Bosonic Quantum Error Correction

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SPICE Workshop on Non-Equilibrium Emergence in Quantum Design
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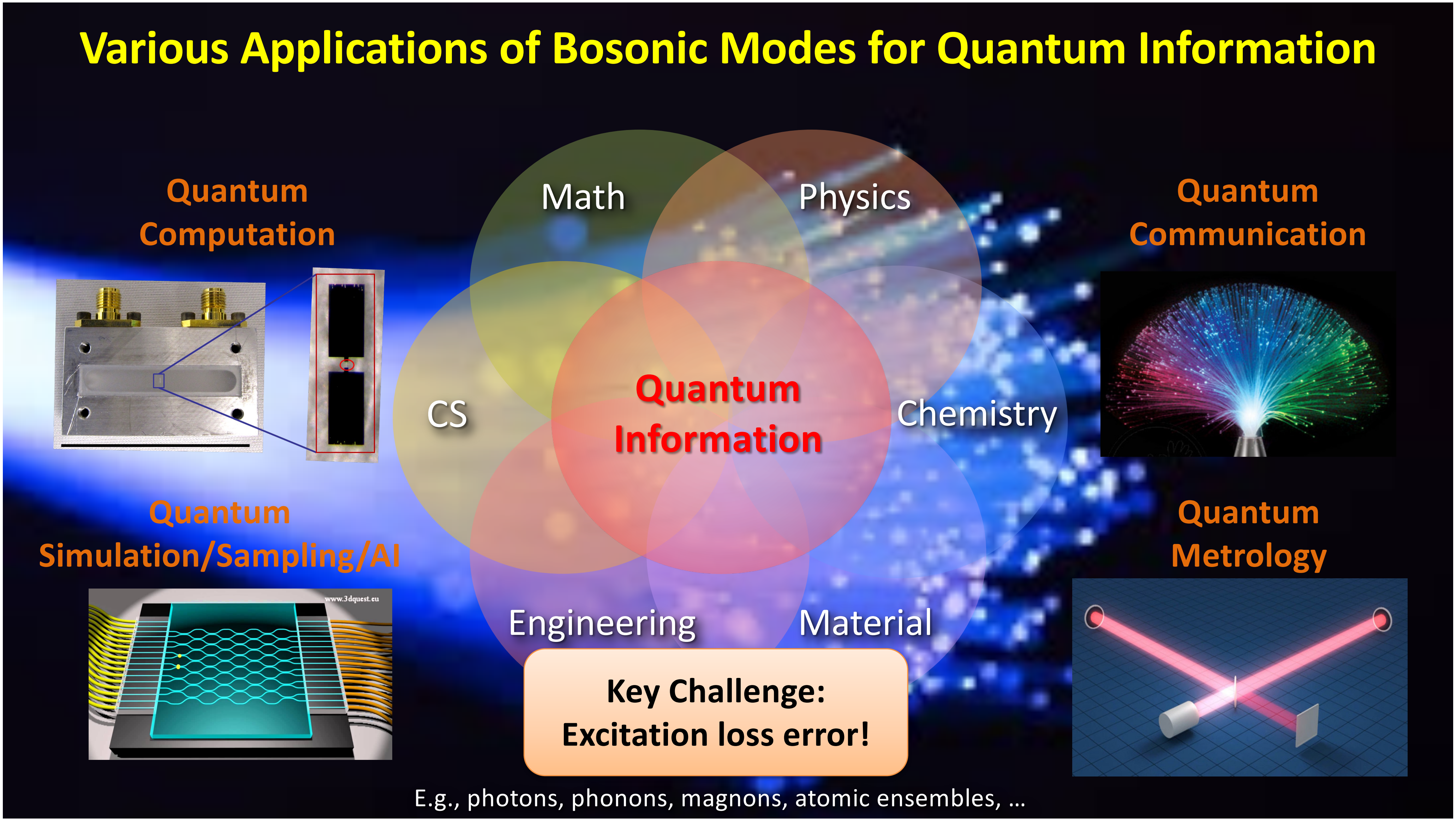
PRITZKER SCHOOL OF MOLECULAR ENGINEERING

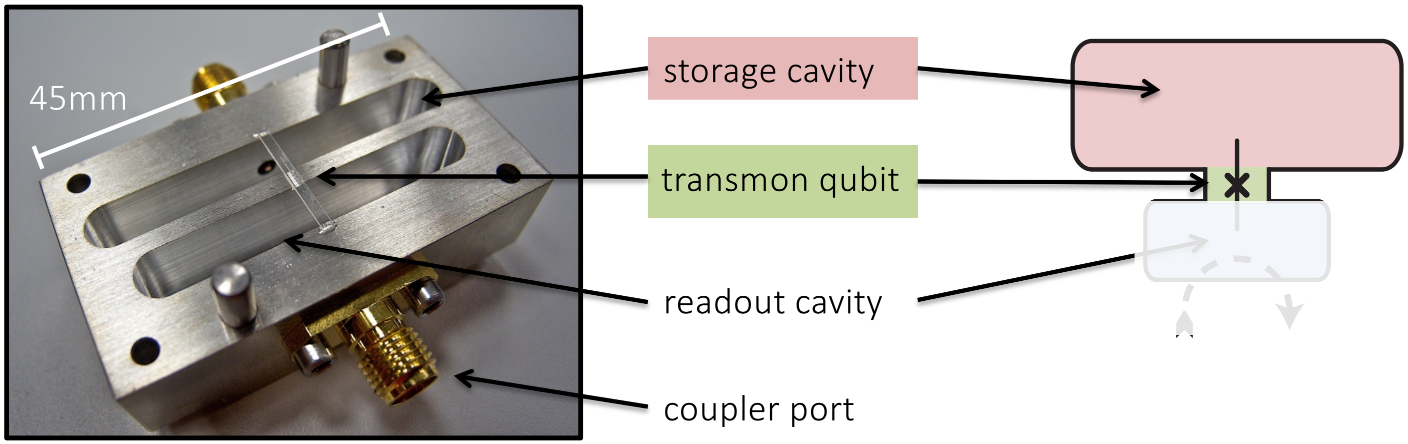

Various Applications of Bosonic Modes for Quantum Information
Quantum Computation
Quantum Communication
Math
Physics
CS
Quantum Information
Chemistry
Engineering
Material
Quantum Simulation/Sampling/AI
www.3dquest.eu
Quantum Metrology
Key Challenge: Excitation loss error!
E.g., photons, phonons, magnons, atomic ensembles, …

Quantum Control of Bosonic Modes

Key Challenges:

- How to achieve **universal control** of individual bosonic modes?
- How to **correct excitation loss errors** for quantum computing?
- How to implement quantum non-demolition (QND) parity measurement?

Superconducting Cavity-Qubit Module



Effective Hamiltonian:

$$H = \omega_s a^\dagger a + \omega_q |e\rangle\langle e| - \chi_s a^\dagger a |e\rangle\langle e|$$

- Long lived SC cavity (T>10 ms >> T_{qubit}~100us)
- Fast QND readout of the qubit (e.g., F>99.5% in 300 ns)
- Adaptive control with FPGA

Strong Dispersive Regime:

$$\chi_s(\sim\text{MHz}) \gg \kappa_s, \gamma_q(\sim\text{kHz})$$

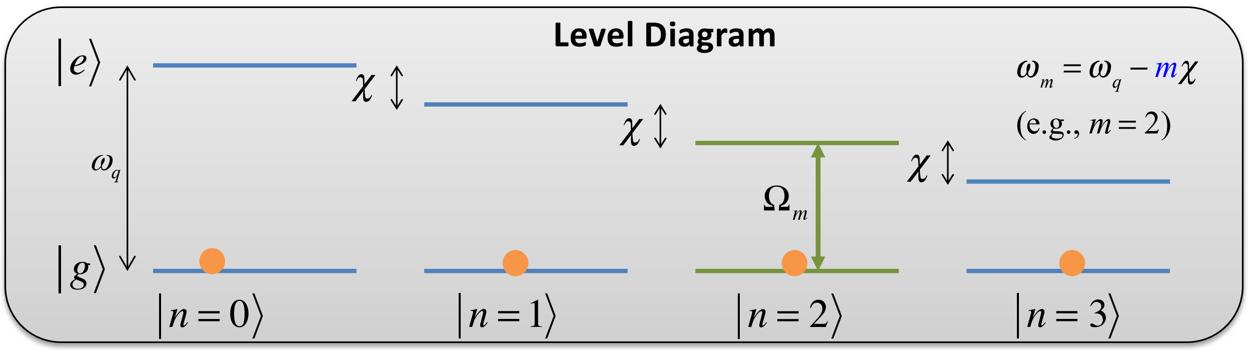
χ_s: dispersive coupling strength
κ_s: photon decay rate
γ_q: transmon decoherence rate

2D device: Nature 445, 515 (2007). 3D device: PRL107, 240501 (2011). Nanomechanical oscillator: Nature 571, 537 (2019); PRX 9, 021056 (2019)

Control of Cavity-Qubit System

$$H = \omega_s a^\dagger a \quad + \varepsilon(t) a^\dagger e^{i\omega_s t} + h.c. \qquad \text{Cavity drive}$$

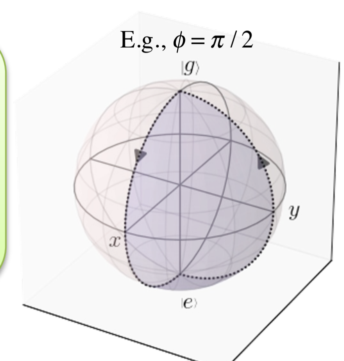
$$+ (\omega_q - \chi_s a^\dagger a)|e\rangle\langle e| \quad + \sum_m \Omega_m(t)|e\rangle\langle g| e^{i(\omega_q - m\chi_s)t} + h.c. \qquad \text{Qubit drive (selective)}$$

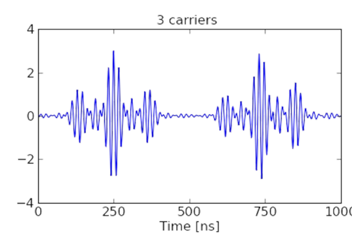


Geometric phase gate

Using two pi-pulses along different axes

$$|\psi'\rangle = \begin{cases} R_0^{(m)}(\pi) R_{\pi+\phi_m}^{(m)}(\pi)|n,g\rangle = e^{i\phi_m}|n,g\rangle & \text{for } n = m \\ |n,g\rangle & \text{for } n \neq m \end{cases}$$





Theory: Krastanov, L.J., et al., PRA 92, 040303; Experiment: Heeres, L.J., Schoelkopf, *et al.* PRL 115, 137002 (2015)

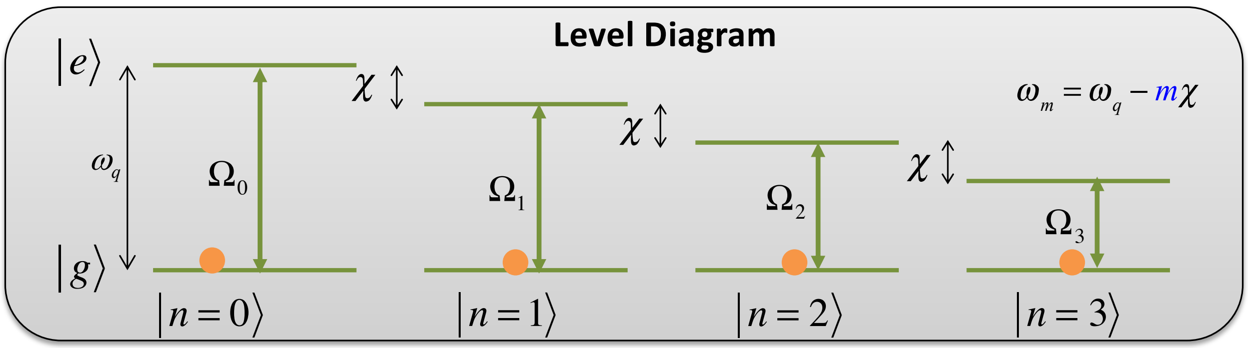
SNAP (Selective Number-dependent Arbitrary Phase) Gate

$$H = \omega_s a^\dagger a + \varepsilon(t) a^\dagger e^{i\omega_s t} + h.c.$$

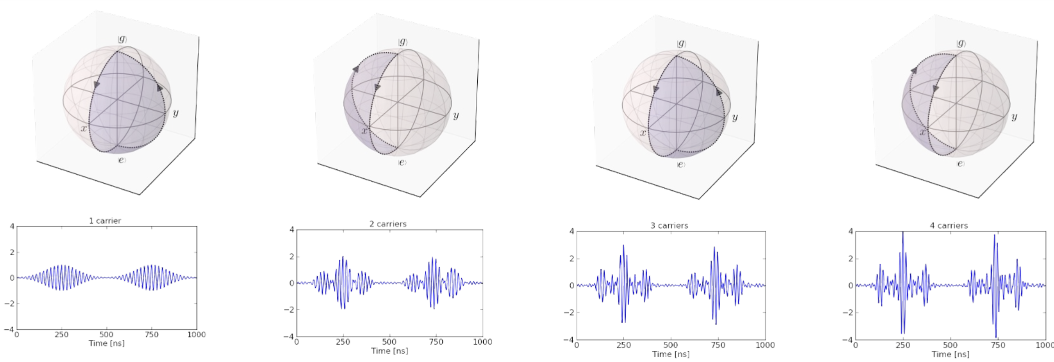
Cavity drive

$$+\left(\omega_q - \chi_s a^\dagger a\right)|e\rangle\langle e| + \sum_m \Omega_m(t)|e\rangle\langle g| e^{i(\omega_q - m\chi_s)t} + h.c.$$

Qubit drive (selective)



Simultaneous Geometric phase gates for all |n>



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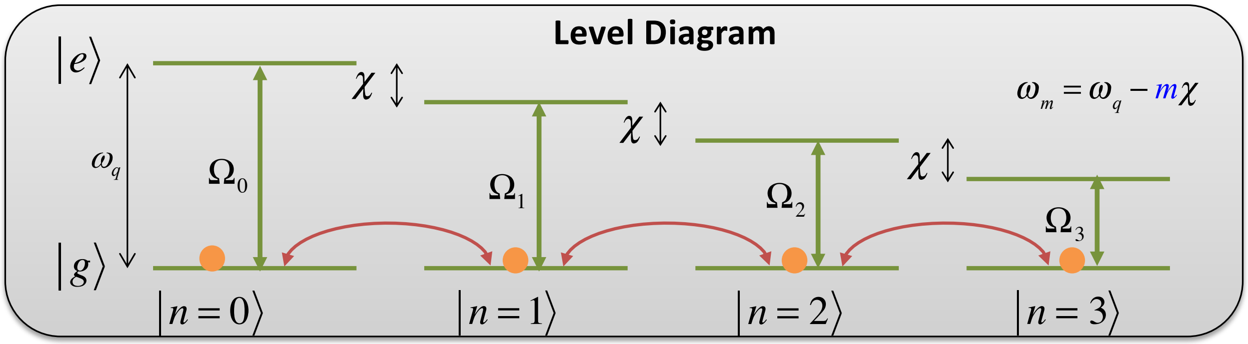

Qubit Drive $|g\rangle \rightarrow |g\rangle$

Simultaneous Geometric phase gate

$$|\psi\rangle = \sum_n c_n |n,g\rangle \longrightarrow |\psi'\rangle = \sum_n e^{i\phi_n} c_n |n,g\rangle$$

SNAP Gate: $$S_{\vec{\phi}} = \sum_n e^{i\phi_n} |n\rangle\langle n|$$

Cavity Drive $|g\rangle\langle g|$

Displace the cavity field with qubit in |g>

$$|\psi\rangle = \sum_n c_n |n,g\rangle \longrightarrow |\psi'\rangle = D(\alpha) \sum_n c_n |n,g\rangle$$

Displacement: $$D_\alpha = \exp\left[\alpha a^\dagger - \alpha^* a\right]$$

Theory: Krastanov, L.J., et al., PRA 92, 040303; Experiment: Heeres, L.J., Schoelkopf, *et al.* PRL 115, 137002 (2015)

SNAP (Selective Number-dependent Arbitrary Phase) Gate

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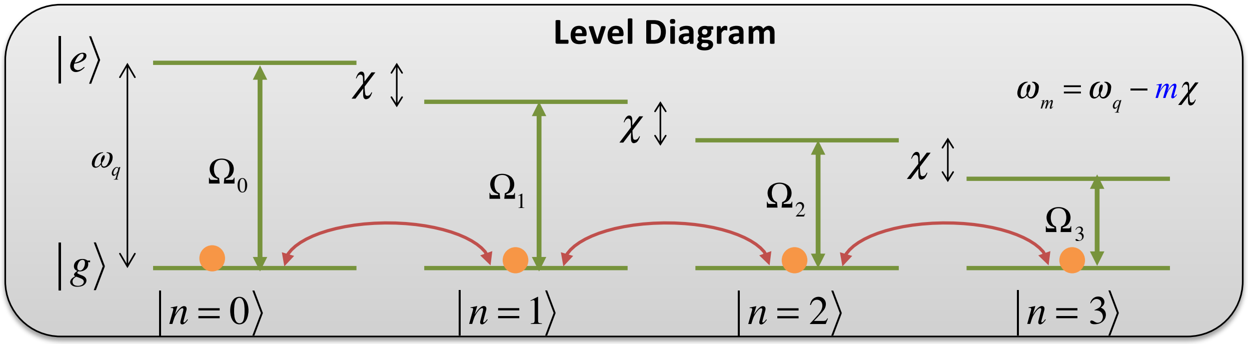
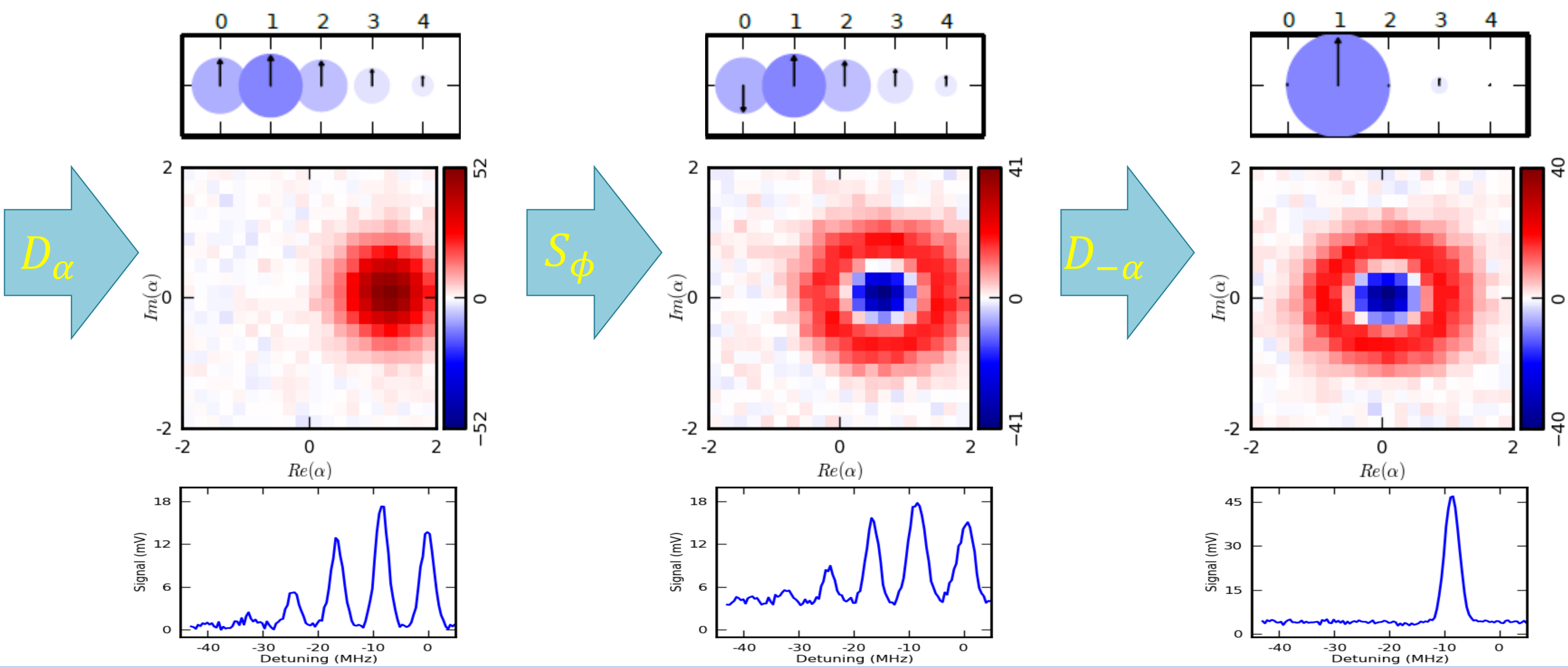


Universal Control of cavity-qubit system with Cavity & Qubit Drives.

Theory: Krastanov, L.J., et al., PRA 92, 040303; Experiment: Heeres, L.J., Schoelkopf, *et al.* PRL 115, 137002 (2015)

Application 1: Arbitrary State Preparation

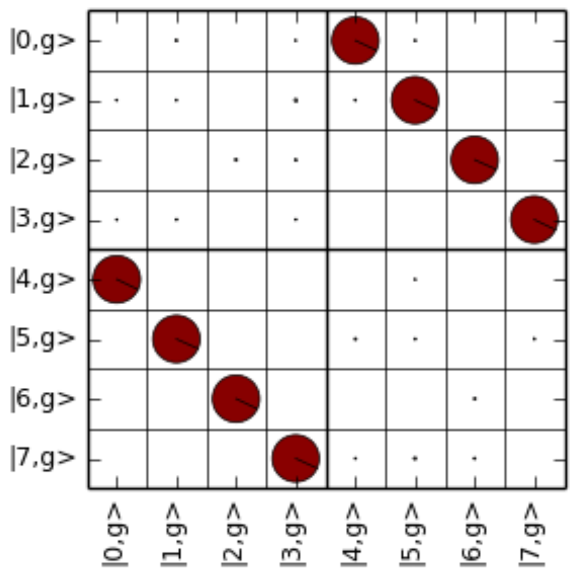
Three-pulse scheme: $\boldsymbol{D_{-\alpha} S_{\phi} D_{\alpha}}|0\rangle \approx |1\rangle$ prepares single photon state deterministically.



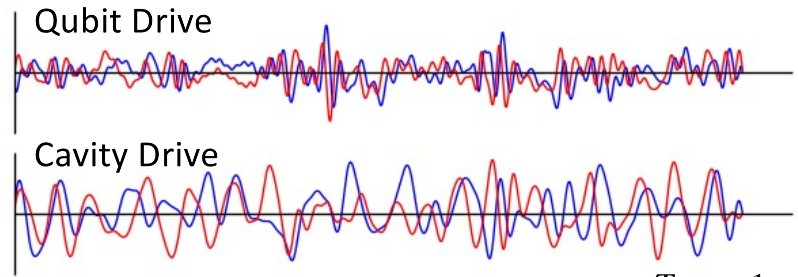
Heeres, L.J., Schoelkopf, *et al.* PRL 115, 137002 (2015)

Application 2: Arbitrary Unitary Control of Bosonic Mode

Target Unitary

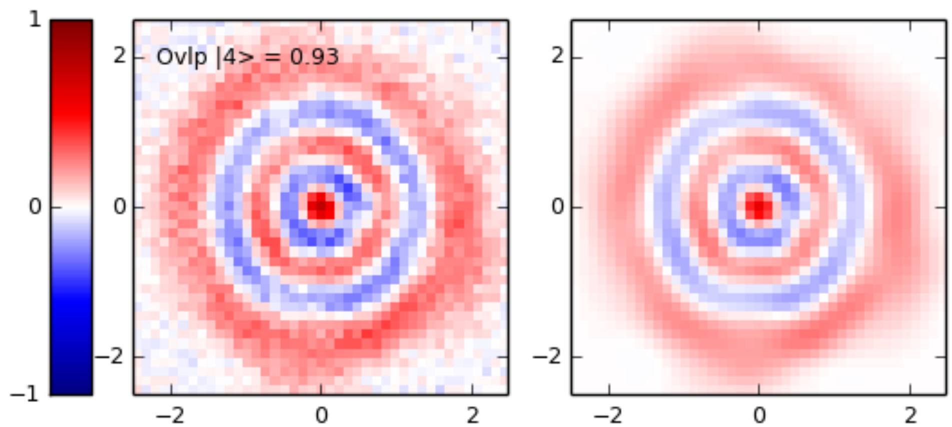


GRAPE Optimized Pulses (simultaneous qubit & cavity drives)



$T_{pulse} \sim 1\mu s$

Tomography

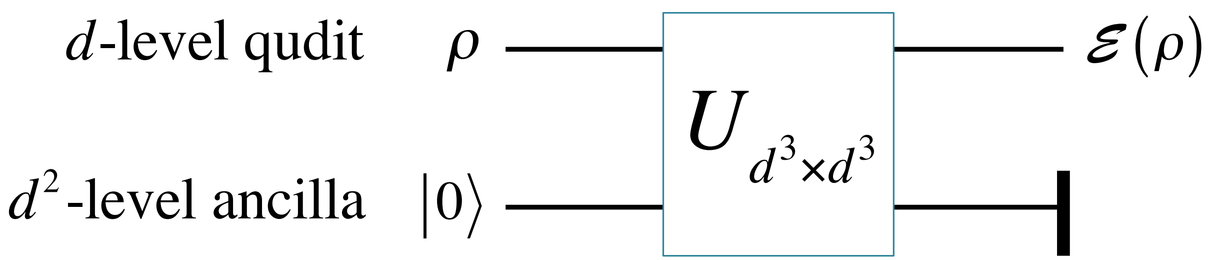


Application 3: Arbitrary Quantum Process

Most general CPTP *(Complete Positive Trace Preserving)* Map:

Kraus representation of CPTP channel

$$\rho \rightarrow \mathcal{E}(\rho) = \sum_{i=1}^{N} K_i \rho K_i^{\dagger} \quad \text{with Kraus rank } N \leq d^2 \text{ and } \sum_{i=1}^{N} K_i^{\dagger} K_i = I.$$



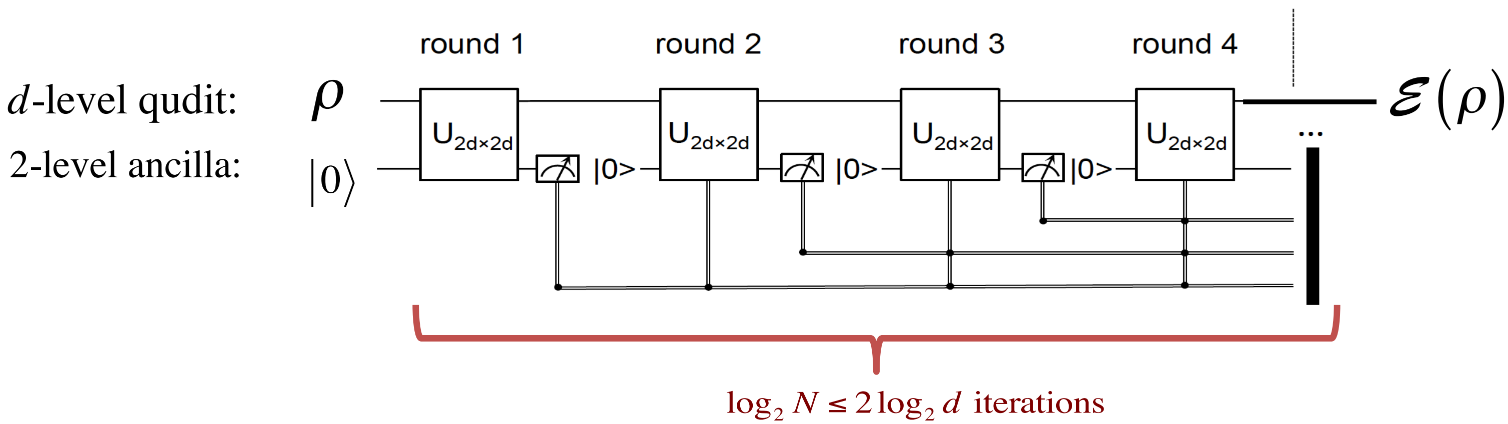
Challenging to use **d^2-level ancilla** and implement $d^3 \times d^3$ large unitary operation.

Application 3: Arbitrary Quantum Channel of d-Level System

Most general CPTP map for d-level system:

$$\rho \to \mathcal{E}(\rho) = \sum_{i=1}^{N} K_i \rho K_i^\dagger \quad \text{with Kraus rank } N \le d^2 \text{ and } \sum_{i=1}^{N} K_i^\dagger K_i = I.$$

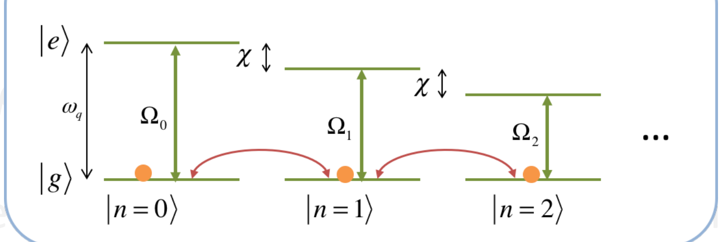
Key Idea: Repeatedly use the 2-level ancilla & adaptive control

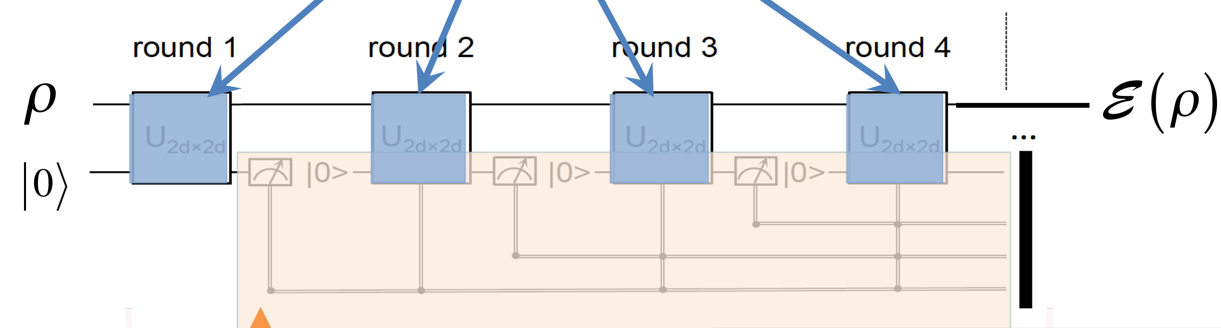


Shen, L.J., et al, PRB 95, 134501 (2017); also: Iten, et al., PRA 95, 052316 (2017); Earlier visionary proposal: Lloyd, Viola, PRA 65, 010101 (2001).

Application 3: Arbitrary Quantum Channel of d-Level System

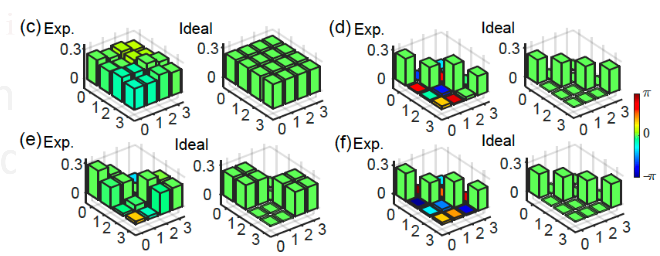
Control Diagram of Cavity-Transmon System





QND Measurement of Transmon & Adaptive Control with FPGA

Experiment @ Tsinghua (d=4, F>90%)



Shen, L.J., et al, PRB 95, 134501 (2017); Experiment at Luyan Sun's group at Tsinghua, Cai, Han, et al., arXiv:2010.11427

Summary of Quantum Control over Bosonic Modes

State Preparation

$$|0\rangle \rightarrow |\phi_{\text{target}}\rangle = U|0\rangle = \sum_{n=0}^{d-1} c_n |n\rangle$$

Unitary Gates

$$\mathcal{U}(\rho) = U\rho U^{\dagger}$$

Quantum Channel

$$\rho \rightarrow \mathcal{E}(\rho) = \sum_{i=1}^{N} K_i \rho K_i^{\dagger}$$

Quantum Error Correction of Bosonic Loss Errors

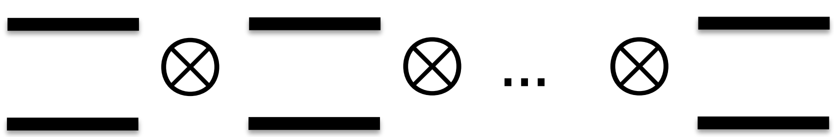
Key Challenges:

- How to achieve **universal control** of individual bosonic modes?
- How to **correct excitation loss errors** for quantum computing?
- How to implement quantum non-demolition (QND) parity measurement?

Multi-Qubit Systems vs Oscillator Systems

Qubits

- Hilbert space consists of tensor products of two-level systems.



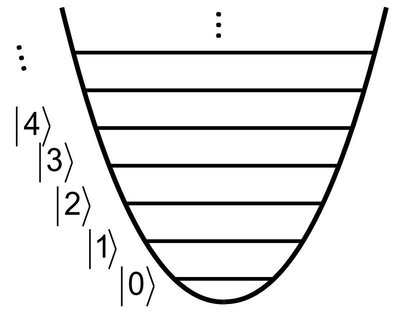
- Dominant errors expressible in terms of the Pauli operators

$$\{\sigma_X, \sigma_Y, \sigma_Z\}$$

- Distance of code: # of tolerable qubit loss
- Hardware inefficient (large overhead)

Oscillators

- Hilbert space is a ladder of Fock states $|n\rangle$ for $n \geq 0$.



- Dominant excitation loss error (and/or dephasing error)

$$\{a^{\ell}\} \text{ with } a|n\rangle = \sqrt{n}|n-1\rangle$$

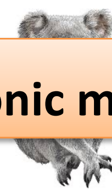
- Distance: # of tolerable excitation loss
- Hardware efficient (e.g., single oscillator)

Zoo of Bosonic Codes

Fock states $|0\rangle, |1\rangle$ only

- **Dual-rail**: Chuang Yamamoto 1995
- **Linear optics**: Knill Laflamme Milburn 2001
- **Parity check**: Ralph Hayes Gilchrist 2005
- Any codes correcting against qubit amplitude damping: [37,64-73] in arxiv:1708.05010v2

Coheren



- **Cat (no loss protec**
- Niset Andersen Cerf 2008
- **Cat**: Leghtas Kirchmair Vlastakis Schoelkopf Devoret Mirrahimi 2013
- **Coherent state constellation (scheme)**: Lacerda Renes Scholz 2016
- **Multimode cat**: Albert Mundhada Grimm Touzard Devoret L.J. 2018

Higher Fock state superpositions

- Chuang Leung Yamamoto 1997
- Wasilewski Banaszek 2007
- **Noon**: Boto Kok Abrams Braunstein Williams Dowling 2000
- **Bin**: Michael Silveri Brierley Albert Salmilehto L.J. Girvin 2016
- χ^2: Niu Chuang Shapiro 2017

Position/momentum eigenstates

- Lloyd Slotine 1998
- Braunstein 1998

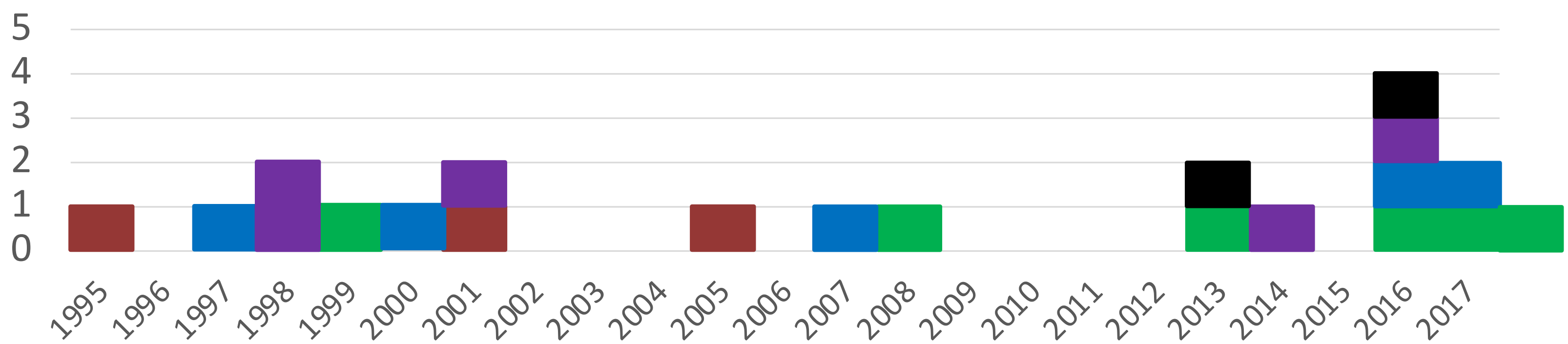
- **Can we use single bosonic mode to correct loss errors?**

Hybrid – multi-qubit codes

- **Hybrid optical qubits**: Lee Jeong 2013
- **Very small logical qubit**: Kapit 2016

Bosonic Codes by year

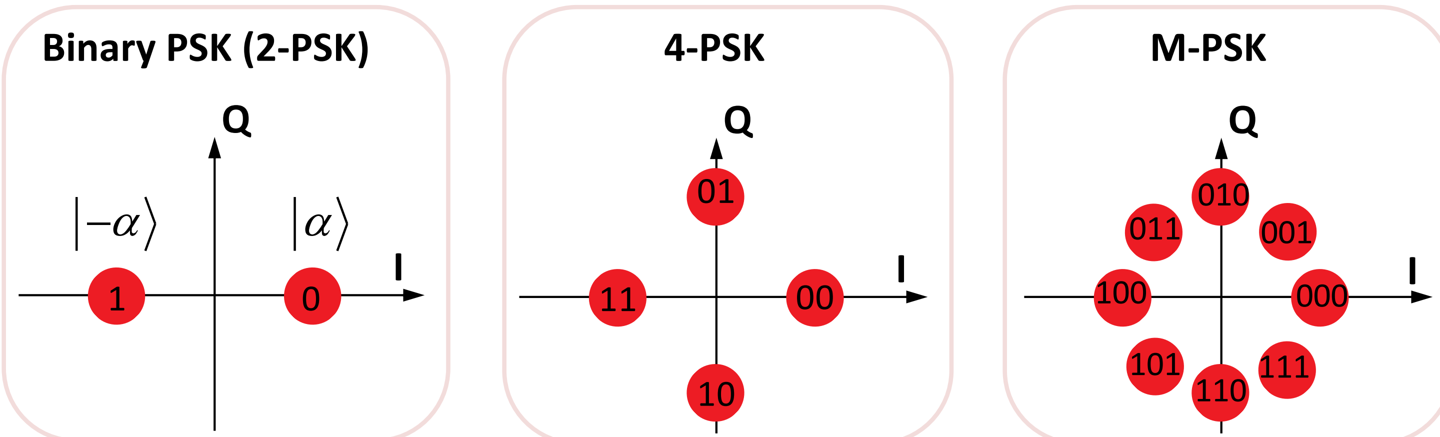
Prepared by: Victor Albert (Zookeeper)



See more at Error Correction Zoom: https://errorcorrectionzoo.org/

Classical Encoding – Phase-Shift Keying (PSK)

- **<u>Idea:</u>** Encode information in the phase of a coherent state, $\alpha = |\alpha|e^{i\phi}$

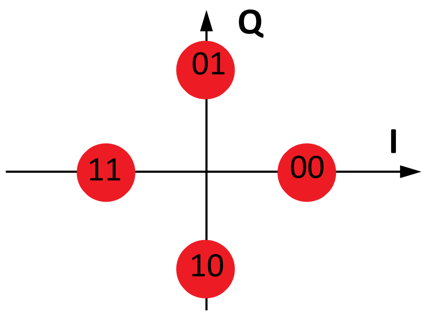


- Encode more information by increasing the number of phase angles
- Widely used in Wifi, Bluetooth, etc.

(Classical) PSK v.s. (Quantum) Cat Codes

Classical Phase-Shift Keying (PSK):

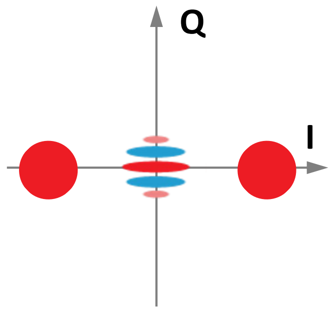
- Convey data by modulating phase of a pulse signal

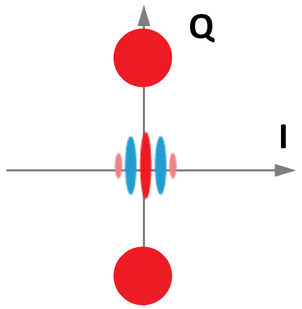


$$\{00, 01, 10, 11\}$$

(Quantum) Cat Codes:

- **Superposition** of coherent states
- Fixed (even) **parity** logical subspace





$$|\tilde{0}_L\rangle = |\alpha\rangle + |-\alpha\rangle \qquad |\tilde{1}_L\rangle = |i\alpha\rangle + |-i\alpha\rangle$$

Leghtas, et al, PRL 111, 120501 (2013); Mirrahimi, Leghtas, Albert, Touzard, Schoelkopf, L.J., Devoret, NJP **16**, 045014 (2014).

Cat Codes

Logical States: $|\tilde{0}_L\rangle = |\alpha\rangle + |-\alpha\rangle$ $\quad |\tilde{1}_L\rangle = |i\alpha\rangle + |-i\alpha\rangle$

within **even** number subspace

Circle of Life

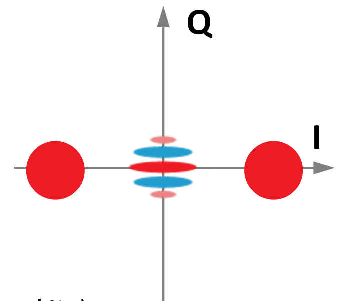
No excitation loss: $|\Psi^{[0]}\rangle = c_g(|\alpha\rangle + |-\alpha\rangle) + c_e(|i\alpha\rangle + |-i\alpha\rangle)$ **even**

1 excitation loss: $|\Psi^{[1]}\rangle = c_g(|\alpha\rangle - |-\alpha\rangle) + ic_e(|i\alpha\rangle - |-i\alpha\rangle)$ **odd**

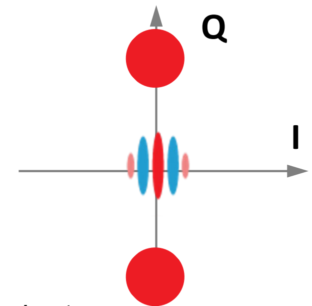
2 excitation loss: $|\Psi^{[2]}\rangle = c_g(|\alpha\rangle + |-\alpha\rangle) - c_e(|i\alpha\rangle + |-i\alpha\rangle)$ **even**

3 excitation loss: $|\Psi^{[3]}\rangle = c_g(|\alpha\rangle - |-\alpha\rangle) - ic_e(|i\alpha\rangle - |-i\alpha\rangle)$ **odd**

4 excitation loss: $|\Psi^{[0]}\rangle = c_g(|\alpha\rangle + |-\alpha\rangle) + c_e(|i\alpha\rangle + |-i\alpha\rangle)$ **even**



$|\tilde{0}_L\rangle = |\alpha\rangle + |-\alpha\rangle$



$|\tilde{1}_L\rangle = |i\alpha\rangle + |-i\alpha\rangle$

Properties:

1. **Detect** single excitation loss by measuring parity
2. Only need to track single photon losses (mod 4)

Correct single loss error!

Leghtas, et al, PRL 111, 120501 (2013); Mirrahimi, Leghtas, Albert, Touzard, Schoelkopf, L.J., Devoret, NJP **16**, 045014 (2014).

Other choices of codes - Kitten/Binomial Codes

Kitten Code: $|0_L\rangle = (|0\rangle + |4\rangle)/\sqrt{2}$

$|1_L\rangle = |2\rangle$

Binomial Code: $|0_L/1_L\rangle = \frac{1}{\sqrt{2^N}} \sum_{p\ even/odd} \sqrt{\binom{N+1}{p}} |p \cdot (S+1)\rangle$

$\boldsymbol{E} = \{I, a\}$

$\boldsymbol{E} = \{I, a, a^2, \cdots, a^S\}$

$$|0_L\rangle = (|0\rangle + |4\rangle)/\sqrt{2}$$
$$|1_L\rangle = |2\rangle$$

a →

$$a|0_L\rangle = \sqrt{2}|3\rangle$$
$$a|1_L\rangle = \sqrt{2}|1\rangle$$

$$c_0 \frac{(|0\rangle + |4\rangle)}{\sqrt{2}} + c_1|2\rangle$$

a →

$$\sqrt{2}(c_0|3\rangle + c_1|1\rangle)$$

Michael, Silveri, Brierley, Albert, Salmilehto, L.J., Girvin, PRX 6, 031006 (2016).

Quantum Error Correction using Cat Code

Key Challenges:

- How to **correct excitation loss errors** for quantum computing?
- How to achieve **universal control** of individual bosonic modes?
- How to implement **quantum non-demolition (QND)** parity measurement?

Recall: Cat Codes

Logical States: $|\tilde{0}_L\rangle = |\alpha\rangle + |-\alpha\rangle$ $\quad |\tilde{1}_L\rangle = |i\alpha\rangle + |-i\alpha\rangle$

within **even** number subspace

Circle of Life

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2 excitation loss: $\left|\Psi^{[2]}\right\rangle = c_g\left(|\alpha\rangle + |-\alpha\rangle\right) - c_e\left(|i\alpha\rangle + |-i\alpha\rangle\right)$ **even**

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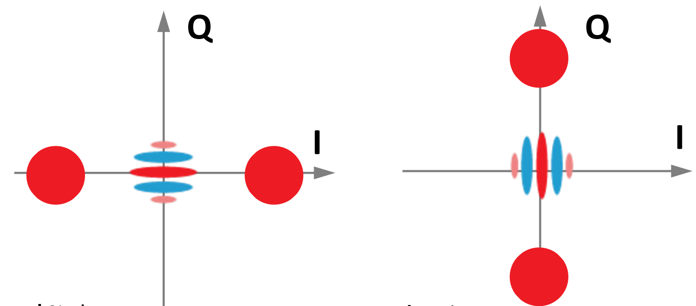
4 excitation loss: $\left|\Psi^{[0]}\right\rangle = c_g\left(|\alpha\rangle + |-\alpha\rangle\right) + c_e\left(|i\alpha\rangle + |-i\alpha\rangle\right)$ **even**



$|\tilde{0}_L\rangle = |\alpha\rangle + |-\alpha\rangle$ $\quad |\tilde{1}_L\rangle = |i\alpha\rangle + |-i\alpha\rangle$

Properties:

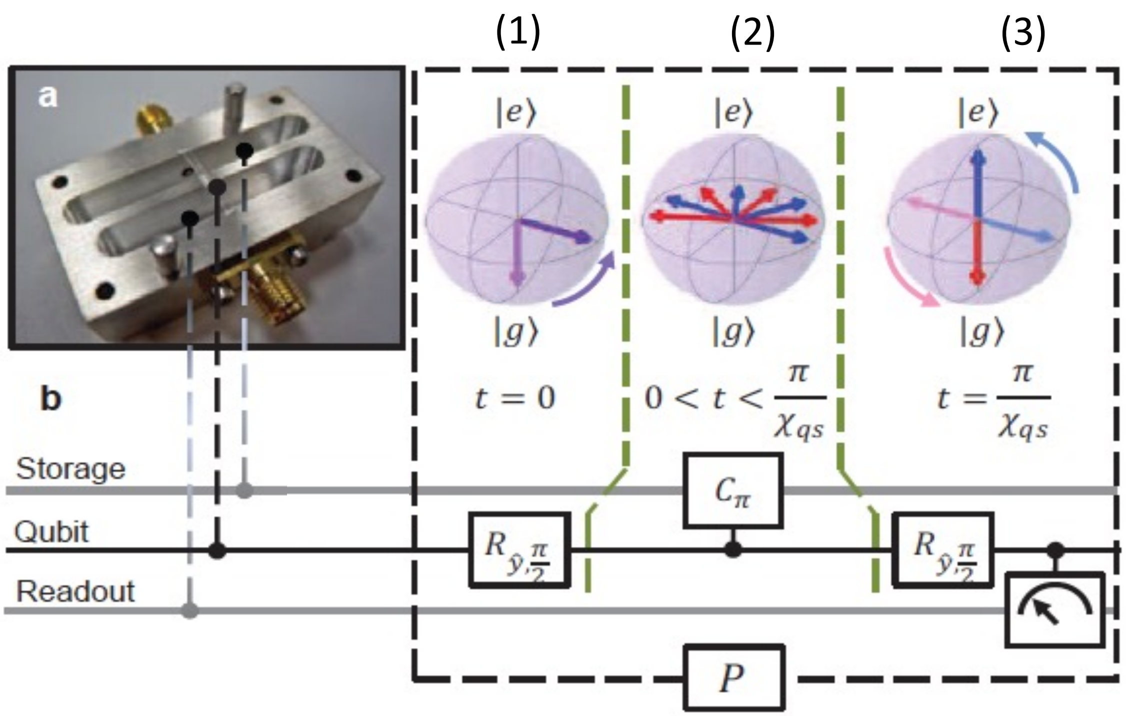
1. **Detect** single excitation loss by measuring parity
2. Only need to track single photon losses (mod 4)

Can correct single loss error!

Question: How to perform QND measurement of parity?

Leghtas, et al, PRL 111, 120501 (2013); Mirrahimi, L.J. et al., NJP **16**, 045014 (2014).

QND Measurement of Photon Number Parity



Procedure:

(1). Transmon initialization: $|g\rangle + |e\rangle$

(2). Phase accumulation:

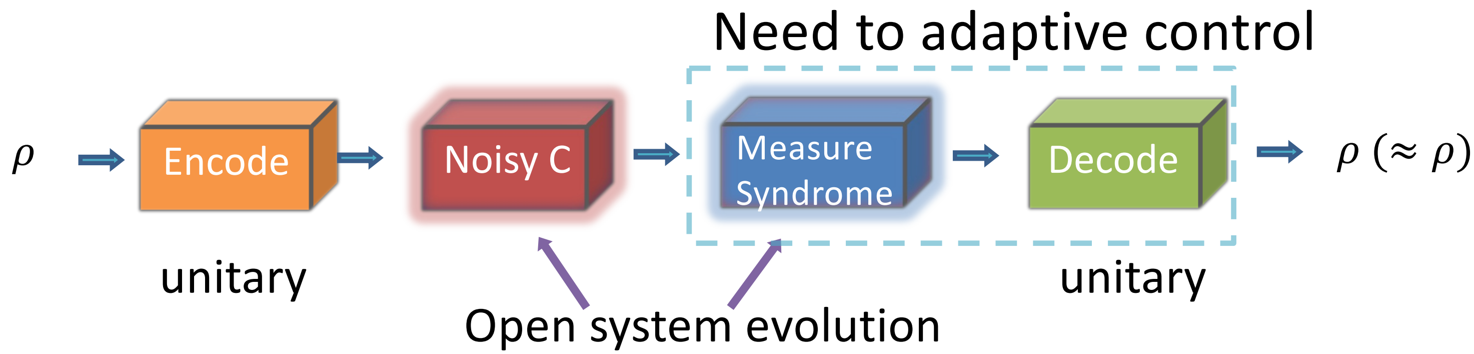
$$H = -\chi a^\dagger a |e\rangle\langle e|$$

$$U\left(T = \frac{\pi}{\chi}\right) = \begin{cases} |g\rangle\langle g| + |e\rangle\langle e| & \text{for even } n \\ |g\rangle\langle g| - |e\rangle\langle e| & \text{for odd } n \end{cases}$$

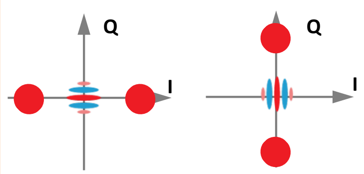
(3). Transmon measurement in

$|g\rangle \pm |e\rangle$ basis

Quantum Error Correction (QEC)

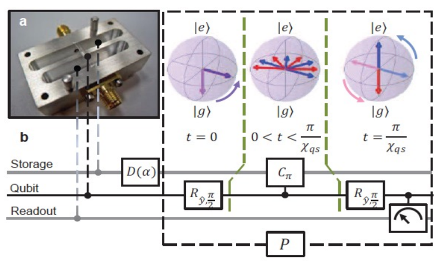


Cat Codes



$|0\rangle_L = |\alpha\rangle + |-\alpha\rangle$ $|1\rangle_L = |i\alpha\rangle + |-i\alpha\rangle$

Parity Monitoring



$$U\left(T = \frac{\pi}{\chi_{qs}}\right) = \begin{cases} |g\rangle\langle g| + |e\rangle\langle e| & \text{for even } n \\ |g\rangle\langle g| - |e\rangle\langle e| & \text{for odd } n \end{cases}$$

Decoding

No photon loss: even

$$|\Psi^{[0]}\rangle = c_g(|\alpha\rangle + |-\alpha\rangle) + c_e(|i\alpha\rangle + |-i\alpha\rangle)$$

1 photon loss: odd

$$|\Psi^{[1]}\rangle = c_g(|\alpha\rangle - |-\alpha\rangle) + ic_e(|i\alpha\rangle - |-i\alpha\rangle)$$

2 photon loss: even

$$|\Psi^{[2]}\rangle = c_g(|\alpha\rangle + |-\alpha\rangle) - c_e(|i\alpha\rangle + |-i\alpha\rangle)$$

3 photon loss: odd

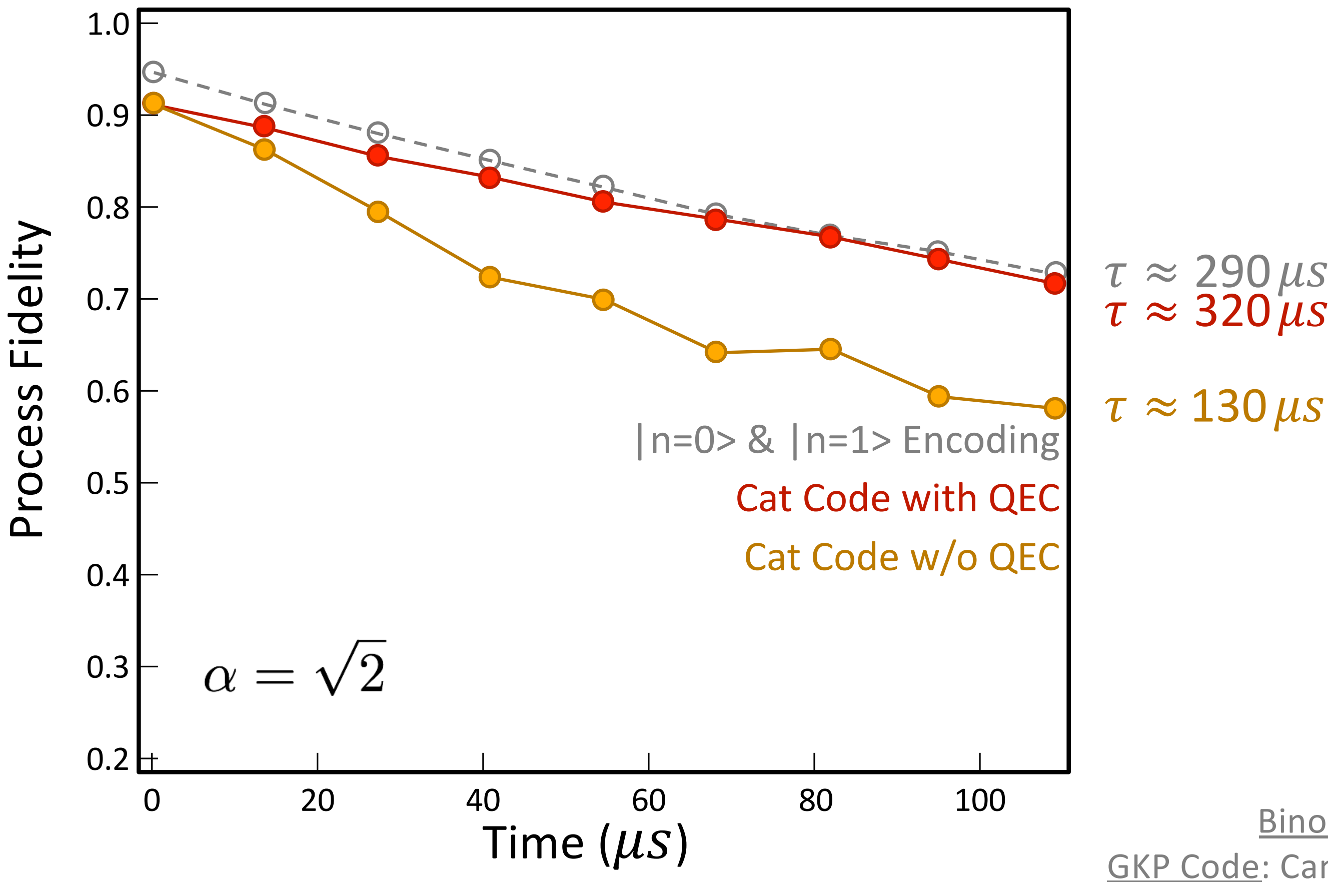
$$|\Psi^{[3]}\rangle = c_g(|\alpha\rangle - |-\alpha\rangle) - ic_e(|i\alpha\rangle - |-i\alpha\rangle)$$

4 photon loss: even

$$|\Psi^{[0]}\rangle = c_g(|\alpha\rangle + |-\alpha\rangle) + c_e(|i\alpha\rangle + |-i\alpha\rangle)$$

Ofek, Petrenko, Heeres, Reinhold, Leghtas, Vlastakis, Liu, Frunzio, Girvin, L.J., Devoret, Schoelkopf, Nature 536, 441 (2016)

QEC Experiment – Reaching Break Even

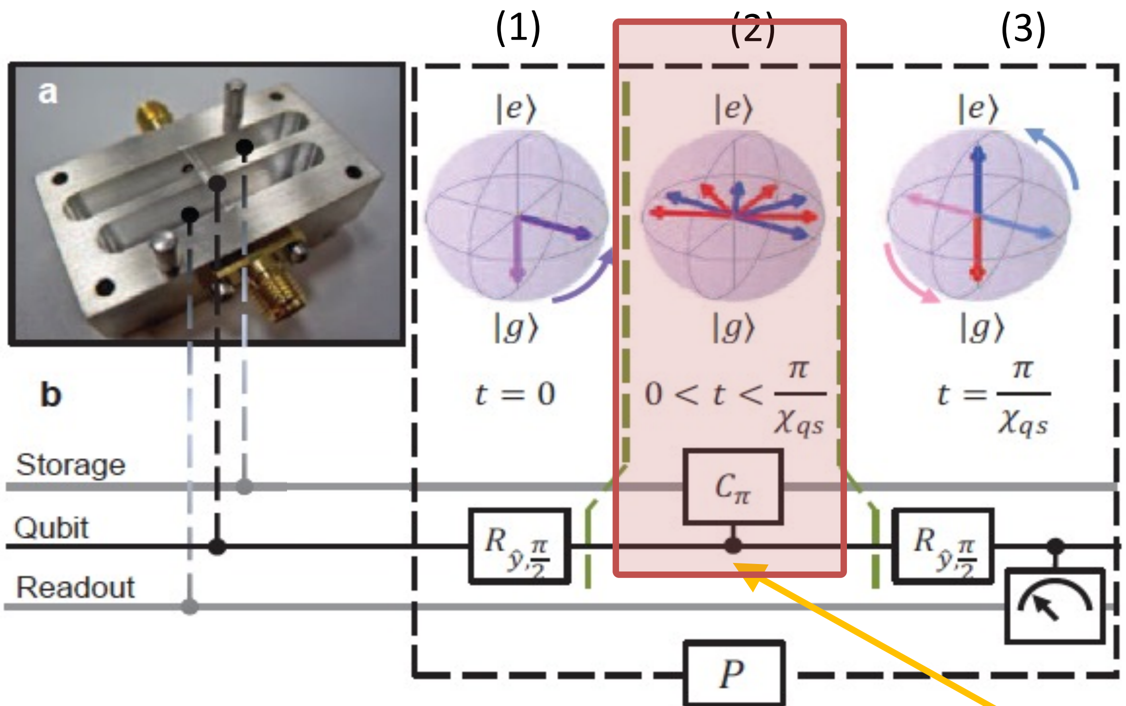


Limited by Transmon decay (prob ~ 10^{-2}) during measurement, causing cavity dephasing

Related Experiments:
Binomial Code: Hu, et al., Nat Phys 15, 503 (2019)
GKP Code: Campagne-Ibarcq, et al., Nature 584, 368 (2020)

Ofek, Petrenko, Heeres, Reinhold, Leghtas, Vlastakis, Liu, Frunzio, Girvin, L.J., Devoret, Schoelkopf, Nature 536, 441 (2016)

QND Measurement of Photon Number Parity



Procedure:

(1). Transmon initialization: $|g\rangle + |e\rangle$

(2). Phase accumulation:

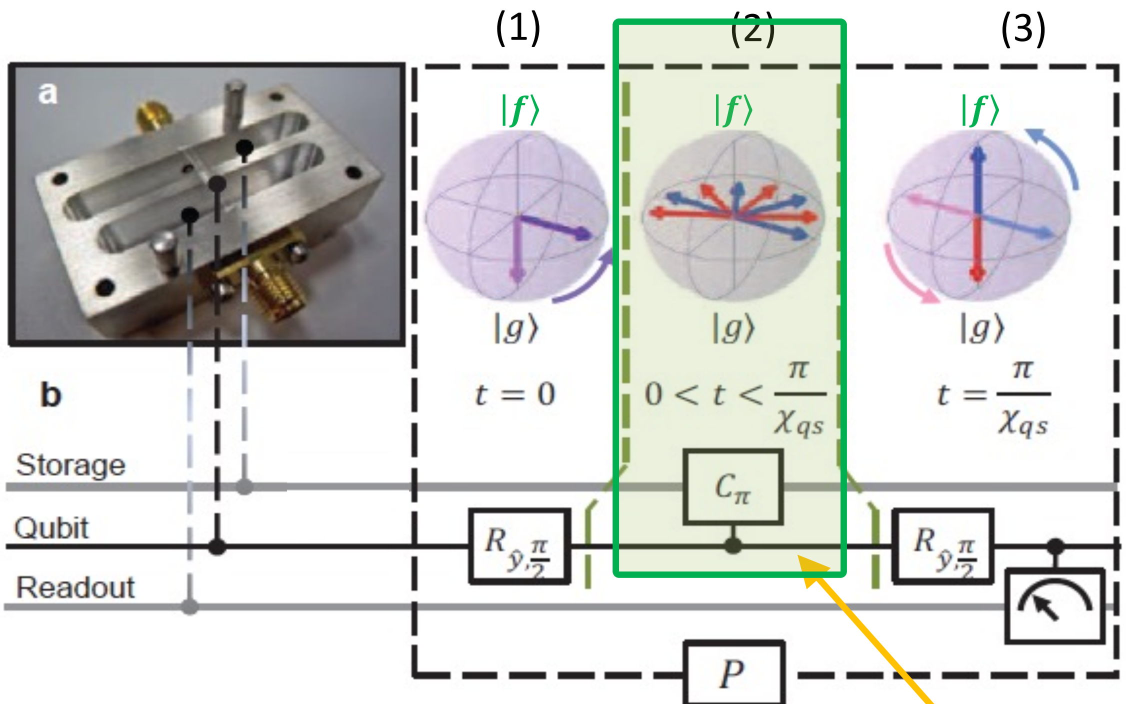
$$H = -\chi a^\dagger a |e\rangle\langle e|$$

$$U\left(T = \frac{\pi}{\chi}\right) = \begin{cases} |g\rangle\langle g| + |e\rangle\langle e| & \text{for even } n \\ |g\rangle\langle g| - |e\rangle\langle e| & \text{for odd } n \end{cases}$$

(3). Transmon measurement in $|g\rangle \pm |e\rangle$ basis

Not Fault Tolerant!
Vulnerable to Transmon Decay ($|e\rangle \rightarrow |g\rangle$)

How to Go beyond Break Even?
-- Use Fault-tolerant Parity Measurement



Key Idea: Error Transparency (Path-Independency):

$$H\ \hat{E}|\phi_{enc}\rangle = \hat{E}\ H|\phi_{enc}\rangle$$

for $\hat{E} = |e\rangle\langle f|$ & $|\phi_{enc}\rangle \in span\{|g\rangle, |f\rangle\}$

Procedure: (Transmon decay: $\hat{E} = |e\rangle\langle f|$)

1. Transmon initialization: $|g\rangle + |f\rangle$
2. Phase accumulation:

$$H = -\chi a^\dagger a(|e\rangle\langle e| + |f\rangle\langle f|)$$

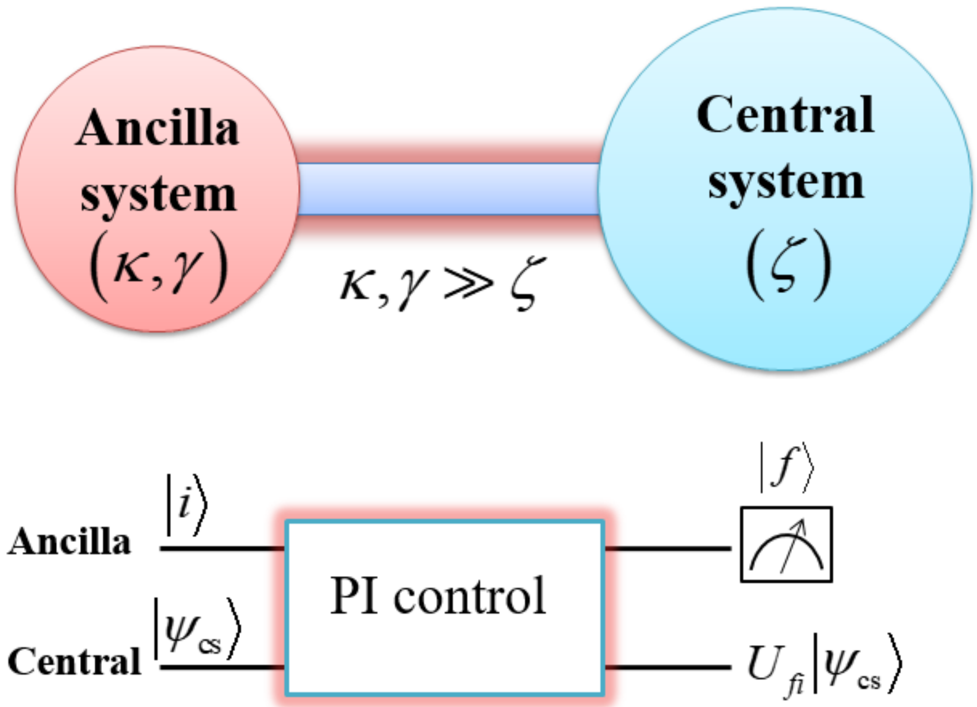
$$U\left(T = \frac{\pi}{\chi}\right) = \begin{cases} |g\rangle\langle g| + |f\rangle\langle f| & \text{for even } n \\ |g\rangle\langle g| - |f\rangle\langle f| & \text{for odd } n \end{cases}$$

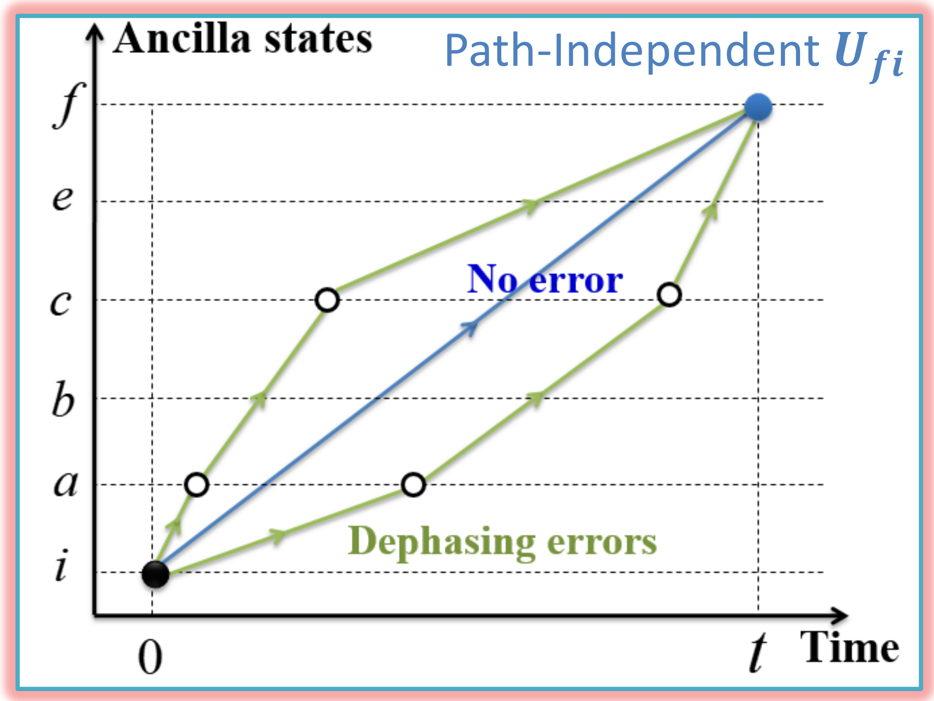
3. Transmon measurement in basis:
   - $|g\rangle \pm |f\rangle$ for even/odd parity
   - $|e\rangle$ for transmon T1 decay, with unitary $U = e^{i\pi a^\dagger a}$ of the cavity.

Fault Tolerant!
Robust against Transmon Decay ($|f\rangle \to |e\rangle$)

FT Measm't: Science 361, 266; FT Ctrl: Nature Phys 16, 822; Error Transparency & Path-Independency: PRL 120, 050503; PRL 125, 110503; arXiv:2201.00360

Use Noisy Ancilla for High Fidelity Operations
-- Path-Independent (PI) Quantum Gates





Note: {Error Transparent Gates} ⊂ {Path Indenpendent Gates}

Experiment: Reinhold, Rosenblum, *et al.*, Nature Physics, 16, 822 (2020) & Theory: Ma, *et al.*, PRL 125, 110503 (2020); arXiv:2201.00360

Ordinary matrix algebra & PI matrix algebra (Isomorphism)

	Ordinary matrix algebra $\mathcal{A}$ (for only the ancilla system):	**PI matrix algebra $\mathcal{A}_{PI} \cong \mathcal{A}$** (for the ancilla and central systems):
Basis	$\mathcal{B}_A = \{\lvert m\rangle\langle n\rvert\}_{m,n=0}^{d-1}$	$\mathcal{B} = \{\lvert m\rangle\langle n\rvert \otimes U_{mn}\}_{m,n=0}^{d-1}$
Jump operator	$(\lvert a\rangle\langle b\rvert)\cdot(\lvert c\rangle\langle d\rvert) = \delta_{bc}\lvert a\rangle\langle d\rvert$	$(\lvert a\rangle\langle b\rvert \otimes U_{ab})\cdot(\lvert c\rangle\langle d\rvert \otimes U_{cd}) = \delta_{bc}\lvert a\rangle\langle d\rvert \otimes U_{ad}$
Hamiltonian	$H_A(t) = \sum_{m,n}\varepsilon_{mn}(t)\lvert m\rangle\langle n\rvert$	$H(t) = \sum_{m,n}\varepsilon_{mn}(t)\lvert m\rangle\langle n\rvert \otimes U_{mn}$
Propagator	$W_A(t_2,t_1) = \sum_{m,n}\xi_{mn}(t_2,t_1)\lvert m\rangle\langle n\rvert$	$W(t_2,t_1) = \sum_{m,n}\xi_{mn}(t_2,t_1)\lvert m\rangle\langle n\rvert \otimes U_{mn}$
Path independence	$\lvert r\rangle\langle a\vert a\rangle\langle b\vert b\rangle\langle c\rvert \cdots \lvert e\rangle\langle i\rvert = \lvert r\rangle\langle i\rvert$	$(\lvert r\rangle\langle a\rvert \otimes U_{ra})\cdot(\lvert a\rangle\langle b\rvert \otimes U_{ab})\cdot(\lvert b\rangle\langle c\rvert \otimes U_{bc}) \cdots (\lvert e\rangle\langle i\rvert \otimes U_{ei}) = \lvert r\rangle\langle i\rvert \otimes U_{ri}$

Quantum Computation with Bosonic QEC

complexity ↑

Fault tolerant quantum computation	**FT SNAP gate: Nature Physics 16, 822 (2020)** **FT parity measurement: Science 361, 266 (2018)**
Algorithms on multiple logical qubits	**Teleported CNOT: Nature 561, 368 (2018)** **e-SWAP gate: Nature 566, 509 (2019)**
Operations on single logical qubits	**Universal gates:** **Nat Comm 8, 94 (2017)**
Logical memory with longer lifetime than phys. qubits	**Break-even:** **Nature 536,441 (2016)**
QND measurements for error correction and control	**QND parity measurement:** **Nature 511, 444 (2014)**

Devoret & Schoelkopf, Science 339, 1169 (2013)

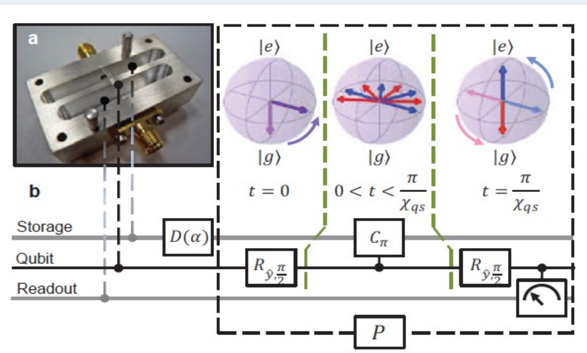
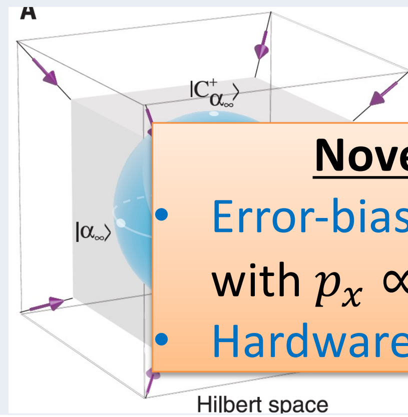
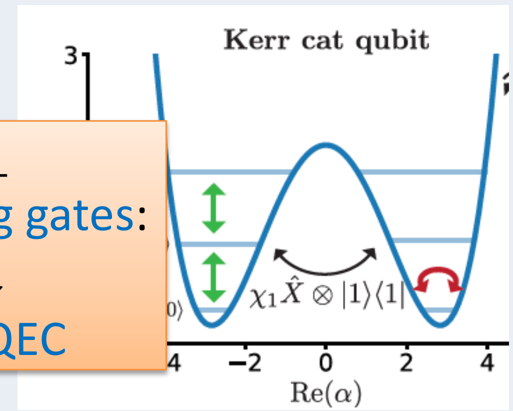
Key Challenges:

- How to achieve control of bosonic modes?
- How to implement QND parity msmt & fault-tolerance?
- *How to couple remote bosonic modes?*
- *How to achieve code-independent coupling gate?*

Ideas:

- ✓ Transmon SNAP gate & optimal control
- ✓ Dispersive Ramsey msmt & Error transparent design
- ✓ *Teleported CNOT gate*
- ✓ *E-SWAP gate*

Various Approaches for Bosonic QEC

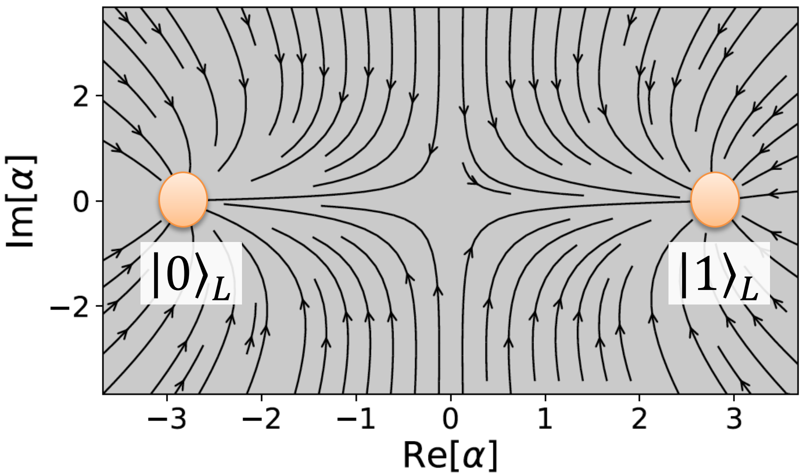
Discrete QEC	Continuous QEC	
Syndrome Measurement	**Engineered Dissipation**	**Hamiltonian Protection**
		
E.g., Cat, Binomial, GKP codes	E.g., Dissipative-cat qubit	E.g., Kerr-cat qubit
Experiments on cat/binomial/GKP codes • Ofek, et al., Nature 536, 441 (2016) • Hu, et al., Nat Phys 15, 503 (2019) • Campagne-Ibarcq, et al., Nature 584, 368 (2020)	Experiments on Dissipative-cat qubit: • Leghtas, et al., Science 347, 853 (2015) • Lescanne, et al., Nature Physics 16, 509 (2020) • Leghtas, MM 22', A28.00004	Experiment on Kerr-cat qubit: • Grimm, et al., Nature 584, 205 (2020)
Real-time adaptive control	Engineer dissipation to remove entropy	Energy gap to suppress noise, no cooling

Novel Feature:

- Error-bias preserving gates: with $p_x \propto e^{-n} \ll p_z$
- Hardware-Efficient QEC

Outlook: *Hardware Efficient* Bias-Preserving QEC

Exponentially suppressed bit-flip errors



Key feature:

- Apply continuous QEC to exponentially suppress bit-flip errors:

$$p_x, p_y \propto e^{-2\alpha^2} \ll p_z$$

- Design universal gates preserving error bias (no mixing p_z into $p_{x,y}$)
- Use repetition-like code to achieve fault-tolerance

Hardware-Efficient Proposals for Fault-Tolerant QC with Bias-Preserving Gates

- Dissipative-cat qubit: Guillaud, Mirrahimi, PRX 9, 0411053 (2019); Chamberland, L.J., et al., PRXQ 3, 010329 (2022)
- Kerr-cat qubit: Puri, L.J., Girvin, et al., Science Advances 6, eaay5901 (2020); Qian, L.J. et al., arXiv:2105.13908
- Colored Kerr-cat qubit: Putterman, Iverson, Xu, L.J., Painter, Brandao, Noh, PRL (accepted), arXiv:2107.09198.
- **Code optimization for biased noise: Xu, Mannucci, Seif, Kubica, Flammia, L.J., arXiv:2203.16486 (2022).**

Lescanne, et al., Nature Physics 16, 509 (2020); Grimm, et al., Nature 584, 205 (2020); Leghtas, MM 22', A28.00004