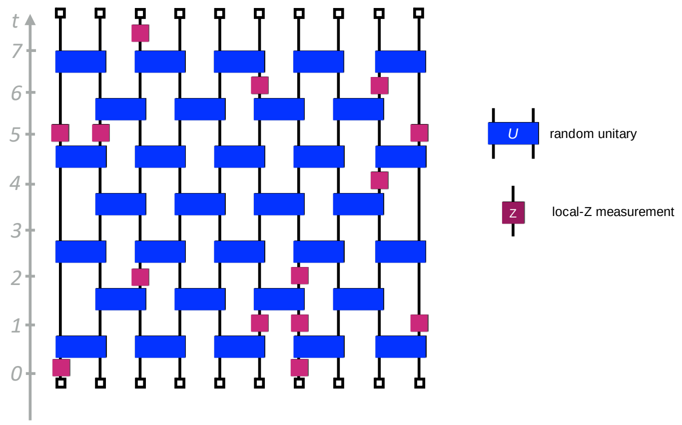
Monitoring Quantum Dynamics

Mainz Conf on Non-Equilibrium Emergence in Quantum Design
6/21/22

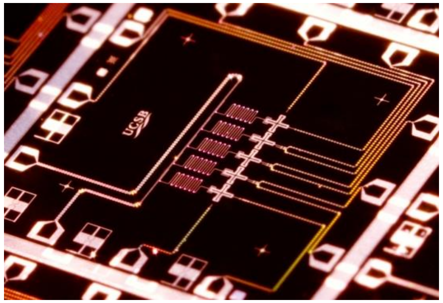
MPA Fisher

This talk: Overview of **“monitored”** non-equilibrium, dynamical, quantum phases/transitions

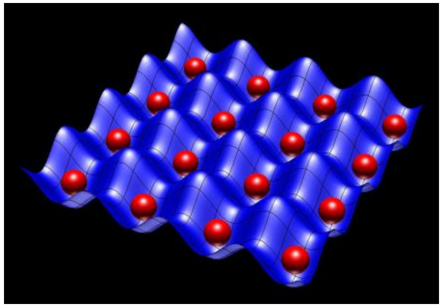


Measurement driven entanglement transitions

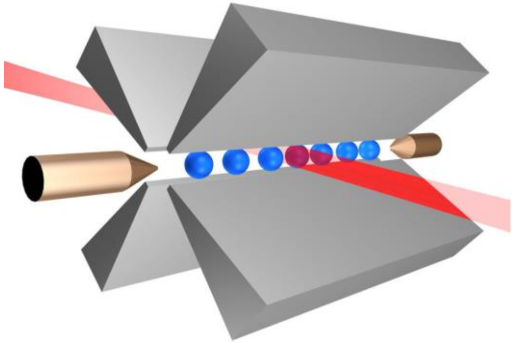
New Experimental Platforms for **many-body** physics: (NISQ computers)



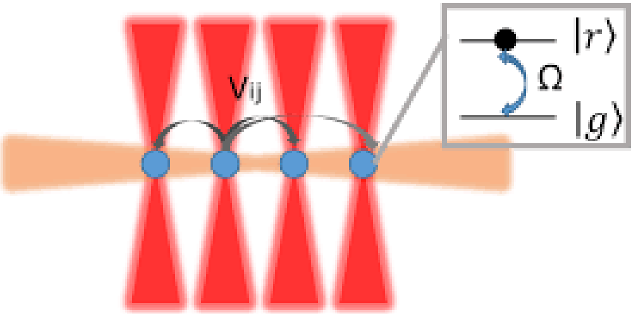
Superconducting QuBit arrays



Ultra-cold atoms



Trapped ions



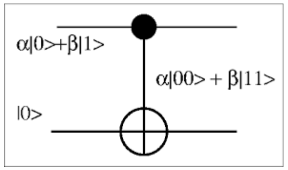
Rydberg atoms

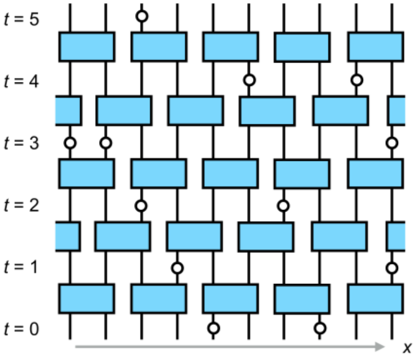
New opportunities for quantum many-body theory

Quantum Hamiltonians;

$$\hat{\mathcal{H}} = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Quantum circuits





Ground states, equilibrium,

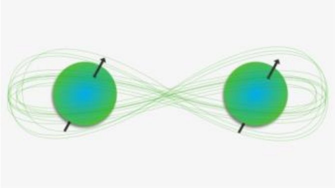
$$\hat{\rho}_{eq} = \frac{1}{Z} e^{-\beta \hat{\mathcal{H}}}$$

Non-equilibrium dynamics,
open quantum systems,
role of measurement

Order parameters

$$\vec{M} = Tr(\hat{\rho}_{eq} \vec{S}_i)$$

Quantum entanglement;
(entanglement entropy)

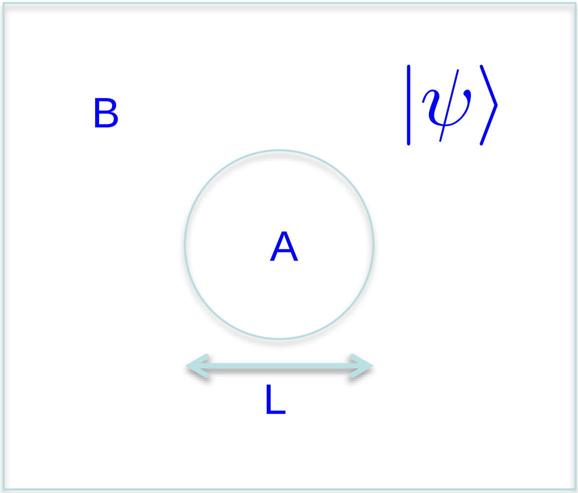


Interest: Quantum phases/transitions driven by measurements
(in open, non-equilibrium systems, in thermodynamic limit)

Common thread: Entanglement entropy

Single eigenstate $|\psi\rangle$

Pure-state density matrix: $\hat{\rho} = |\psi\rangle\langle\psi|$



Spatial Bi-partition: Regions A and B

Reduced density matrix in A

$$\hat{\rho}_A = Tr_B(\hat{\rho})$$

Entanglement entropy:

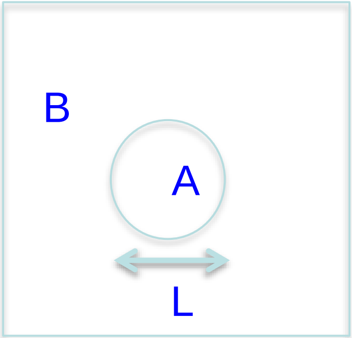
$$S_A(L) = -Tr_A(\hat{\rho}_A \ln \hat{\rho}_A)$$

Scaling of Entanglement entropy in equilibrium

Ground states:

Gapped - ***area law entanglement entropy***:

$$S_A(L) \sim L^{d-1} \sim |\partial A|$$



Ground states manifest spatial locality

Excited eigenstates with finite energy-density

Volume-law entanglement entropy

$$S_A(L) = sL^d \sim |A|$$

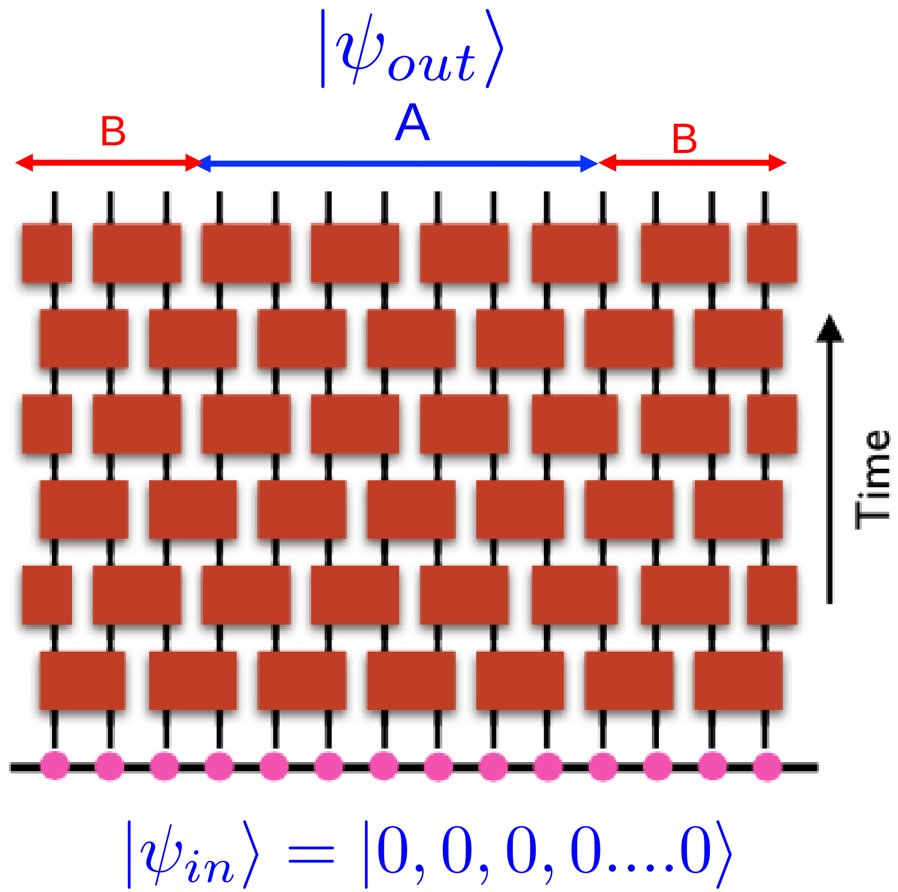
Finite energy-density eigenstates are non-local

Entanglement dynamics out of equilibrium in closed system

- Quantum circuit with L qubits:
- Initial state: unentangled product state
- Evolve Qubits w/ (random) 2-qubit unitary gates
- Run quantum circuit for long times

- **Entanglement spreads ballistically, into maximal entropy state**

$$S_A^{out} = \log 2 \times L_A$$



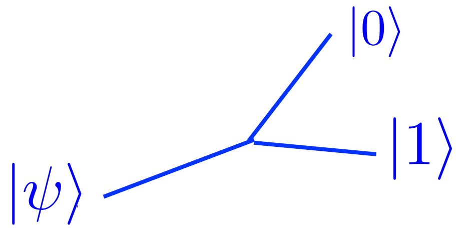
Nahum, Ruhman, Vijay, Haah (2017)

How to control entanglement growth? With measurements!

Quantum Measurements and Quantum trajectories

Measure Z-component of single qubit:

$$|\psi\rangle = a|0\rangle + b|1\rangle$$



Projective measurement: wavefcn collapse

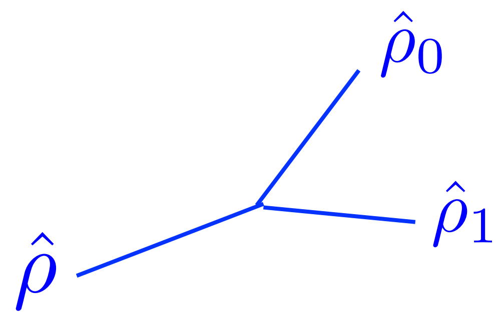
$$|\psi\rangle \to \hat{P}_\alpha|\psi\rangle/\sqrt{p_\alpha}$$

$$p_\alpha = \langle\psi|\hat{P}_\alpha|\psi\rangle \qquad \alpha = 0, 1$$

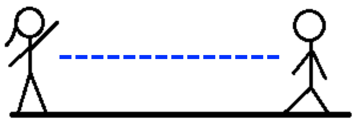
Quantum trajectories:
A monitored circuit

Measurements via density matrices

$$\hat{\rho} \to \hat{\rho}_\alpha = \hat{P}_\alpha \hat{\rho} \hat{P}_\alpha / p_\alpha \qquad \alpha = 0, 1$$



Entanglement and measurements



Alice and Bob share a Bell pair

Alice measures Z-comp of her spin

Before measurement

$$S_A = S_B = \log 2$$

After measurement

$$\overline{S_A} = \sum_{\alpha=0,1} p_\alpha S_A^\alpha = 0$$

$$\overline{S_A^{after}} \leq S_A^{before}$$

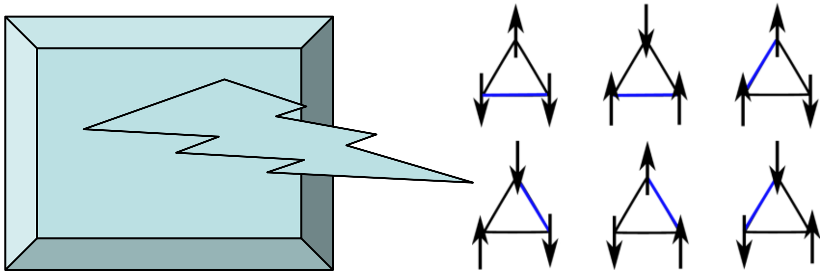
(Local) Measurement induce disentanglement

w/ measurements have an **Open system**

Open Quantum Systems

Two classes:

System coupled to a **bath (environment)**

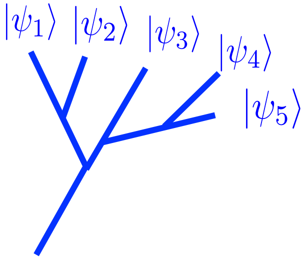


- Initial pure density matrix becomes mixed
- Environment “measures” system, but results lost
- **Decoherence**
- Dynamics of density matrix evolves w/ (e.g.) Lindblad equation

$$\partial_t \hat{\rho} = -i[\hat{\rho}, \hat{H}] + \hat{\mathcal{L}}_D(\hat{\rho})$$

System is **monitored** by an “observer”

- Initial pure state is measured and stays pure
- ”Observer” keeps track of measurements
- Wavefunction evolves as a pure state
- Dynamics described in terms of ensemble of (wavefunction) quantum trajectories



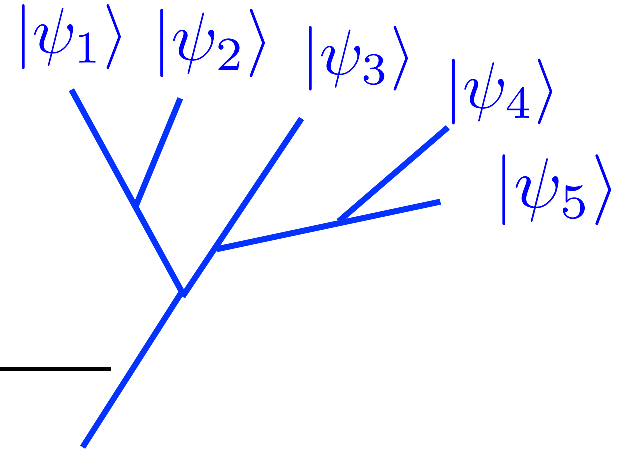
Ensemble of Quantum Trajectories

Decoherence: Trajectory averaged density matrix

$$\overline{\rho} = \sum_m p_m |\psi_m\rangle\langle\psi_m| \qquad \langle\hat{\mathcal{O}}\rangle = Tr(\overline{\rho}\hat{\mathcal{O}})$$

Average observables, linear in density matrix

Quantum effects (largely) washed out



Monitored Systems

Measurement conditioned density matrix, and reduced density matrix

$$\hat{\rho}_m = |\psi_m\rangle\langle\psi_m|$$

$$\hat{\rho}_A^m = Tr_B(\hat{\rho_m})$$

Entanglement entropy of wavefunction trajectories is non-linear in the density matrix

$$S_A^m = -Tr\hat{\rho}_A^m \log \hat{\rho}_A^m$$

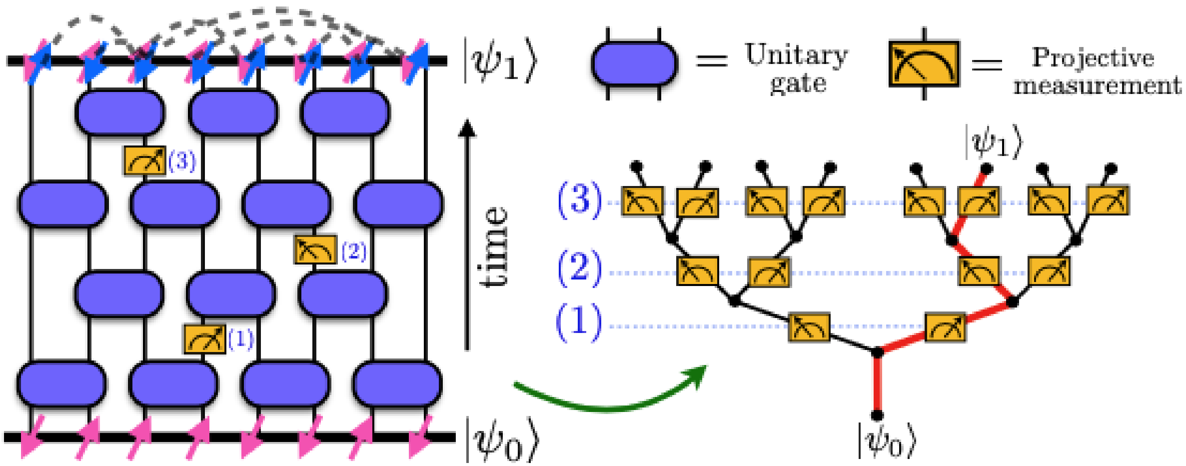
Average entropy over quantum trajectories $\overline{S}_A = \sum_m p_m S_A^m$

Monitored Quantum trajectories reveal entanglement phases/transitions

Extended systems w/ measurements (“monitored”)

“Hybrid Quantum Circuit”

w/ both unitary and measurement gates



- Unitary evolution induces entanglement growth
- QuBit Measurements induce disentanglement

Explore competition between unitary evolution and measurements

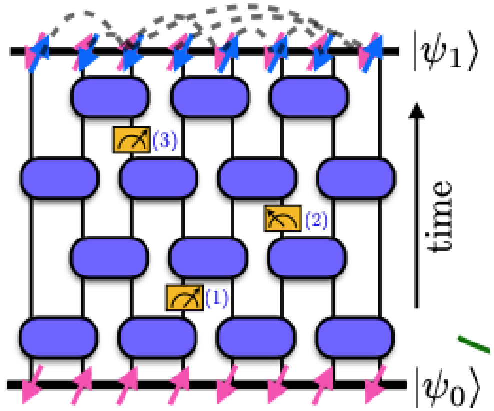
(by following wavefcn quantum trajectories)

- Li, Chen, MPAF (2018/2019)
- Skinner, Ruhman, Nahum (2018)
- Chan, Nandkishore, Pretko, Smith (2018)
- Choi, Bao, Qi, Altman (2019)
- Gullans, Huse (2019)
- Many more...

Non-unitary "Hybrid" Quantum Circuit

- Randomly chosen 2-Qubit unitaries
- Single qubit measurements made with probability, p
- Run circuit dynamics to long times
- Compute pure-state bipartite entanglement entropy

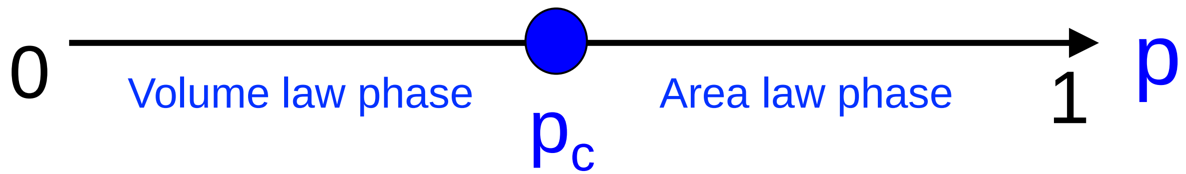
$$S_A(t) = -Tr_A(\hat{\rho}_A \log \hat{\rho}_A)$$

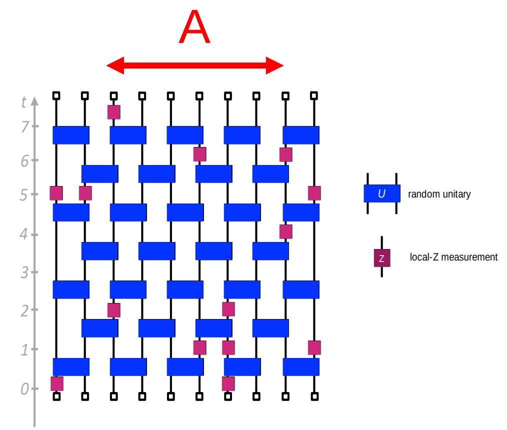


Phase Diagram?

Single parameter $p \in [0, 1]$

- p=0; No measurement, Volume law entanglement $S_A \sim |A|$
- p=1; Measure every Qubit, no entanglement (area law) $S_A \sim |\partial A|$
- Transition at p=p_c ?





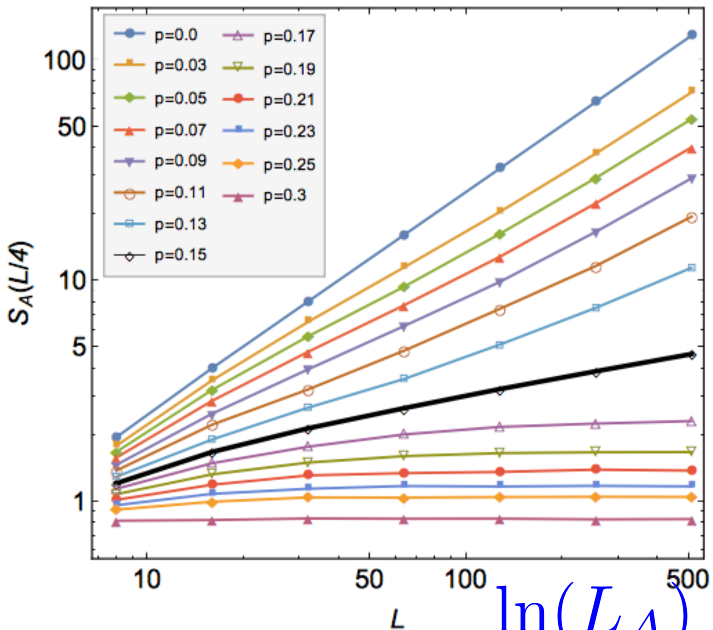
Entanglement transition in Monitored Dynamics

Entanglement Transition in Hybrid Clifford circuit

w/ pure initial state, long-time steady-state of Clifford circuit, 500 qubits

Li, Chen, MPAF (2018)
Skinner, Ruhman, Nahum (2018)

$\ln(S_A)$

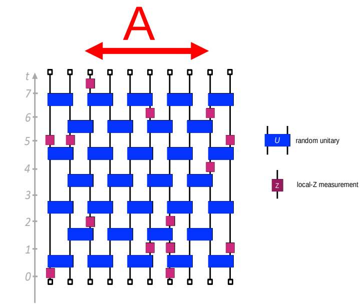


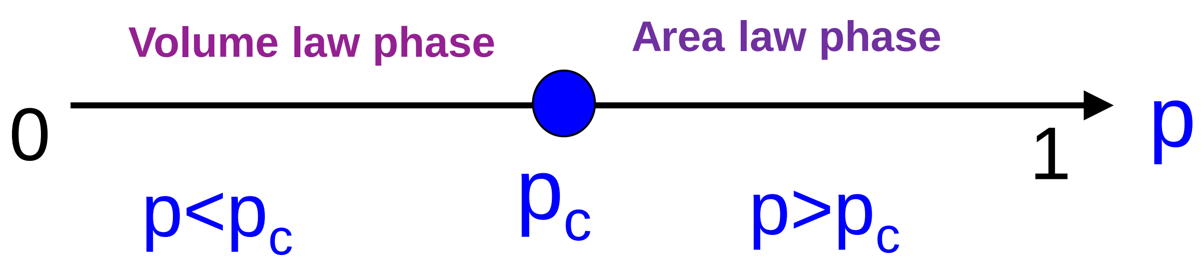
$\ln(L_A)$

Volume law entanglement

Increasing measurement rate

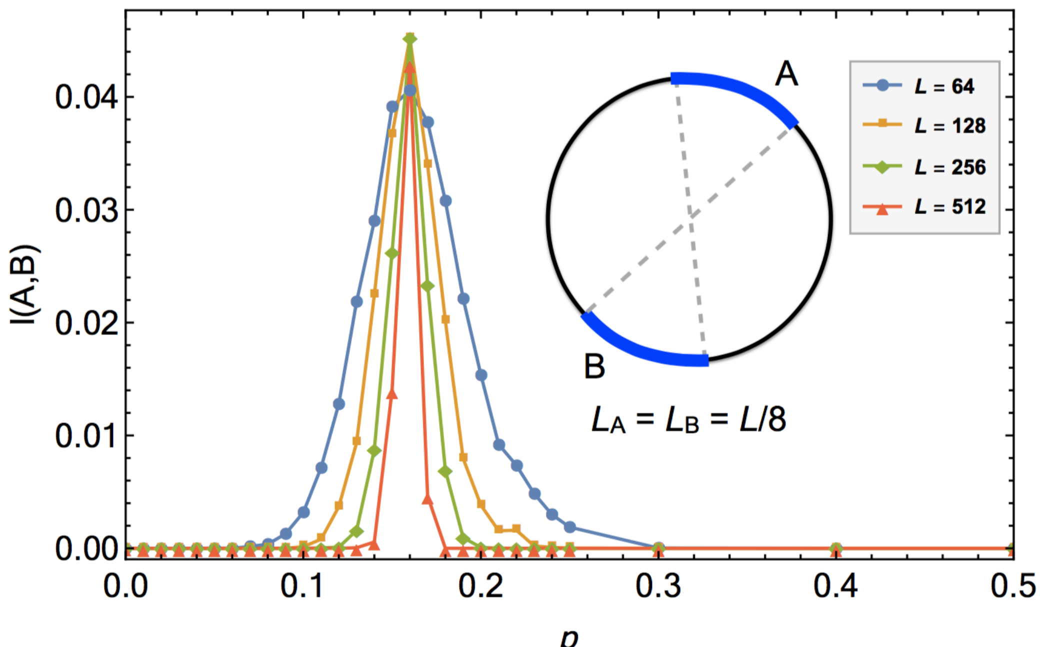
Area law entanglement





Mutual Information: Locates transition

$$I_{AB} = S_A + S_B - S_{AB}$$

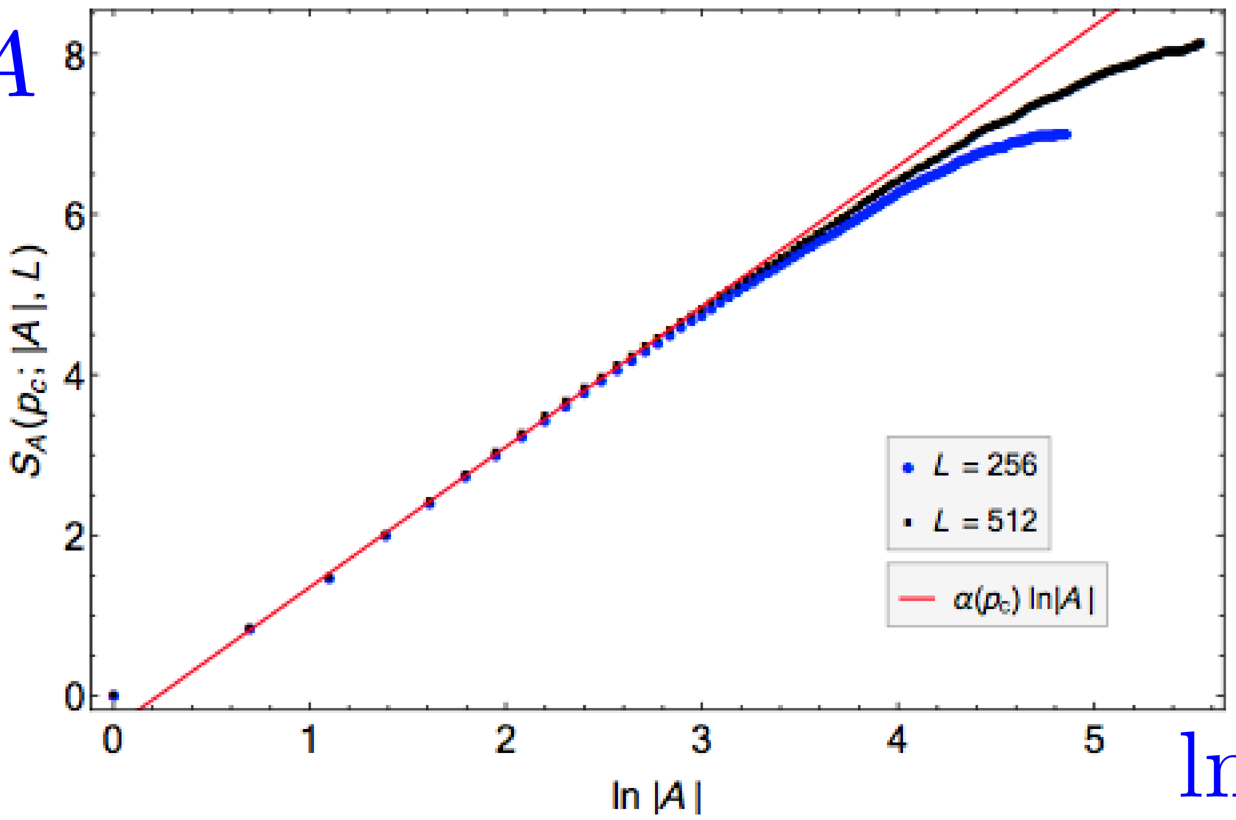


$$I_{AB}(L \to \infty) = \begin{cases} 0; & p \neq p_c \\ \text{const}; & p = p_c \end{cases}$$

Log Scaling at Criticality ($p=p_c$)

$$S_A(L_A) = \alpha_c \log(L_A) \qquad \alpha_c \approx 1.6$$

S_A



$\ln(L_A)$

Resembles 1+1 CFT ground state

Conformal Symmetry at criticality ($p=p_c$)

Li, Chen, Ludwig, MPAF (2020)

$$I_{AB} = S_A + S_B - S_{AB}$$

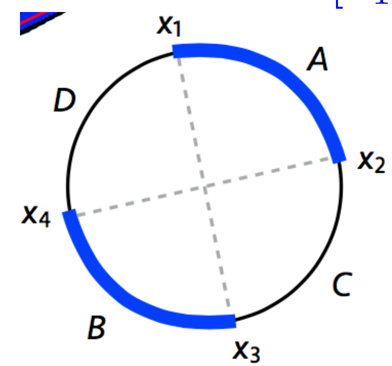
If have underlying conformal field theory, then mutual information depends only on the cross ratio

$$I_{AB} = f(\eta)$$

$$\eta \equiv \frac{x_{12}x_{34}}{x_{13}x_{24}}$$

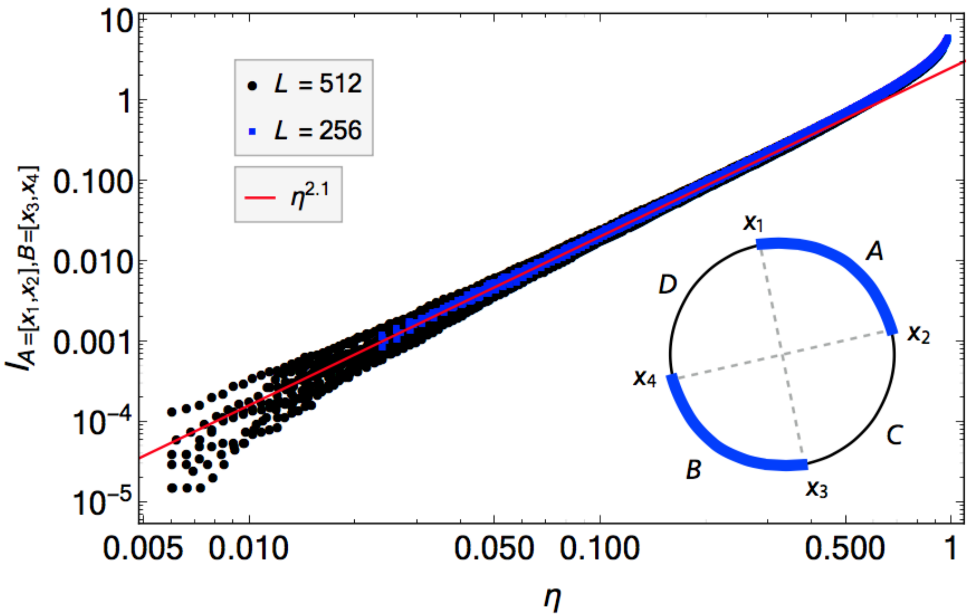
$$x_{ij} = \frac{L}{\pi}\sin\left(\frac{\pi}{L}|x_i - x_j|\right)$$

$A = [x_1, x_2]$



$B = [x_3, x_4]$

I_{AB}



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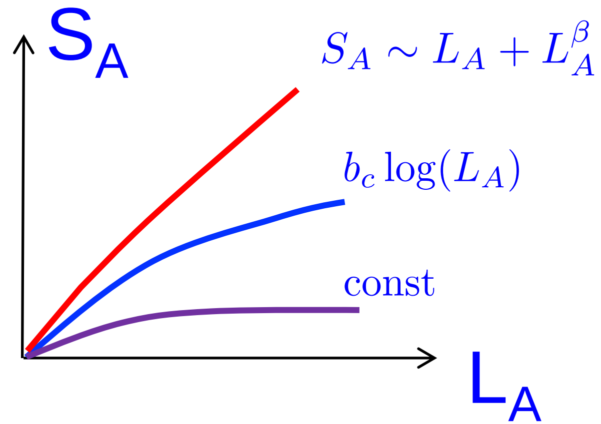
Nature of the volume law phase?

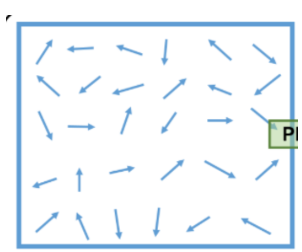
"Background" in volume law phase
(Clifford numerics)

$$S_A \sim L_A + L_A^{\beta}$$

$$\beta \approx 1/3$$



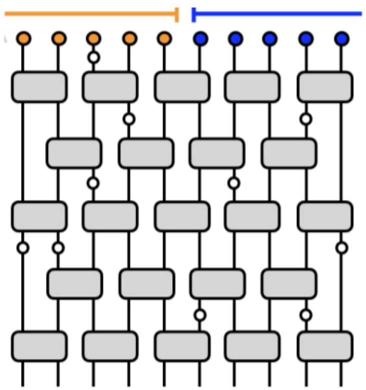
"Understand" via mapping to stat mech model

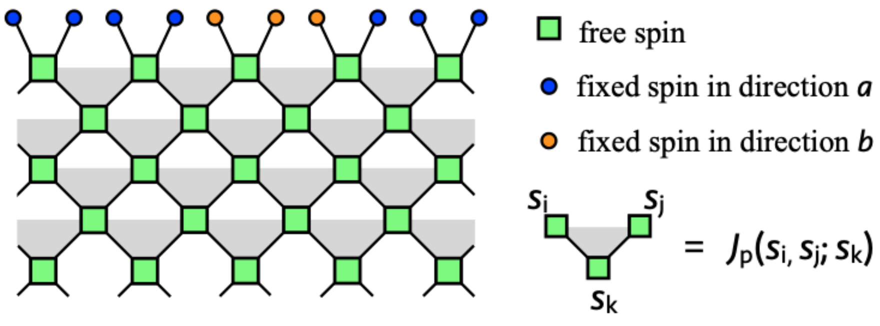


Mapping to Stat Mech (spin) model

Random Haar circuit w/ measurements mapped to 2d "Generalized Potts" model (in space-time)

T. Zhou, A. Nahum (2018)
Jian, You, Vasseur, Ludwig (2019)
Bao, Choi, Altman (2019)



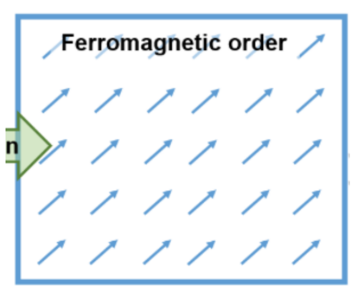


Phases in Stat mech model

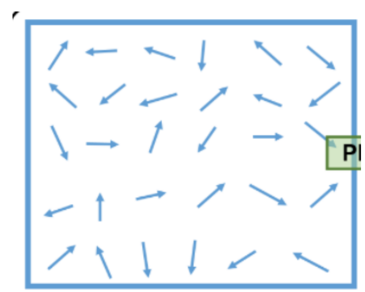
0 — p_c — 1 → p

Volume law phase | Area law phase

"Ordered" phase



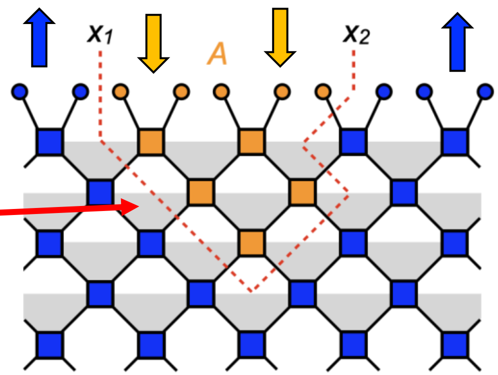
"Paramagnetic" phase



Entanglement entropy from Stat Mech model

$S_A = F_A =$ free energy cost for changing boundary conditions in region A

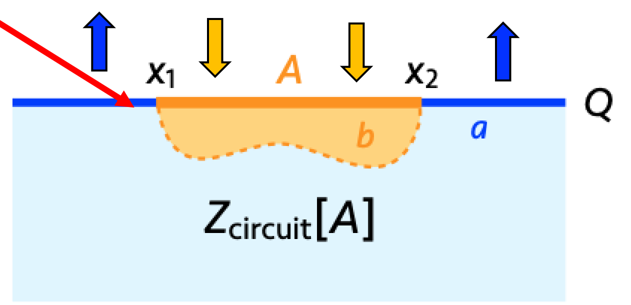
Volume law (ordered) phase:



Expect an “entanglement domain wall”

Surface tension gives

$$S_A = F_A \approx \sigma L_A$$



Area law (paramagnetic) phase: $S_A = F_A = O(1)$

Domain walls have proliferated (zero surface tension)

Fluctuations of Entanglement domain wall

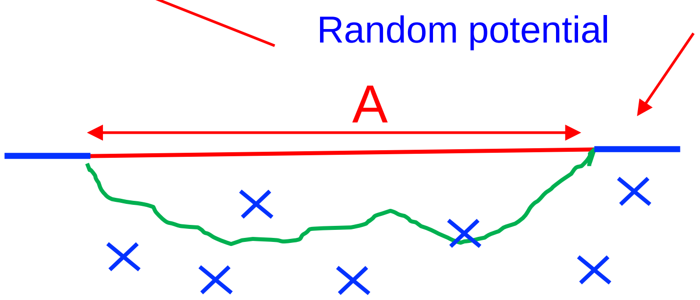
Stat mech model requires a "replica limit": $m \to 0$

m=1 spin model is "clean" Ising model
m=2,3,... entanglement domain wall splits into m domain walls with an attractive interaction

Map to Directed Polymer in a Random Environment (DPRE)

M. Kardar, Nucl. Phys. B (1987)

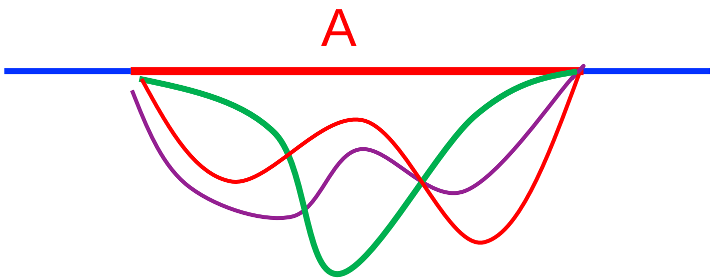
$$Z_{DP}(L_A) = e^{-\sigma L_A} \int Dy(x) e^{-\sigma \int_0^{L_A} [(\partial_x y)^2 + V(x,y)]}$$



Average free energy over disorder, using replica trick

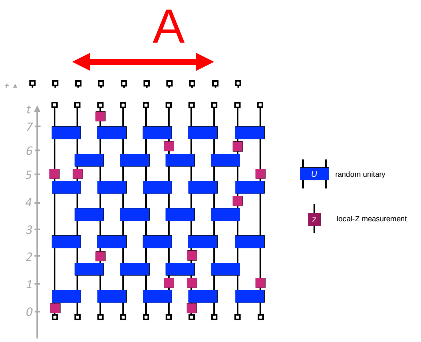
$$\langle F_A \rangle = -\langle \ln Z_{DP} \rangle = \lim_{m \to 0} \frac{1}{m} [\langle Z_{DP}^m \rangle - 1]$$

Average free energy is m-directed polymers w/ an attractive interaction, in replica limit,
$m \to 0$

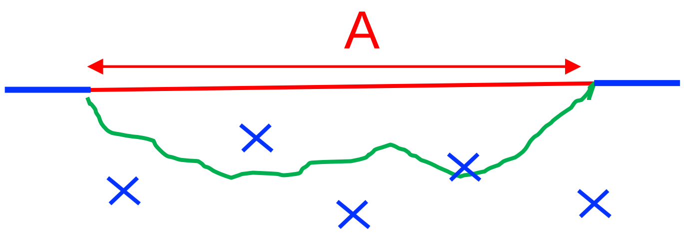


Entanglement entropy in volume law phase of hybrid circuit given by free energy of DPRE

Li, Vijay, MPAF (2021)



Entanglement in random hybrid circuit



DPRE Free energy

Average over (Haar) unitary gates

Replica trick

$$\langle S_A \rangle \approx \langle F_A \rangle$$

Universal critical exponents for DPRE

Subdominant free energy corrections;

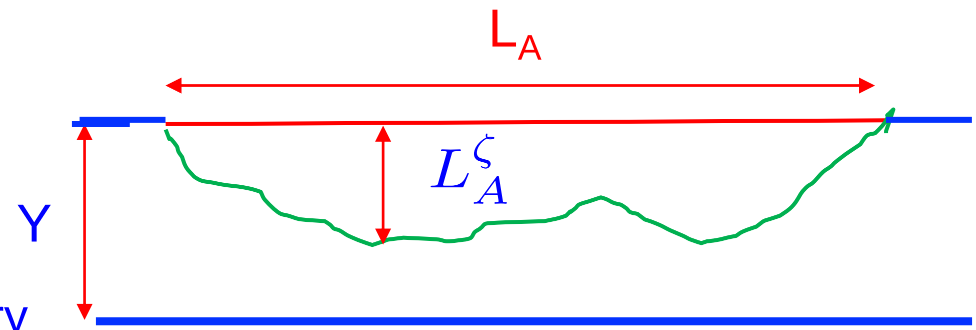
$$F_A^{sub} = 2\langle F_A \rangle - \langle F_{2A} \rangle = bL_A^{\beta}$$

$$\beta = 1/3 \qquad \zeta = 2/3$$

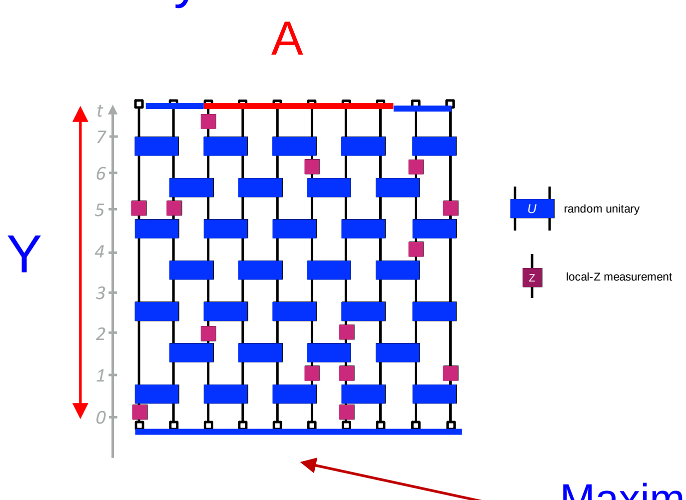
Finite size scaling in confined geometry

$$F_A^{sub}(Y) = L_A^{\beta} \Phi(Y L_A^{-\zeta})$$

Wandering Exponent of DPRE



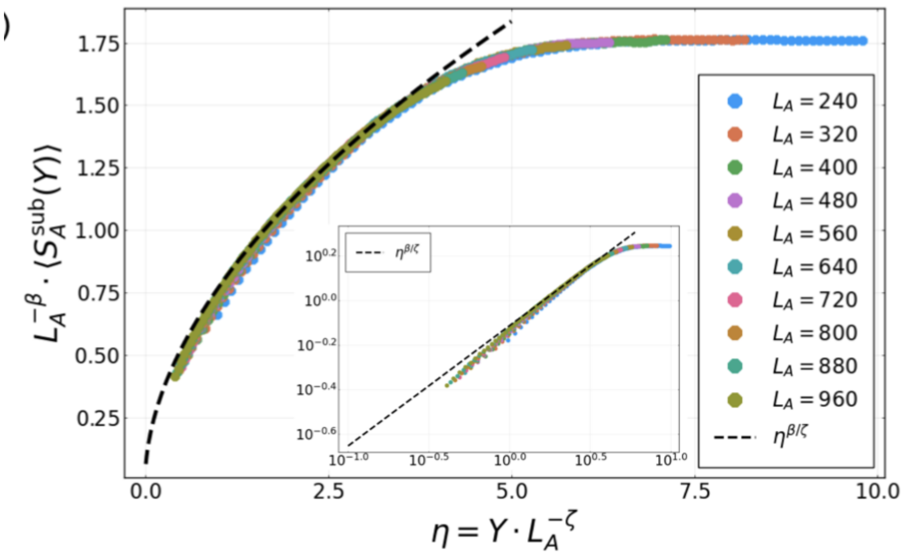
Clifford hybrid circuit in confined geometry



Maximally mixed initial state $$\hat{\rho}_0 = \frac{1}{2^L} \hat{1}$$

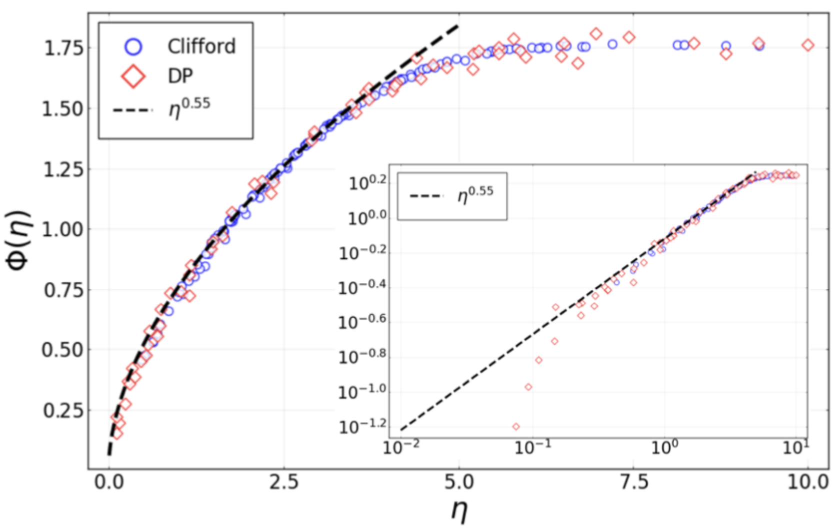
Clifford hybrid circuit (volume law) versus DPRE

Sub-dominant Entanglement-entropy in random hybrid Clifford circuit



$$S_A^{sub}(Y) = L_A^{\beta}\Phi(Y L_A^{-\zeta})$$

Sub-dominant free-energy for DPRE



$$F_A^{sub}(Y) = L_A^{\beta}\Phi(Y L_A^{-\zeta})$$

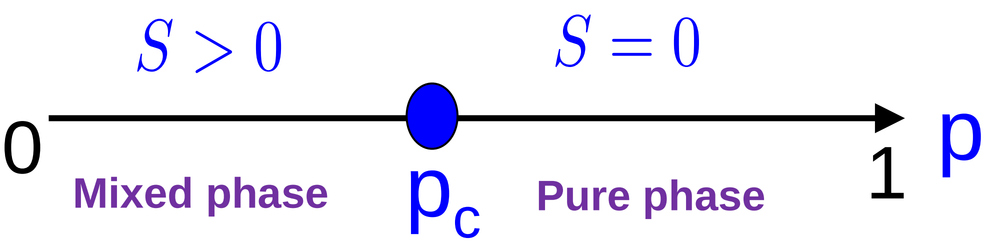
Purification Transition in hybrid Clifford circuit

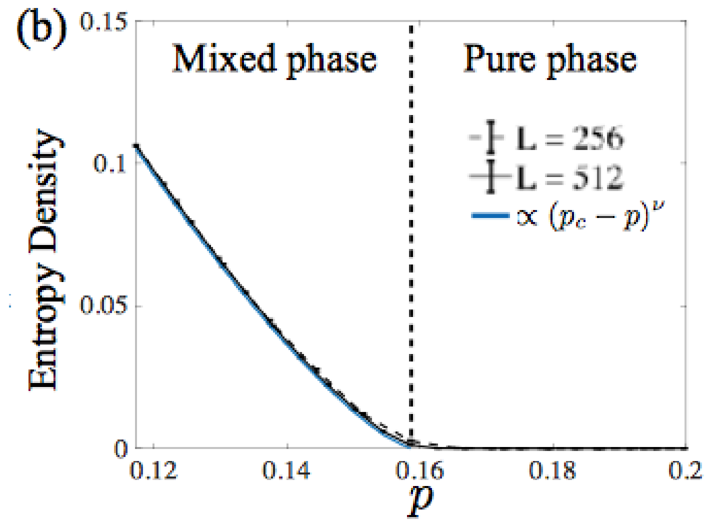
Gullans, Huse (2019)

Start dynamics in maximally mixed state, run for $t \approx cL$ $\quad \hat{\rho}(t=0) = \frac{1}{2^L}\hat{1}$

Compute **thermal entropy** at time t

$$S_t = -Tr(\rho_t \ln \rho_t)$$





Purification transition = entanglement transition

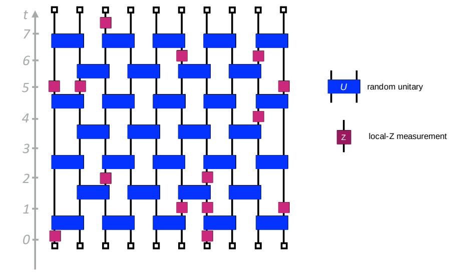
mixed ρ_0 $\qquad$ pure $\rho_0 = |\psi_0\rangle\langle\psi_0|$

Volume-law (mixed) phase as QECC

Gullans, Huse 2019
Choi, Bao, Qi, Altman 2019

Volume law phase is encoder of quantum info
Unitaries scramble (and hide) quantum information from measurements

Start w/ maximally mixed state
Measurements tend to purify, but volume law phase stays mixed



$$\hat{\rho}(t=0) = \frac{1}{2^L}\hat{1}$$

Dynamically generate (L,k,d) stabilizer QECC

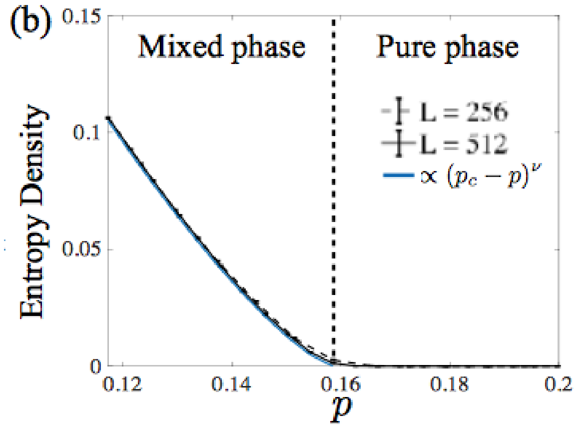
Li, MPAF 2021

L=number of physics qubits
k=number of logical qubits

Code rate k/L is finite in volume law phase

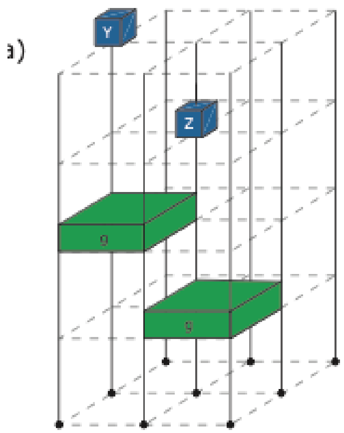
Code distance (shortest logical operator) $d \sim L^{\beta}$

Decoding? Very challenging



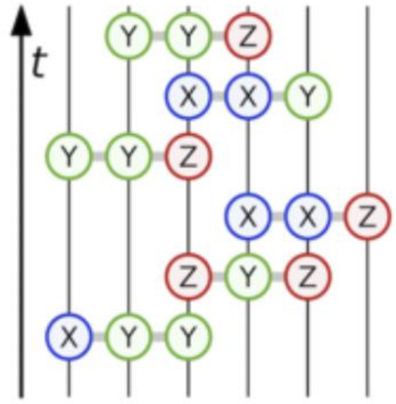
“Enriched” phases in “hybrid” circuits

Stabilize 2d Toric code



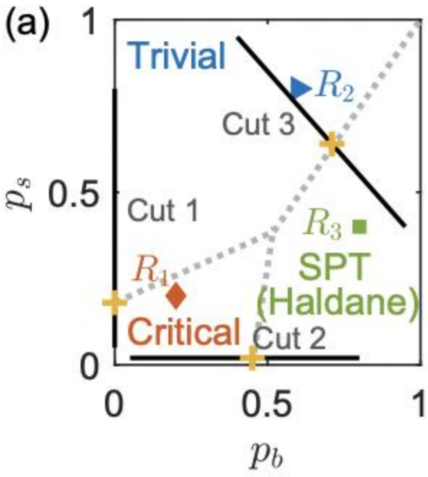
Lavasani, Alavirad, Barkeshli ’20

Measurement-only models



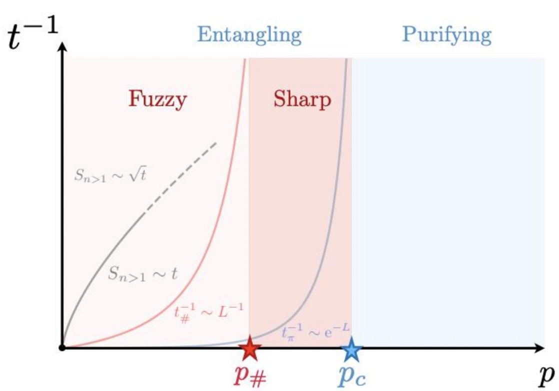
Ippoliti, Gullans, *et. al.* ’20

Symmetry enriched phases



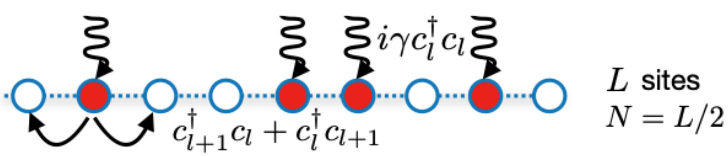
Bao, Choi, Altman ’21

U(1) Symmetric dynamics



Agrawal *et. al.* ’21

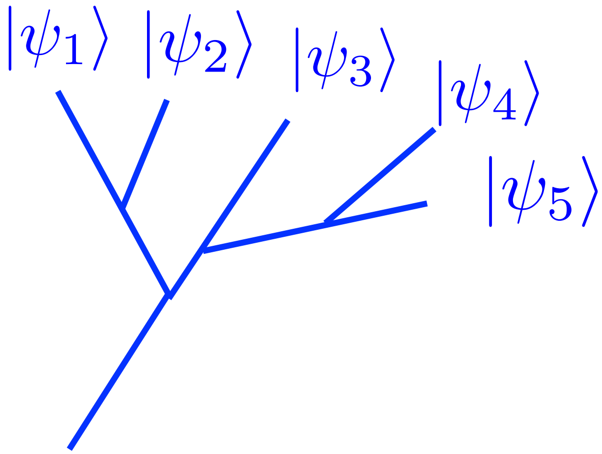
Entanglement transition in monitored free fermion chain



Alberton, Buchold, Diehl ‘21

Experimental Access?

Quantum trajectories reveal phases/transitions



But, averaging over quantum trajectories washes out all effects

$$\hat{\rho}_{mixed} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

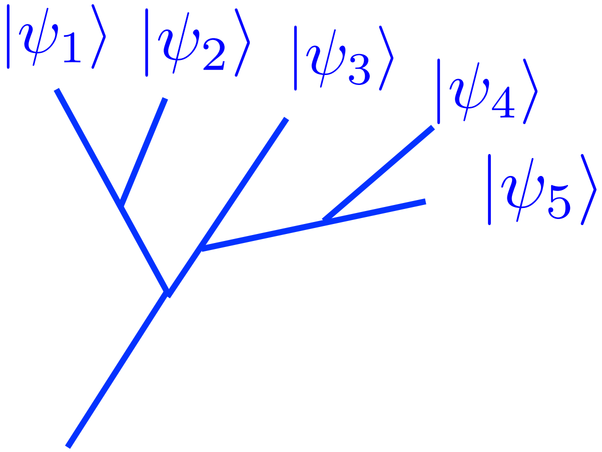
- Multiple copies of **same pure state** $|\psi\rangle$ are required to measure the density matrix (via tomography) and extract the entanglement entropy
- Post selection on O(Lt) measurement outcomes to get copies? Must choose among 2^{Lt} possible (random) outcomes, to get each copy

Overcoming “Post-selection”?

- Accessing via local probe, plus using measurement outcomes to “decode” via **active feedback** (cf active QECC)

  Gullans, Huse ’19
  Noel, Niroula, Zhu *et. al. Ion trap experiment* ’20

  Dehgani et. al. arXiv:2204.10904
  Neural network decoders for measurement induced phase transitions



- Space-time duals of unitary dynamics, which looks like unitary plus measurements

  Ippoliti, Rakovszky, Khemani ’21
  Lu, Grover ’21

- Employ Clifford circuits: Measurement outcomes *can* be forced, using error correction to “undo” a wrong measurement outcome, then try again

- Brute force (for “smaller” circuits)!!

  A. Minnich et al

Recent experiment arXiv:2203.04388

Experimental Realization of a Measurement-Induced Entanglement Phase Transition on a Superconducting Quantum Processor

Jin Ming Koh,[1] Shi-Ning Sun,[2] Mario Motta,[3] and Austin J. Minnich[2,*]

[1] *Division of Physics, Mathematics and Astronomy, California Institute of Technology, Pasadena, California 91125, USA*

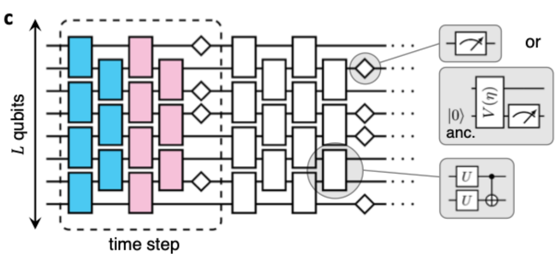
[2] *Division of Engineering and Applied Science, California Institute of Technology, Pasadena, CA 91125, USA*

[3] *IBM Quantum, IBM Research Almaden, San Jose, CA 95120, USA*

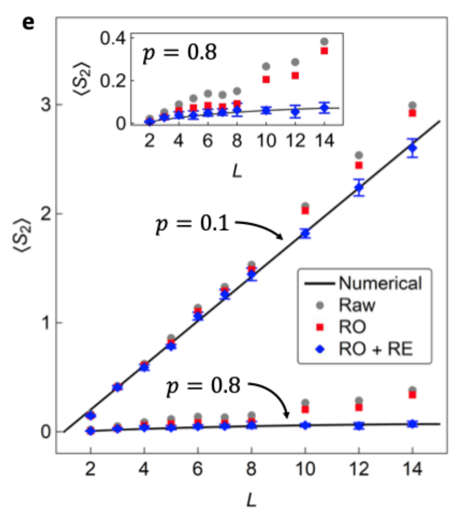
Circumvented the post-selection challenge by brute force

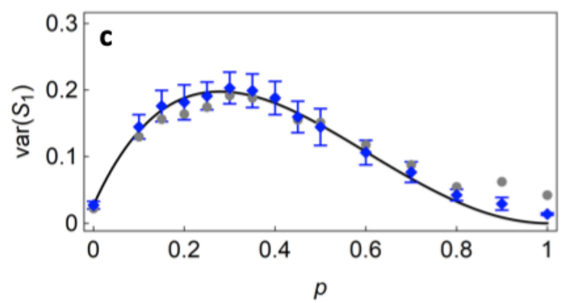
~ 5200 hardware device-hours over multiple IBM quantum processors

Measured density matrix via tomography



2 qubit gates: CX plus random single qubit rotations



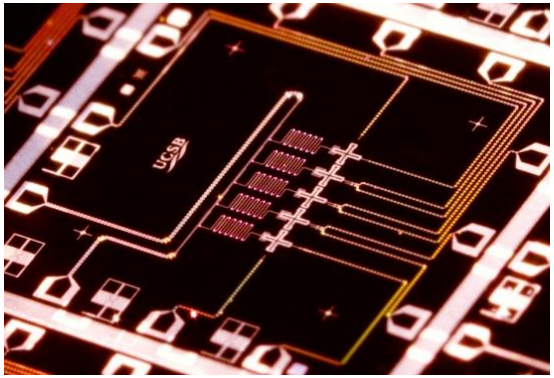


L=4,5

New Opportunities in NISQ era

Quantum Many-body theory

Quantum information theory



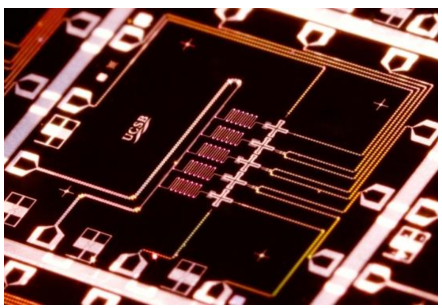
NISQ computers

Non-equilibrium dynamics of
monitored quantum systems

Quantum Interactive dynamics

Measure
(monitor)

Measurement results



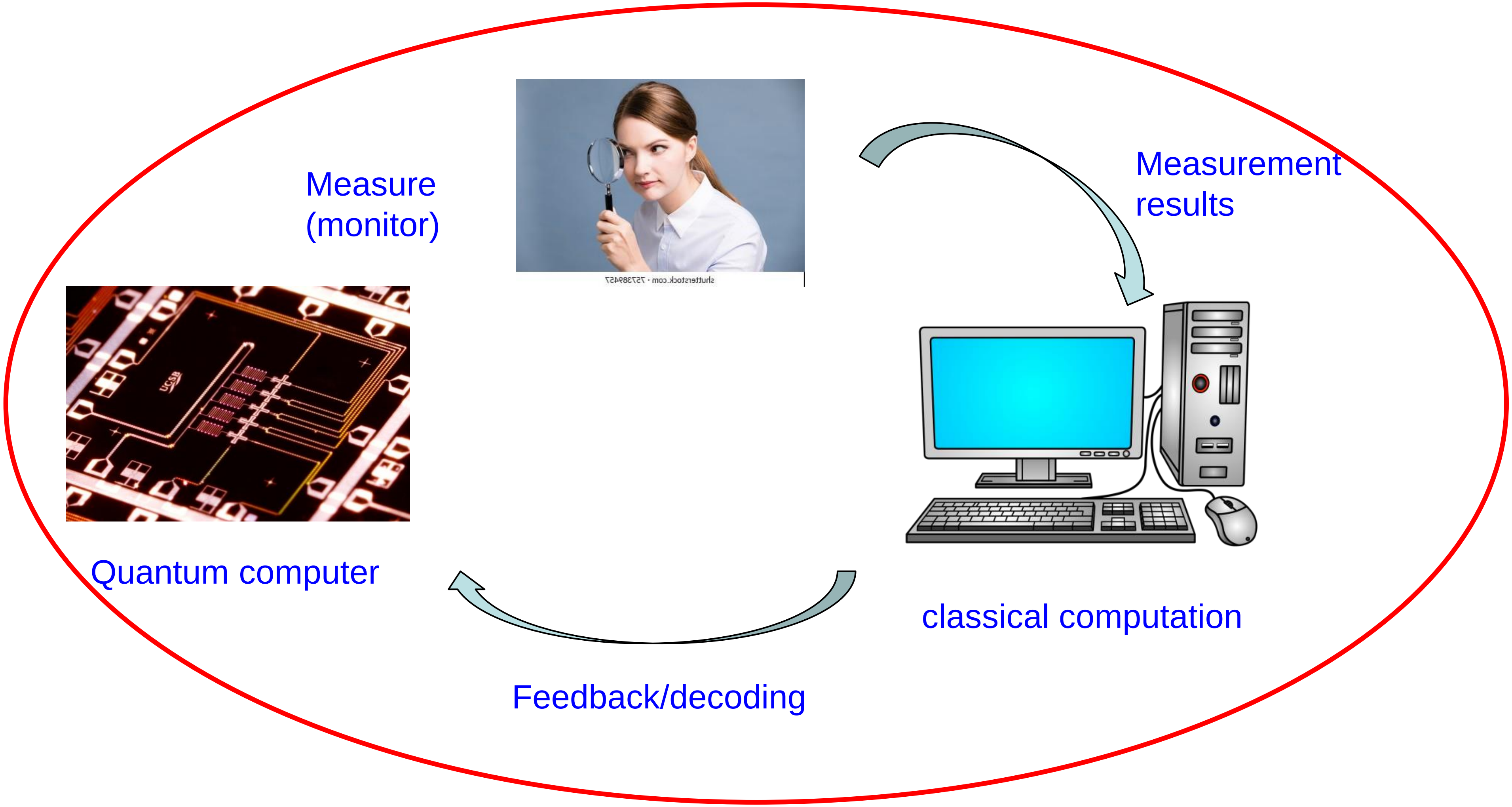
Quantum computer

classical computation

Feedback/decoding

Novel Quantum Dynamical Phases?

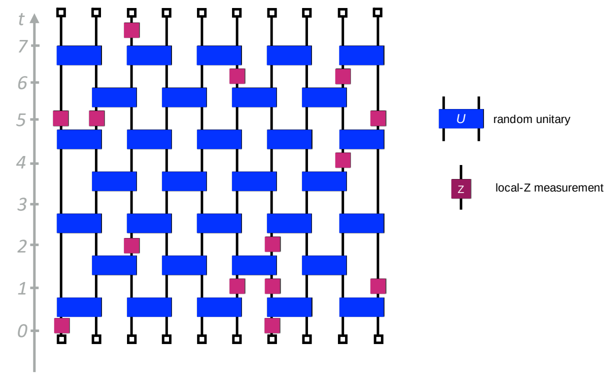
(beyond active error correction…)



Summary: Entanglement Transitions

Quantum Entanglement Transitions
in "monitored" systems:

Competition between unitary induced entanglement and measurement induced disentanglement

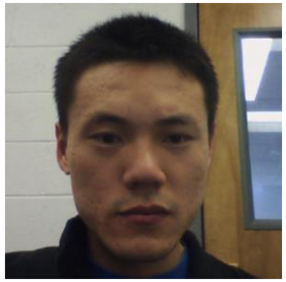


Many Open Questions being explored:

- CFT for 1d entanglement transition?
- U(1) symmetry: Stat mech model for entanglement transition?
- Decoding the Measured-induced phase transition?
  ("beating" post-selection)
- Novel Quantum Dynamical phases in Monitored systems w/ active feedback?

Thanks to my collaborators!

Yaodong Li



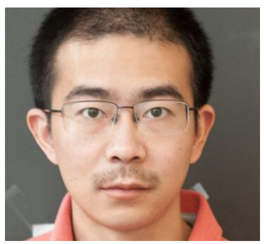
Xiao Chen

Sagar Vijay

Andreas Ludwig

Andrew Lucas

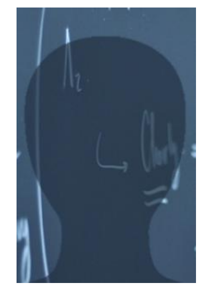
Tim Hsieh



Tianci Zhou

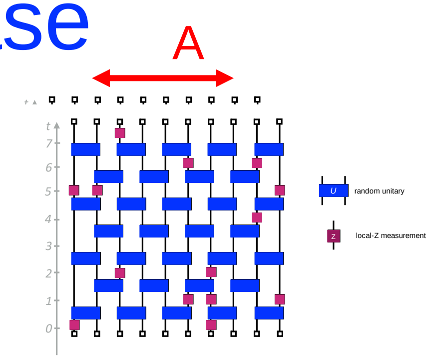
Zhi-Chen Yang

Romain Vasseur



Shengqi Sang

Stability of the Volume law phase



A layer of unitaries increases S_A only at the boundary,

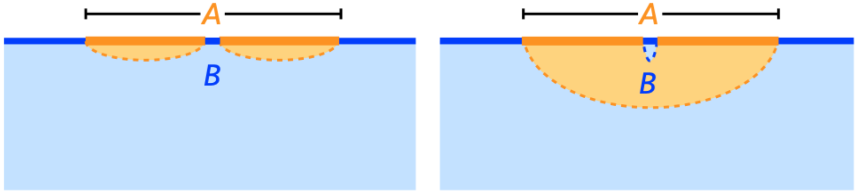
$$\Delta S_A^U \sim |\partial A| \sim O(1)$$

Naively, a layer of measurements decreases S_A throughout region A;

$$-\Delta S_A^m \sim O(L_A)$$

Suggests that measurements swamp unitaries, destroying volume law phase

Chan, Nandkishore, Pretko, Smith (2018)



But, compute decrease in S_A due to a single measurement, a distance x from boundary

Due to background entropy in volume law phase – Clifford and DPRE both give:

$$-\delta S_A^m(x) \sim x^{-\alpha}; \quad \alpha = 1.25$$

Total drop in entropy due to a layer of measurements:

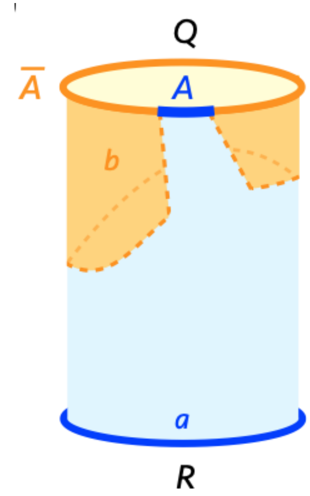
$$-\Delta S_A^m \approx \int_1^{L_A} dx \frac{1}{x^{1.25}} \sim O(1)$$

Volume-law phase is (self-consistently) stable to measurements

Code distance from "decoupling"

Y. Li, MPAF (2020)

$Q = A \cup \bar{A}$



Mutual information between sub-system A and reference qubits (R)

$$I_{A,R} = S_A + S_R - S_{AR}$$

Theorem: $I_{A,R} =$ # of logical operators that can be localized on sub-system A

Corollary:

$I_{A,R} = 0$ implies $|A| < d_{cont}$

d_{cont} = "Contiguous" Code distance (shortest logical operator) $d_{cont} \geq d$

Sub-system A decoupled from environment, and environment gives no information on sub-system A (protection from errors)

Code distance for Clifford circuit

Li, MPAF (2020)
M. Ippoliti et. al. (2020)

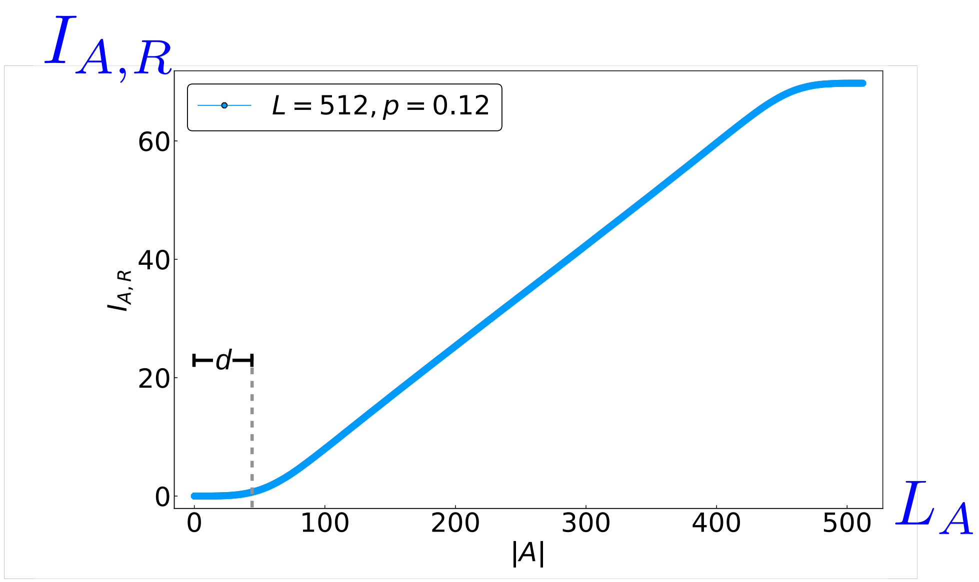
Mutual information numerically
for Clifford circuit

Extract code distance from condition;

$$I_{A,R} \approx 0; \quad |A| < d_{cont}$$

Code distance scaling w/ L

$$d_{cont} \sim L^{0.38}$$



Clifford numerics

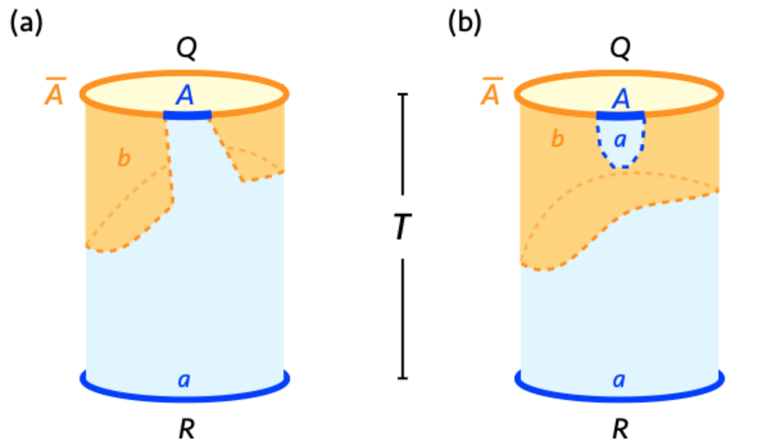
Code distance from entanglement domain walls

2 competing domain wall configs

(a) dominates for $|A| > d$

(b) dominates for $|A| < d$

Equating two gives d



$I_{A,R} > 0$ $\quad$ $I_{A,R} = 0$

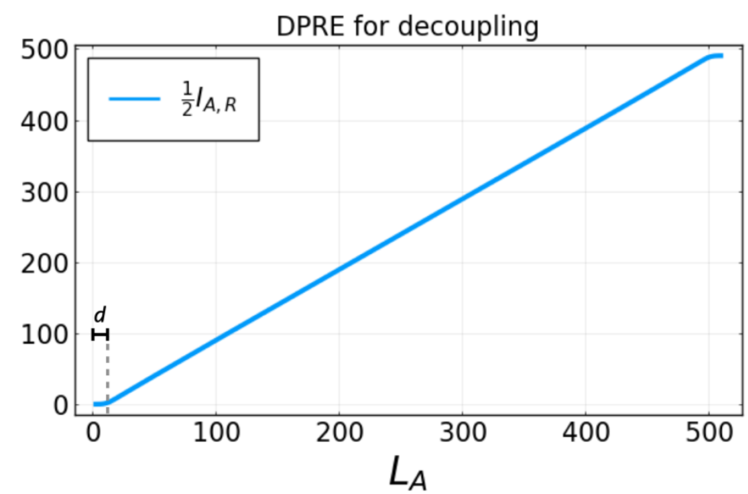
DPRE Scaling with L

$d_{DPRE} \sim L^{\beta}; \quad \beta = 1/3$

Agrees w/ Clifford numerics

$d_{cont} \sim L^{0.38}$

$I_{A,R}$



DPRE numerics $\quad$ L_A