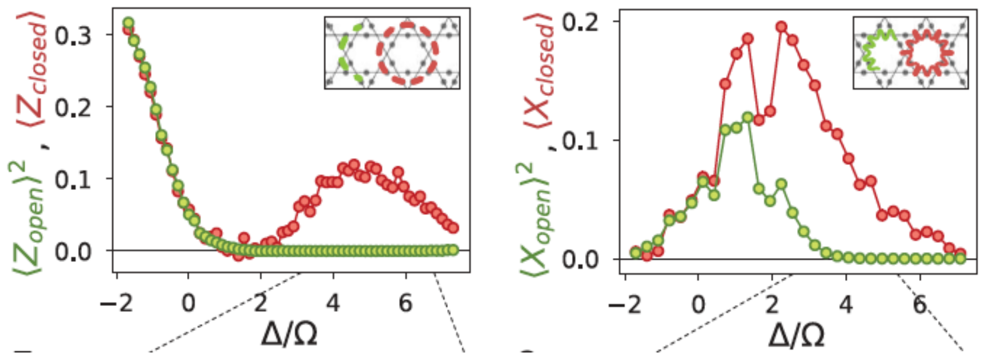
GORDON AND BETTY MOORE FOUNDATION

New platforms offer access to previously hidden aspects of many body systems

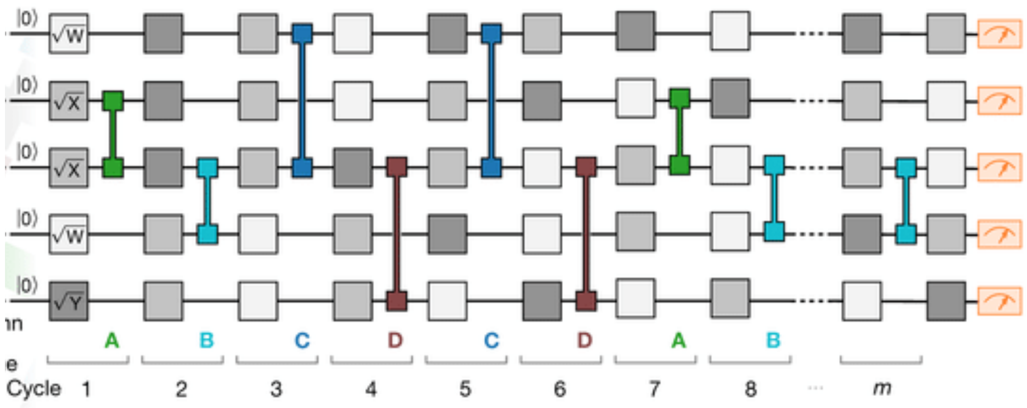
Entanglement entropy:
Greiner Group (2018)



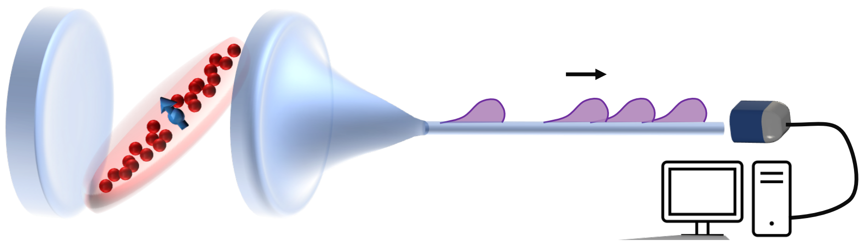
String correlators (topo. order):
Lukin, Greiner, Vuletic groups (2020)



Quantum measurement and scrambling:
Google team (Nature 2019)



Continuous **measurement**:



Measurement in quantum mechanics

$$|\psi\rangle \mapsto \frac{\hat{P}_\mu|\psi\rangle}{\sqrt{\langle\psi|\hat{P}_\mu|\psi\rangle}}$$

Quantum Collapse can destroy quantum correlations:

$$\frac{1}{\sqrt{2}}|\uparrow\downarrow\rangle - \frac{1}{\sqrt{2}}|\downarrow\uparrow\rangle$$

Measurement in quantum mechanics

$$|\psi\rangle \mapsto \frac{\hat{P}_\mu|\psi\rangle}{\sqrt{\langle\psi|\hat{P}_\mu|\psi\rangle}}$$

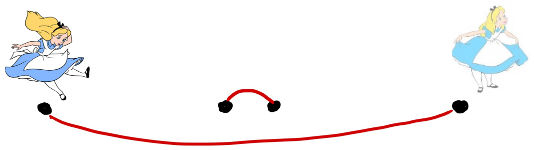
Measurements can also create new larger scale quantum correlations.

Quantum teleportation:

Measurement in quantum mechanics

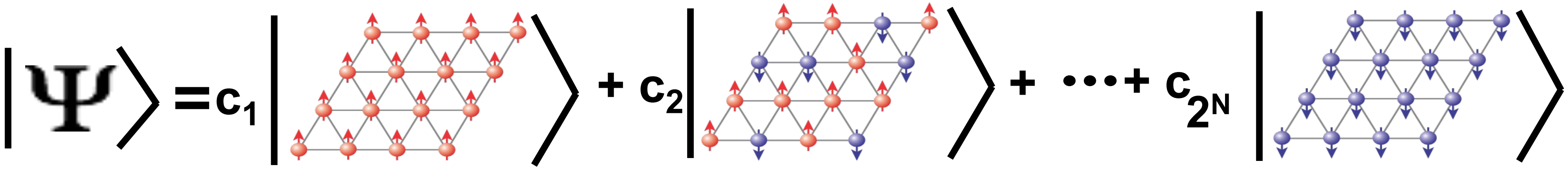
$$|\psi\rangle \mapsto \frac{\hat{P}_\mu|\psi\rangle}{\sqrt{\langle\psi|\hat{P}_\mu|\psi\rangle}}$$

- Measurements can destroy quantum correlations
- But can also create larger scale correlations



How do these effects manifest in many-body systems?

New phenomena from partial measurement of correlated states?

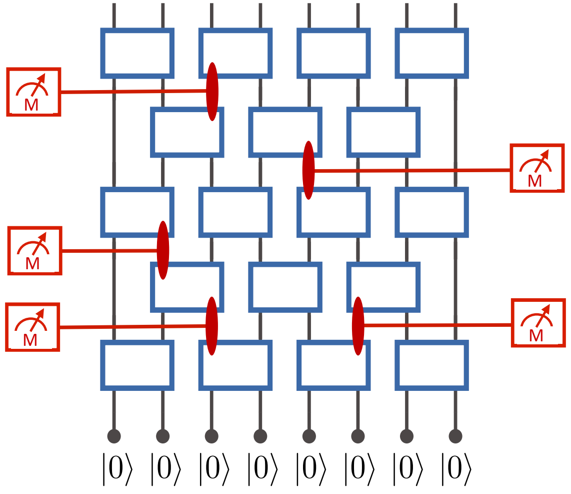


Previous work: measurement induced transitions in quantum circuits

1. Measurement induced phase transition in Hybrid quantum circuits.

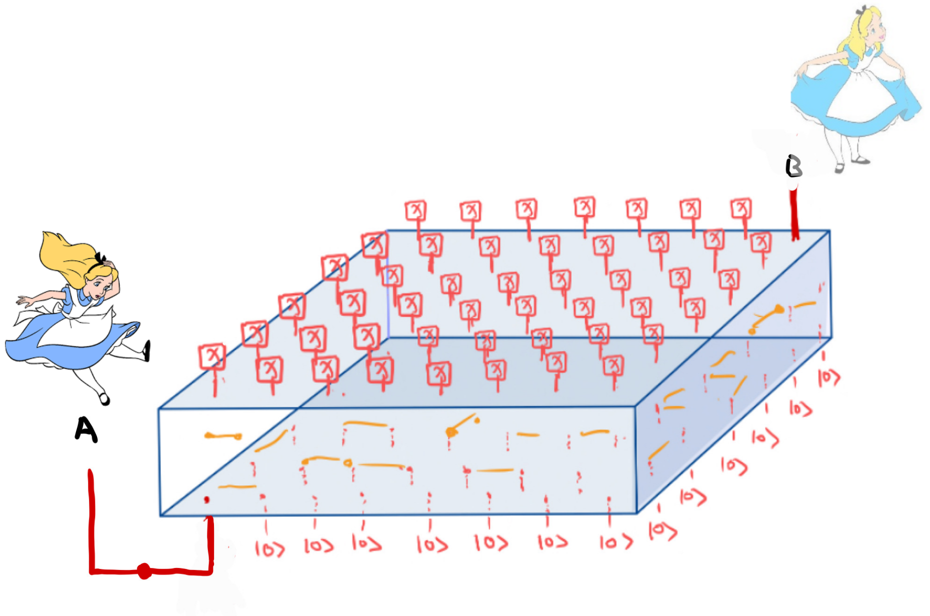
   Skinner, Ruhman & Nahum PRX 2019 Li, Chen & Fisher PRB 2019;
   Choi, Bao & EA, PRL 2020; Gullans & Huse PRX 2020 …

   Experiments: Noel et. al. Nature 2021; Koh et. al. arXiv:2203.04338



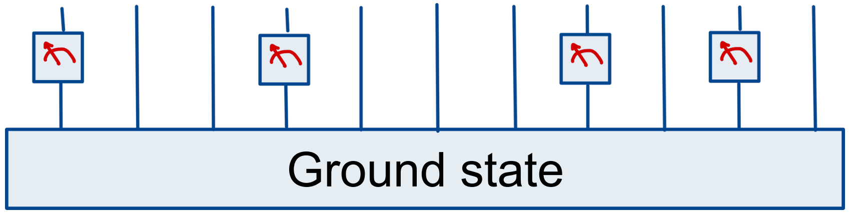
2. Finite time teleportation transition

   Bao, Block and EA arXiv:2110.06963

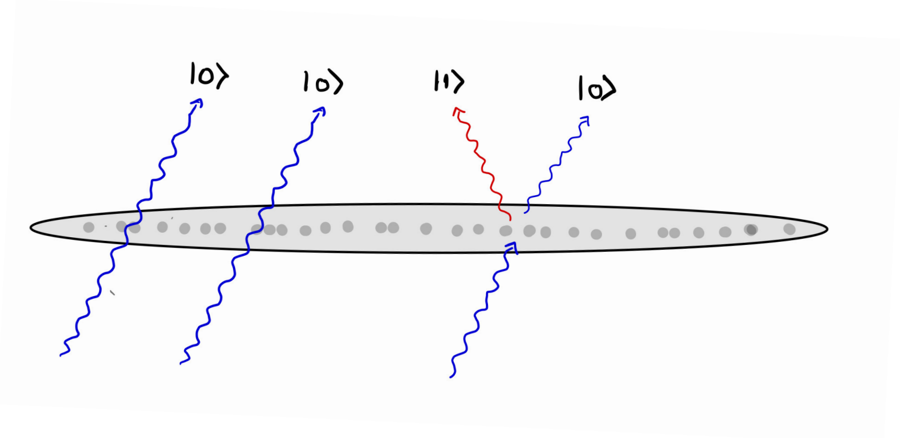


This talk

1. How do partial measurements of the ground state change the long-distance correlations in that state?



Example 1: Critical 1d quantum gas

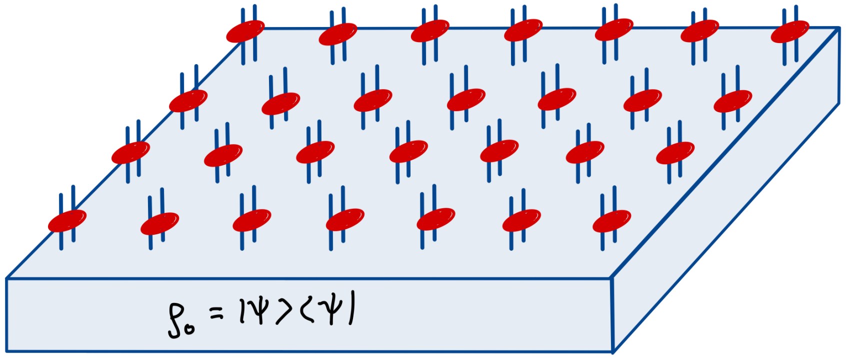


Example 2: Critical 1d transverse-field Ising model

2. Phase transition induced by a local quantum channel (no measurements)

Topological mixed states and phase transitions

Yimu Bao, Ruiha Fan and Ashvin Vishwanath and EA



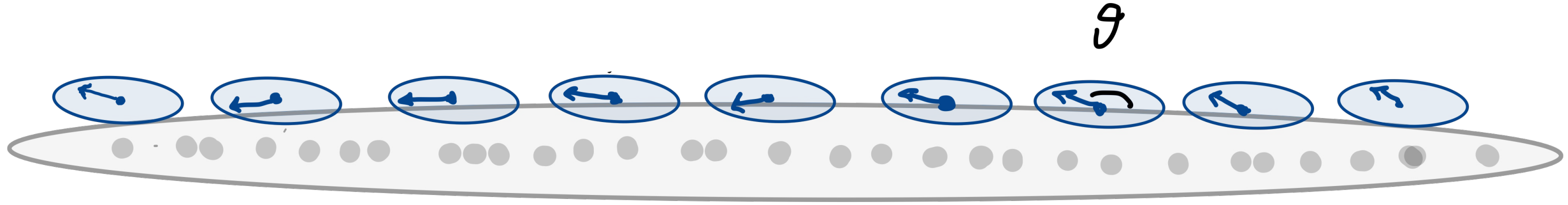
How do measurements affect quantum critical correlations?

Example 1: One-dimensional quantum liquids

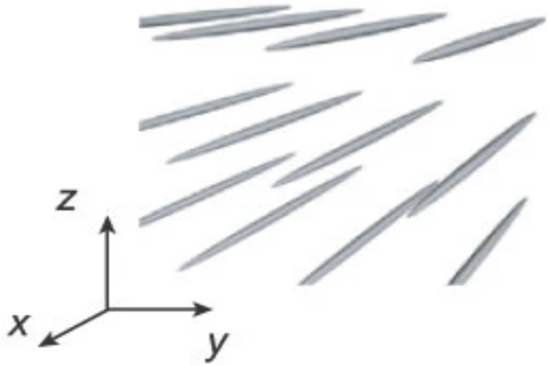
- Universal long wavelength description: Luttinger liquid

Action of phase fluctuations: $$S = \frac{K}{2\pi}\int dx d\tau \left[\dot{\theta}^2 + (\nabla\theta)^2\right]$$

For bosons: $$\psi^\dagger \sim e^{-i\theta}$$



Realized with ultracold atoms in optical lattices:



Paredes et. al.
(I. Bloch group) Nature 2004

How do measurements affect quantum critical correlations?

Example 1: One-dimensional quantum liquids

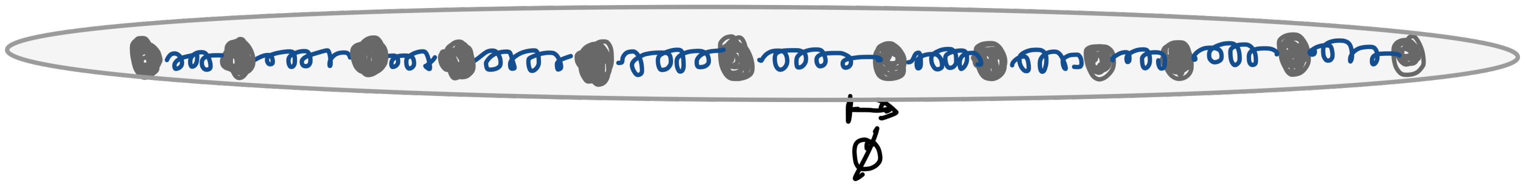
- Universal long wavelength description: Luttinger liquid

Dual action for the particle position fluctuations:

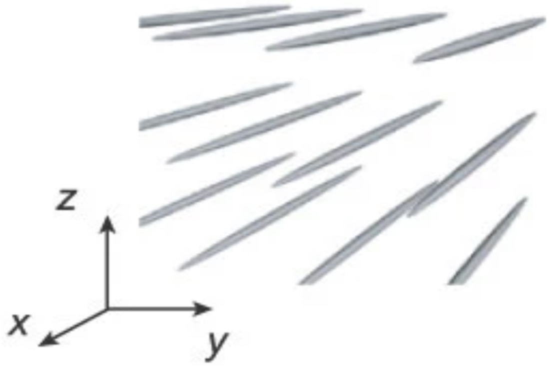
$$S = \frac{1}{2\pi K}\int dx d\tau \left[\dot{\phi}^2 + (\nabla\phi)^2\right]$$

Closely related to density fluctuations:

$$\delta n(x) \approx -\frac{1}{\pi}\nabla\phi(x) + \frac{1}{\pi}\cos[2\pi\rho_0 x - 2\phi(x)]$$



Realized with ultracold atoms in optical lattices:



Paredes et. al.
(I. Bloch group) Nature 2004

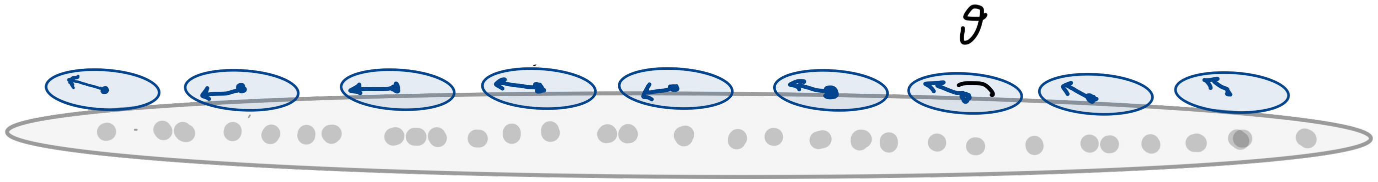
How do measurements affect quantum critical correlations?

Example 1: One-dimensional quantum liquids

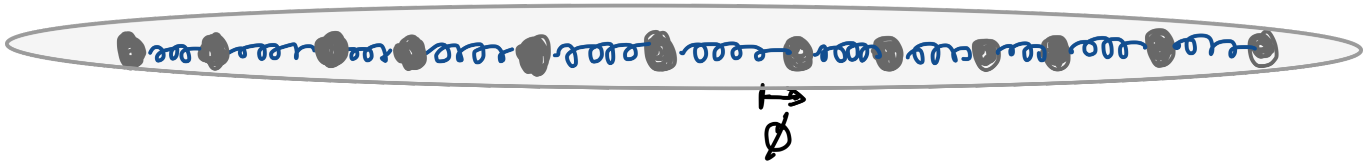
- Universal long wavelength description: Luttinger liquid

Dual descriptions in terms of phase or density fluctuations

$$S = \frac{K}{2\pi}\int dx d\tau \left[\dot{\theta}^2 + (\nabla\theta)^2\right]$$



$$S = \frac{1}{2\pi K}\int dx d\tau \left[\dot{\phi}^2 + (\nabla\phi)^2\right]$$

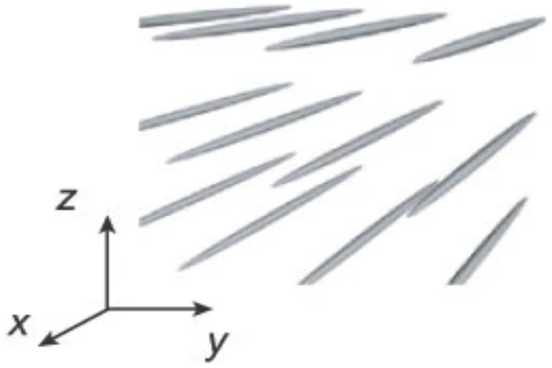


K=1 corresponds to non interacting fermions or hard-core bosons.
K<1: fermions with repulsive interactions or bosons with power law interaction

How do measurements affect quantum critical correlations?

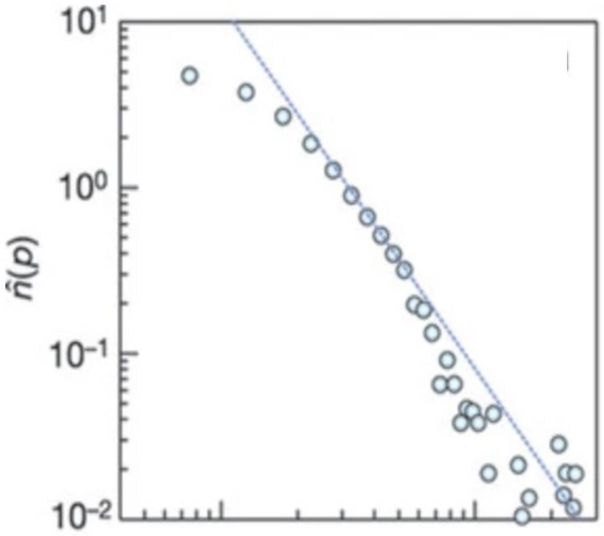
Example 1: One-dimensional quantum liquids

Realized with ultracold atoms in optical lattices:



➡ Quantum critical states with continuously tunable exponents.

Paredes et. al.
(I. Bloch group) Nature 2004



Density correlations:

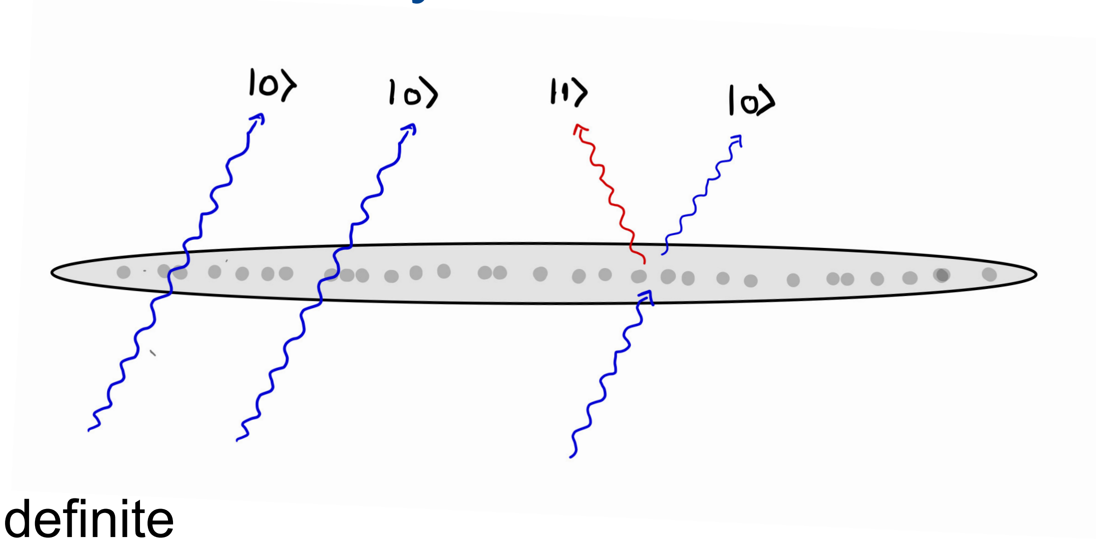
$$\langle \Psi_{gs} | \delta n(x) \delta n(0) | \Psi_{gs} \rangle \sim c_1 \left(\frac{1}{x}\right)^2 + c_2 \cos(2\pi\rho_0 x) \left(\frac{1}{x}\right)^{2K}$$

$\langle \nabla\phi(x)\nabla\phi(0)\rangle$ $\qquad$ $\langle e^{i(2\phi(x)-2\phi(0))}\rangle$

Phase correlations:

$$\langle \Psi_{gs} | \psi^\dagger(x)\psi(0) | \Psi_{gs} \rangle \sim \langle e^{i(\theta(x)-\theta(0))}\rangle \sim \left(\frac{1}{x}\right)^{\frac{1}{2K}}$$

Quantum Non-Demolition Measurement: Homodyne detection



- Weak measurement of the density everywhere: probe light interacts weakly with the particles.
- Measure polarizations of photons at different locations.
- Outcome |1> = particle found at this location
  Outcome |0> = “No click”. Particle occupation remains indefinite

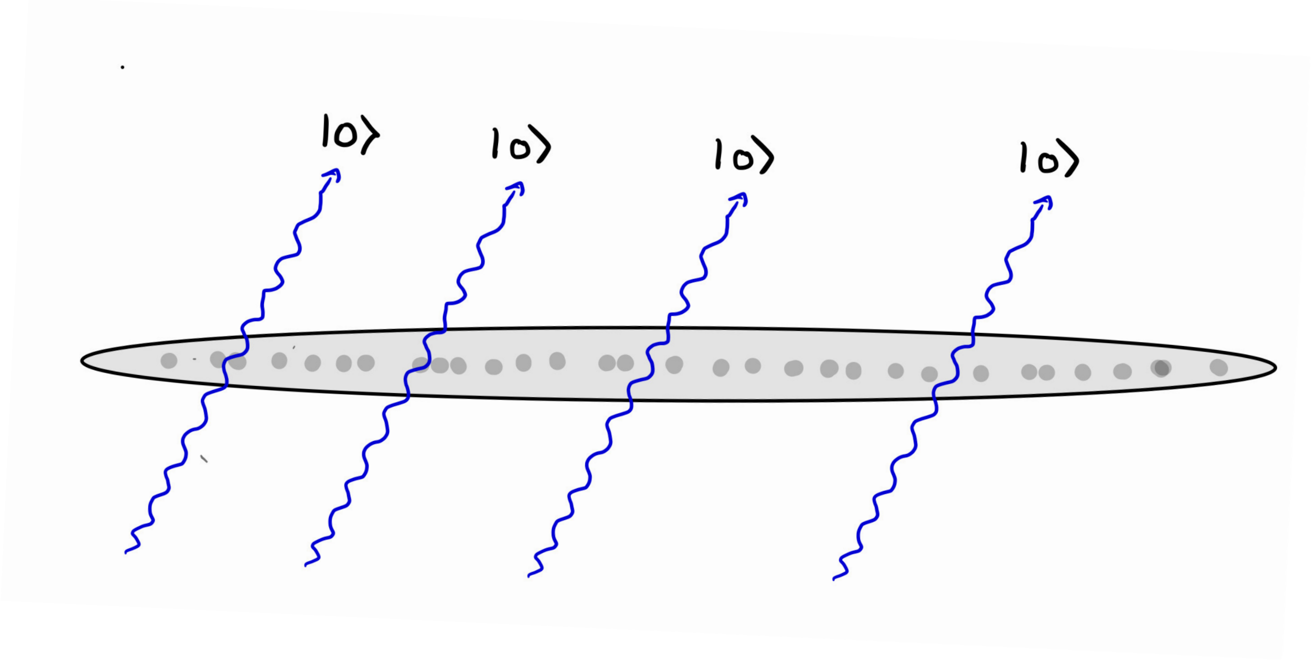
How does the partial/weak measurement affect the critical correlations?

$$\langle \Psi_{\rm gs}| P_m n(x) n(0) P_m |\Psi_{\rm gs}\rangle \overset{?}{\sim} c_1 \left(\frac{1}{x}\right)^2 + c_2 \cos(2\pi\rho_0 x) \left(\frac{1}{x}\right)^{2K}$$

$$\langle \Psi_{\rm gs}| P_m \psi^\dagger(x) \psi(0) P_m |\Psi_{\rm gs}\rangle \overset{?}{\sim} \left(\frac{1}{x}\right)^{\frac{1}{2K}}$$

Important: there is no dynamics. Perform measurements then evaluate correlations in the output state.

Simplest case: post-select on the null measurement outcome (no clicks)

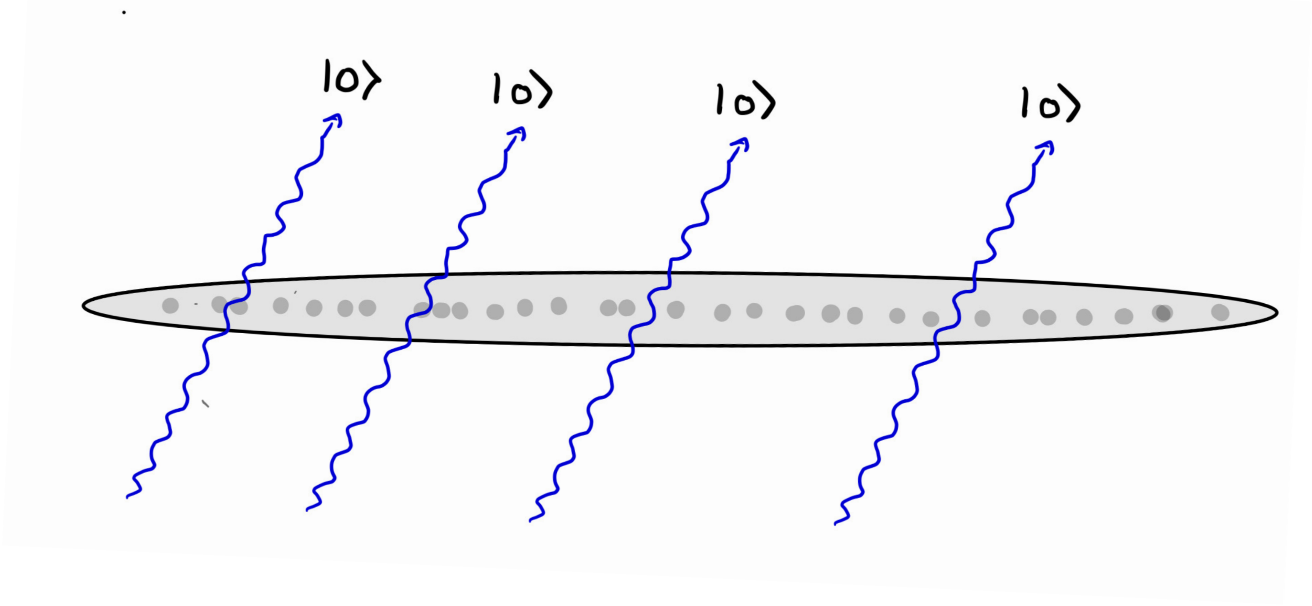


<u>The no click state:</u> $$|\Psi_{\rm nc}\rangle = e^{-\int dx v(x) n(x)}|\Psi_{\rm gs}\rangle$$

If the measurement strength *v*(x) is oscillating in space at a wavelength commensurate with the particle density, then we can represent it in terms of the long-wavelength fields:

$$|\Psi_{\rm nc}\rangle = e^{-v\int dx\, \cos[2\phi(x)]}|\Psi_{\rm gs}\rangle$$

Simplest case: post-select on the null measurement outcome (no clicks)

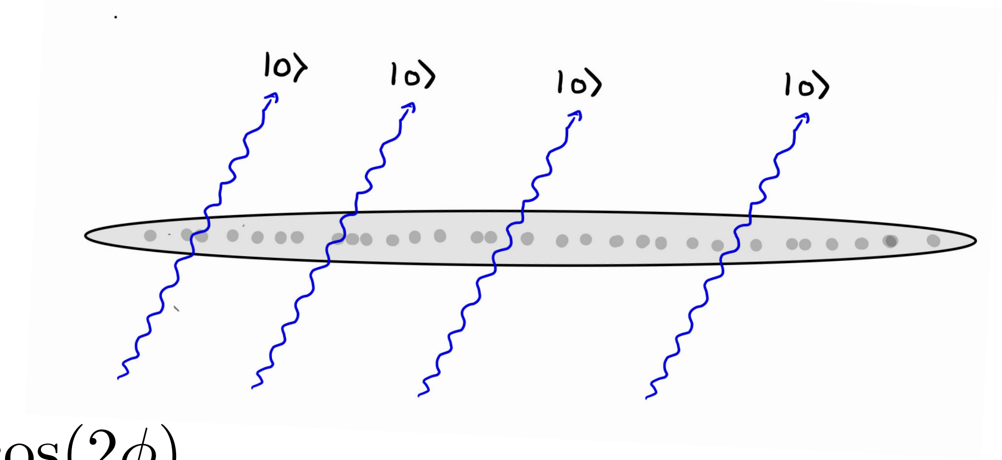


Correlations in the no click state:

$$\langle n(x)n(0)\rangle_{\rm nc} = \lim_{\beta\to\infty} \langle \Psi_{\rm ref}| e^{-\beta H_{\rm LL}} e^{-v\int dx\,\cos(2\phi)} n(x)n(0) e^{-v\int dx\,\cos(2\phi)} e^{-\beta H_{\rm LL}} |\Psi_{\rm ref}\rangle$$

$$= \int \mathcal{D}\phi\, e^{-S_{\rm nc}[\phi]}\, \delta n(x)\delta n(0)$$

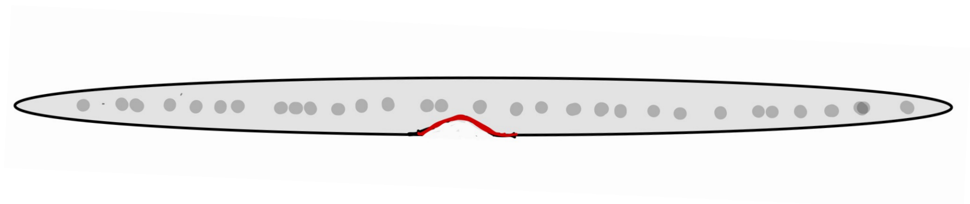
The "no click" action



$$\langle \delta n(x)\delta n(0)\rangle_{\rm nc} = \int \mathcal{D}\phi \, e^{-S_{\rm nc}[\phi]} \, \delta n(x)\delta n(0)$$

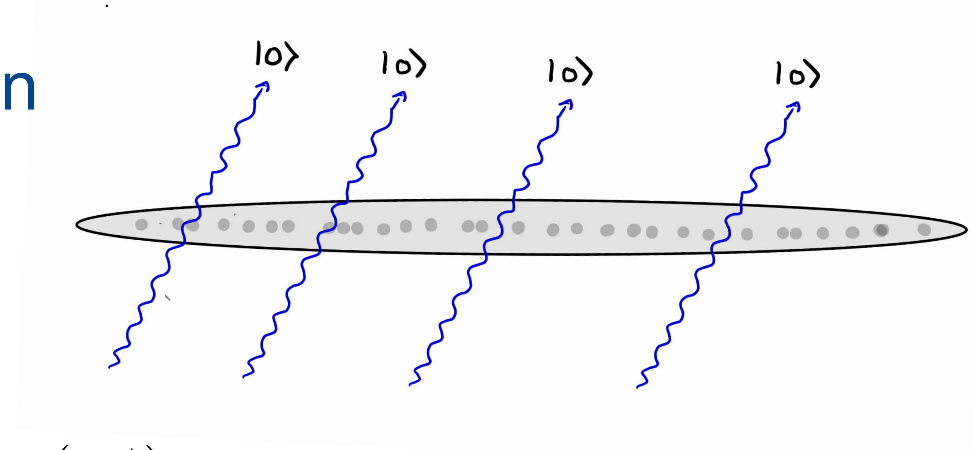
$$S_{\rm nc} = \frac{1}{2\pi K}\int dx d\tau \left[(\partial_\tau \phi)^2 + (\partial_x \phi)^2\right] - v\int dx \cos(2\phi)$$

Compare to the well-known problem of a single impurity in a Luttinger liquid
[Kane and Fisher PRL 1992]



$$S_{\rm imp} = \frac{1}{2\pi K}\int dx d\tau \left[(\partial_\tau \phi)^2 + (\partial_x \phi)^2\right] - v\int d\tau \cos(2\phi)$$

The “no click” action

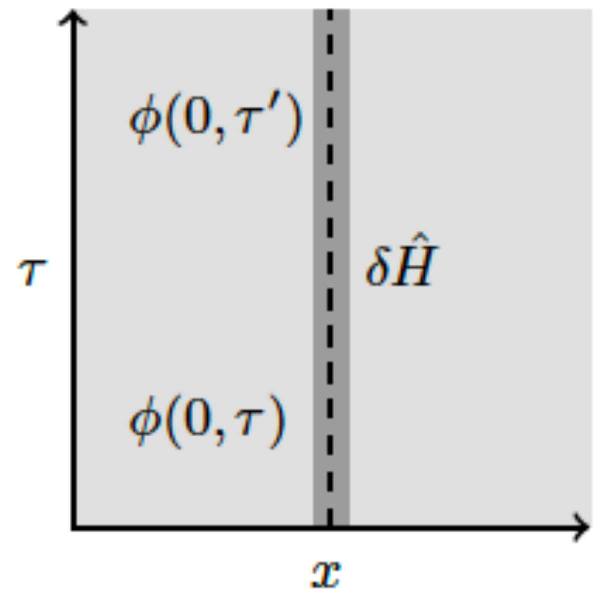


$$\langle n(x)n(0)\rangle_{\rm nc} = \int \mathcal{D}\phi e^{-S_{nc}}\, n(x)n(0)$$

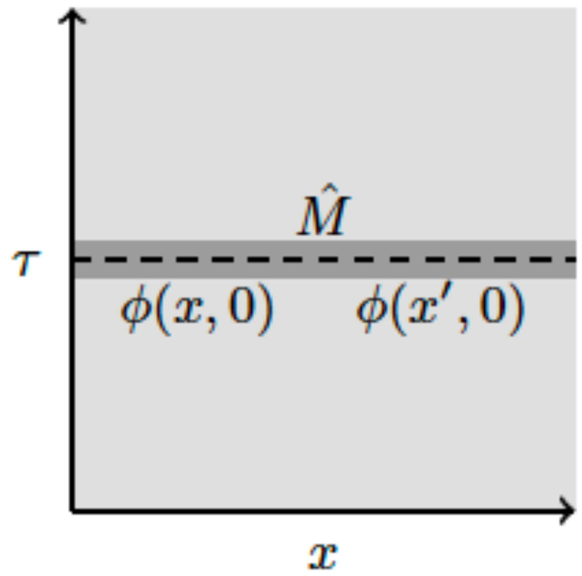
$$S_{\rm nc} = \frac{1}{2\pi K}\int dx d\tau \left[(\partial_\tau\phi)^2 + (\partial_x\phi)^2\right] - v\int dx \cos(2\phi)$$

= Wick rotated impurity problem

impurity

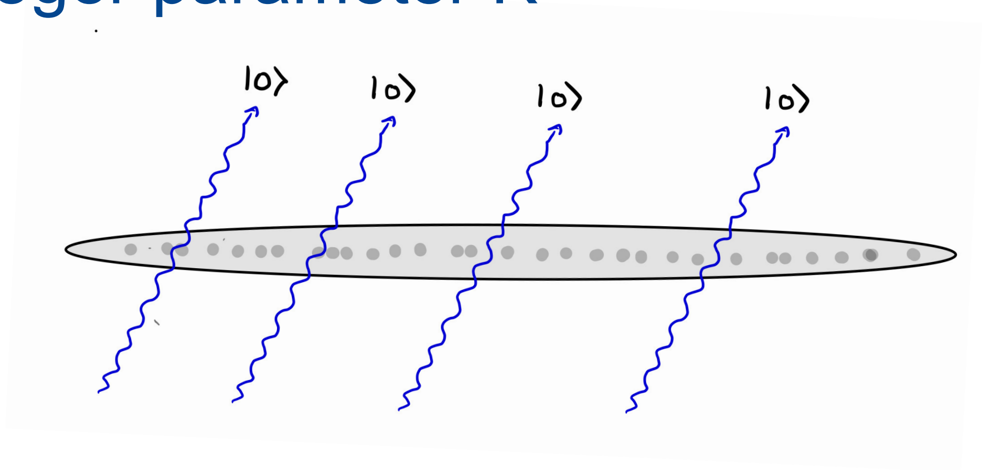


measurements



Phase transition tuned by the Luttineger parameter K

$$S_{\rm nc} = \frac{1}{2\pi K}\int dx d\tau \left[(\partial_\tau\phi)^2 + (\partial_x\phi)^2\right] - v\int dx \cos(2\phi)$$



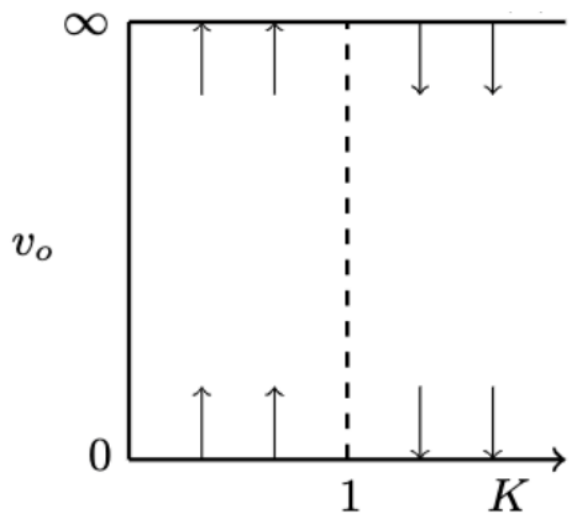
Scaling of the measurements : $\frac{dv}{d\ell} = (1-K)\, v$

$K > 1$ Measurements are **irrelevant**

Long distance correlations unaffected for any finite measurement strength

Phase: $\langle e^{i[\hat\theta(x)-\hat\theta(0)]}\rangle \sim x^{-1/(2K)}$

Smooth component of the density: $\langle \nabla\hat\phi(0)\nabla\hat\phi(x)\rangle \sim x^{-2}$



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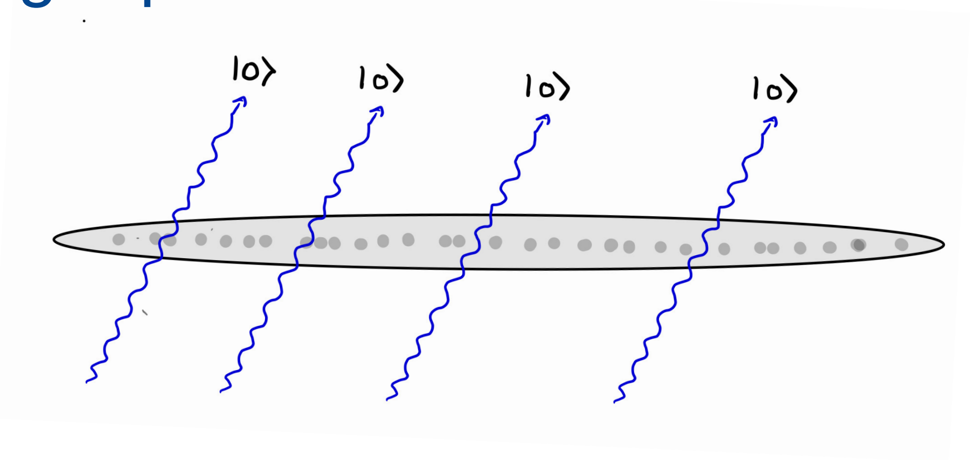


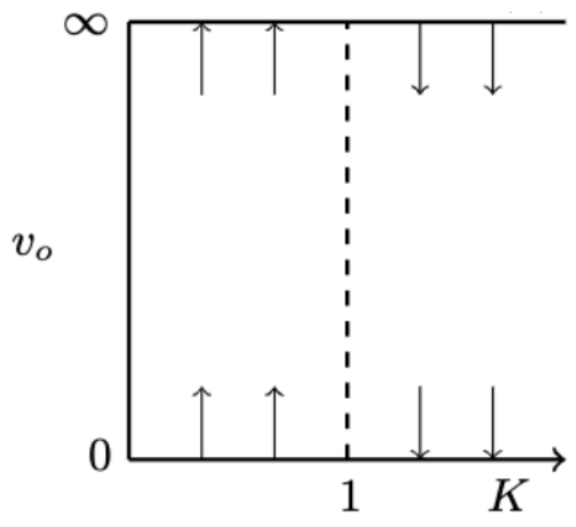
Scaling of the measurements : $\frac{dv}{d\ell} = (1-K)\,v$

$K < 1$ Measurements are **relevant**

Non perturbative effect on long distance correlations:

$$\langle e^{i[\hat\theta(x)-\hat\theta(0)]}\rangle \sim x^{-1/(2K)} \rightarrow x^{-1/K}$$

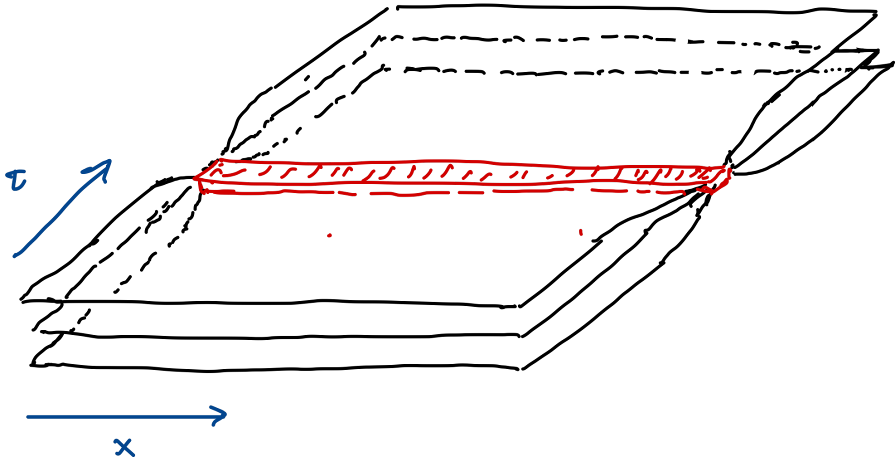
$$\langle \nabla\hat\phi(0)\nabla\hat\phi(x)\rangle \sim x^{-2} \rightarrow x^{-2/K}$$



Performing local measurements has a highly non-local effect on the quantum state!

Ensemble of measurement outcomes

To implement the average over measurement outcomes we need to introduce N replicas taking the limit N -> 0 at the end:

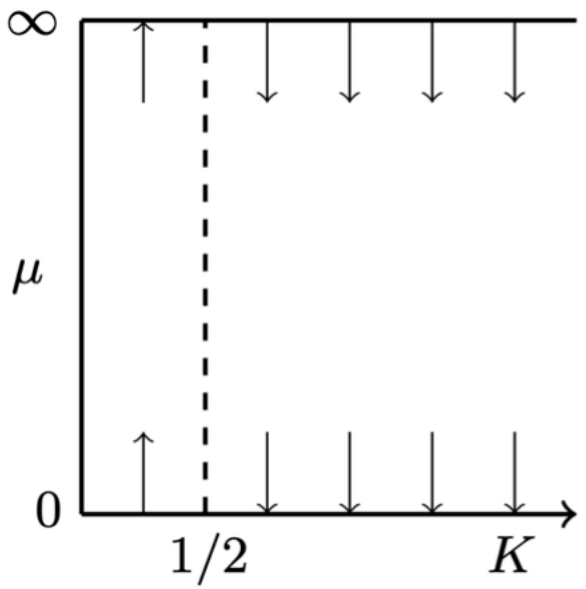


$$s_N[\{\varphi_\alpha\}] = \sum_{\alpha=1}^{N} s[\varphi_\alpha] - \frac{\mu}{2N\pi^2}\int dx \sum_{\alpha<\beta} \cos[2(\varphi_\alpha - \varphi_\beta)]$$

The random measurement outcomes couple the replicas attempting to lock them to each other.

Need to look at quantities that are non-linear in the density matrix, e.g.:

$$C^{(2)}(r) \equiv \sum_m p_m |\langle \psi_m | e^{i(\theta(r)-\theta(0))} | \psi_m \rangle|^2$$



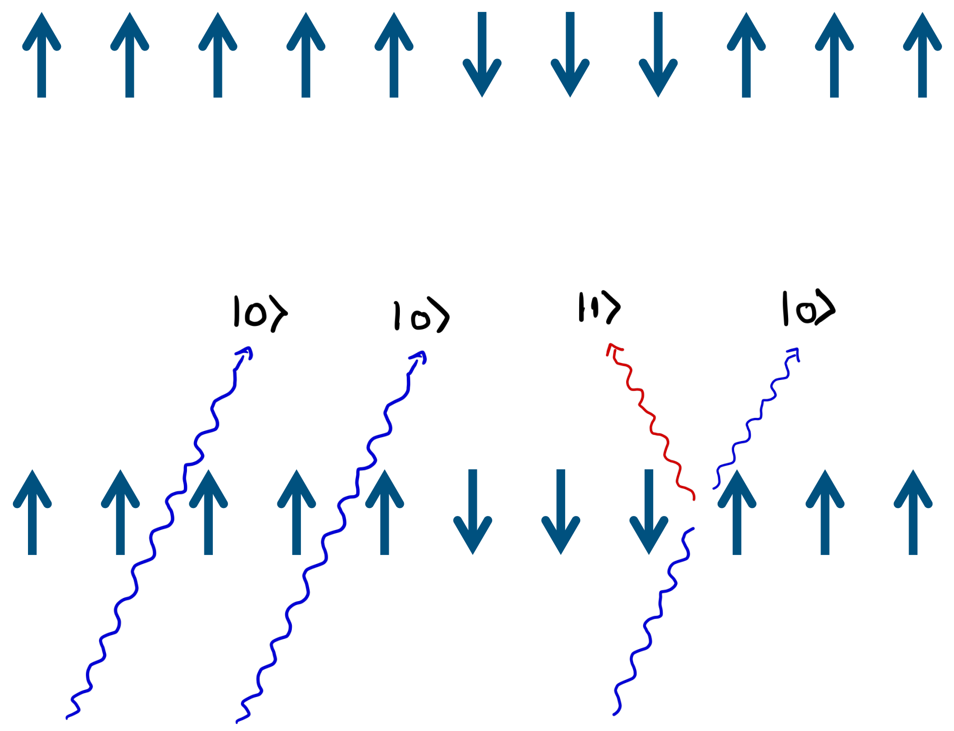
Perturbative RG reveals a transition at K=1/2 in this case.

Example 2: critical transverse field Ising model

$$H = -\sum_i (Z_i Z_{i+1} + X_i)$$

Order parameter correlations: $\langle Z_{j+r} Z_j \rangle \sim x^{-1/4}$

QND measurements $Z_i Z_{i+1}$ (or of X_i):



$Z_i Z_{i+1}$: outcome |1> detects a domain wall.

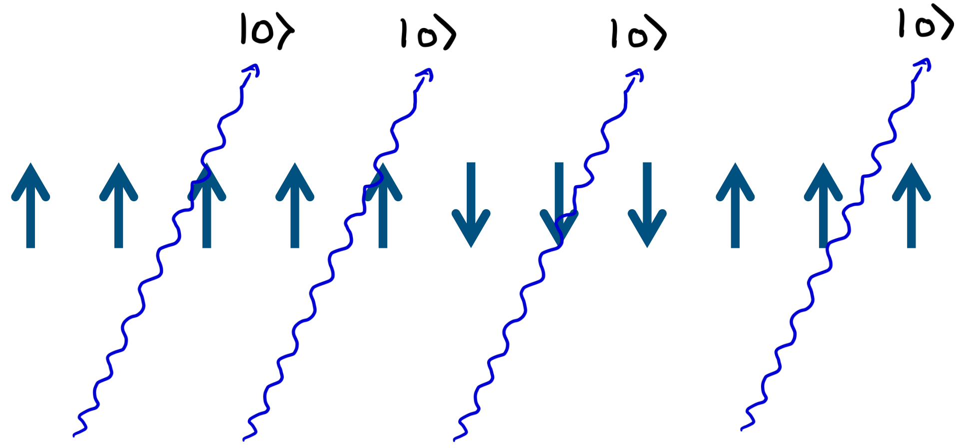
X_i : outcome |1> odd parity $(\uparrow - \downarrow)_i$

What are the correlations immediately after applying the weak measurements?

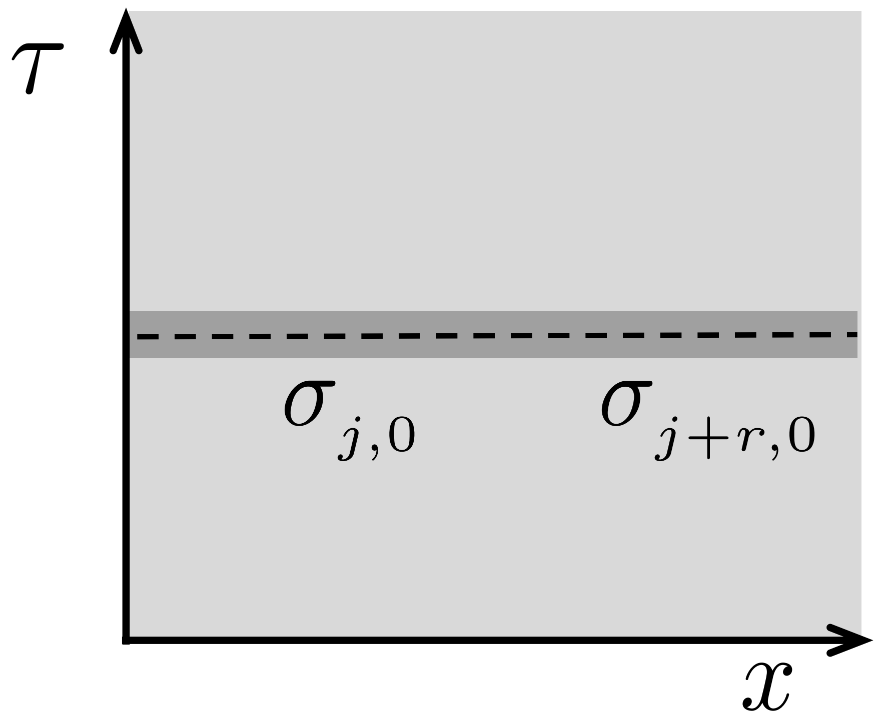
Simplest case: postselect on “no click”

The general scheme for calculating the correlations in the no click state is the same:

$$\langle Z_{j+r} Z_j \rangle_{\rm nc} = \frac{1}{Z_{\rm nc}} \sum_{\{\sigma\}} e^{-S_{\rm nc}[\sigma]} \sigma_{j,0} \sigma_{j+r,0}$$



No click action = Critical 2d Ising model with a line defect having either weaker or stronger bonds than the bulk

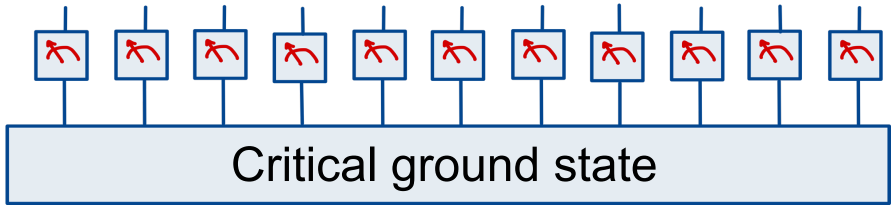


Scaling analysis shows that such a defect is exactly marginal at the Ising critical point.

Prediction: critical exponent for decay of the Ising correlations is tuned continuously by varying the measurement strength !

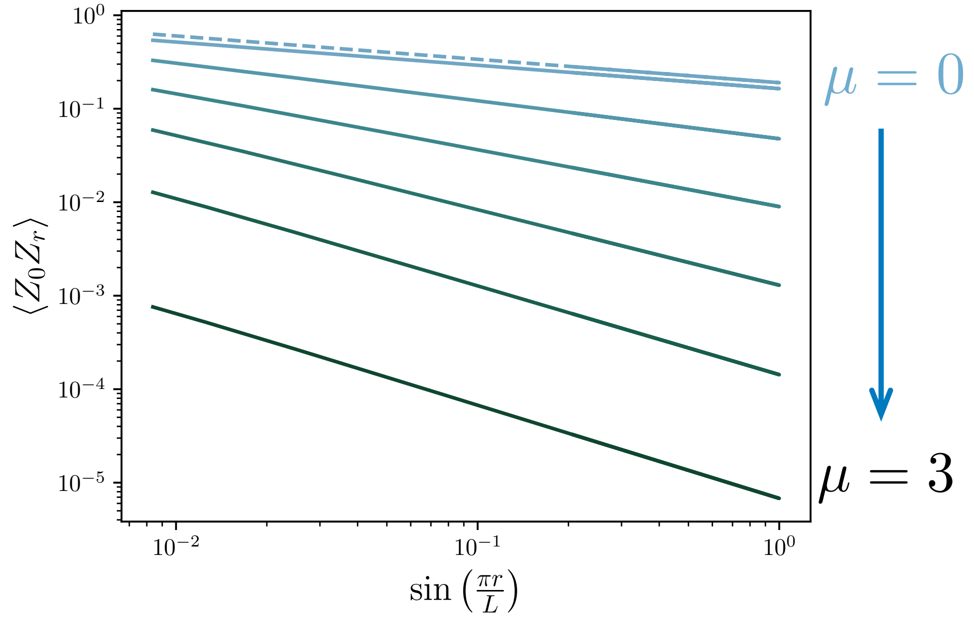
Numerical confirmation through mapping to free fermions

$e^{-\mu \hat{M}_i}$ →

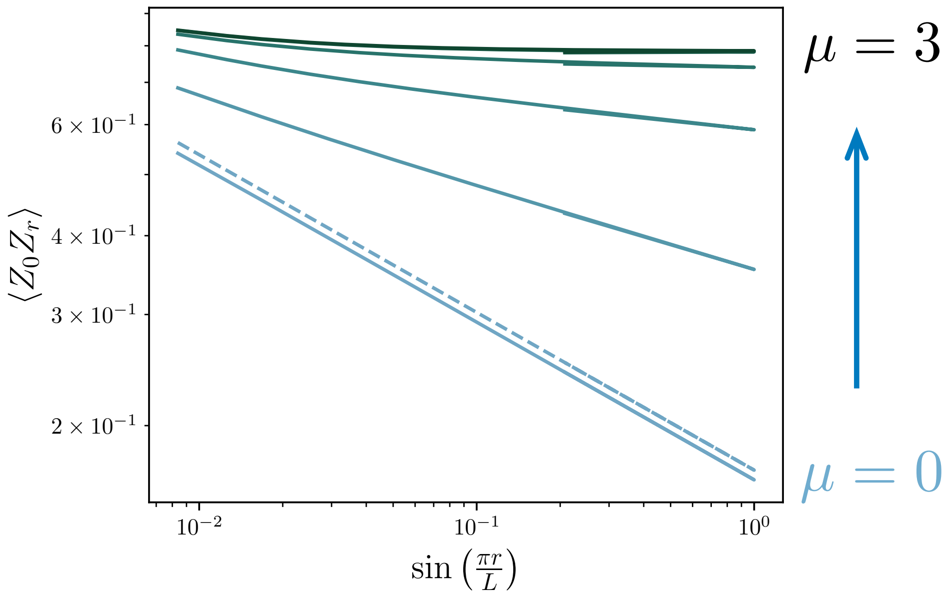


Behavior of Ising correlations $\langle Z_j Z_{j+r} \rangle_{\mathrm{nc}}$ after "no-click" measurements:

$\hat{M}_i = X_i$

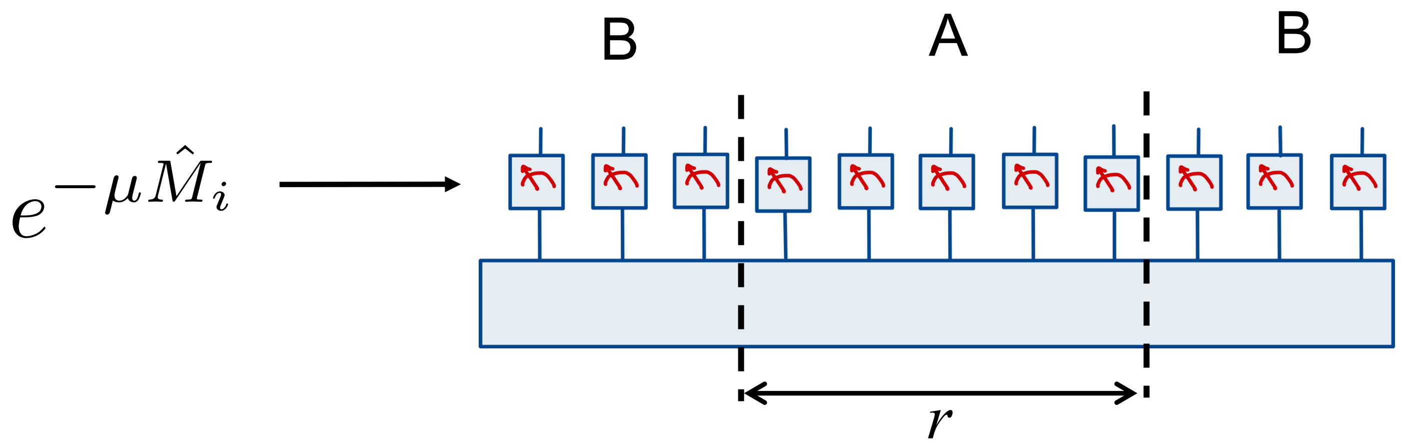


$\hat{M}_i = Z_i Z_{i+1}$

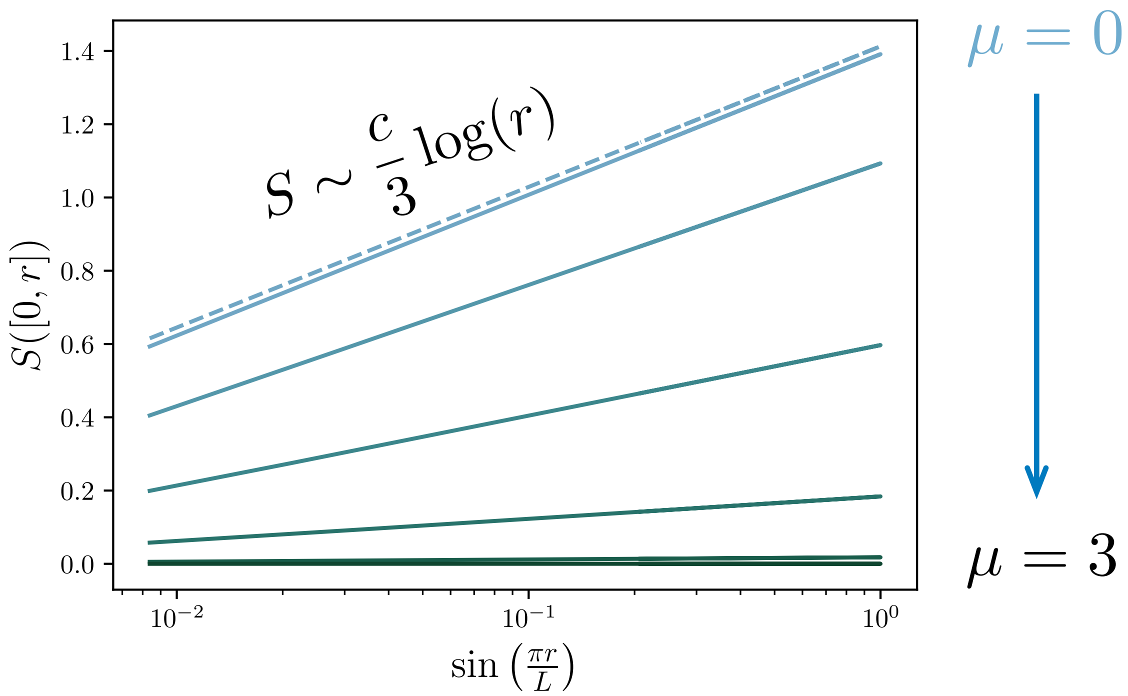


Order parameter correlation exponent tuned continuously by the measurement strength

Effect of measurement on the subsystem entanglement entropy

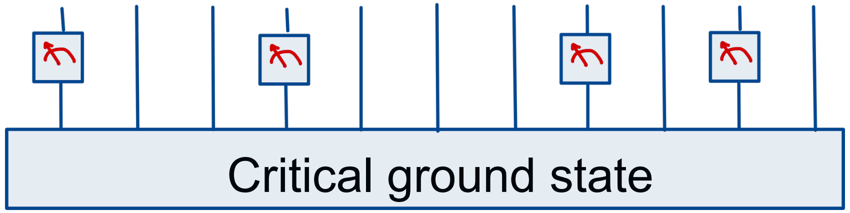


The effective central charge is changing continuously with X measurement strength.

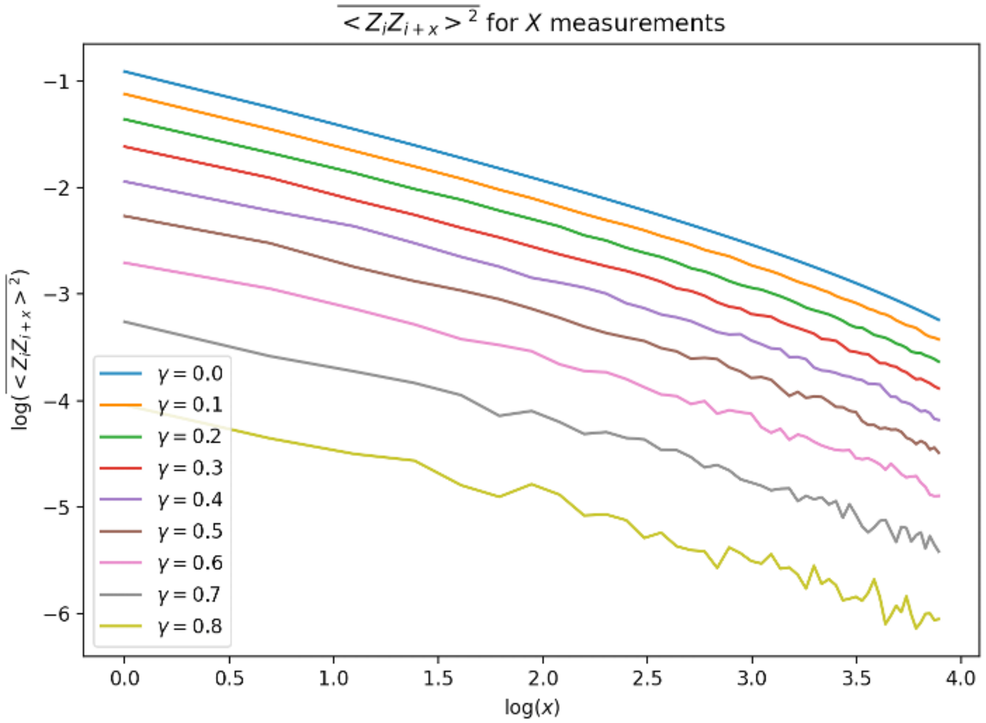


Need a theory of the entanglement entropy!

Ensemble of measurement outcomes

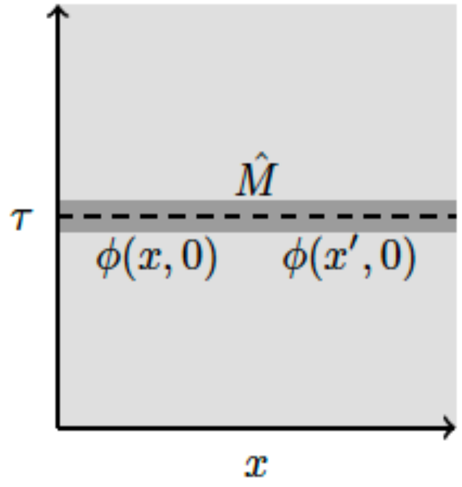


Averaging over measurement outcomes leads to coupling of different replicas at τ=0



- Measurements are an irrelevant perturbation
- No change in the decay of correlations at long distances regardless of measurement strength

Summary and Outlook of this part

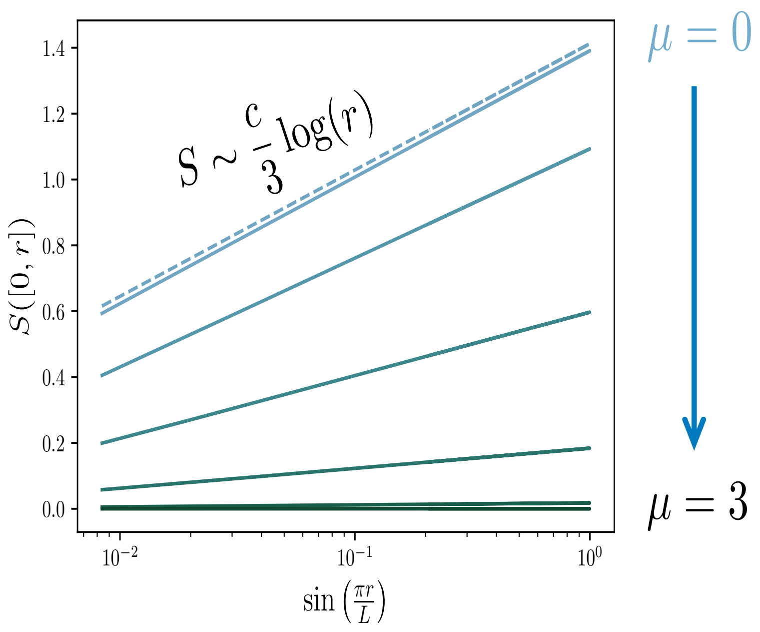


The effect of measurements of a many-body state in d dimensions is mapped to the effect of a d dimensional defect in a d+1 dimensional stat-mech model.

Open qustions

- Use feedback on measurement outcome to alleviate post-selection ?

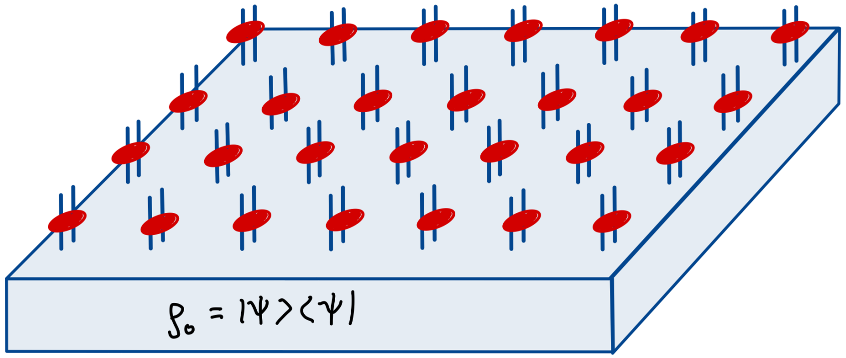
- Theory of the entanglement entropy following partial measurement. Explain the continuously varying of the effective central charge in the Ising case



- Phase transitions tuned by decoherence/errors instead of measurements?

Part 2 : Effect of a local decoherence channel on a topological state: Mixed topological phases and phase transitions

With: Yimu Bao, Ruiha Fan and Ashvin Vishwanath



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Now operate on ρ_0 with a local quantum channel, such as

$$\rho \to (1-\gamma)\rho + \gamma\, X_i \rho X_i$$

Is there topological order in the mixed state? A critical γ ?
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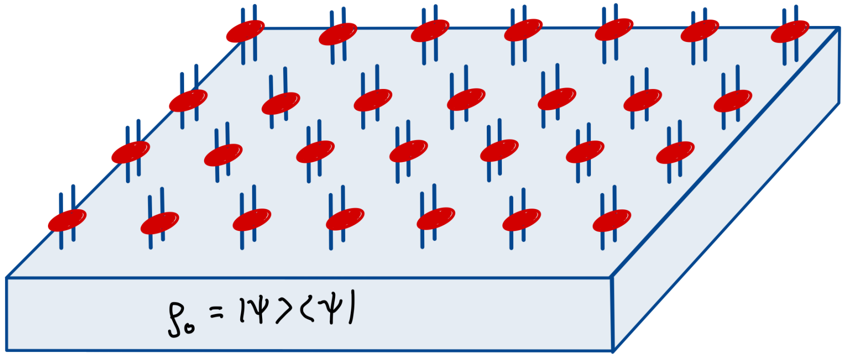

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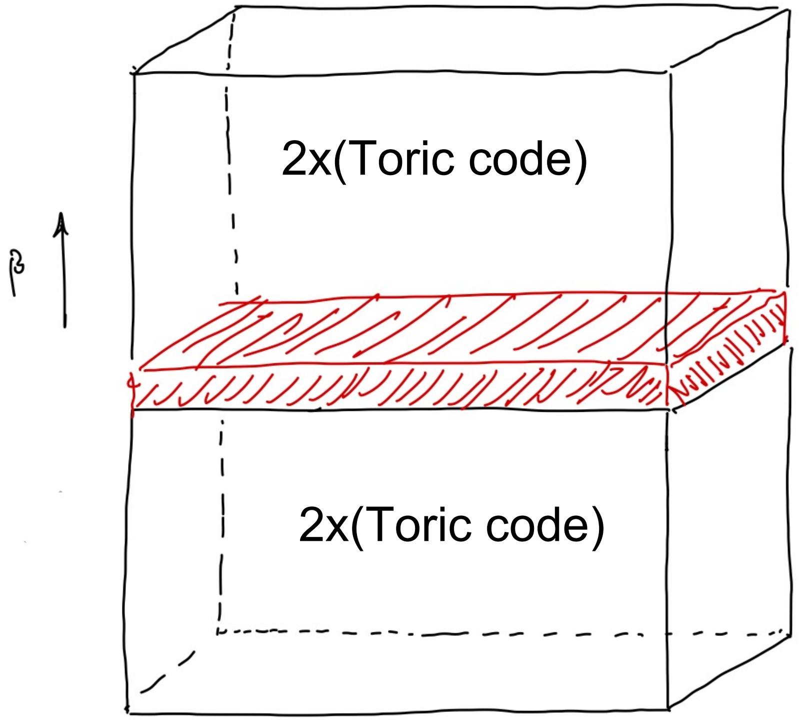

Two independent copies of the toric code are coupled by the channel at $\tau=0$. Leads to condensation of anyon pairs on the 2d defect

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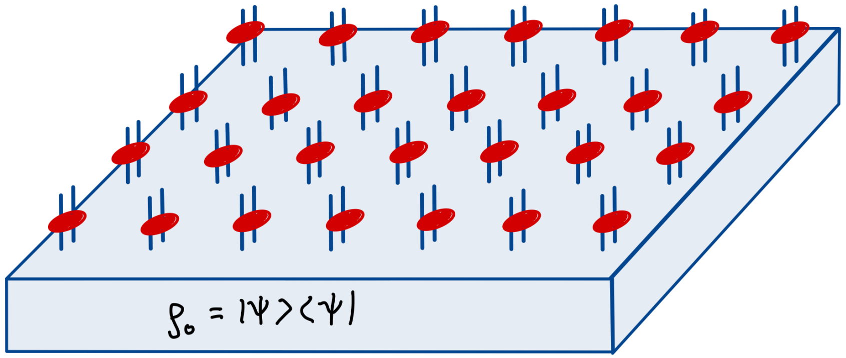

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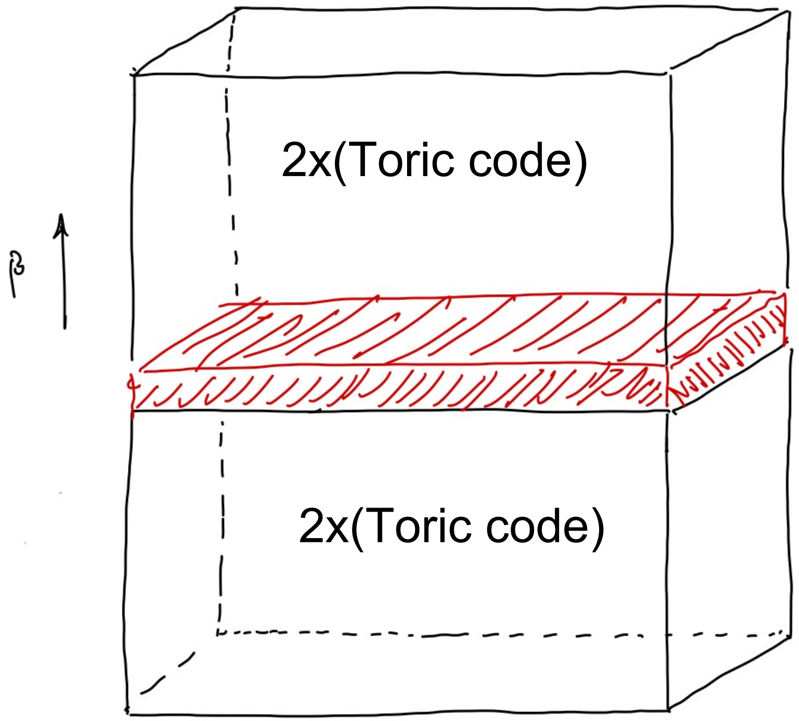

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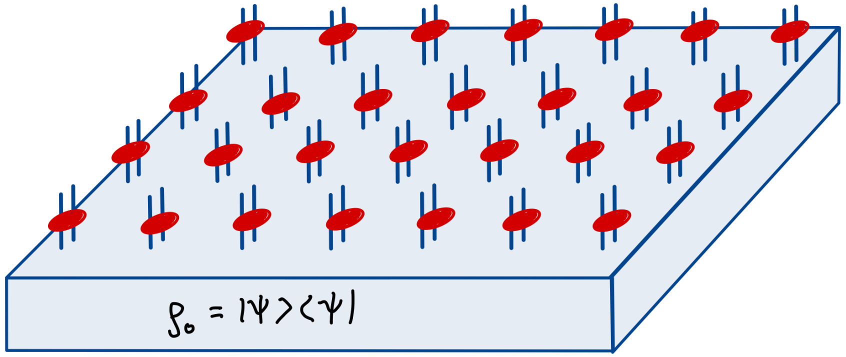

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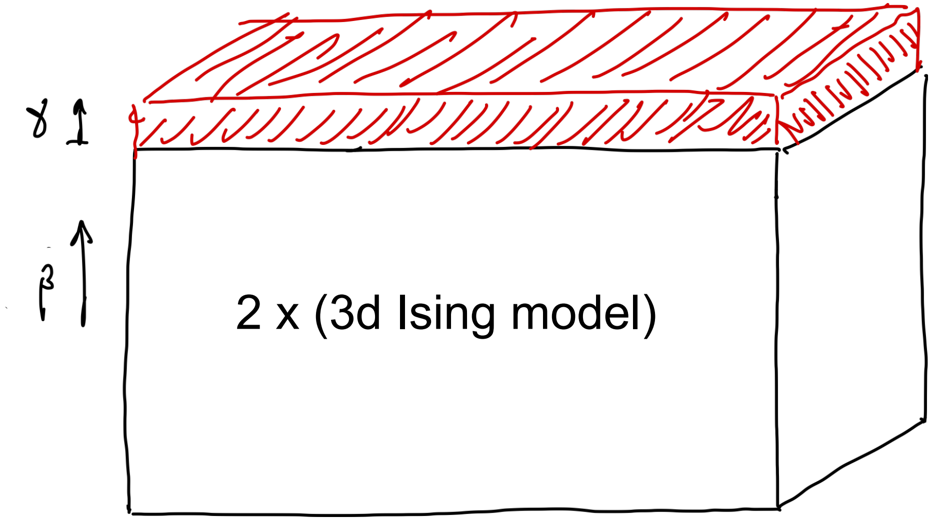
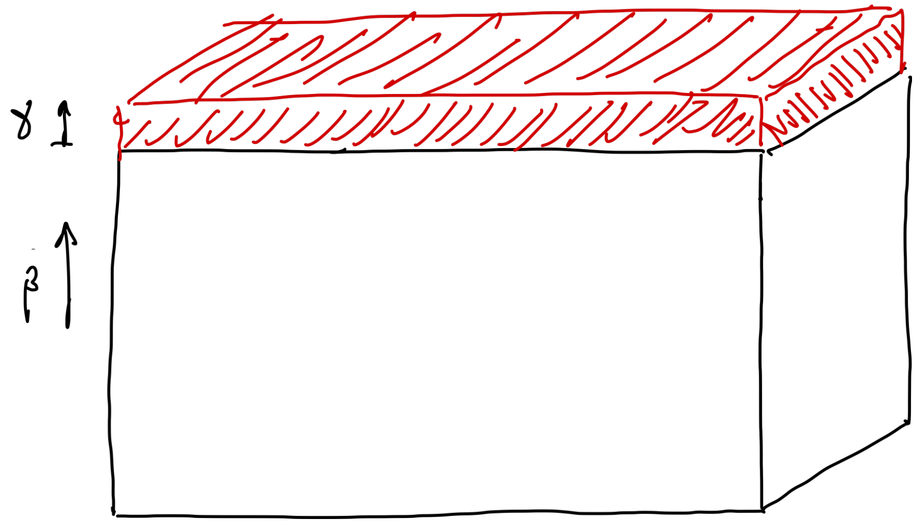


Only the quantum channel at the top boundary couples the two Ising models

The decoherence channel induces a boundary phase transition at a critical value of γ

When strong enough, the term $\gamma \sum_{\langle ij \rangle} Z_i^1 Z_j^1 Z_i^2 Z_j^2$

leads to spontaneously broken symmetry on boundary, diagnosed by establishment of long range order in the order parameter $Z_i^1 Z_i^2$



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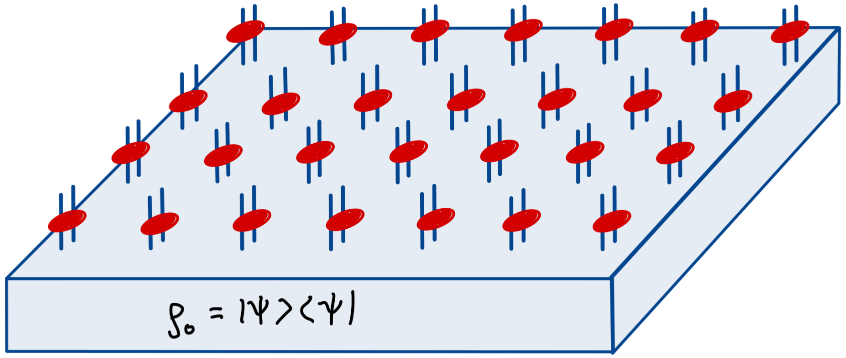
With: Yimu Bao, Ruiha Fan and Ashvin Vishwanath



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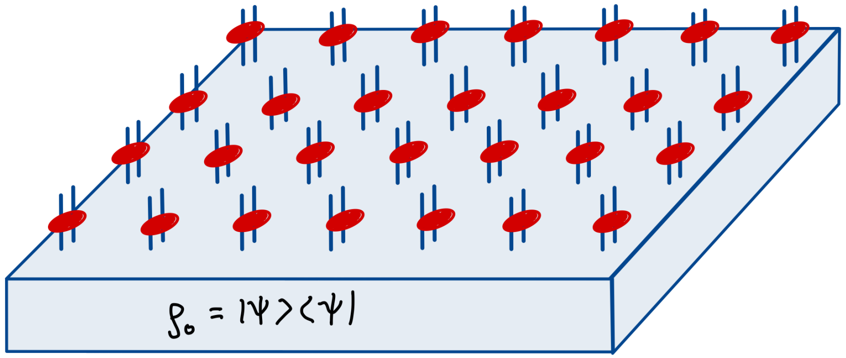

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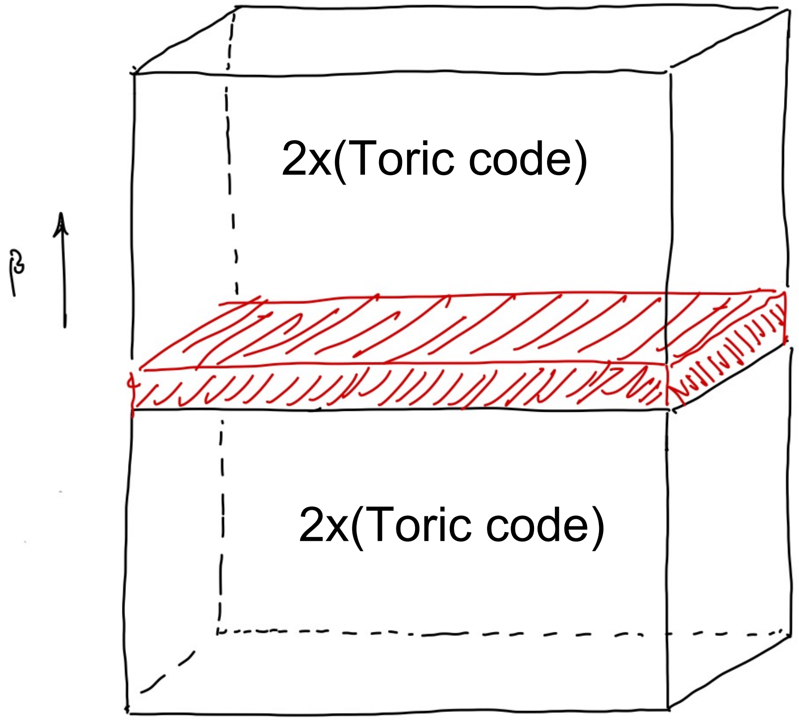

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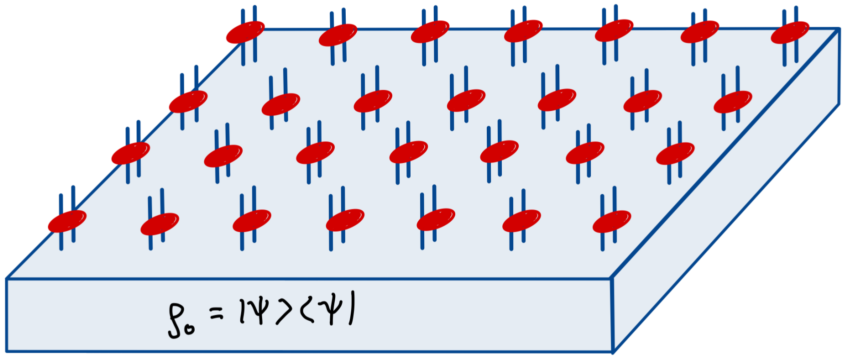

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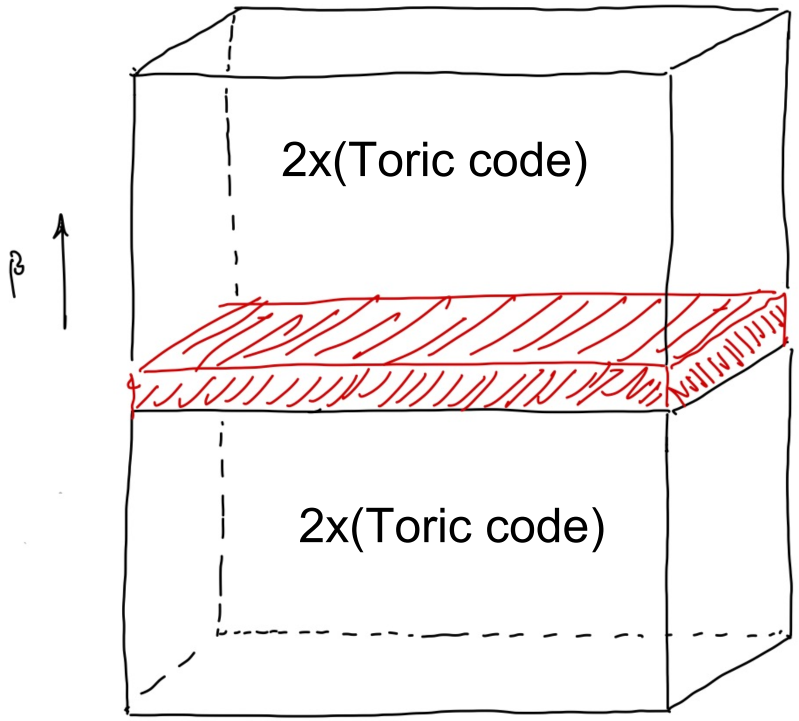

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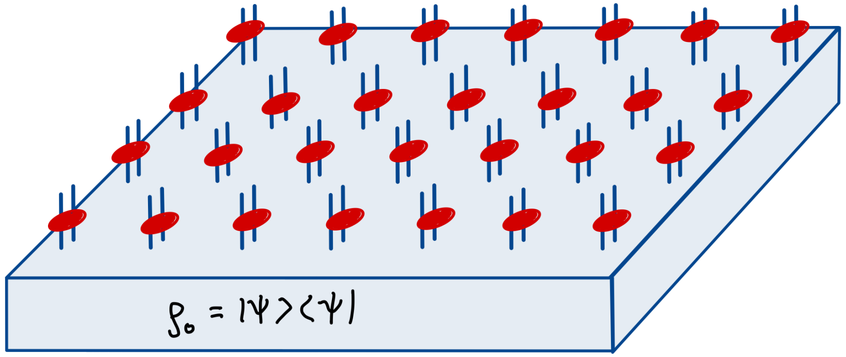

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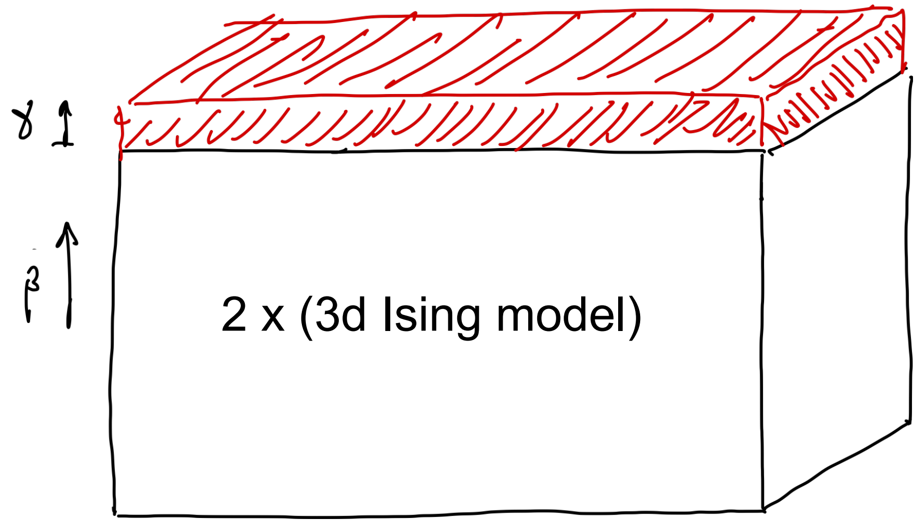

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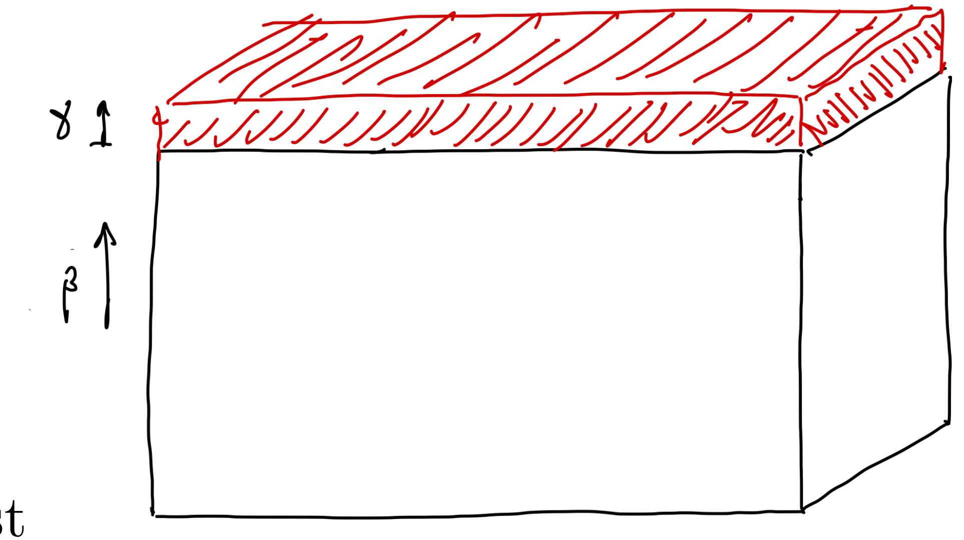

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