SYMMETRY BREAKING AND ENTANGLEMENT TRANSITIONS IN DRIVEN-DISSIPATIVE SYSTEMS

(…with implementation in a trapped ion chain)

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X. Turkeshi, R. Fazio, and M. Dalmonte, Phys. Rev. B **98**, 102, 014315 (2020)
X. Turkeshi, A. Biella, R. Fazio, M. Dalmonte, and M. Schirò, Phys. Rev. B **103**, 224210 (2021)
P. Sierant, G. Chiriacò, F.M. Surace, S. Sharma, X.Turkeshi, M. Dalmonte, R. Fazio, and G. Pagano, Quantum **6**, 638 (2022)
S. Sharma, X. Turkeshi, R. Fazio, and M. Dalmonte, SciPost Phys. Core 023 (2022)
X. Turkeshi, R. Fazio, M. Dalmonte, and M. Schirò, Phys. Rev. B (Letter), in press; arXiv:2111.03500

Driven-dissipative systems

Competition between unitary dynamics and the effect of an external environment in a many-body system

$$\dot{\rho} = -i[\mathcal{H}, \rho] + \mathcal{D}[\rho]$$

Competition between *many-body ordering* and *local dissipation*

See e.g. the review: L. Sieberer, M. Buchhold, S. Diehl, Rep. Progr. Phys. **79**, 096001 (2016)

Steady state

$$\rho_s = \rho(t \to \infty) \longrightarrow \sum_\alpha p_\alpha |\chi_\alpha\rangle\langle\chi_\alpha|$$

Symmetry breaking

(dissipative phase transitions)

$$\langle \hat{O} \rangle_s \begin{cases} = 0 \\ \neq 0 \end{cases}$$

- mixed
- non-equilibrium

correlations

vs

quantum correlations

(entanglement)

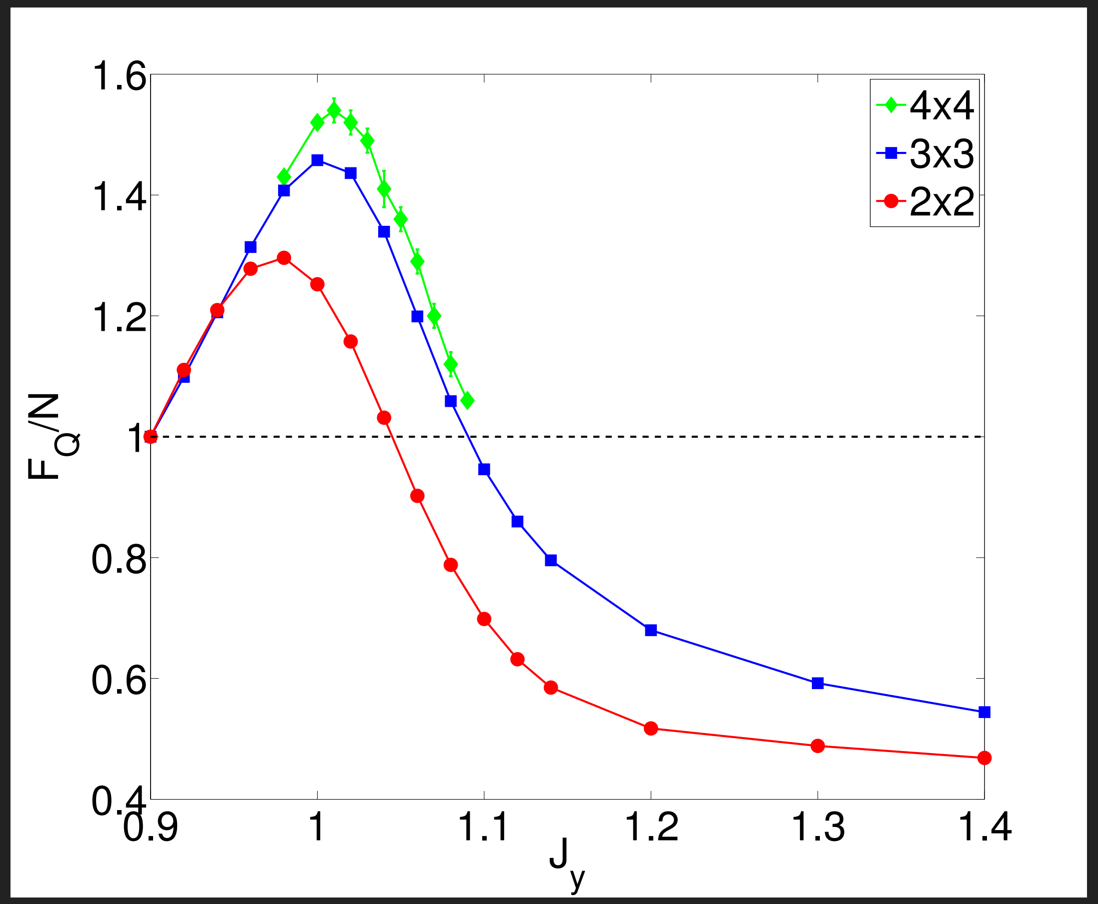
Entanglement in the steady state

- Entanglement content in ρ_s

C. Joshi, F. Nissen, J. Keeling, Phys. Rev. B **88**, 060835 (2013)
P. Calabrese, J. Cardy, E. Tonni, J. Phys. A **48**, 015006 (2015)
P. Hauke, M. Heyl, L. Tagliacozzo, P. Zoller, Nat. Phys. **12**, 778 (2016)
….
….

Quantum Fisher information (measuring the multipartite entanglement) as a function of the coupling shows a critical divergence

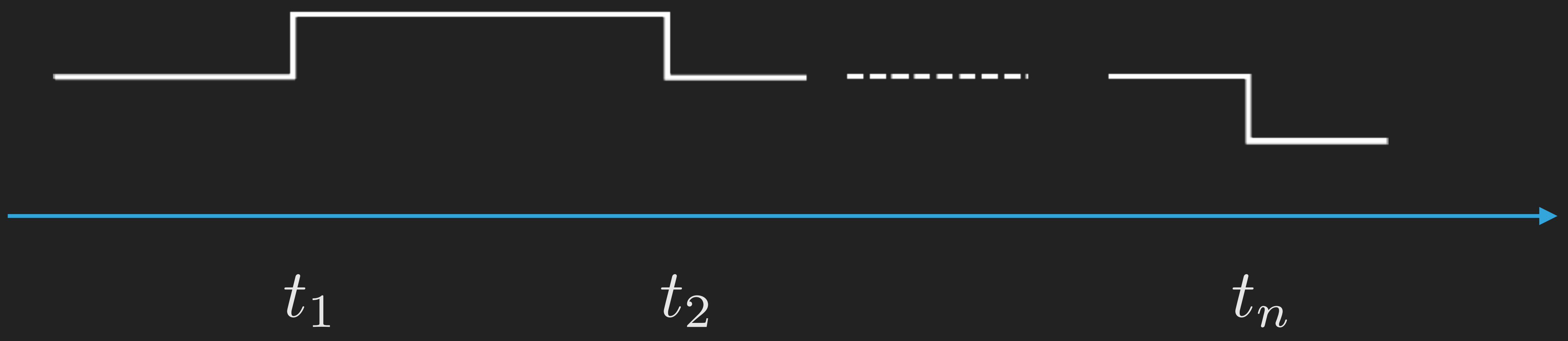
Two-dimensional lattices of spins interacting via an anisotropic Heisenberg Hamiltonian and subject to incoherent spin flips



Rota, F. Storme, N. Bartolo, R. Fazio, C. Ciuti, Phys. Rev. B **95**, 134431 (2017)

- Unravelling and entanglement content in single trajectories

$$|\psi(t), \mathcal{R}(t)\rangle$$



Competition between unitary dynamics and "non-unitary maps" may also reflect in a different dynamics of quantum correlations

Entanglement Transitions

Y. Li, X. Chen, and M. Fisher, Phys. Rev. B **98**, 205136 (2018)
B. Skinner, J. Ruhman, and A. Nahum, Phys. Rev. X **9**, 031009 (2019)
M. J. Gullans, and D. A. Huse, Phys. Rev. X **10**, 041020 (2020)
Y. Bao, S. Choi, and E. Altman, Phys. Rev. B **101**, 104301 (2020)
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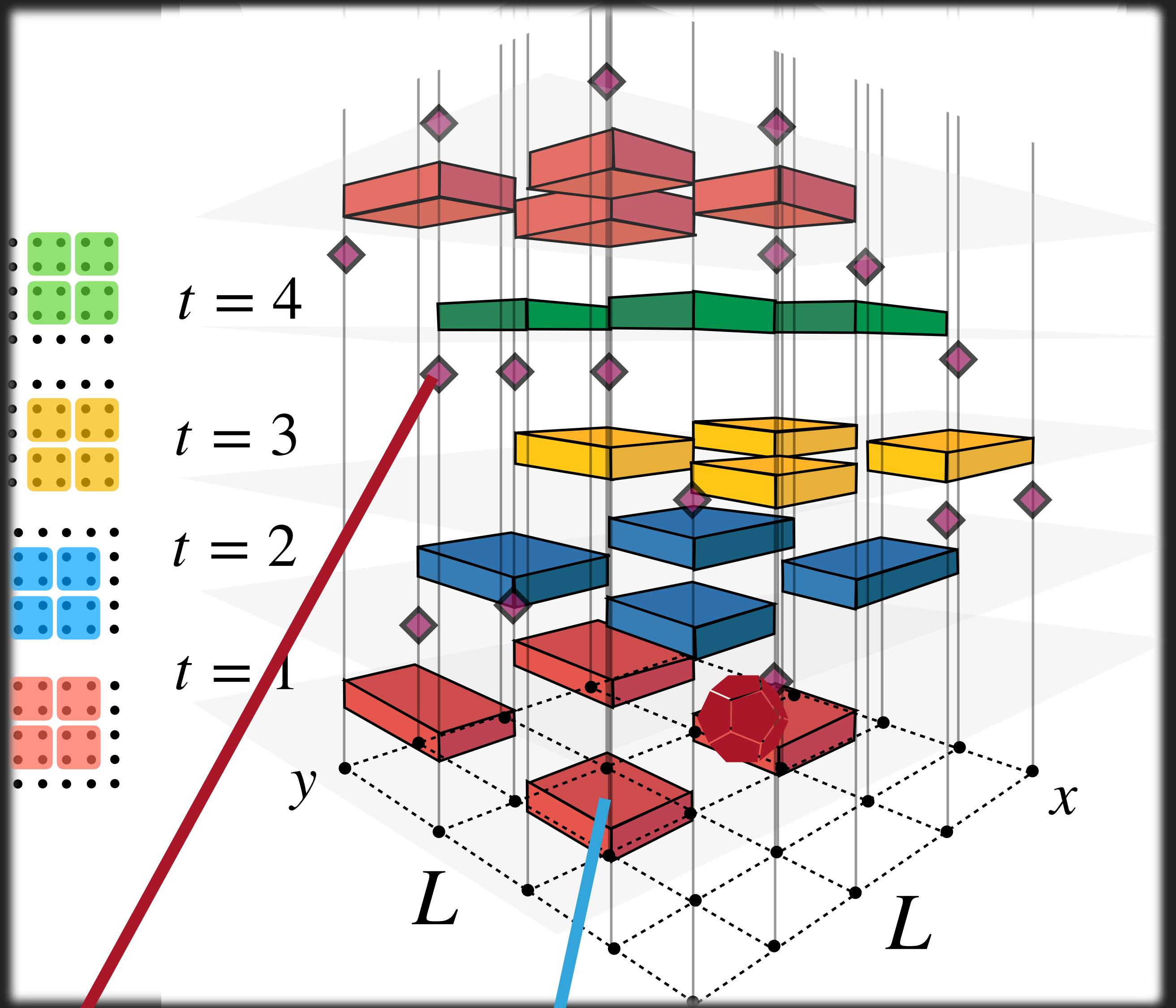


Unitary (2-qubit) gates interrupted by local measurements

Unitary evolution leads to volume-law in the entanglement

Non-unitary local operations favour a separable state (area - law)

Our work

- Driven-dissipative model with spontaneous symmetry breaking and entanglement transitions

While both are typically equally driven by measurement (both are generally suppressed by the measurement processes performed by the environment), they do have very different sensitivity to changes in the coherent dynamics:

- Possible realisation in experiments with trapped ions

The Model

$$\mathcal{H} = \underbrace{\sum_{ij} J_{ij}\sigma_i^x\sigma_j^x}_{\mathcal{H}_J} \underbrace{- h\sum_i \sigma_i^z}_{\mathcal{H}_h}$$

$$J_{ij} \sim \frac{J}{|i-j|^{\alpha}}$$

$\alpha < 1$ long-range coupling may lead to ordering in the x-direction

Short-range interaction - no symmetry breaking

The Model

The non-unitary consists in a local resetting where spins are independently reset to the down state with probability p

... corresponding to the single-site Kraus operators

$$K_0 = \sqrt{p} |\downarrow\rangle\langle\downarrow|$$

$$K_1 = \sqrt{p} |\downarrow\rangle\langle\uparrow|$$

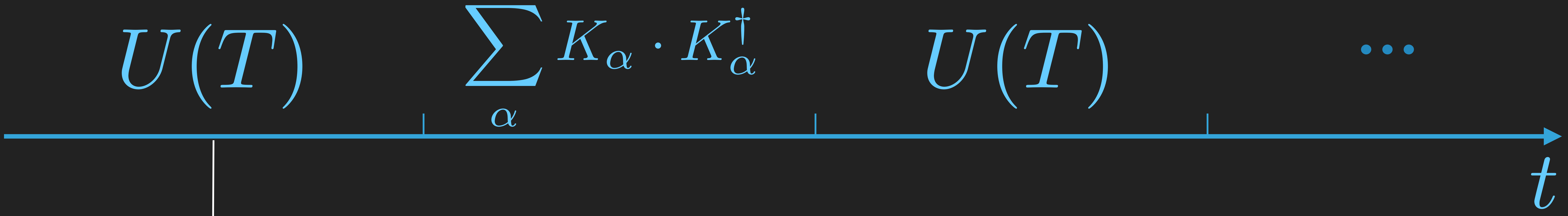
$$K_2 = \sqrt{1-p}\ \mathbb{I}$$

$$|\psi'\rangle = \frac{K_\mu|\psi\rangle}{\sqrt{\langle\psi|K_\mu^\dagger K_\mu|\psi\rangle}}.$$

with probability given by the Born rule

$$\mathcal{P}(\mu) = \langle\psi|K_\mu^\dagger K_\mu|\psi\rangle$$

The Dynamics



$$U(T) = e^{-i\mathcal{H}_h T} e^{-i\mathcal{H}_J T}$$

Averages

$$\overline{\langle \hat{O} \rangle_{\mathcal{R}}}$$

average over trajectories

Quantum average for a given trajectory

“Advantages”

It allows us to tune both the range of interactions and the strength of measurements at the same time

Secondly, this type of transition can be experimentally investigated in a realistic trapped-ion setup (local resetting can be implemented with optical pumping)

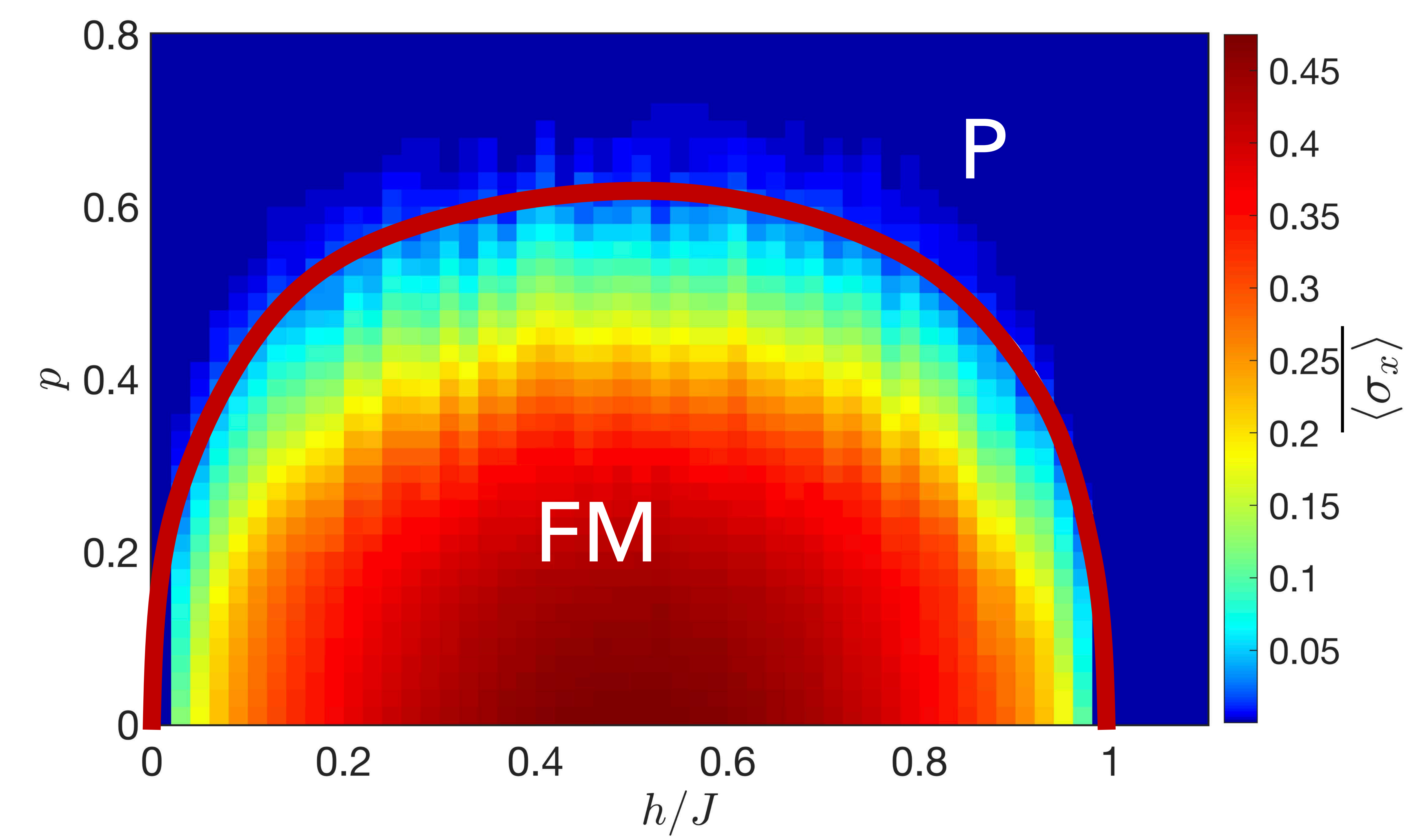
Long-range - DPT

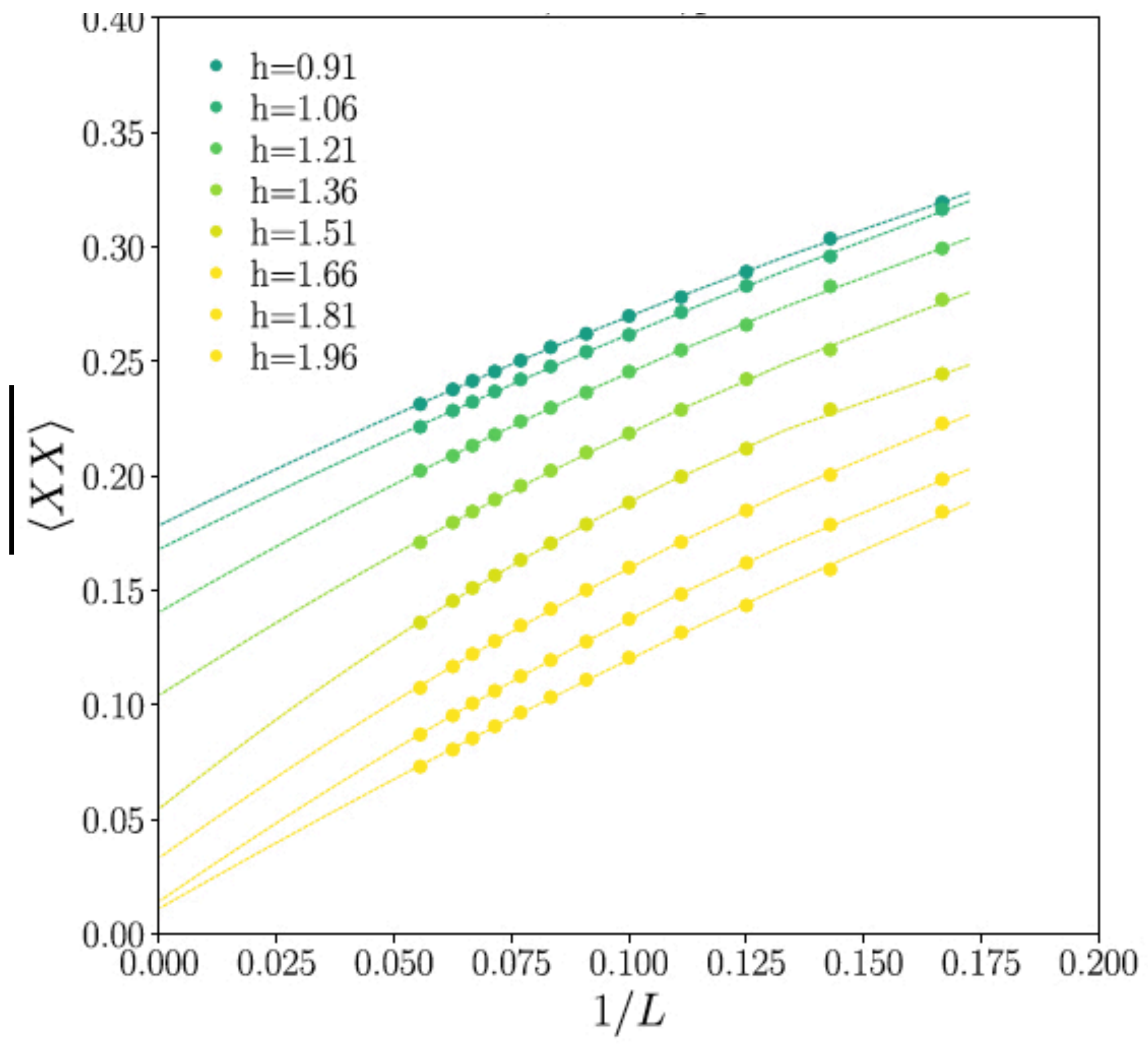
Scaling of the x-x correlations in finite-size systems

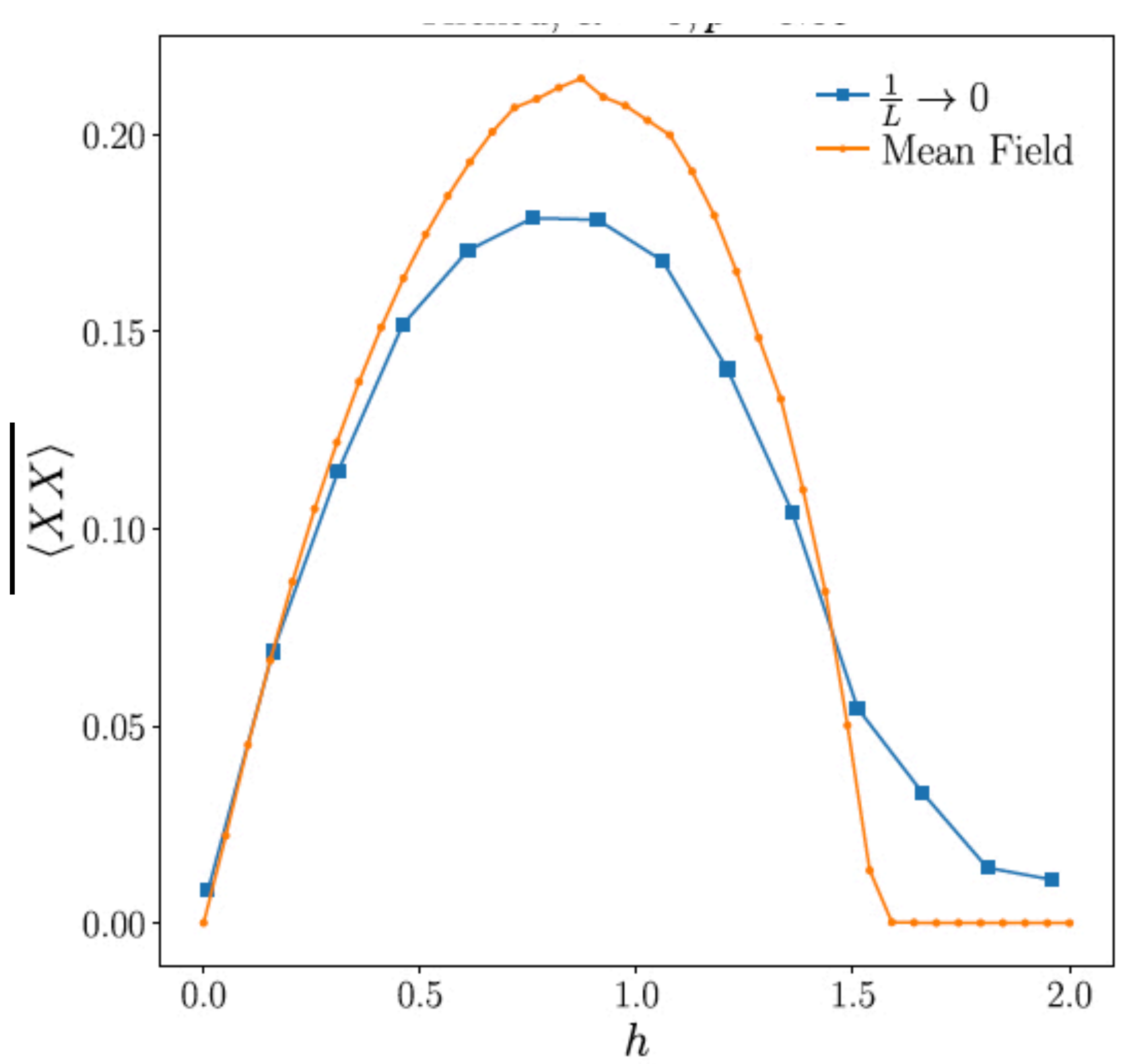
$$\hat{X} = \frac{1}{L}\sum_i \sigma_i^x$$

For sufficiently long-ranged interactions mean-field approx is accurate.

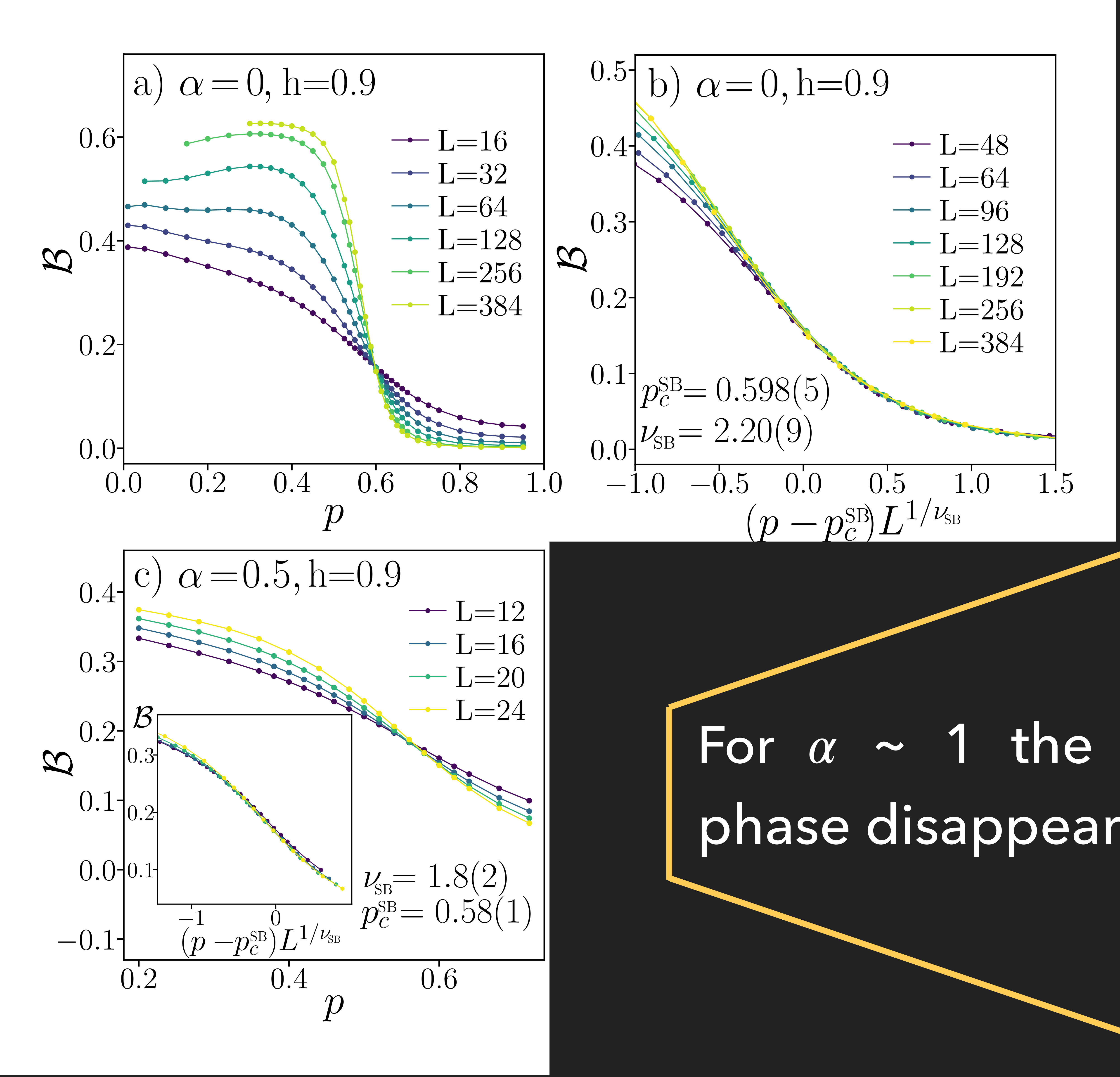
A steady state for times t > max{2L, 10/*p*}







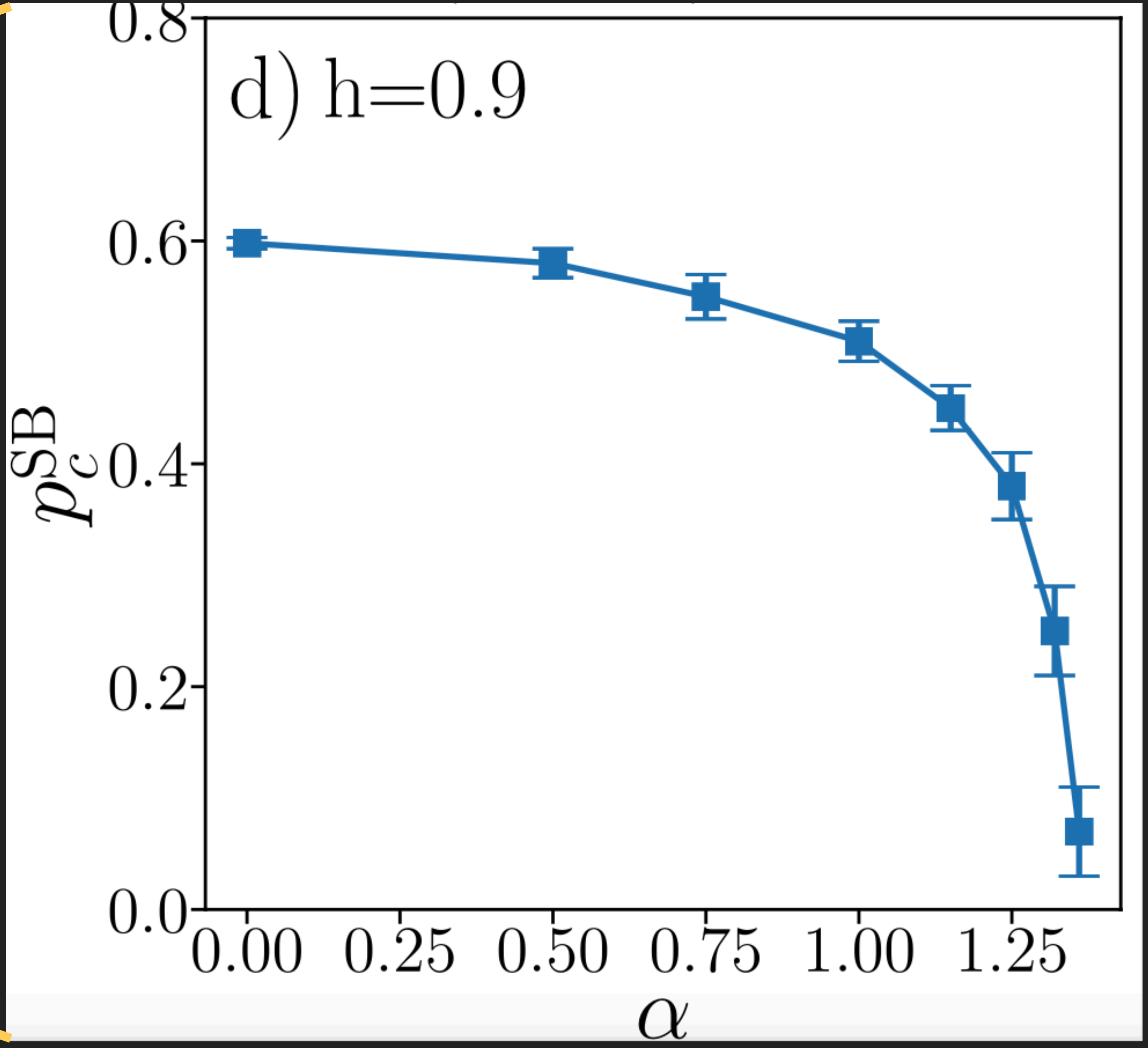
Binder cumulant



$$\mathcal{B} = 1 - \frac{\mathrm{Tr}\left(\rho_{ss}(\sum_{i=1}^{L}\sigma_i^x)^4\right)}{3\left(\mathrm{Tr}(\rho_{ss}(\sum_{i=1}^{L}\sigma_i^x)^2)\right)^2}$$

$$\mathcal{B} = f[(p-p_c^{SB})L^{1/\nu_{SB}}]$$

For α ~ 1 the ordered phase disappears

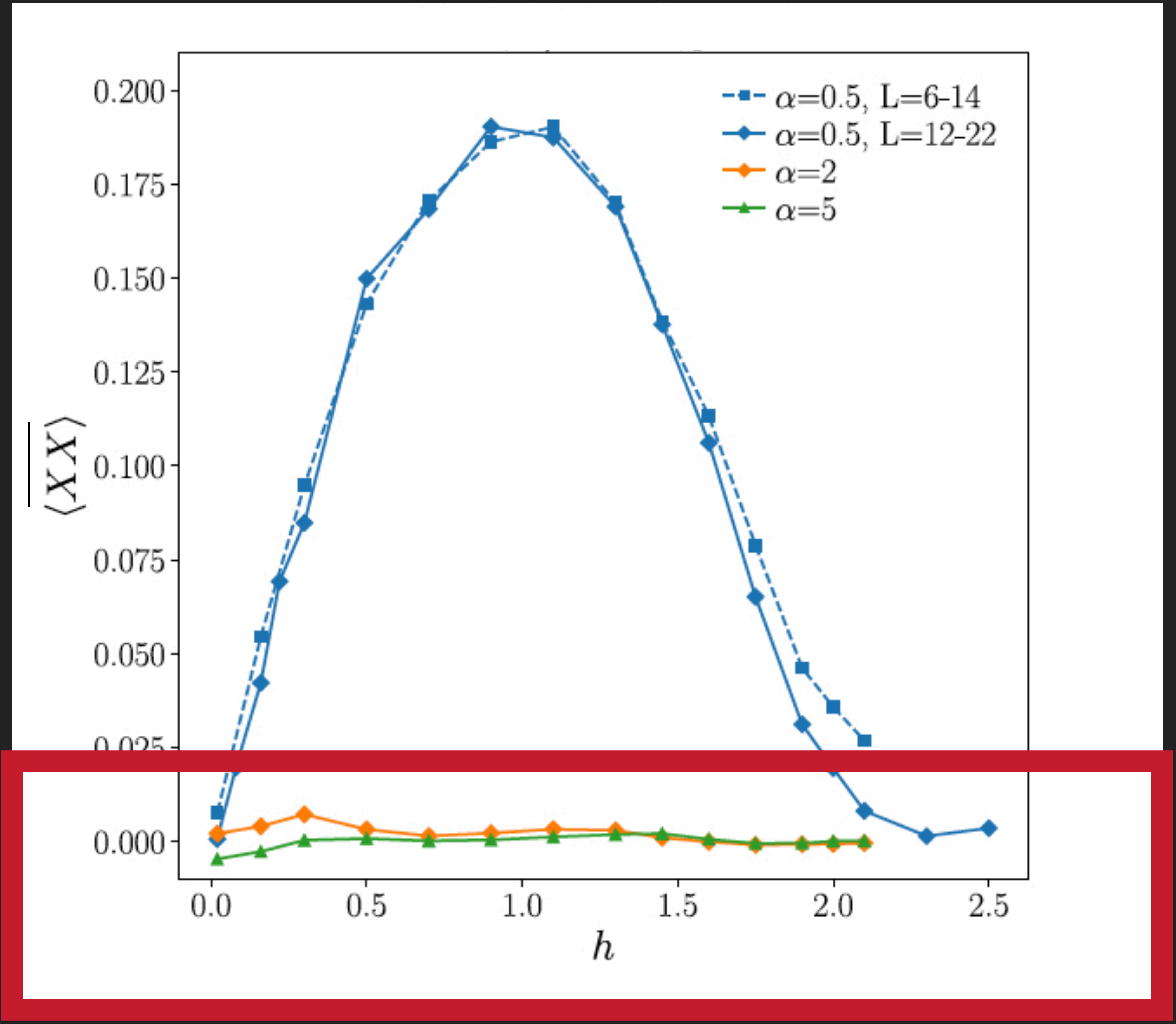


Short-range

For short-ranged interactions symmetry breaking in the steady state is forbidden and

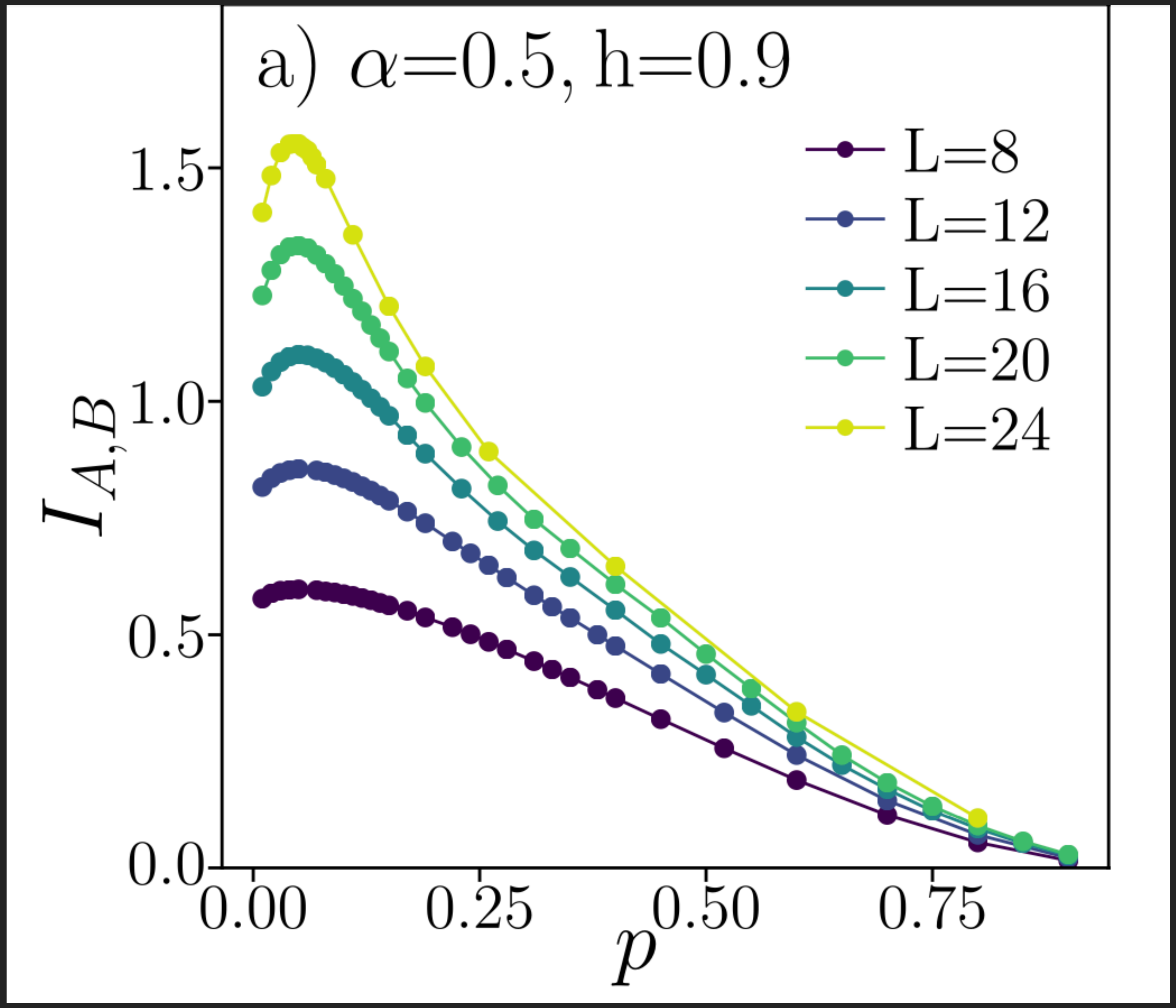
$$\overline{< \sigma_x >_{ss}} = 0$$

A transition in the entanglement behaviour is expected



Entanglement transition

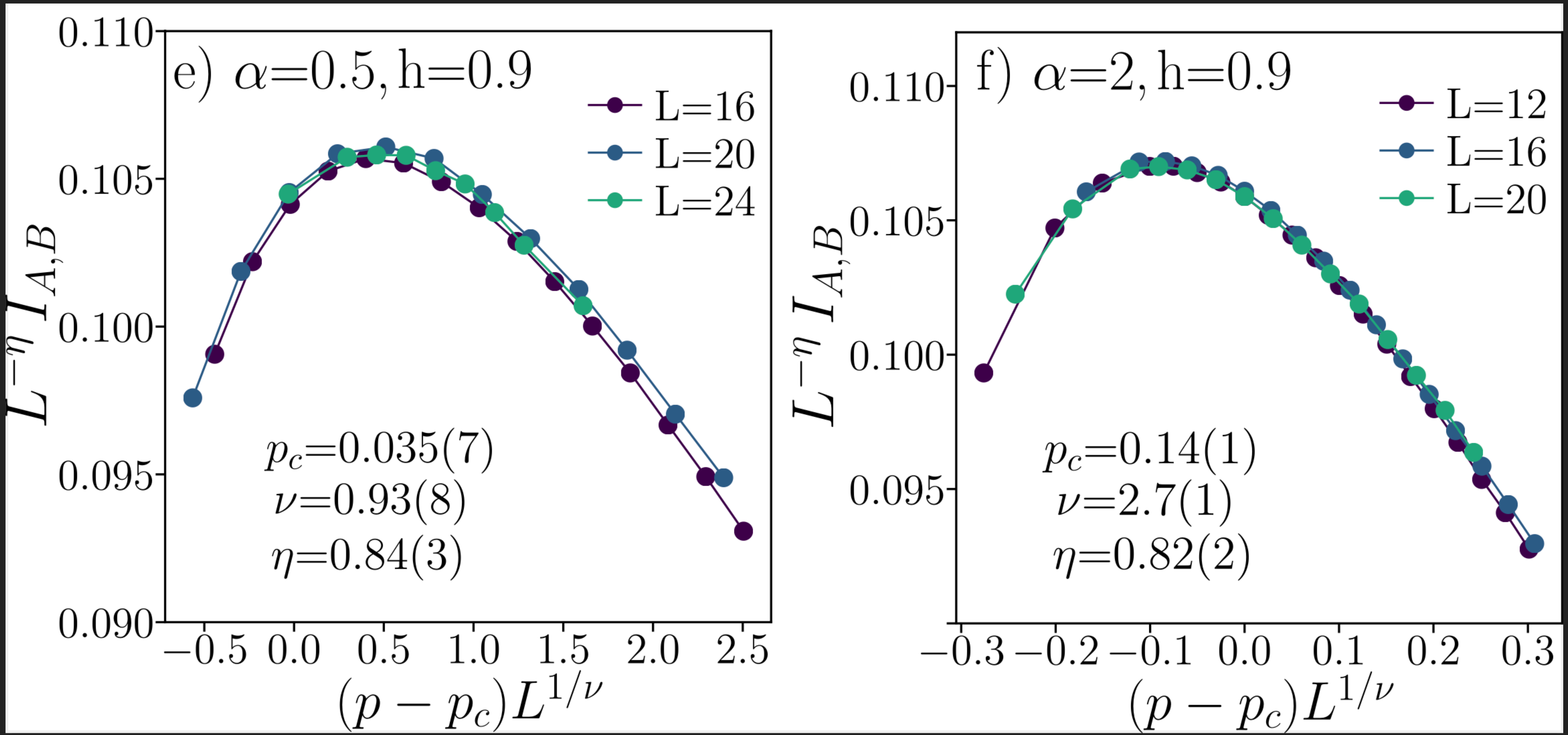
$$I_{A,B} = S(A) + S(B) - S(A+B)$$



$|A| = |B| = L/4$

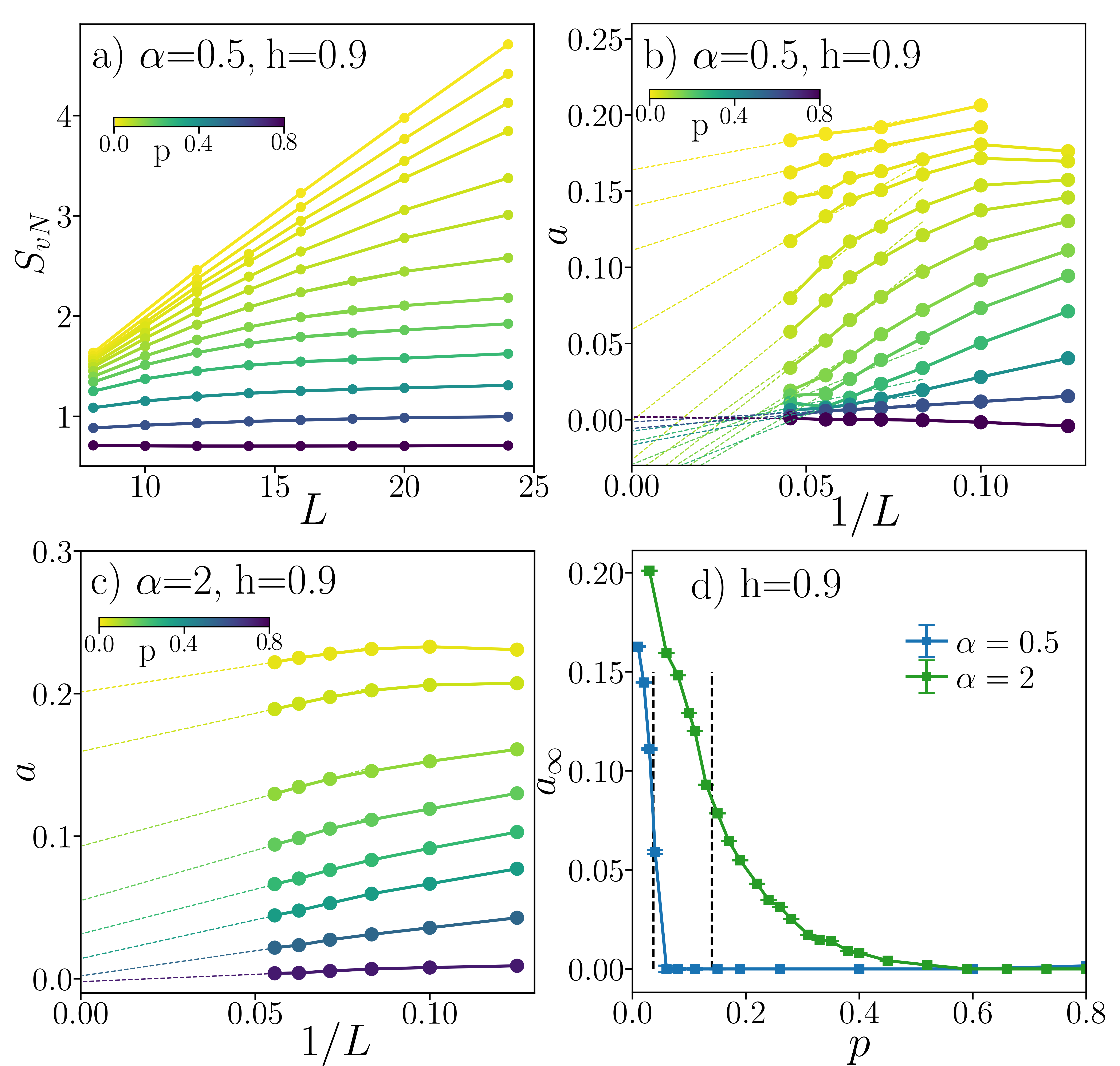
The mutual information is expected to have a maximum at the entanglement transition

$$I_{A,B} = L^{\eta} f[(p - p_c)L^{1/\nu}]$$



Entanglement transition

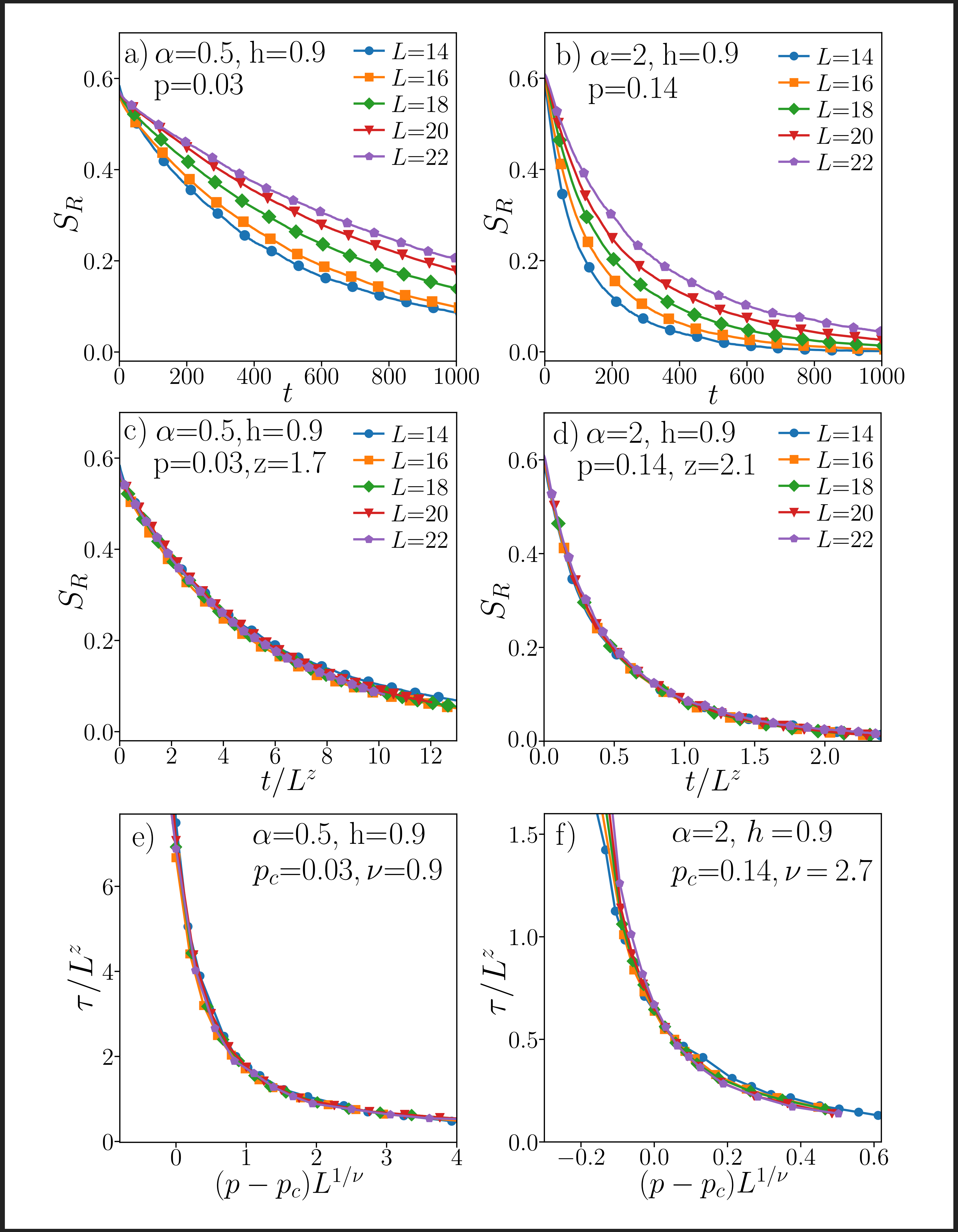
$|A| = L/2$



$$S(A) = a_\infty L + \dots$$

- The extrapolation is difficult due to the sizes available
- Good indication of a transition in agreement with what observed with the mutual information
- Impossible to distinguish between area and sub-volume behaviour

Entanglement transition



Probing the entanglement phase transition with a single reference qubit

M. J. Gullans and D. A. Huse, Phys. Rev. Lett. **125**, 070606 (2020)

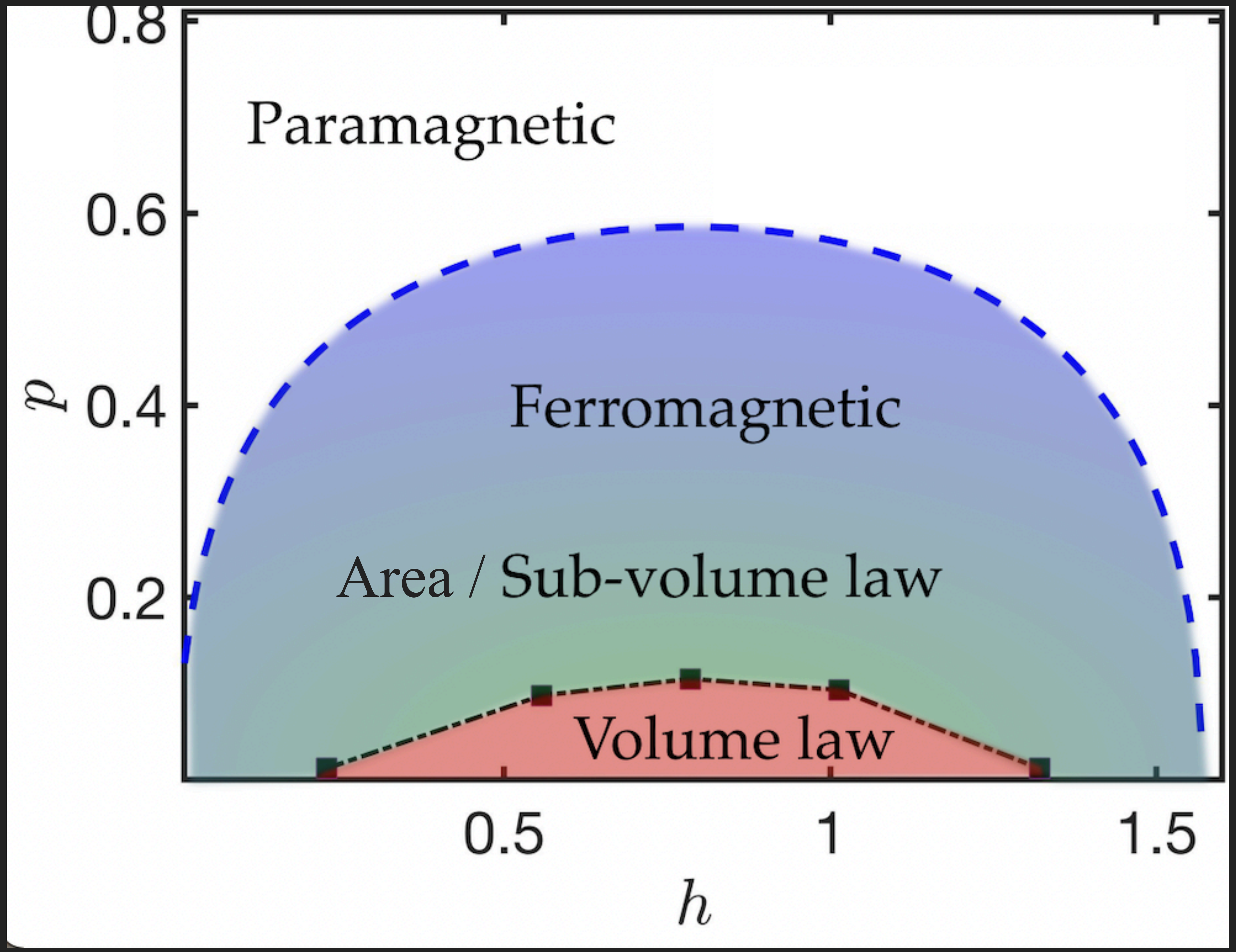
S_R is the entanglement entropy between a reference spin (initially strongly entangled with the rest of the chain) and the system.

The decay in time of S_R becomes slower on increasing the system size and universal function of t/L^z

Finite size scaling of the time when $S_R(\tau) = 0.15$

$$\tau = L^z g[(p-p_c)L^{1/\nu}]$$

Phase Diagram



A region of coexistence of the ordered phase and the area law phase appears for intermediate values of *p*.

Qualitative diagram of the interplay between ordering and entanglement transition as a function of the range of interactions α and the resetting probability p

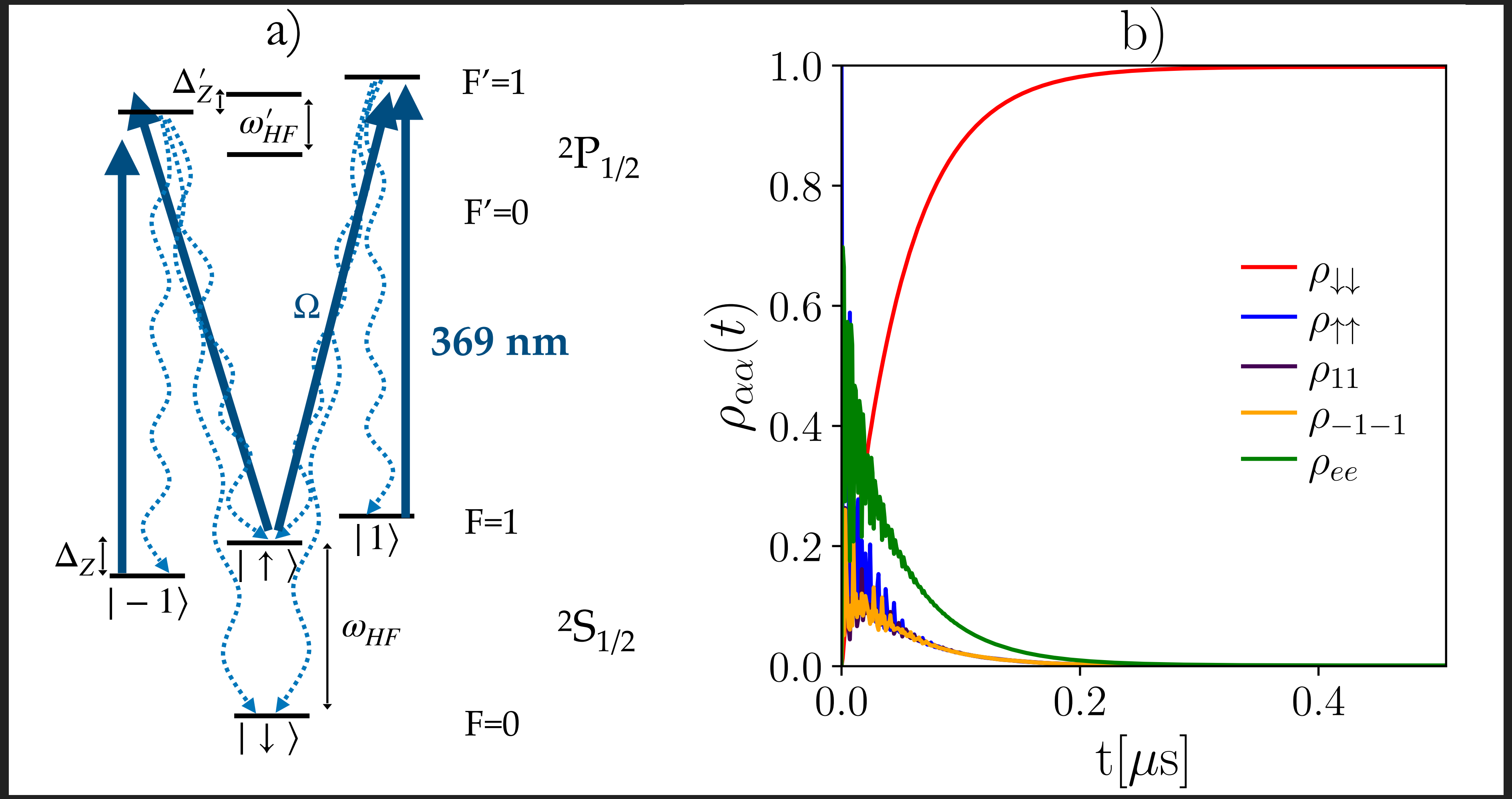
EXPERIMENTAL REALIZATION WITH TRAPPED IONS

C. Monroe *et al*, Rev. Mod. Phys. **93**, 025001 (2021)

Trapped-ion systems can directly realize long-range interacting spin models by off-resonantly coupling pseudo-spin degrees of freedom (detailed below) to the motional collective modes stemming from ion-ion Coulomb interactions.

The power-law exp can be tuned by changing the parameters of the trap.

Kraus operation via optical pumping



The symmetry-breaking transition can be observed by averaging over quantum trajectories. ET can be observed only by measuring properties of the quantum state, postselection necessary

Effect of noise:

- DPT: shrinks the ordered phase
- ET: the area-law a transient

Summary

- Analysis of a model described by a time-periodic Lindblad equation, showing both symmetry breaking and entanglement transitions in the steady state.
- (As expected) DPT and ET not connected
- Similar features expected in a generically periodic dynamics as well as in the case with time-translational invariant eqs.
- Possible realisation in experiments with trapped ions