SPICE workshop
June 21, 2022
Ingelheim, Germany

Universität zu Köln

Measurement Induced Phases and Phase Transitions in Fermion Chains

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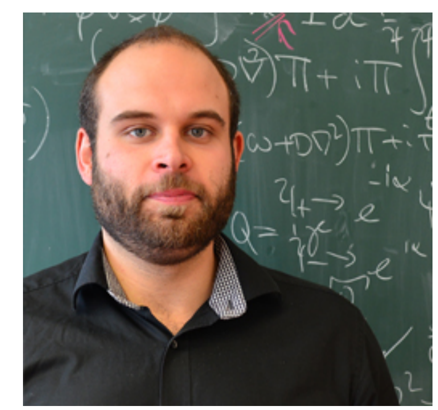
Thomas Müller



Michael Buchhold

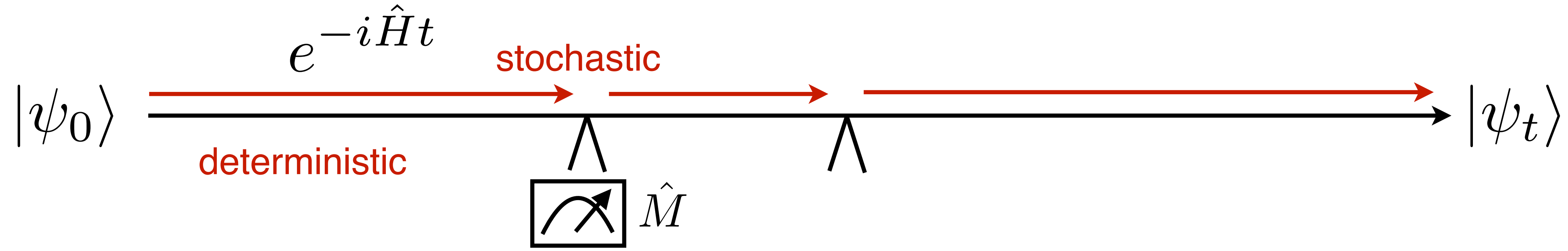
€ € € :

DFG Deutsche Forschungsgemeinschaft

erc European Research Council
Established by the European Commission

Measurement Induced Phase Transitions (MITs)

Small quantum systems: Measurements



Competition for $[\hat{H}, \hat{M}] \neq 0$ —> many-body systems: quantum phase transitions

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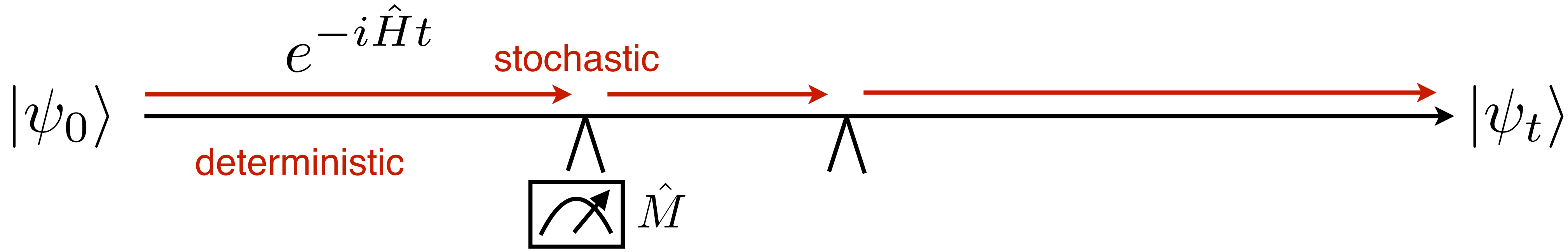
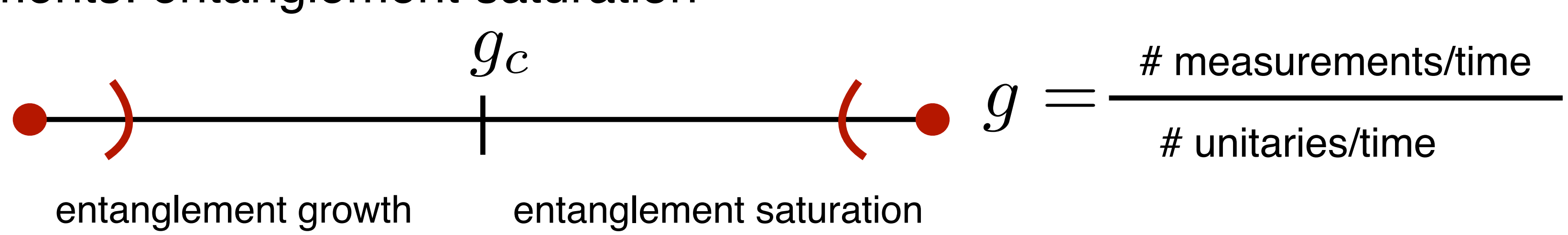


Competition for $[\hat{H}, \hat{M}] \neq 0$ —> many-body systems: quantum phase transitions

MITs as entanglement transitions Skinner, Ruhman, Nahum PRX (2019) Li, Chen, Fisher, PRB (2018, 2019)

- Hamilton dynamics: entanglement growth
- local measurements: entanglement saturation



Measurement Induced Phase Transitions (MITs)

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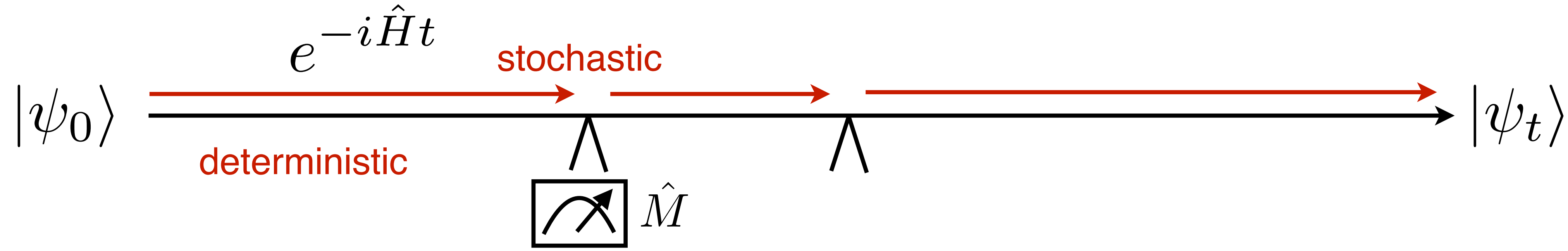

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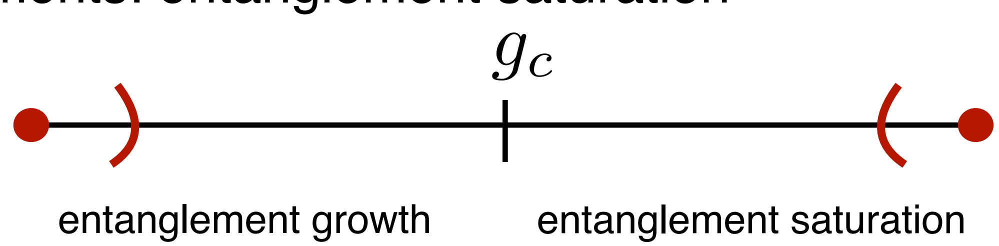
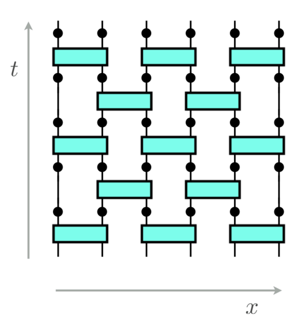


$$g = \frac{\text{\# measurements/time}}{\text{\# unitaries/time}}$$

- two well-studied classes ("Gaussian" polynomial complexity)
  - random Clifford circuits

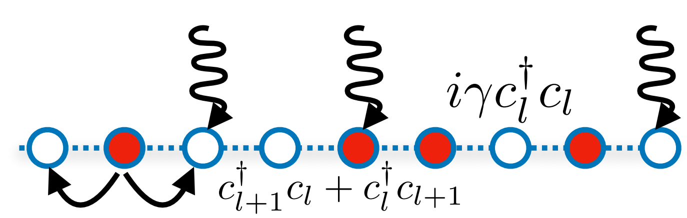


Choi, Bao, Qi, Altman, PRL (2020)
relation to quantum error correction
Gullans, Huse, PRX (2020)
purification transitions
Jian, You, Vasseur, Ludwig, PRB (2020)
relation to stat mech models

➡ volume-to-area law entanglement transition
$S \sim L$

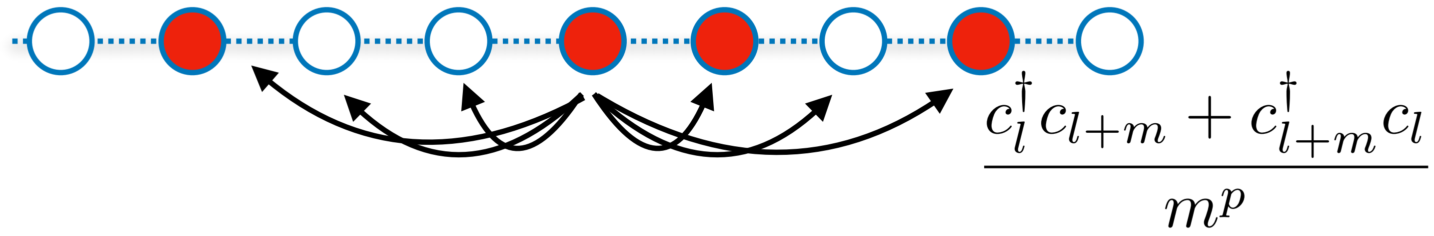
  - monitored free fermions

Alberton, Buchhold, SD, PRL (2021)
Biella, Schiro, Quantum (2021)
Turkeshi, Piroli, Schiro, arxiv (2022)

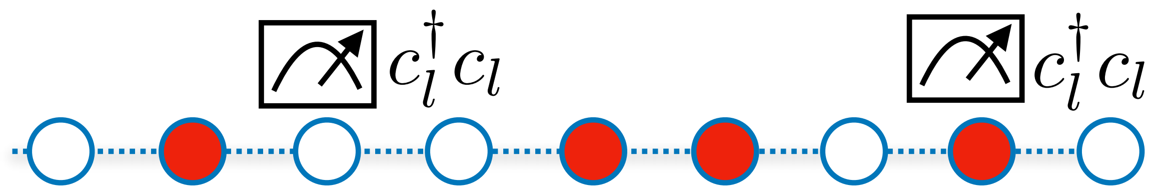


➡ log-to-area entanglement transition
$S \sim \log L$

Entanglement Phase Transitions in Monitored Fermion Chains

Hamiltonian: 

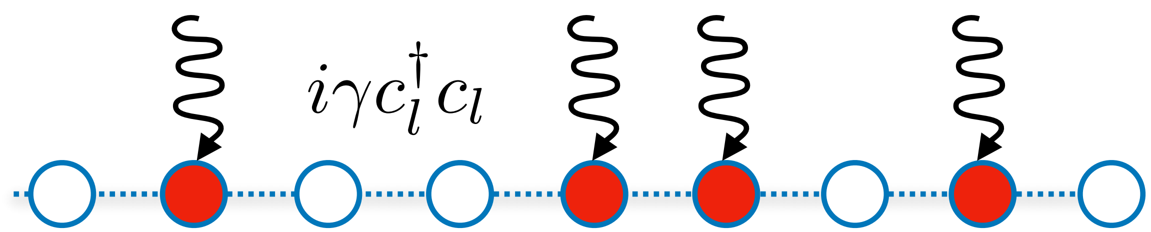
scrambling

Measurement: 

pinning

Alberton, Buchhold, SD, PRL (2021); Buchhold, Minoguchi, Altland, SD, PRX (2021)

Müller, SD, Buchhold, PRL (2022)

Decoherence: 

statistical uncertainty

Ladewig, SD, Buchhold, arXiv:2203.00027 PRR in print

new types of entanglement transition
analytical understanding: Keldysh-replica field theory
outlook: observability in Gaussian complexity class

Hamilton and measurement dynamics

- evolution equation: stochastic Schrödinger equation

Belavkin, Phys. Lett A (1989); Gisin, Percival, JPA (1993)
Jacobs, Steck, Contemp. Phys. (2006)

$$d|\psi_t\rangle = dt(-i\hat{H} - \tfrac{\gamma}{2}\sum_l \hat{M}_l^2|\psi_t\rangle + \sum_l dW_l \hat{M}_l|\psi_t\rangle \qquad \hat{M}_l = \hat{n}_l - \langle\hat{n}_l\rangle_t$$

Gaussian white noise

- each wave function is a random object —> ensemble average trivial in stationary state

$$\rho_t = \overline{|\psi_t\rangle\langle\psi_t|} = \sum_{\text{outcomes}} |\psi_t\rangle\langle\psi_t| \sim \mathbf{1} \qquad \longleftrightarrow \qquad \partial_t\hat{\rho}_t = -i[\hat{H},\hat{\rho}_t] - \gamma\sum_l[\hat{n}_l,[\hat{n}_l,\hat{\rho}_t]]$$

maximally mixed / infinite temperature

quantum master equation

- use state dependent observables instead:

$$F(\overline{\hat{\rho}}) \neq \overline{F[\hat{\rho}]}$$

Cao, Tilloy, De Luca, SciPost (2019)
Zaballo et. al. PRB (2020)

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Gaussian white noise (← dW_l)

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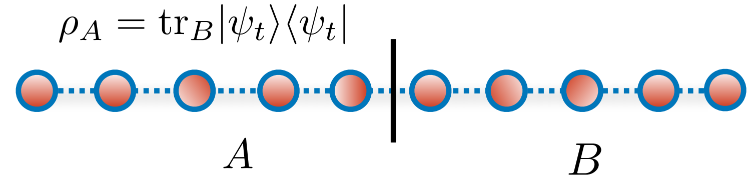
- examples:
  - von Neumann entanglement entropy

$$\overline{S_{vN}(l, L)} = \overline{\langle \log(\rho_A) \rangle}$$

$\rho_A = \mathrm{tr}_B |\psi_t\rangle\langle\psi_t|$



arbitrarily high power of state projector

$|\psi_t\rangle\langle\psi_t|$

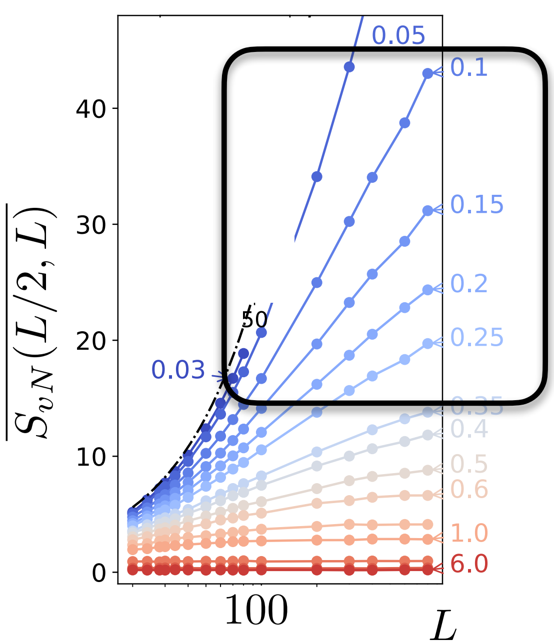
  - correlation function

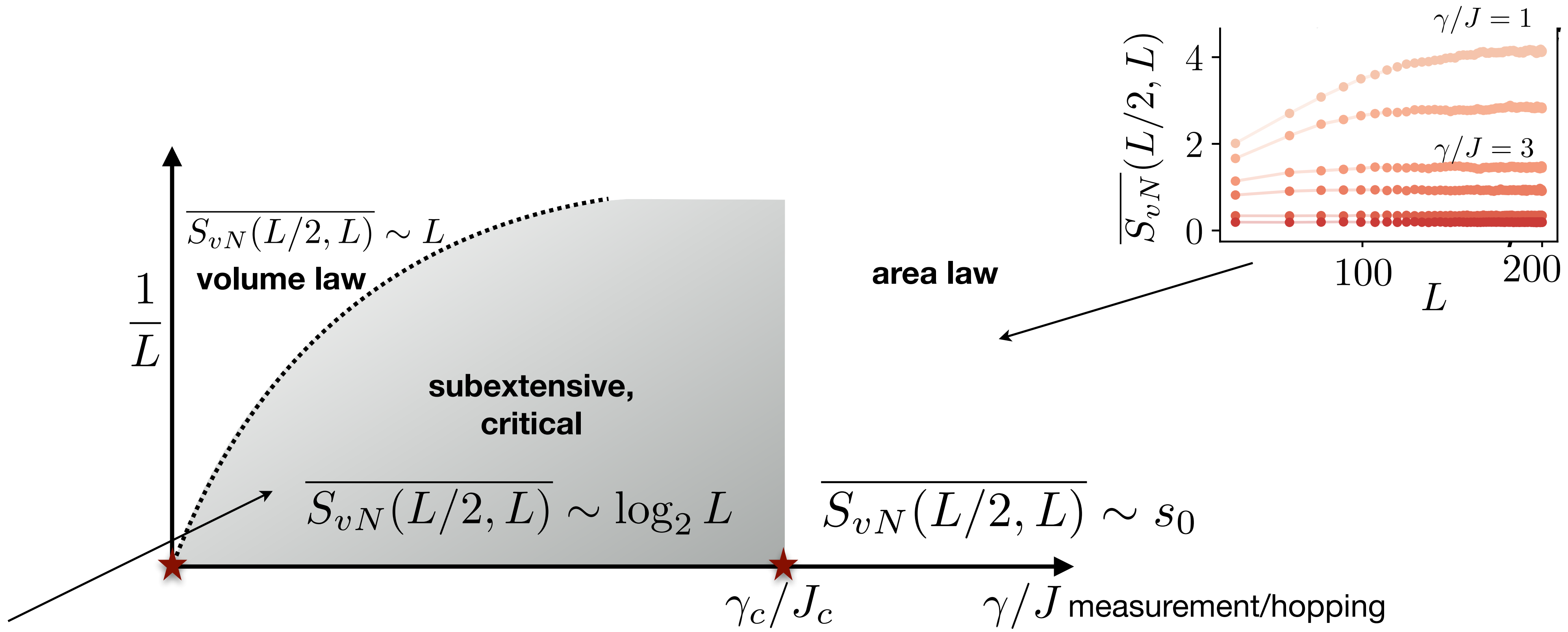
$$C_{ij} = \overline{\langle \hat{n}_i \rangle \langle \hat{n}_j \rangle}$$

quadratic in state projector

Trajectory Ensemble Phase Diagram (nn hopping J)

Alberton, Buchhold, SD, PRL (2021)





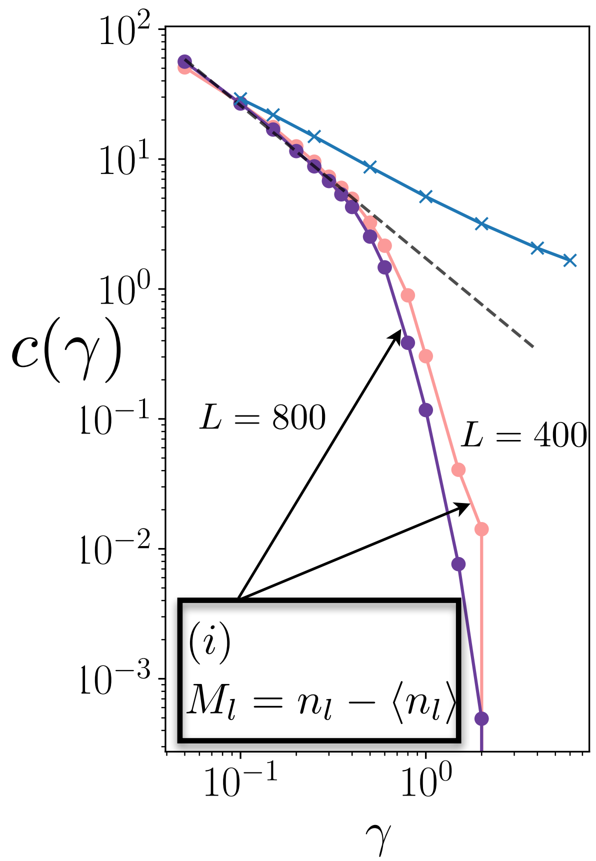
➡ extended critical phase at intermediate monitoring

critical phase but no transition for non-unitary circuits: Chen, Li, Fisher, Lucas PRR (2020)

Characterizing the Weak Monitoring Phase & Phase Transition

- effective central charge $c(\gamma)$

$$\overline{S_{vN}(l,L)} = \frac{c(\gamma)}{3}\log_2\left[\frac{L}{\pi}\sin\left(\frac{\pi l}{L}\right)\right] + s(\gamma)$$



➡ sudden jump reminiscent of BKT

measurement-induced BKT:
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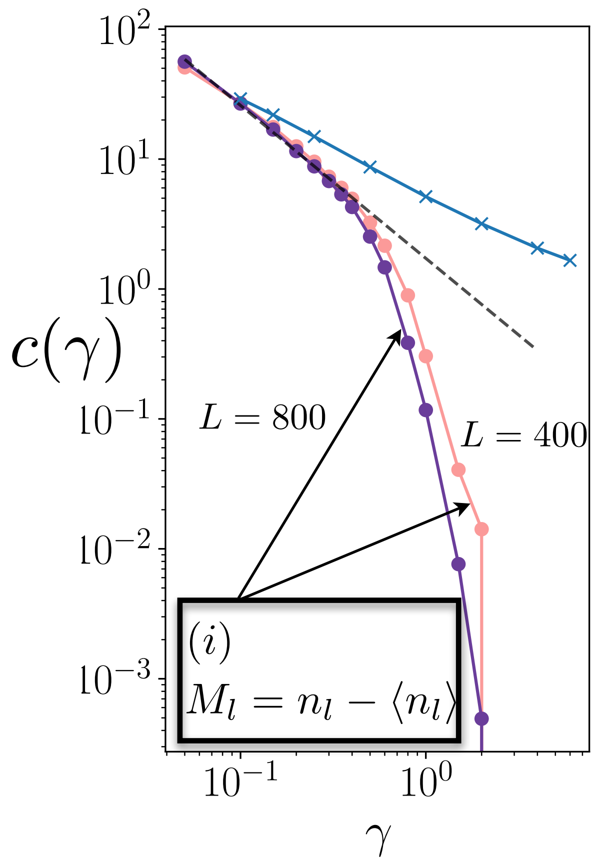

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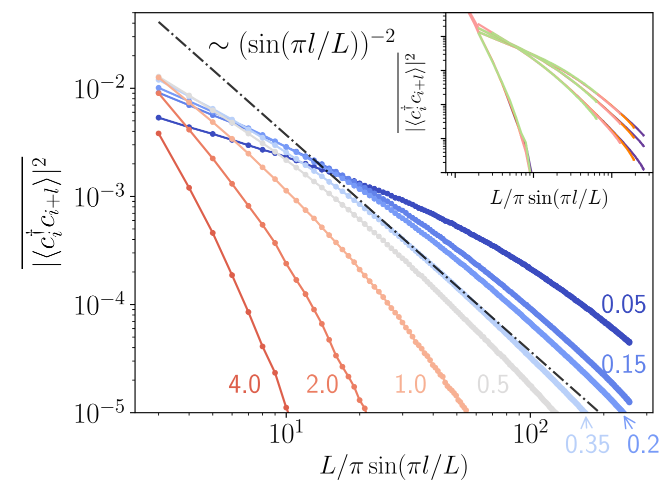
- extended criticality: Connected correlation function

$$C_{i,i+l} = \overline{\langle \hat{n}_i \rangle \langle \hat{n}_{i+l} \rangle} - \overline{\langle \hat{n}_i \hat{n}_{i+l} \rangle}$$

$$\sim l^{-2}$$



$$C_{i,i+l} \sim \begin{cases} 0 & \text{for } H = 0 \\ \exp(-l/\xi) & \text{for } \gamma \gg J \\ l^{-2} & \text{for } \gamma \ll J \\ l^{-1} & \text{for } \gamma = 0 \end{cases}$$

➡ correlation functions equally characterize the transition

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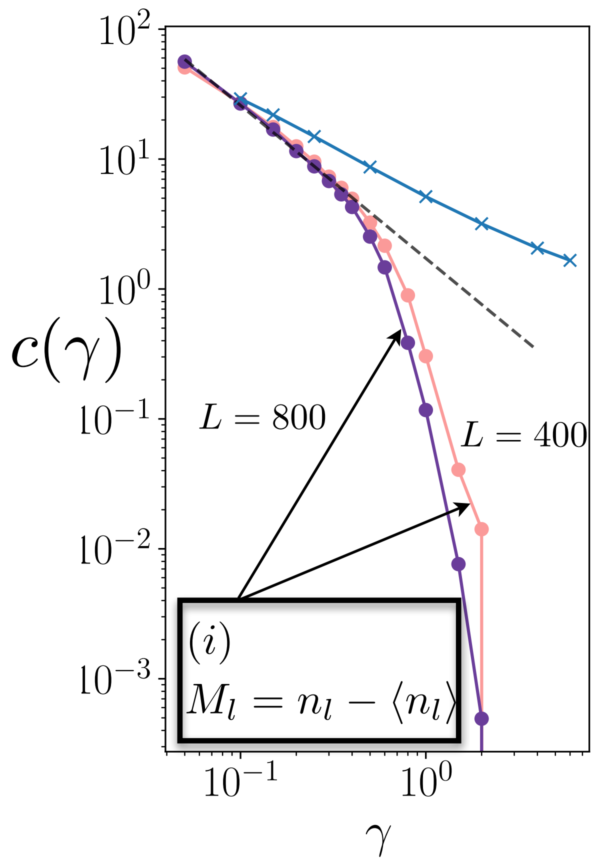

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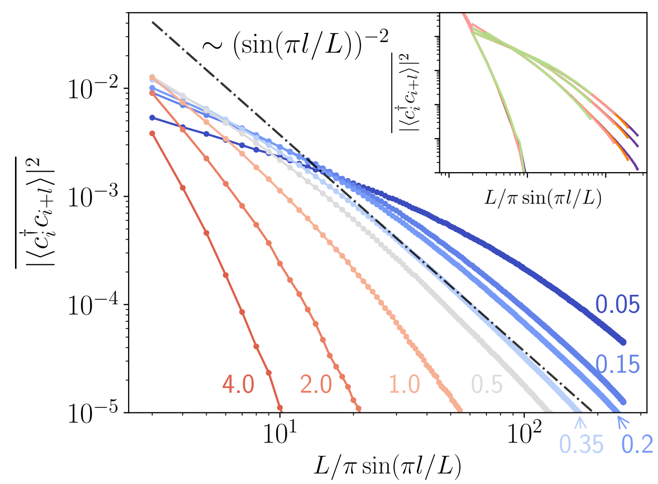

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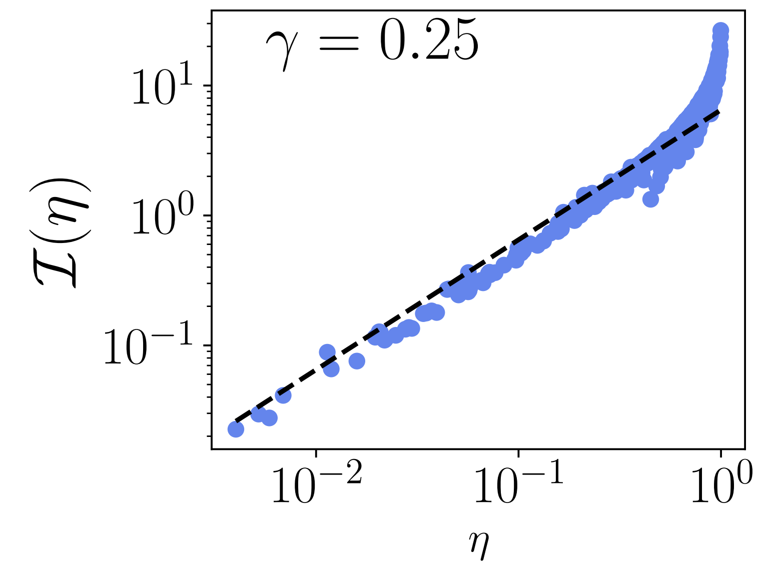
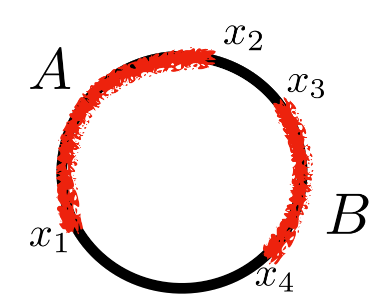
- mutual information



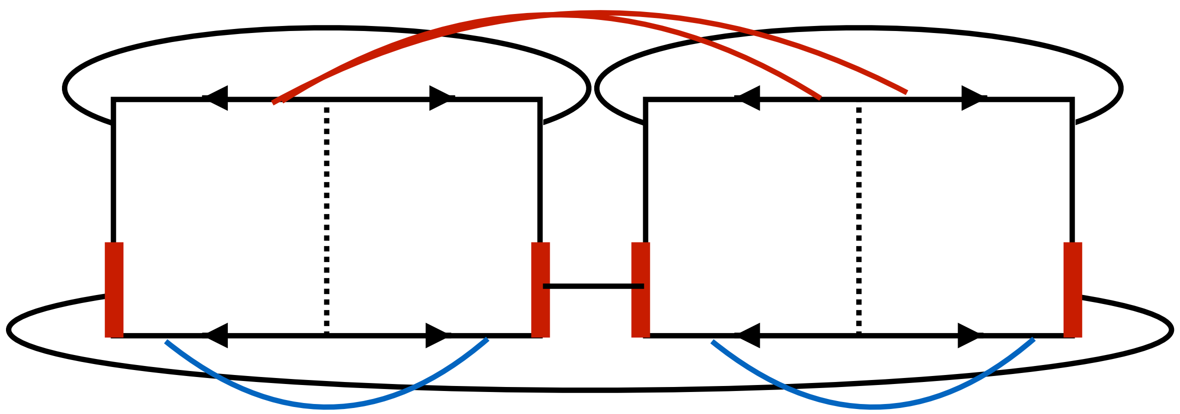


$$\eta = \frac{x_{12}x_{34}}{x_{13}x_{24}}$$

➡ signalling conformal invariance

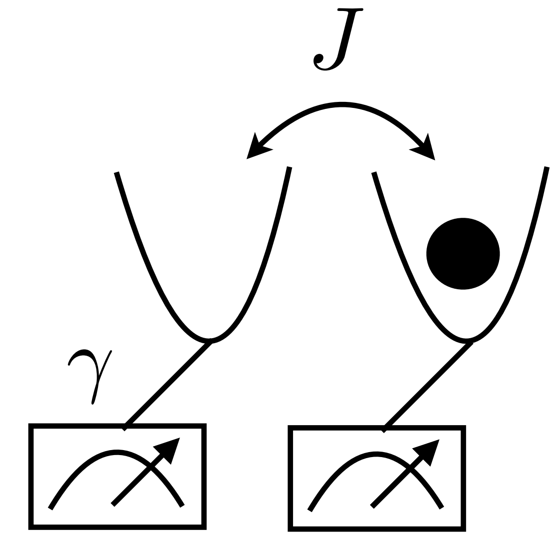
conformally invariant critical point:
Nahum et al. PRX (2019); Li Chen Fisher PRB (2019); Jian et al. PRB (2020);

Effective Replica Field Theory
for
Measurement Induced Phase Transitions



M. Buchhold, Y. Minoguchi, A. Altland, SD, PRX 11, 041004 (2021)

Pinning picture: Toy model



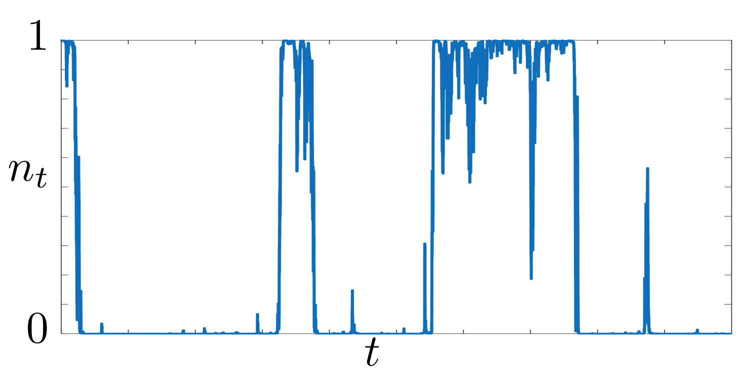
- toy model: trajectory evolution of single fermion on two sites

$$|\psi_{t+dt}\rangle = |\psi_t\rangle - idt\hat{H}_{\text{eff}}|\psi_t\rangle + \sum_{l=1}^{2} dW_l \left(\hat{n}_l - \langle \hat{n}_l \rangle_t\right) |\psi_t\rangle$$

$$\hat{H}_{\text{eff}} = \hat{H} - i\hat{K} \qquad \hat{H} = -J\left(c_1^\dagger c_2 + h.c.\right) \qquad \hat{K} = \frac{\gamma}{2}\sum_{l=1}^{2}\left(\hat{n}_l - \langle \hat{n}_l \rangle_t\right)^2$$

➡ H=0: collapse into dark state at long times $\hat{n}_l|\psi_t\rangle = \langle \hat{n}_l \rangle |\psi_t\rangle \Longrightarrow n_l = 0, 1$

- strong monitoring $J/\gamma \ll 1$



➡ pinning to measurement eigenstate

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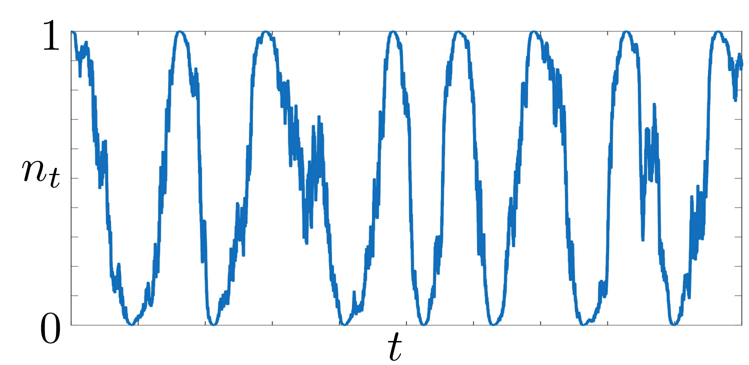

➡ vanishing time spent in eigenstate

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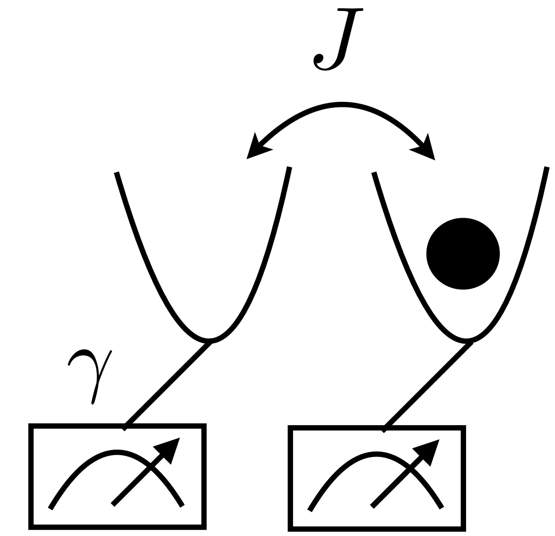

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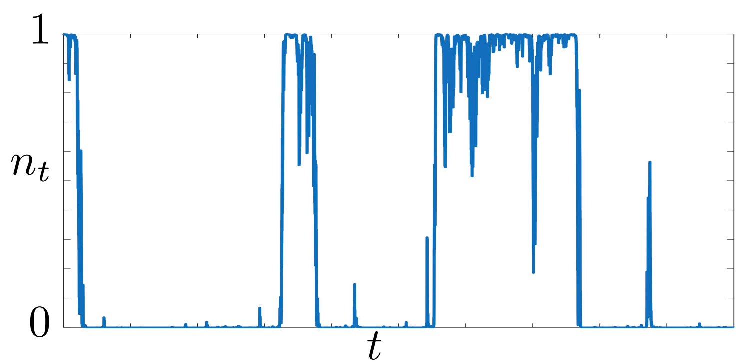

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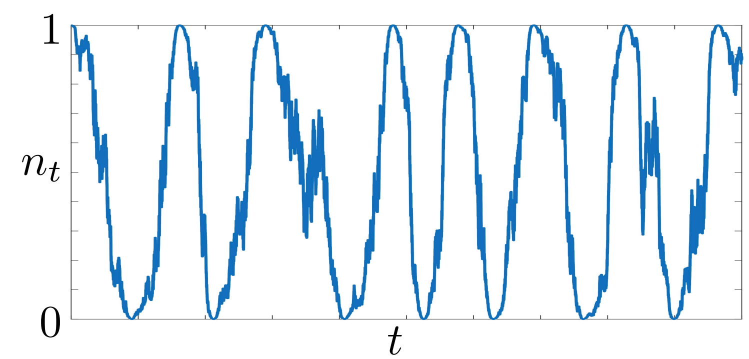
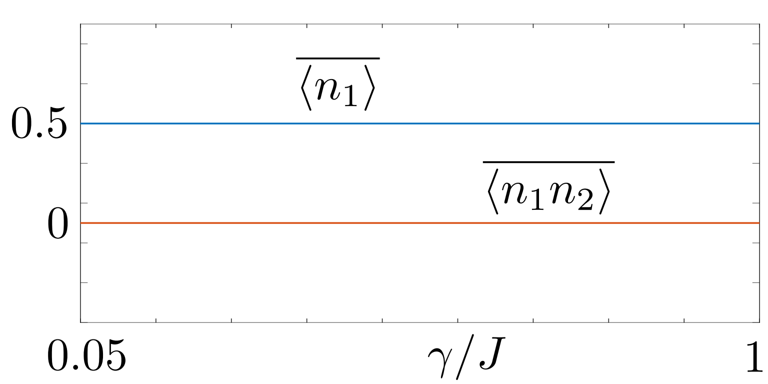
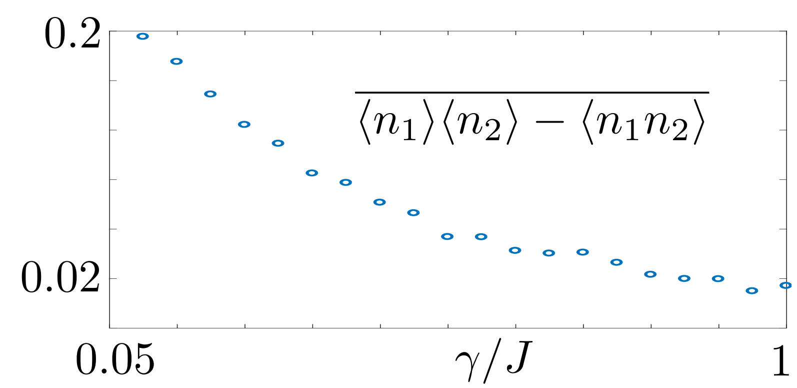


➡ vanishing time spent in eigenstate

- invisible in linear averages



- seen in averaged trajectory covariance matrix



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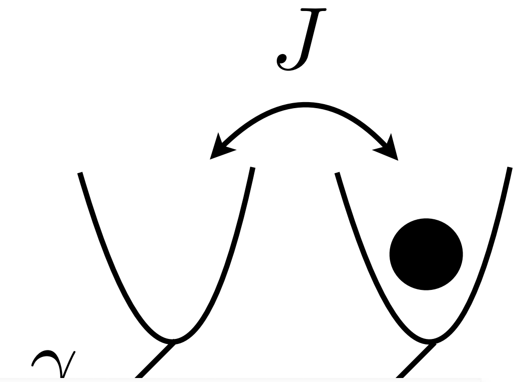

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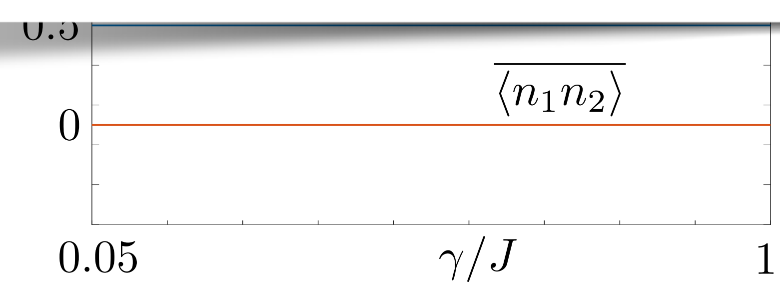
2

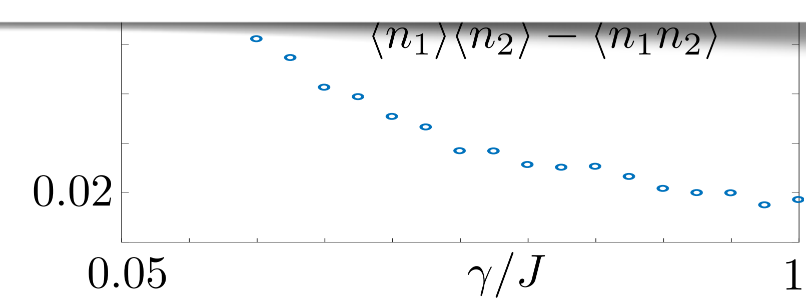
guiding picture and practical approach:

- thermodynamic limit: pinning quantum phase transition at sharply defined point
  - ➡ Minimal continuum model in (1+1) dimensions)
- signalled in state dependent ‘observables’, like the covariance matrix
  - ➡ Replica construction

main insight:

➡ pinning transition in replica degrees of freedom in BKT universality class





Continuum (1+1) dimensional Model

- model obtains from naive continuum limit and bosonization of lattice fermion model

fermionic variant $\longrightarrow$ bosonized variant

- Hamiltonian: massless Dirac fermions $\hat{\Psi}_x = (\hat{\psi}_{R,x}, \hat{\psi}_{L,x})^T$ Luttinger liquid

$$\hat{H} = iv \int_x \hat{\Psi}_x^\dagger \sigma_z \partial_x \hat{\Psi}_x \longrightarrow \hat{H} = \frac{v}{2\pi} \int_x [(\partial_x \hat{\theta}_x)^2 + (\partial_x \hat{\phi}_x)^2]$$

phase density

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phase $\qquad$ density

- measurement operators: current and vertex operators

rate γ_1 : $\hat{O}_{1,x} = \Psi_x^\dagger \Psi_x = \hat{J}_x^{(0)} \longrightarrow \hat{O}_{1,x} = -\frac{1}{\pi} \partial_x \hat{\phi}_x$ linear gapless

rate γ_2 : $\hat{O}_{2,x} = \Psi_x^\dagger \sigma_x \Psi_x \longrightarrow \hat{O}_{2,x} = m \cos(2\hat{\phi}_x)$ nonlinear

m: $\mathcal{O}(1)$

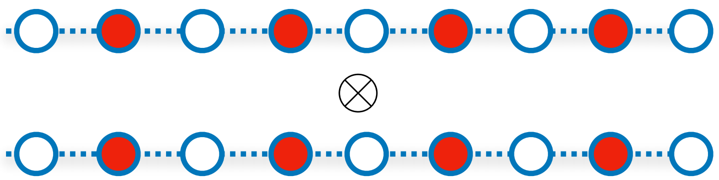
common eigenstates: $\hat{\phi}_x |\Psi_D\rangle = \phi_x |\Psi_D\rangle$

- stabilize product dark states: exactly local
- realize competition: do not commute with H (phase fluctuations)

Replica Approach (n=2)

- Access state-dependent observables, e.g. covariance matrix $$C_{xy} = \overline{\langle \hat{n}_x \hat{n}_y \rangle} - \overline{\langle \hat{n}_x \rangle \langle \hat{n}_y \rangle}$$

- Introduce replicas in Hilbert space $$|\Psi_t\rangle = |\psi_t^{(1)}\rangle \otimes |\psi_t^{(2)}\rangle =$$



- All quadratic-in-state observables encoded in

$$\rho^{2\mathrm{R}} = \overline{|\Psi_t\rangle\langle\Psi_t|}$$

➡ linear statistical average of replica density matrix

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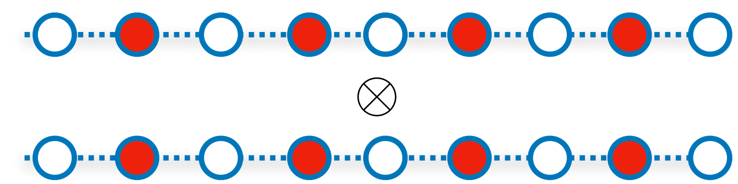

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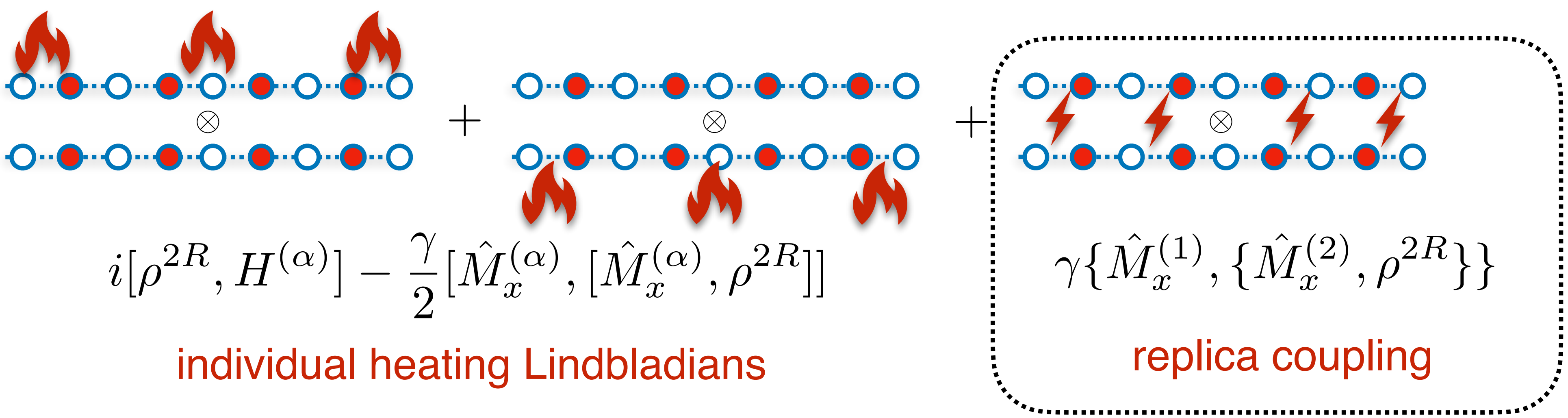
➡ linear statistical average of replica density matrix

- Quantum master equation (truncate coupling to ρ^{3R})

$$\partial_t \rho^{2\mathrm{R}} =$$



$$i[\rho^{2R}, H^{(\alpha)}] - \frac{\gamma}{2}[\hat{M}_x^{(\alpha)}, [\hat{M}_x^{(\alpha)}, \rho^{2R}]] + \gamma\{\hat{M}_x^{(1)}, \{\hat{M}_x^{(2)}, \rho^{2R}\}\}$$

individual heating Lindbladians

replica coupling

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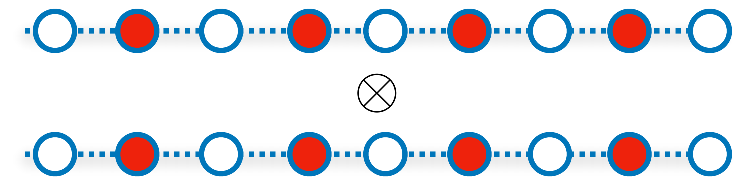
- Access state-dependent observables, e.g. covariance matrix $C_{xy} = \overline{\langle \hat{n}_x \hat{n}_y \rangle} - \overline{\langle \hat{n}_x \rangle \langle \hat{n}_y \rangle}$

- Introduce replicas in Hilbert space $|\Psi_t\rangle = |\psi_t^{(1)}\rangle \otimes |\psi_t^{(2)}\rangle =$



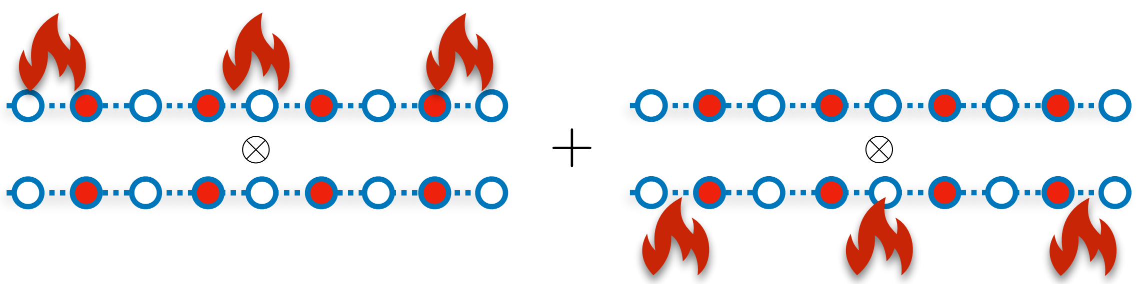
- All quadratic-in-state observables encoded in

$$\rho^{2\mathrm{R}} = \overline{|\Psi_t\rangle\langle\Psi_t|}$$

➡ linear statistical average of replica density matrix

- Quantum master equation (truncate coupling to ρ^{3R})

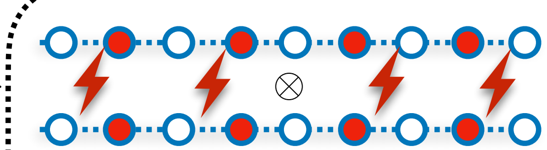
$\partial_t \rho^{2\mathrm{R}} =$



$$i[\rho^{2R}, H^{(\alpha)}] - \frac{\gamma}{2}[\hat{M}_x^{(\alpha)}, [\hat{M}_x^{(\alpha)}, \rho^{2R}]]$$

individual heating Lindbladians

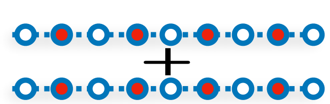
$+$



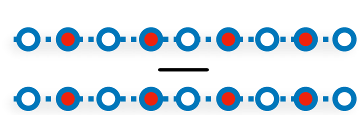
$$\gamma\{\hat{M}_x^{(1)}, \{\hat{M}_x^{(2)}, \rho^{2R}\}\}$$

replica coupling

- New degrees of freedom



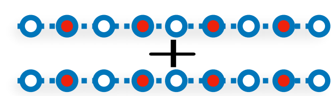
$: \hat{\phi}^{(a)} = \hat{\phi}^{(1)} + \hat{\phi}^{(2)}$ average coordinate



$: \hat{\phi}^{(r)} = \hat{\phi}^{(1)} - \hat{\phi}^{(2)}$ replica fluctuations

Boson Replica Quantum Master Equation

- New degrees of freedom



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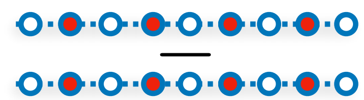

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➡ Master equation becomes separable (exact for Gaussian dynamics, useful more generally)

- Average coordinate: heating to infinite temperature

$$\partial_t \rho^{(a)} = i[\rho^{(a)}, H^{(a)}] + \frac{2\gamma}{\pi} \sum_l \left(\partial_x \hat{\phi}^{(a)} - \overline{\langle \partial_x \hat{\phi}^{(a)} \rangle}\right) \rho^{(a)} \left(\partial_x \hat{\phi}^{(a)} - \overline{\langle \partial_x \hat{\phi}^{(a)} \rangle}\right)$$

← only jump term!

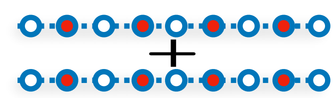
- Relative coordinate: cooling/damping into dark state

$$\partial_t \rho^{(r)} = i[\rho^{(r)}, H^{(r)}] - \frac{\gamma}{\pi} \sum_l \left\{ (\partial_x \hat{\phi}^{(r)})^2, \rho^{(r)} \right\}$$

← no jump term!

Boson Replica Quantum Master Equation

- New degrees of freedom



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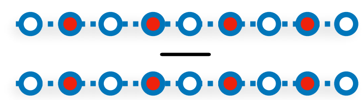

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- Present model: non-Hermitian Schrödinger equation for relative coordinate

$$\partial_t |\psi_t^{(r)}\rangle = -i H_{\text{eff}} |\psi_t^{(r)}\rangle$$

➡ cooling into dark state

$$H_{\text{eff}} = \frac{\nu}{2\pi} \int_x (\partial_x \hat{\theta})^2 + (1 - i\eta^2)(\partial_x \hat{\phi})^2 \; - i \frac{\gamma m}{\pi} \int_x [1 - \cos(\sqrt{8} \hat{\phi}_x)]$$

effect of non-linearity

➡ non-Hermitian Sine-Gordon: pinning via cos term, depinning via theta term

➡ extract physics in path integral approach

Phase diagram

➡ Sine-Gordon action with complex coefficients

Fendley, Saleur, Zamolodchikov,
International Journal of Modern Physics (1993)

$$S = \int_{t,x} \left\{ \frac{K}{16\pi} \left[\frac{1}{\eta} (\partial_t \phi)^2 - \eta (\partial_x \phi)^2 \right] - i\lambda \cos(\phi) \right\}$$

➡ RG flow: standard KT flow with complex K, λ

- flow modified at short distance

➡ shift of phase border

- standard BKT flow at long distance

➡ same long wavelength properties

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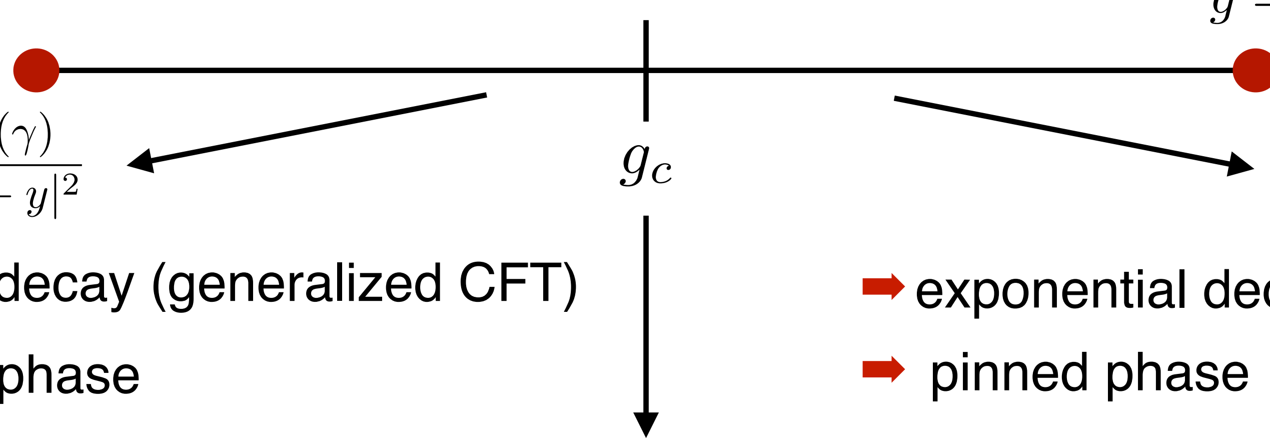
- resulting phase diagram:

$g = \gamma/J$



$C_{xy} = \frac{C(\gamma)}{|x-y|^2}$

➡ algebraic decay (generalized CFT)

➡ depinned phase

$C_{xy} \sim e^{-\frac{|x-y|}{\xi}}$ $\quad \xi^{-1} = \log(2\pi\gamma/v)$

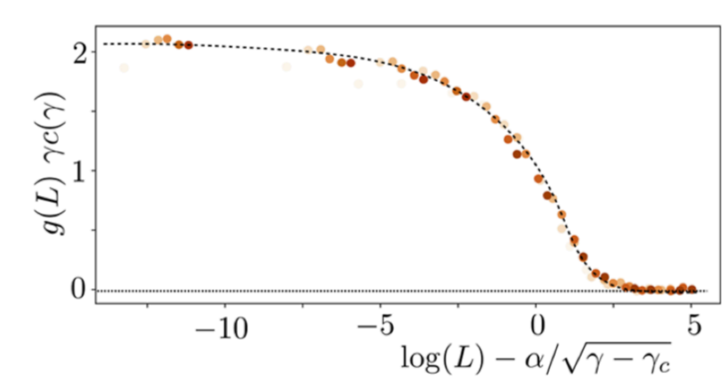
➡ exponential decay

➡ pinned phase

➡ phase transition in the BKT universality class

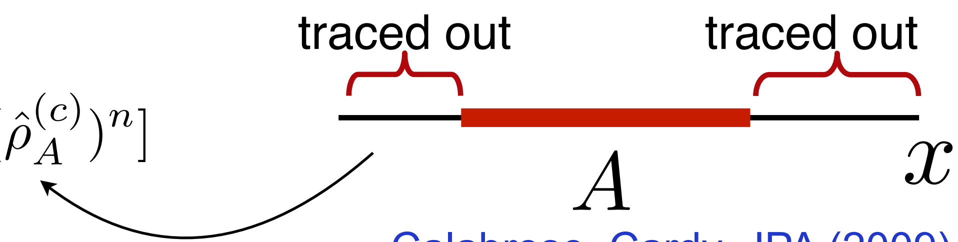
➡ all in line with numerics for lattice fermions

scaling collapse of c



Entanglement Entropies: n-Replica Keldysh approach

- Rényi entropy $S_n(L) = \frac{1}{1-n}\overline{\log Z_A(n,\{dW\})},\ Z_A(n,\{dW\}) \equiv \mathrm{tr}[(\hat{\rho}_A^{(c)})^n]$
- von Neumann entropy: $n \to 1$



Calabrese, Cardy, JPA (2009)

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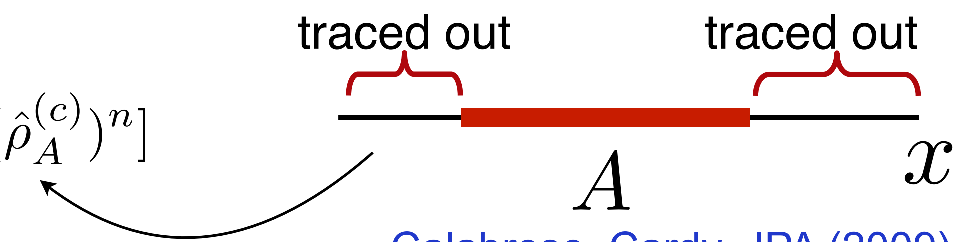

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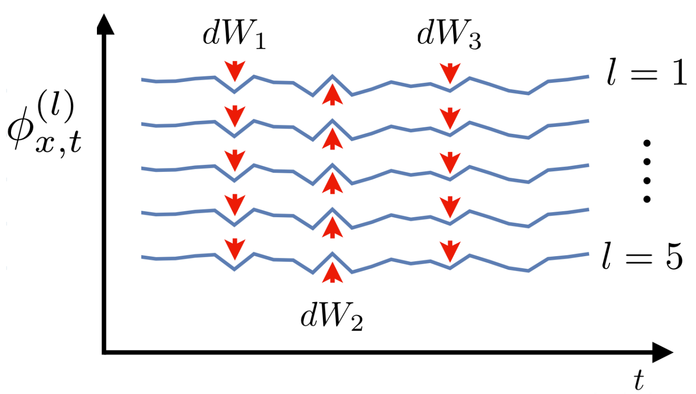
- approach: Keldysh replica field theory for n replicas (entropies via modified boundary conditions)

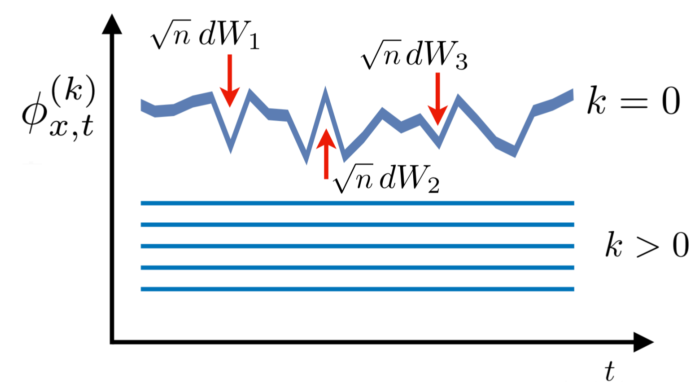
ground states: Casini, Fosco, Huerta, J. Stat. Mech. (2005)

- decoupling of center-of-mass and relative modes (exact for Gaussian states -> good away from transition)

replica space picture



reciprocal replica space picture



- 1 mode heats up (noisy)
- n-1 modes cool down (noiseless)

→

- noisy contribution A independent
- all A dependence in noiseless modes!

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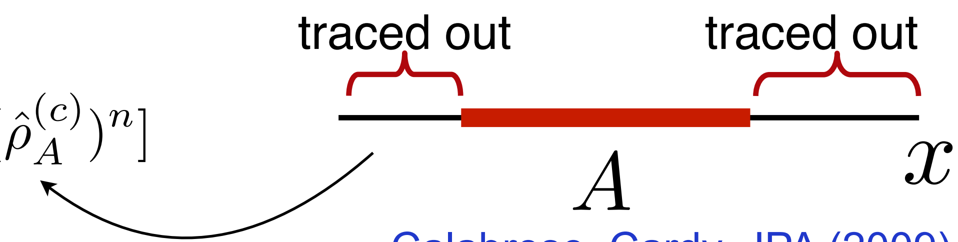

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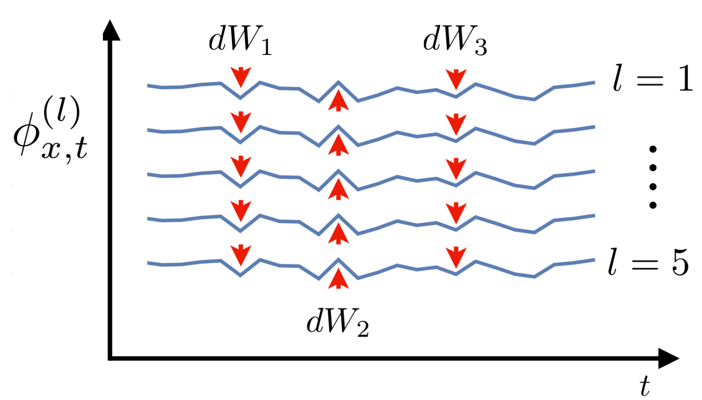

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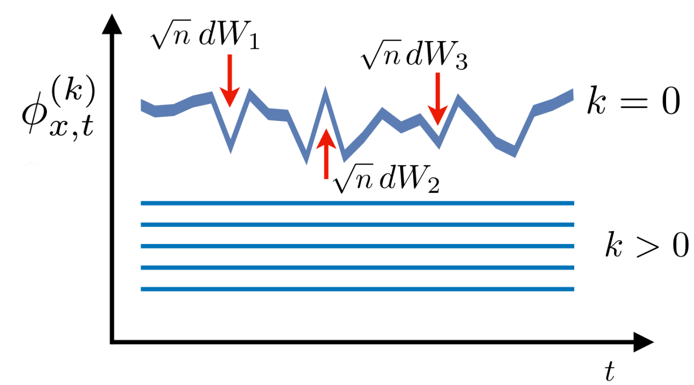

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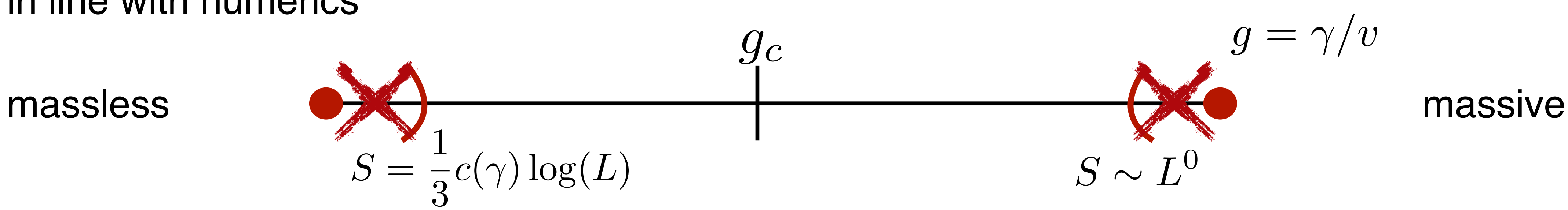
➡ Rényi entropy calculation as for ground states

➡ results in line with numerics



Long range models: numerics and analytics

T. Mueller, SD, M. Buchhold PRL 128, 010605 (2022)

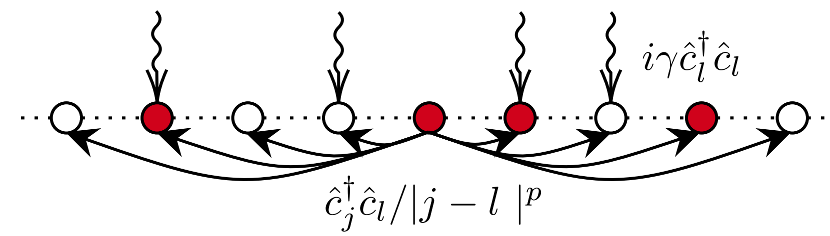
Minato et al, ibid 010603; Block et al. ibid 010604

- long ranged hopping model

$$\hat{H}_{LR} = \sum_{l \neq m} \frac{\hat{c}_l^\dagger \hat{c}_m}{|l-m|^p} \qquad 1 < p \leq \infty$$

superext. nn hopping

$$1 > 1/p \geq 0$$



- new scaling behavior & new phase transition

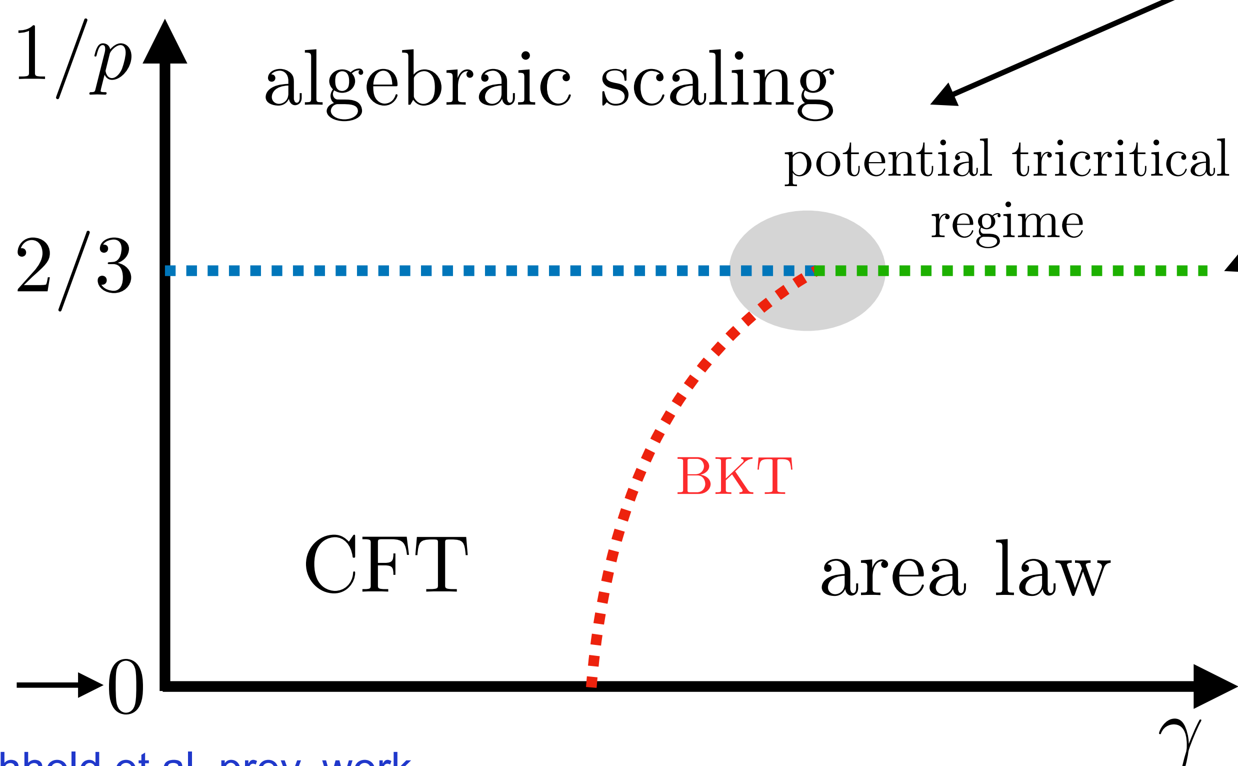
pinning of phase fluctuations

enhanced density fluctuations

critical point at p = 3/2

long range

short range



Buchhold et al. prev. work

striking parallel to ground state phase diagram of long range Luttinger liquids: Maghrebi et al., PRL (2017)

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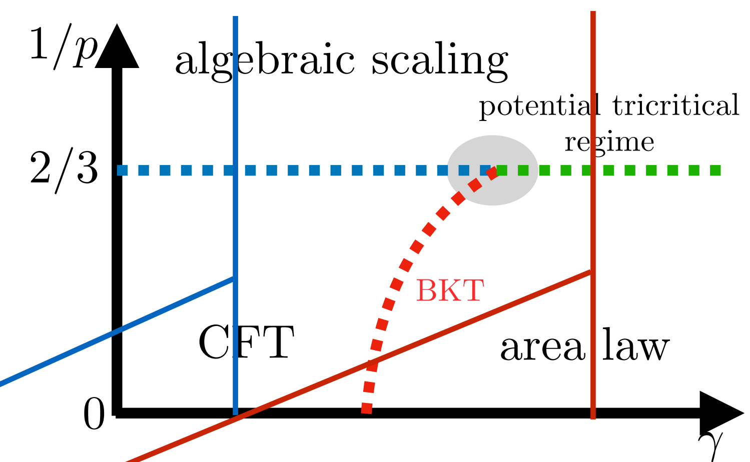
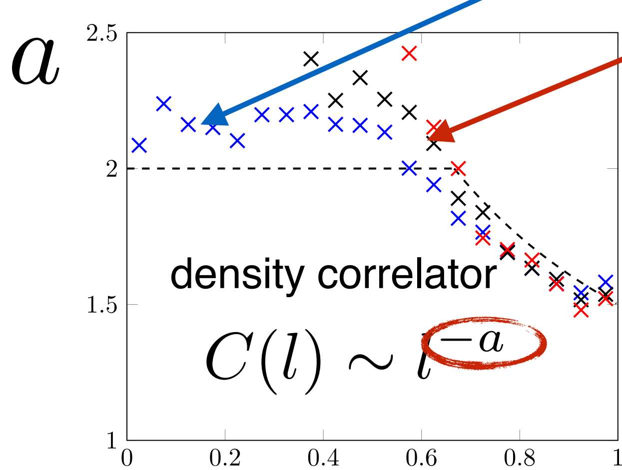
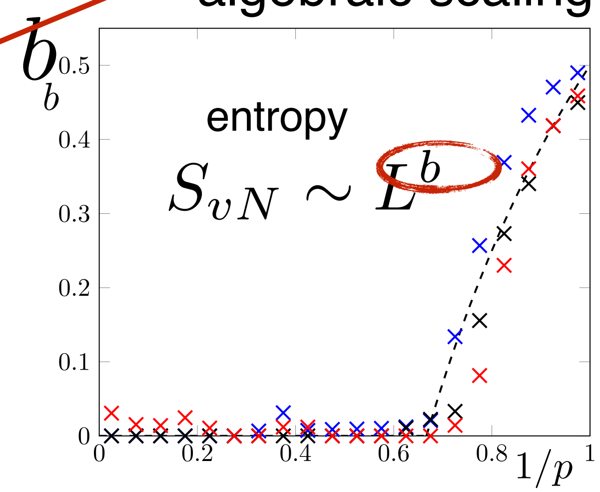


enhanced density fluctuations:
smaller decay exponent



algebraic scaling



field theory predictions:

$$C(l) \sim \begin{cases} l^{-2} & p > 3/2 \\ l^{-(p+1/2)} & 1 < p < 3/2 \\ \text{ill-defined} & p < 1 \end{cases} \qquad a$$

$$S_{vN}(l) \sim \begin{cases} \log_2 l & p > 3/2 \\ l^{3/2-p} & 1 < p < 3/2 \\ \text{ill-defined} & p < 1 \end{cases} \qquad b$$

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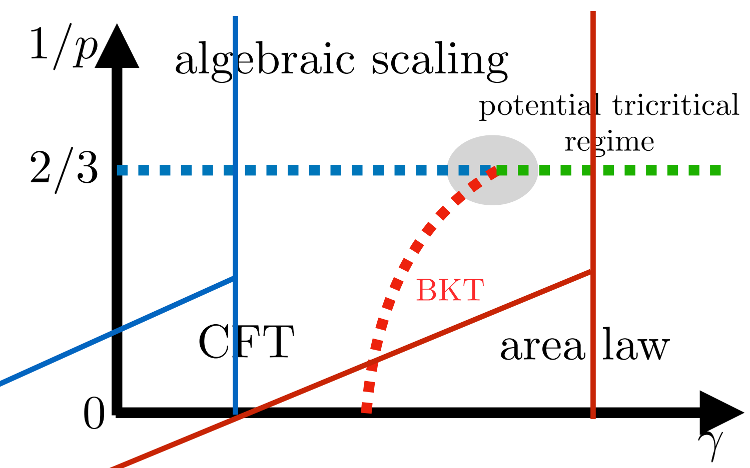

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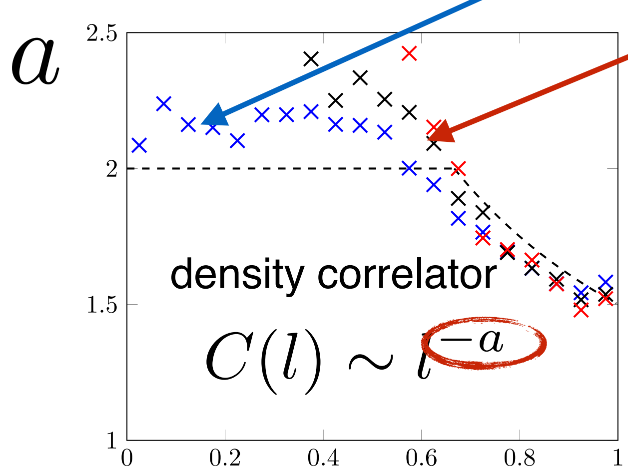

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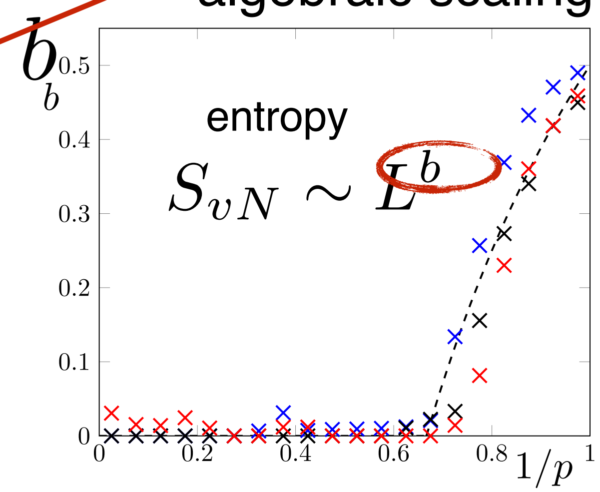

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b

- non-Hermitian long-range Hamiltonian

$$H_{\text{eff}} = H_{\text{eff}}^{(\text{short-range})} + i\Delta \int_{x,y} \frac{\cos(\hat{\theta}_x - \hat{\theta}_y)}{|x-y|^{2p}}$$

- exponent 2p: effect of noise
- exponent p in no-click evolution (incompatible with numerics)

➡ rare vs. typical trajectories

Robustness against imperfect readout / decoherence

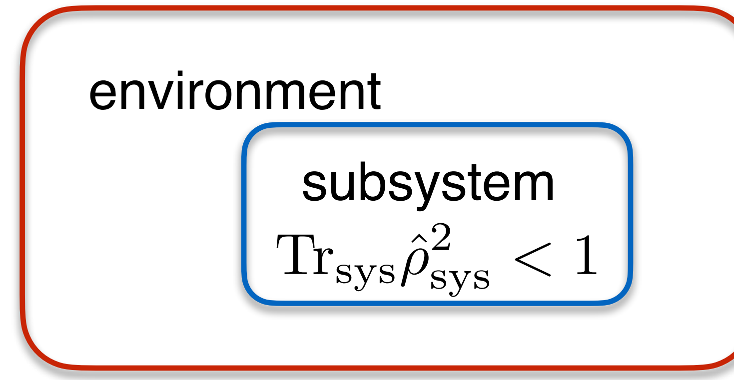
B. Ladewig, SD, M. Buchhold, arXiv (2022), to appear in PRR

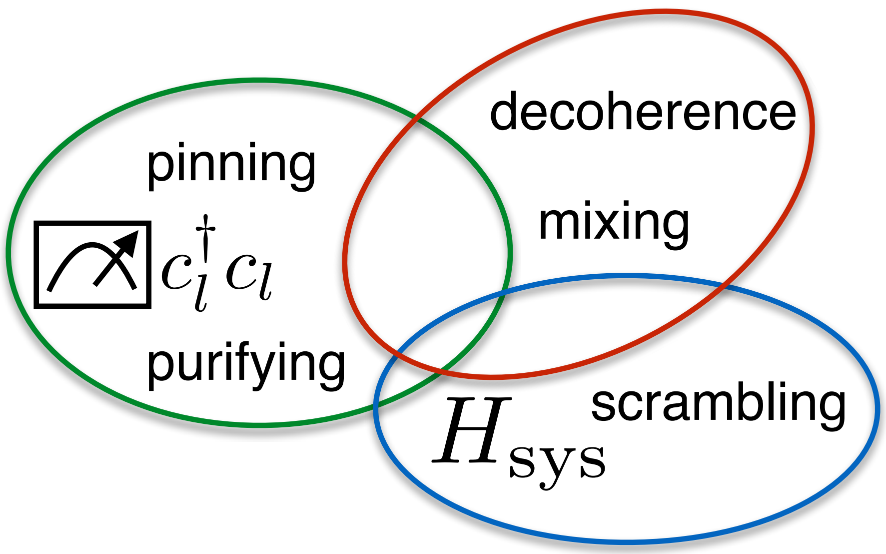
circuits: Li, Fisher, arxiv (2021); Bao, Choi, Altman, Ann. Phys. (2021)

- motivation
  - imperfect read out $c_l^\dagger c_l = ?$
  - coupling to environment $H_{\text{sys-bath}} = \sum_l n_l \hat{E}_l$

➡ common formulation in terms of incoherent stochastic Schrödinger equation

➡ complex interplay





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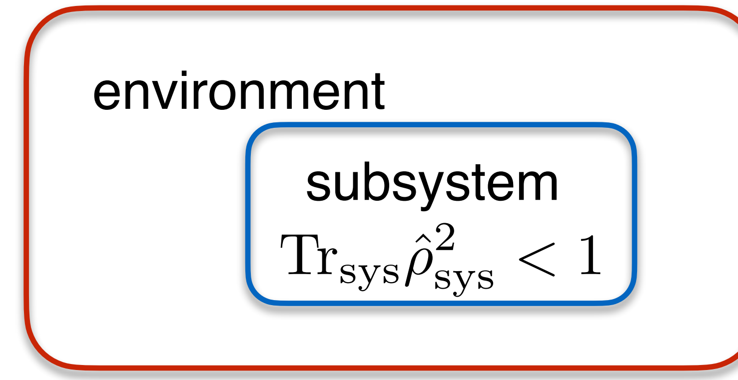

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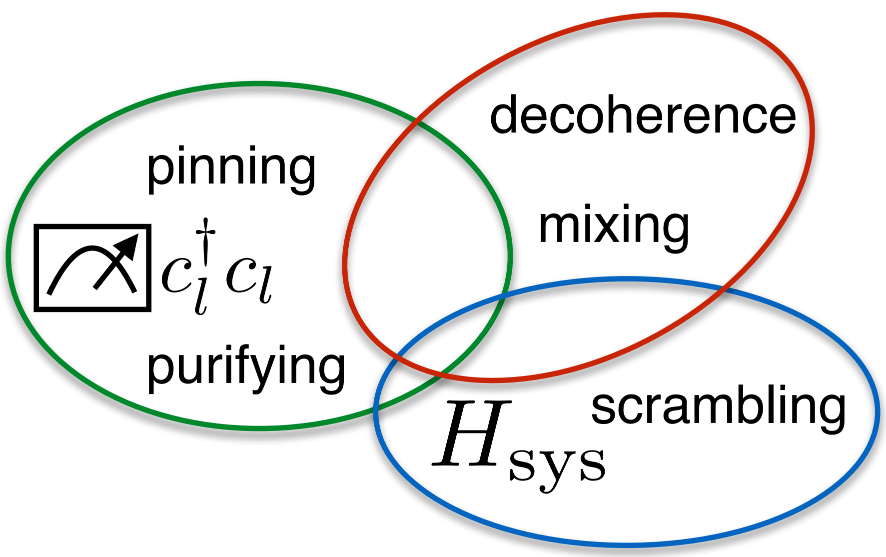

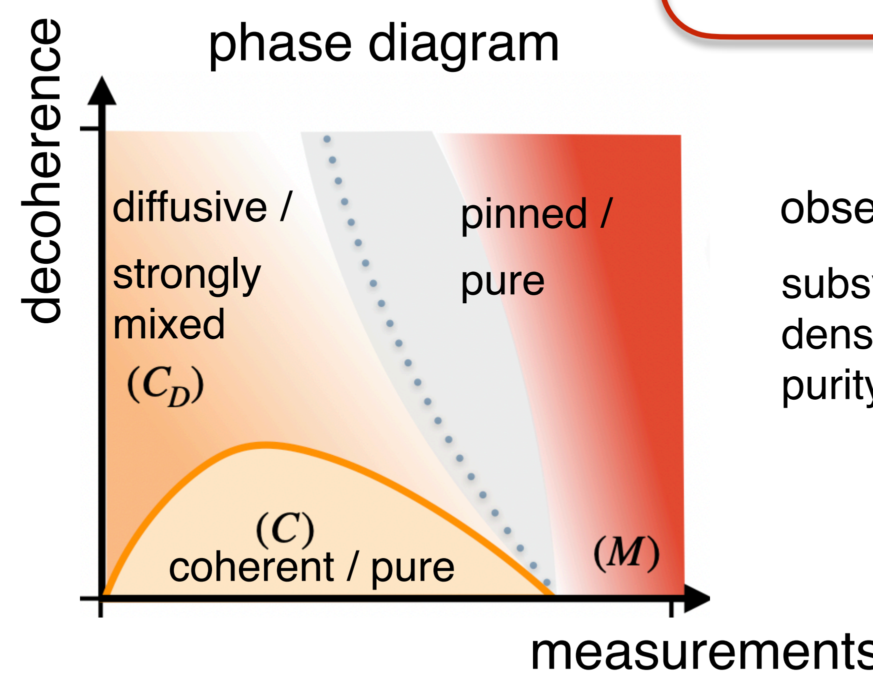

observables:

subsystem parity, density correlations, purity

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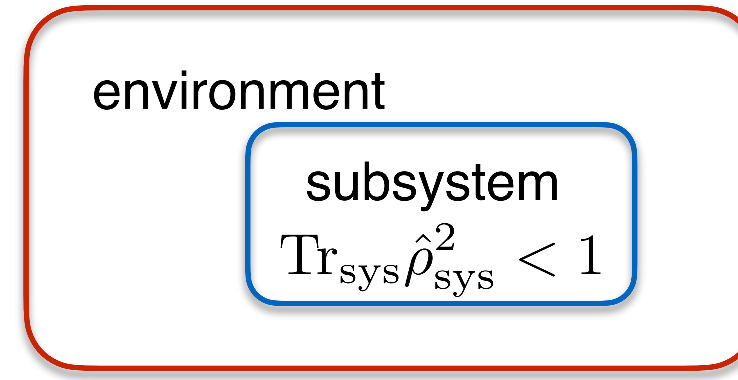

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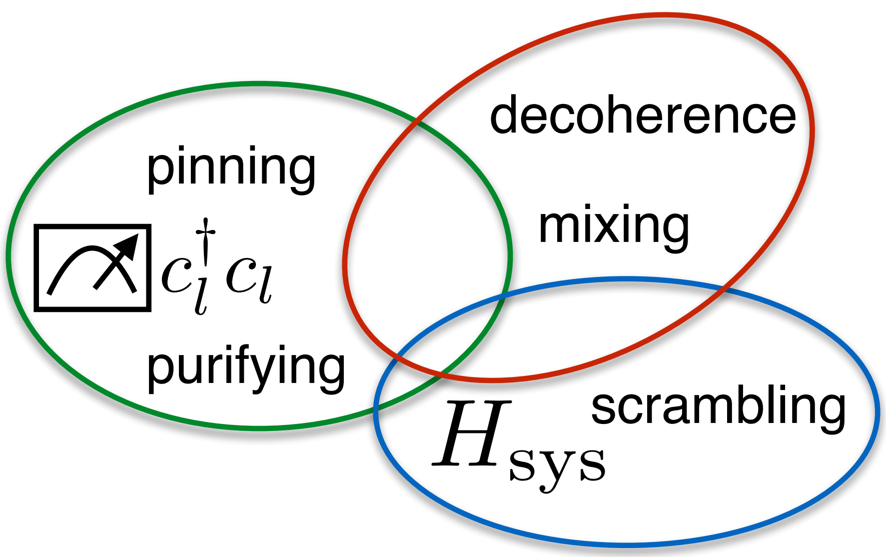

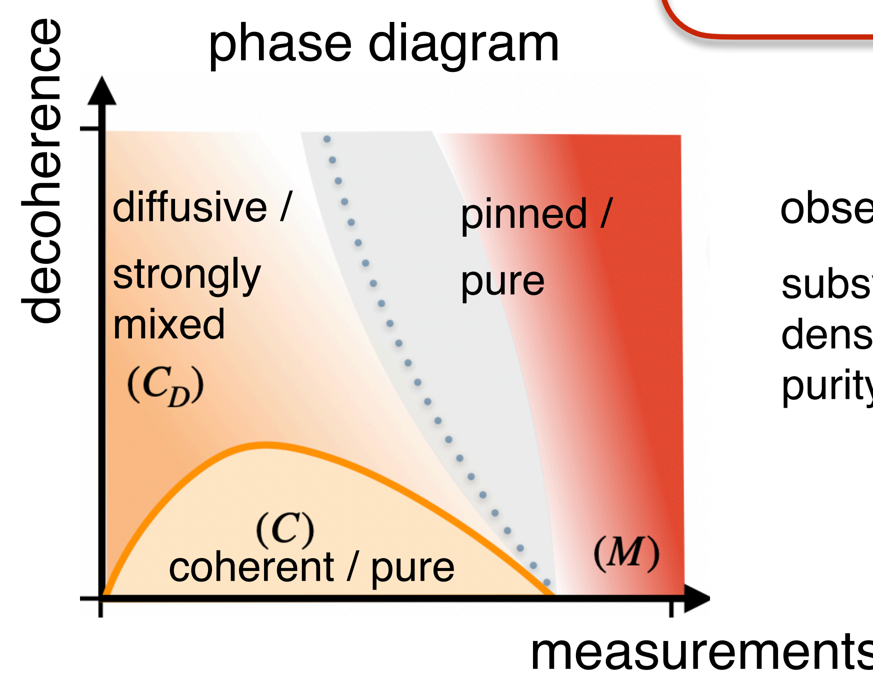

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- combined numerics / replica field theory approach

$$\partial_t \rho = -i \left(H_{\mathrm{eff}} \rho - \rho H_{\mathrm{eff}}^\dagger \right) + \int_x [\partial_x \phi, [\partial_x \phi, \rho]]$$

cooling heating

emergent scales:

spectral gap

temperature gap

➡ three Gaussian fixed point theories

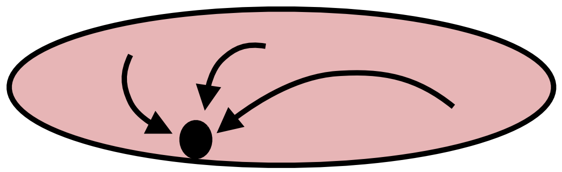
➡ robustness: extended area of critical log-phase

➡ gapless nature persists to large decoherence for small measurement rate

Observability of the transition? Feedback scenario

way out for Cliffords: Gullans, Huse, PRL (2020); exp: C. Noel et al. Nat Phys. (2021)

- Postselection exponentially hard
- Do preselection instead:
    - break degeneracy in measurement outcomes
    - steer system into representative state in Hilbert space (dark state)



➡ pull the transition to the level of standard observables (linear in state)

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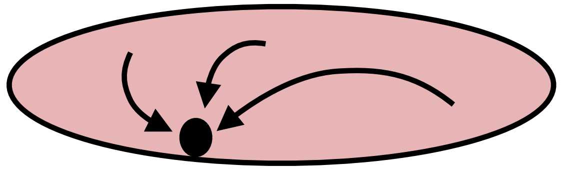

➡ pull the transition to the level of standard observables (linear in state)

- Feedback procedure:
  - track measurement outcomes $I_s = \langle \hat{n}_s \rangle + \frac{dW_s}{2\gamma dt}$
  - condition Hamiltonian on outcomes (follow evolution by simulation - efficient in Gaussian class - free fermions, stabilizers)

$$\hat{H}(\{I_s\}) \to \hat{H}(\{\langle \hat{n}_s \rangle\}) = \sum_l t_l(\{\langle \hat{n}_s \rangle\})\hat{c}^\dagger_{l+1}\hat{c}_l + h.c.$$

state dependent hopping

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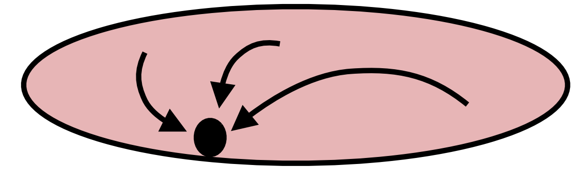

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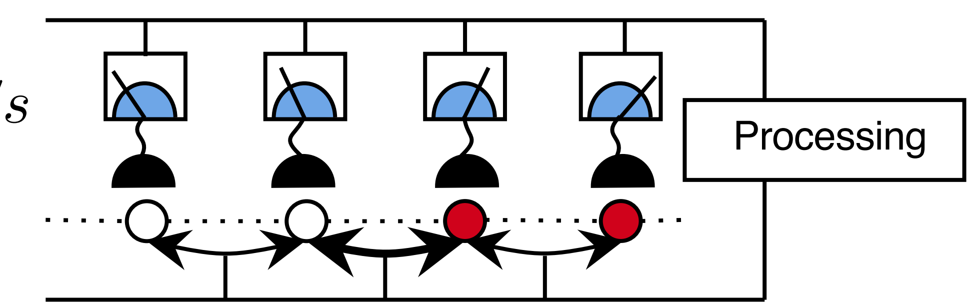
- Feedback procedure:
  - track measurement outcomes $I_s = \langle \hat{n}_s \rangle + \frac{dW_s}{2\gamma dt}$
  - condition Hamiltonian on outcomes (follow evolution by simulation - efficient in Gaussian class - free fermions, stabilizers)

$$\hat{H}(\{I_s\}) \to \hat{H}(\{\langle \hat{n}_s \rangle\}) = \sum_l t_l(\{\langle \hat{n}_s \rangle\}) \hat{c}^\dagger_{l+1} \hat{c}_l + h.c.$$

state dependent hopping

- condition: hoppings such that there is a unique dark state

$$\hat{H}(\{\langle \psi_T | \hat{n}_s | \psi_T \rangle\}) = 0 \quad \& \quad \hat{M}_s |\psi_T\rangle = (\hat{n}_s - \langle \hat{n}_s \rangle)|\psi_T\rangle = 0 \, \forall s$$



for one representative measurement outcome, e.g.

$$|\psi_T\rangle = |1010\ldots\rangle \qquad \text{(charge density wave)}$$

➡ strong monitoring: evolution directed towards one specific state
➡ weak monitoring: steering fails

Feedback induced entanglement transition: Phenomenology

➡ same phenomenology in non-linear in state observables

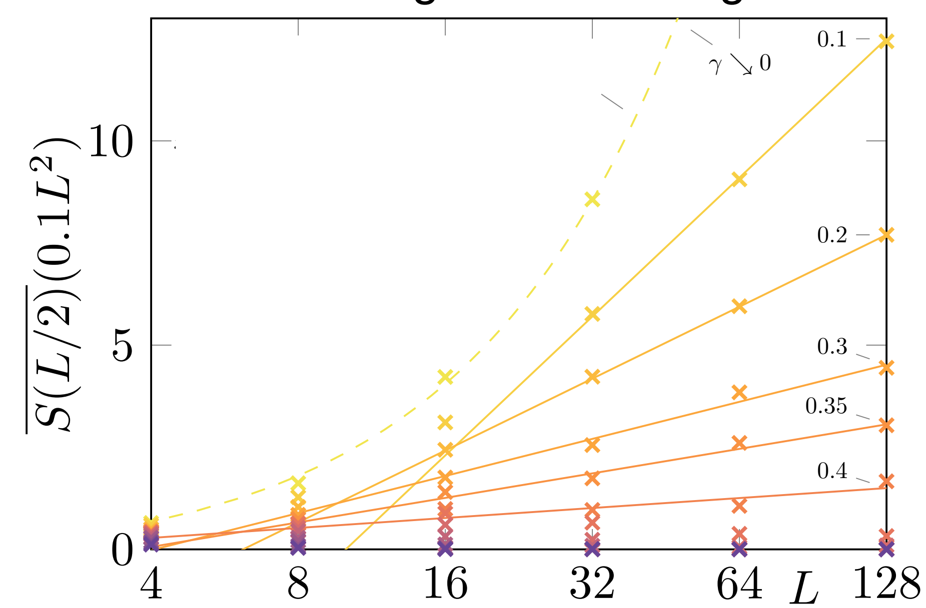
$\gamma < \gamma_c$

- log entanglement
- long-range correlations
- exponentially long time to target

$\gamma > \gamma_c$

- area law entanglement
- exp. decay of correlations
- algebraic time to target

entanglement scaling



Feedback induced entanglement transition: Phenomenology

➡ same phenomenology in non-linear in state observables

➡ transition visible in ordinary qm observables (linear in state/single replica)

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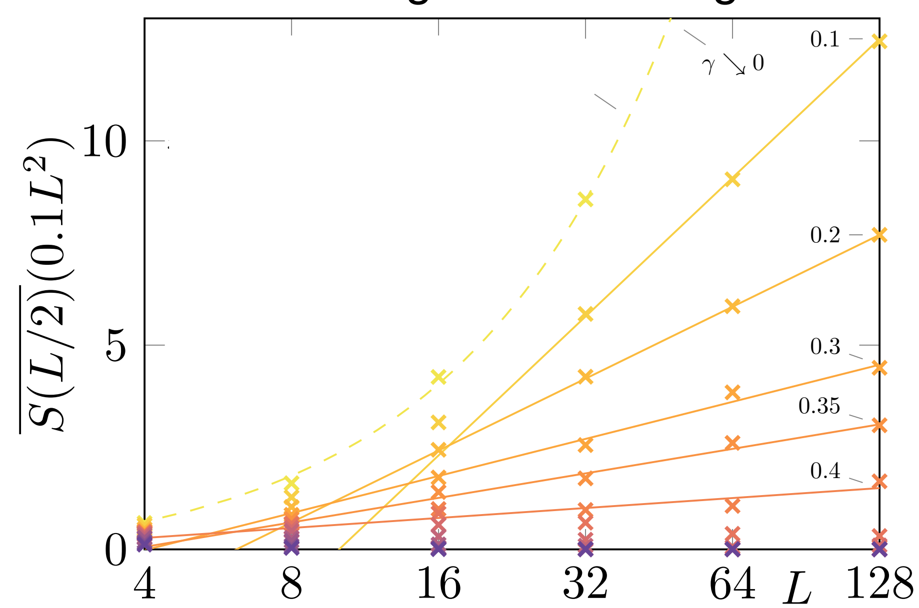
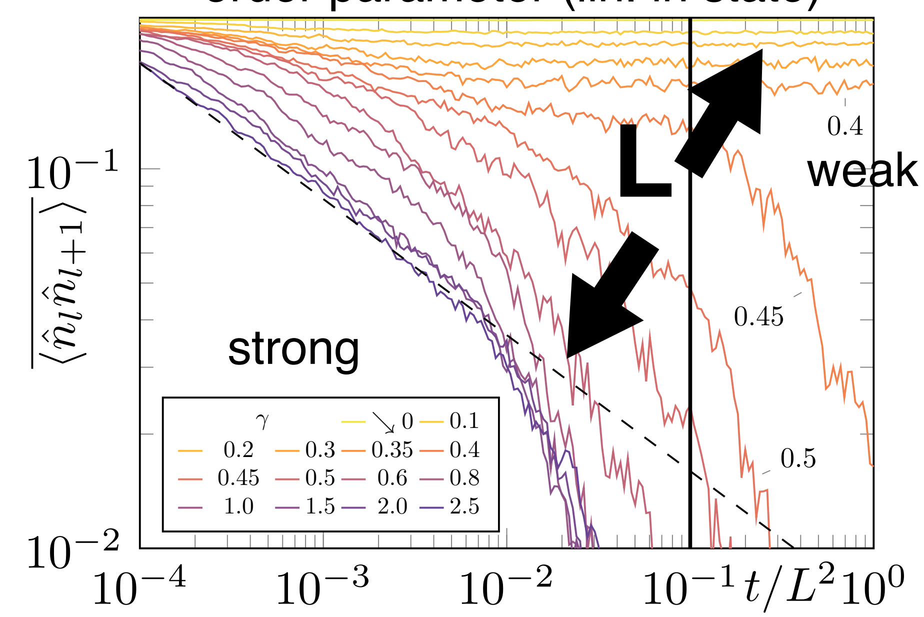


ordinary charge density wave order parameter (lin. in state)



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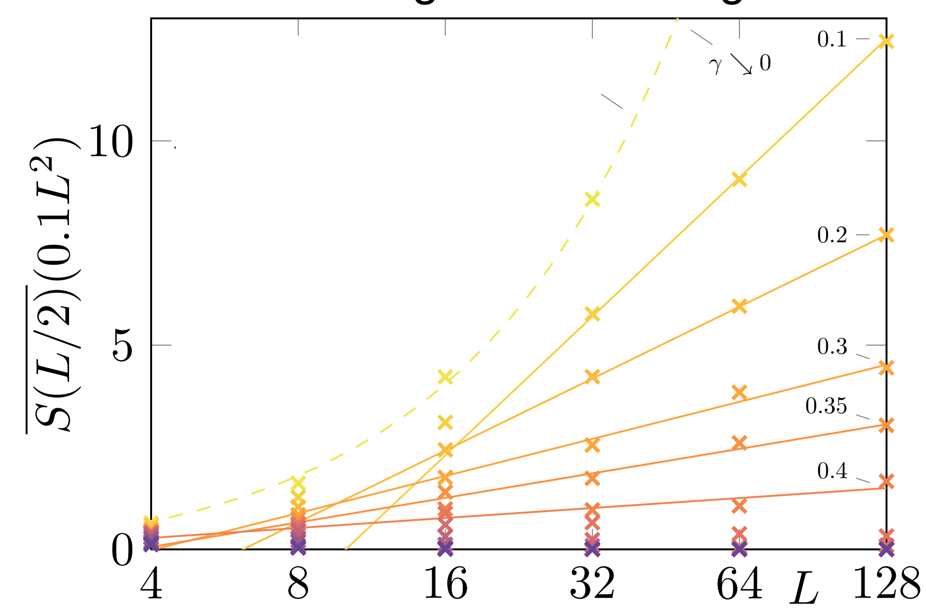
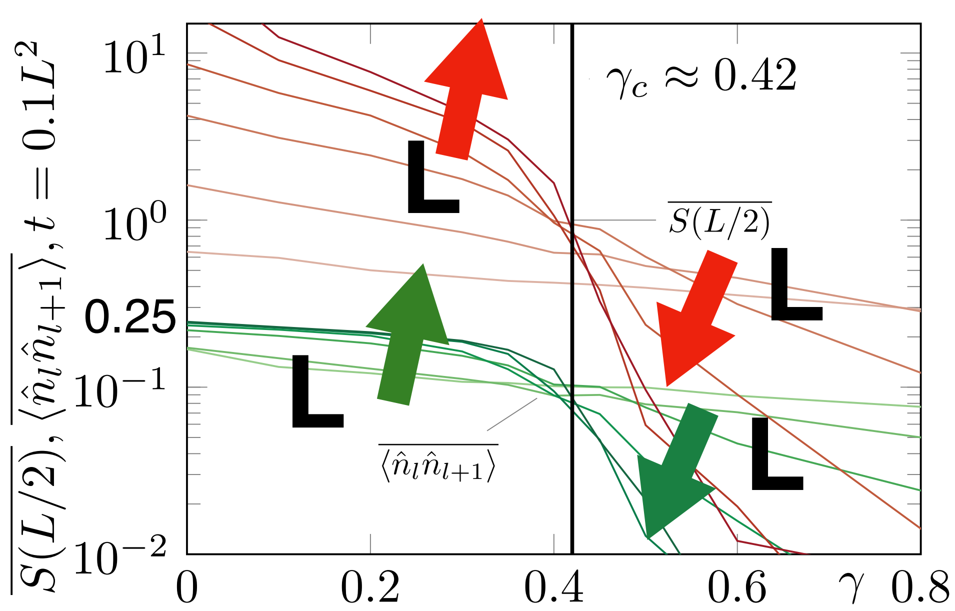
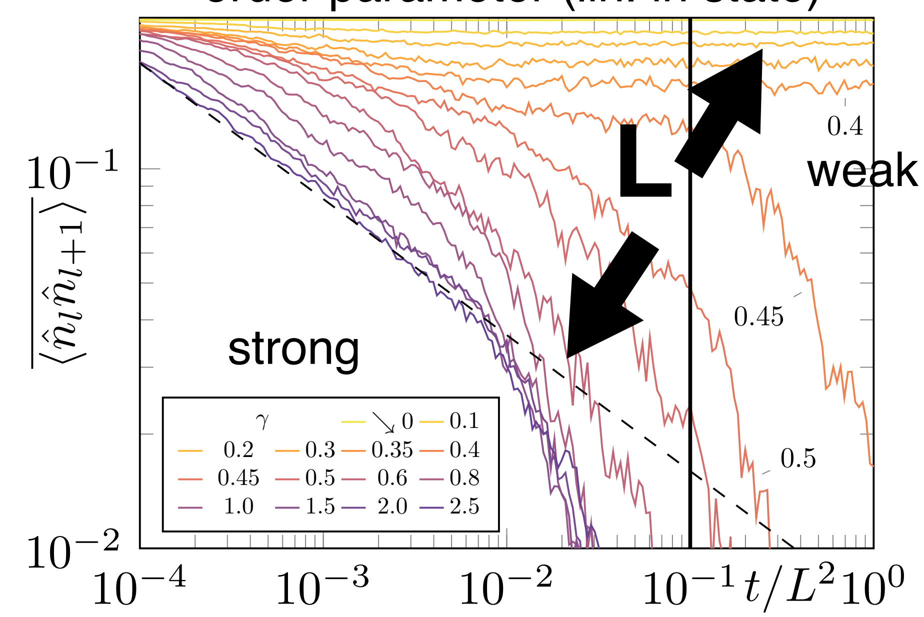


ordinary CDW order parameter

entanglement entropy



ordinary charge density wave order parameter (lin. in state)

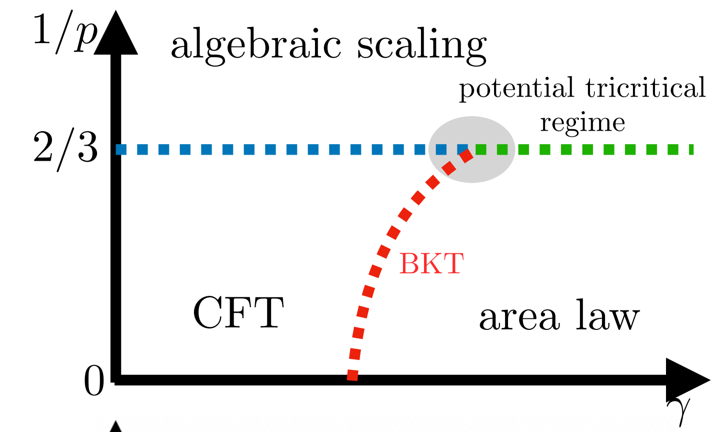


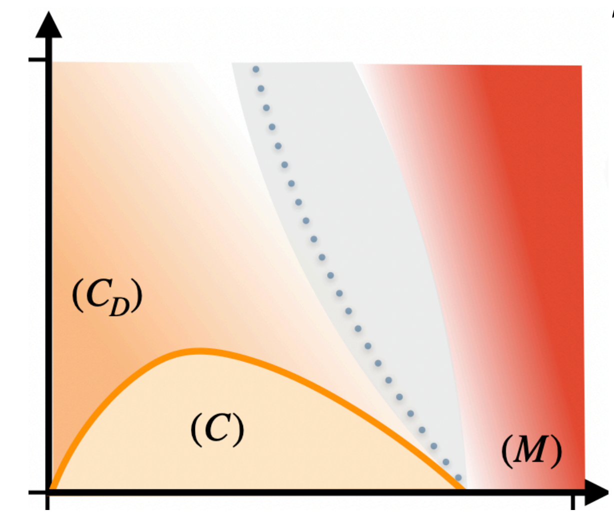
- ➡ coinciding critical point in single-replica correlator and entanglement entropy (all replicas)
- ➡ single run sufficient to detect phase transition

Conclusions & Outlook

Results:

- various measurement induced phases in monitored fermions
  - measurement dominated area law phase
  - log-phase
  - subvolume algebraic scaling phase
  - robustness against imperfect measurements/decoherence
- common theoretical description in terms of Keldysh replica field theory





Directions:

- detection: classical vs. quantum feedback w/o classical readout? beyond 'Gaussian' class? universality classes? relation to dark state / absorbing state transitions?
- phase transitions in no-click evolution (rare events) vs. trajectory ensemble (typical events)

  Biella, Schiro, Quantum (2021); Turkeshi et al. PRB (2021) Gopalakrishnan Gullans, PRL (2021)
- integrability breaking in measurement induced phase transitions

  O. Lunt, A. Pal, PRR (2020)