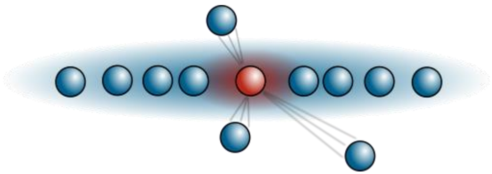
PHYSIK

TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Equilibrium-, non-equilibrium & steady-state properties of quantum impurities in 1D Bose gases



Michael Fleischhauer

University of Kaiserslautern

SFB/TR 185

Non-Equilibrium Emergence in Quantum Design, Ingelheim, 21.-23.06.2022

Thanks to:

Martin Will

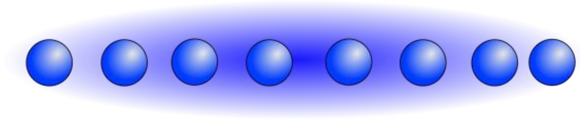
Jonas Jager

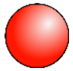
Gregori Astrakharchik

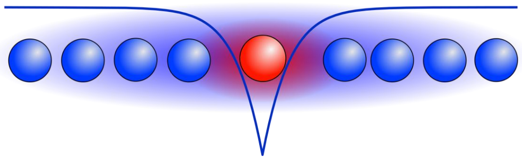
Jamir Marino

Deutsche Forschungsgemeinschaft DFG

Bose-polaron in strong coupling regime







$$\underbrace{\hat{H} = \int dx\, \hat{\phi}^\dagger(x)\Big[-\frac{\partial_x^2}{2m} + \frac{g_{\mathrm{BB}}}{2}\hat{\phi}^\dagger(x)\hat{\phi}(x)\Big]\hat{\phi}(x)}_{} + \underbrace{\frac{\hat{p}^2}{2M}}_{} + \underbrace{g_{\mathrm{IB}}\int dx\, \hat{\phi}^\dagger(x)\,\delta\big(x-\hat{r}\big)\,\hat{\phi}(x)}_{}$$

homogeneous Bose gas
2nd quantized

single impurity
1st quantized

interaction

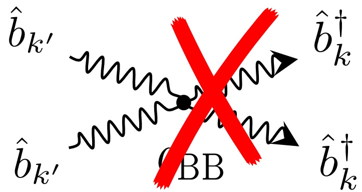
- weakly interacting Bose gas $\gamma \leq 1 \;\; (\xi\, n > 1)$
- heavy impurity $M \geq m$

- strong-coupling regime
  $g_{\mathrm{IB}} \gg g_{\mathrm{BB}}\, \xi\, n$

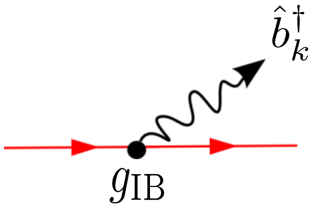
(extended) Fröhlich model

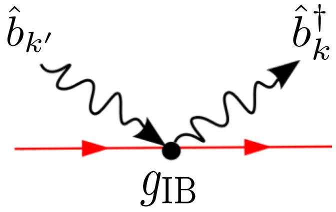
1. Bogoliubov theory of Bose gas w/o impurity

$$\hat{\phi}(x) = \sqrt{n} + \hat{\xi}(x)$$

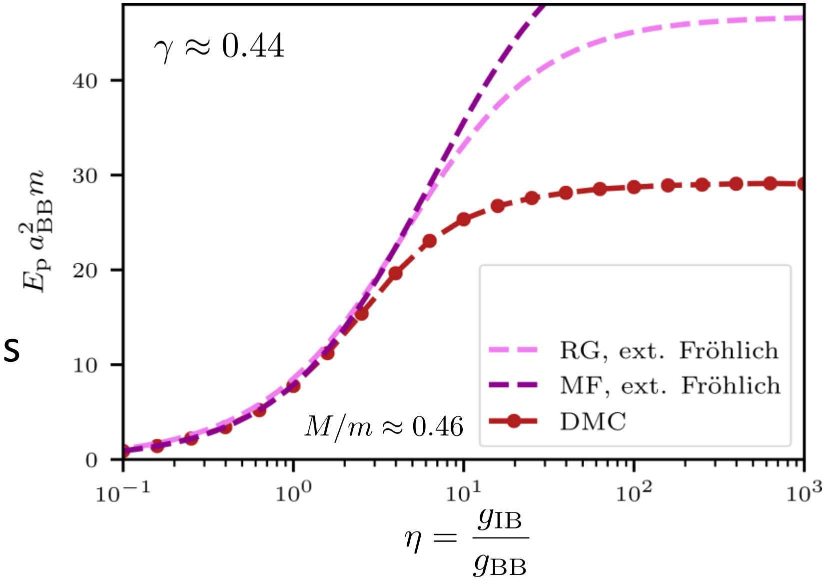


2. impurity: scattering & generation of phonons





polaron energy:

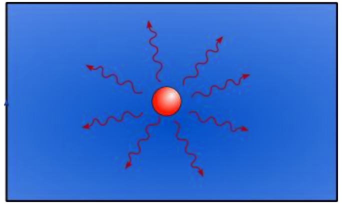


Grusdt *et al.* New J.Phys.19, 103035 (2017)
(*experiment:* Catani *et al.* Phys.Rev.A 85, 023623 (2012))

Bose-polaron in strong coupling regime

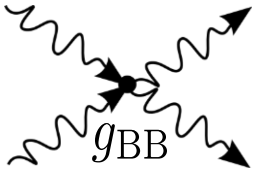
extended Fröhlich model

$$\hat{\phi}(x) = \sqrt{n} + \hat{\xi}(x)$$



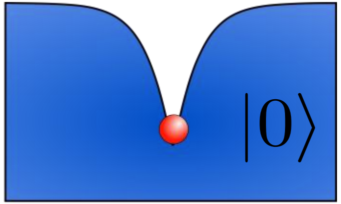
impurity generates phonons

$$\overline{n}_{\mathrm{ph}} \sim g_{\mathrm{IB}}^2 n$$



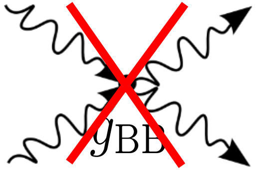
deformed condensate quantum corrections

$$\hat{\phi}(x) = \phi(x) + \hat{\xi}(x)$$

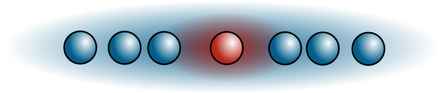


phonons in deformed condensate

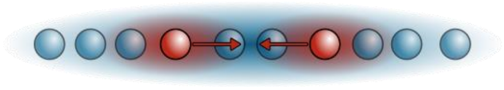
$$\overline{n}_{\mathrm{ph}} = 0$$



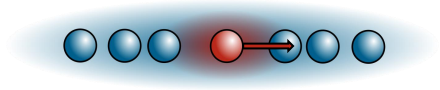
Spin models using Rydbergs



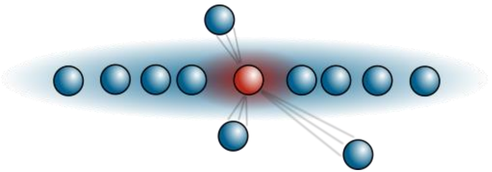
- Condensate deformation and modified phonons



- Polaron interactions and bi-polarons

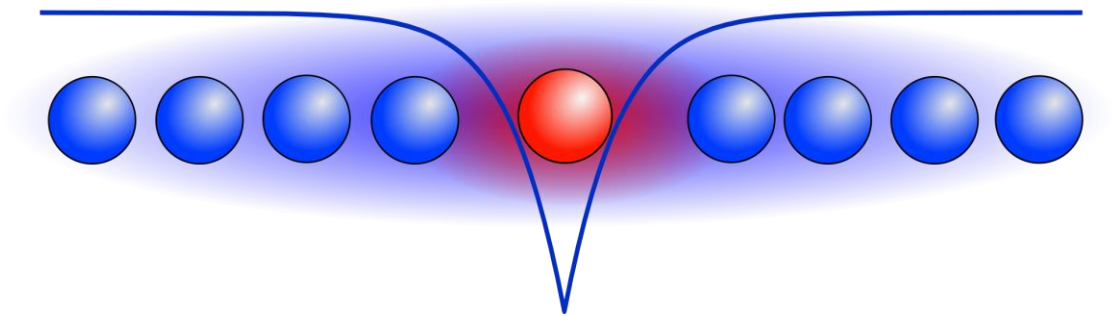


- Adiabatic & quench dynamics of a quantum impuritiy



- Control of superfluid flow by noisy impurities

Single Bose-polaron



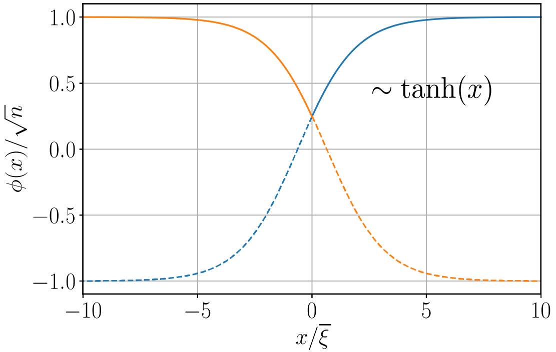
Jonas Jager, Ryan Barnett, Martin Will and Michael Fleischhauer
Phys. Rev. Res. **2**, 033142 (2020)

Bogoliubov with deformed phonons

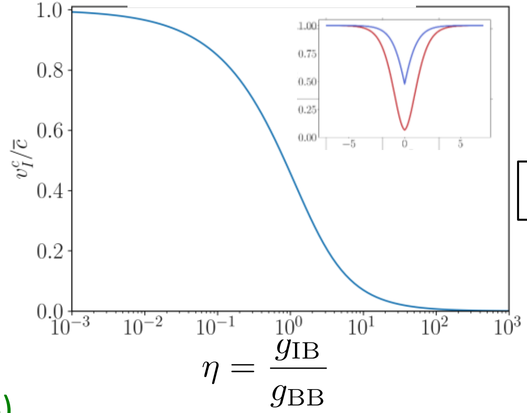
- **Mean-field:** $\hat{\phi}(x) = \phi(x)$ $\quad \tilde{m} = \frac{mM}{m+M}$

$$\left[-\frac{\partial_x^2}{2\tilde{m}} + \frac{i}{M}\left(p - P_{\mathrm{BEC}}\right)\partial_x + g\left|\phi(x)\right|^2 + g_{\mathrm{IB}}\delta(x) - \mu\right]\phi(x) = 0$$

$\Rightarrow$ solved by two cut grey solitons (2 solutions)

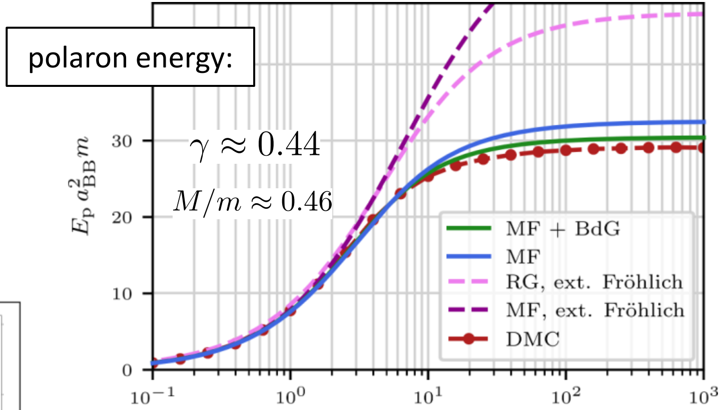


critical momentum

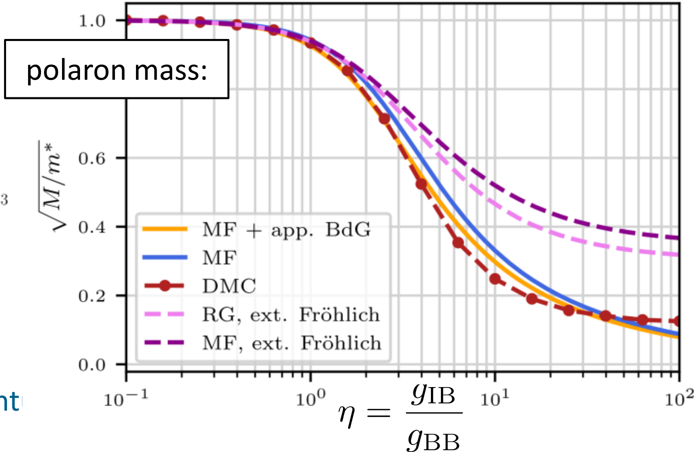


Hakim Phys.Rev.E **55**, 2835 (1996)

- **BdG:** $\hat{\phi}(x) = \phi(x) + \hat{\xi}(x)$

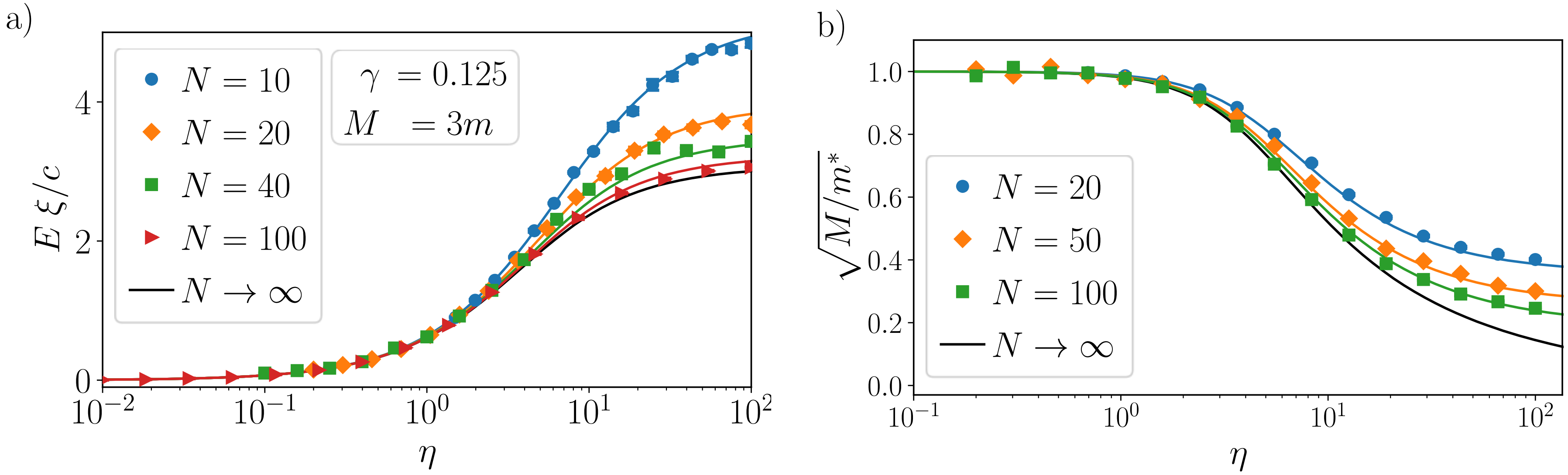


$p = 0$

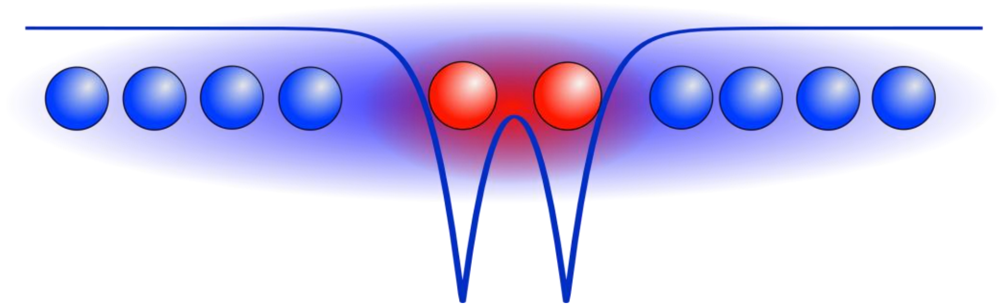


Bogoliubov with deformed phonons

- Finite-size effects:



Polaron interaction & Bi-polaron



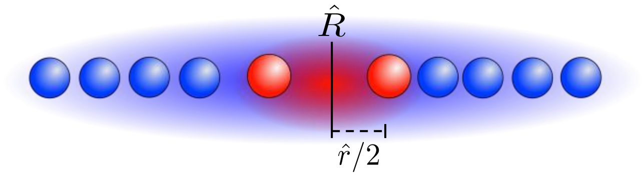
Martin Will, Grigori Astrakharchik and Michael Fleischhauer
Phys. Rev. Lett. **127**, 103401 (2021)

Polaron interaction & Bi-polaron

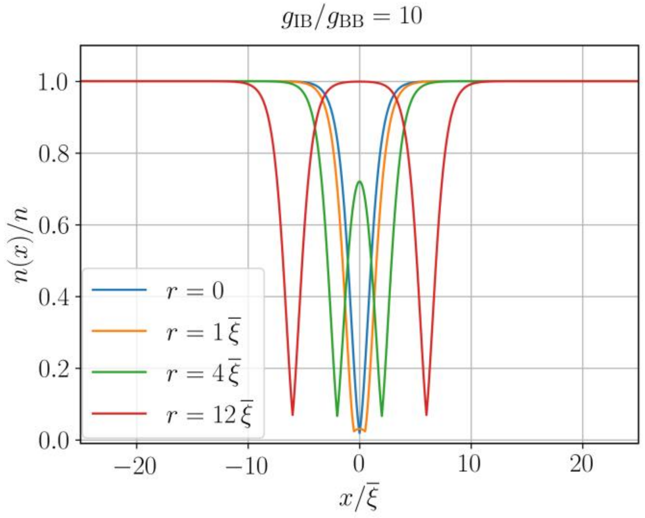
relative impurity coordinates: $\hat{p},\ \hat{r}$

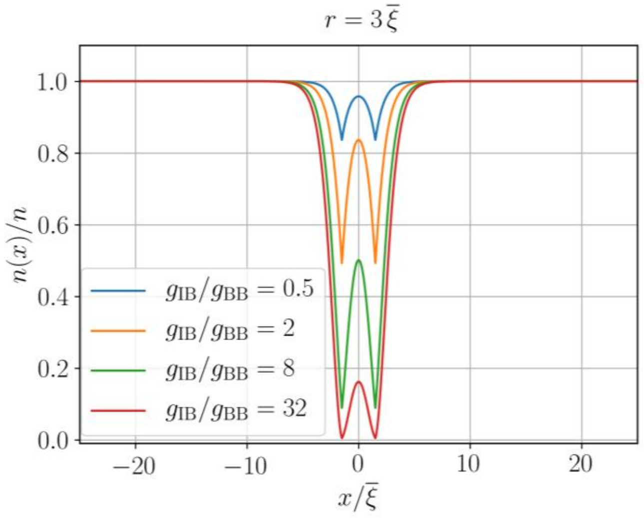
center of mass: $\hat{P},\ \hat{R}$

- Lee-Low-Pines removes $\hat{P}$
- Born-Oppenheimer: $M \gg m$

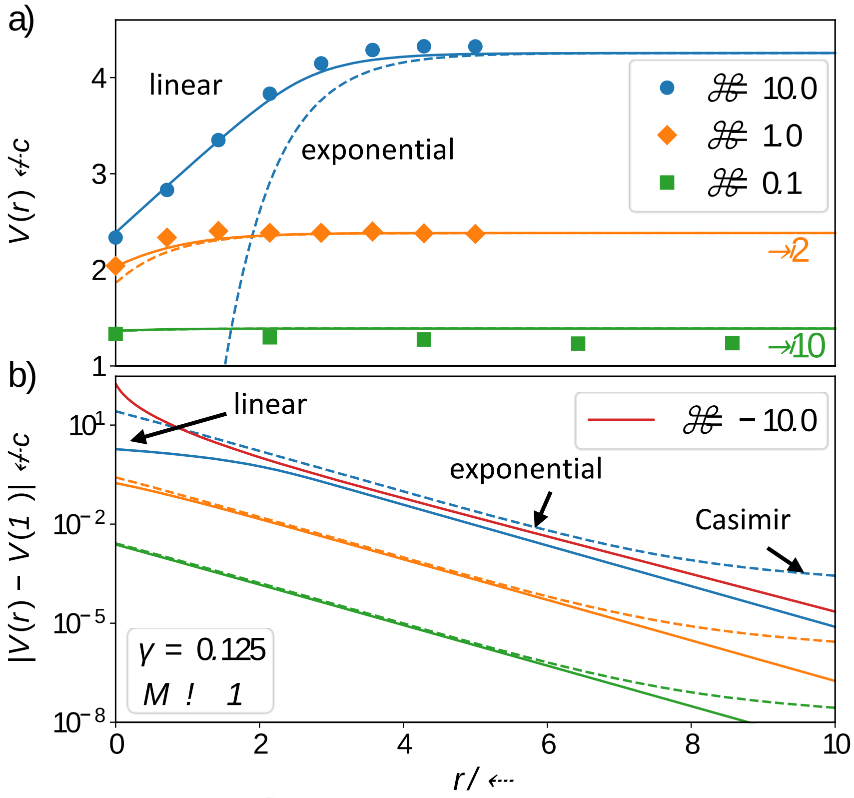


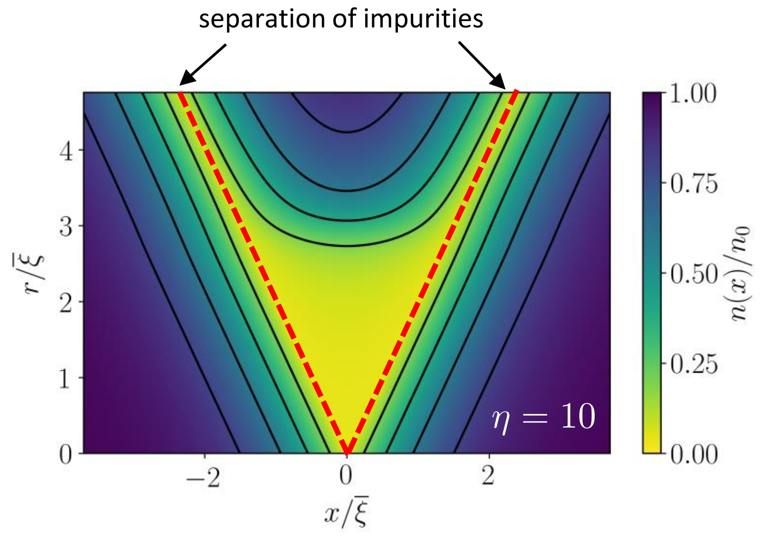
Mean-field for $P = 0$:





interaction potential $M \gg m$





density of condensate

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correction to Born-Oppenheimer

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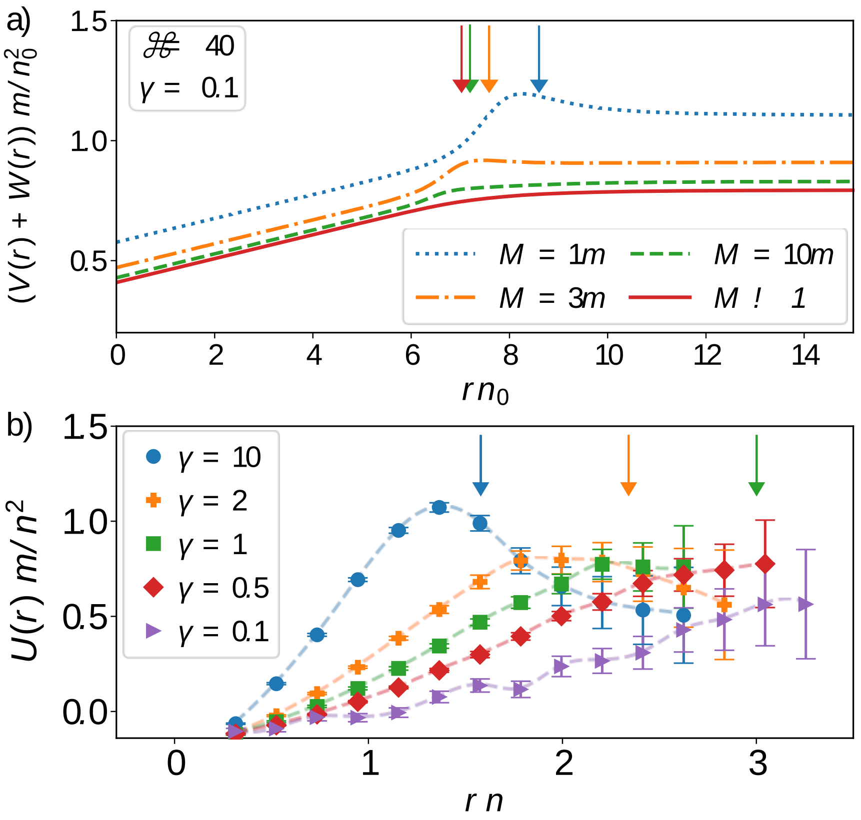
$$\left[-\frac{1}{M}\partial_r^2 + V(r) + W(r) - E\right]\Psi(r) = 0$$

- **additional potential**

$$W(r) = \frac{1}{M}\int dx\, |\partial_r \phi(x,r)|^2$$

potential maximum

$$r_{\max} = \frac{\pi}{\sqrt{2m_r\mu}}$$



QMC

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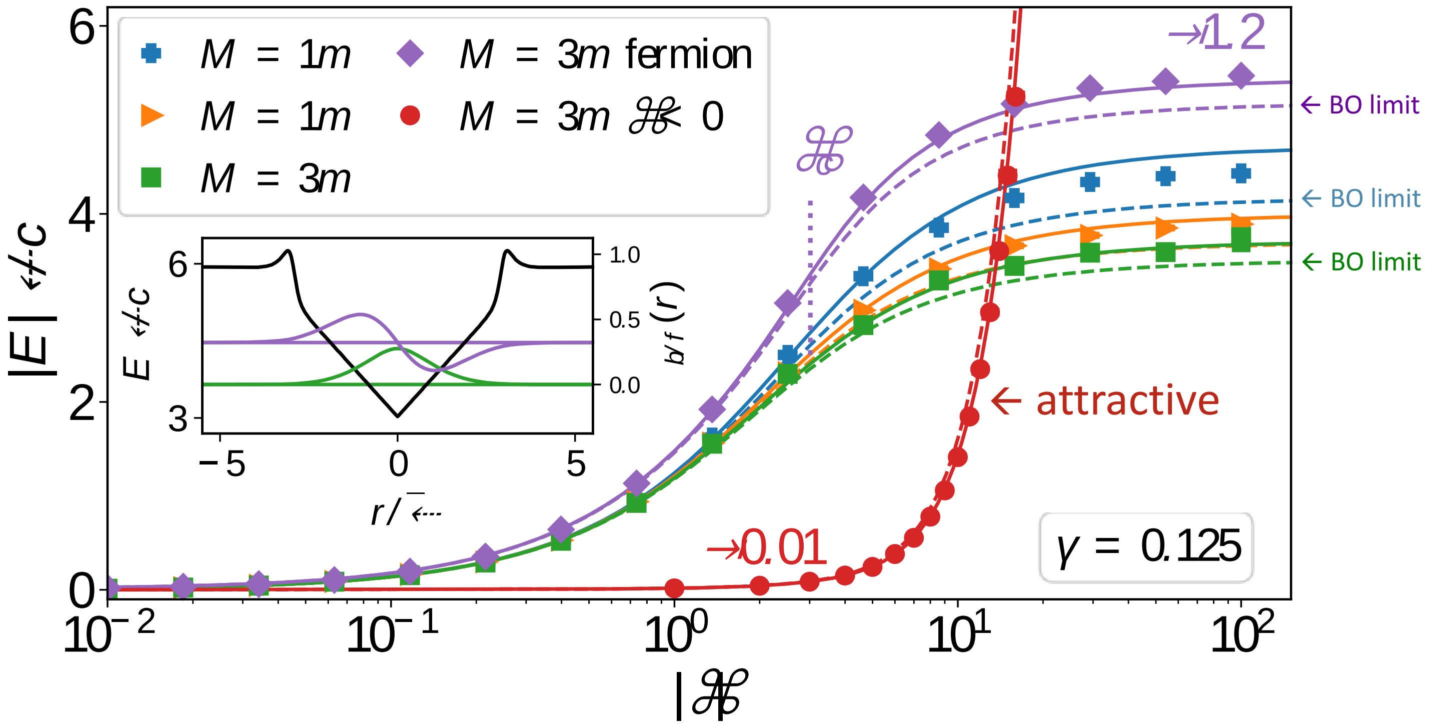
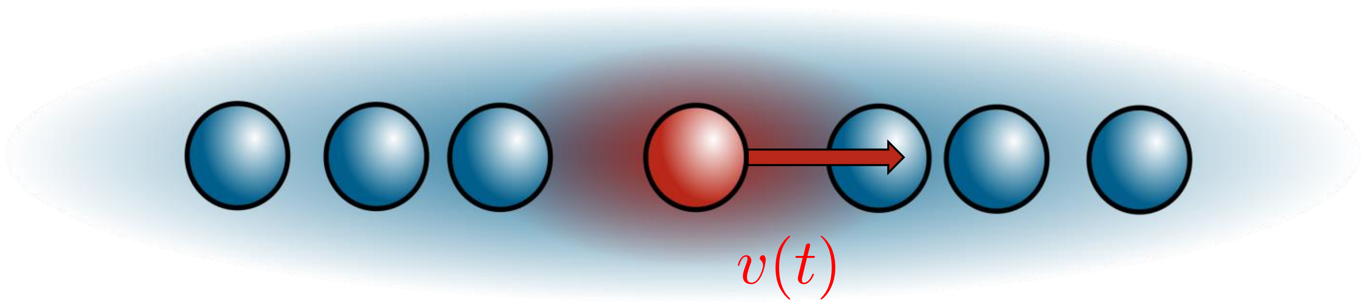
Bi-polaron bound states



Formation of Bose polaron



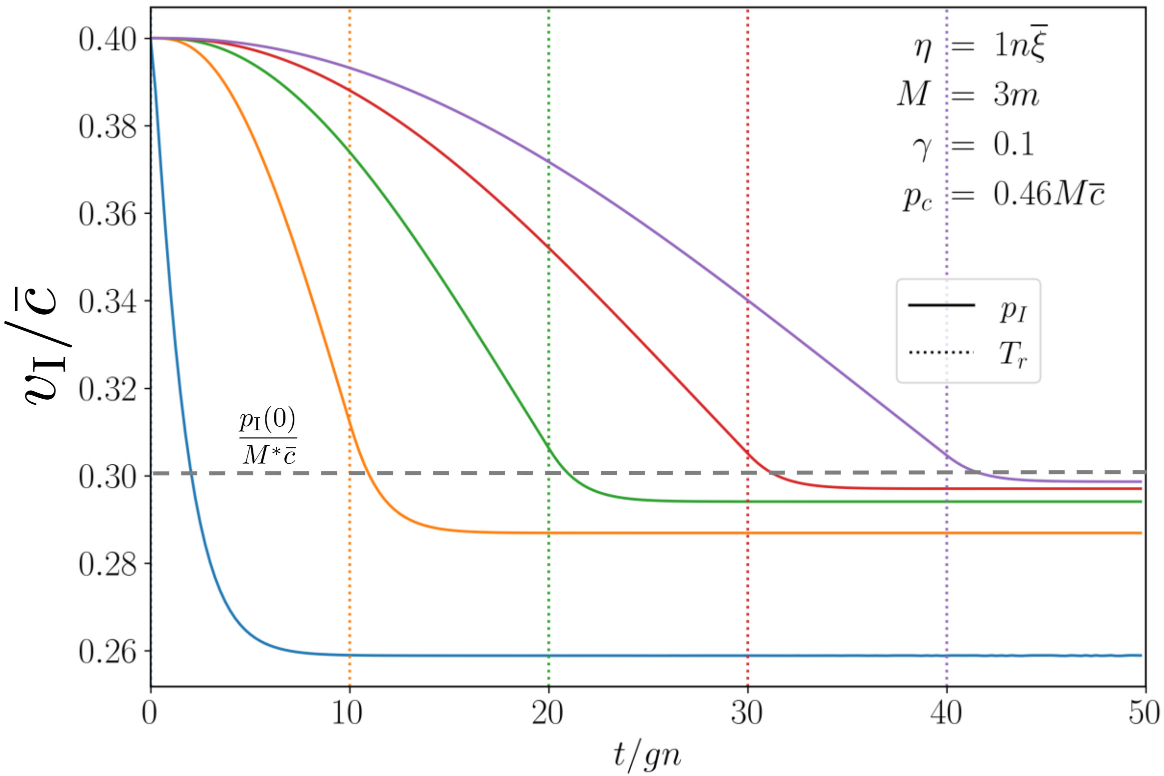
Martin Will and Michael Fleischhauer

in preparation

Adiabatic switch-on of $g_{\mathrm{IB}}(t)$

$v_{\mathrm{I}} < c$

linear ramp of $g_{\mathrm{IB}}(t)$
in time T_c

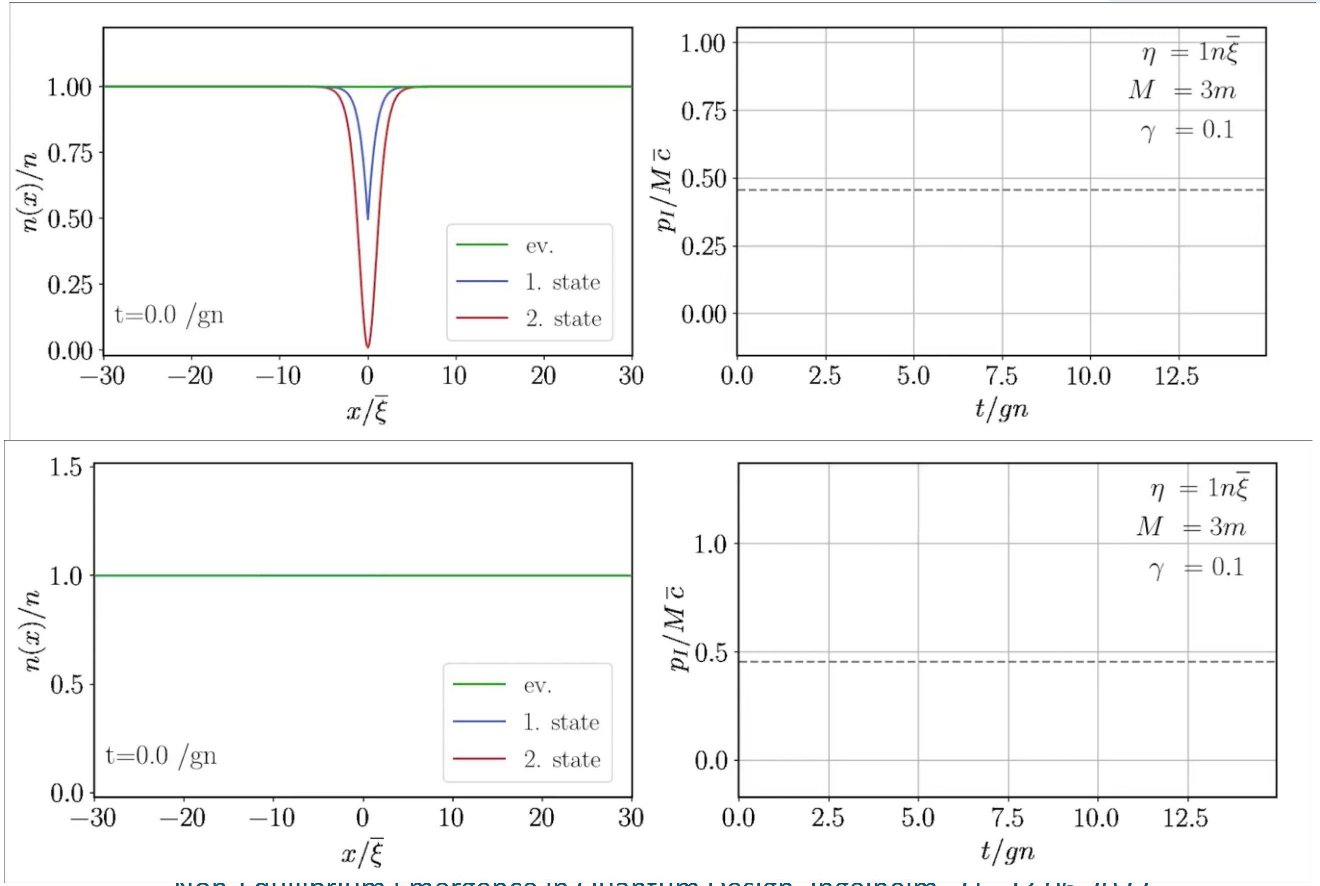


Dynamics after quench of g_{IB}

$$M = 3\,m$$

$$p_I(0) = 0.1 M\,\bar{c}$$

$$p_I(0) = 1.3\,M\,\bar{c}$$

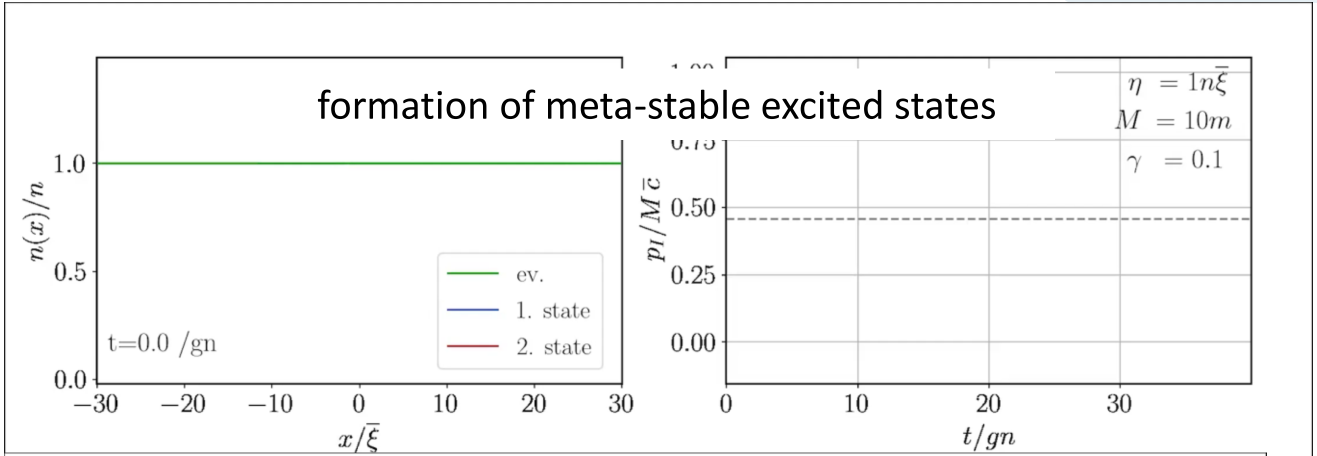


Dynamics after quench of g_{IB}

$$M = 10\,m$$

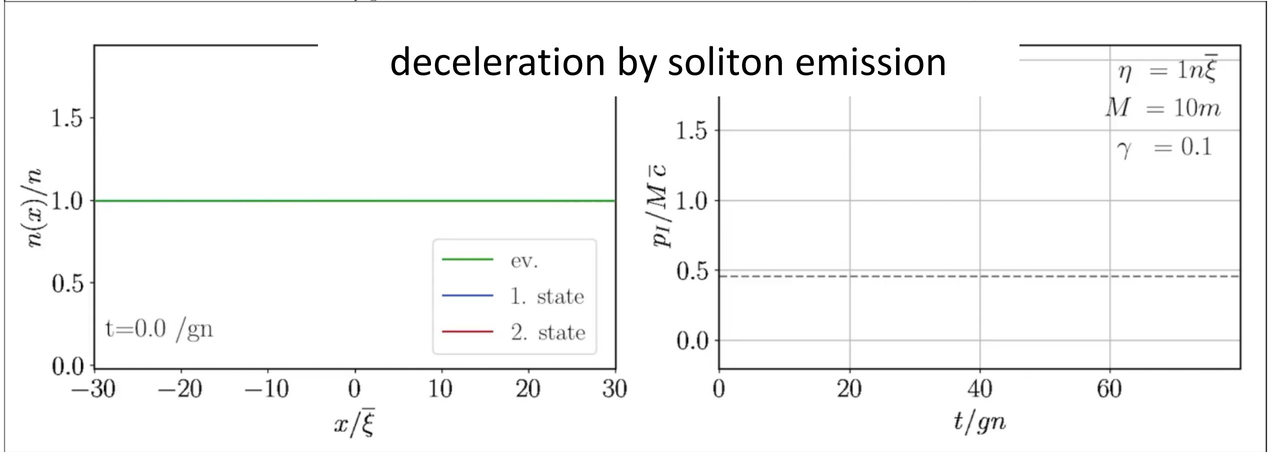
$$p_I(0) = 1\,M\,\bar{c}$$

formation of meta-stable excited states



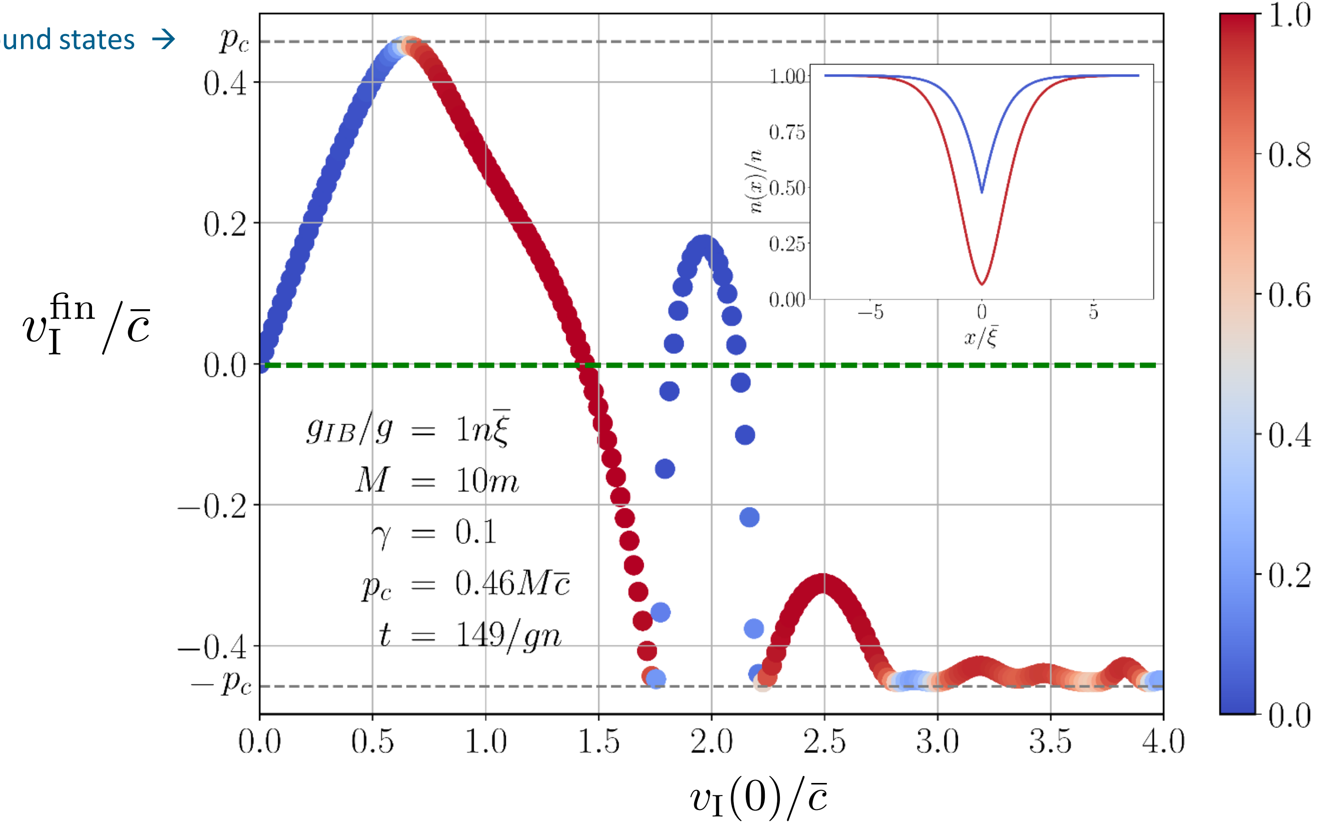
$$p_I(0) = 2\,M\,\bar{c}$$

deceleration by soliton emission

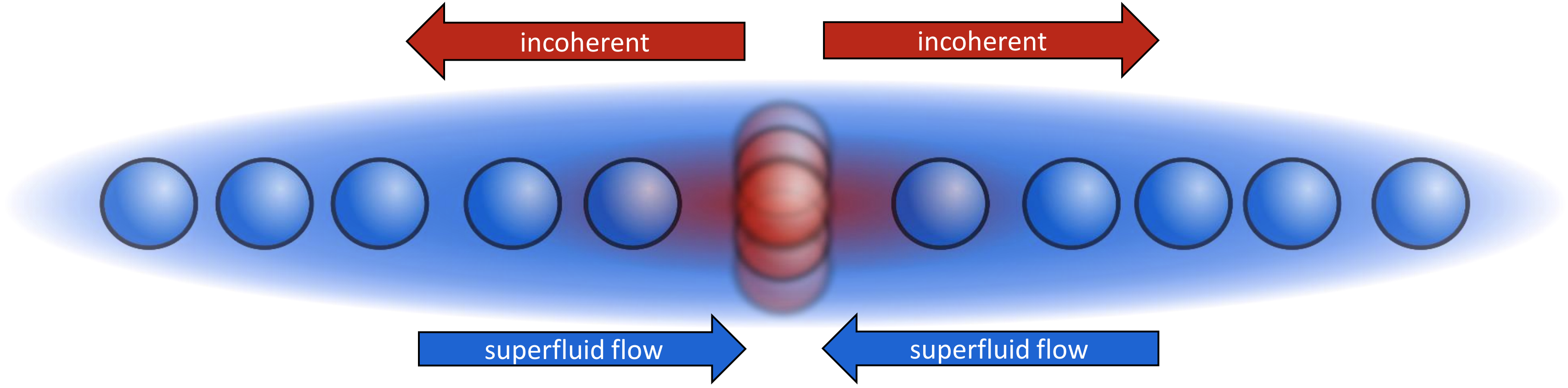


final impurity momentum

existence of bound states →



Controlling currents with noisy impurities



Martin Will, Jamir Marino and Michael Fleischhauer

in preparation

D. A. Zezyulin, V. V. Konotop, G. Barontini and H. Ott, Phys. Rev. Lett. **109**, 020405
D. Sels and E. Demler, Ann. Phys. **412**, 168021 (2020)

Stochastic mean-field

$$i\partial_t \Phi(x,t) = \Big[-\frac{1}{2m}\partial_x^2 + iv\partial_x + g|\Phi(x,t)|^2 + \eta(t)V(x) \Big] \Phi(x,t)$$

Random variable: $\overline{\eta(t)} = 0 \qquad \overline{\eta(t)\eta(t')} = \delta(t-t')$

- Stratonovich stochastic differential equation:

$$d\Phi(x,t) = -i\Big[-\frac{1}{2m}\partial_x^2 + iv\partial_x + g|\Phi(x,t)|^2 \Big] \Phi(x,t)dt - i\Phi(x,t)V(x)dW$$

Stochastic increment: $dW = \eta(t)dt$

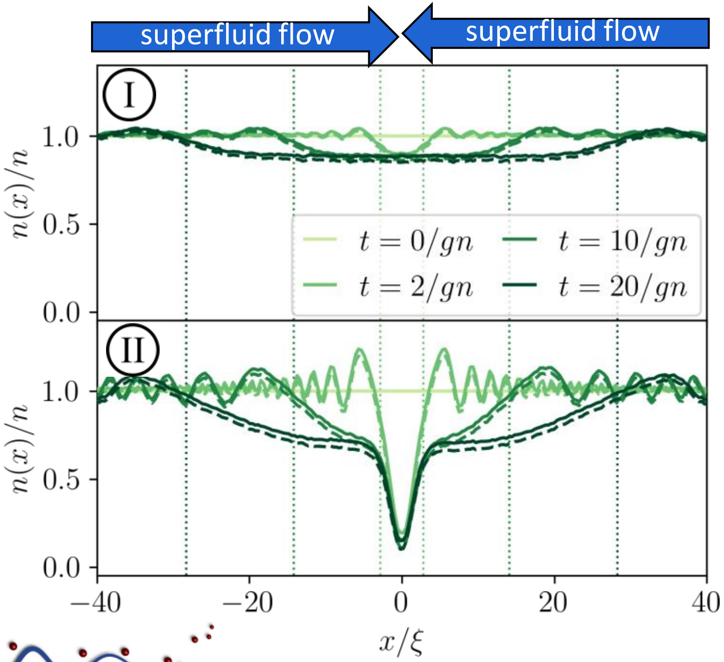
Noise correlated with field: $\overline{\Phi(x,t)\, dW} \neq 0$

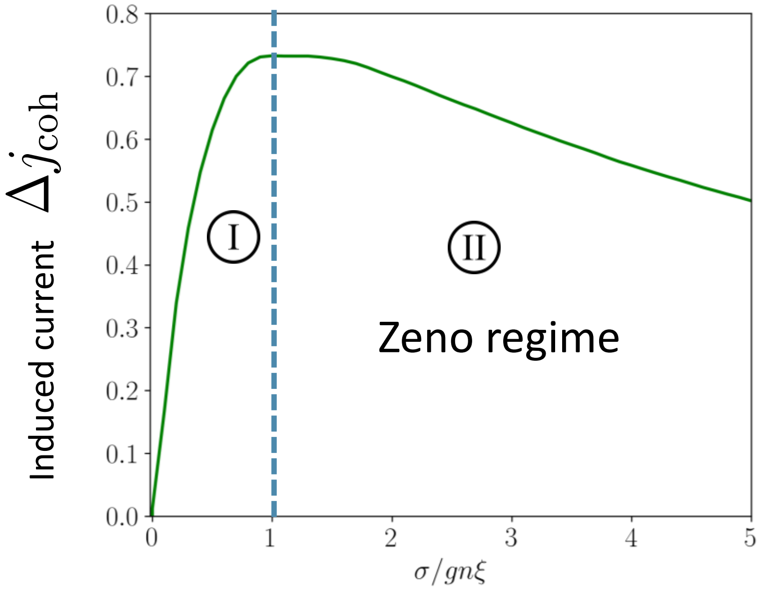
Stationary impurity $v = 0$

- average field (superfluid amplitude)

local loss

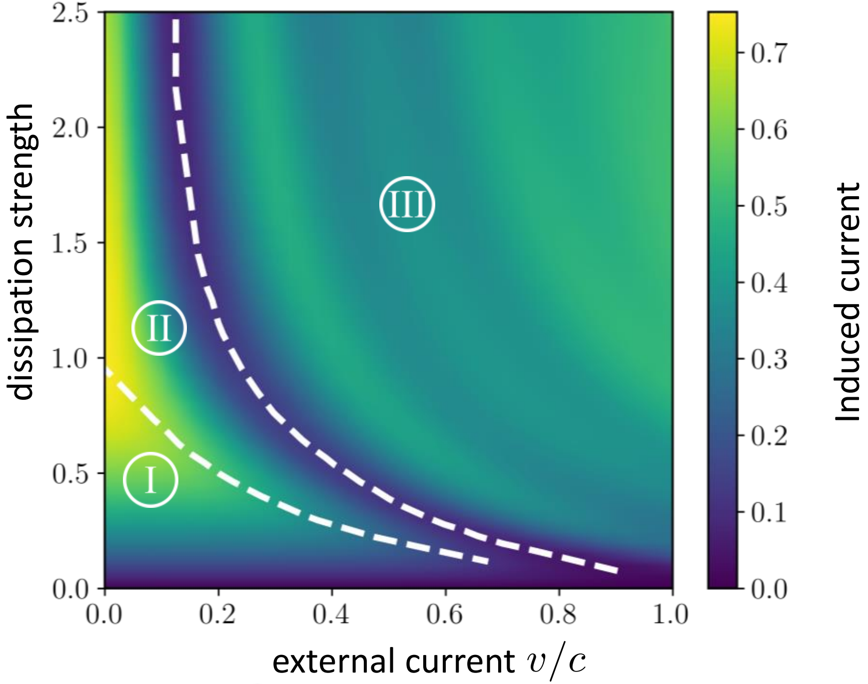
$$i\partial_t \overline{\Phi(x,t)} = -\frac{\partial_x^2}{2m}\overline{\Phi(x,t)} + g\overline{|\Phi(x,t)|^2\Phi(x,t)} - \frac{i}{2}V(x)^2\overline{\Phi(x,t)} \qquad V(x)^2 = 2\sigma\delta(x)$$

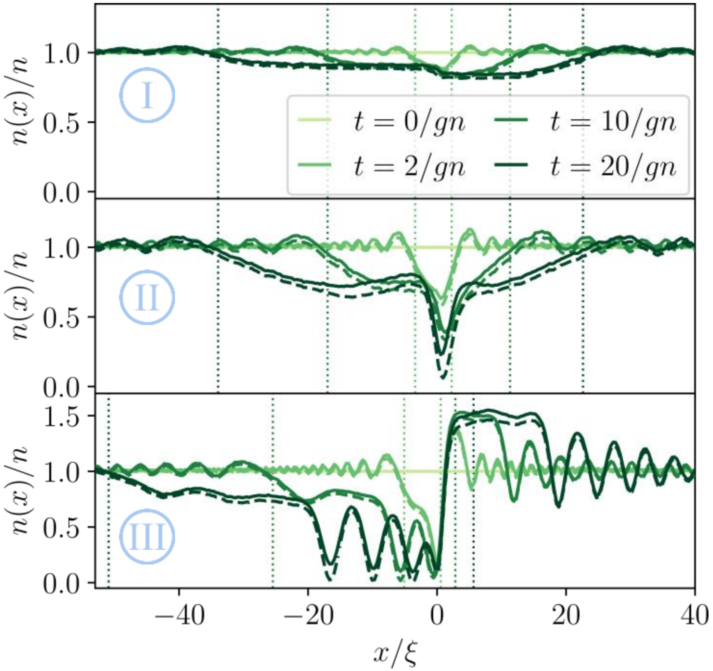




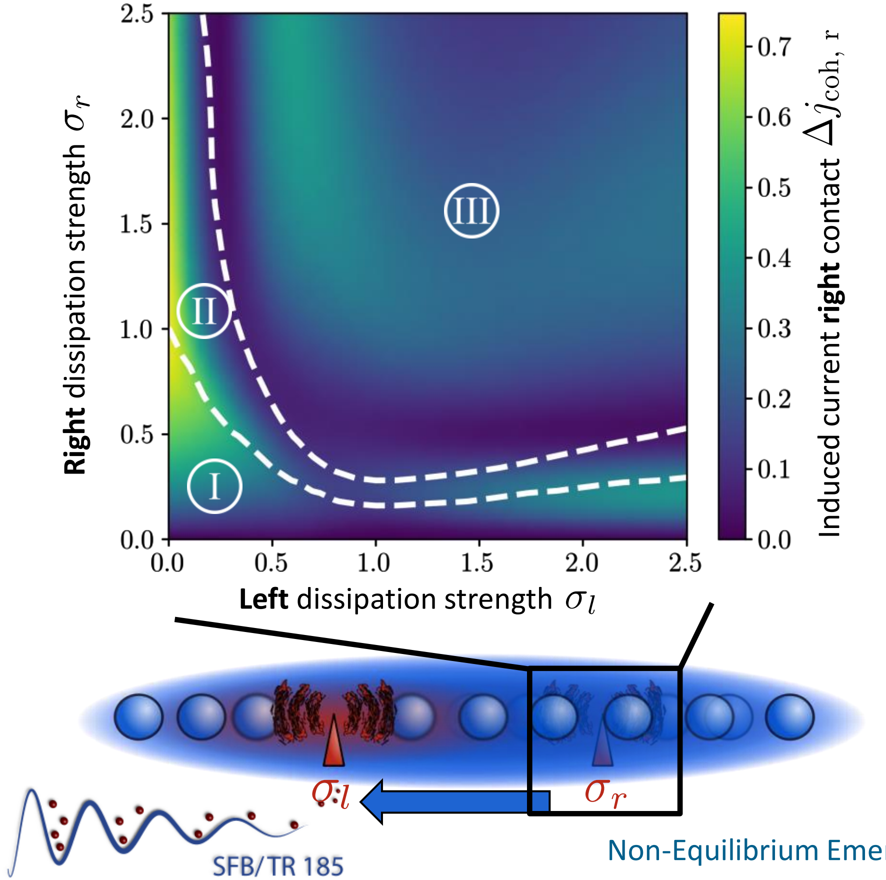
Impurity in external current $v \neq 0$

- New soliton regime III

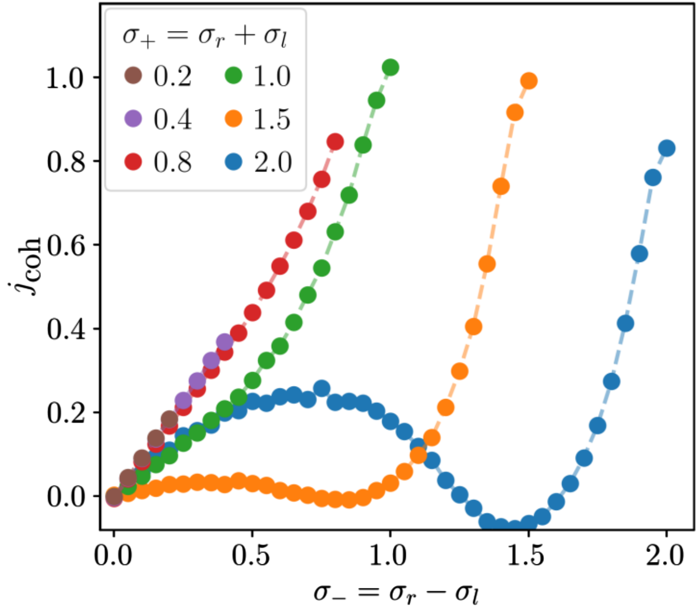




Control of superfluid flow by two impurities



Current **between** contacts



Black line:

Single contact at dissipation strength σ_-

Summary

Bose polaron

- Backaction of impurity to condensate: deformed condensate and modified Bogoliubov phonons

Bi-polaron

- BO & beyond BO: linear binding potential on short distances

Dynamics of impurity injected into BEC

- slow down also for $v < c$ even in adiabatic regime
- slow down by (i) density wave emission (ii) soliton emission
- momentum reversal possible $|p_{\text{fin}}| \leq p_c$

Noisy impurity

- Control of superfluid and incoherent flow
- Different dynamical regimes: (i) linear response (ii) Zeno / negative differential conductivity (iii) soliton regime

$$
\begin{aligned}
V(r,\eta) =& g n^2 r \left(\frac{1}{2} - \frac{4+2\nu}{3(\nu+1)^2} \right) \\
&+ \frac{4}{3} g n^2 \bar{\xi} \frac{1}{\sqrt{1+\nu}} \Bigg\{ 2\,\mathrm{E}\big(\mathrm{am}(u,\nu),\nu\big) - \frac{3\sqrt{\nu}}{2+2\nu} \mathrm{cd}(u,\nu) \left[1 + \nu + \frac{2}{3} \nu \, \mathrm{cd}(u,\nu)^2 \right] \\
&+ \sqrt{2\nu+2} - \frac{\nu}{1+\nu} \mathrm{cd}(u,\nu) \, \mathrm{sn}(u,\nu) \left[\nu \, \mathrm{cd}(u,\nu)^2 + 2\nu + 1 \right] \Bigg\}
\end{aligned}
$$