

# Driving Exchange Mode Resonance as Adiabatic Quantum Motor with 100% Mechanical Efficiency

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Junyu Tang  
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Reference:

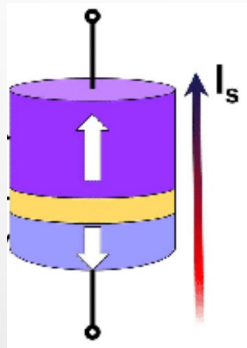
J. Tang and R. Cheng, Phys. Rev. B **106**, 054418 (2022)

Funding



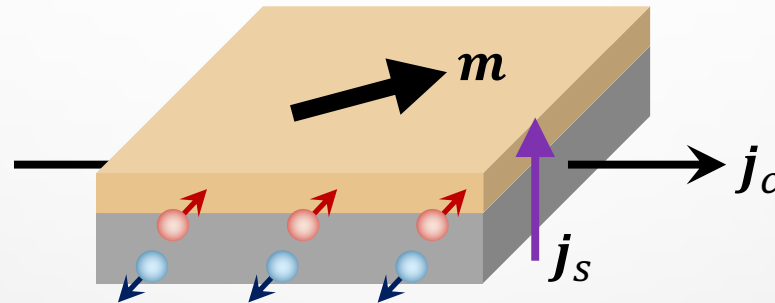
# Current-Induced Torque

From spin valve to spin Hall device



$$P_M \sim \frac{\alpha M_s}{\gamma} |\dot{m}^2|$$

$$P_J \sim \bar{j}_c^2 / \sigma \text{ with } \sigma = \sigma_0 + \sigma_{SMR}$$



$$\eta = \frac{P_M}{P_J} \ll 1$$

## 1. Joule heating

2. Spin Hall angle:  $\theta_{SH} = j_s / j_c$

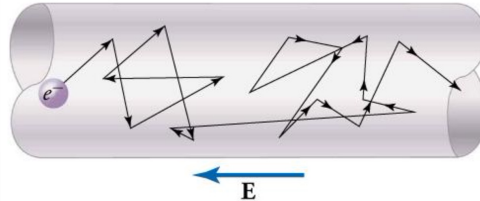
3. Spin diffusion & Backflow

$$g'_r = \frac{g_r}{1 + \frac{2\lambda e^2}{h\sigma} g_r \coth \frac{d_N}{\lambda}}$$

renormalized  
spin-mixing conductance

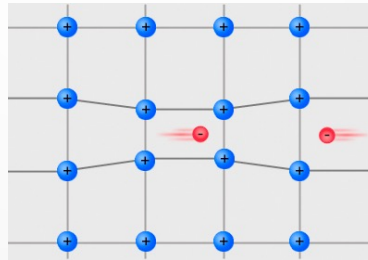
# Physical Currents

- Dissipative Current



$$\mathbf{j}_c = \sigma \mathbf{E}$$

- Superconducting Current



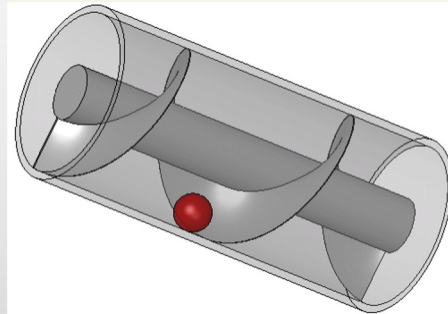
persistent current

$$(T < T_c)$$

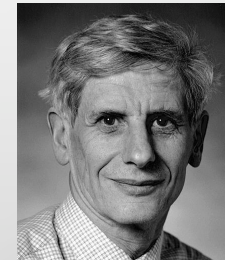
- Adiabatic Current

$$\mathbf{j} = \int dk \vec{\Omega}_{kt}$$

$$\rightarrow \mathbf{j} \sim \dot{\mathbf{m}}$$



One cycle:  $Q = \int dt \int dk \vec{\Omega}_{kt}$



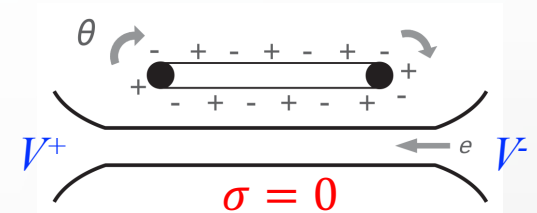
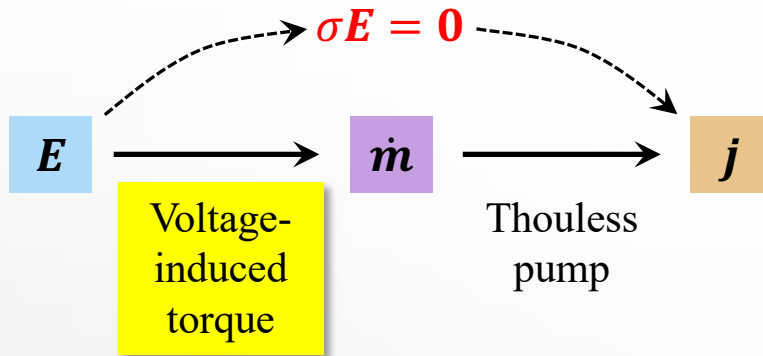
David J. Thouless  
(1934 - 2019)



Nobel Prize in  
Physics **2016**  
(1/2 Share)

# Adiabatic Quantum Motor

- Reversed operation of a Thouless pump

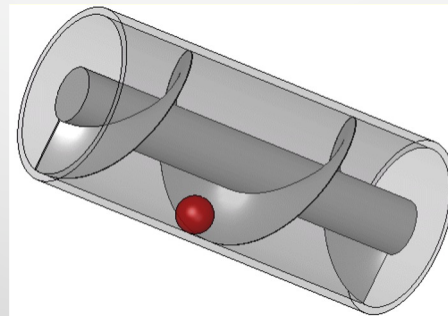


Bustos-Marún, Refael & von Oppen, Phys. Rev. Lett. **111**, 060802 (2013)

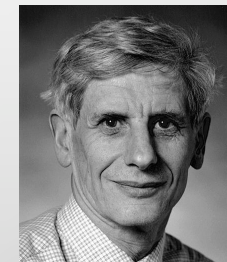
$$P_j = \langle \mathbf{E} \cdot \mathbf{j} \rangle \text{ all converted to } P_j = \alpha \langle \dot{\mathbf{m}}^2 \rangle$$

$$\mathbf{j} = \int dk \vec{\Omega}_{kt}$$

$$\rightarrow \mathbf{j} \sim \dot{\mathbf{m}}$$



$$\text{One cycle: } Q = \int dt \int dk \vec{\Omega}_{kt}$$



David J. Thouless  
(1934 - 2019)



Nobel Prize in  
Physics **2016**  
(1/2 Share)

# Adiabatic Quantum Motor

- Voltage-induced torque

*Sundaram-Niu* equation [PRB **59**, 14915 (1999)]

$$\dot{\mathbf{r}} = \frac{\partial \varepsilon}{\partial \mathbf{k}} + \dot{\mathbf{k}} \cdot \vec{\Omega}_{kk} + \dot{\mathbf{r}} \cdot \vec{\Omega}_{rk} - \Omega_{kt}$$

$$\dot{\mathbf{k}} = -e\mathbf{E} - \dot{\mathbf{k}} \cdot \vec{\Omega}_{kr} - \dot{\mathbf{r}} \cdot \vec{\Omega}_{rr} + \Omega_{rt}$$

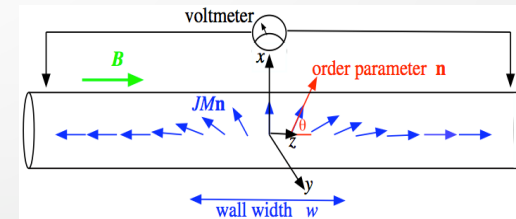
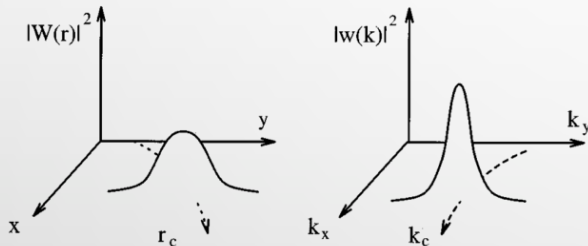


Ganesh Sundaram



Qian Niu

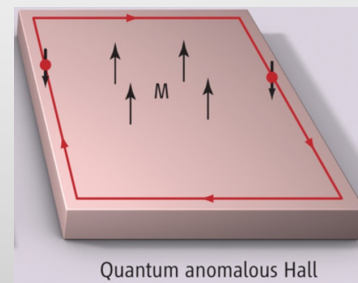
Semi-classical wavepacket



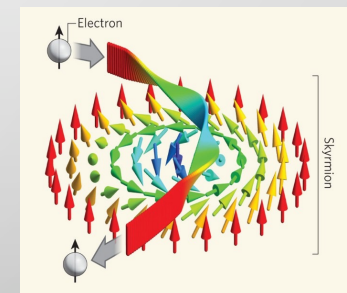
Spin-motive force

$$\vec{\Omega} = \begin{bmatrix} \Omega_{kk} & \Omega_{kt} & \Omega_{kr} \\ \Omega_{tk} & \Omega_{tt} & \Omega_{tr} \\ \Omega_{rk} & \Omega_{rt} & \Omega_{rr} \end{bmatrix}$$

uniform magnetic order



Quantum anomalous Hall



Topological Hall effect

# Adiabatic Quantum Motor

- Voltage-induced torque

Coupled *Sundaram-Niu* & *LLG* equations

$$\dot{\mathbf{k}} = -e\mathbf{E} \quad (\hbar = 1)$$

$$\dot{\mathbf{r}} = \frac{\partial \varepsilon}{\partial \mathbf{k}} + \dot{\mathbf{k}} \cdot \vec{\Omega}_{kk} + \dot{\mathbf{m}} \cdot \vec{\Omega}_{mk}$$

$$\mathbf{m} \times \dot{\mathbf{m}} = \frac{\partial \varepsilon}{\partial \mathbf{m}} + \dot{\mathbf{k}} \cdot \vec{\Omega}_{km} + \dot{\mathbf{m}} \cdot \vec{\Omega}_{mm}$$



Ganesh Sundaram



Qian Niu

Voltage-induced torque

Renormalization of Damping

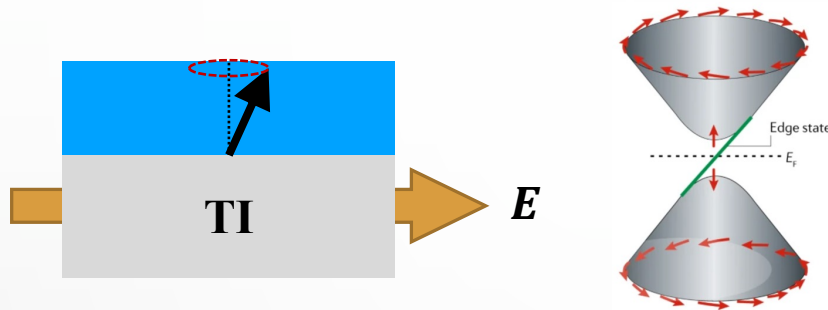
$$\dot{\mathbf{m}} = \gamma \left( e\mathbf{E} \cdot \int \vec{\Omega}_{km} [dk] - \dot{\mathbf{m}} \cdot \int \vec{\Omega}_{mm} [dk] \right) \times \mathbf{m}_i + \dots$$

$$\vec{\Omega} = \begin{bmatrix} \Omega_{kk} & \Omega_{km} & \mathbf{0} \\ \Omega_{mk} & \Omega_{mm} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

$$\mathbf{j} = -e\dot{\mathbf{m}} \cdot \int \vec{\Omega}_{mk} [dk]$$

Thouless pumping  
(adiabatic current)

# Voltage-Induced Torque



$$H^{\text{SOT}} \sim \text{sng}(m_z)E$$

**Not from edge state**

Xiao, Xiong & Niu, PRB **104**, 064433 (2021)

Xiong & Niu, PRB **98**, 035123 (2018)

Arrachea & von Oppen, Phys. E **74**, 596 (2015)

Voltage-induced torque

Renormalization of Damping

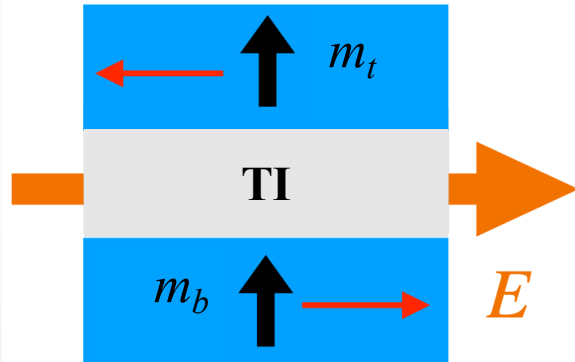
$$\dot{m} = \gamma \left( eE \cdot \int \vec{\Omega}_{km} [dk] - \dot{m} \cdot \int \vec{\Omega}_{mm} [dk] \right) \times m_i + \dots$$

$$\vec{\Omega} = \begin{bmatrix} \Omega_{kk} & \Omega_{km} & 0 \\ \Omega_{mk} & \Omega_{mm} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$j = -e\dot{m} \cdot \int \vec{\Omega}_{mk} [dk]$$

Thouless pumping  
(adiabatic current)

# Voltage-Induced Torque



$$H_0 = \hbar v_f (k_x \tau_z \otimes \sigma_y - k_y \tau_z \otimes \sigma_x) + A(\mathbf{k}) \tau_x \otimes I + J_{sd} (\mathbf{m}^t \cdot \boldsymbol{\sigma}) \oplus (\mathbf{m}^b \cdot \boldsymbol{\sigma}),$$

$$A(\mathbf{k}) = b_0 + b_1 (k_x^2 + k_y^2)$$

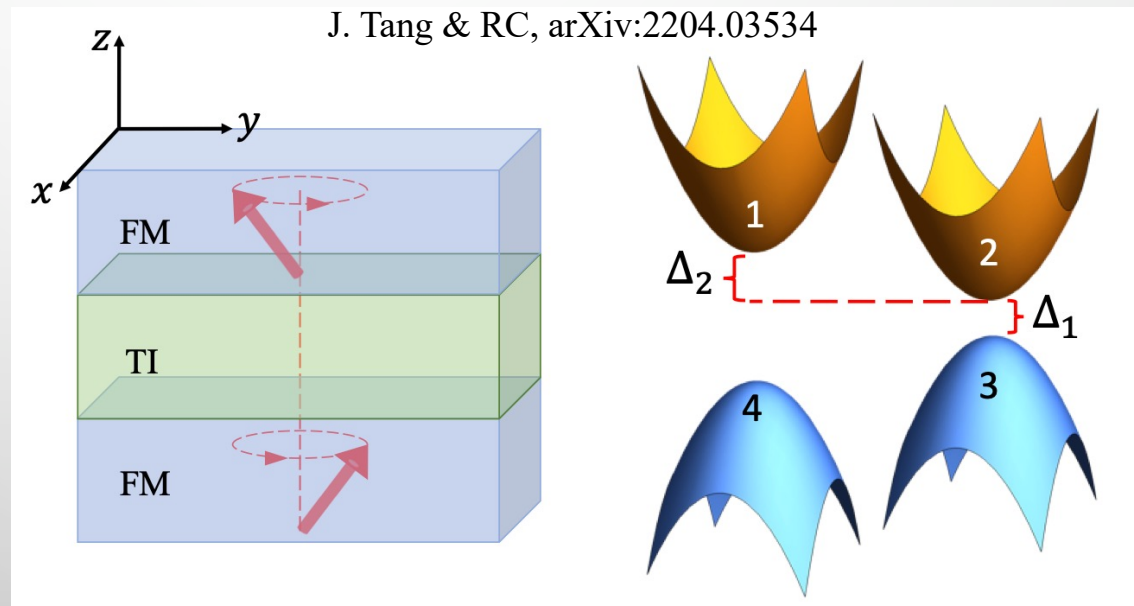
$$H_t^{\text{SOT}} = -H_b^{\text{SOT}}$$

Exchange mode

$$f_r = f_A + f_E$$

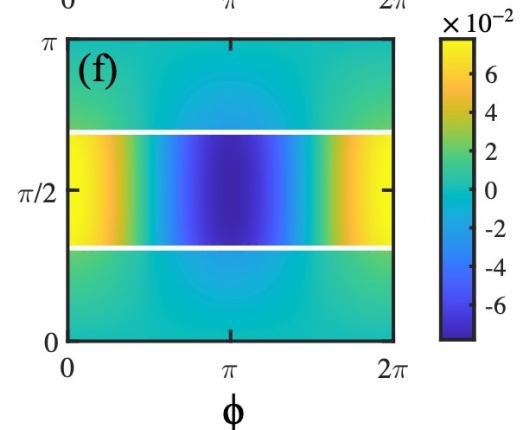
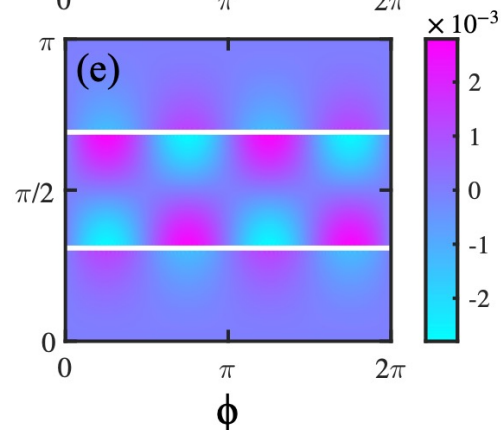
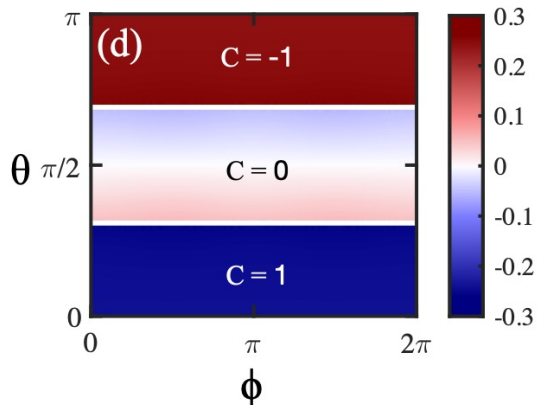
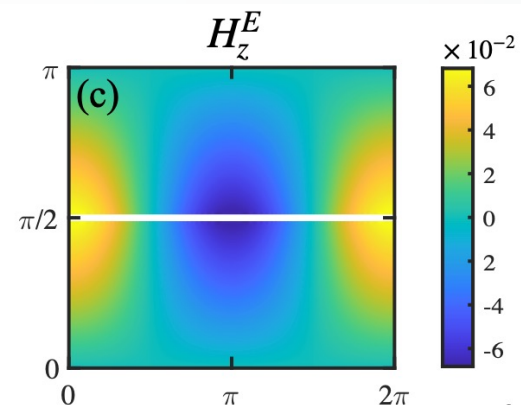
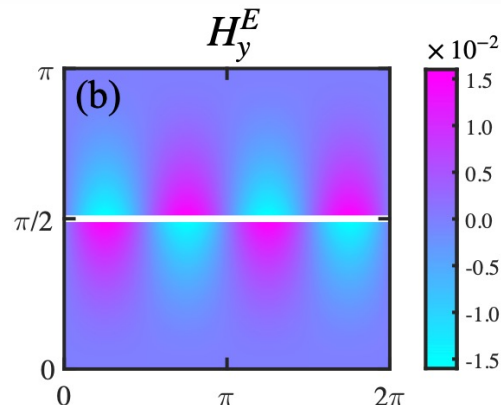
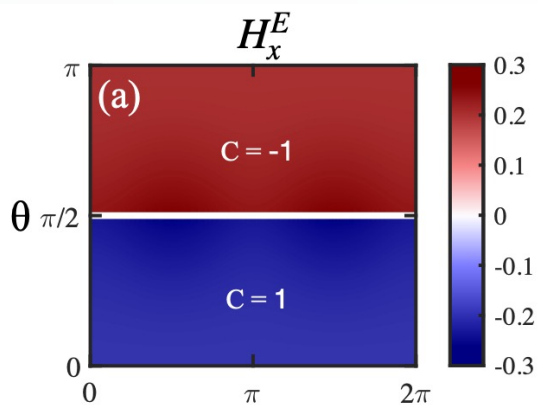
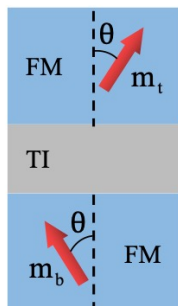
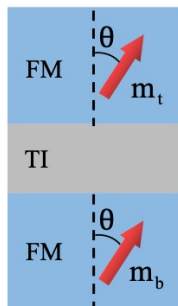
can be 100 GHz

Li *et al.*, PRB **91**,  
014427 (2015);  
Son Ho & Jalil, AIP Adv. **7**,  
055926 (2017)



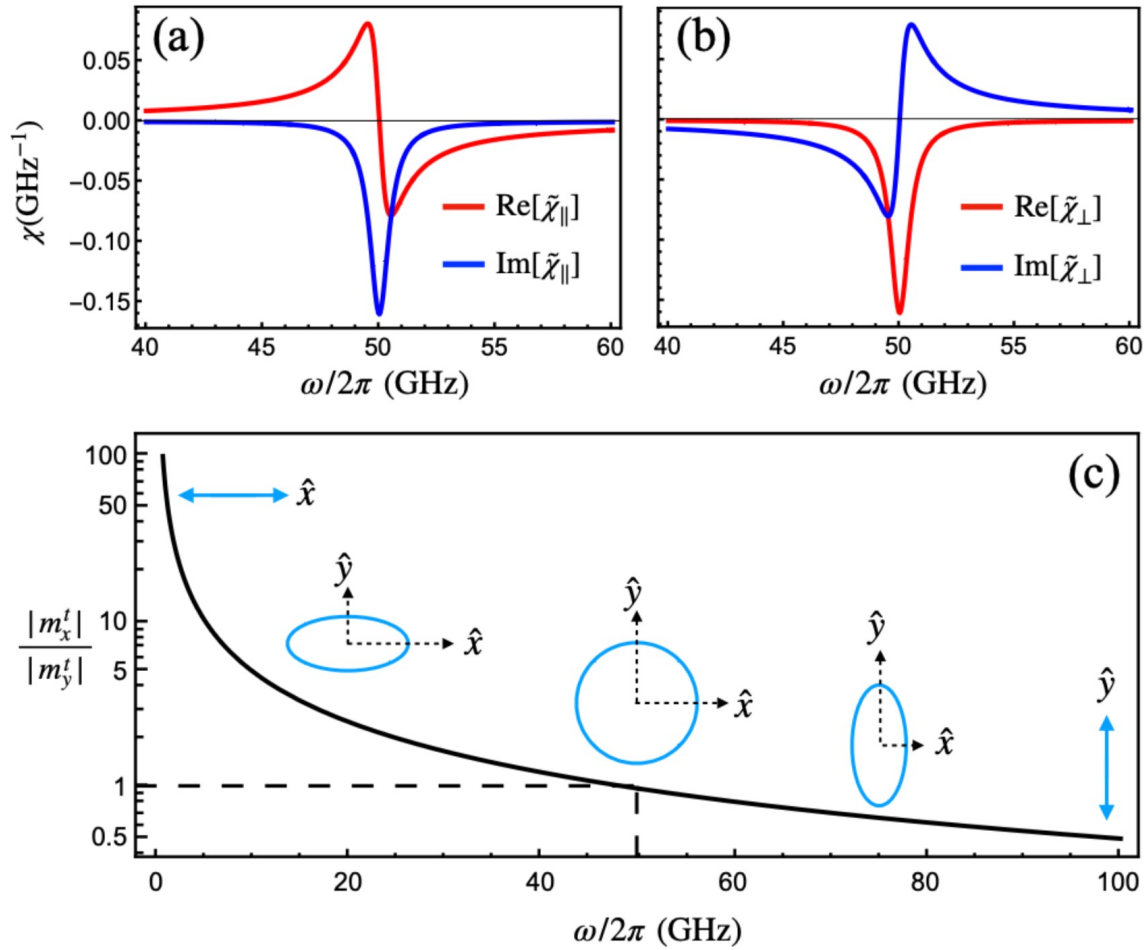


# Exchange Mode Resonance



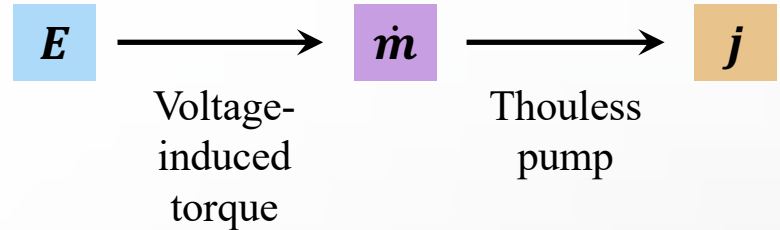
# Exchange Mode Resonance

$$\tilde{m}_x^{t/b} = \pm \tilde{\chi}_{\parallel} \tilde{H}_x^{\text{SOT}} \quad \text{and} \quad \tilde{m}_y^{t/b} = \pm \tilde{\chi}_{\perp} \tilde{H}_x^{\text{SOT}}$$



# Effective Admittance

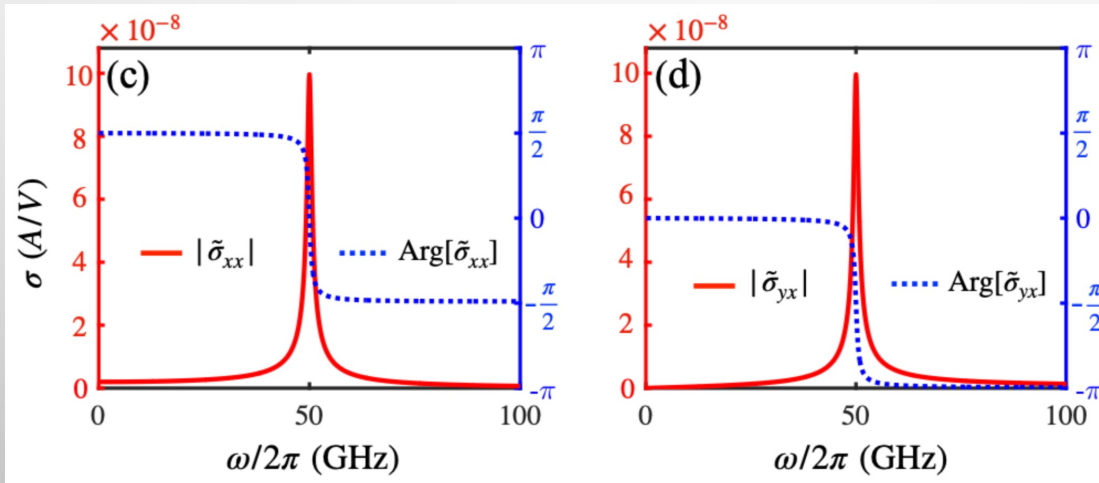
For AC drive:  $\mathbf{E}(t) = \hat{\mathbf{x}}E_0e^{i\omega t}$



Insert LLG  $\dot{\mathbf{m}}_i = -\gamma\mathbf{m}_i \times \vec{\Omega}_{k\mathbf{m}_i} \cdot \frac{e\mathbf{E}}{\hbar} + \dots$

into  $\mathbf{J} = -\frac{e}{(2\pi)^2} \sum_{i=t,b} \int \dot{\mathbf{m}}_i \cdot \vec{\Omega}_{\mathbf{m}_i\mathbf{k}} d^2k$

$$J_\mu(\omega) = \sigma_{\mu\nu}(\omega)E_\nu(\omega)$$



- Transverse:
 
$$\sigma_{xy}(\omega_r) \ll \sigma_{\text{QAH}}$$
- Longitudinal:
 
$$\sigma_{xx}^0 \approx 0$$

$$\sigma_{xx}(\omega) \text{ significant}$$

# Effective Admittance

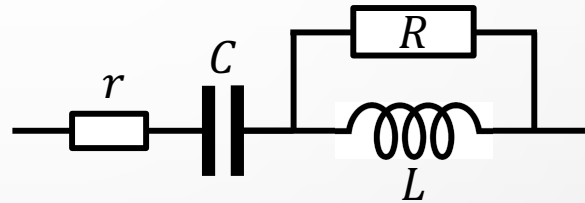
Impedance  $Z_{\text{sys}} = 1/\sigma_{xx}$

$$Z_{\text{sys}} = Z_m \frac{(\alpha\omega - i\omega_r)^2 + \omega^2}{\omega(\alpha\omega - i\omega_r)}$$

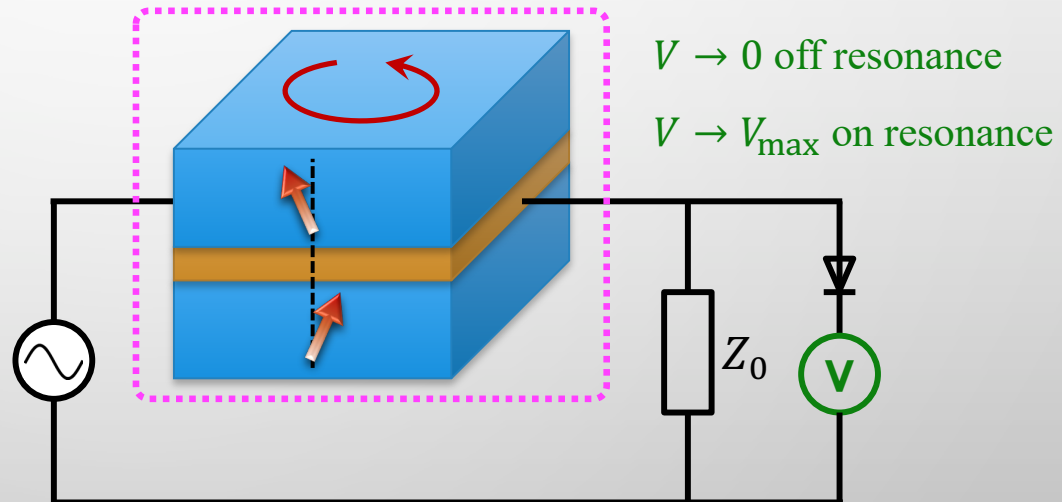
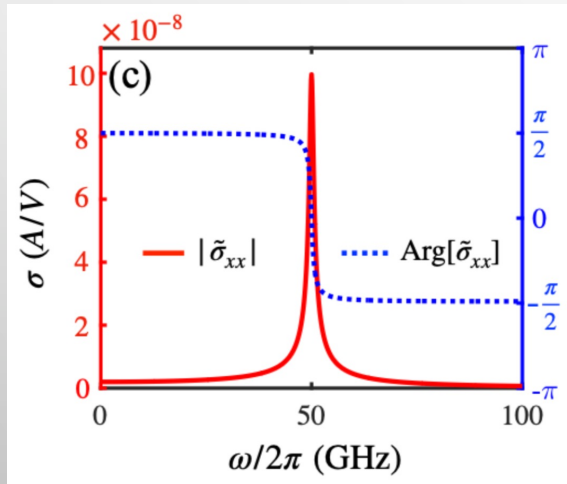
$$= Z_m \left( \alpha + \frac{\omega_r}{i\omega} \right) + \frac{Z_m}{\left( \alpha + \frac{\omega_r}{i\omega} \right)}$$

Setting  $Z_{\text{eff}}(\omega) = Z_{\text{sys}}(\omega)$  solves

$$R = \frac{Z_m}{\alpha}, \quad r = Z_m\alpha, \quad L = \frac{Z_m}{\omega_r}, \quad C = \frac{1}{Z_m\omega_r},$$



$$Z_m^{-1} = \bar{\Omega}_{k_x m_x^t}^2 \zeta w/l \text{ about } 5 \times 10^8 \Omega$$



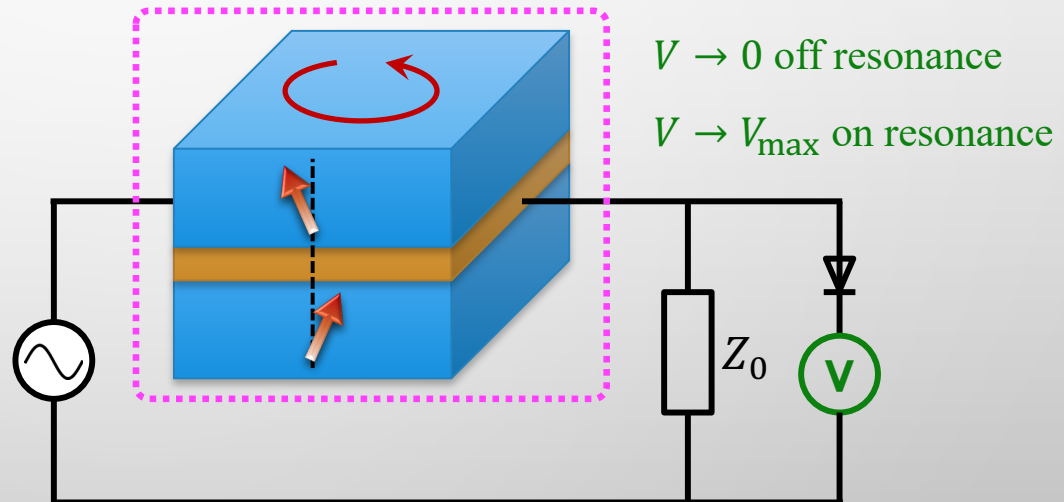
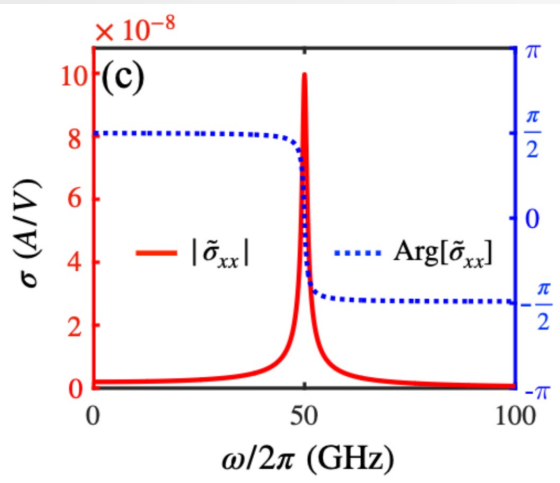
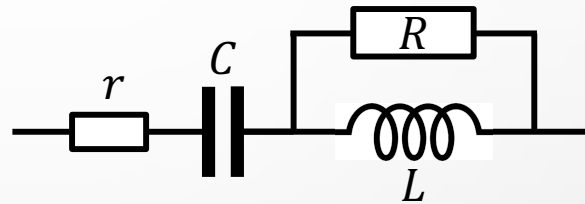
# Mechanical Efficiency

$$P_M = \alpha \frac{E_x^2 w l \zeta}{2} \bar{\Omega}_{k_x m_x}^{-2} \omega^2 [|\tilde{\chi}_{\parallel}(\omega)|^2 + |\tilde{\chi}_{\perp}(\omega)|^2],$$

$$P_J = \frac{E_x^2 w l \zeta}{2} \bar{\Omega}_{k_x m_x}^{-2} \omega |\text{Im} [\tilde{\chi}_{\parallel}(\omega)]|,$$

$$\eta = \frac{P_M}{P_J} = 1,$$

$$\zeta = \frac{2e^2}{N_z \hbar S a_0^2}$$

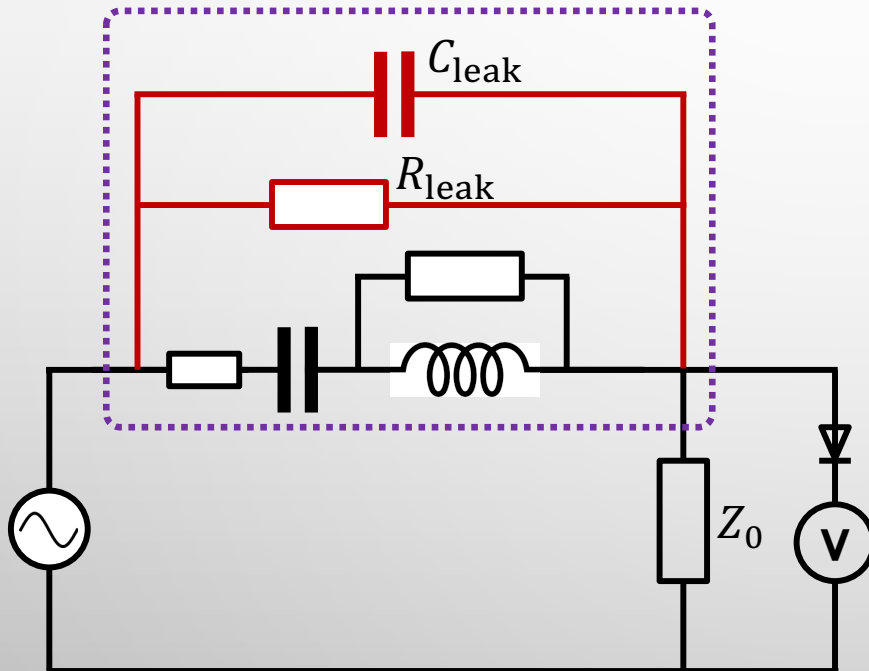


# Mechanical Efficiency

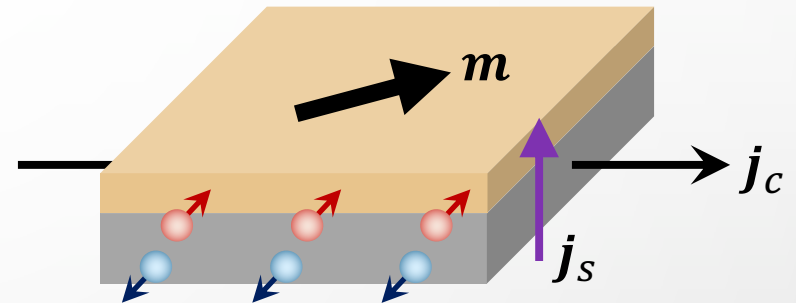
$$\eta = \frac{P_M}{P_M + P_{\text{leak}}}$$

Still above 50% for imperfect TI

$$P_{\text{leak}} = E_x^2 l^2 / 2R_{\text{leak}}$$



- Comparing with current-driven systems



$$\eta = \frac{P_M}{P_J} = \frac{\alpha \sigma \hbar \theta_s^2 \xi^2}{S e^2} \left( \frac{a_0^3}{d_n d_m} \right) \omega^2 (|\tilde{\chi}_\perp|^2 + |\tilde{\chi}_\parallel|^2)$$

$$\xi = \frac{\lambda G_r \tanh \frac{d_n}{2\lambda}}{\sigma + 2\lambda G_r \coth \frac{d_n}{2\lambda}}$$

$\eta$  on the order of 1%

# Oersted Field

## Current-driven system

**Metal**  
 $J_c = \sigma E$

charge-spin conversion

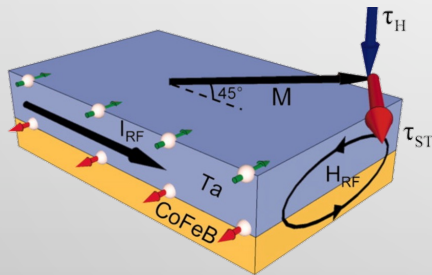
$$J_c \rightarrow J_s$$

$$\tau_{ST} = \nabla \cdot J_s$$

**current-induced torque**

Oersted field

$$\nabla \times H = J_c$$



Liu *et al.*, Science 2012

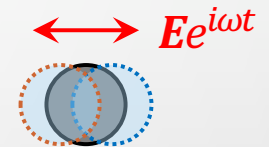
## Voltage-driven system

**Insulator**  
 $J_c = 0$  but  $J_D = \epsilon_0 \epsilon_r \dot{E}$

Oersted field

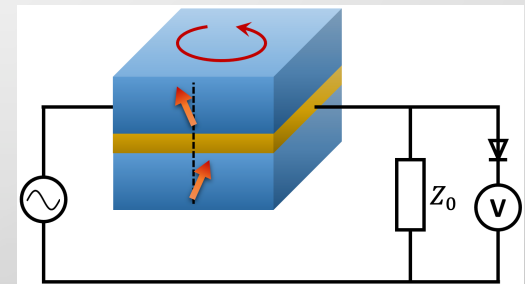
$$\nabla \times H = J_D$$

bound charge vibration



**voltage-induced torque**

$$\tau_H \ll \tau_{ST}$$



J. Tang & RC, arXiv:2204.03534

# Conclusions & Outlook

- Operating F/TI/F as adiabatic quantum motor can achieve 100% mechanical efficiency
  - Voltage-induced torque can drive the exchange mode resonance
  - Effective circuit found for the proposed device
  - Oersted field effect is negligible
- 
- Under investigation: generalizing into intrinsic antiferromagnetic topological insulators