Dynamics, Complexity, and Entanglement of Noisy Random Circuits

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Collaborations and References

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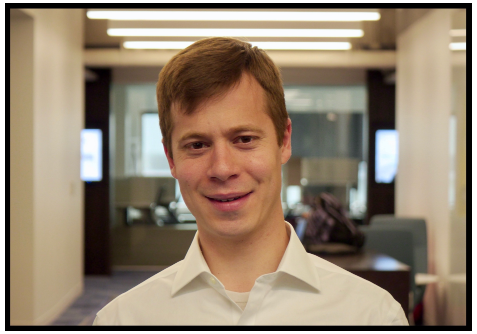


Bill Fefferman

Reference: A. Deshpande, P. Niroula, O. Shtanko, A. V. Gorshkov, B. Fefferman, and M. J. Gullans, arXiv:2112.00716, QIP2022

Discussions: Alex Dalzell

Related work: A. Dalzell, N. Hunter-Jones, and F. G. S. L. Brandao, arxiv:2111:14907, QIP2022

Ongoing work on classical simulations also with Brayden Ware

Noisy circuits without error correction

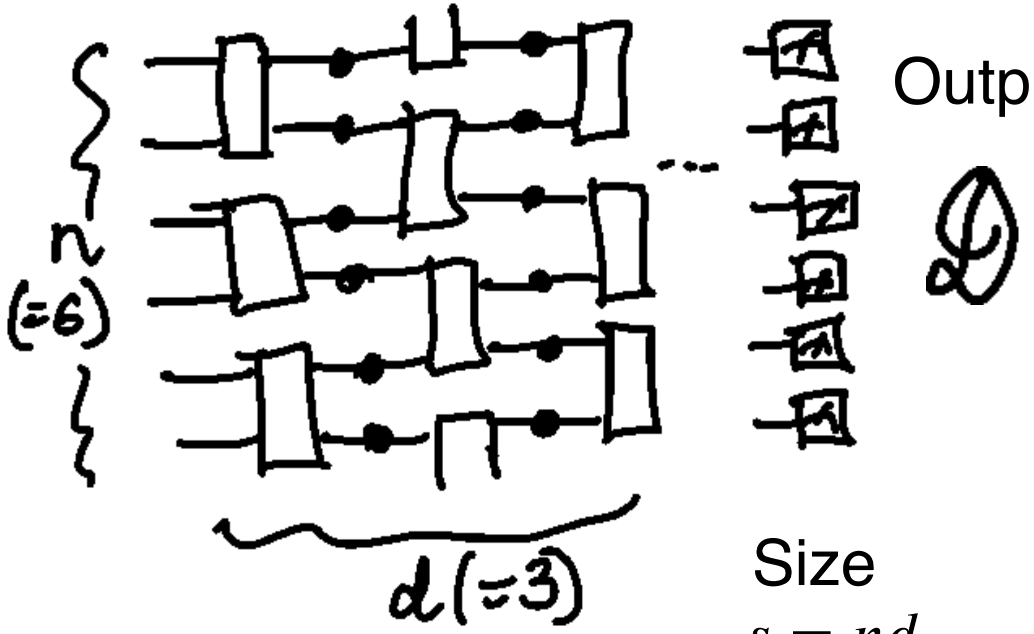
- Noisy, intermediate-scale, quantum (NISQ) era: tradeoff between scale (system size) and low noise
  - Currently can achieve one or the other, but hard to do both
- Goal: quantify this tradeoff and witness the “quantum signal”
- Has a direct bearing on algorithms: informs us which algorithms can work in practice
- We study random circuits
  - quantum computational advantage
  - benchmarking noise
  - modeling near-term algorithms
  - chaotic & scrambling dynamics
  - exponentially long growth of complexity

Model - Parallel circuit architecture

- Haar-random gates
- Noise model: single-qubit Pauli noise after every time step:

$$\mathscr{E}[\rho] = \sum_{P_i \in P} p_i P_i \rho P_i$$

- Noise strength: $$\sum_{P_i,\, P_i \neq I} p_i = \gamma$$
- Intuition: Pauli noise generally adds entropy, which we are not removing



Output

$\mathcal{D}$

Size

$s = nd$

- Study convergence properties of output distribution with respect to uniform distribution
- <u>Total variation distance (TVD) to uniform</u> and <u>anticoncentration</u>

Distance to uniform distribution

- Prior work studied upper bounds on $\delta := \left|\left|\mathscr{D} - \mathscr{U}\right|\right|$ in different settings
- Aharonov *et al.*[1] (foundational work on fault-tolerance): with depolarizing noise, δ decays exponentially for any arbitrary circuit: $\delta \leq 2^{-\Omega(\gamma d)}$.
- Gao and Duan[2]: for random circuits and slightly more general Pauli noise, $\mathrm{E}_C[\delta] \leq 2^{-\Omega(\gamma d)}$
- Boixo et al.[3,4] numerically observed a faster decay to uniform (of the form $2^{-\Omega(\gamma s)}$) for small sizes
- What about lower bounds? Not known so far for random circuits

[1] Aharonov et al., arXiv:quant-ph/9611028 [2] Gao and Duan, arXiv:1810.03176 [3] Boixo et al., Nat. Phys.14, 595-600 (2018) [4] Boixo et al., arXiv:1708.01875

Lower bound on TVD

- Our result: for arbitrary local Pauli noise, $\mathrm{E}_C[\delta] \geq e^{-O(d)}$

Depth dependence of distance to uniformity is tight.

- Disproves the $e^{-\Omega(\gamma s)}$ possibility for large depth/noise
  - Matches result of Dalzell *et al.*[1] who showed $e^{-\Omega(\gamma s)}$ in limit $\gamma \to 0$
- Our result applies in the physically relevant limit of $\gamma = \Theta(1)$

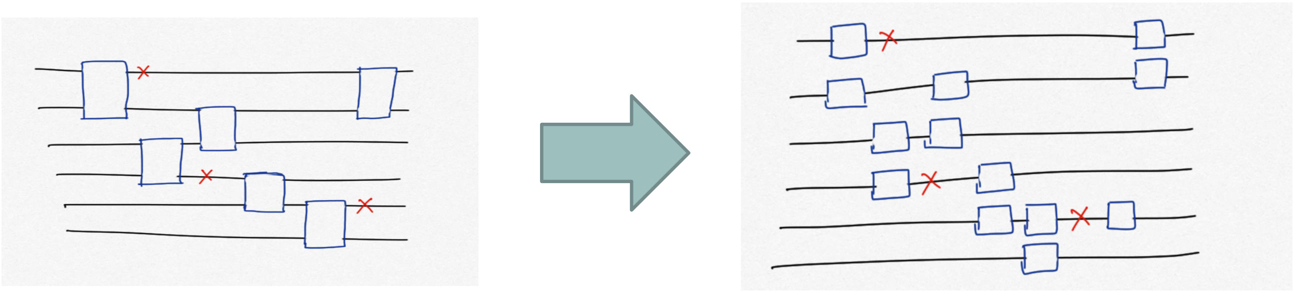
[1] Dalzell, Hunter-Jones, Brandão, arXiv:2111.14907

Intuition for lower bound

- By the data processing inequality, it suffices to consider a single-qubit density matrix ρ on 1 site at the output
- Show that ρ is far-enough away from $\mathbb{I}/2$
    - Works because we can consider second moments and replace Haar-random gates by Cliffords
- At constant depth, this density matrix cannot “know” how many other qubits there are, so no n dependence at constant d.

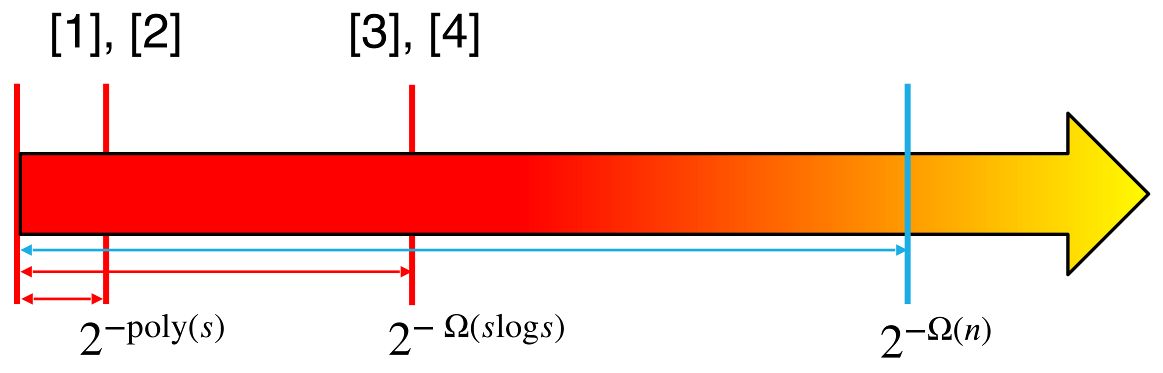
Upper bound on total-variation distance

- For Pauli noise with heralded error locations, we prove an upper bound on TVD of the form $\mathbb{E}_B[\delta] \leq \text{poly}(n)\exp(-\Omega(d))$
- Implications for measurement-induced entanglement transitions[1] - Rules out a phase transition with unknown measurement outcomes
- Method is based on a mapping from replica averages of quantum circuits to a statistical mechanics model for Ising spins[2]
  - Proof idea: recursively construct a new circuit composed solely of single-qubit gates and dephasing events



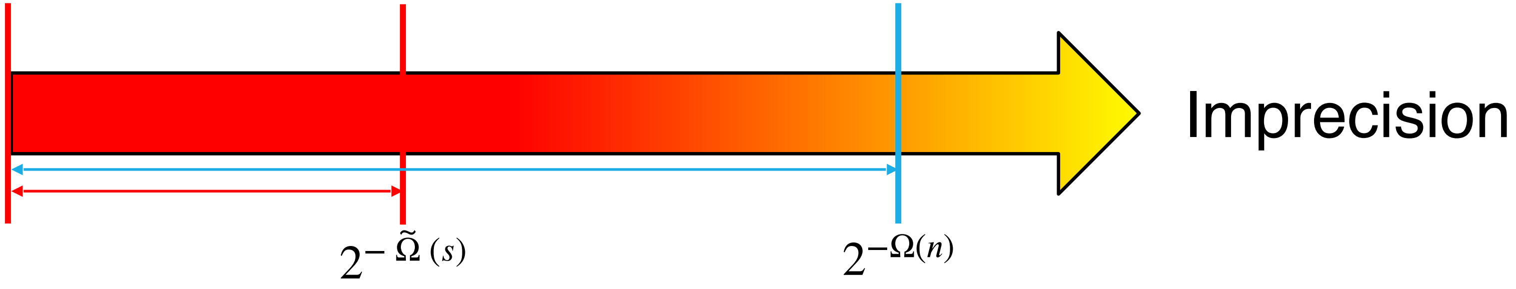
Implication in quantum advantage: Context

- Quantum advantage via random circuit sampling, assuming a complexity-theory conjecture.
- Suffices to show that approximating output probability $p_{00\ldots0}$ of random circuits to imprecision 2^{-n} is hard.
- 2^{-n} comes from Hilbert space dimension



Recall: $s = nd$

Tightness of quantum advantage: Results

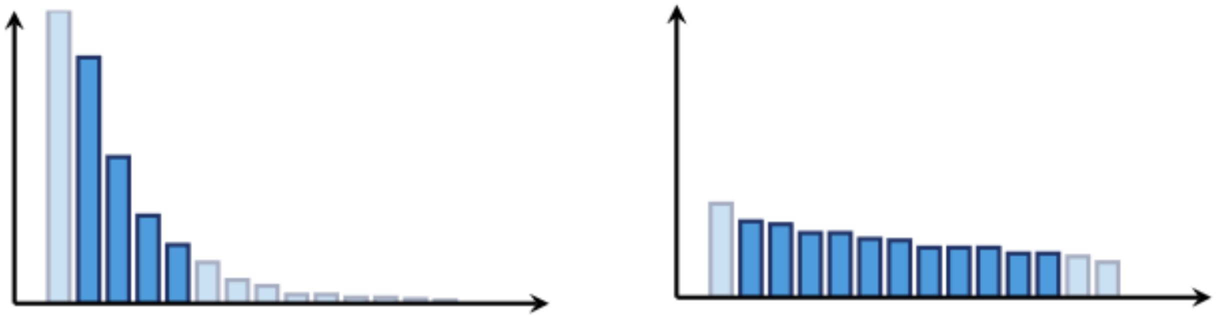


- Is hardness with $2^{-\tilde{\Omega}(s)}$ imprecision optimal?
- One way of probing: hardness results also valid for noisy circuits! Ask whether these are tight for noisy circuits
- Because of convergence to uniform, trivial algorithm "guess 2^{-n}" can in principle work well.
- If δ behaved as $2^{-\Theta(s)}$, then hardness results optimal
- This is not the case $\Rightarrow$ methods can be pushed further! ☺

Anticoncentration

- A property cited in the past as an important measure for quantum advantage. Measures how "spread out" a distribution is.

Output mass mostly distributed on few outcomes



Not anticoncentrated

Anticoncentrated

Output mass well distributed on almost all outcomes

- Formally, a distribution is anticoncentrated if there are constants c, a, such that $\Pr_{C}\left[p_{00\ldots0} \geq \frac{c}{2^n}\right] \geq a$
- Not inherently a hardness property (neither necessary nor sufficient for hardness of sampling)
- Can be a useful property when combined with other properties

Anticoncentration and depth dependence

- Physics intuition: Probes thermalization time of random circuits
- Follows from approximate 2-design property[1-3], but not necessary[4]
- State of the art result[4]: 1D log-depth circuits anticoncentrate
- Can these results be improved further?
- We show: for $d = o(\log n)$, $\Pr_B \left[p_{00\ldots0} \geq \frac{c}{2^n} \right] = o(1)$

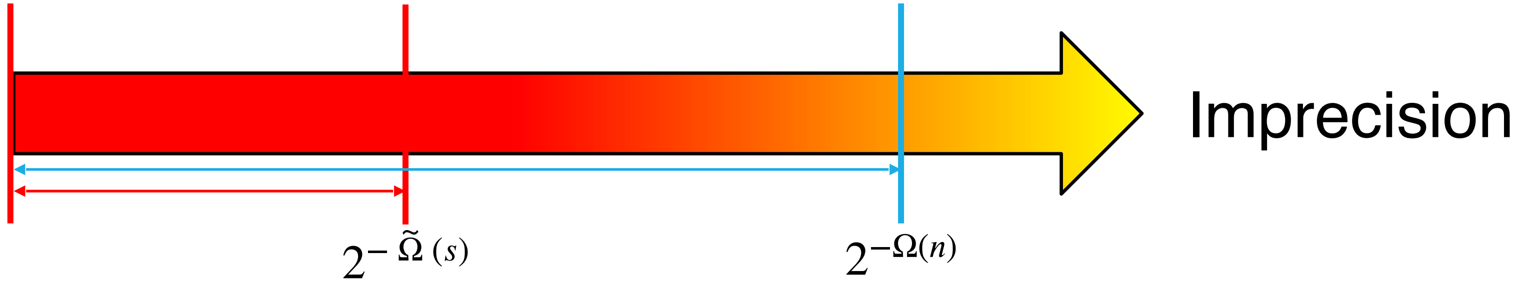
Severe lack of anticoncentration at sub-logarithmic depth

- Applies to both noisy and noiseless random circuits.

[1] Harrow & Low, Comm. Math. Phys. (2009)
[2] Brandão, Harrow, & Horodecki, Commun. Math. Phys. (2016)
[3] Harrow and Mehraban, arXiv:1809.06957
[4] Dalzell, Hunter-Jones, Brandão, PRX Quantum 3, 010333 (2022)

Application: constant-depth circuits

- Napp et al.[1]: it can be average-case hard to compute $p_{00\ldots0}$ to error 2^{-s}, but average-case easy to compute to 2^{-n}.



- Small constant depth: uses a matrix-product-states-based "space-evolving block decimation" (SEBD) algorithm
- Our results $\Rightarrow$ don't need the above
- Trivial algorithm "output 0" works well to compute $p_{00\ldots0}$ on average: already achieves error smaller than 2^{-n}.

Not a sampling algorithm!

[1] Napp et al., arXiv:2001.00021

Summary

- Lower (and upper) bounds on how fast noise takes you to uniform
  - $E_B[\delta] \sim \exp[-\Theta(d)]$
  - Tight w.r.t. depth dependence

- Anticoncentration for both noisy and noiseless random circuits
  - Sub-log depth: poorly anticoncentrated
  - Log-depth and larger: anticoncentrated

Outlook

- Tightness of current proof techniques for hardness
  - No obvious hurdle right now in improving techniques
- Typicality of δ? We have computed $\mathbb{E}_C[\delta]$
  - Can show typicality in some cases
- Implications for noise-induced barren plateaus[1]
  - At $\log n$ depth, enough information in output distribution to distinguish from uniform
- Log depth is the sweet spot!
- Classical simulations of noisy circuits might be efficient at low precision - implications for benchmarking and fault-tolerant simulations

[1] Wang et al., Nat. Comms. 12, 6961 (2021)

Thank you!