Gauge theory origin of Rydberg quantum spin liquids

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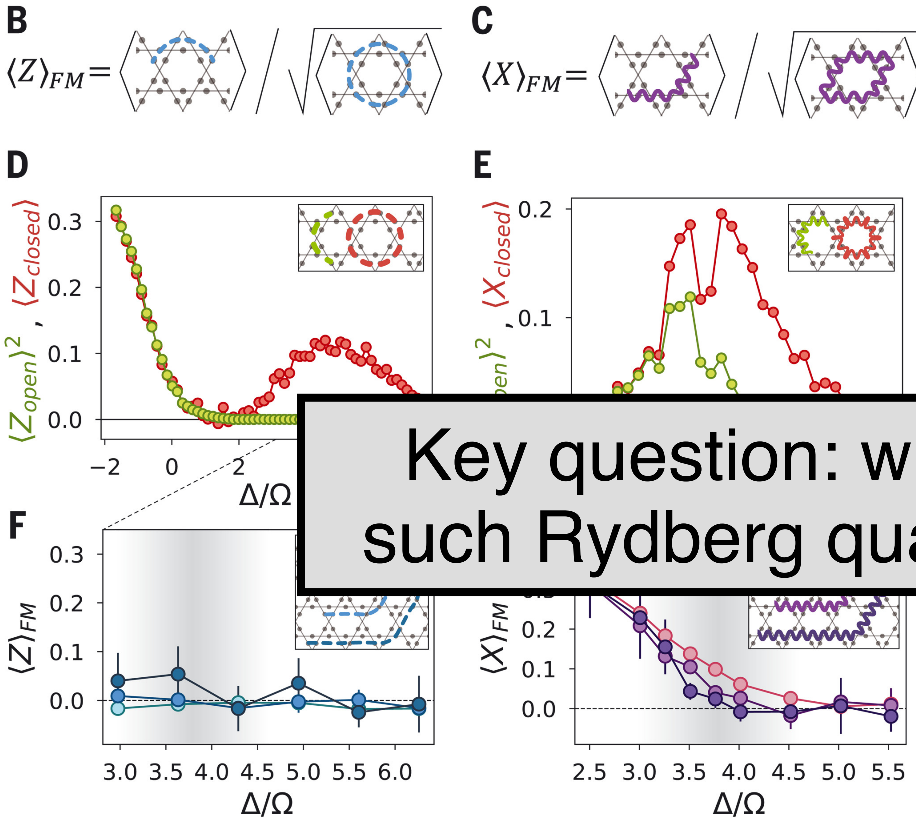
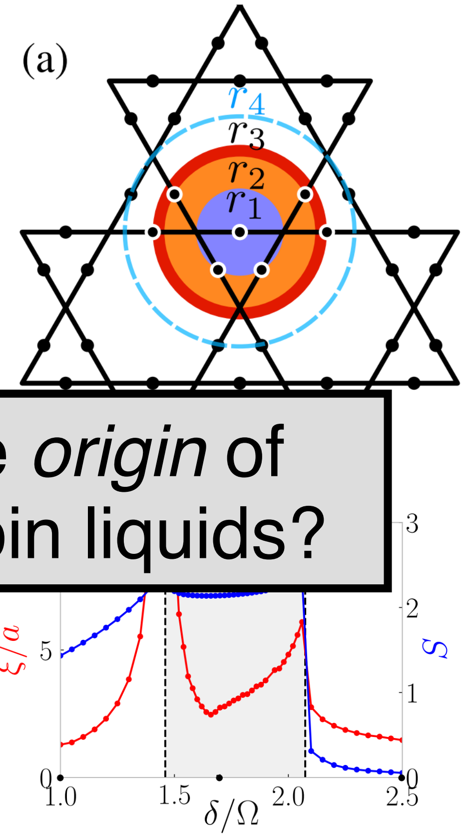
SISSA

Joint work with R. Andreoni, A. Angelone, **F. Surace and P. Tarabunga.**

arXiv:2205.13000

Deconfined phases of matter in quantum simulators





Key question: what is the *origin* of such Rydberg quantum spin liquids?

Exp: Science 374, 1242 (2021);

Numerics: Phys. Rev. X 11, 031005 (2021)

Why is there a spin liquid?

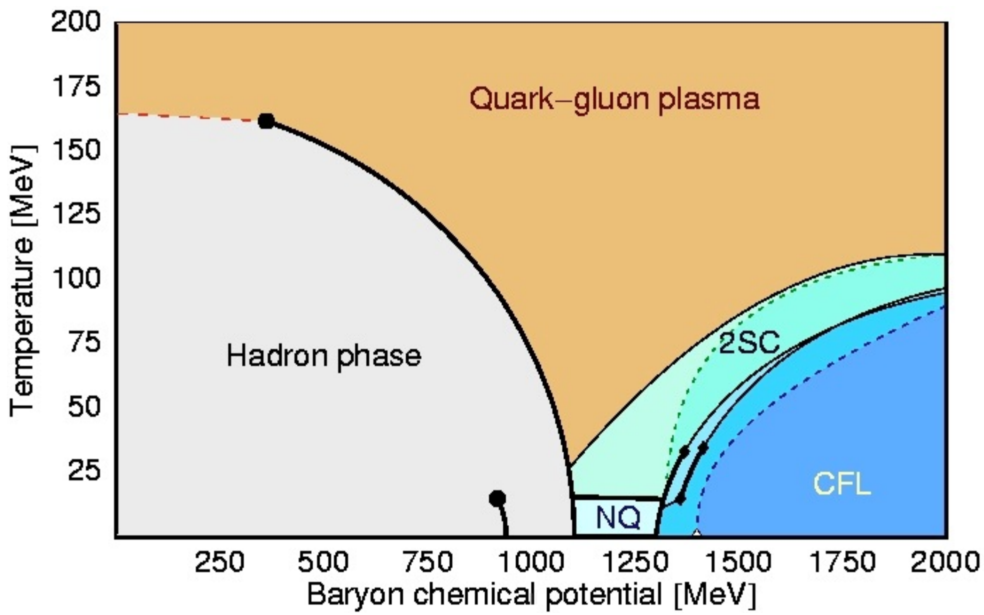
Motivations:

- Insights on the microscopics
- A new class of interacting field theories? Or just a “lucky” model?
- Better understanding of diagnostics

Results:

- New class of **exactly solvable models** with dual Z2 gauge theory description and *two-body* Hamiltonians
- **Broad** class of realistic models! (Rydberg “blessing”)
- Unclear physical interpretation of real space loop order parameters

My very personal motivation



- We need a mechanism to achieve the continuum limit -> a *microscopic interaction* that induces deconfinement

Bañuls et al., EPJD 2021; Zohar et al., RPP 2015; MD and Montangero, CRP 2016

many other details under the carpet…

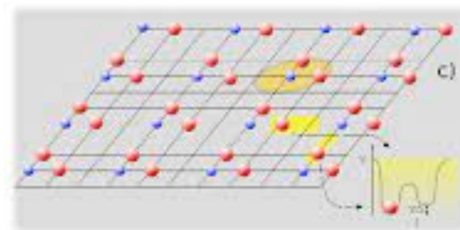
Outline

- The challenge: why is it so hard to realize a (deconfined) gauge theory in experiments
  - Older works on Rydberg gauge theories
- Origin of Rydberg quantum spin liquids
  - Exact mapping: from Z2 LGT to blockaded XY models
  - Numerical simulations
  - Non-local order parameters?
- (If time allows) new tools for interactions engineering of Rydberg spin liquids

Kaleidoscope of the Rydberg - gauge theory connection

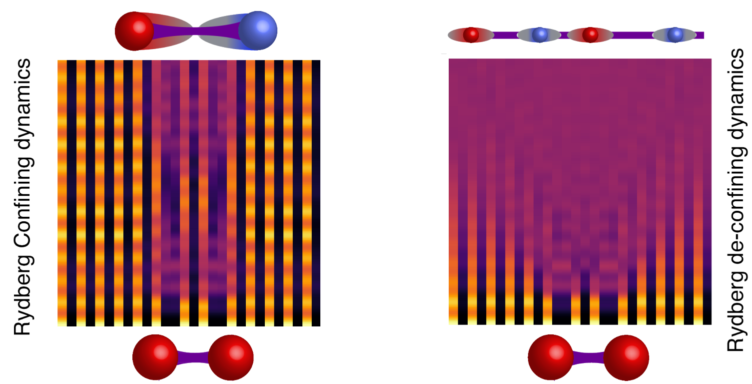
Digital works

Weimer et al., Nat. Phys. 2010
Tagliacozzo et al., Nat. Comm. 2013 (**SU(2)**), Ann. Phys. 2013



Analog 1D

Surace et al., PRX 2020
Notarnicola et al. PRR 2021
Exp: Bernien, Nature 2017 (large scale quantum simulation of the Schwinger model)



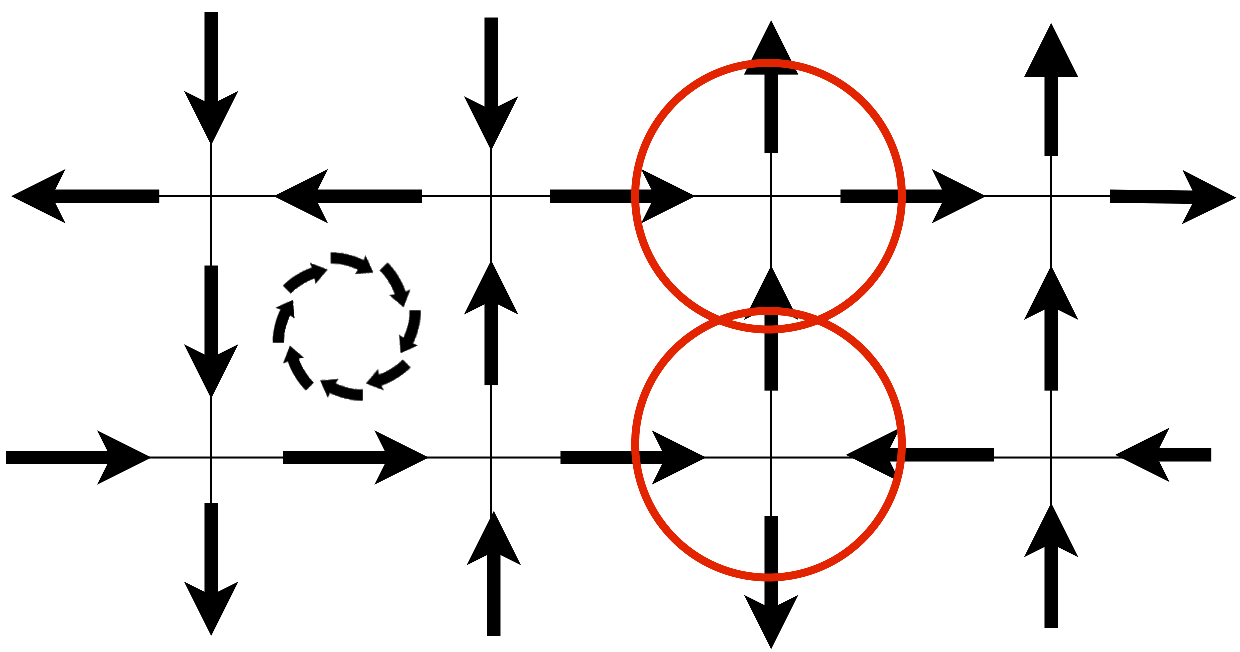
Other works on atoms, ions, and circuit QED reviewed in
Bañuls et al., EPJD 2021; Zohar et al., RPP 2015; MD and Montangero, CRP 2016

Beyond 1D: the example of spin ice

Non-trivial dynamics has to satisfy Ice rules

Flip os 1, 2 or 3 spins are not allowed!

4-spin (*plaquette* moves / ring exchange) are allowed!



Minimal Hamiltonian for quantum Ice

$$H = J \sum_{j,i,k,\ell \in \square} S_j^+ S_i^- S_k^+ S_\ell^-$$

N.B.: close relation with the toric code, which is also a gauge theory!

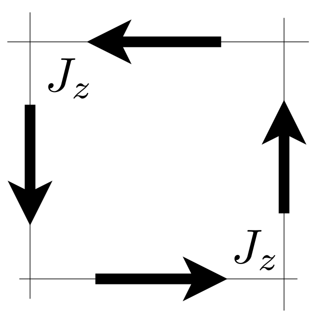
Rydberg blockade and gauge theories - II

Step 1: impose **gauge invariance via energy punishment - Ising interactions**

$$H_0 = J_z(\sum_{j\in +} S_j^z)^2$$

Step 2: generate **dynamics in perturbation theory - various ways**

$$H_1 = J_\perp \sum_{<i,j>} S_j^+ S_i^-$$



$$H_{ef} = J \sum_{j,i,k,\ell\in\square} S_i^+ S_j^- S_k^+ S_\ell^-, \quad J \simeq \frac{J_\perp^2}{J_z}$$

Rydberg blockade and gauge theories - III

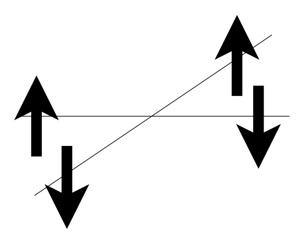
Goal Hamiltonian

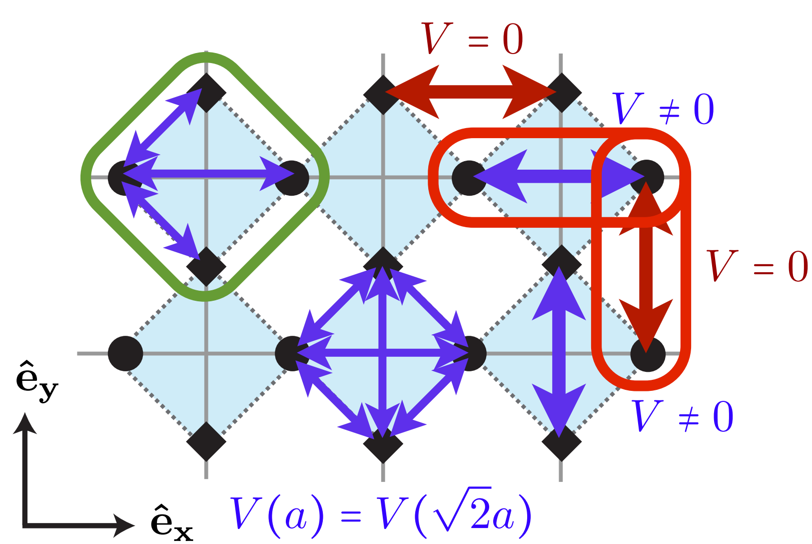
$$H = J_z \sum_{i,j\in+} S_i^z S_j^z + J_\perp \sum_{i,j\in+} \left(S_i^+ S_j^- + \text{h.c.}\right)$$

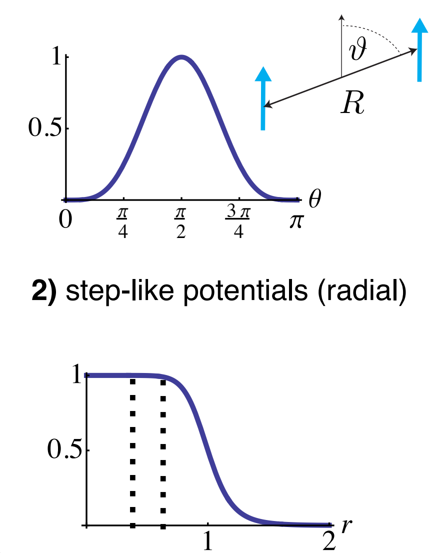
1)Interactions are **plateau-like**

2)Strength depends on **orientation**

2 up, 2 down







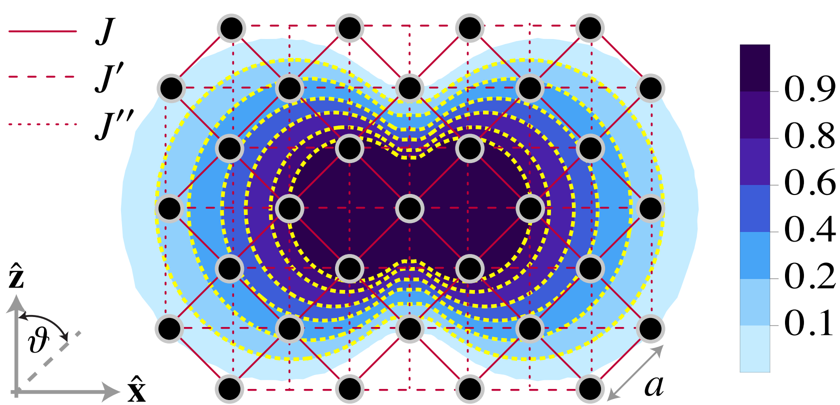
3) two *"species/labels"*: ◆ and ●

Physical incarnations: Rydberg atoms in optical lattices - Enabling elements

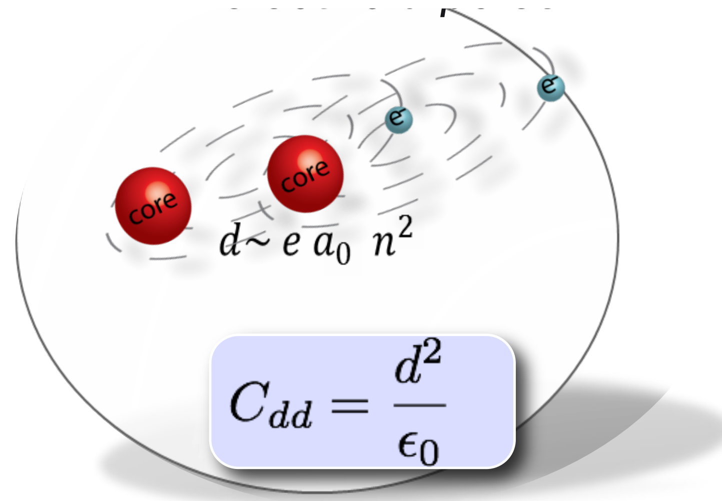
Goal Hamiltonian

$$H = J_z \sum_{i,j\in +} S_i^z S_j^z + J_\perp \sum_{i,j\in +} \left(S_i^+ S_j^- + \text{h.c.}\right)$$

Enabling elements: **dressing techniques + Rydberg p-states**



Enormous long-range dipolar and VdW interactions!



^{87}Rb

n=43S

$r \simeq 0.16\mu$m

$\tau \simeq 100$ms

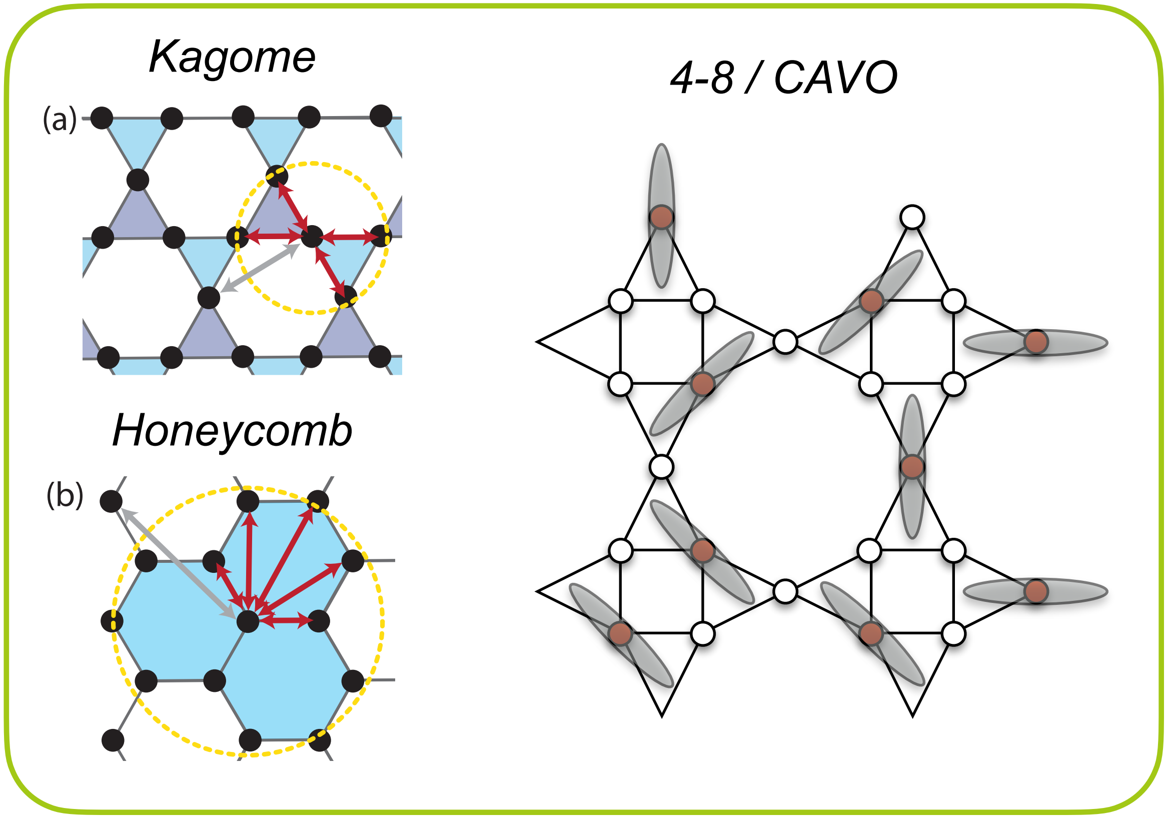
Experiments in Paris, Stuttgart, Pisa, Madison, Munich, Kaiserslautern, Dresden, Nottingham, Innsbruck,…

A.W. Glätzle, MD, R. Nath, I. Rousochatzakis, R. Moessner and P. Zoller, PRX 4, 041037

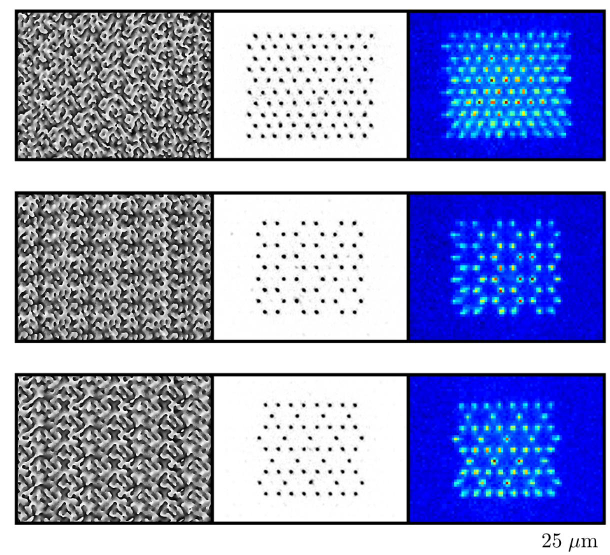
Physical incarnations: Rydberg atoms in optical lattices - s-states

What can be done with ***isotropic interactions*** (e.g. s-states)?

Quantum dimer models!



Nogrette et al., PRX2014 (Palaiseau)



A.W. Glätzle, MD, R. Nath, I. Rousochatzakis, R. Moessner and P. Zoller, PRX 4, 041037;
A.W. Glätzle, MD, R. Nath, C. Gross, I. Bloch and P. Zoller, PRL 114, 173002

However….

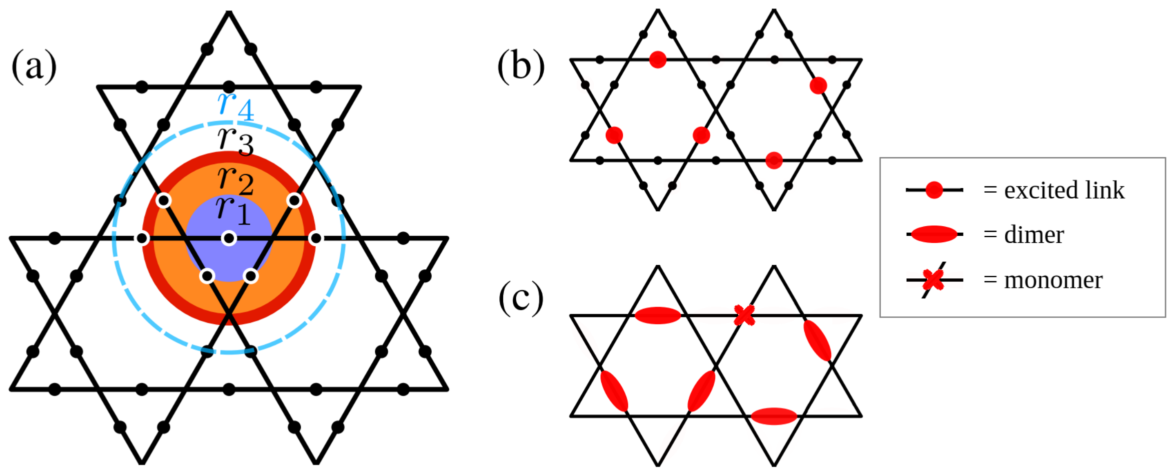
A lot of interesting confined phases, but **no** deconfinement….

Reason: no way to get a constrained gauge theory on a *non-bipartite* lattice with isotropic interactions!!!

Example: theories such as the Balents-Fisher-Girvin model require interactions that are incompatible with Rydberg

New route: Higgs theories

Key Insight in Phys. Rev. X 11, 031005 (2021): utilize Higgs fields to deconfine!



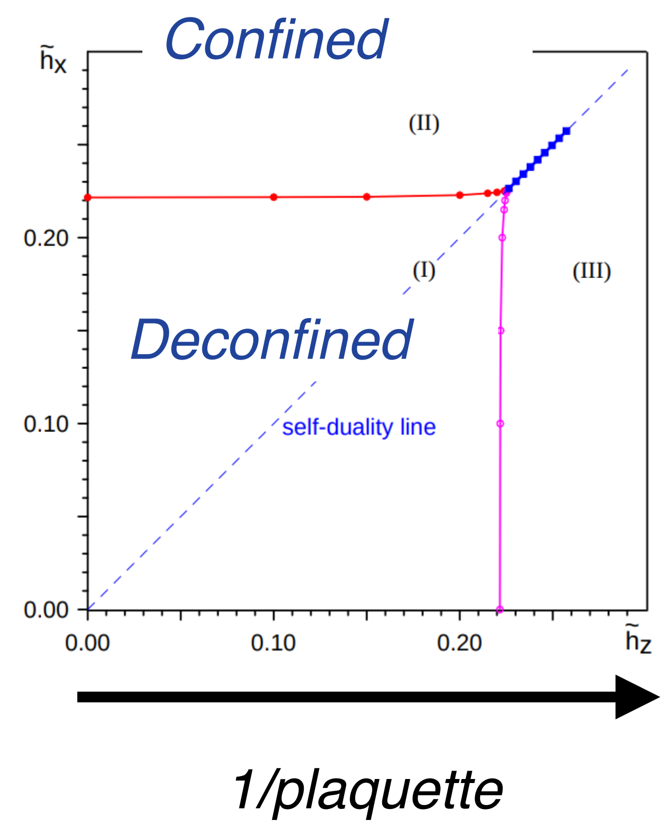
First proposed in New J. Phys. 19 (2017) 063038 in the context of U(1) + staggered fermions

Origin of spin liquid?

Proposed description:

- U(1) quantum link + Higgs on the Kagome
- No magnetic field terms

No deconfinement!
(In the continuum limit)



...

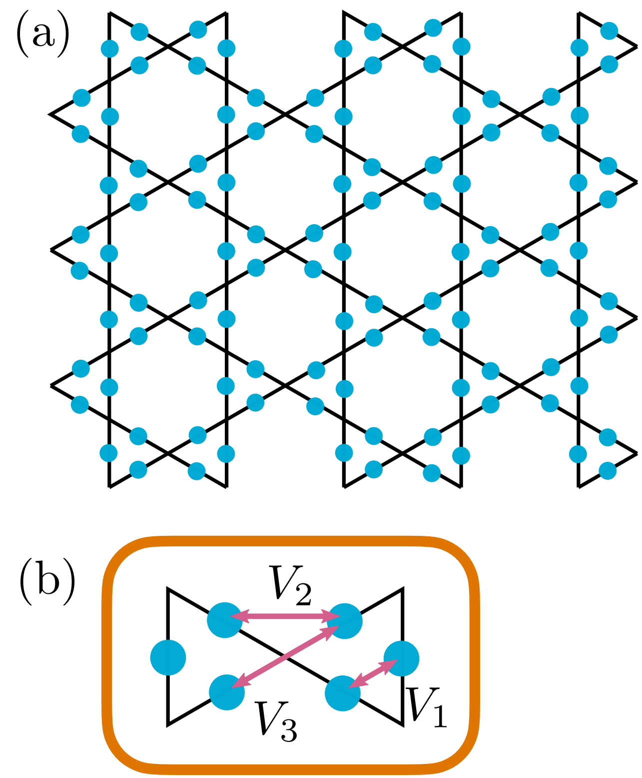
See: Fradkin and Shenker, 1979; more recent works by Vicari, Bonatti, Prokofef et al., Somoza et al.,

Main idea

Identify novel degrees of freedom that map to a solvable model

$$H_0^{\mathrm{Ryd}} = -g\sum_j (b_j + b_j^\dagger) - g\sum_{\langle i,j\rangle}(b_i^\dagger b_j + b_j^\dagger b_i) - 4W\sum_j n_j + 4W\sum_{\langle\langle i,j\rangle\rangle} n_i n_j + 4W\sum_{\langle\langle\langle i,j\rangle\rangle\rangle} n_i n_j$$

$$H_1^{\mathrm{Ryd}} = -h\sum_j (b_j + b_j^\dagger) + h\sum_{\langle ij\rangle}(b_i^\dagger b_j + b_j^\dagger b_i) + 4\lambda\sum_j n_j$$



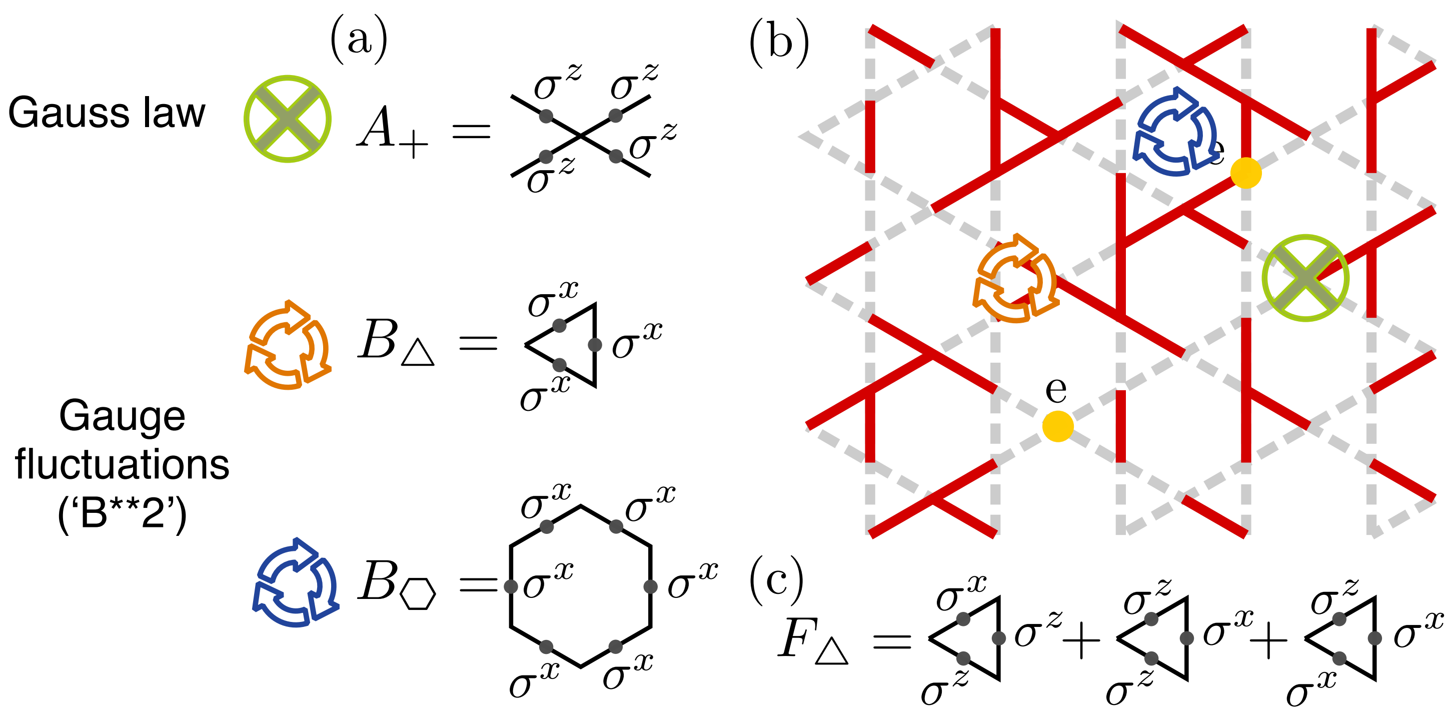
Single spin terms

Spin-exchange

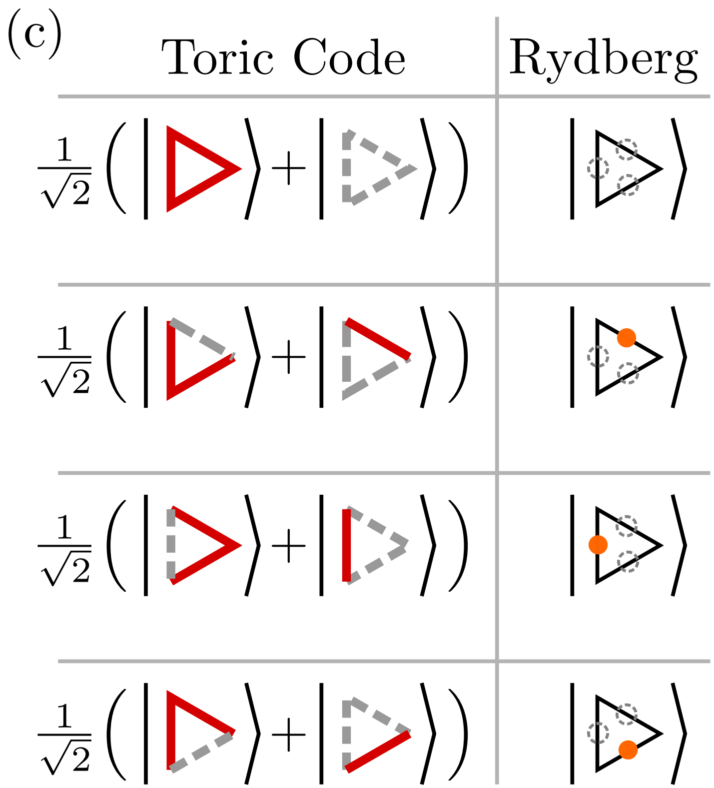
Main idea

Mapped onto a Z2 + Higgs / a *Kagome toric code*:

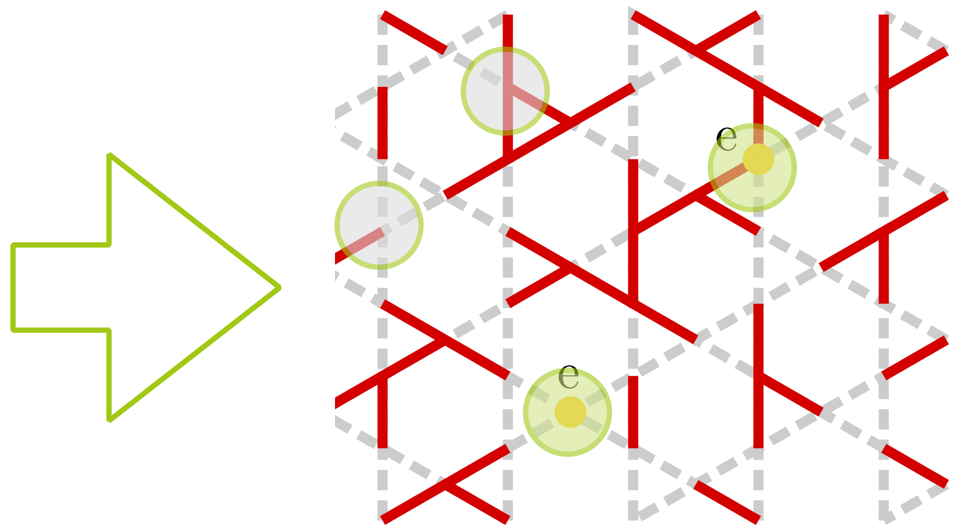
$$H_0^{\mathrm{TC}} = W\sum_{+} A_{+} - J_1\sum_{\triangle} B_{\triangle} - J_2\sum_{\hexagon} B_{\hexagon} - g\sum_{j}\sigma_j^x \qquad H_1^{TC} = -h\sum_{\triangle} F_{\triangle} + \lambda\sum_{\langle i,j\rangle}\sigma_i^z\sigma_j^z$$



Hilbert space mapping



Odd gauge: no charges



Even gauge: charge-1 Higgs

Are we actually gaining anything?

The mapping works under the assumption that: $J_1 \to \infty,\ J_2 = 0$

$$-J_1 \sum_{\triangle} B_{\triangle} - J_2 \sum_{\hexagon} B_{\hexagon}$$

Exact RVB state* for local, (sign-problem free), two-body interacting Hamiltonian!

* mappable to RVB as found in Giudici, Lukin, Pichler, 2201.04034

Exact RVB ground state - quick "proof"

$$H_0^{\mathrm{TC}} = W \sum_{+} A_{+} - J_1 \sum_{\triangle} B_{\triangle} - J_2 \sum_{\hexagon} B_{\hexagon} - g \sum_{j} \sigma_j^x$$

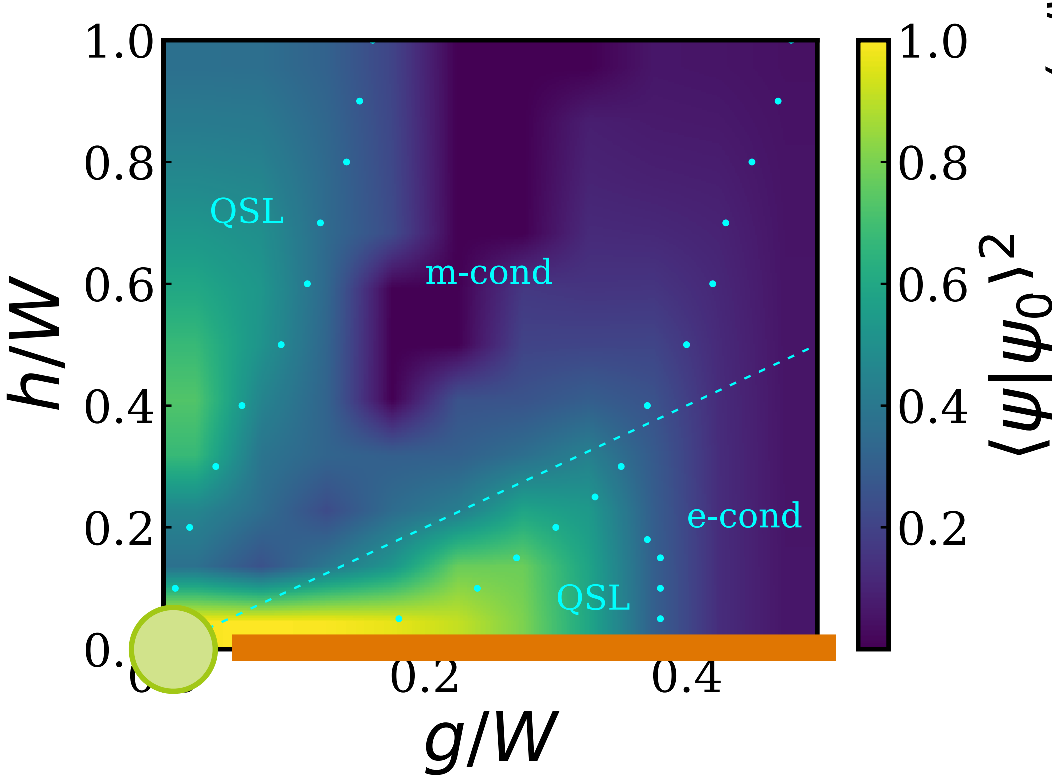
Equal weight superposition of all possible dimer coverings (no quarks)

ut weird as
needed

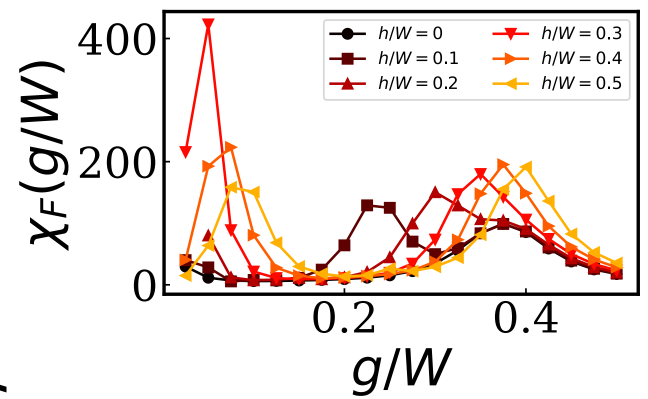
Message 1: **blockaded models are dual to gauge theories with native strong gauge fluctuations** - ideal for deconfinement!

Stability?

Overlap with exact RVB, 36 site system on a torus



Fidelity susceptibility

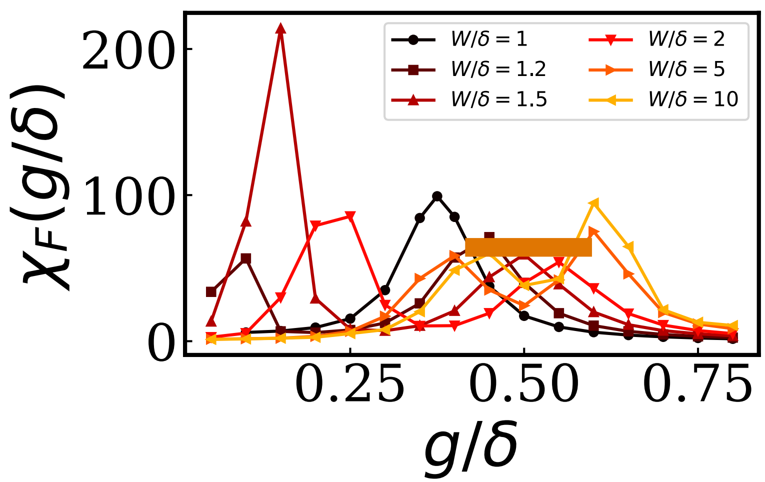


Exactly soluble

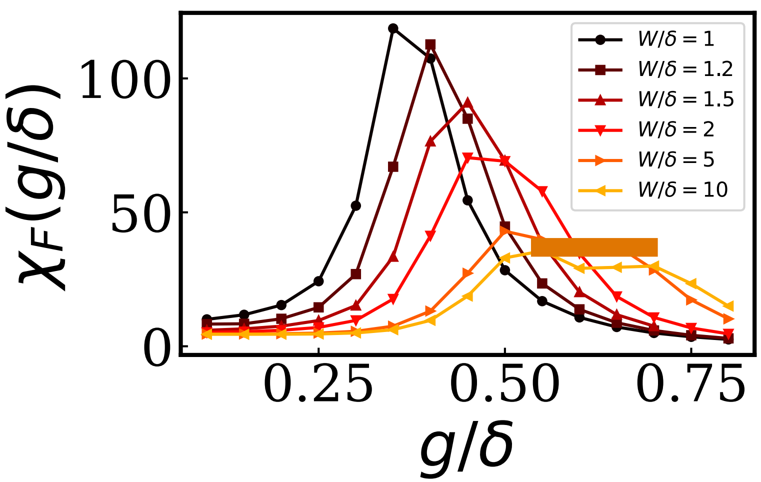
Efficient cluster algorithm (Phys. Rev. E 66, 066110 (2002).

Towards the Ising limit

XY limit



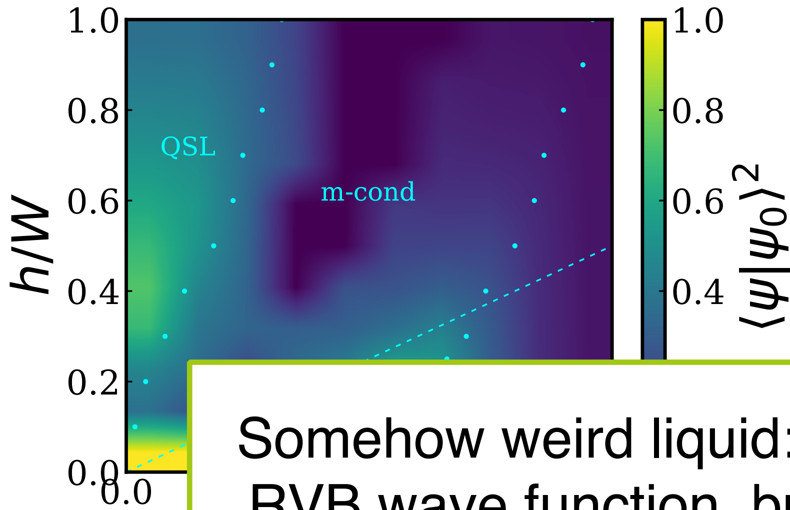
Ising limit



old

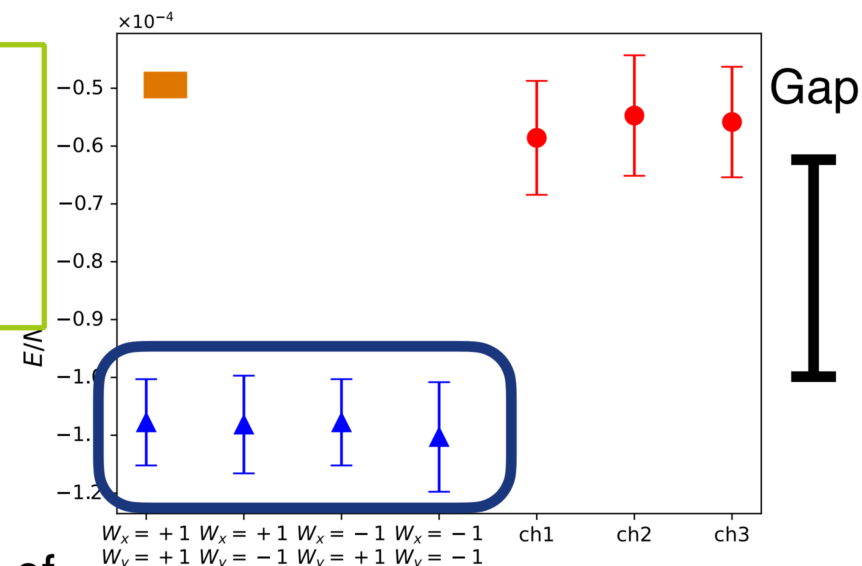
Message 2: "**adiabatic" continuity of exact RVB to realistic models** within a gauge theoretic description

Monte Carlo spectroscopy



Somehow weird liquid: **exact** RVB wave function, but **tiny** topological energy scales

SSE simulations, beta = 128

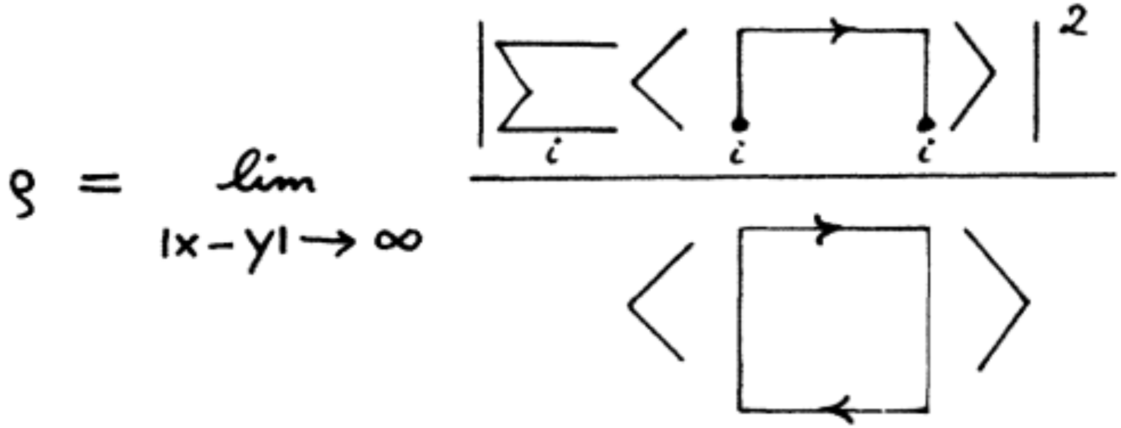


Field theory reasoning: very strong breaking of Lorentz invariance

Degenerate GS manifold

Non-local order parameters

In Phys. Rev. X 11, 031005 (2021) and Science 2021:



Real space correlations inspired by Fredenhagen-Marcu order parameter (vanish if charged asymptotic states exist)

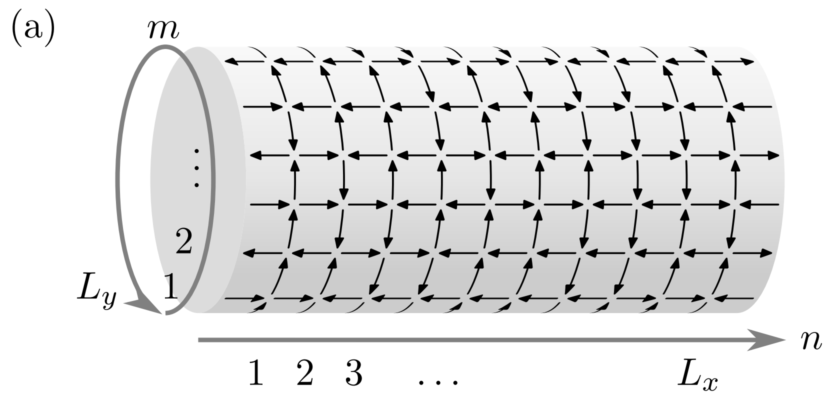
Map exactly onto Z2-Higgs correlations on the RVB!!

However, not the original FM (defined in imaginary time) - Can we really trust that?

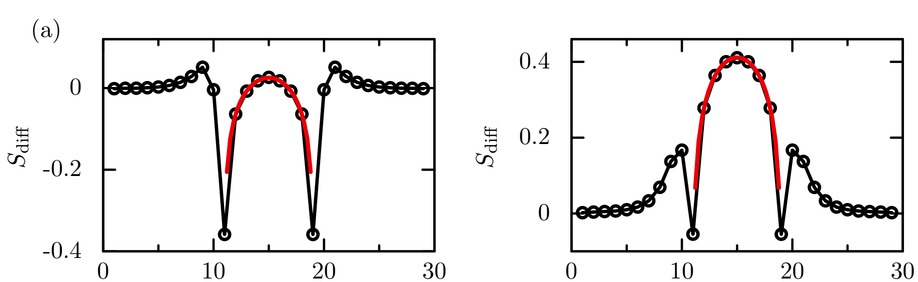
Lesson: spectacular failure of real space Wegner-Wilson loops in spin ice

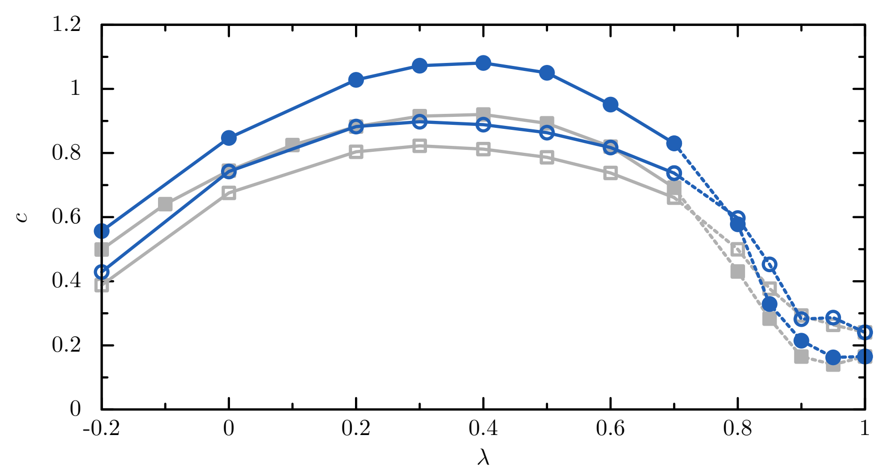
Gauge invariant MPS for cylinders up to 600 spins (30x10)

SciPost Phys. 6, 028 (2019)



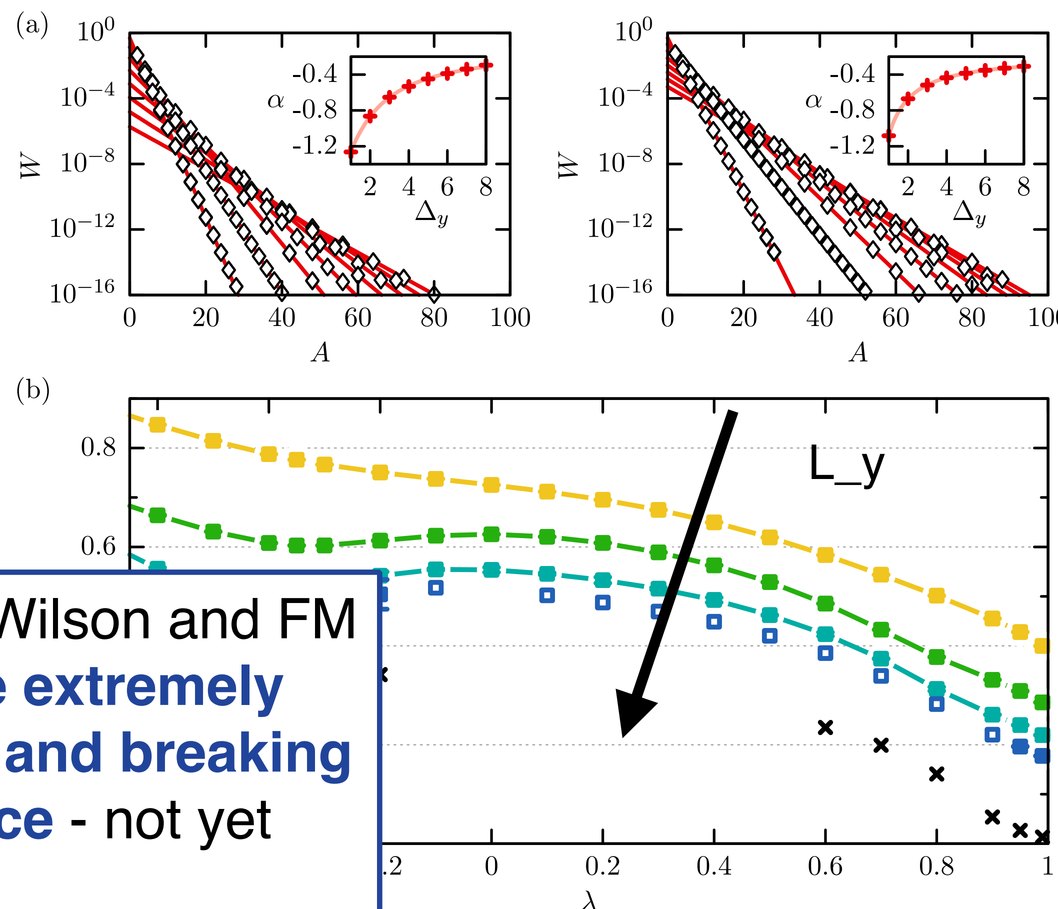
Problem 1: string excitations not matching Lüscher scaling





Lesson: spectacular failure of real space Wegner-Wilson loops in spin ice

Problem 2: Wilson loop at finite size fails spectacularly



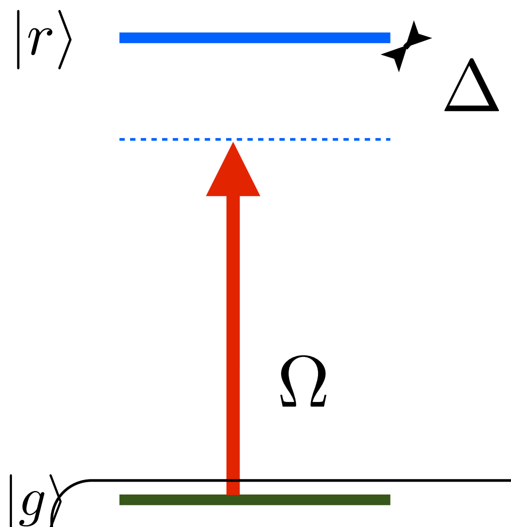
- Huge finite size effects even for very large cylinders
- Even Creutz ratios do not help
- Unreliable even for

Message 3: real space Wilson and FM order parameters **are extremely sensitive to finite size and breaking of Lorentz invariance** - not yet reliable?

We

Banerjee et al., PRB 2013

Better shaped potentials? Use dressing!



$$\frac{\Omega}{|\Delta|} \ll 1$$

New ground state inherits **partially Rydberg character**

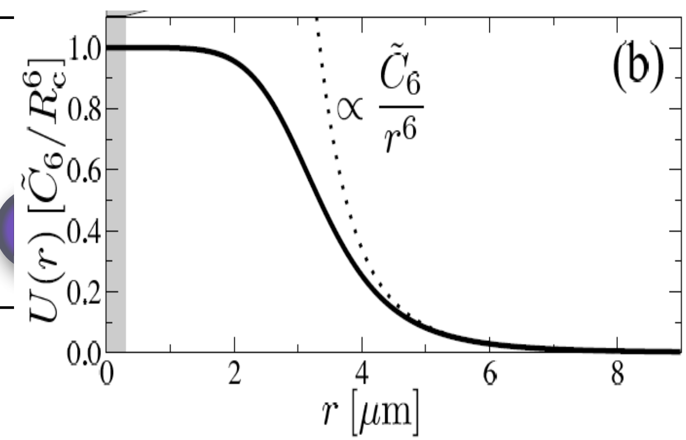
$$|\tilde{g}\rangle = |g\rangle + \frac{\Omega}{2|\Delta|}|r\rangle$$

$$U(r) = \frac{\Omega^2}{8\Delta^2(2\hbar\Delta - V_R(r))}$$

Before Dressing

$$V_R(r) = C_6/r^6$$

Van der Waals interaction

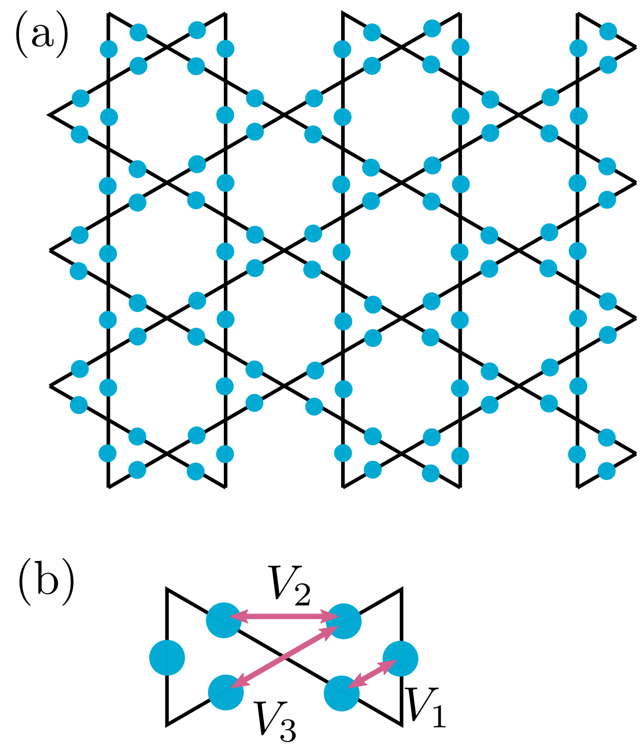


Henkel et al., Pupillo et al., PRL **104**, Honer et al, PRL**105**, ...

Other things for discussions

- Implementation of solvable models: utilize double-dressing!

$$V(r) = \frac{V^{(1)}}{1+(r/r_1)^6} + \frac{V^{(12)}}{1+(r/r_{12})^6}$$



Conclusions

- Basic mechanism for topological QSL: (i) blockade + (ii) balanced combination of XY and transverse field
- Message to HEP: **Exactly solvable models** with dual Z2 gauge theory description / *plaquette for free* via the Rydberg blockade
- *Two-body* Hamiltonians with exact **RVB liquid** ground state
- **Broad** class of models! (Rydberg “blessing”)
- Unclear physical interpretation of real space loop order parameters

What's next?

Open points:

- all these models have no sign problem - efficient cluster algorithms? Very important to address nature of the order parameters
- Can we further improve correlation length and stability using 'smart' engineering?
- role of long range couplings not fully understood
- new class of local models - maybe also interesting for CM materials?

ICTP and SISSA

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Thank you!

arXiv:2205.13000

Other Rydberg approaches to gauge theories: PRX 4, 041037 (2014), PRL 114, 173002 (2015)