Quantum Steering:
passive vs. active navigation in Hilbert space

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Measurement…

can we harness the measurement's backaction for something good?
i.e. quantum steering → quantum state engineering?

our wish list:

- steer towards a desired target state
- arbitrary (unknown to us) initial state
- robustness against perturbation
- no control on the time of operation
- target state: needs not be a ground state

PRINCIPLES OF QUANTUM STEERING

system-
detector
Hamiltonian

system
density matrix

system-

blind steering : an instance of passive steering

(the steps of the protocol are pre-determined)

- blind steering
- post-selective steering
  (exponentially small success probability
- active feedback steering

example: **Spin ½ : steer towards** $S_z = \uparrow$

System $|\psi_{sys}\rangle$
} coupled by H_{S-D}
Detector $|d_i\rangle$

$$U_{S-D} = \exp(-i\Delta t \cdot H_{S-D}) \Rightarrow |\Psi\rangle = U_{S-D} |\psi_{sys}\rangle \otimes |d_i\rangle$$

arbitrary state

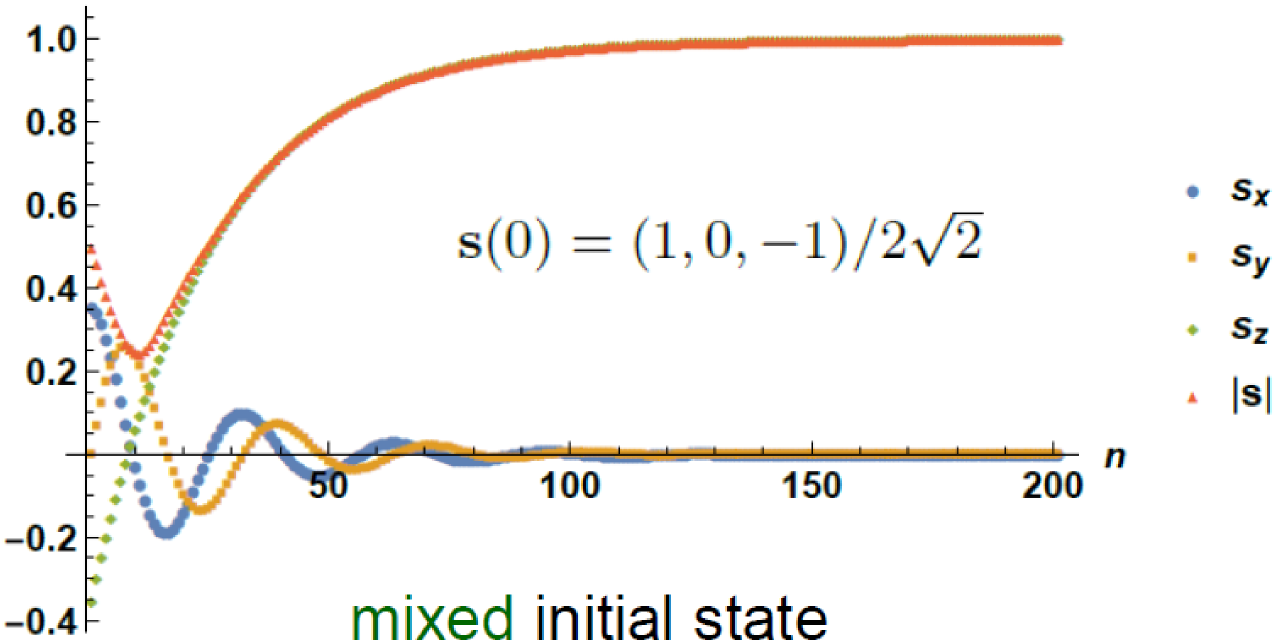
$$|d_i\rangle = |\uparrow\rangle_d$$

$$H_{\text{s-d}} = J(|\downarrow_{\text{d}}\rangle\langle\uparrow_{\text{d}}|) \otimes (|\uparrow_{\text{s}}\rangle\langle\downarrow_{\text{s}}|) + \text{h.c.}$$
$$= J\left(\sigma_{\text{d}}^{-}\sigma_{\text{s}}^{+} + \sigma_{\text{d}}^{+}\sigma_{\text{s}}^{-}\right),$$

$$\rho_s(n+1) = \text{Tr}_d[U_{s-d}(\rho_s(n) \otimes \mathbb{P}_d)U^{\dagger}_{s-d}]$$

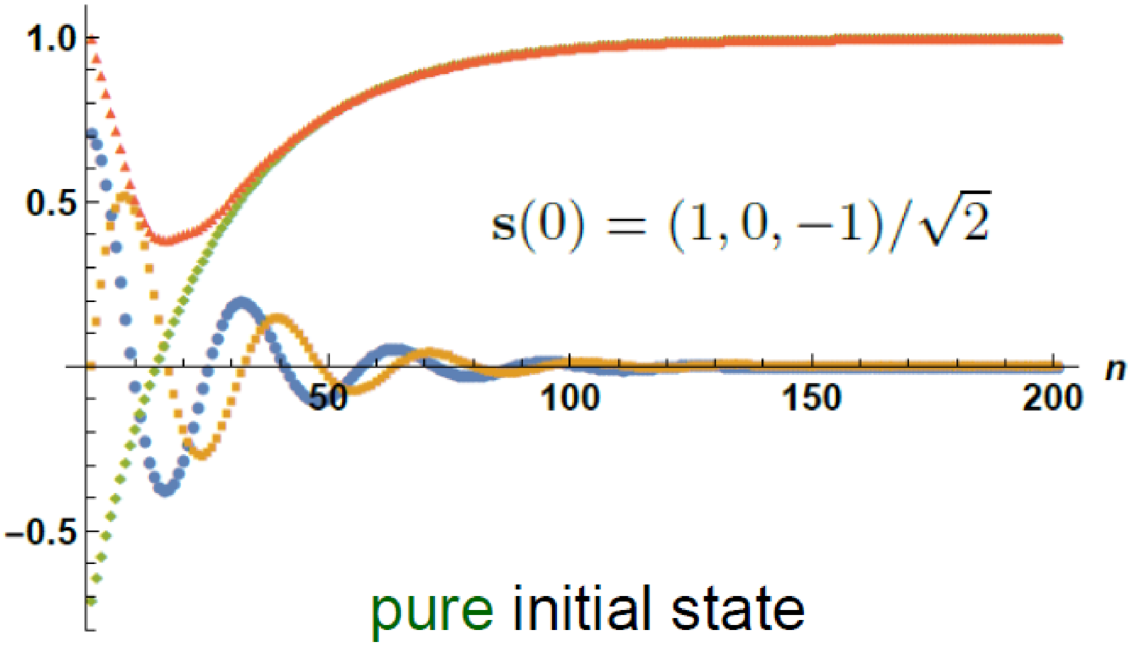
Spin ½

System density matrix after n^th iteration: $\rho_s(n) = (\mathbb{I}_2 + \mathbf{s}(n) \cdot \boldsymbol{\sigma}_s)/2$



$\mathbf{s}(0) = (1, 0, -1)/2\sqrt{2}$

mixed initial state



$\mathbf{s}(0) = (1, 0, -1)/\sqrt{2}$

pure initial state

example:

steering towards a weakly entangled many-body state…

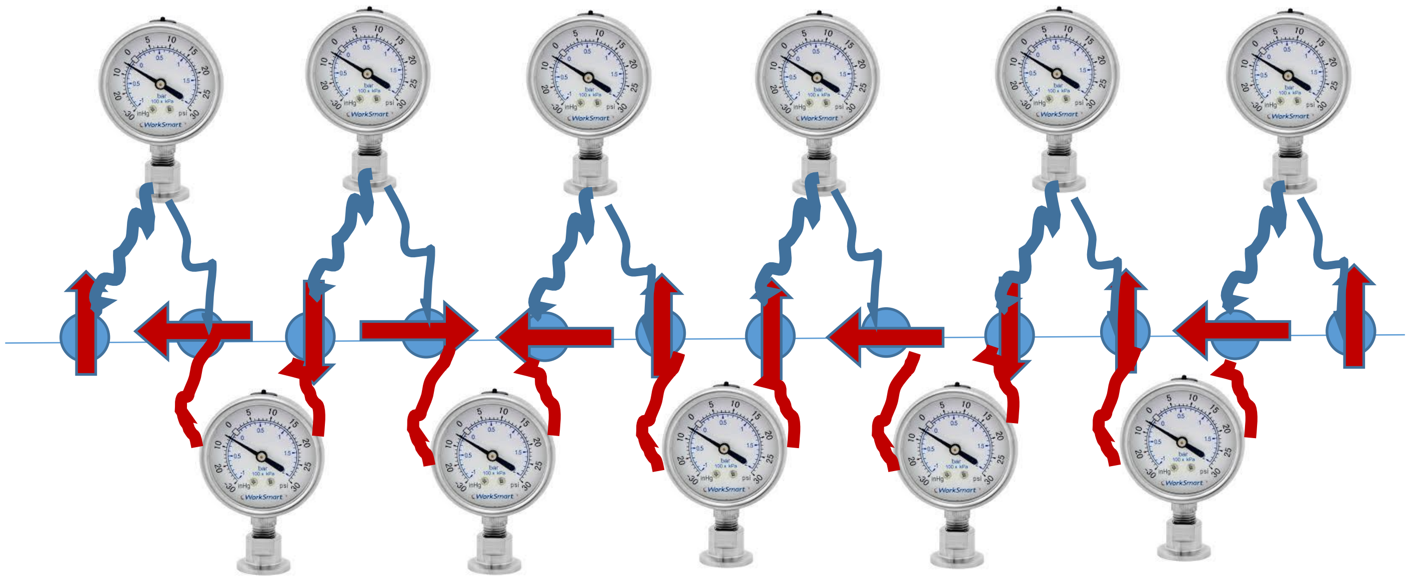
AKLT chain

$$H_{AKLT} = \sum_{1}^{N} P^{(l,l+1)}_{S^{\text{total}}=2}$$

ground state: each bond has zero weight in the $S^{\text{total}} = 2$ sector

steering towards an AKLT gorund state

- locality
- comlexity of H_{S-D} does not increase with N
- # detectors extensive



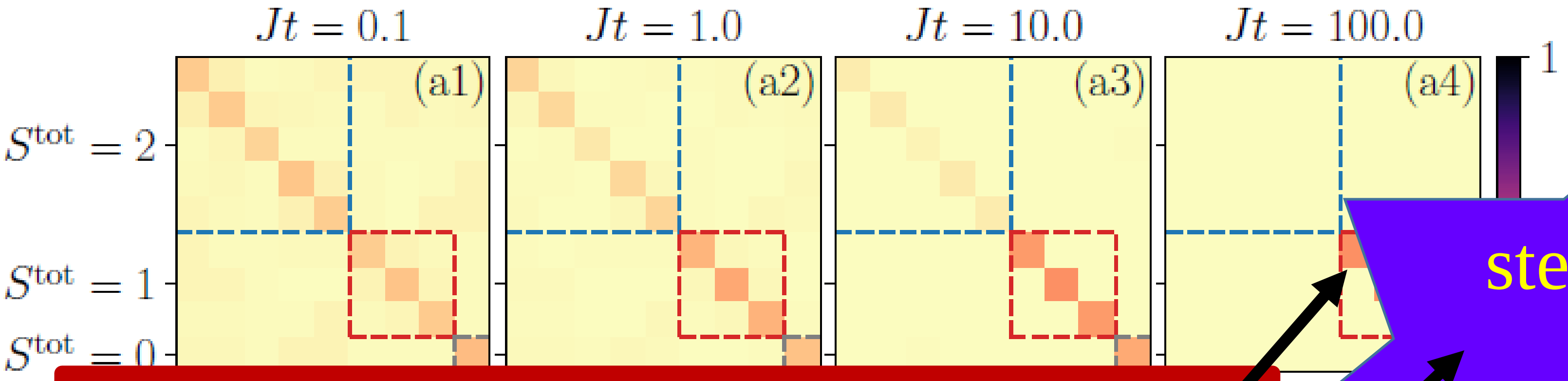
$$H_{\text{s-d}} = J \sum_{i=1}^{5} \left(\sigma^{-}_{\mathrm{d}_i^{(\ell,\ell+1)}} \otimes U_i^{(\ell,\ell+1)} + \sigma^{+}_{\mathrm{d}_i^{(\ell,\ell+1)}} \otimes (U_i^{(\ell,\ell+1)})^{\dagger} \right),$$

$$\begin{aligned}
U_1^{(\ell,\ell+1)} &= (S_\ell^- - S_{\ell+1}^-) P_\ell^+ P_{\ell+1}^+ \propto (|1,1\rangle\langle 2,2|)_{(\ell,\ell+1)}, \\
U_2^{(\ell,\ell+1)} &= \left(\mathbb{I}_9 - \frac{S_\ell^- S_{\ell+1}^+}{2}\right) P_\ell^+ P_{\ell+1}^0 - \left(\mathbb{I}_9 - \frac{S_\ell^+ S_{\ell+1}^-}{2}\right) P_\ell^0 P_{\ell+1}^+ \propto (|1,1\rangle\langle 2,1|)_{(\ell,\ell+1)}, \\
U_3^{(\ell,\ell+1)} &= \left(\mathbb{I}_9 - \frac{(S_\ell^- S_{\ell+1}^+)^2}{4}\right) P_\ell^+ P_{\ell+1}^- - \left(\mathbb{I}_9 - \frac{(S_\ell^+ S_{\ell+1}^-)^2}{4}\right) P_\ell^- P_{\ell+1}^+ + (S_\ell^+ S_{\ell+1}^- - S_\ell^- S_{\ell+1}^+) P_\ell^0 P_{\ell+1}^0 \propto (|1,0\rangle\langle 2,0|)_{(\ell,\ell+1)} \\
U_4^{(\ell,\ell+1)} &= \left(\mathbb{I}_9 - \frac{S_\ell^+ S_{\ell+1}^-}{2}\right) P_\ell^- P_{\ell+1}^0 - \left(\mathbb{I}_9 - \frac{S_\ell^- S_{\ell+1}^+}{2}\right) P_\ell^0 P_{\ell+1}^- \propto (|1,-1\rangle\langle 2,-1|)_{(\ell,\ell+1)}, \\
U_5^{(\ell,\ell+1)} &= (S_\ell^+ - S_{\ell+1}^+) P_\ell^- P_{\ell+1}^- \propto (|1,-1\rangle\langle 2,-2|)_{(\ell,\ell+1)}.
\end{aligned}$$

compare to Lindblad: U plays the role of jump operators

steering towards an AKLT ground state

does a steering on a bond deter/compete with steering on another bond?



contrary to a single pair:

the asymptotic state of each pair is unique

employ the $\left|S^{\text{total}}, S^{\text{total},z}\right\rangle$ basis

$\langle 1, s_1 | \rho_s^{pair}(t \to \infty) | 1, s_1 \rangle = 2/9$

$\langle 0,0 | \rho_s^{pair}(t \to \infty) | 0,0 \rangle = 1/3$

no off-diagonals

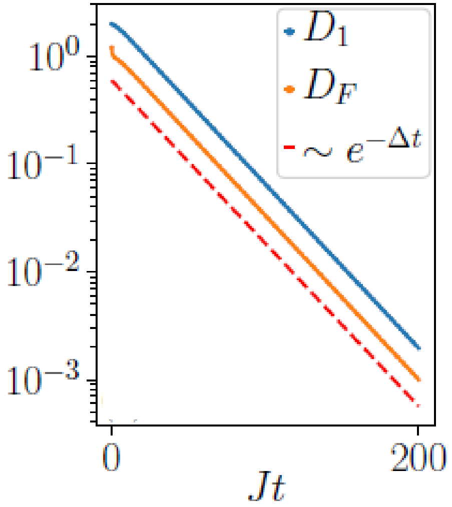
steering out of $S^{\text{total}} = 2$ is not unique

steering towards an AKLT ground state

many-body yet local steering

Frobenius norm $D_F(\mathrm{t}) = \sqrt{Tr_s[\rho_s(t) - \rho_{s=\mathrm{target}}]^2}$

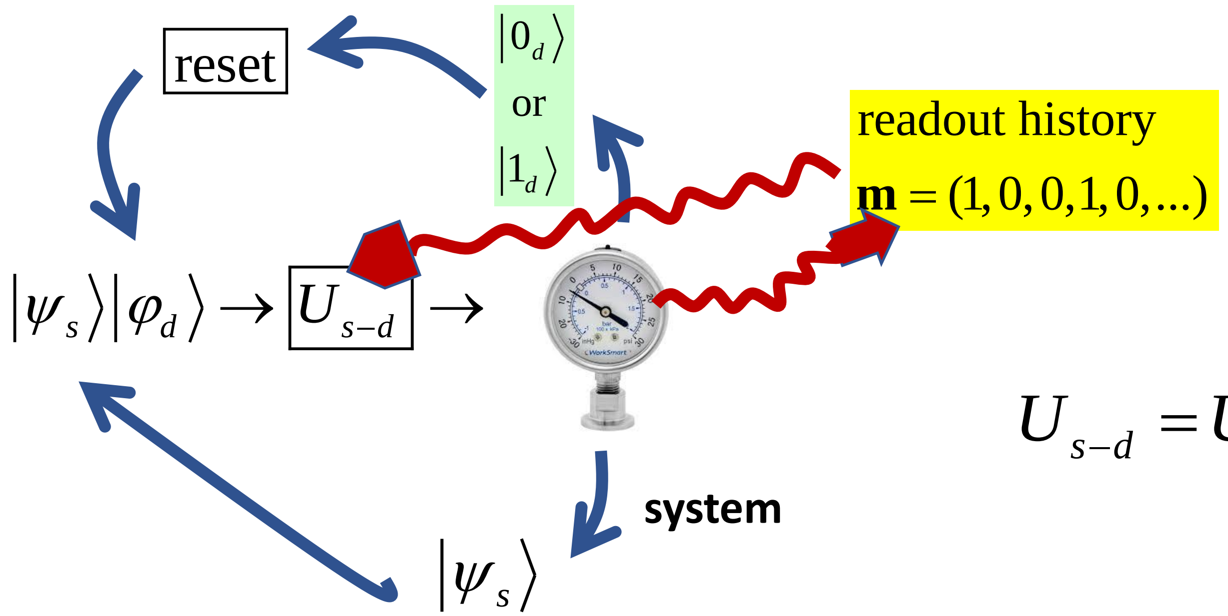
trace norm $D_1(t) = Tr_s[\sqrt{(\rho_s(t) - \rho_{s=\mathrm{target}})^2}]/2$



S. Roy, J. Chalker, I. Gornyi, YG 2020

ACTIVE STEERING...

active feedback steering : steering



$$U_{s-d} = U_{s-d}(\mathbf{m})$$

?? how to use this to improve the protocol ??
(runtime, fidelity…)

non-trivial feedback: coordinated steering

Element 1: multiple $H^{(\alpha)}_{s-d}$

$$[H^{(\alpha)}_{s-d}, H^{(\beta)}_{s-d}] \neq 0$$

to use in $U_{s-d} = e^{-i H^{(\alpha)}_{s-d} \cdot \delta t}$

Element 2: a policy;

$\vec{m}$ = readout history $\Rightarrow$ choose $H^{(\alpha)}_{s-d}$ to act with.

global

quantum compass: cost function policy

introduce a cost function, $r(\rho_s)$ to be minimized in the protocol.

Example: *infidelity* $r^{(\text{inf})}$ of the system's state ρ_s to the target state $\left|\psi_s^{(\text{target})}\right\rangle$

$$r^{(\text{inf})}(\rho_s) = 1 - \left\langle \psi_s^{(\text{target})} \middle| \rho_s \middle| \psi_s^{(\text{target})} \right\rangle$$

note: if we know the initial state of the system we know ρ_s hence we can calculate $r^{(\text{inf})}$

Objective: find a sequence $(H_{s-d}^{\alpha 1}, H_{s-d}^{\alpha 2}, H_{s-d}^{\alpha 3}, \ldots)$ that brings the system to the global minimum of $r^{(\text{inf})}(\rho_s)$ in the fastest expected time

$\Rightarrow$ greedy version (cheaper): the fastest expected reduction of the cost function in a single measurement step

NON-COMMUTING STEERING :weakly entangled state (AKLT)

- AKLT state is the ground state of the parent AKLT Hamiltonian:

$$H_{\text{AKLT}} = \sum_i H_{i,i+1} = \sum_i \left[\overrightarrow{S}_i \cdot \overrightarrow{S}_{i+1} + \frac{1}{3} \left(\overrightarrow{S}_i \cdot \overrightarrow{S}_{i+1} \right)^2 \right]$$

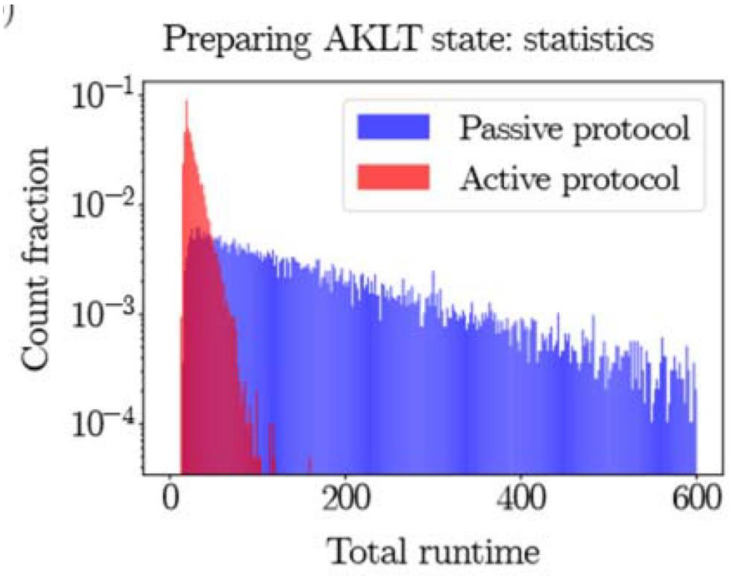
- With 4 local ground states $|\phi_a^{(j)}\rangle$ and 5 excited states $|\theta_a^{(j)}\rangle$, make a coupling family:

$$V(\mathbf{c}, i) = |\phi_4^i\rangle\langle\theta_5^i| + \sum_{\alpha,\beta=1,..4} c_{\alpha\beta} |\phi_\alpha^i\rangle\langle\theta_\beta^i|$$

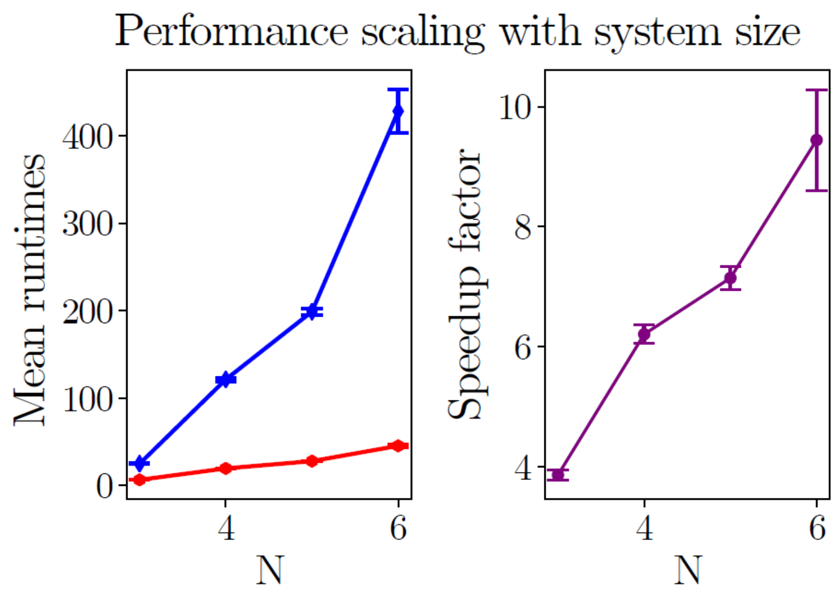
$$H_{s,d} = V_s \sigma_d^+ + V_s^\dagger \sigma_d^-$$

- $V(\mathbf{c}, i)$ allow preparing the ground state passively by choosing one random $\mathbf{c}$

- **Navigation strategy: iterate cyclically over location *i* and optimise the matrix c; this is to be compared with no optimisation over c.**



time to infidelity 0.01



global cost function, e.g., $r^{(\text{inf})}(\rho_s)$, may be fundamentally flawed

target state $|0,0,0,0,..\rangle$ (product state)

initial state $|1,1,1,1,1,..\rangle$

sequence $|1,1,1,1,1,..\rangle \rightarrow |1,0,0,1,1..\rangle \rightarrow |1,0,0,1,0,..\rangle \rightarrow$

orthogonal to $|0,0,0,0..\rangle$

$r^{(\text{inf})} = 1$ for most states $\Rightarrow$ flat landscape

" Anderson orthogonality catastrophe"

local cost function

<u>local Reduced Density Matrix</u>

$$|\psi_s\rangle \rightarrow \rho_s^{(\alpha)} \equiv \mathrm{tr}_{sys\backslash\alpha}\left[|\psi_s\rangle\langle\psi_s|\right]$$

active steering: a Quantum State Machine

steering transformation $\Lambda^{click}, \Lambda^{n-c}$ + basis (vertices) $\Rightarrow$ quantum graph

(we will denote only Λ^{click}, Λ^{n-c} is implicit)

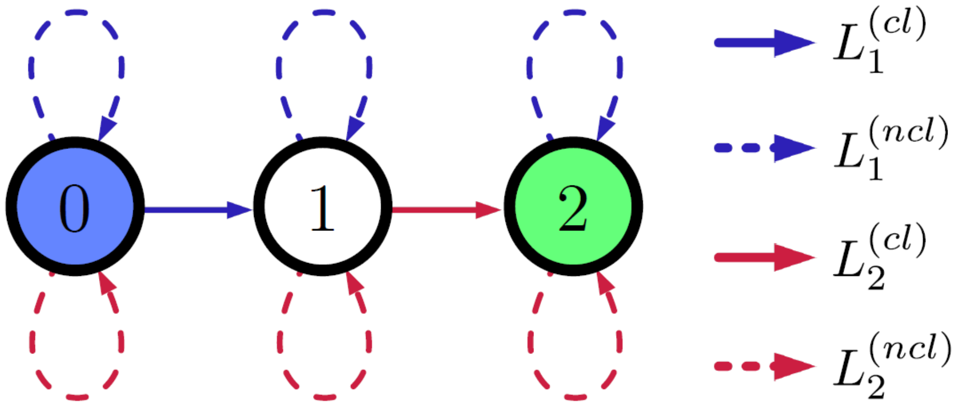
$$\Lambda^{c}[|\phi_{\alpha}\rangle] = \sum_{\beta} L_{\alpha\beta} |\phi_{\beta}\rangle$$

complex amplitudes

$$\left.\begin{array}{l}\text{variety of possible steering H}\\ \text{different readouts}\end{array}\right\} \Rightarrow \text{different graphs}$$

Quantum State Machine (QSM) for active steering:
combining all graphs to a single "colored" graph

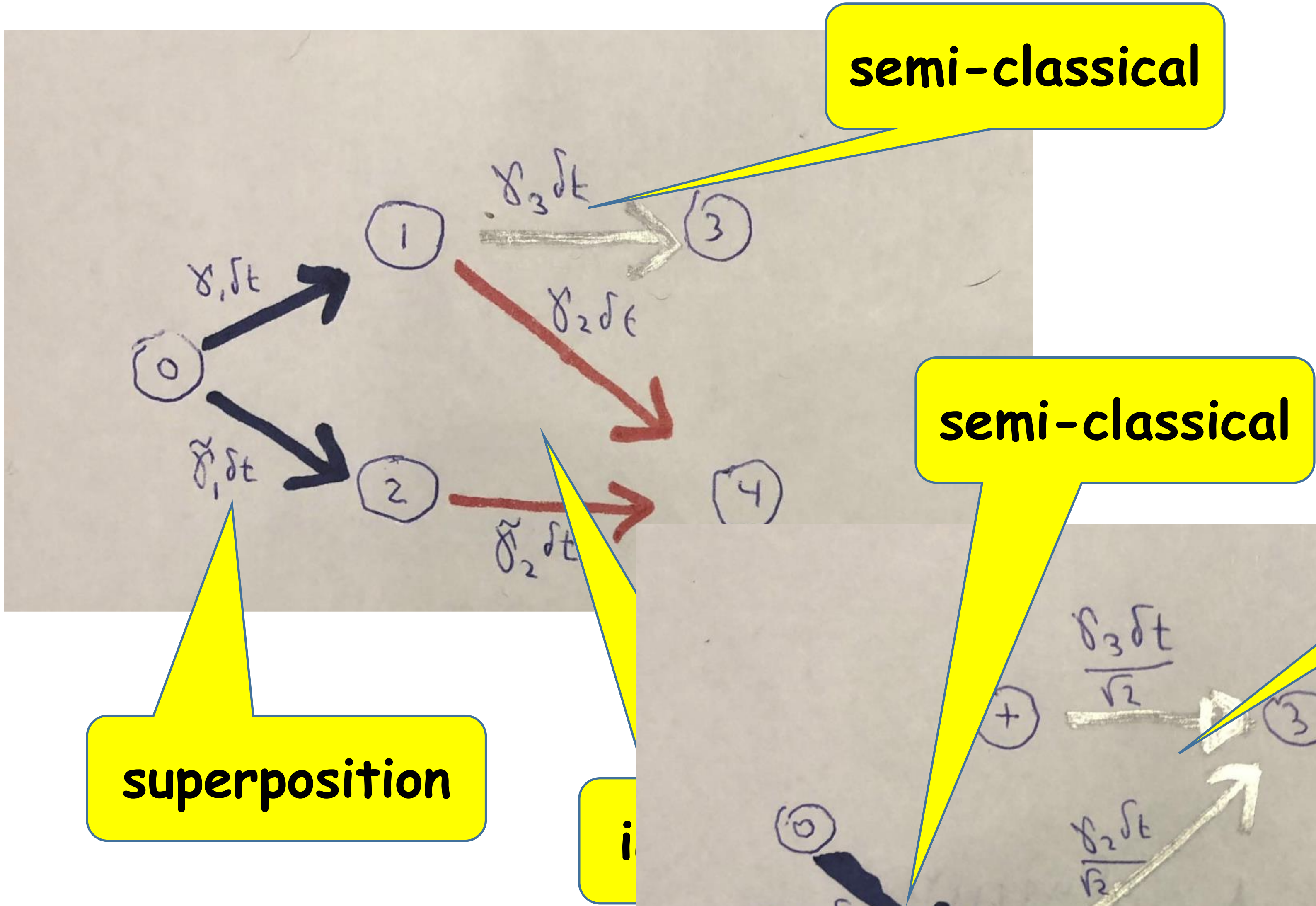
- Quantum State Machine (QSM) is a coloured multigraph representing entire coupling set $\{V_s(\mathbf{p})\}$.
- In simplest ("classical") cases, optimal active decision directly follows from QSM
- This "classicality" is basis-dependent and generally does not occur

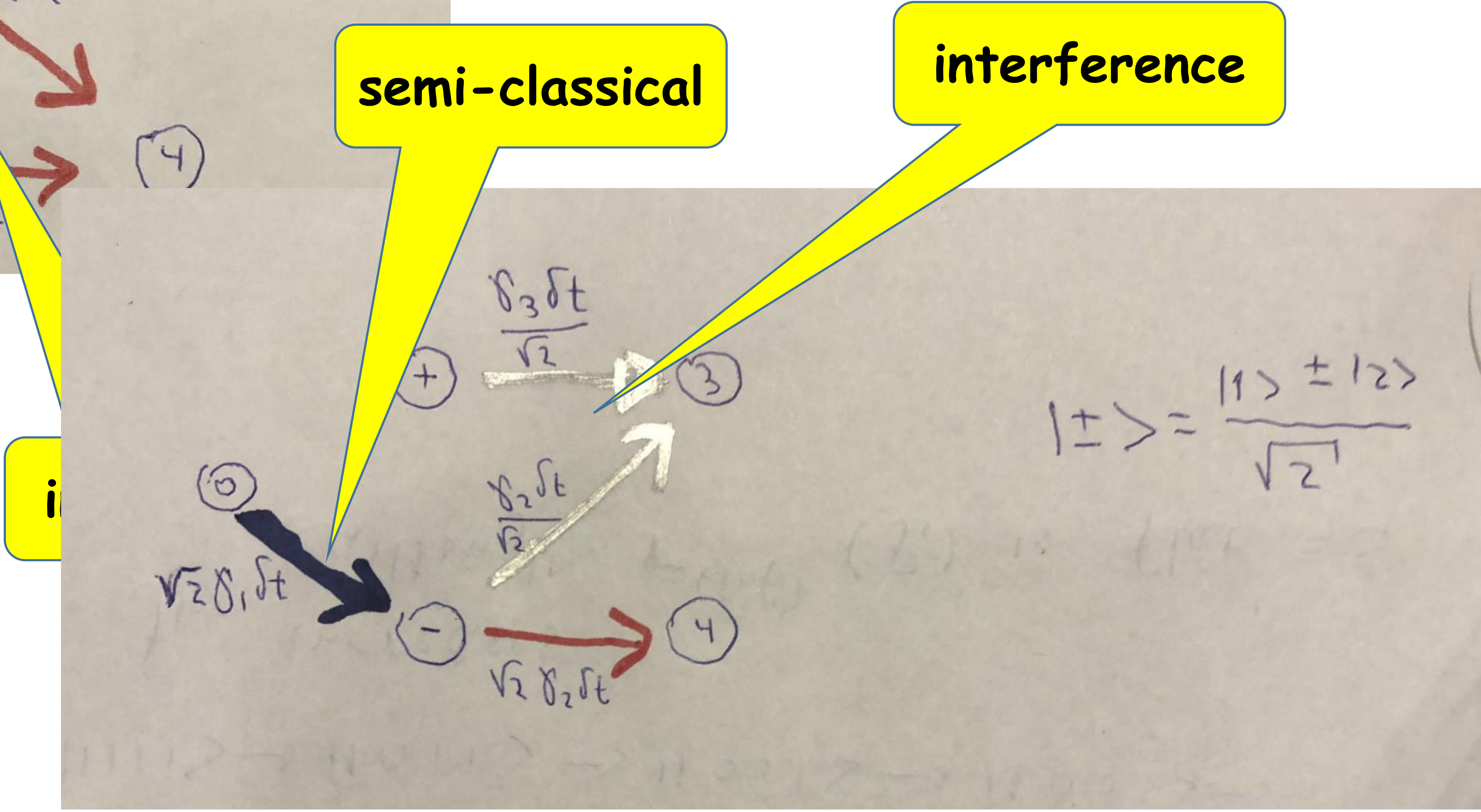


Example of a classical Quantum State Machine

Optimal decision-making here is straightforward

irreducible
quantum block



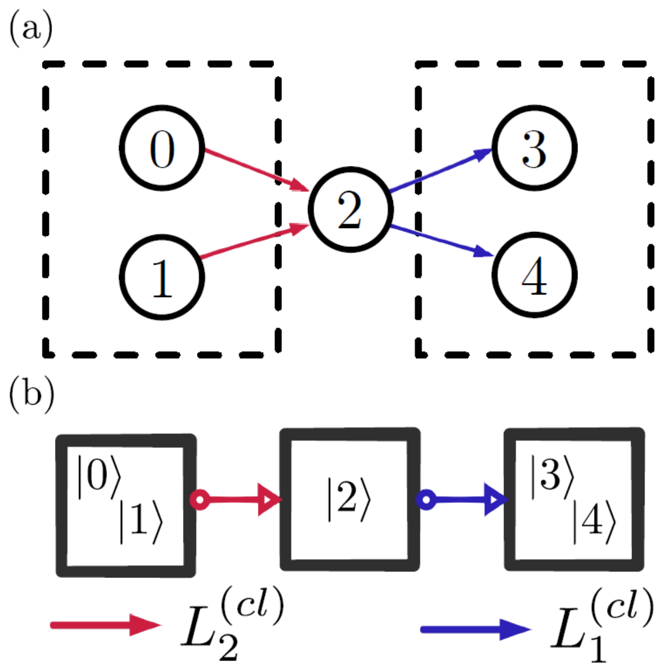


Quantum State Machine mapping

Coarse-graining and semiclassical QSM

- QSM with “quantum” subgraphs cannot be used for optimising the feedback policy in a classical way

- Instead, QSM is first to be coarse-grained into a `semi-classical’ graph



Semiclassical coarse-graining of a QSM

active steering: a Quantum State Machine: the W-state

a 3-qubit state :

$$W = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$$

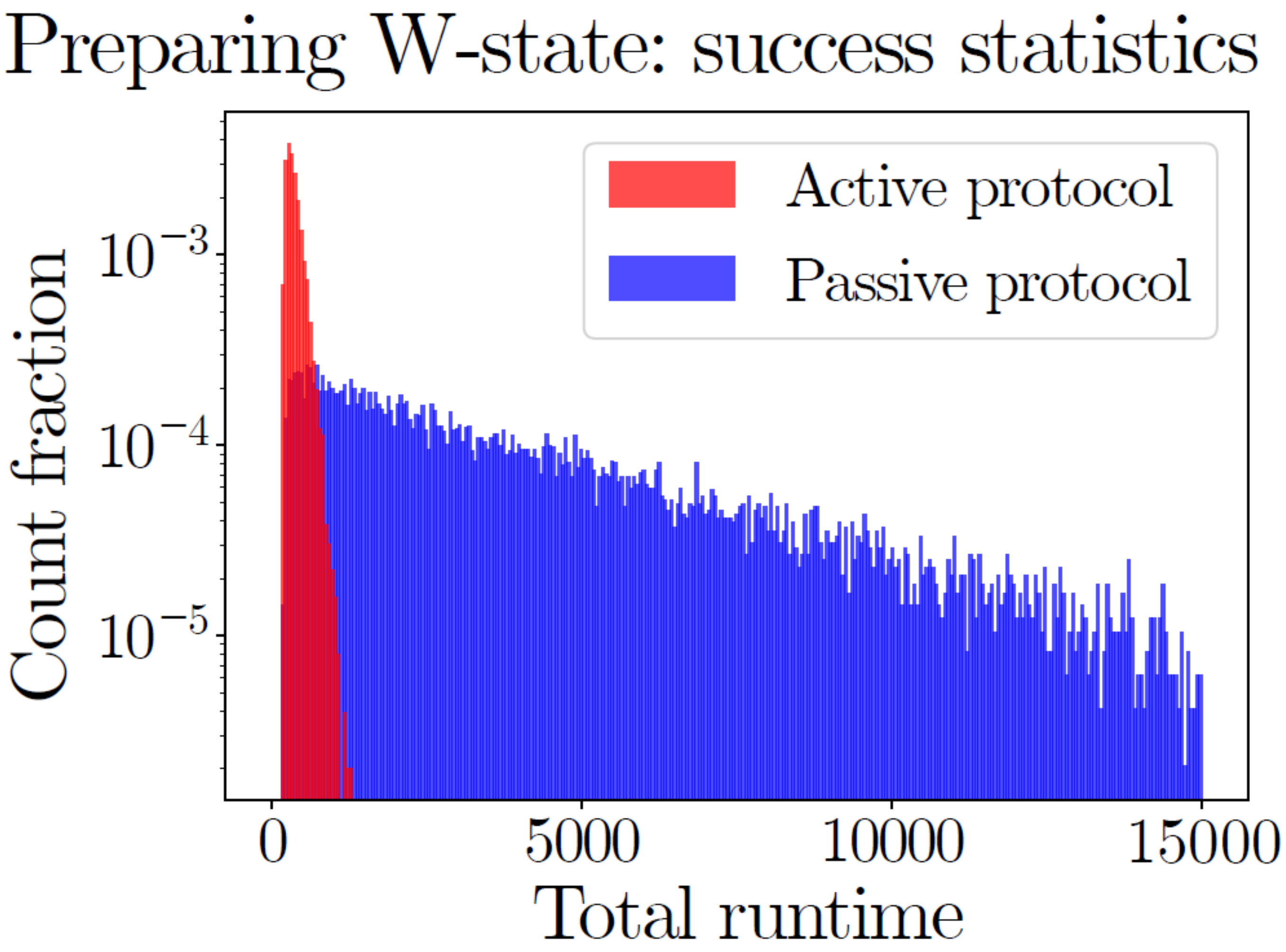
initial state :

$$\overset{ABC}{|000\rangle}$$

possible steering operators :

$$V_1 = \sigma_A^+ - \sigma_C^+,$$
$$V_2 = \sigma_A^- \sigma_C^-,$$
$$V_3 = \sigma_A^- \sigma_B^+ - P_A^0 P_B^1,$$
$$V_4 = \sigma_B^+ \sigma_C^- - P_B^1 P_C^0,$$

$$H_{s-d} = V_i \sigma_d^+ + h.c.$$


Preparing W-state: success statistics
Active protocol
Passive protocol
Count fraction
10^{-3}
10^{-4}
10^{-5}
0
5000
10000
15000
Total runtime

Future Challenges

- scale up system sizes (eg AKLT)
- local cost function (milestone trajectory)
- role of frustration
- state manipulation / logical gates / error correction

…

THANK YOU