Dynamics of coherence in measurement induced phase transitions

And its role in quantum communication

TopDyn DYNAMICS AND TOPOLOGY

TRR 306 QuCoLiMa Quantum Cooperativity of Light and Matter

Shane P. Kelly

With Jamir Marino[1], Mathew Fisher[2],
Ulrich Poschinger[1], and Ferdinand Schmidt-Kaler[1]

1) Mainz
2) Santa Barbara

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Measurement induced entanglement transitions

Weak Measurements

Volume law entanglement

$p_m < p_c$

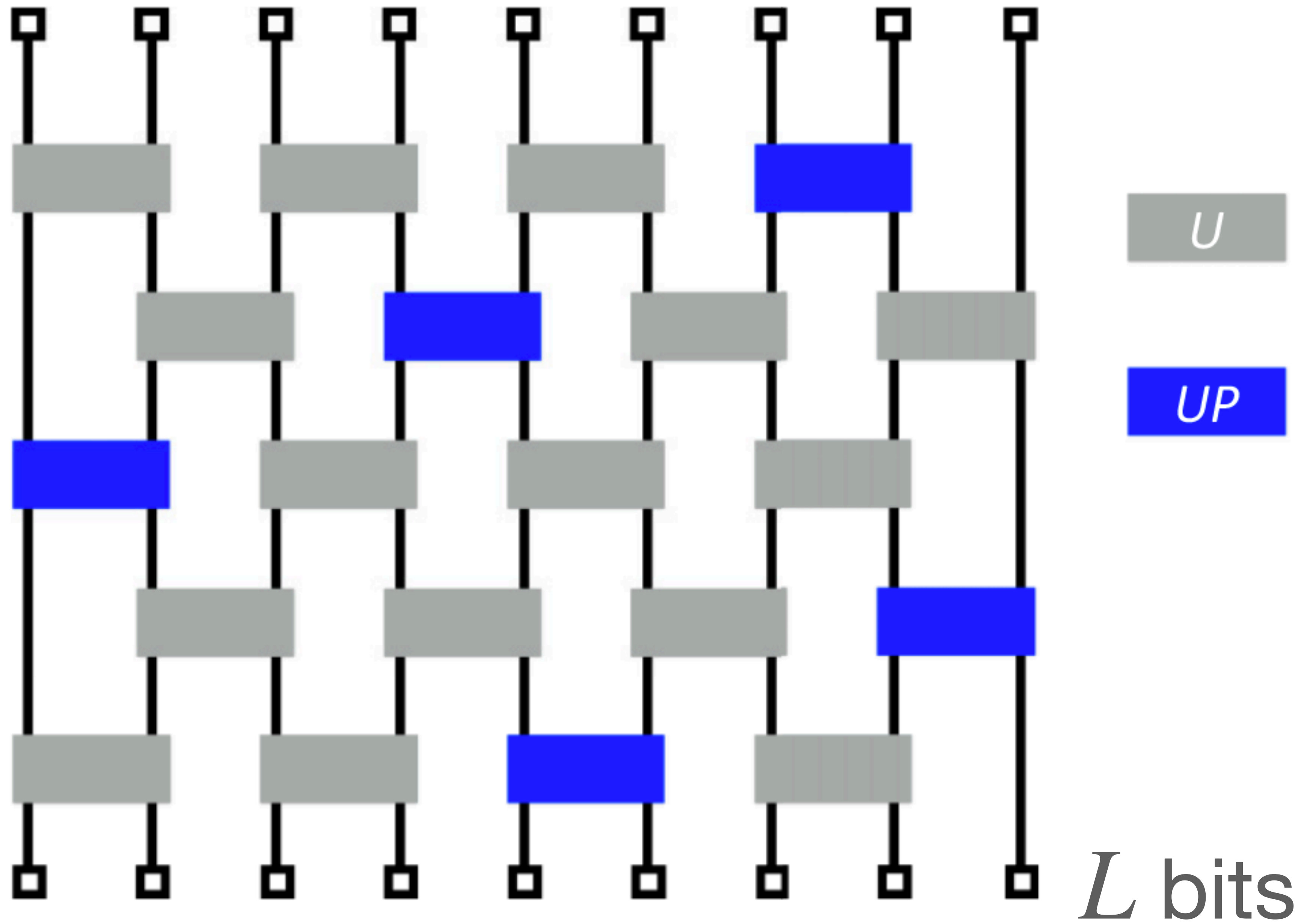
Strong Measurements

Area law entanglement

$p_m > p_c$

Criticality

$p_m = p_c$



$|\psi\rangle \to U|\psi\rangle$

$|\psi\rangle \to U\frac{P|\psi\rangle}{\|P|\psi\rangle\|}$

L bits

$\rho_0 = \left|0\right\rangle\left\langle 0\right|$

Circuit of unitaries

and

measurements

rate p_m

Image from PRB.98.205136 Li, Chen, Fisher

Measurement induced purification transition

Weak Measurements

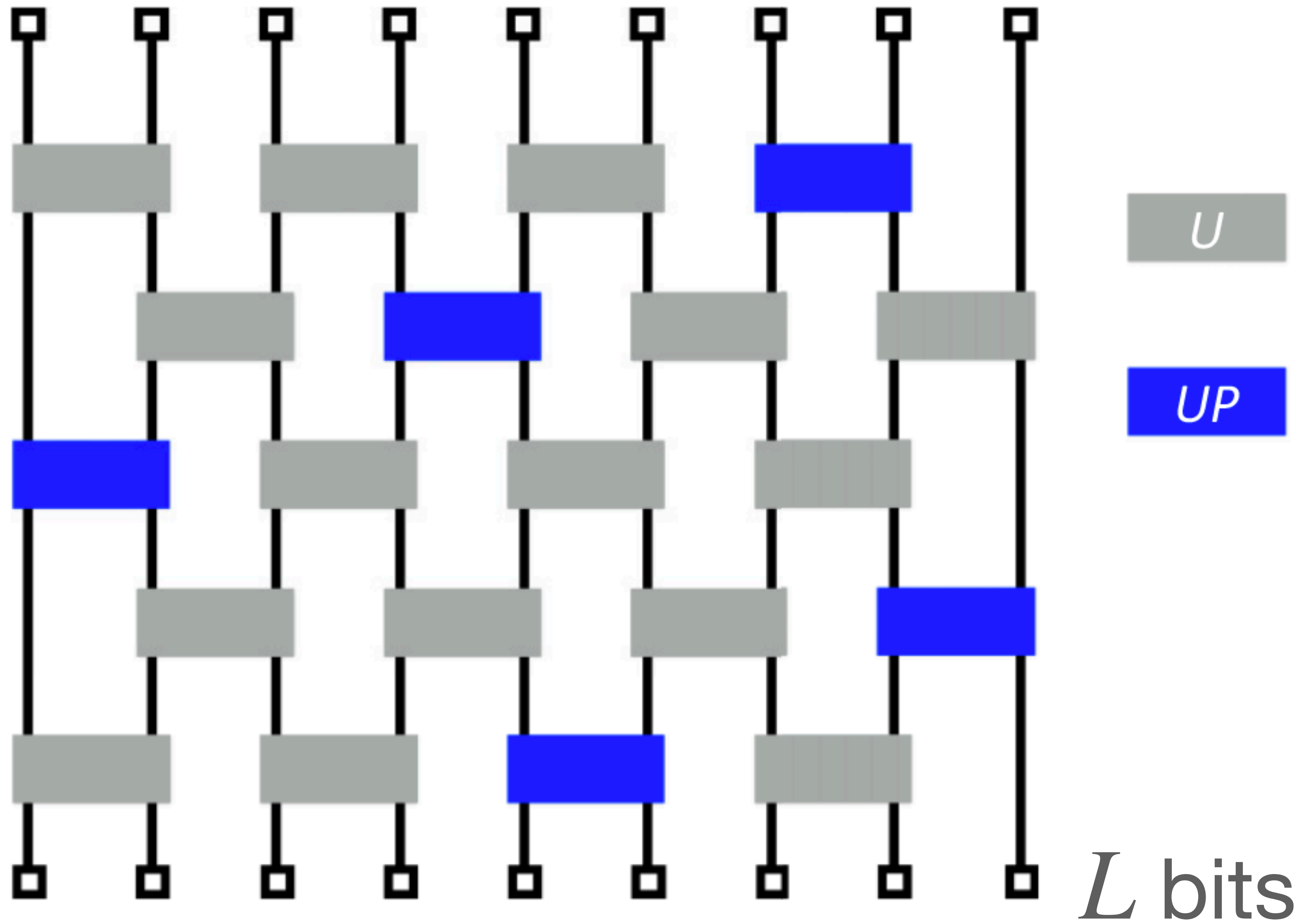
Mixed State

$$p_m < p_c$$

$$S(\rho_T) = Tr[\rho_T \log \rho_T] > 0$$

Criticality

$$p_m = p_c$$



$|\psi\rangle \to U|\psi\rangle$

$|\psi\rangle \to U\frac{P|\psi\rangle}{\|P|\psi\rangle\|}$

L bits

$$\rho_0 = \frac{1}{2^L}$$

Strong Measurements

Pure state

$$p_m > p_c$$

$$S(\rho_T) = 0$$

Image from PRB.98.205136 Li, Chen, Fisher

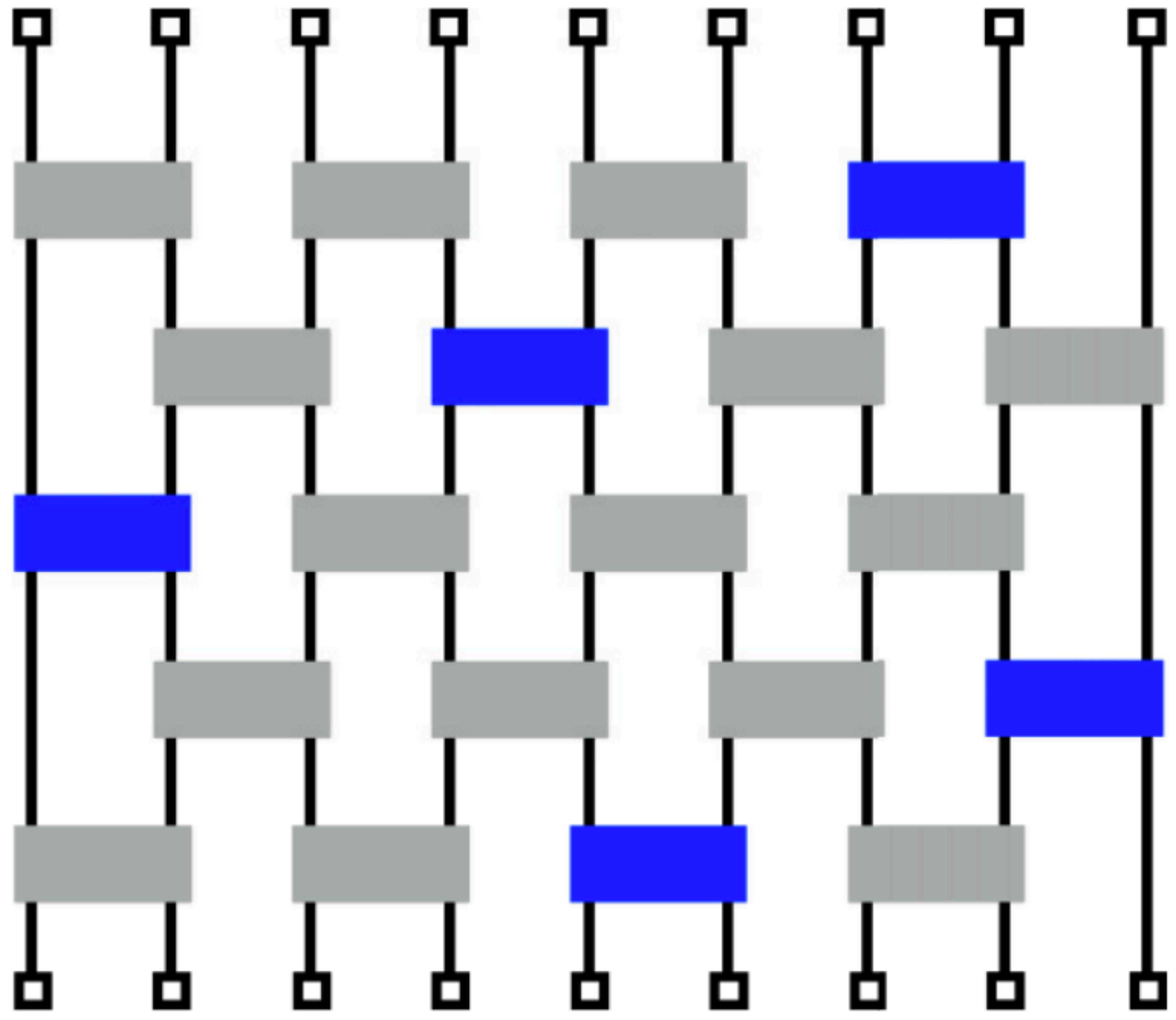
Measurement induced memory transition

Can Bob try to destroy Alice's state?

Bob

Tries to destroy
diary by shining light

Can Alice reconstruct
$|\psi_k\rangle$ from collected light and final state?



$$|\psi_k\rangle \otimes |0\rangle^{\otimes L-k}$$

Alice

Alice collects light
and applies unitaries

Alice encodes diary
in her quantum system

Measurement induced memory transition

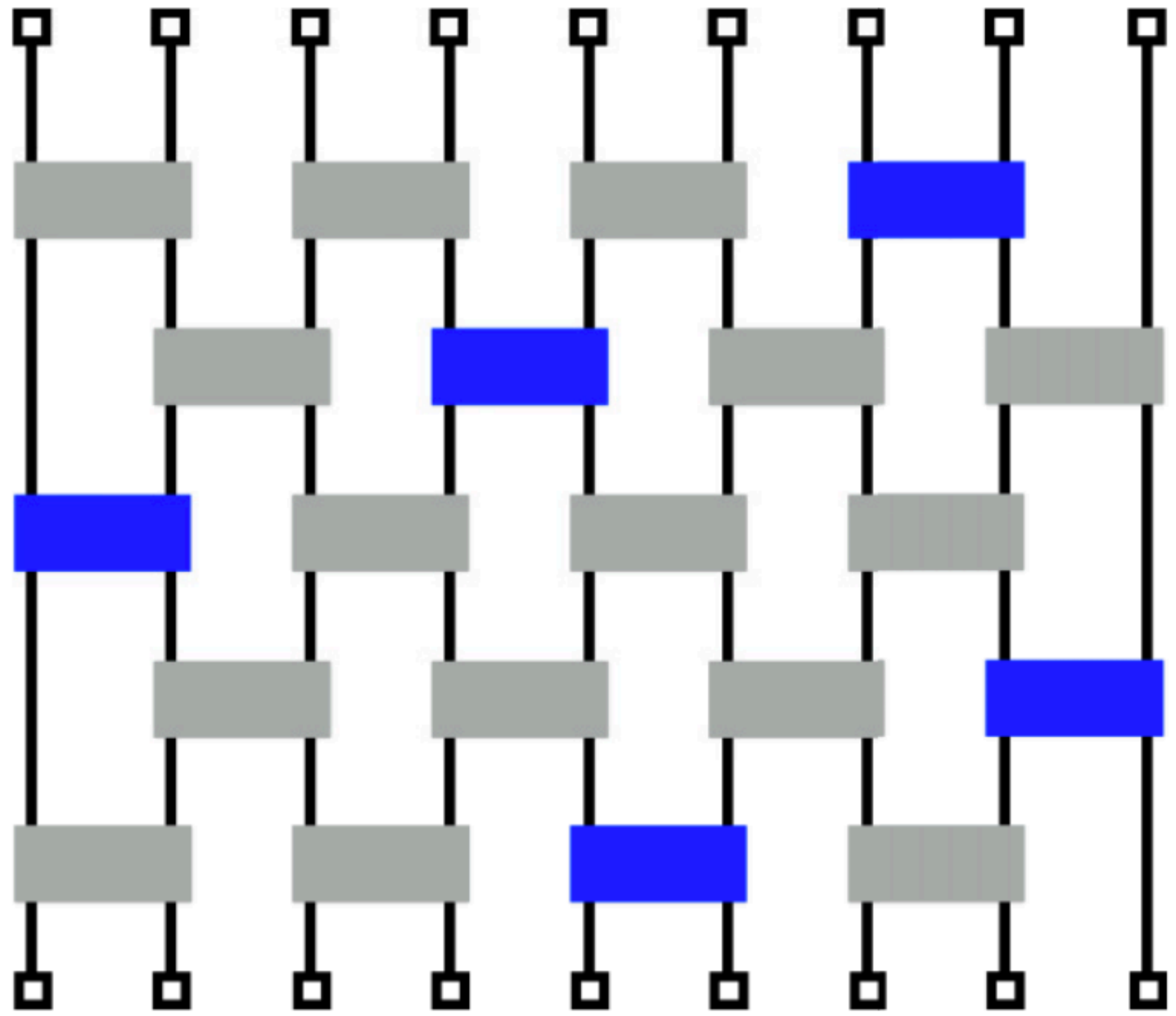
Weak Measurements

Finite Channel Capacity $C_s = S_T$

Alice **can** recover $\left|\psi_k\right\rangle$

Diary of size $k = C_s < L$ Qubits

State $\left|\psi_k\right\rangle$



$\left|\psi_k\right\rangle \otimes \left|0\right\rangle^{\otimes L-k}$

Strong Measurements

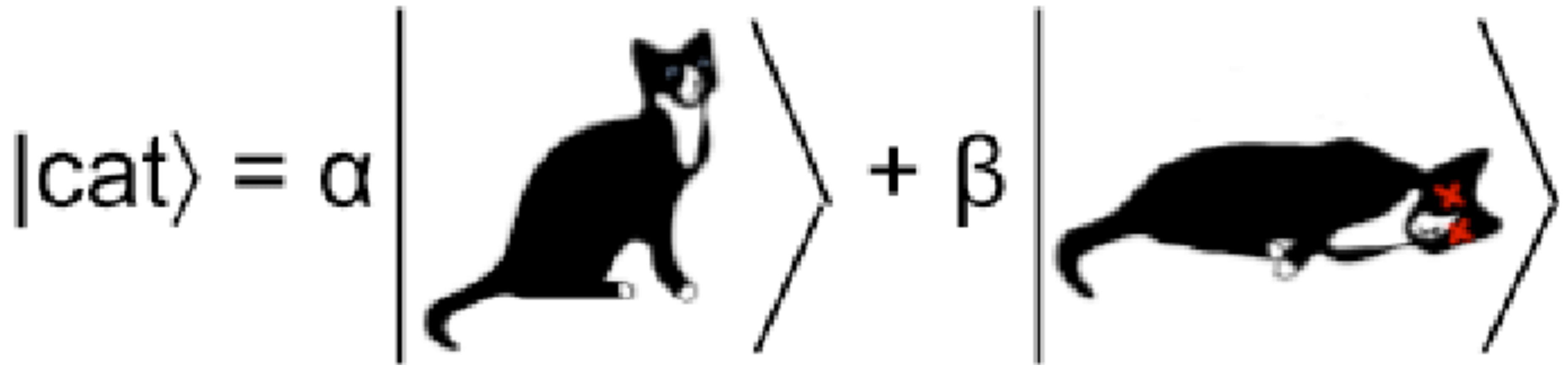
Zero Channel Capacity $C_s = 0$

Alice **can't** recover $\left|\psi_k\right\rangle$

Image from PRB.98.205136 Li, Chen, Fisher

What if Alice is constrained in her ability to generate coherence?

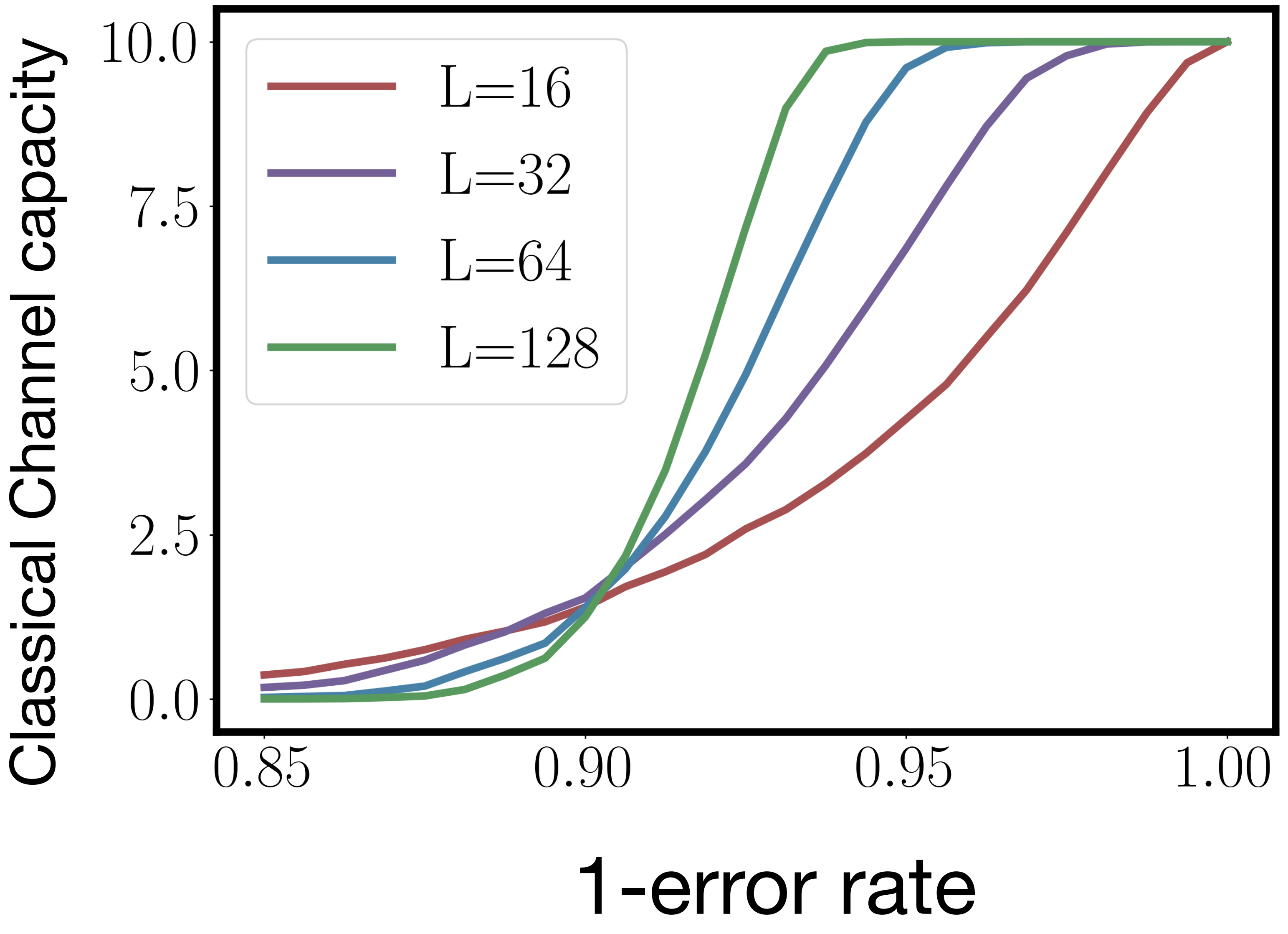
Coherence: Quantum uncertainty in a given basis



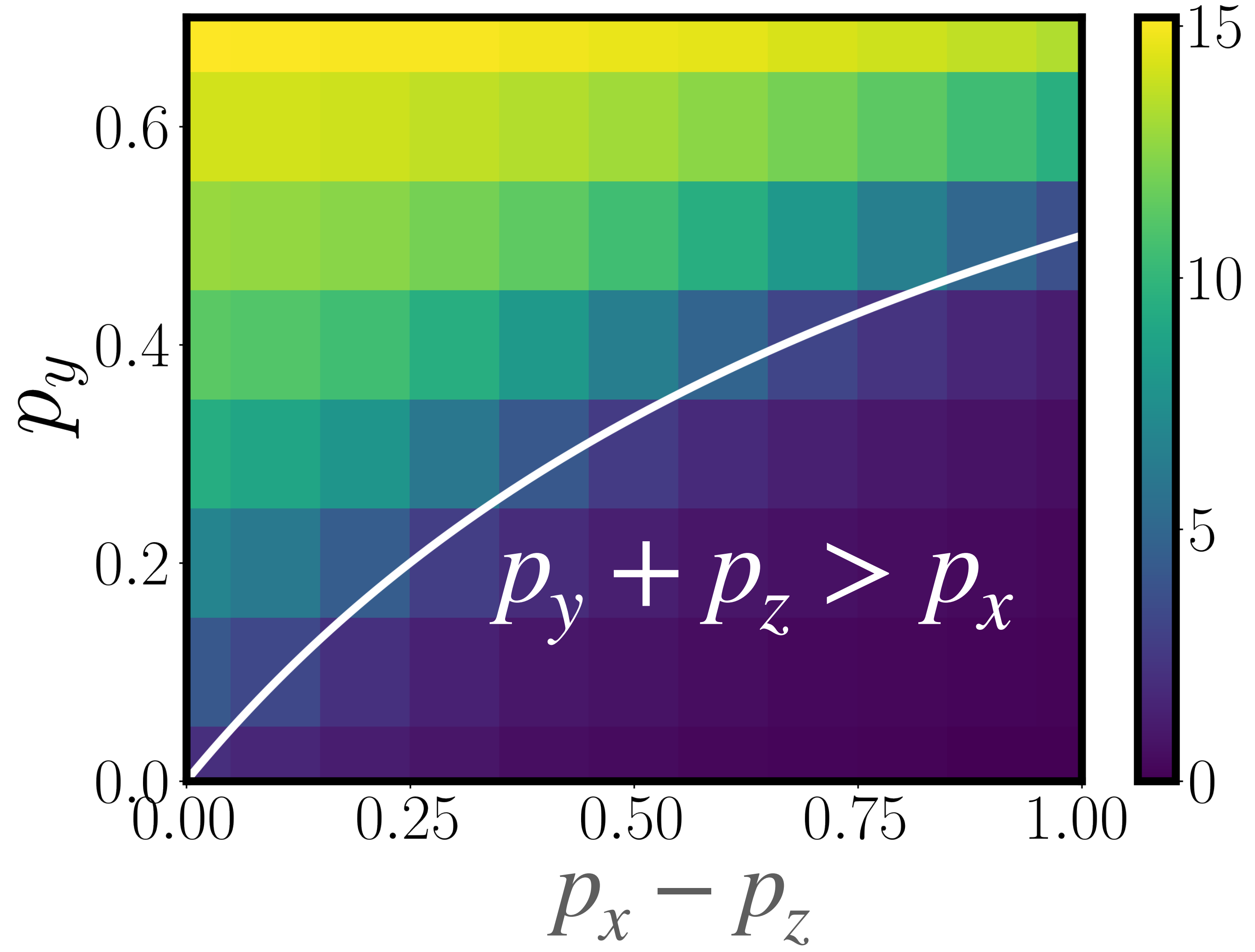
Teaser: The utility of coherence to Alice

Coherence = Quantum uncertainty (entropy) of a given basis

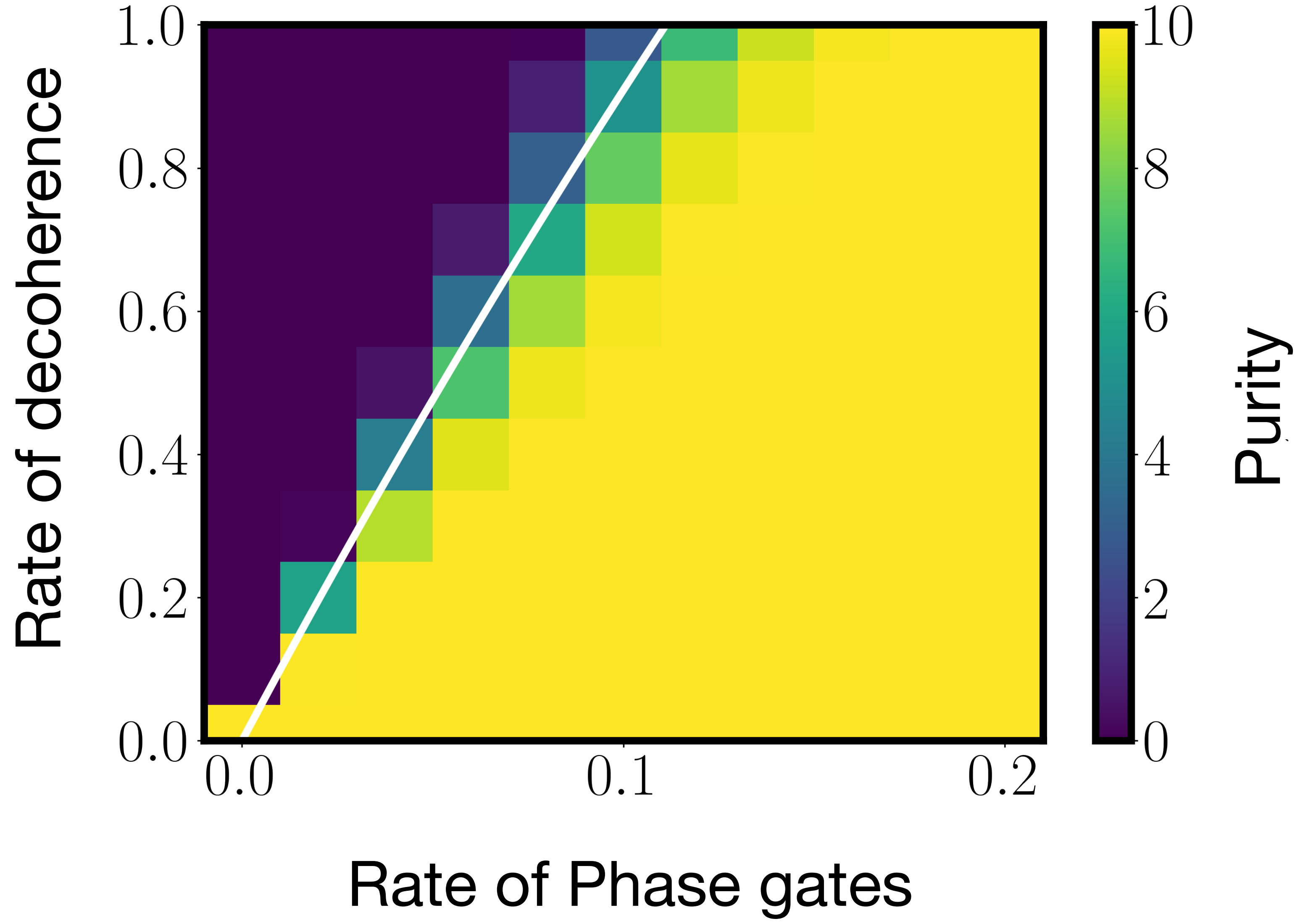
Coherence free :
Alice protects only classical diary



Measurements (bob) **generating coherence:**
entanglement transitions



Unitaries generating coherence:
Alice protects quantum diary

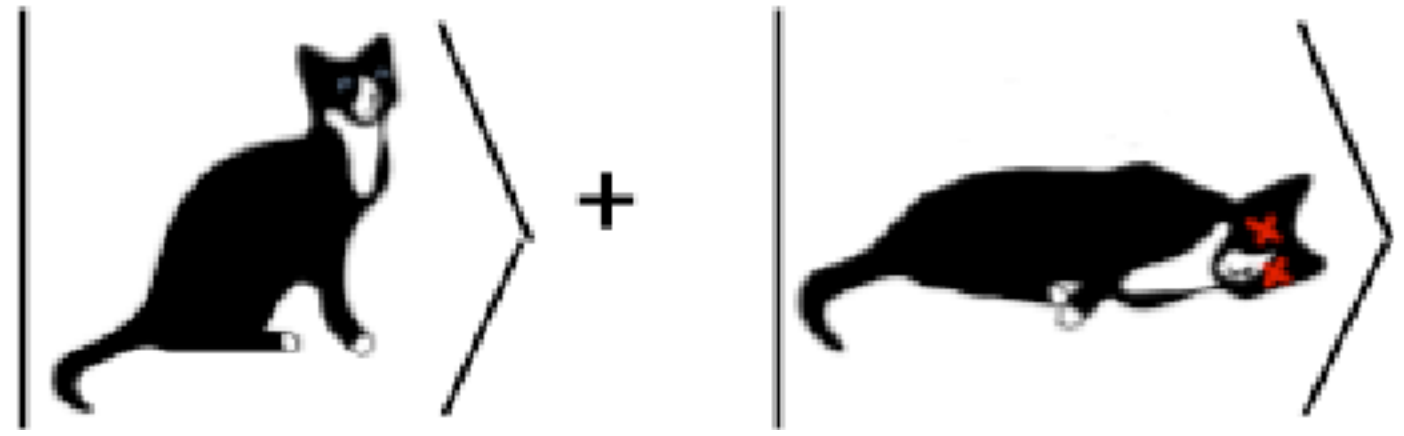


We quantify coherence requirement for quantum error correction

$$d < \max_{|\psi\rangle \in \mathscr{C}} C(|\psi\rangle, D)$$

Quantum coherence

A natural perspective on “what is quantum uncertainty?”

$|cat\rangle =$ 

$$\rho_c = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \textbf{v.s.} \qquad \rho_q = |cat\rangle\langle cat| = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}!$$

- ρ_c is classically uncertain about cat vitality
- ρ_q has zero classical uncertainty
  - physical state known perfectly

Basis specific!

Relative entropy of coherence

How to quantify quantum uncertainty of basis D?

- Projectors of basis D : $P_d = \left| d \rangle \langle d \right|$

Answered with resource theory![1)]

Similar to relative entropy of entanglement

- Relative entropy of coherence:

$$C(\rho, D) = S\left(\sum_d P_d \rho P_d\right) - S(\rho)$$

Coherence = Entropy of diagonal elements − entropy of quantum states

= Uncertainty of basis states − uncertainty about which pure state

1) Baumgratz, Cramer, Pleno PRL 2014

Coherence

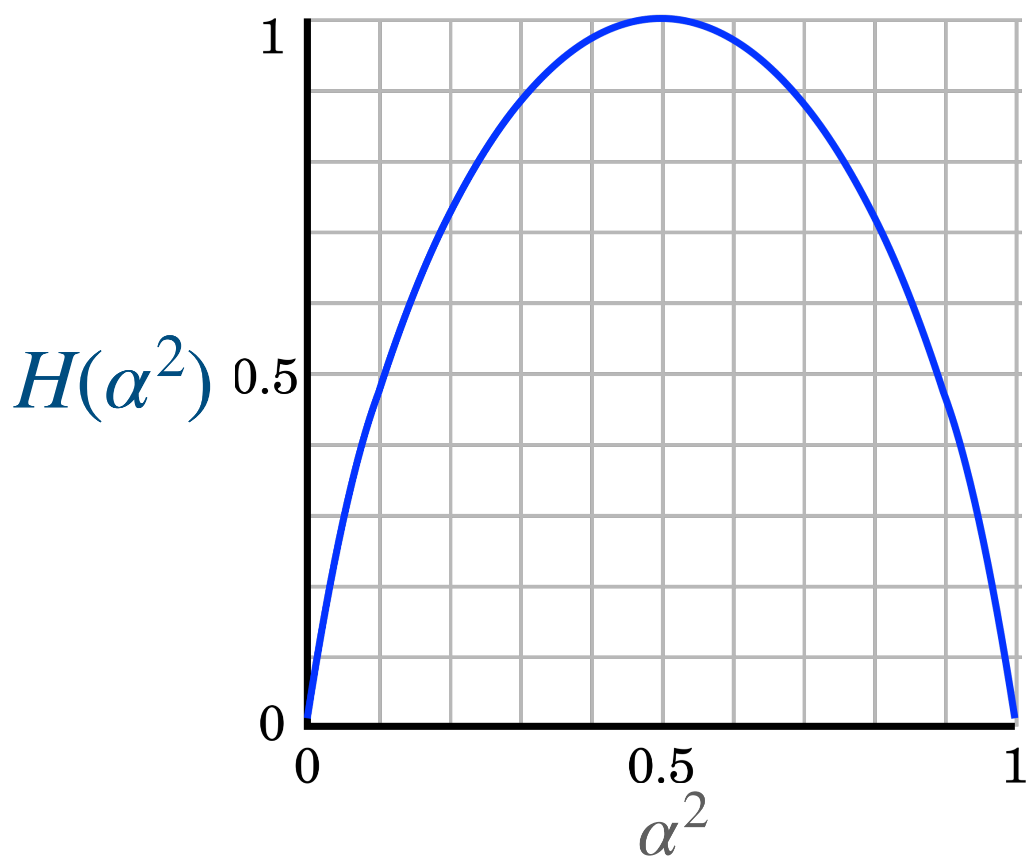
Single bit example

$$C(\rho, D) = S\left(\sum_d P_d \rho P_d\right) - S(\rho)$$

- Example state $\alpha \left|0\right\rangle + \sqrt{1-\alpha^2}\left|1\right\rangle$

$$\rho = \begin{bmatrix} \alpha^2 & \alpha\sqrt{1-\alpha^2} \\ \alpha\sqrt{1-\alpha^2} & 1-\alpha^2 \end{bmatrix}$$

$$C(\rho, X) = S\left(\sum_d P_d \rho P_d\right) = H(\alpha^2)$$



- Incoherent density matrix $C(\rho, X) = 0$
  - $\rho = \sum_d P_d \rho P_d = \rho_d = \begin{bmatrix} \alpha^2 & 0 \\ 0 & 1-\alpha^2 \end{bmatrix}$

What is quantum coherence?

For multi qubit state state

$$C(\rho, D) = S\left(\sum_d P_d \rho P_d\right) - S(\rho)$$

- Take product state which is
  - $M = L - N_x - N_z$ mixed bits
  - Eigenstate of X_i for $i = 1 \ldots N_x$
  - Eigenstate of Z_i for $i = (N_x + 1) \ldots N_x + N_z + 1$

$$S(\rho) = M$$

$$S\left(\sum_z P_z \rho P_z\right) = N_x + M \qquad C_z = N_x$$

$$S\left(\sum_x P_x \rho P_x\right) = N_z + M \qquad C_x = N_z$$

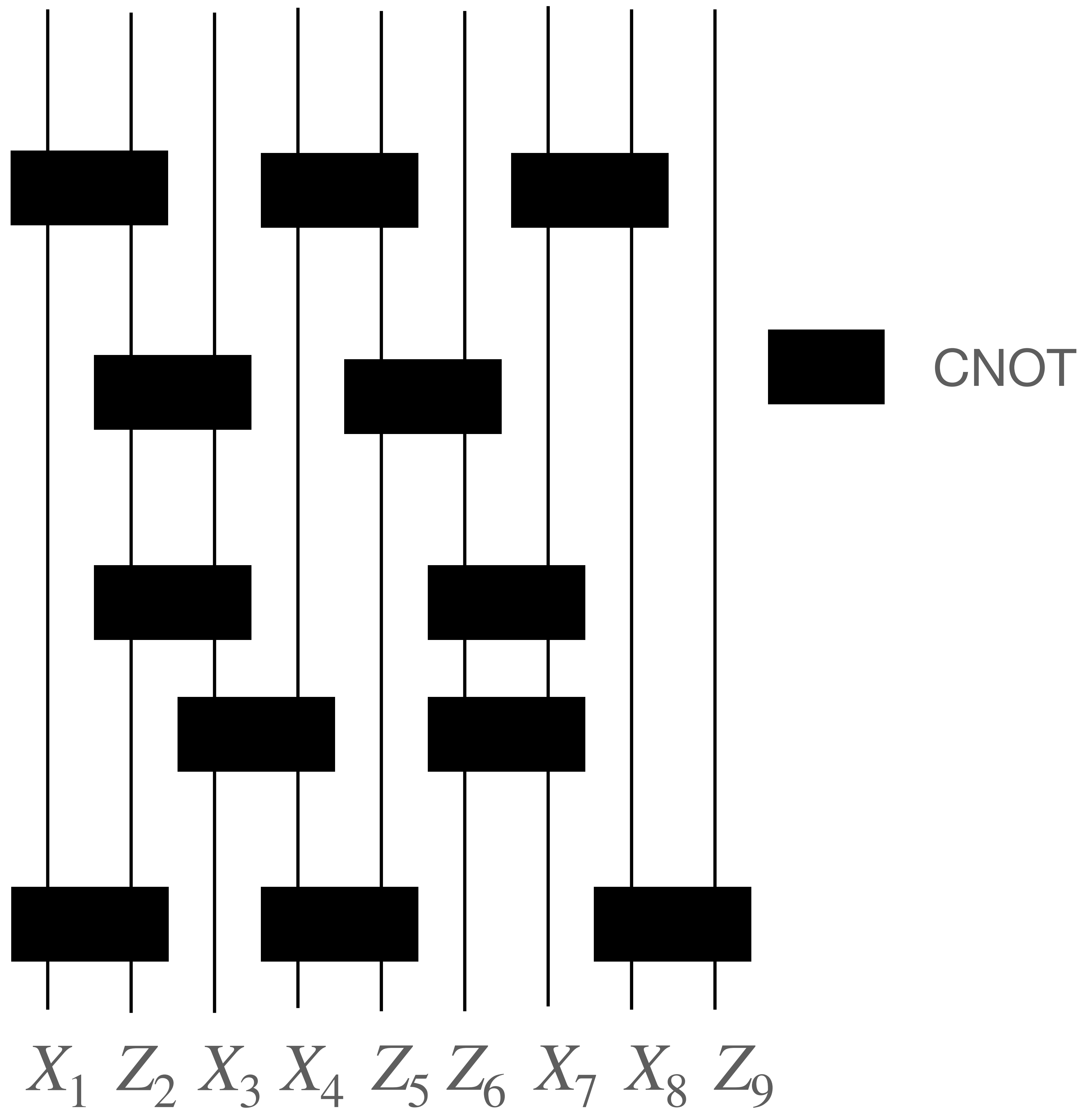
CNOTs can’t generate coherence in X or Z basis

CNOTs don't generate coherence

- CNOT gate:
  - X basis: $|x_1, x_2\rangle \rightarrow |x_1, x_2 \otimes x_1\rangle$
  - Z basis: $|z_1, z_2\rangle \rightarrow |z_1 \otimes z_2, z_2\rangle$
- Coherence is conserved in both bases:

C_x: coherence in X basis $\qquad \partial_t C_x(t) = 0$

C_z: coherence in Z basis $\qquad \partial_t C_z(t) = 0$



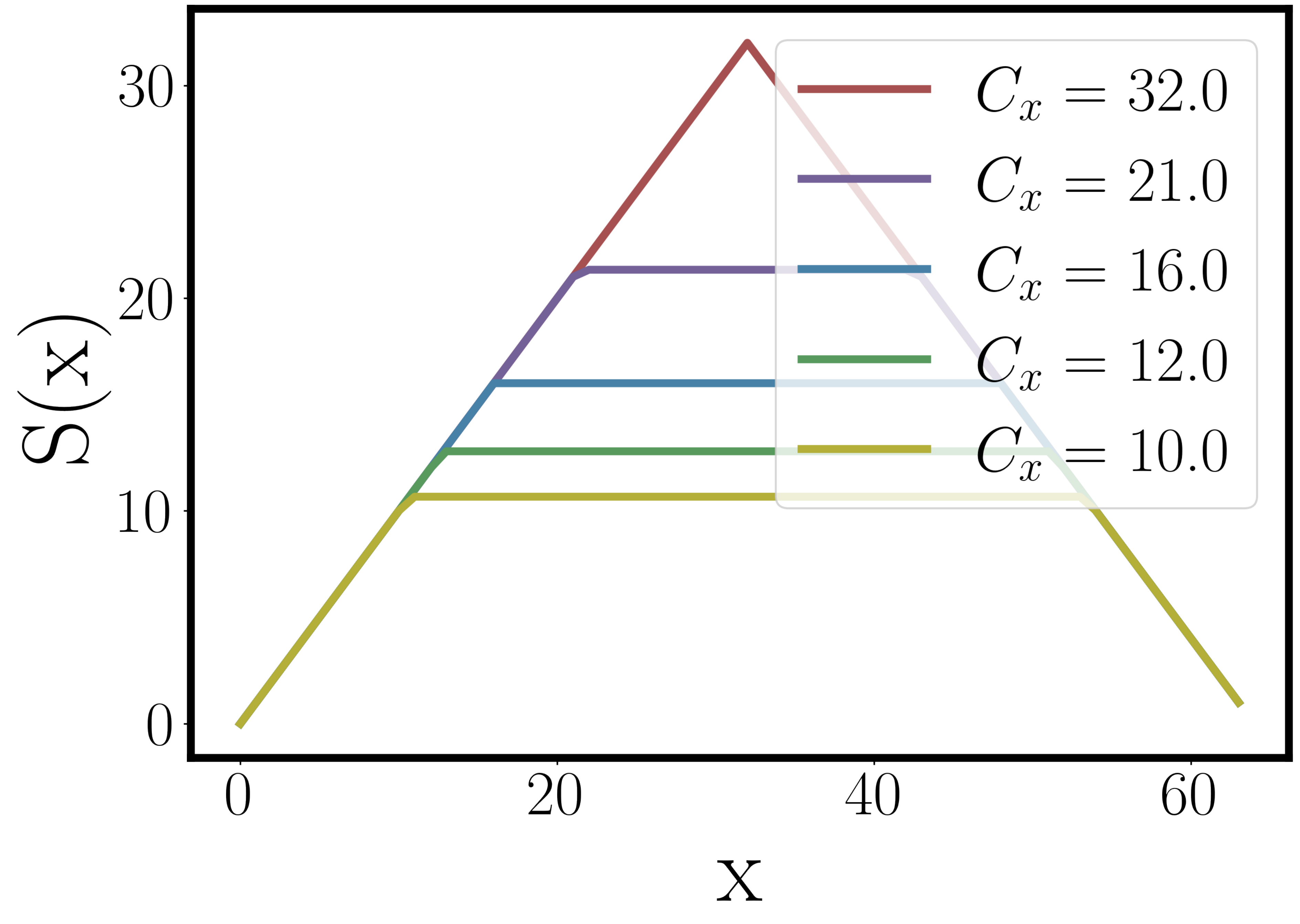
Unitary only dynamics

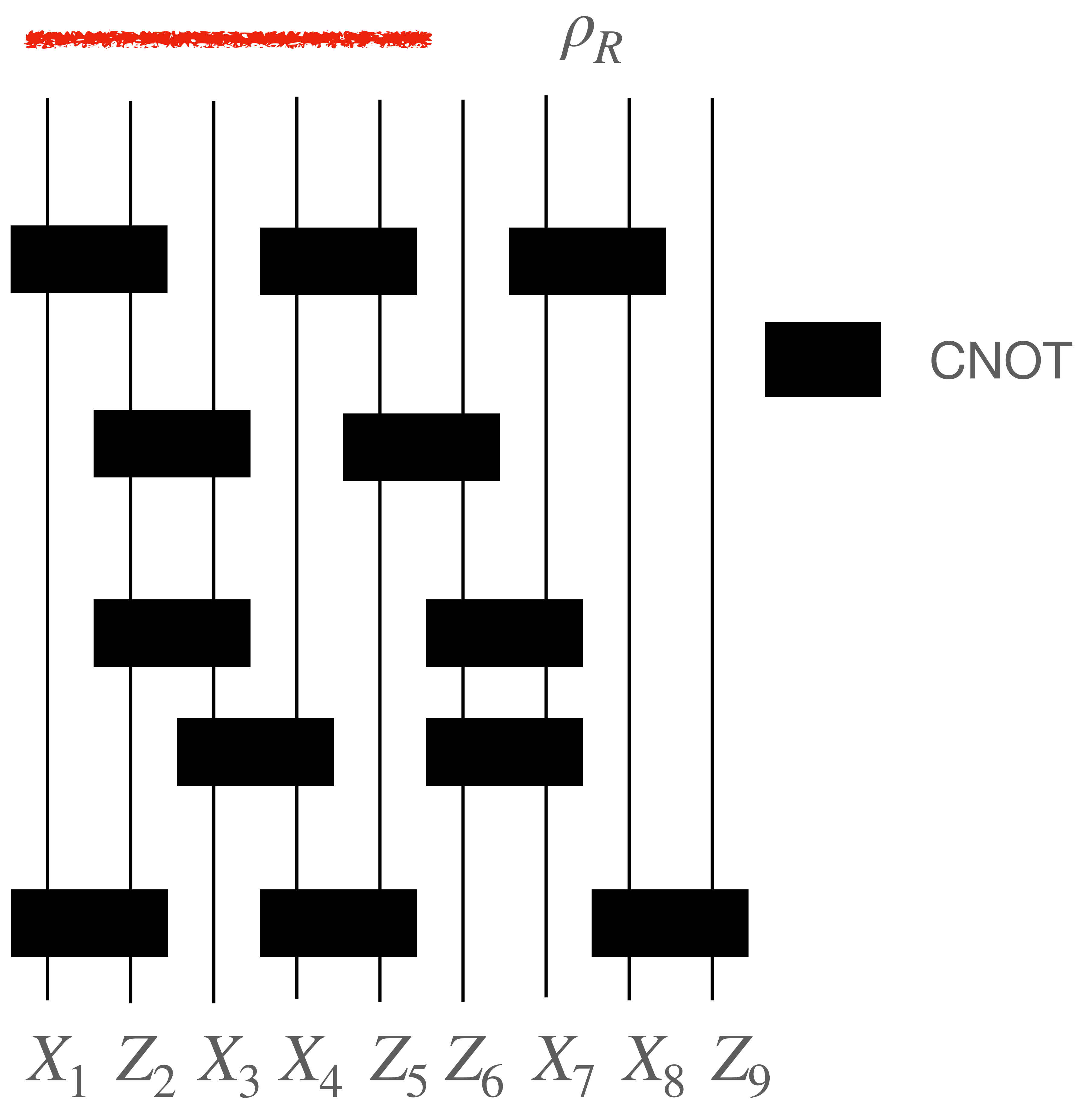
Volume law v.s. area law depends on coherence!

Coherence bounds entanglement[1)]

Of full state | between any partition of bits

$$\min(C_x, C_z) > S(\rho_R)$$





1) Straightforward to prove for pure state, similar results in *Mekala and Sen PRA (2021)*

Coherence free circuits

Coherence free dynamics

Evolution in classical **computational basis** X

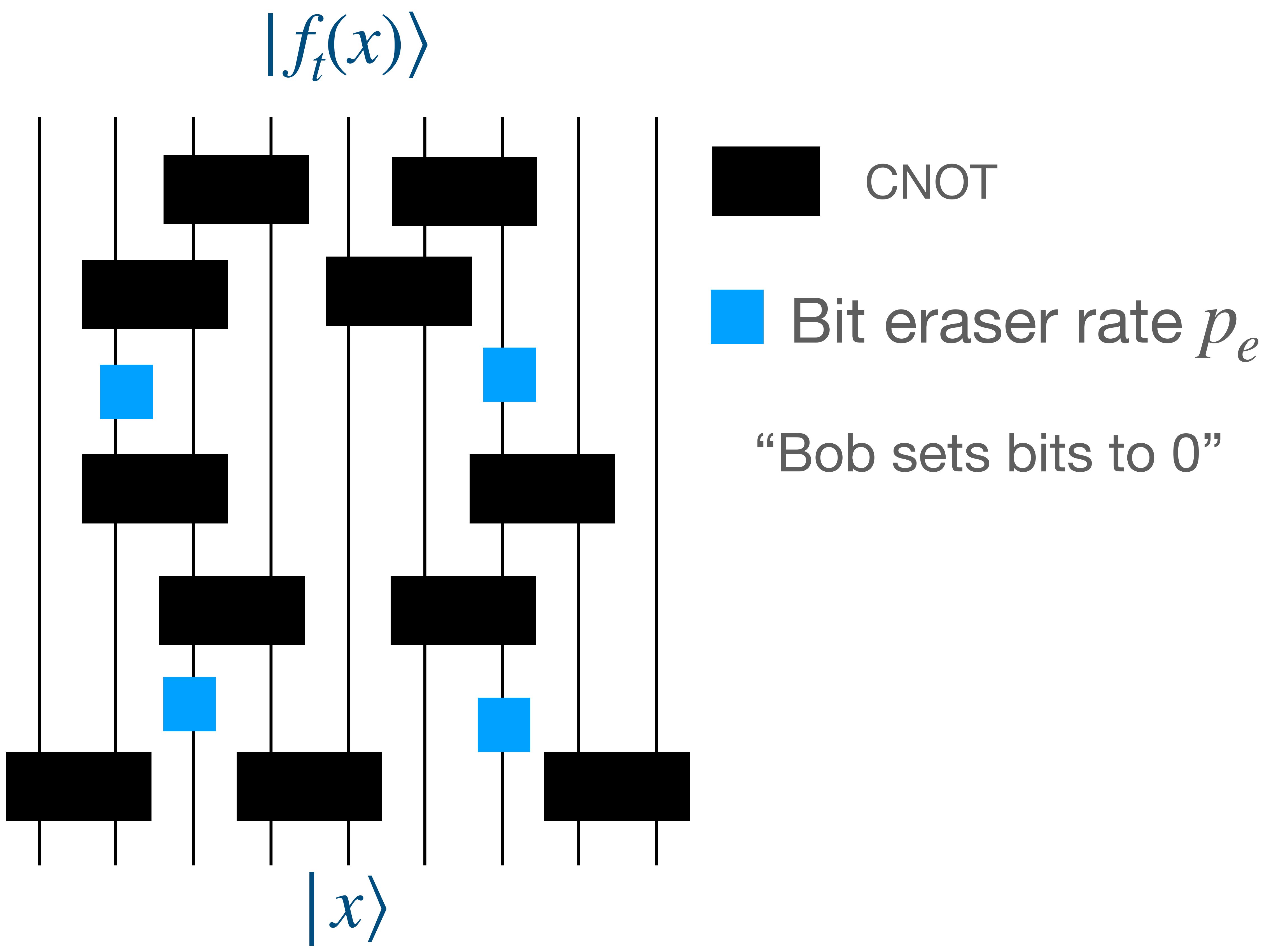
- Random CNOTs $|x_1, x_2\rangle \rightarrow |x_1, x_2 \otimes x_1\rangle$
- Bit eraser $|x_1\rangle \rightarrow |0\rangle$

Circuit maps X **basis** states to X **basis** states

$$C_x(t) = 0$$

Is $f_t(x)$ invertible?

Can Alice recover a classical diary?



Similar:

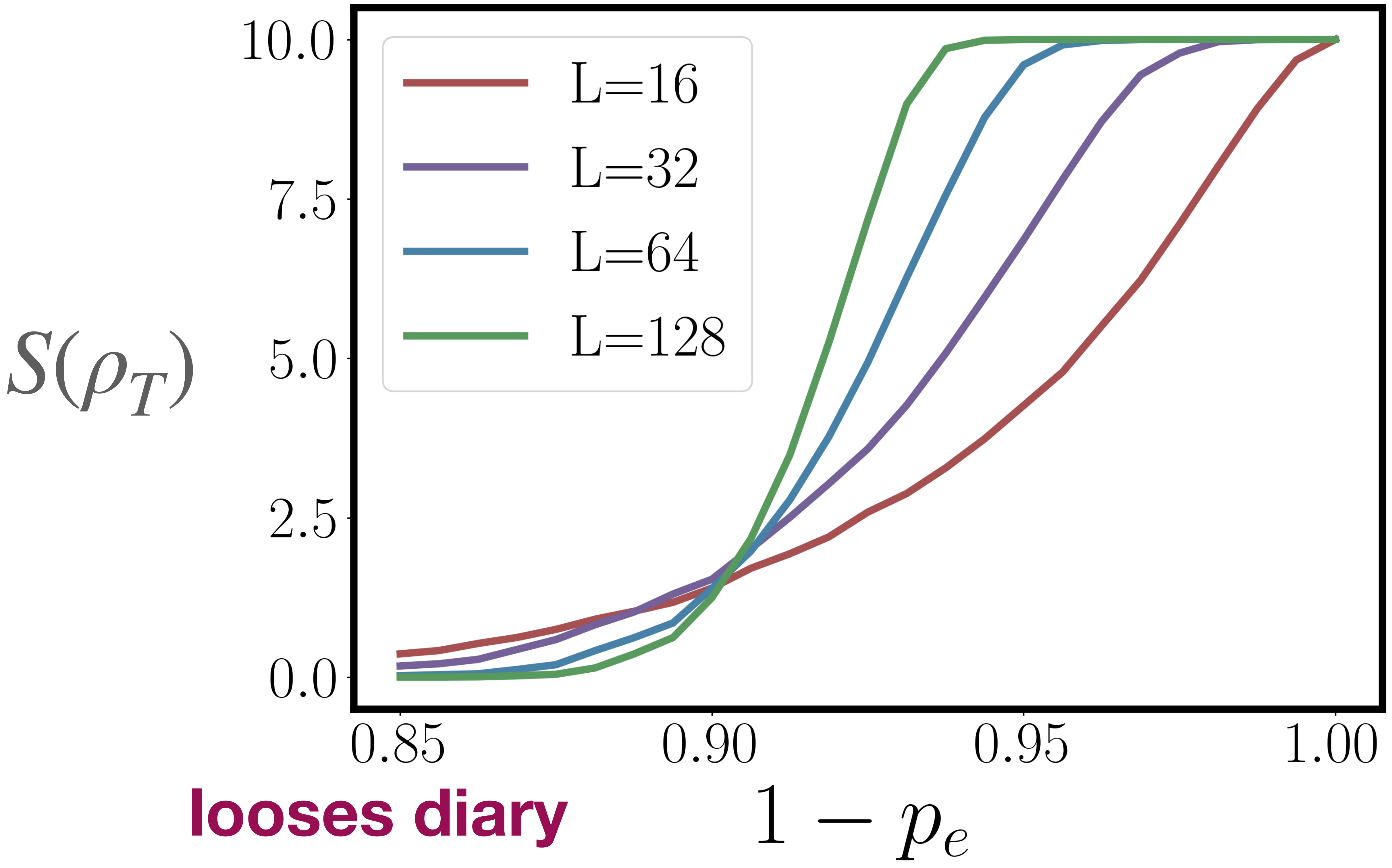
1) Lyons, Choi, Altman APS 2022

2) Knolle group arXiv:2203.11303 2022

Purification transition

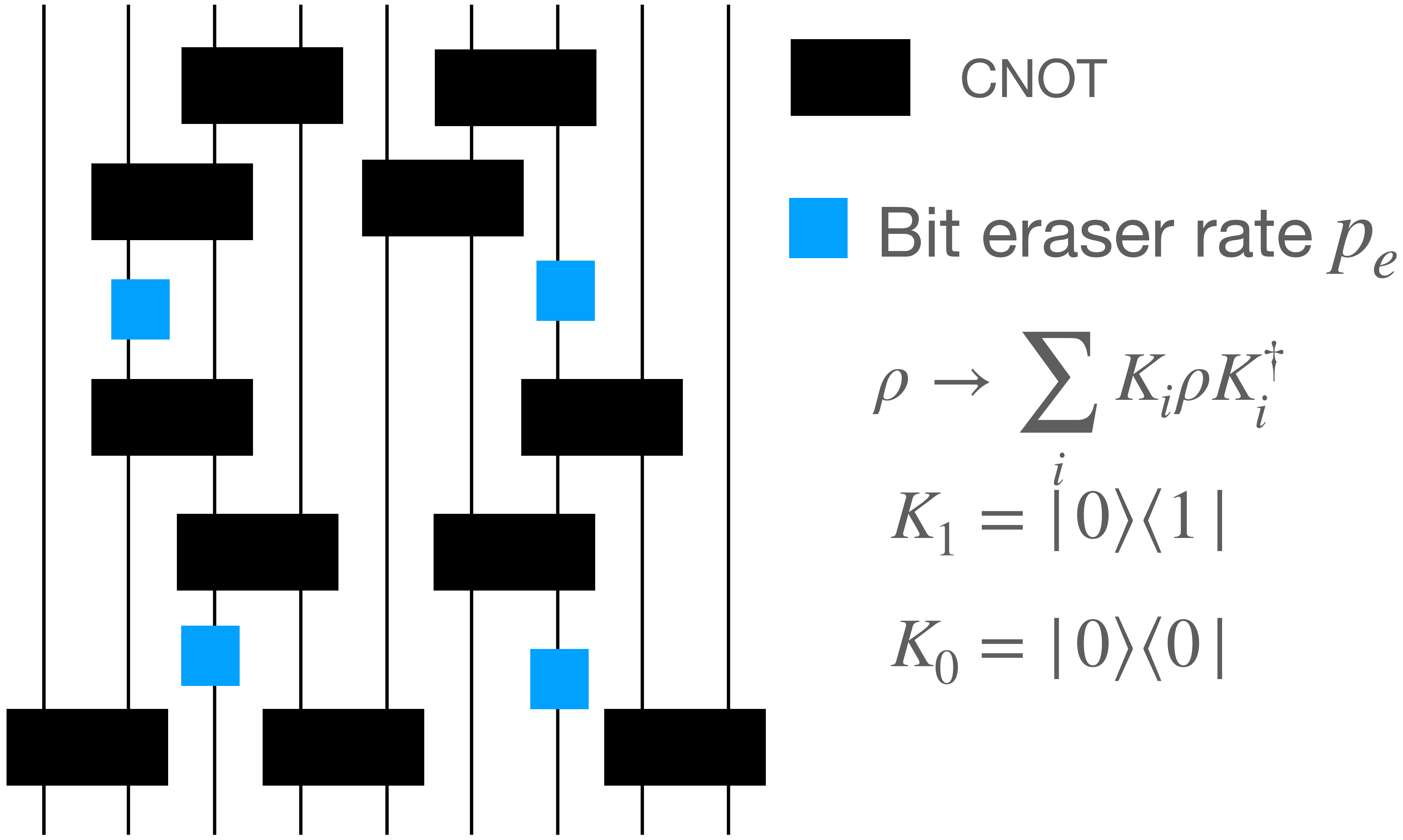
Can Alice protect a classical diary?

saves diary



looses diary

$$S(\rho_T) = ?$$



$$\rho \to \sum_i K_i \rho K_i^\dagger$$

$$K_1 = |0\rangle\langle 1|$$

$$K_0 = |0\rangle\langle 0|$$

$S(\rho_0) = 10$ $C_x = 0$

Similar:

1) Lyons, Choi, Altman APS 2022

2) Knolle group arXiv:2203.11303 2022

Can this classical channel protect quantum info?

Coherence non-generating dynamics

Same dynamics-different initial state/observable

- Encode a quantum state $\left|\psi_0(x)\right|^2 = \left|\mathscr{C}\right|^{-1/2}$

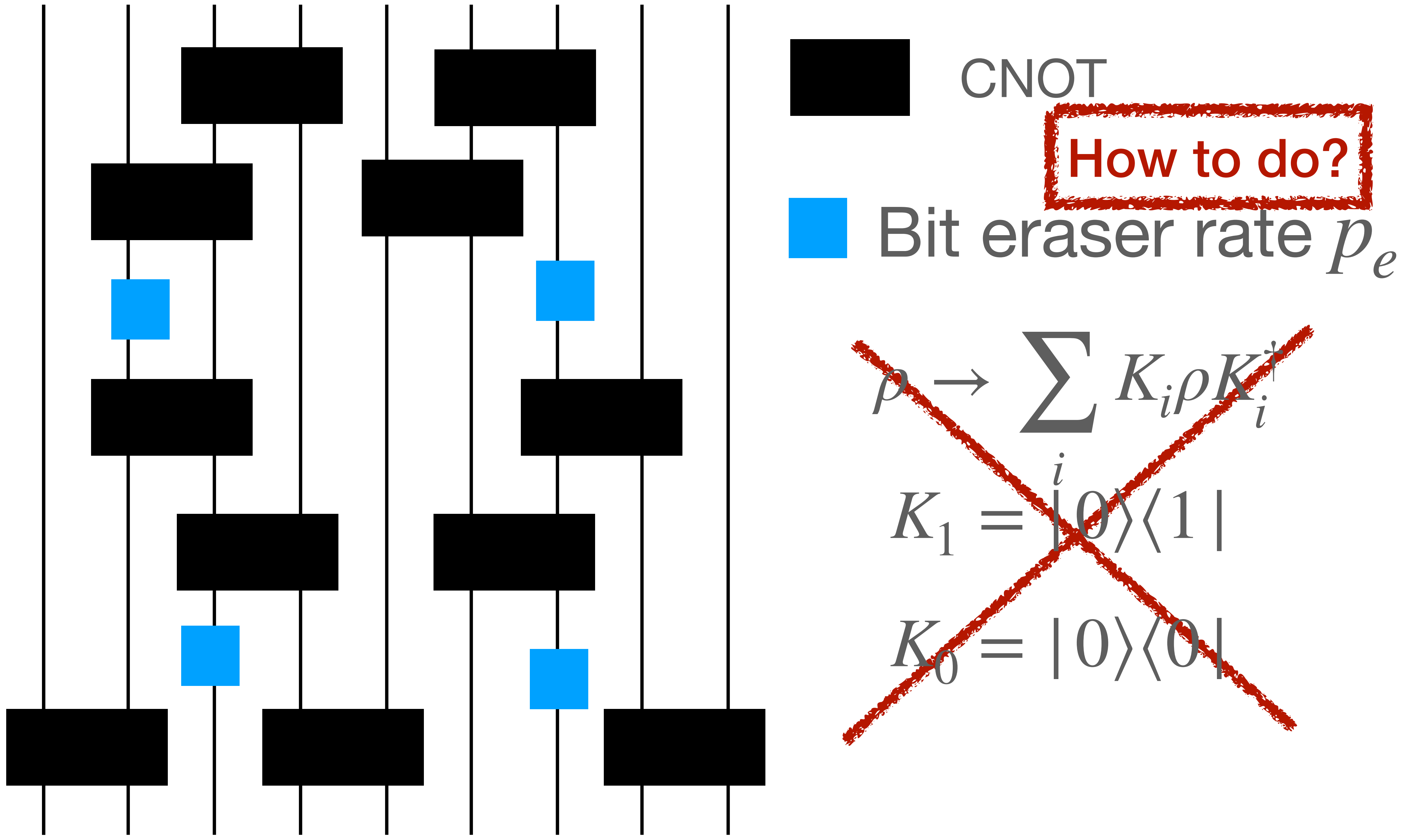
$$\left|\psi_0\right\rangle = \sum_{x\in\mathscr{C}} \psi_0(x)\,|x\rangle$$

- State is coherent

$$C_x\left(|\psi_0\rangle\right) = \log(|\mathscr{C}|) = 10$$

- Can Alice recover phase of $\psi_0(x)$?

? $\left|\psi_0\right\rangle = \sum_{x\in\mathscr{C}} \psi_0(x)\left|f_t(x)\right\rangle$?



$$\left|\psi_0\right\rangle = \sum_{x\in\mathscr{C}} \psi_0(x)\,|x\rangle$$

How to implement classical bit eraser

“Simple” eraser

“Clever” eraser

$C_X = 1$

$|00\rangle + |11\rangle$

Measure X_i and flip

$|00\rangle$ $\quad$ $|10\rangle$

$C_X = 0$

Coherence destroyed

Measure Z_i and rotate

$\big(|0\rangle + |1\rangle\big) \otimes |0\rangle$ $\quad$ $\big(|0\rangle - |1\rangle\big) \otimes |0\rangle$

$C_X = 1$

Coherence preserved

$$\left|\psi_0\right\rangle = \sum_{x\in\mathscr{C}} \psi_0(x)\,|x\rangle$$

$$\left|\psi_t\right\rangle = \left|f_t(x')\right\rangle$$

Alice looses quantum diary

$$\left|\psi_t\right\rangle = \sum_{x\in\mathscr{C}} \psi_0(x)(-1)^{g(x,z_i)}\left|f_t(x)\right\rangle$$

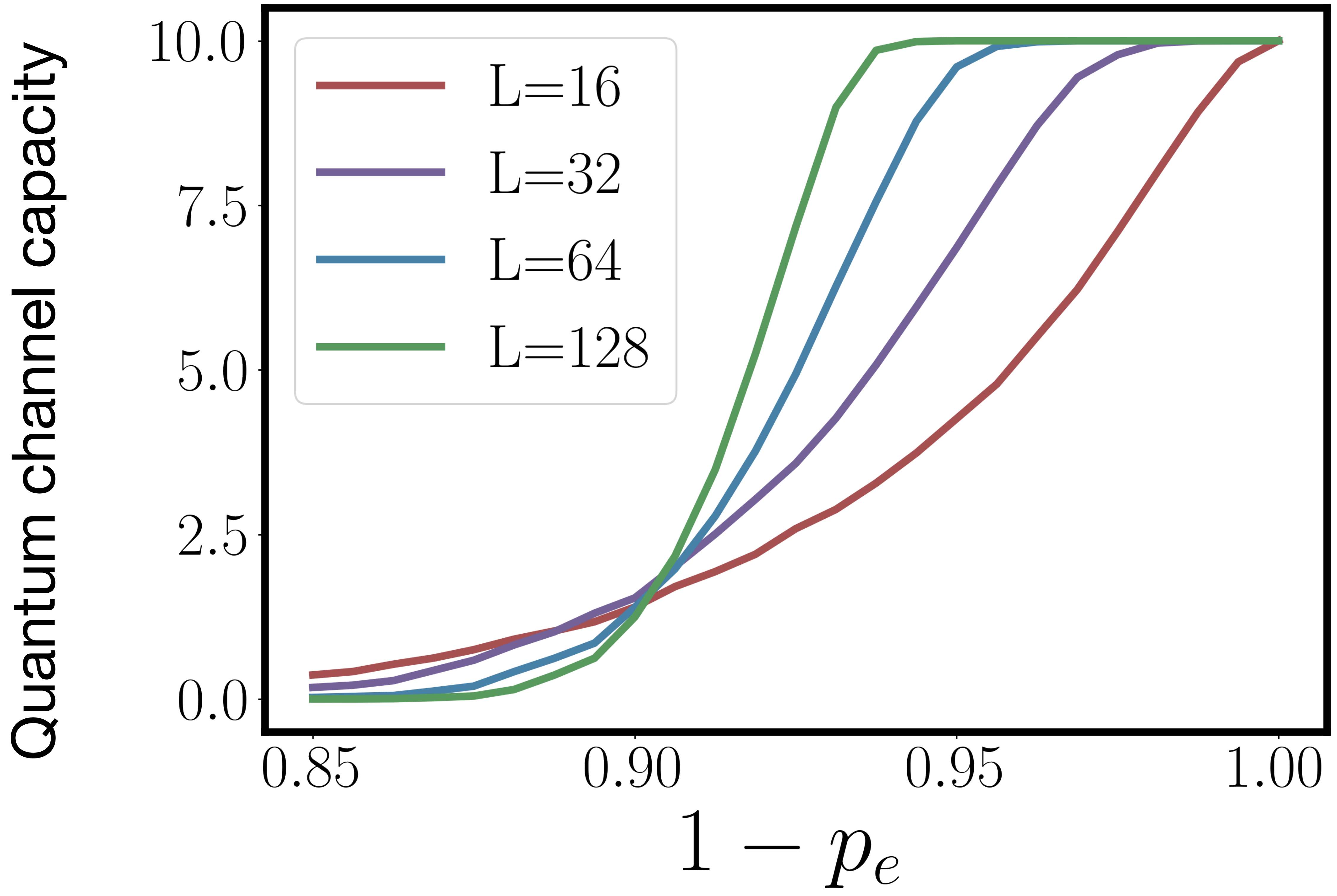
Alice preserves quantum diary

if f_t reversible

(i.e. finite classical channel capacity)

Transition in quantum channel capacity

Can Alice protect a quantum diary?



Bit eraser must preserve coherence!

Bob must only measure Z

How to turn a classical communication channel quantum?

Classical to quantum channels

Lessons from quantum error correction codes

- Classical code:
  - Can correct bit flip errors (coherence preserving errors)
  - Can't correct phase flips (coherence destroying errors)
- CSS quantum stabiliser codes
  - Code constructed from 2 "compatible" classical codes
  - In asymptotic code performance: all stabiliser codes $\sim$ CSS codes
  - Difficult: producing good CSS codes

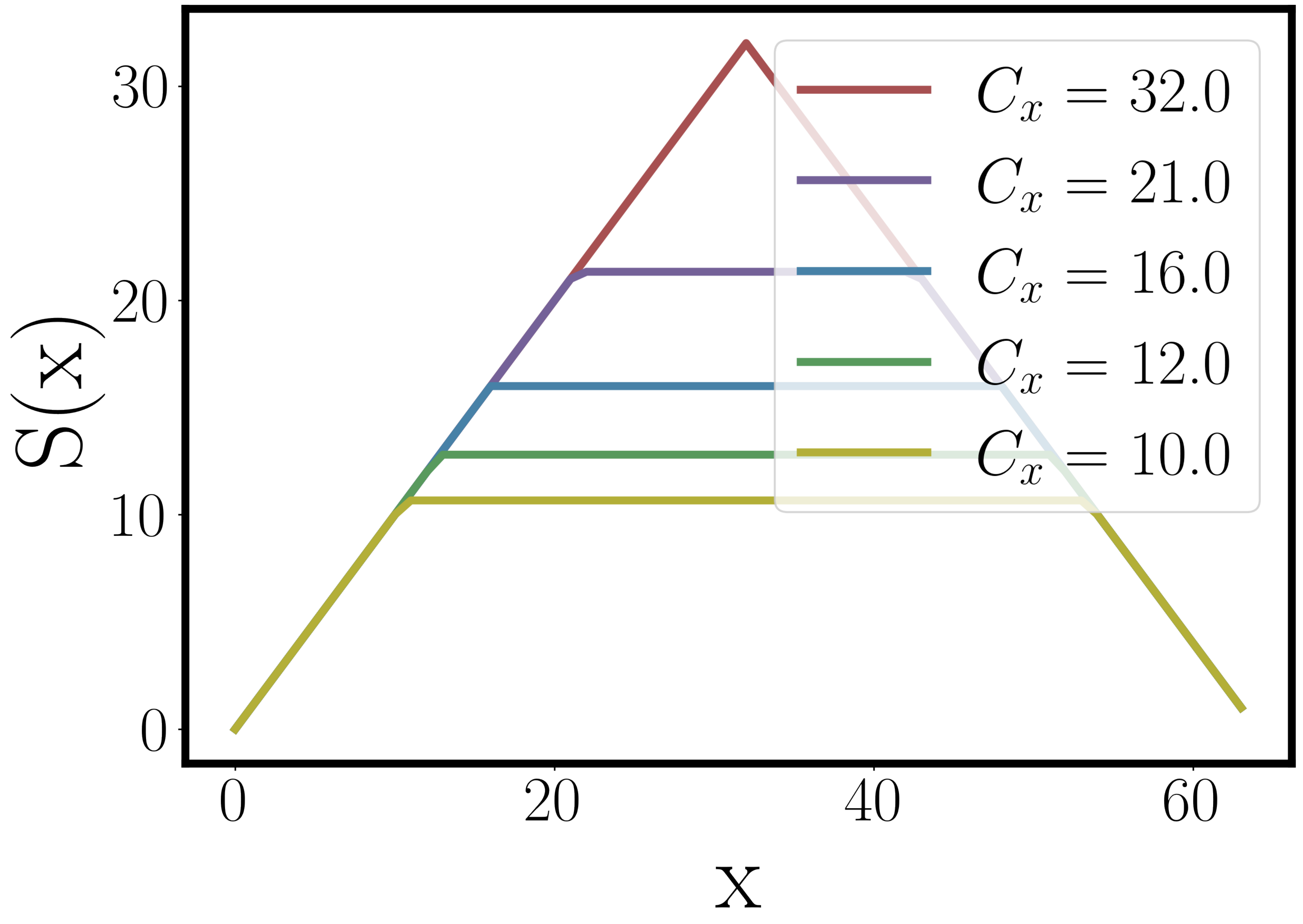
Is there something similar Alice can do to turn her classical channel quantum?

But first, we will focus on entanglement and coherence dynamics

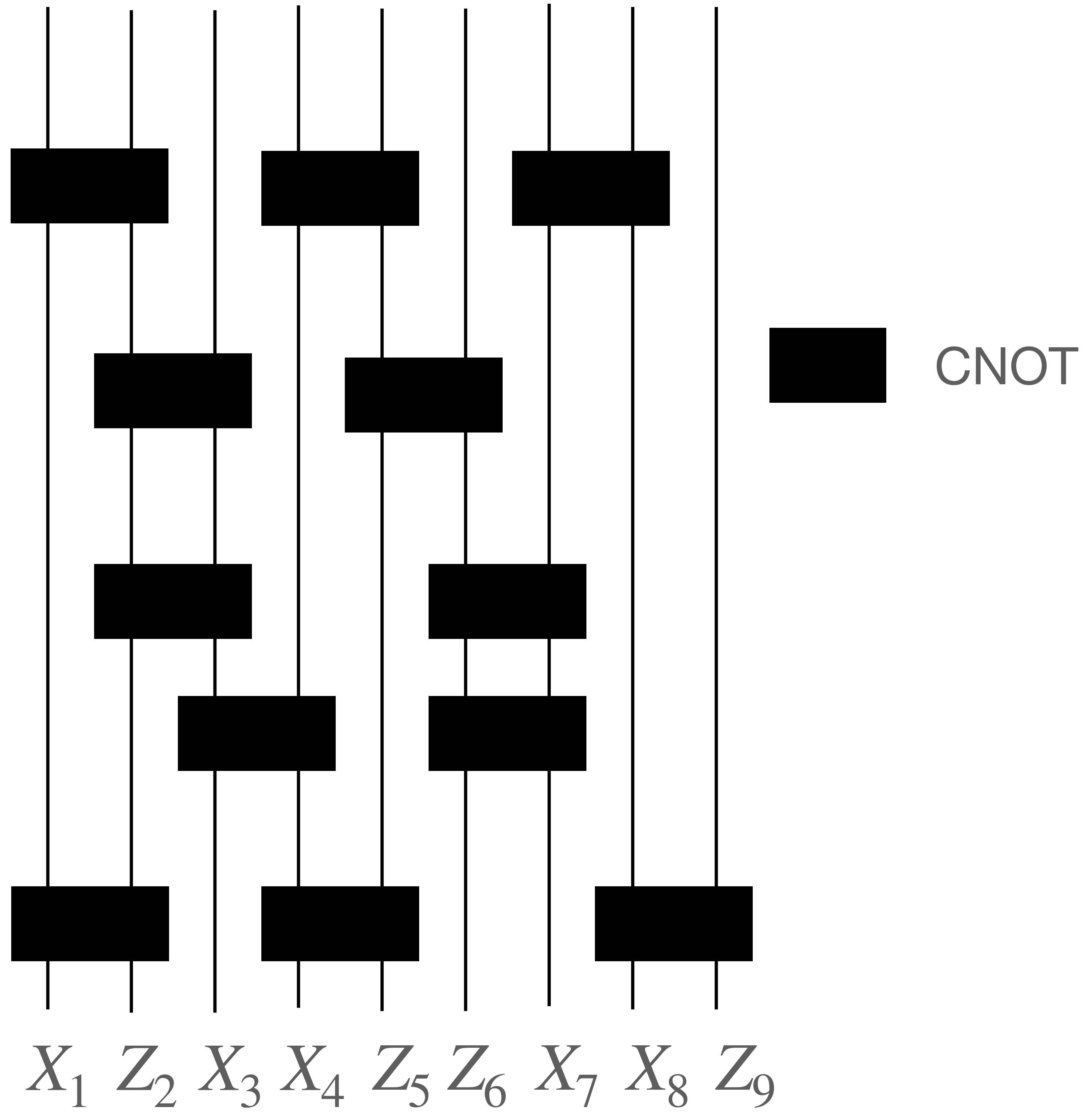
Volume v.s. area law depends on coherence

Coherence bounds entanglement[1)]

$$\min(C_x, C_z) > S(\rho_r)$$



Volume law ~ Alice saves quantum diary

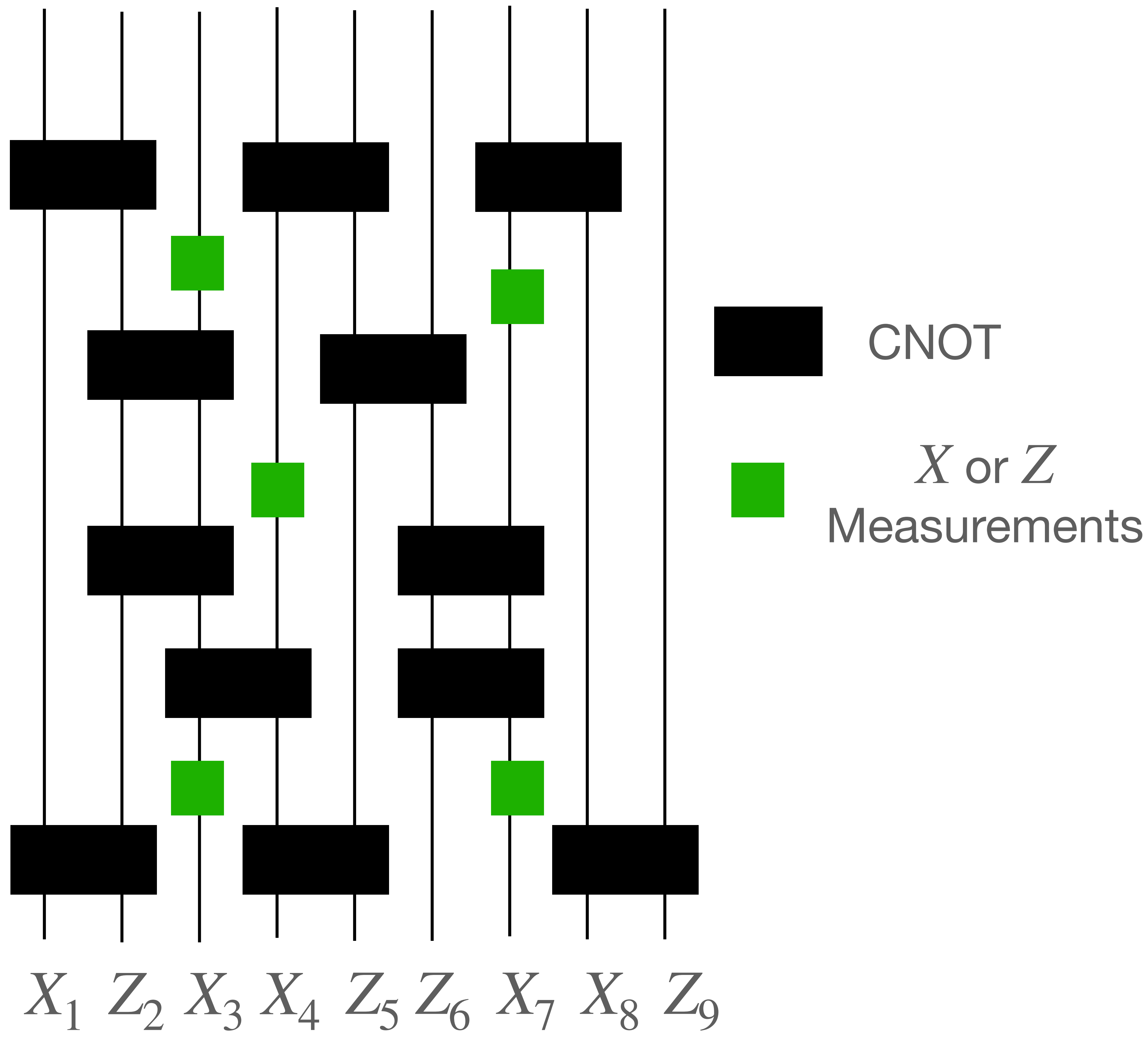


1) Straight forward to prove for pure state, similar results in *Mekala and Sen PRA (2021)*

Measurements generate coherence dynamics

X and Z measurements (errors)

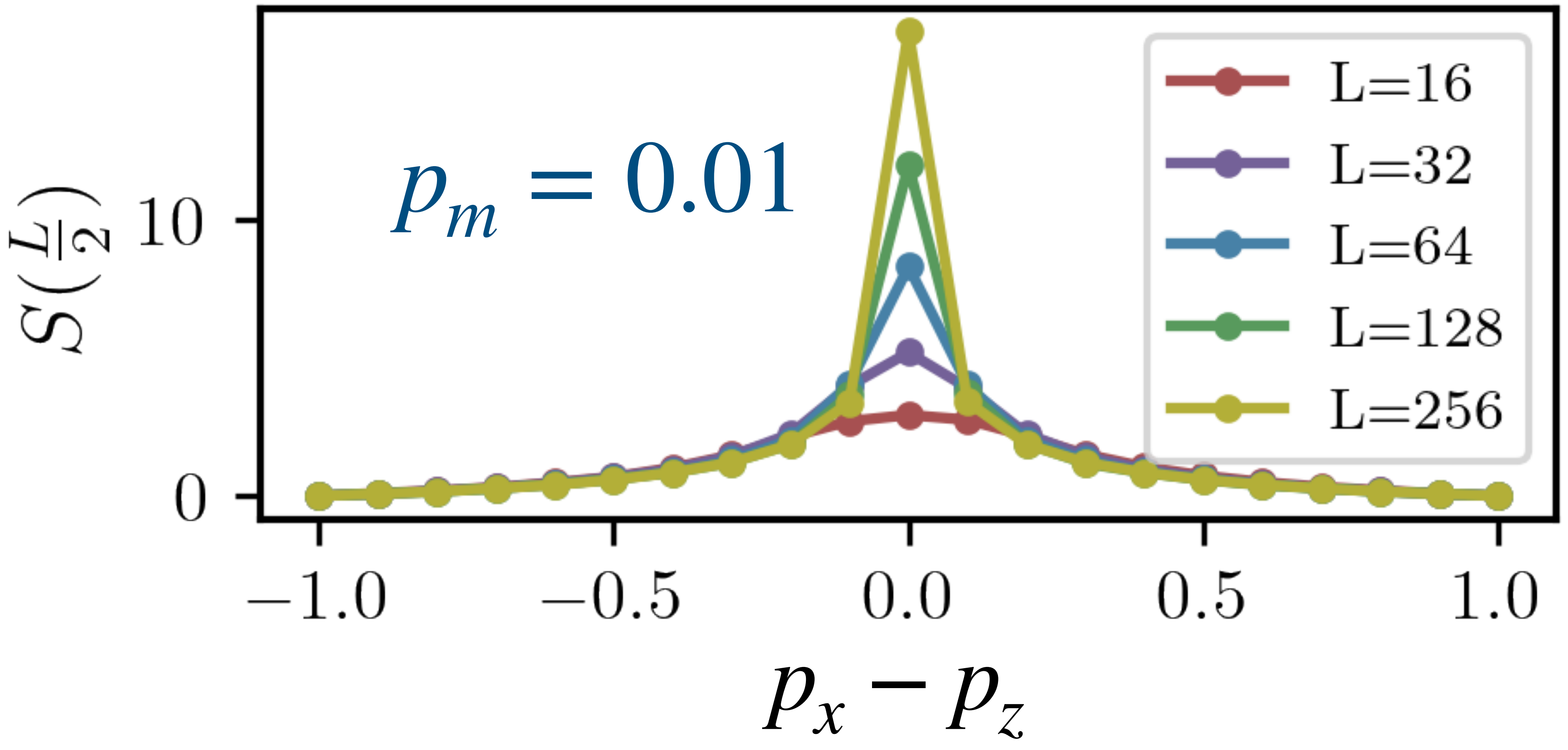
- Total rate of measurements p_m
- Measure Z rate $p_m(1 - p_x)$
    - Generate C_x , destroy C_z
- Measure X at rate $p_m p_x$:
    - Generate C_z , destroy C_x
- Coherence evolves to some steady state determined by p_m and p_x

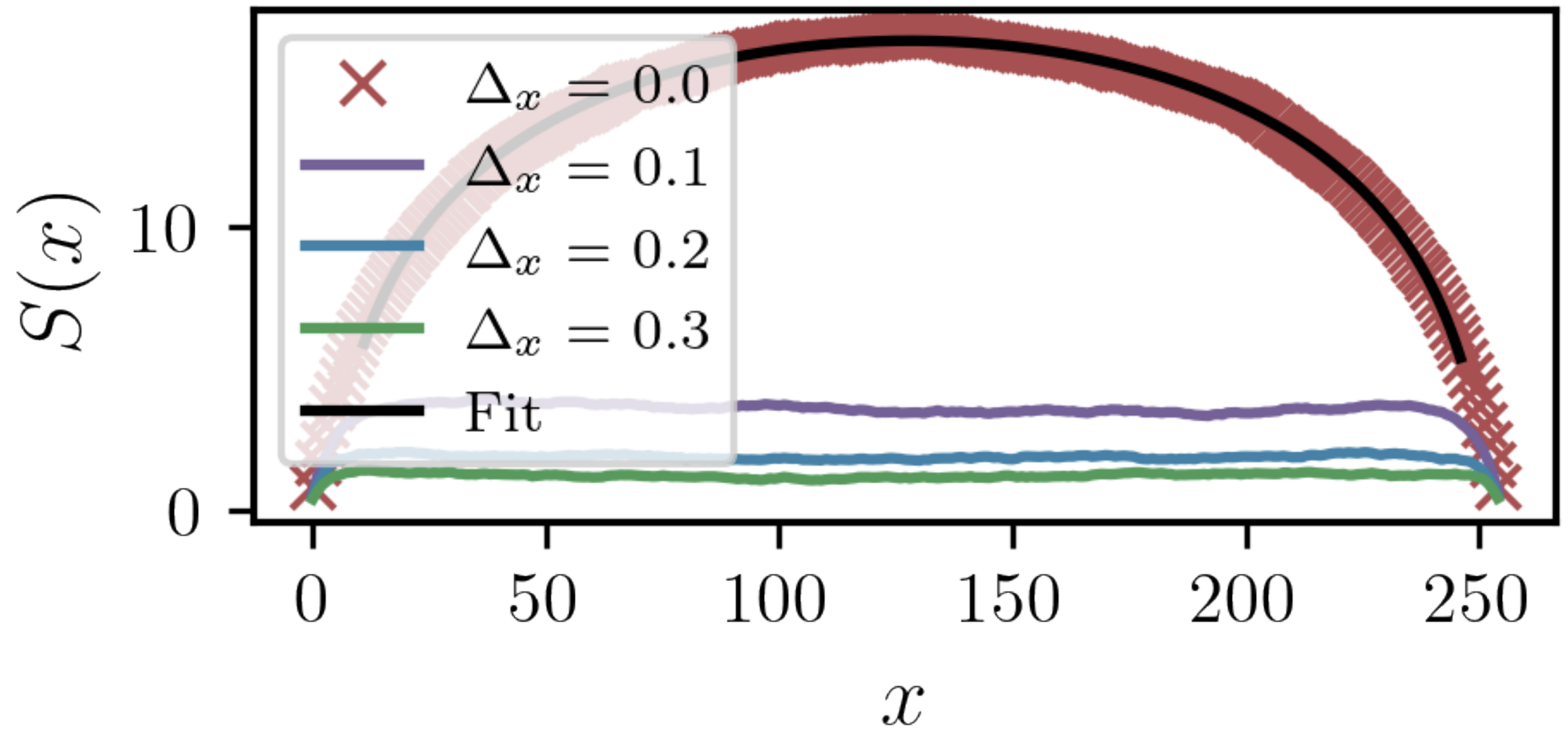


Measurements generate coherence dynamics

X and Z measurements (errors)

- Total rate of measurements p_m
- Measure Z rate $p_m(1-p_x)$
- Measure X at rate $p_m p_x$:
- Small p_m coherence dynamics: Markovian
    - $\partial_t \langle C_x \rangle \approx p_z - p_x$
    - $min(C_x, C_z) \to 0$





Interpretation using evolving code spaces

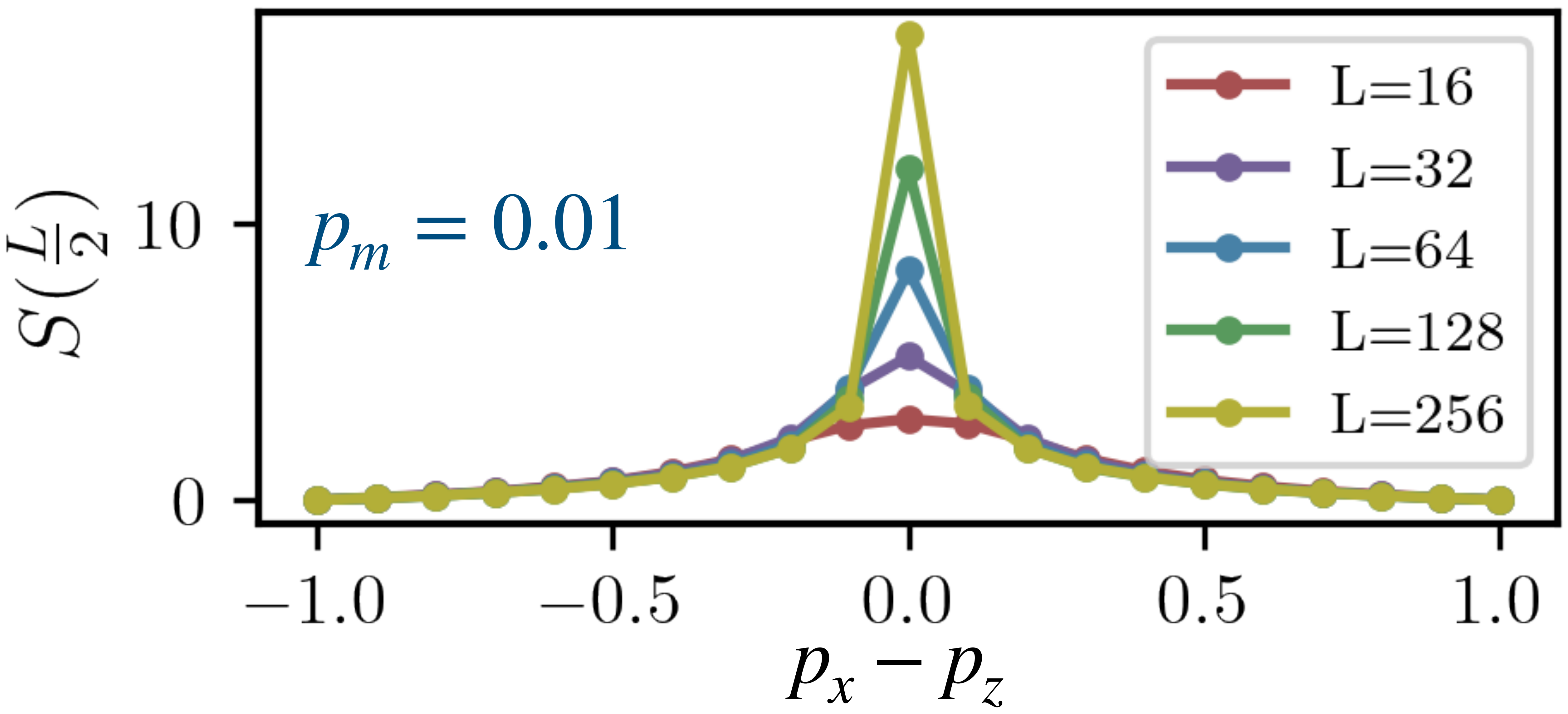
Competing Classical Codes

Generalisation from CSS code won't work for Alice

$p_z > p_x$

$\partial_t \langle C_z \rangle \approx p_x - p_z$

$\langle C_z \rangle \to 0$



$p_x > p_z$

$\partial_t \langle C_x \rangle \approx p_z - p_x$

$\langle C_x \rangle \to 0$

Classical in Z

$k_z \to 0$ and $k_x \to L$

"can't protect bit errors"

Number of logical qubits in CSS code:

$k = k_x + k_z - L$

Classical in X

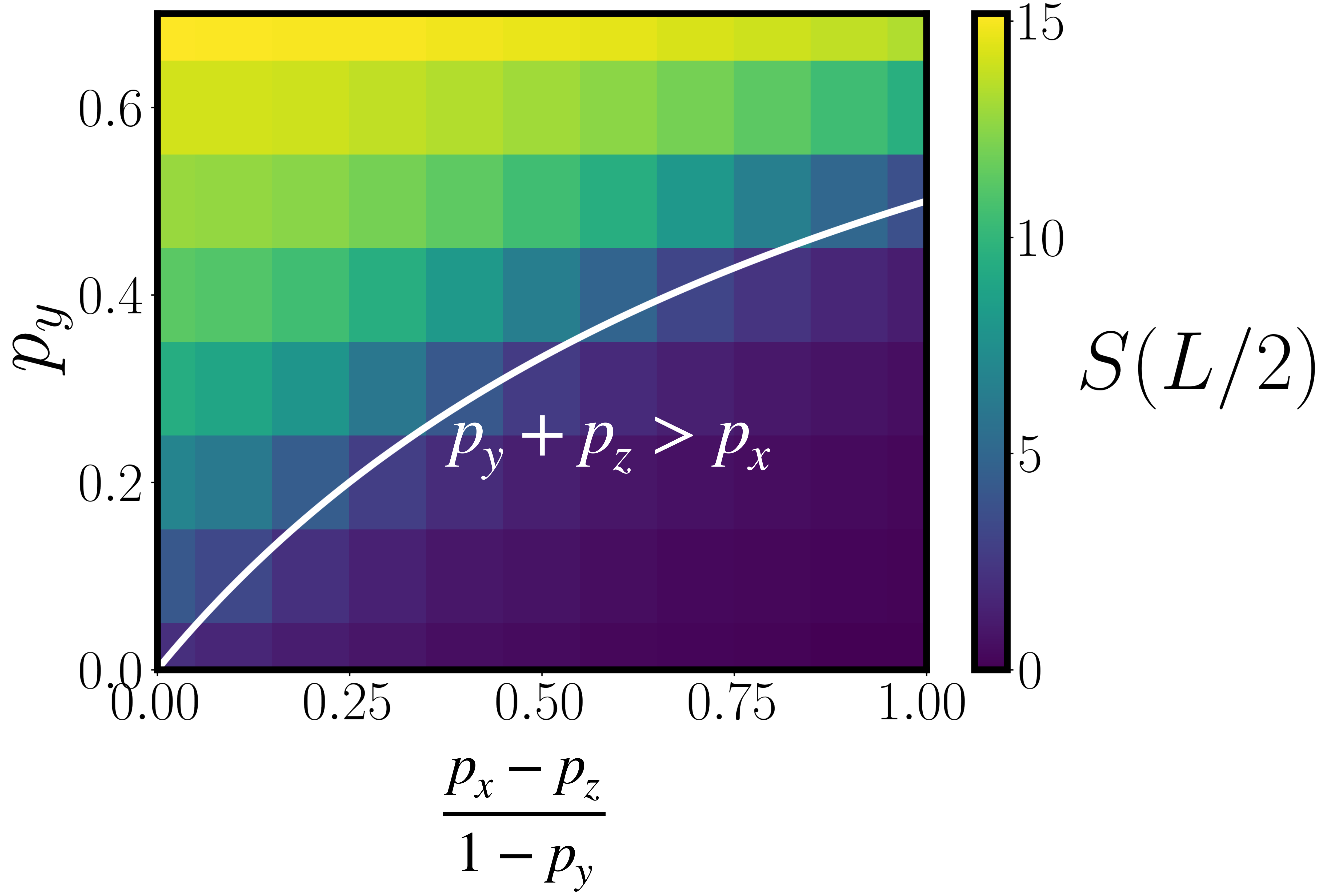
$k_x \to 0$ and $k_z \to L$

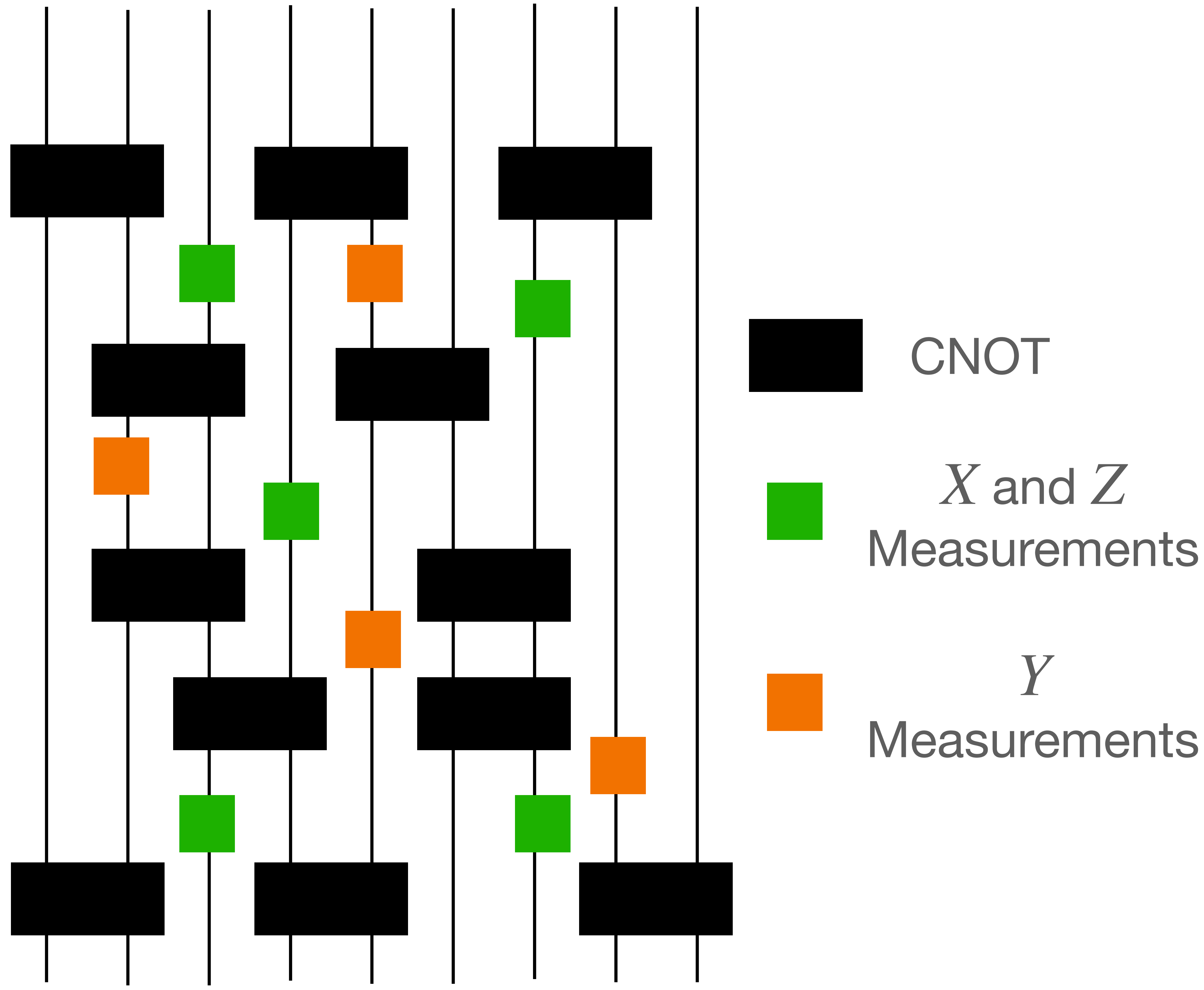
"can't protect phase errors"

Coherence controlled entanglement transitions

What if we add Y measurements?

$$\partial_t \langle C_x \rangle \approx p_y + p_z - p_x > 0$$



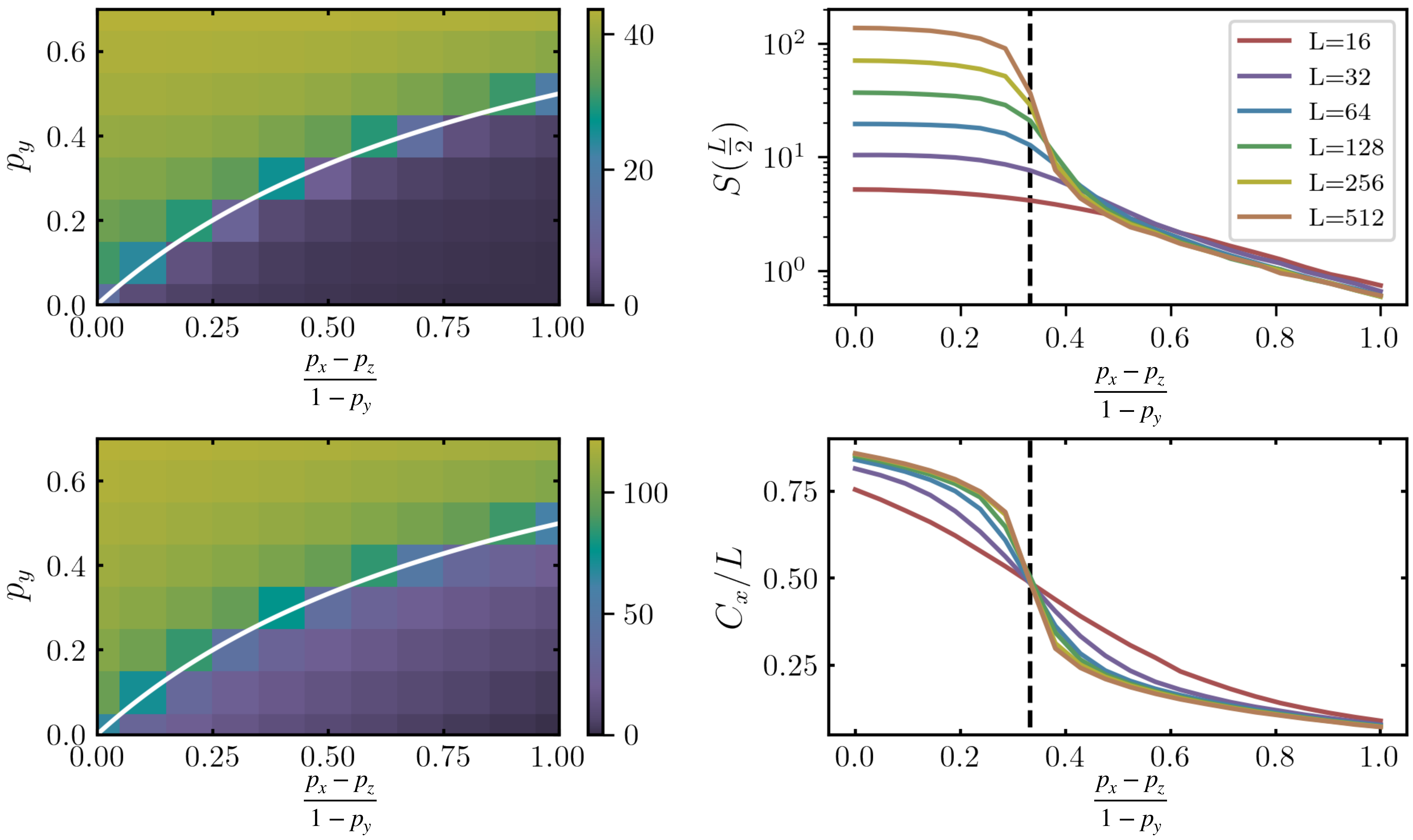


Controlling entanglement transition with coherence

$$\partial_t \langle C_x \rangle \approx p_y + p_z - p_x > 0$$

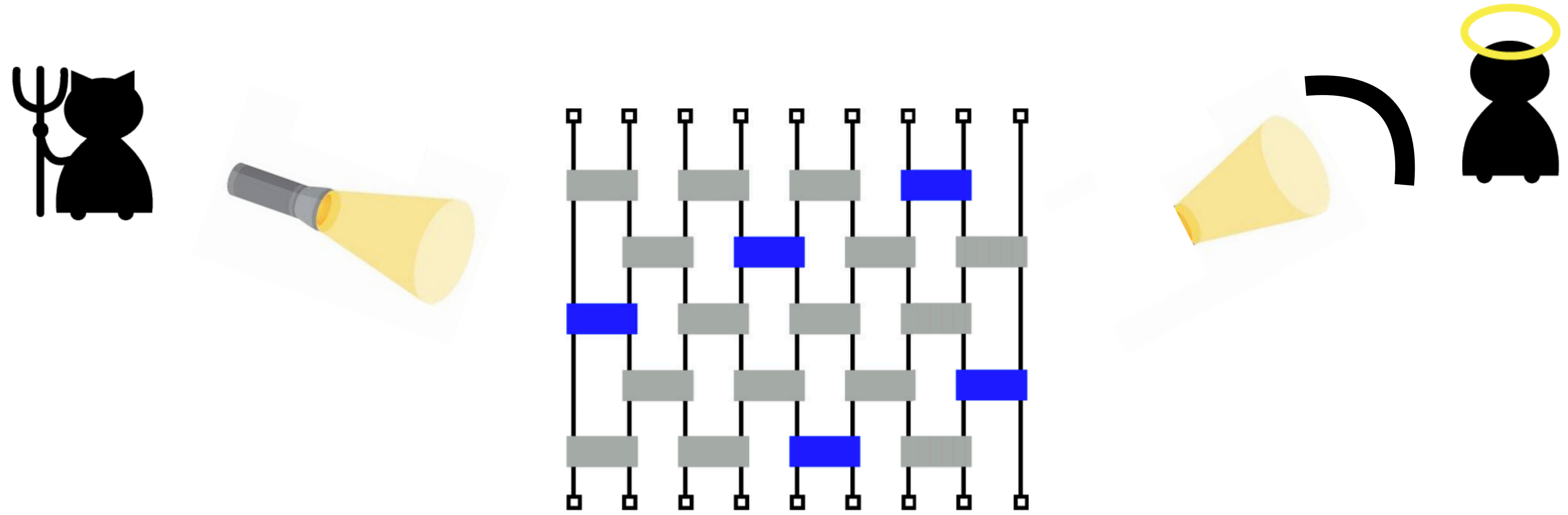
Entanglement:

Coherence:



That's entanglement, what about channel capacity?

What if Alice can only preform unitaries?

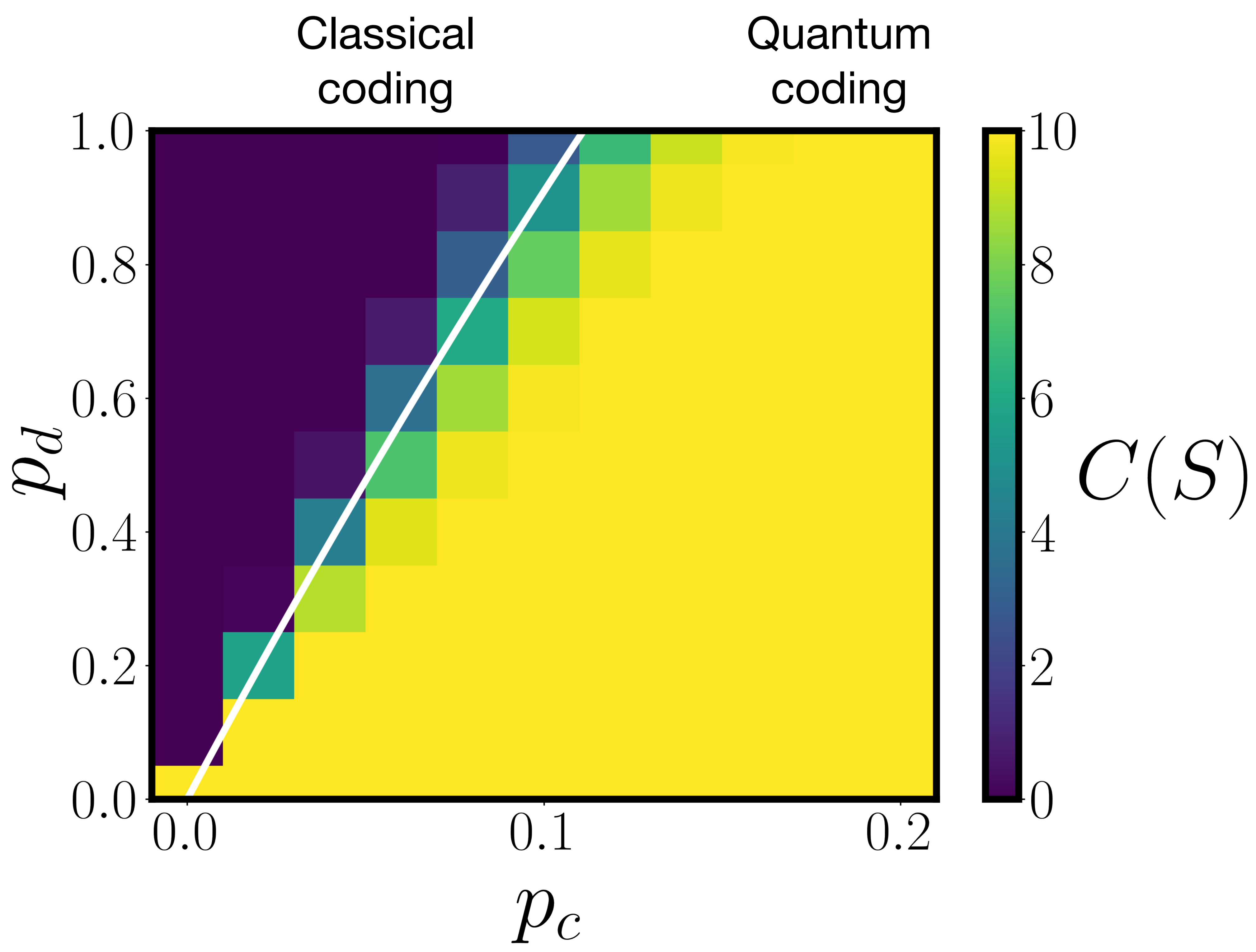


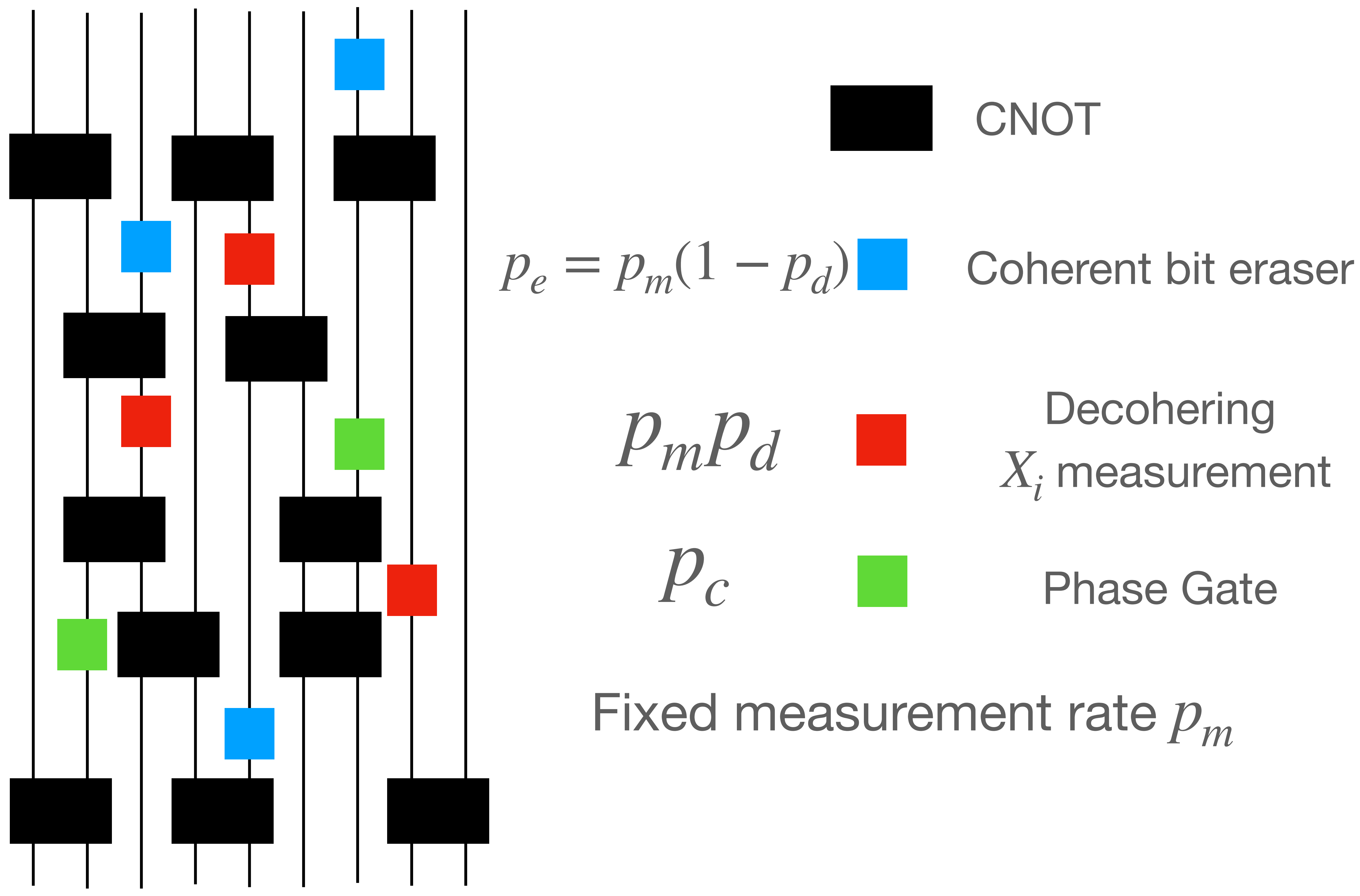
- Y measurement → volume law phase
  - Possible for Alice to protect diary
  - Requires Bob to be limited in light he shines (only Y measurements)
- What unitaries does Alice require to protect any light Bob could shine?
  - Phase gate $U_i = e^{i\frac{\pi}{2}(Z_i+1)}$
  - Generically increases C_x

How to protect Alice’s quantum diary?

Coherence generating element

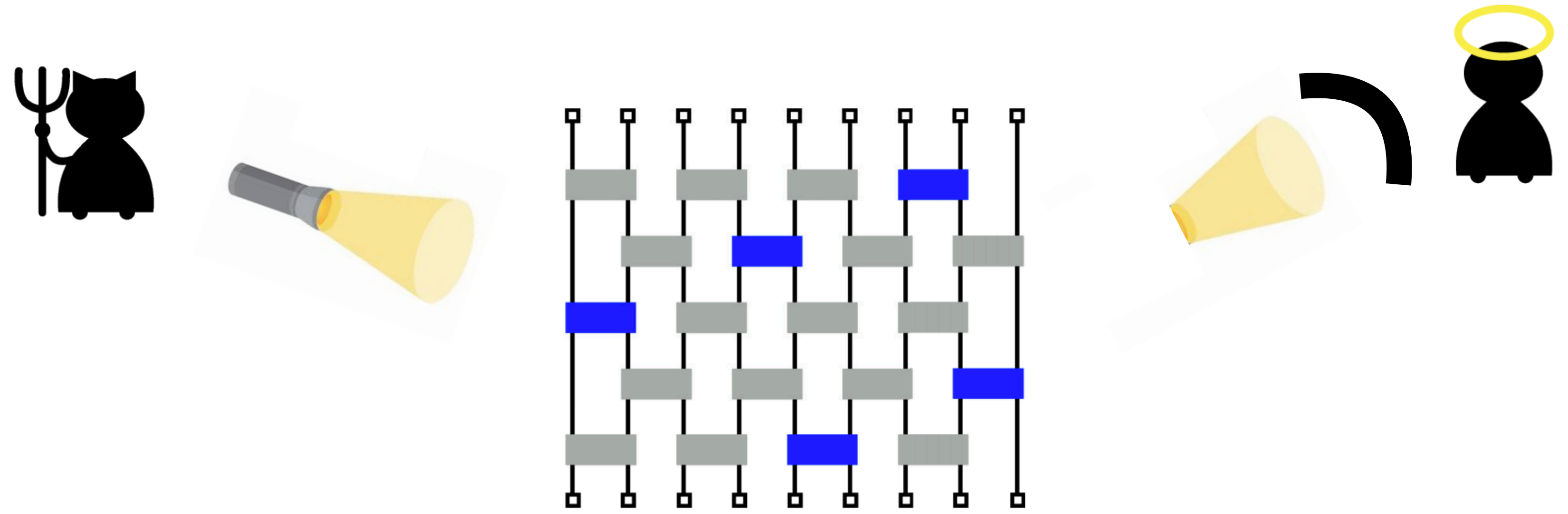
- Phase gate $U_i = e^{i\frac{\pi}{2}(Z_i+1)}$



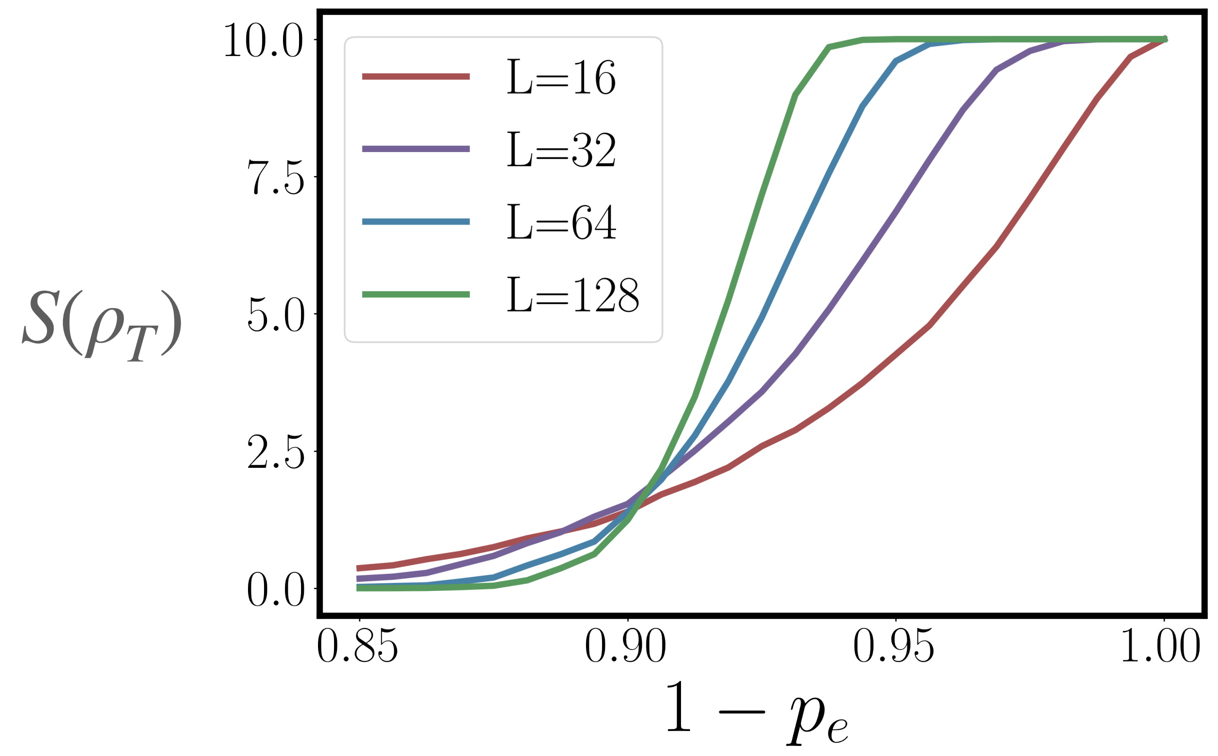


Protecting Alices diaries

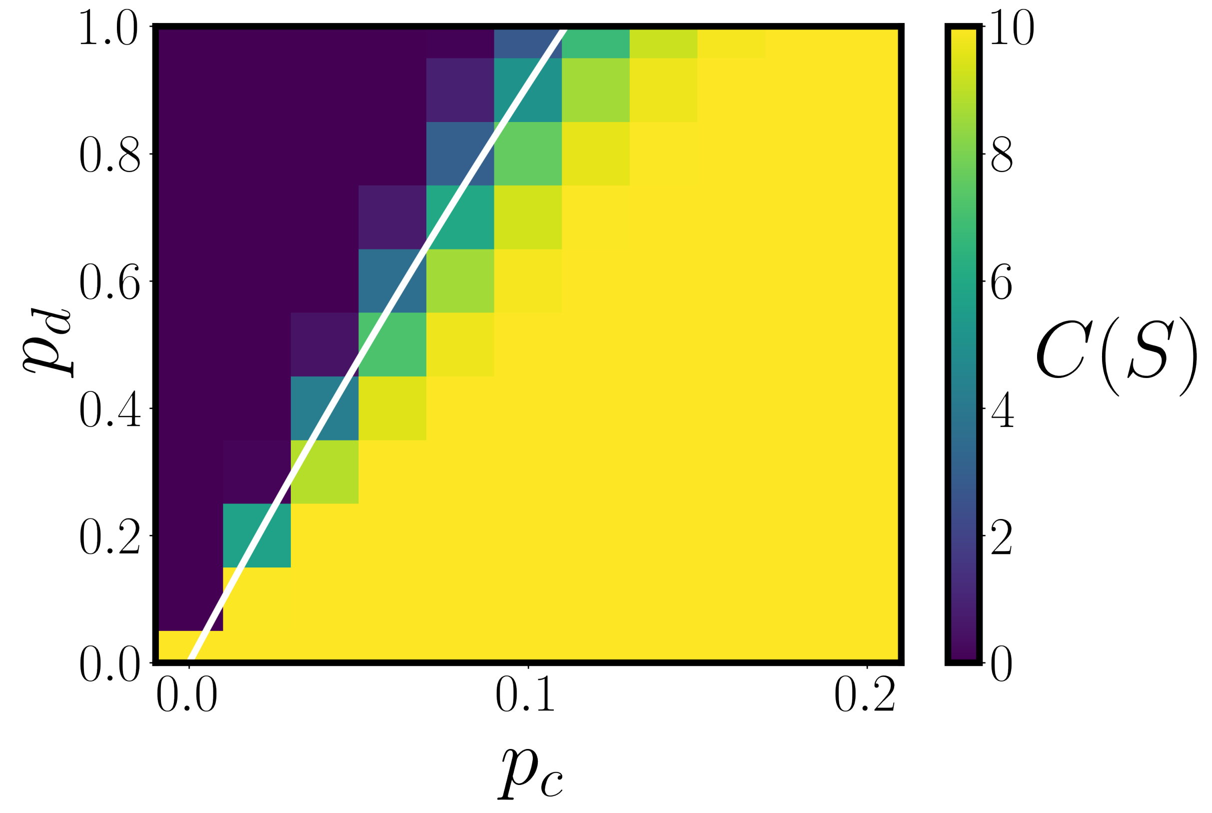
The role of coherence



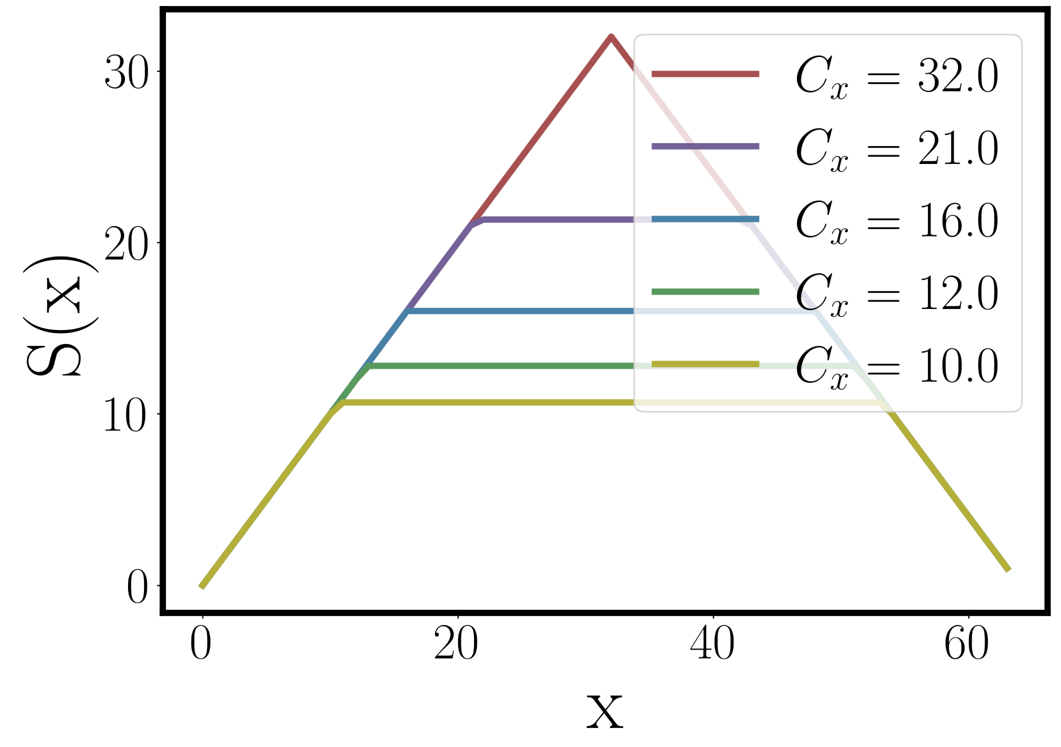
Coherence free circuits - Alice protects classical diary



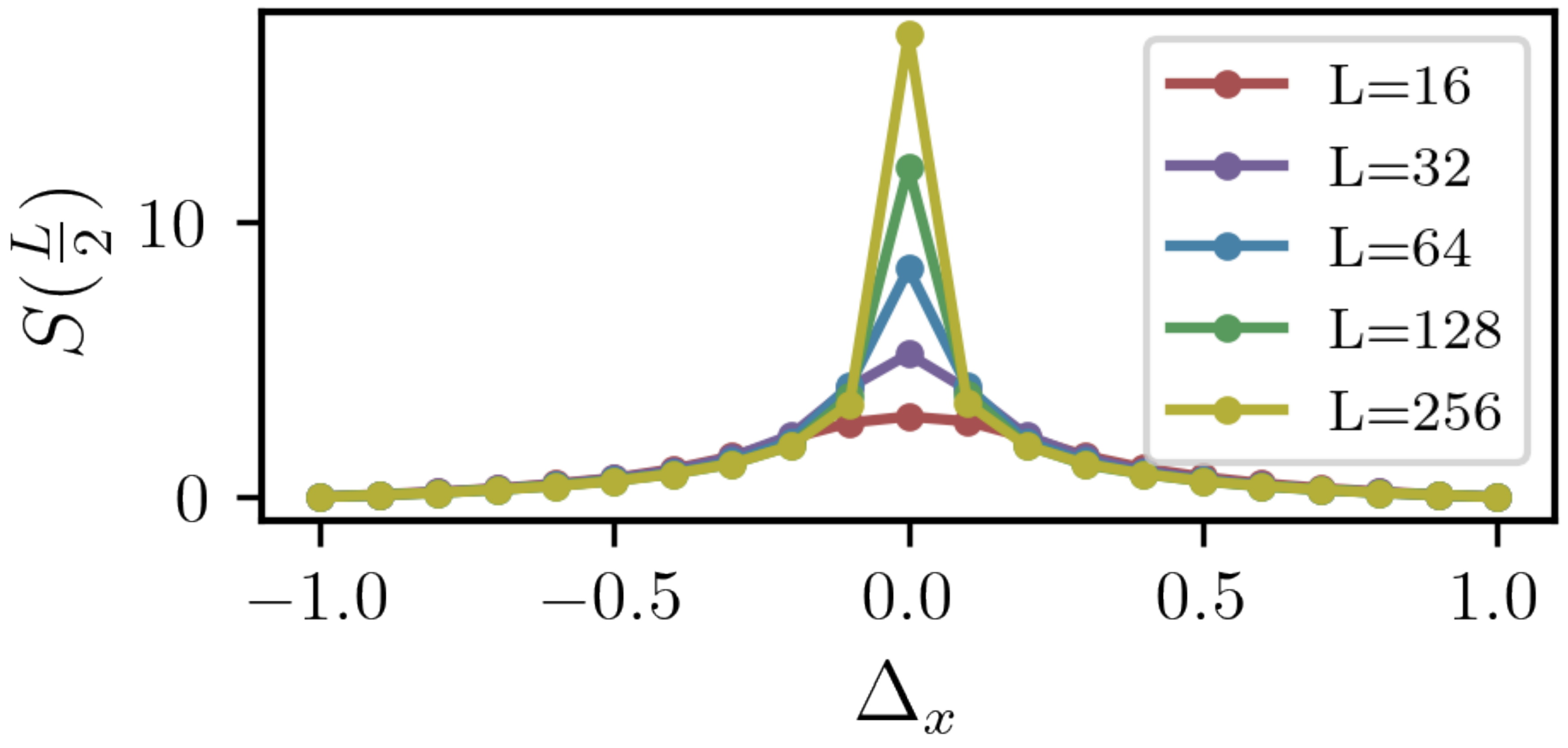
Alice must dynamically regenerate coherence



Coherence bounds entanglement



Alice can’t use CSS code approach



Coherence requirements for error correction

Coherence requirement for error correction
Code distance bound by coherence

- $[n, k, d]$ stabiliser code
- Take any single qubit subspace of the code $\quad |\psi\rangle = \psi_1 |c_1\rangle + \psi_2 |c_2\rangle$
- Coherence in any local Pauli basis D bounds code distance:

$$d < \max_{\psi_1,\psi_2} C(|\psi\rangle, D)$$

- Applied to CSS code

$$\max_{\psi_1,\psi_2} C(|\psi\rangle, X) = \max C_x = L - k_z + 1 > d$$

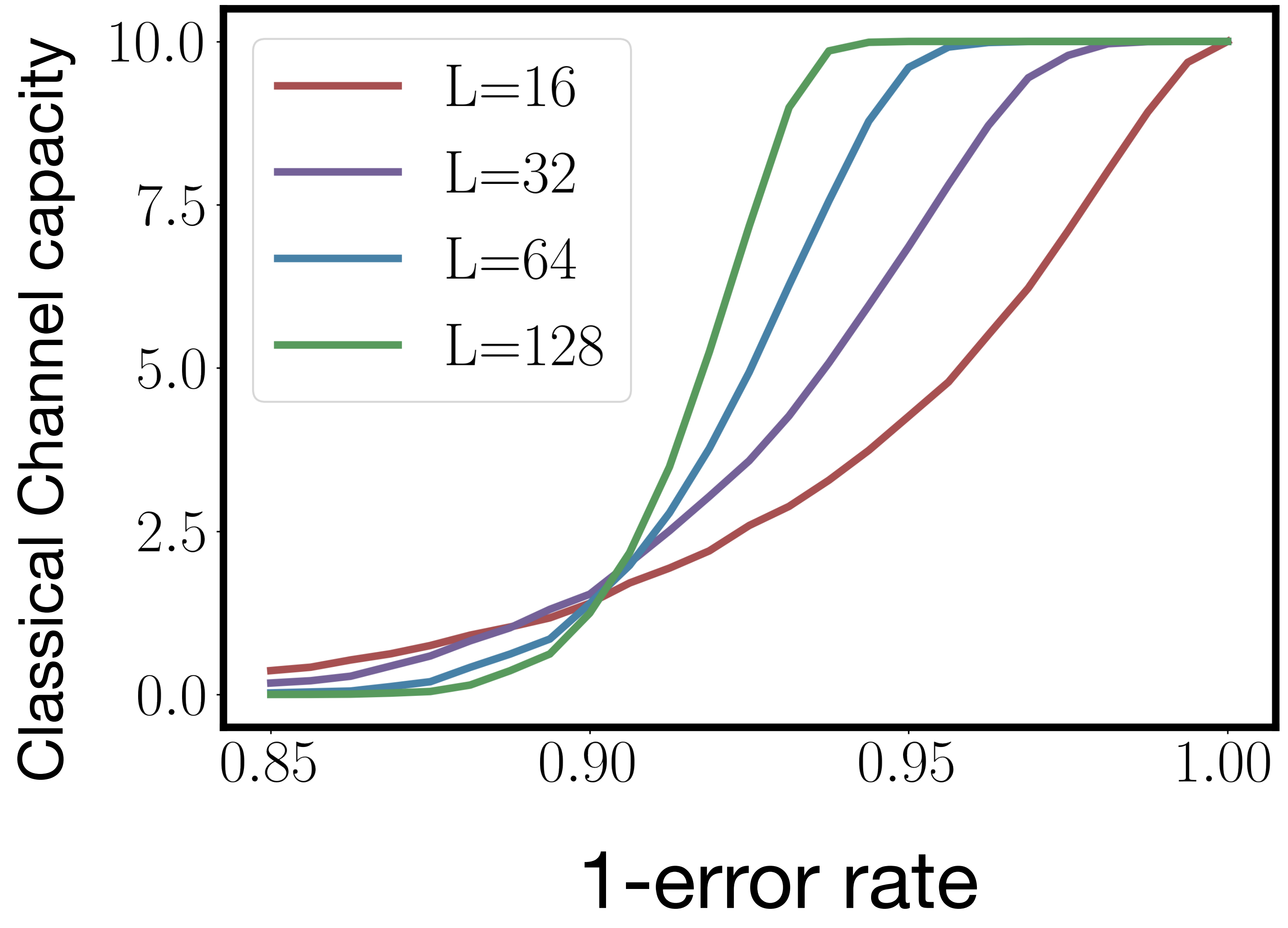
Classical Singleton bound code space defined by H^z

- Similar for C_z: $L - k_x + 1 > d$

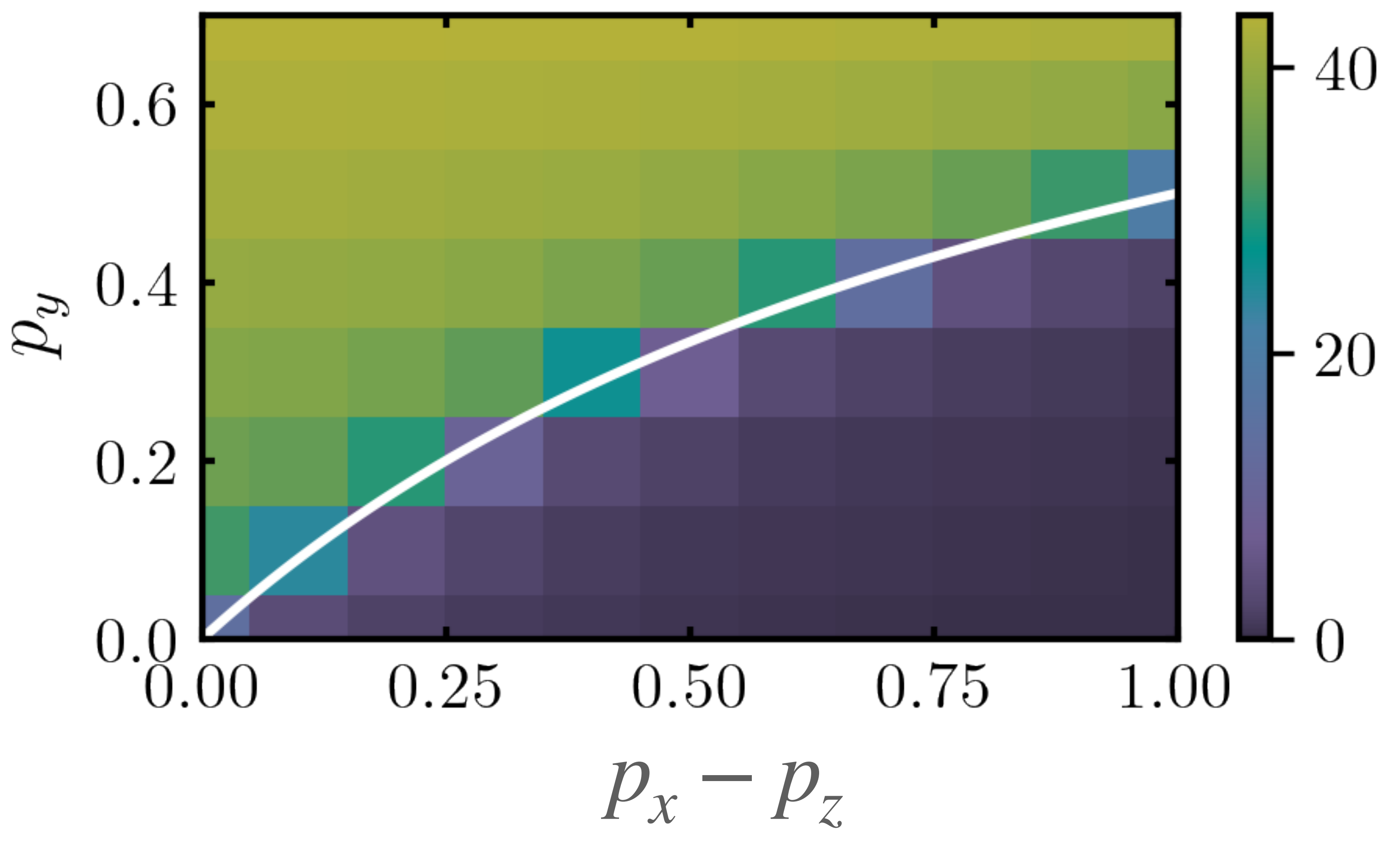
The role of coherence

Conclusions

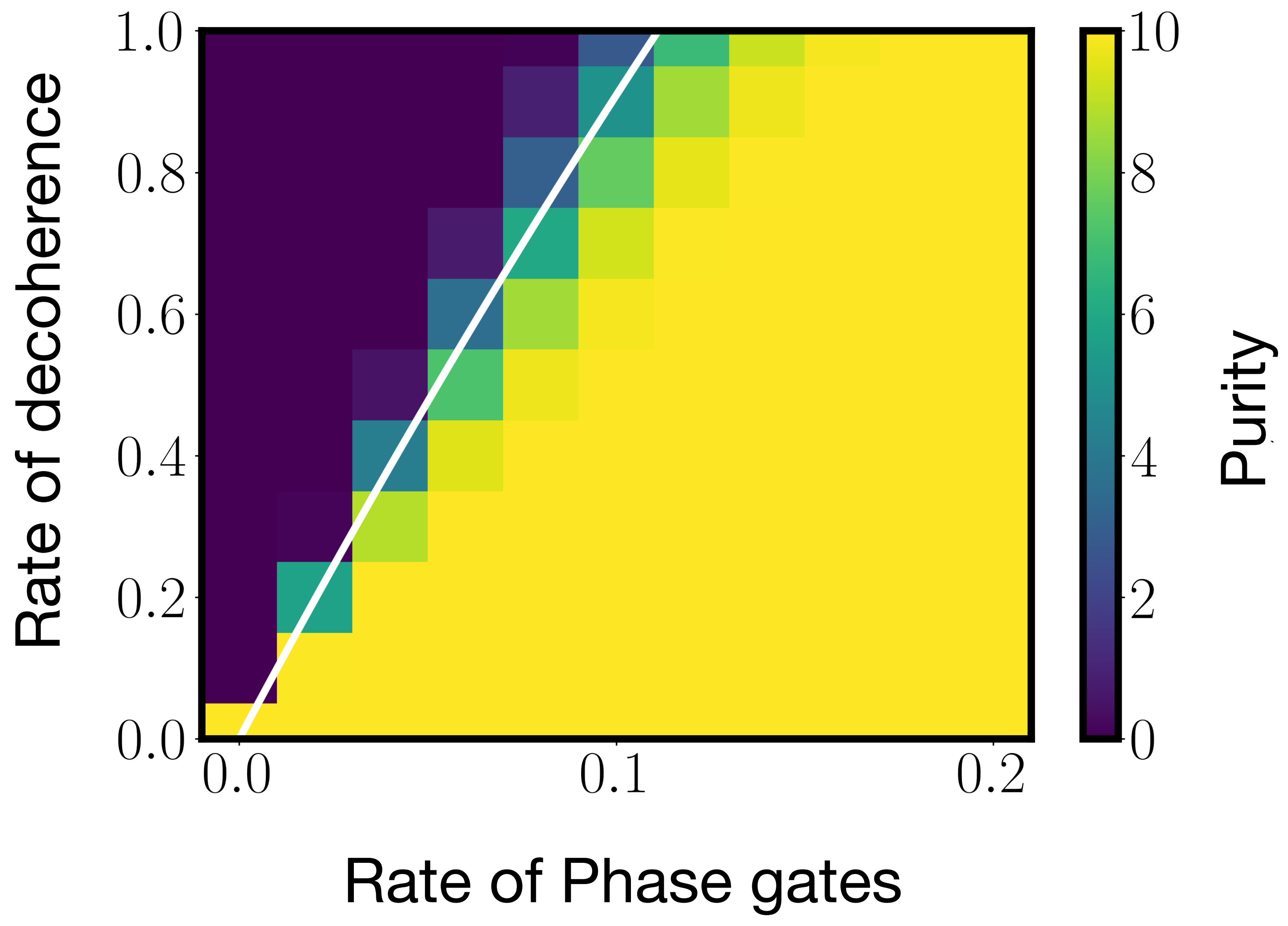
Coherence free :
Alice protects only classical diary



Measurements (bob) **generating coherence:**
entanglement transitions



Unitaries generating coherence:
Alice protects quantum diary



We quantify coherence requirement for quantum error correction

$$d < \max_{|\psi\rangle \in \mathscr{C}} C(|\psi\rangle, D)$$