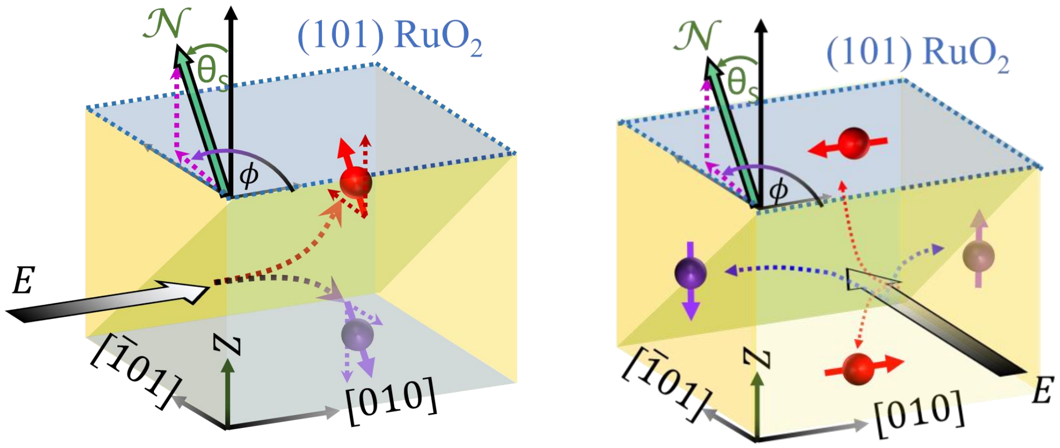
Generation of tilted spin current by the collinear antiferromagnet RuO_2



A. Bose et. al. Nat. Electron. 5, 267 (2022)
A. Bose & D. C. Ralph. Research Briefings
www.nature.com/articles/s41928-022-00758-2

2023 May
Altermagnet workshop

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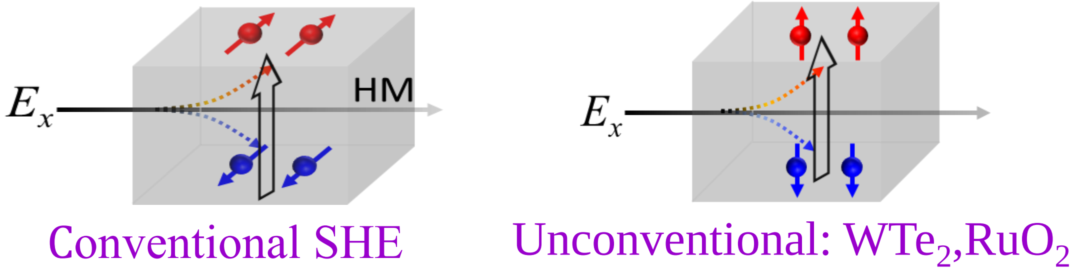
SPIN PHENOMENA INTERDISCIPLINARY CENTER
SPIN+X SFB/TRR 173 Kaiserslautern • Mainz
Alexander von Humboldt Stiftung/Foundation

Outlines

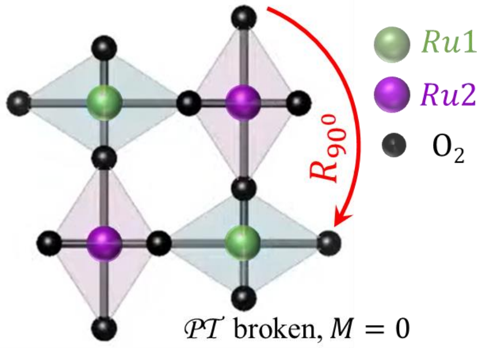
Background and motivation

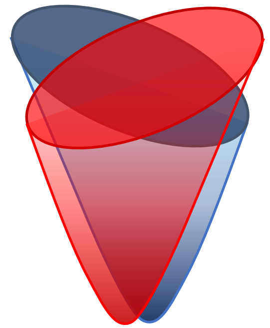
- Conventional vs unconventional spin current



Band structure of RuO_2

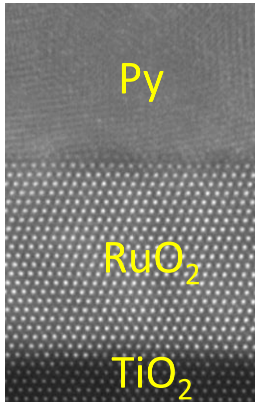
- Spin-split bands contributing to ***exotic electronic transport properties*** (unique property of an altermagnet)

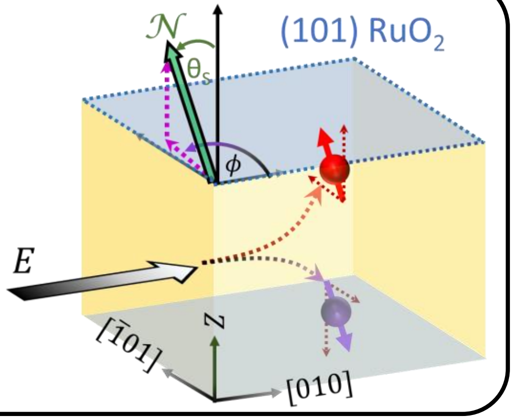




Experimental detection of to *exotic electronic transport properties* via electrical measurements

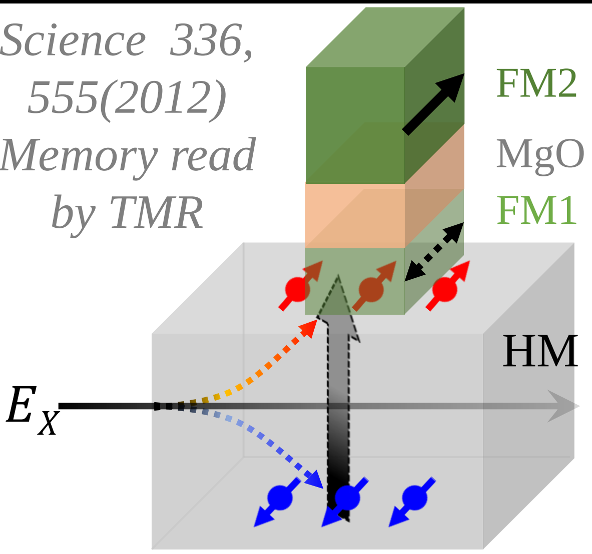
- Strategies
- Measurement techniques (ST-FMR, In-plane 2nd harmonics Hall)
- Results and summary





Spin-orbit torques

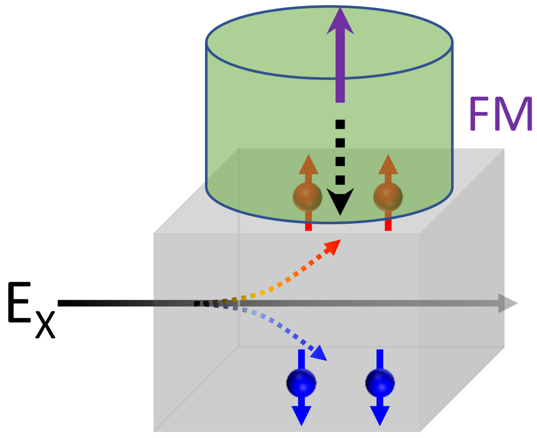
Science 336, 555(2012)
Memory read by TMR



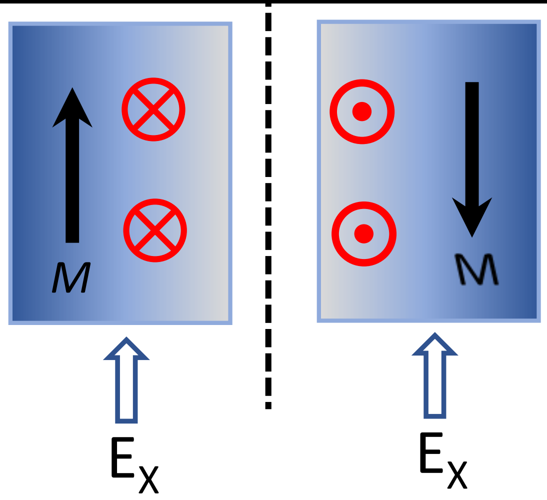
Spin Hall effect (SHE)

LLGS Equation
(JMMM 159, L1 (1996))

$$\frac{d\hat{m}}{dt} = -\gamma(\hat{m}\times\vec{H}_{net}) + \alpha\gamma(\hat{m}\times\vec{H}_{net}\times\hat{m}) + \Gamma_{FL}(\hat{m}\times\sigma_y) + \Gamma_{DL}(\hat{m}\times(\sigma_y\times\hat{m}))$$



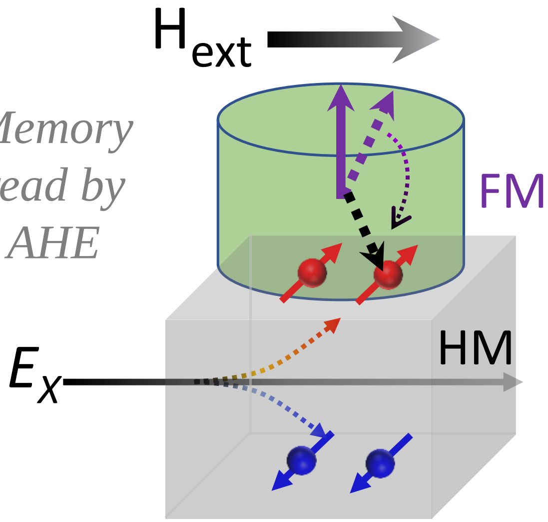
Unconventional spin-current (op-DL)



Mirror symmetry ⊥ to E_X to be broken

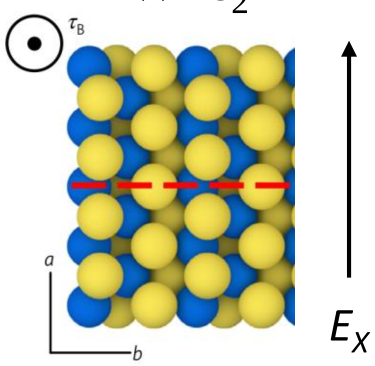
H_{ext}

Memory read by AHE



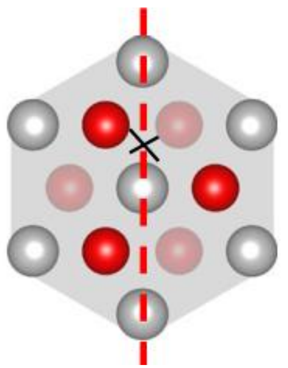
Nature 476, 189 (2011), PRL 109, 096602 (2012)

WTe_2



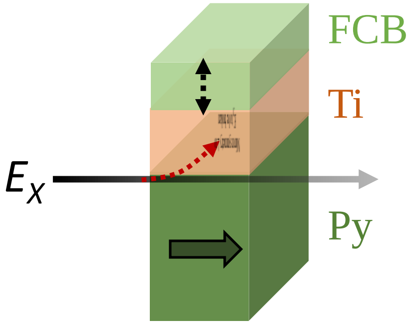
Nat. Phys. 13, 300 (2017)

CuPt

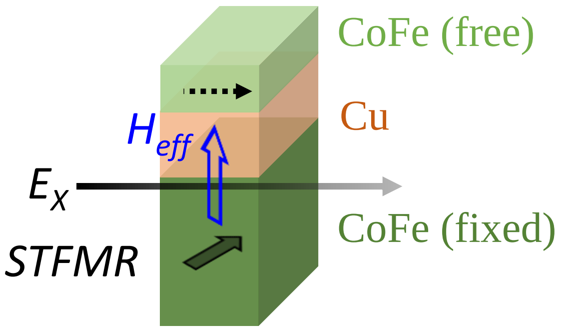


Nat. Nano. 16, 277 (2021)

M-dependent SHE / Magnetic SHE/ Interface generated J_S
(RMP 91, 035004 (2019))

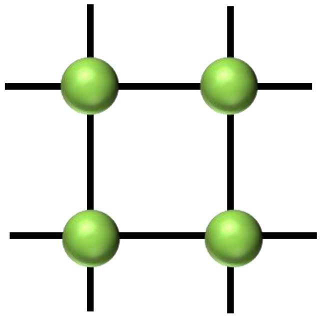


Nat. Mater. 17, 509 (2018)

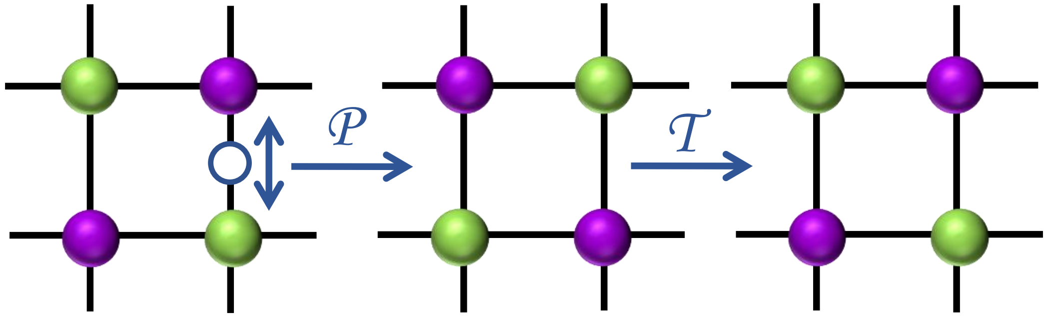


A. Bose et. al. PRAppl 9, 064026 (2018)

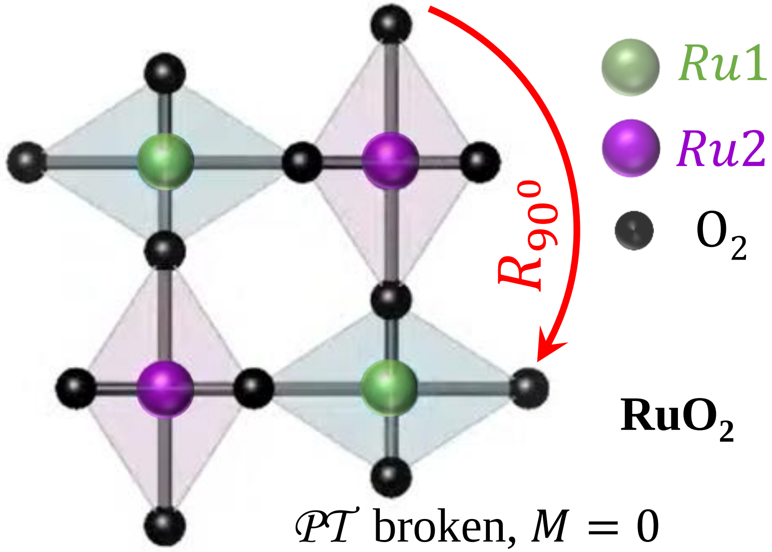
Altermagnetic RuO_2: spin-split bands (*Phys. Rev. X. 12, 031042 (2022)*)



FM ($M \neq 0$)



Conventional (Kramer's) AFM ($M = 0$)

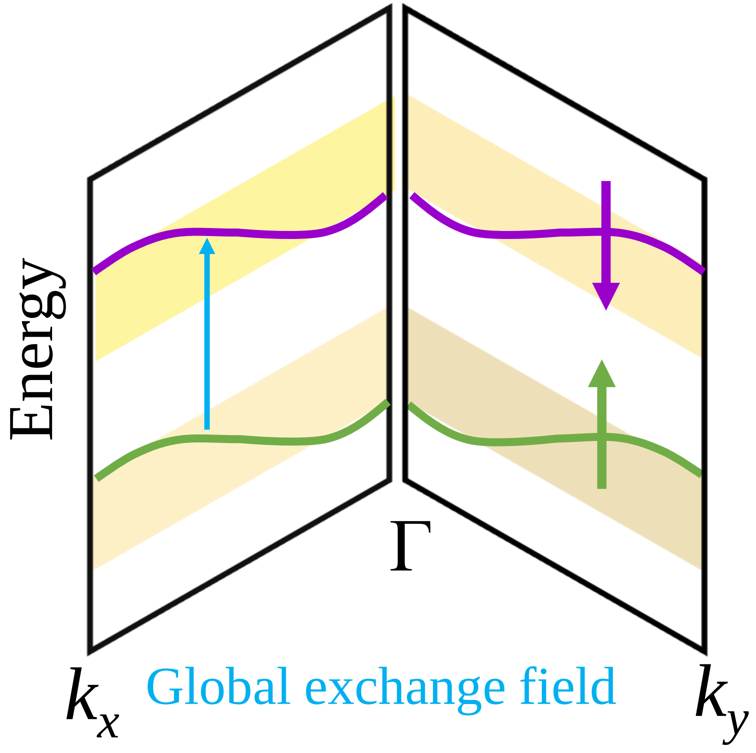


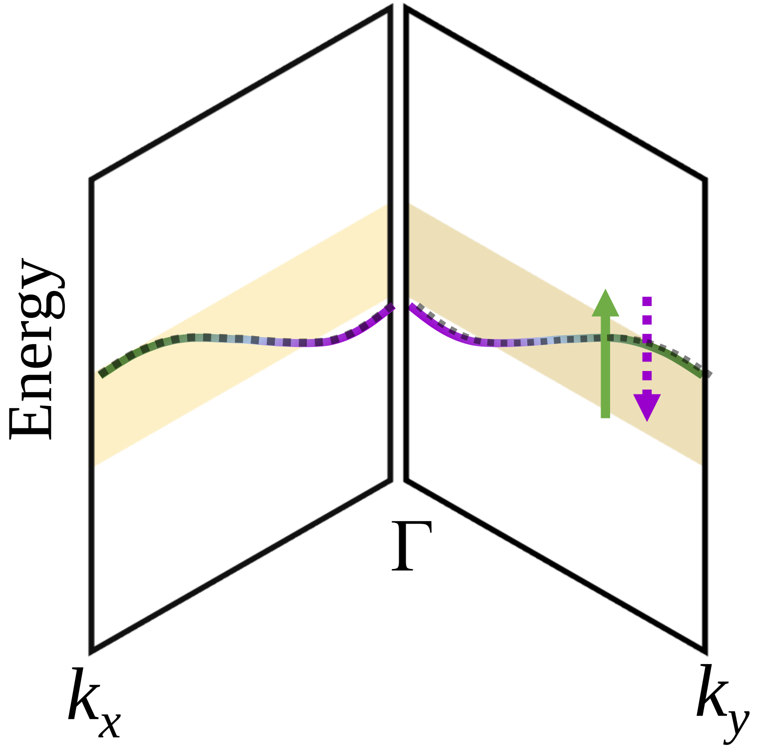
Kramer 1930's

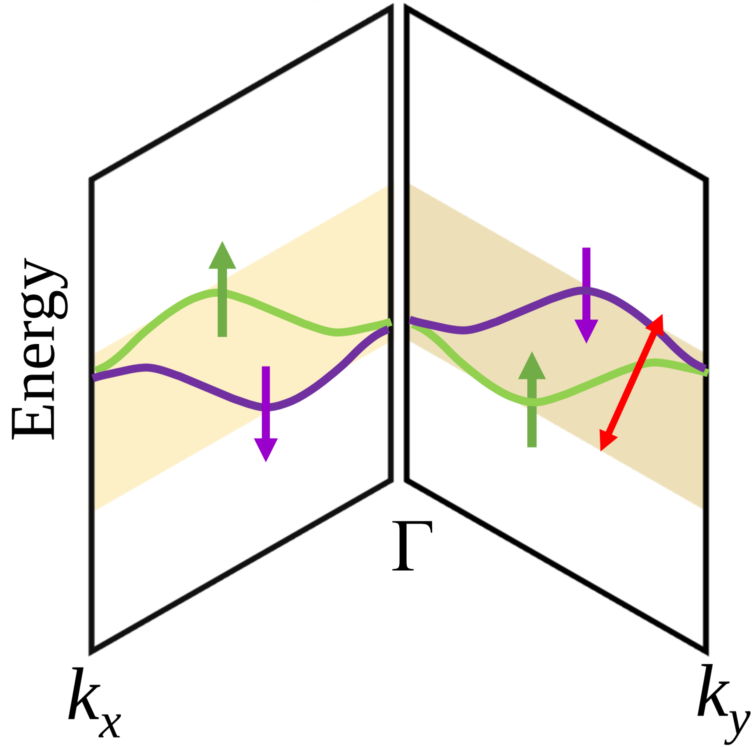
𝒫𝒯 transform: 𝒫𝒯ε(↑,k)=ε(↓,k)
𝒫𝒯 symmetry: 𝒫𝒯ε(↑,k)=ε(↑,k)

→ ε(↑,k)=ε(↓,k)

𝒫𝒯 broken, $M = 0$



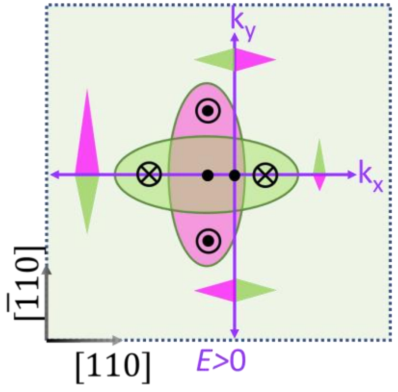




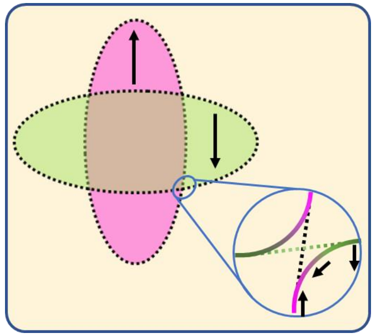
(1) Electrostatic crystal field (k-dep. Splitting)

Electronics with the altermagnet (RuO_2) (*Phys. Rev. X 12, 040501 (2022)*)

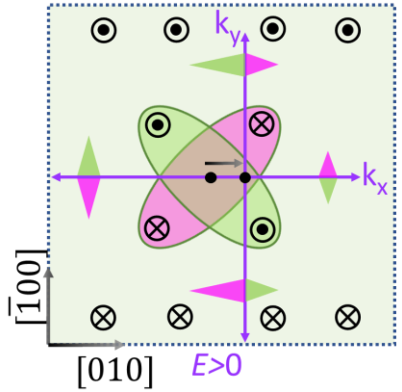
(1) Longitudinal time odd spin-polarized current by the real space collinear AFM (strongly crystal axis dependent)
Theory: Phys Rev X 12, 011028 (2021)



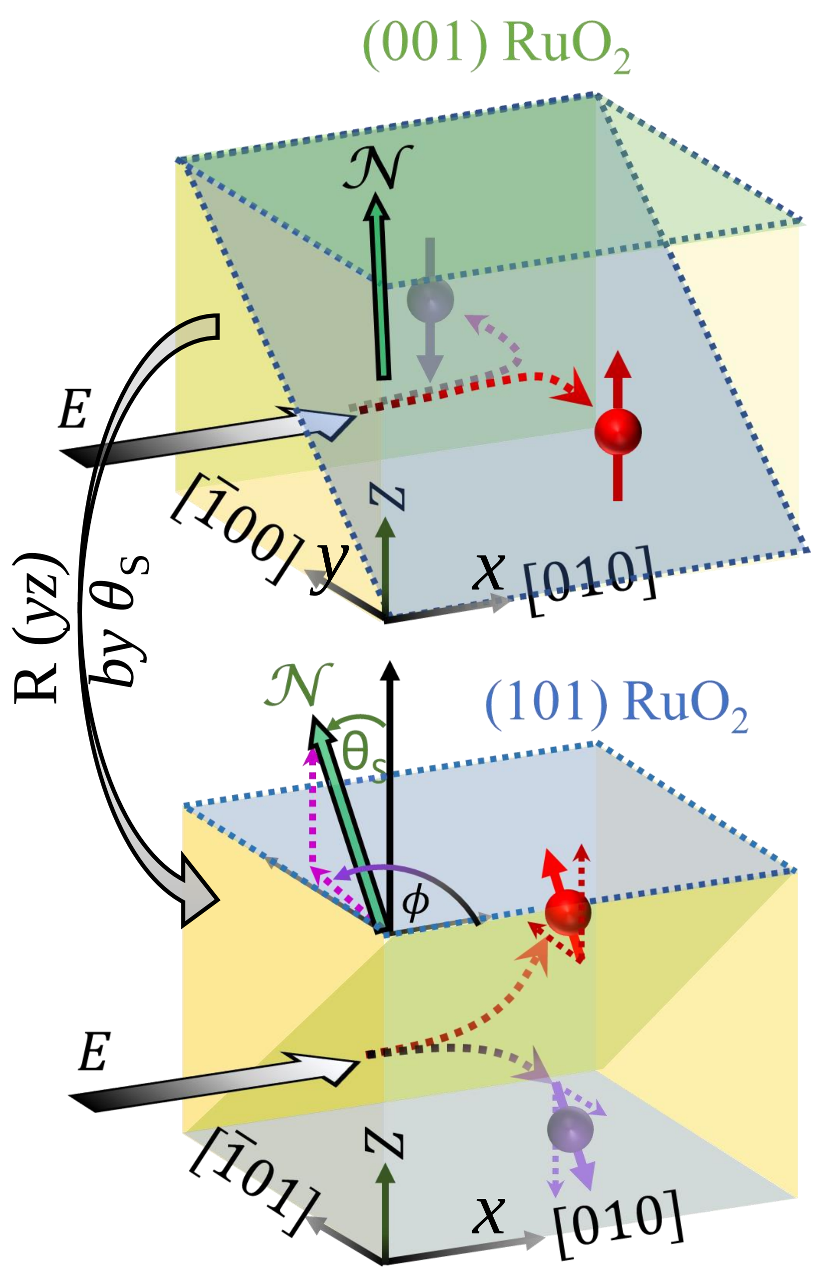
(2) Anomalous Hall effect by the real space collinear AFM (strongly crystal axis dependent)
Theory: Sci Adv. 6, eaaz8809 (2020)
Exp: Nat. Electron5, 735 (2022), arXiv:2012.15651

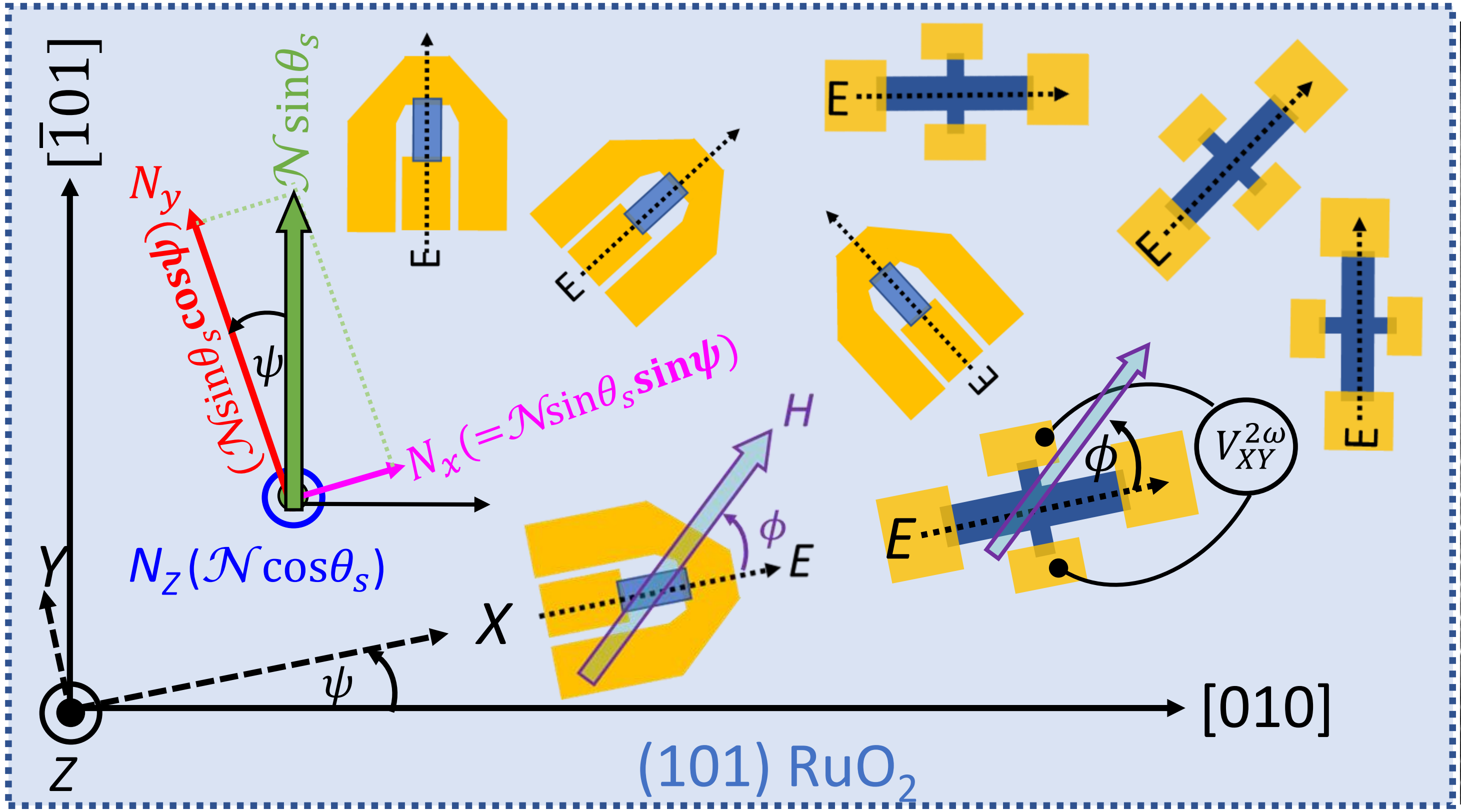


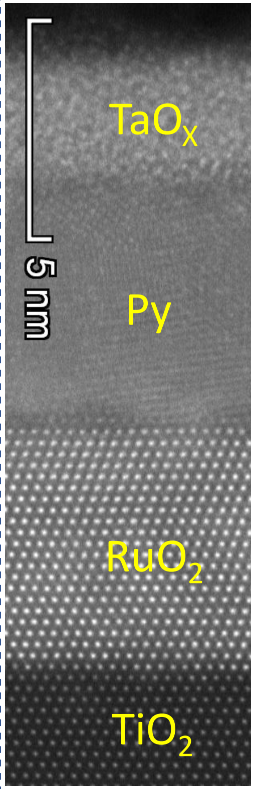
(3) Unconventional time-odd spin-Hall current even in the absence of SOC determined by *N*-vector (strongly crystal axis dependent)
Theory: Phys. Rev Lett 126, 127701 (2021)
Exp: A. Bose. et. al. Nat. Electron. 5, 267 (2022)

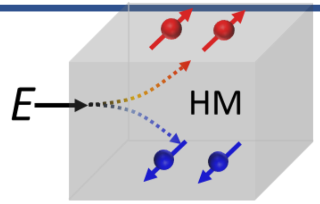


Tilted spin-current vs. unconventional torque (*Nat. Elecl.5, 267 (2022)*)

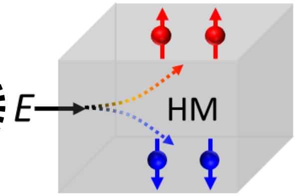




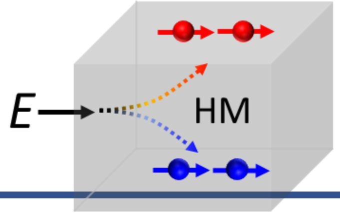




$$\xi^{Y}_{DL,E} = C_1.\ \sin\theta_s(\sin\theta_s \cos\psi)\cos\psi + C_0$$



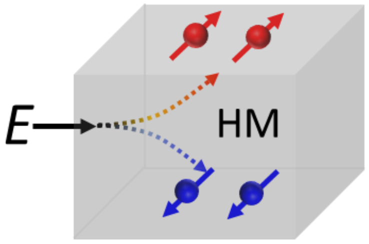
$$\xi^{Z}_{DL,E} = C_1.\ \sin\theta_s\ (\cos\theta_s)\cos\psi$$



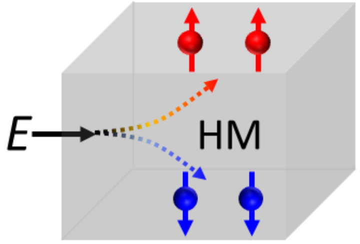
$$\xi^{X}_{DL,E} = C_1.\ \sin\theta_s(\sin\theta_s \sin\psi)\cos\psi$$

Time even vs time odd transverse spin currents

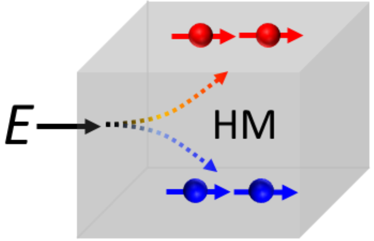
$\xi_{DL}^{Y} \propto \cos^2\psi$

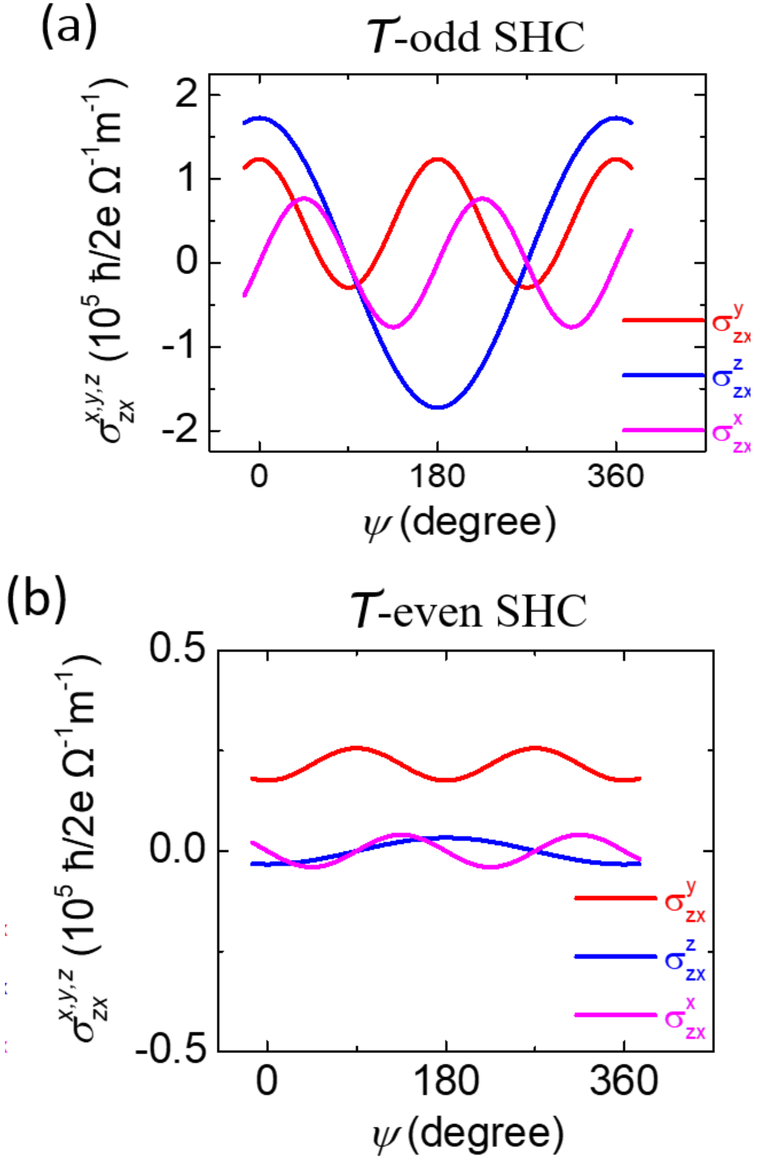


$\xi_{DL}^{Z} \propto \cos\psi$



$\xi_{DL}^{X} \propto \sin\psi\cos\psi$





$\mathcal{T}$-odd $J_S >> \mathcal{T}$-even J_S

$$\sigma_{ij}^{k} = -\frac{e\hbar}{\pi}\int \frac{d^3\vec{k}}{(2\pi)^3}\sum_{n,m}\frac{\Gamma^2 \mathrm{Re}\left(\langle n\vec{k}|J_i^k|m\vec{k}\rangle\langle m\vec{k}|v_j|n\vec{k}\rangle\right)}{\left[(E_F - E_{n\vec{k}})^2 + \Gamma^2\right]\left[(E_F - E_{m\vec{k}})^2 + \Gamma^2\right]}$$

$\Gamma \approx 50\ eV$ for $\rho = 270\ \mu\Omega - cm$
$\Gamma \approx 25\ eV$ for $\rho = 140\ \mu\Omega - cm$

$\sigma_{ij}^{k}(\Gamma \approx 25) \approx 2\sigma_{ij}^{k}(\Gamma \approx 50)$

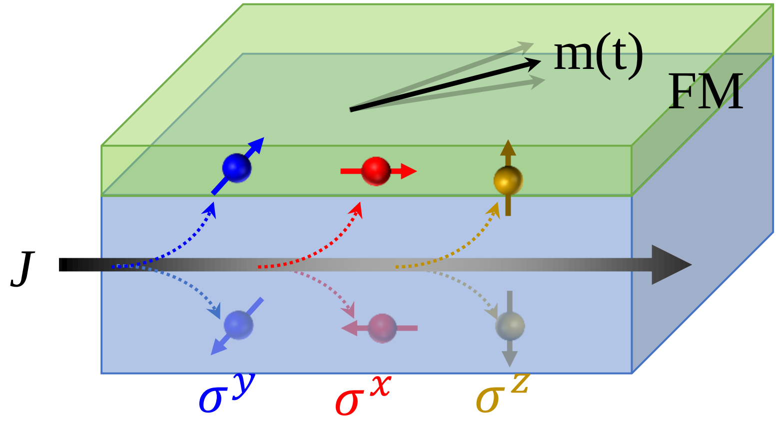
Strong temperature dependence

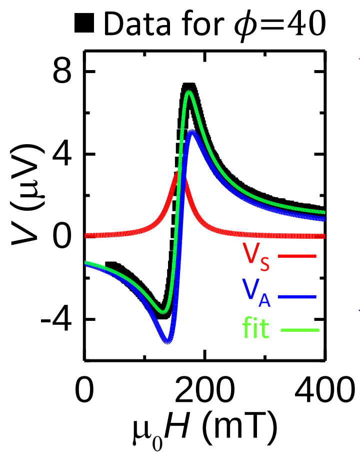
$$\sigma_{ij}^{k} = -\frac{2e}{\hbar}\int \frac{d^3\vec{k}}{(2\pi)^3}\sum_{n'\neq n}\frac{\mathrm{Im}\left(\langle n\vec{k}|J_i^k|n'\vec{k}\rangle\langle n'\vec{k}|v_j|n\vec{k}\rangle\right)}{(E_{n\vec{k}} - E_{n'\vec{k}})^2}$$

Spin-torque measurements by ST-FMR

$\Gamma_{DL} \rightarrow \left(m \times \sigma_{z,x}^{X,Y,Z} \times m\right)$ $\Gamma_{FL} \rightarrow \left(m \times \sigma_{z,x}^{X,Y,Z}\right)$ *6 types of torques*

$I_{ac} \rightarrow J_{as}(t) \rightarrow$ ac torques $\rightarrow m(t) \rightarrow R_{xx}(t) \rightarrow V \sim \langle I_{ac}(t).R_{xx}(t) \rangle$





$$V_S = S\left(\frac{\Delta^2}{(H-H_0)^2+\Delta^2}\right)$$

In-plane torques

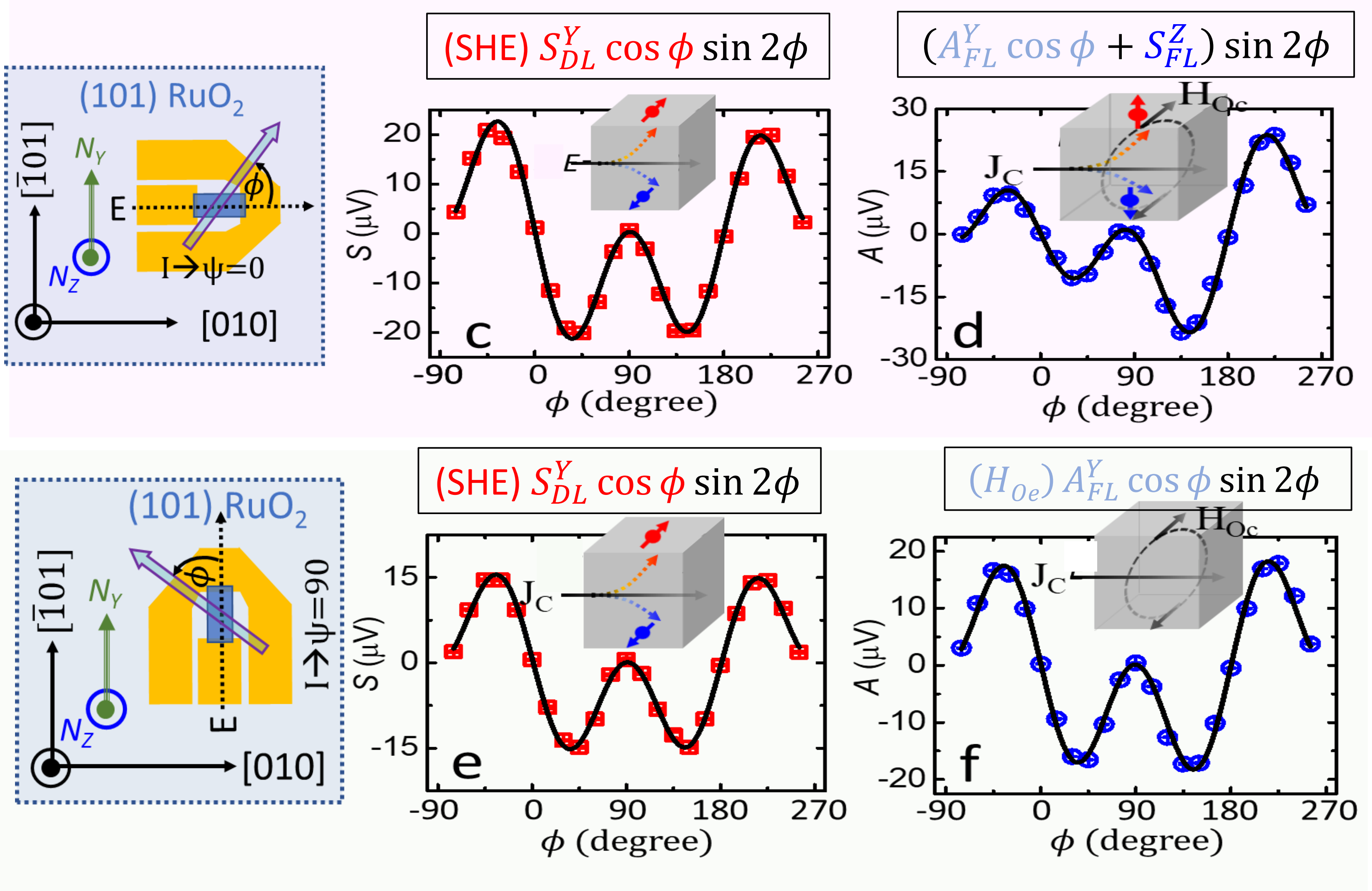
$$V_A = A\left(\frac{(H-H_0)\Delta}{(H-H_0)^2+\Delta^2}\right)$$

Out-plane torques

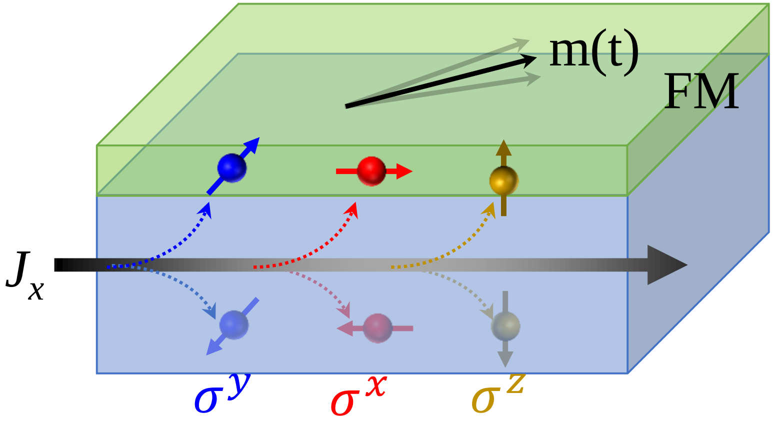
Angular ST-FMR ($\phi \rightarrow 0$ to 360 degree)

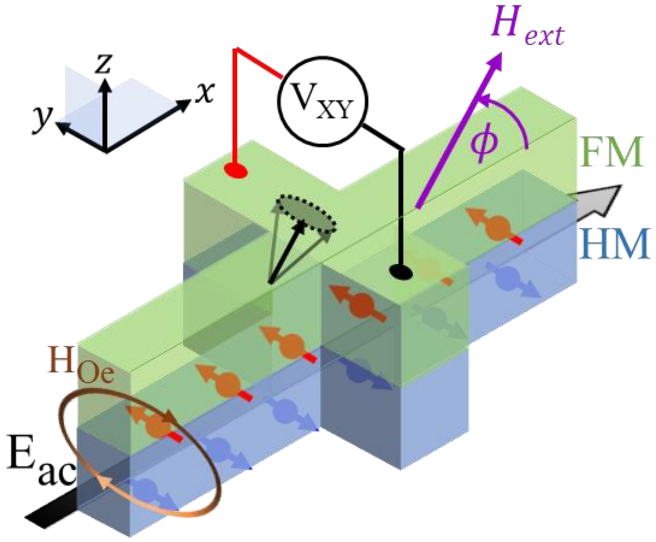
$$S = (S_{DL}^{Y}\cos\phi + S_{DL}^{X}\sin\phi + S_{FL}^{Z})\sin 2\phi$$

$$A = (A_{FL}^{Y}\cos\phi + A_{FL}^{X}\sin\phi + A_{DL}^{Z})\sin 2\phi$$



Spin-torque measurements by 2nd harmonic Hall





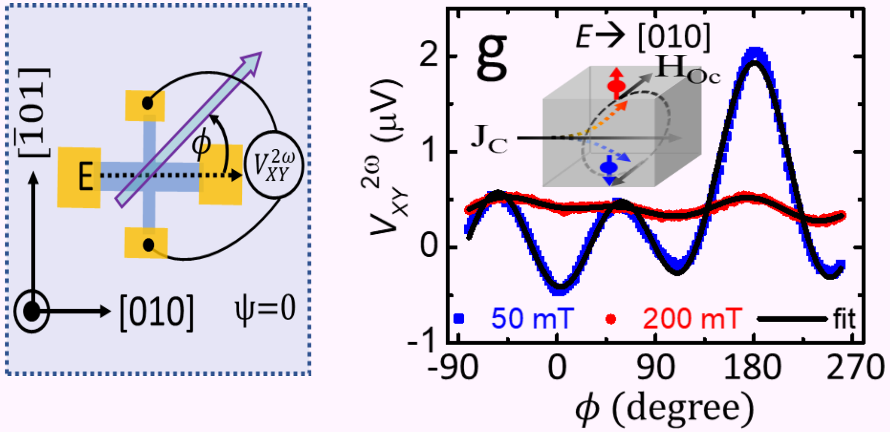
$\Gamma_{DL} \rightarrow \left(m \times \sigma_{z,x}^{X,Y,Z} \times m\right)$ $\Gamma_{FL} \rightarrow \left(m \times \sigma_{z,x}^{X,Y,Z}\right)$

$I_{ac} \rightarrow J_{as}(t) \rightarrow m(t) \rightarrow R_{xy}(t) \rightarrow V \sim \langle I_{ac}(t).R_{xy}(t)\rangle$

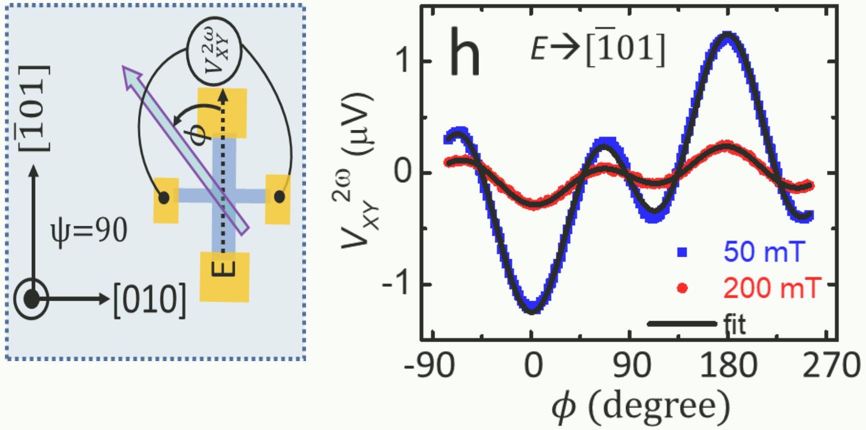
$$V_{XY}^{2\omega} \rightarrow \begin{cases} D_{DL}^{Y} \cos\phi + F_{FL}^{Y} \cos\phi \cos 2\phi \\ +D_{DL}^{X} \sin\phi + F_{FL}^{X} \sin\phi \cos 2\phi \\ +D_{DL}^{Z} \cos 2\phi + F_{FL}^{Z} \end{cases}$$

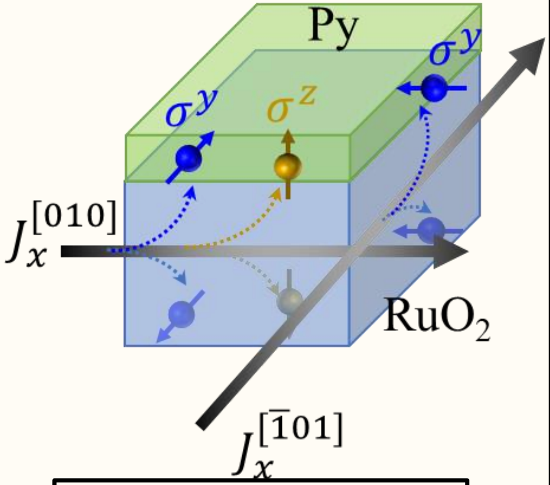
$V_{STT} \rightarrow$ field-dependent of coefficients (D)

$D_{DL}^{Y} \cos\phi + F_{FL}^{Y} \cos\phi \cos 2\phi + \boldsymbol{D_{DL}^{Z} \cos 2\phi}$



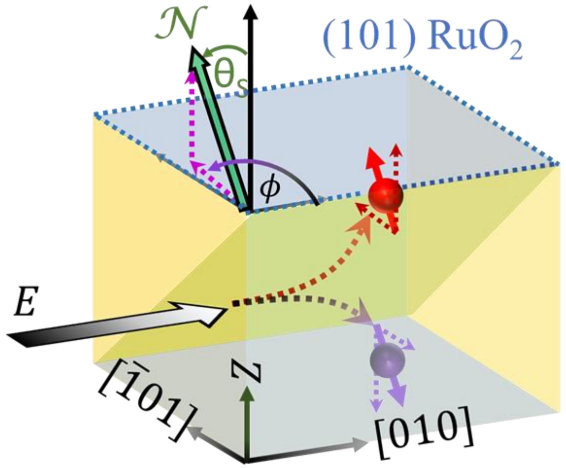
(SHE) $D_{DL}^{Y} \cos\phi + F_{FL}^{Y} \cos\phi \cos 2\phi$ (H_{Oe})

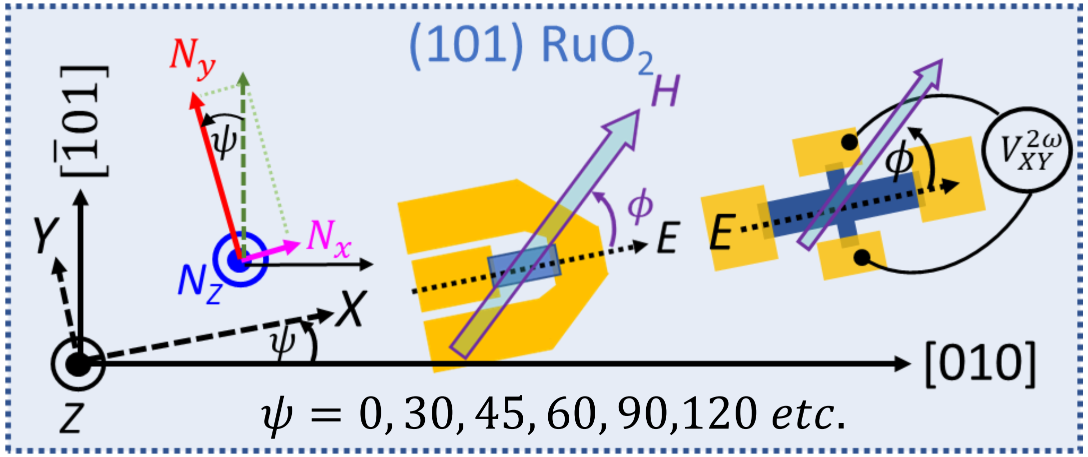


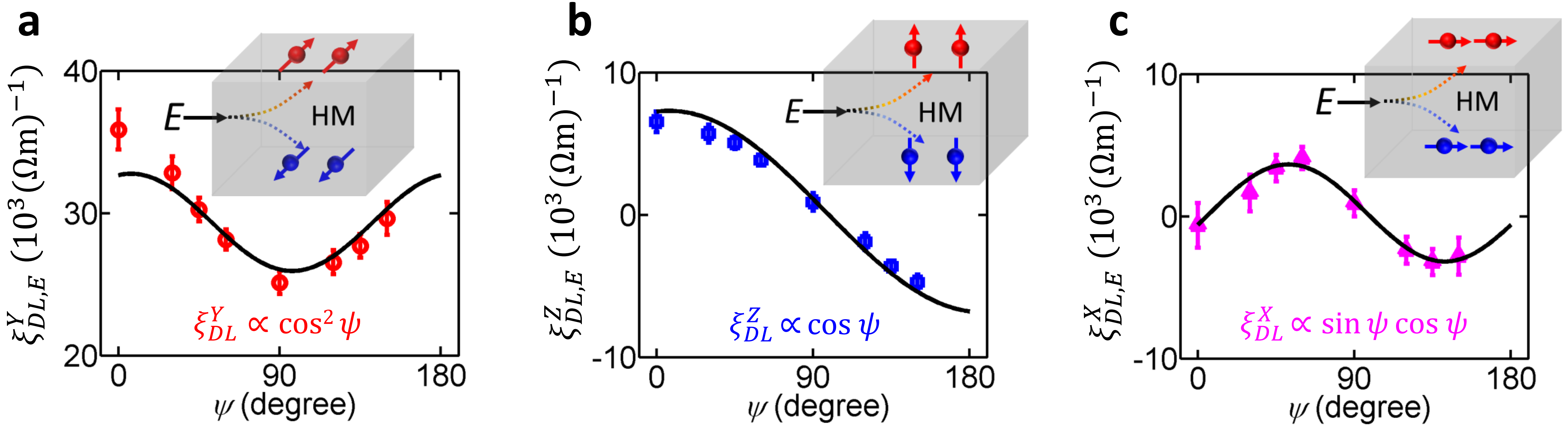


ST-FMR & SHH are consistent

Experimental detection of the tilted spin current in RuO_2

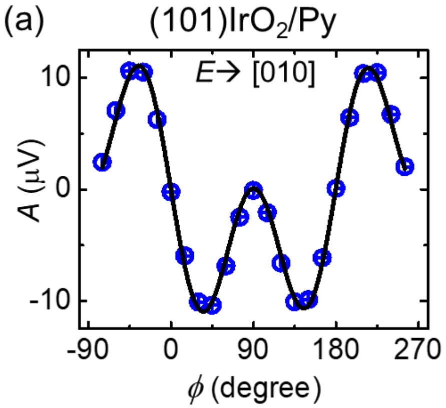






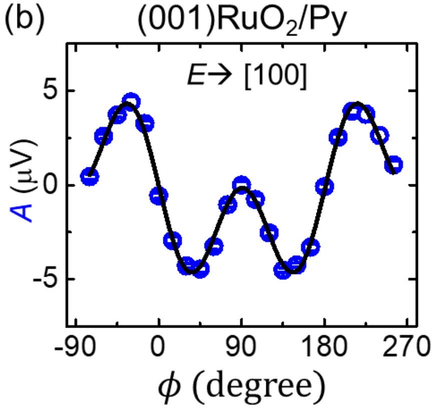
The tilted spin-current is a consequence of the novel spin-split bands of the emerging altermagnet

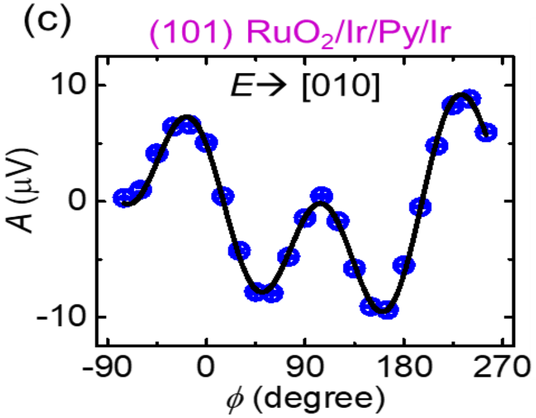
Additional experiments

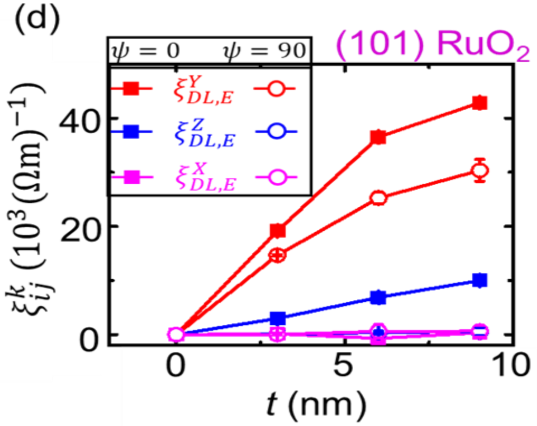


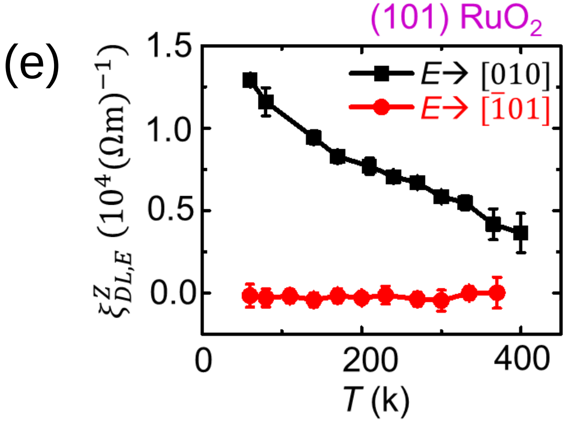
Isostructural

	RuO_2	IrO_2
a	4.543	4.545
b	4.543	4.545
c	3.140	3.190



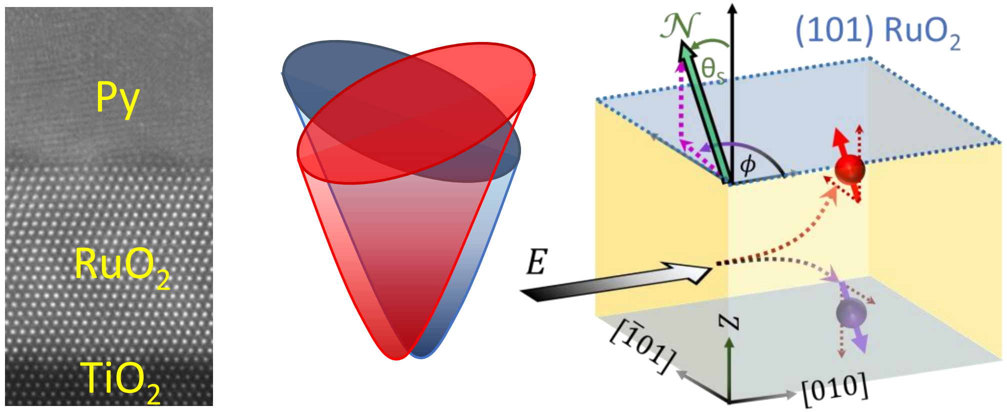






Summary

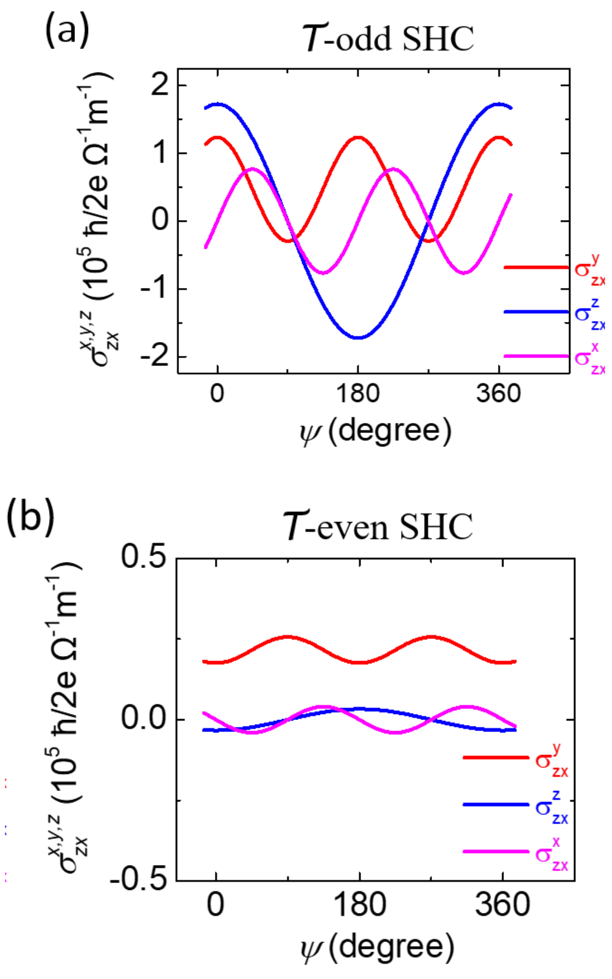
1. Isostructural (101) IrO_2 cannot produce op-DLT that (101) RuO_2 exhibits suggesting the importance of AF-ordering.
2. Strong dependence of OP-DLT with crystal axis and crystal planes.
3. Bulk origin of the spin current from RuO2 thickness dependence and Ir spacer insertion.
4. Signature of T-odd spin current from strong temperature dependence of op-DLT.



A. Bose et. al. Nature Elecltron.5, 267 (2022)

Acknowledgement

THEORY



$\mathcal{T}$-odd J_S >> $\mathcal{T}$-even J_S

H. Nair, N. Schreiber, D. Schlom (RuO_2)
and J. Sun, J. Nelson (IrO_2)
Cornell, Material Science, Phys

D.-F. Shao, E. Tsymbal, (DFT for RuO_2)
University of Nebraska–Lincoln

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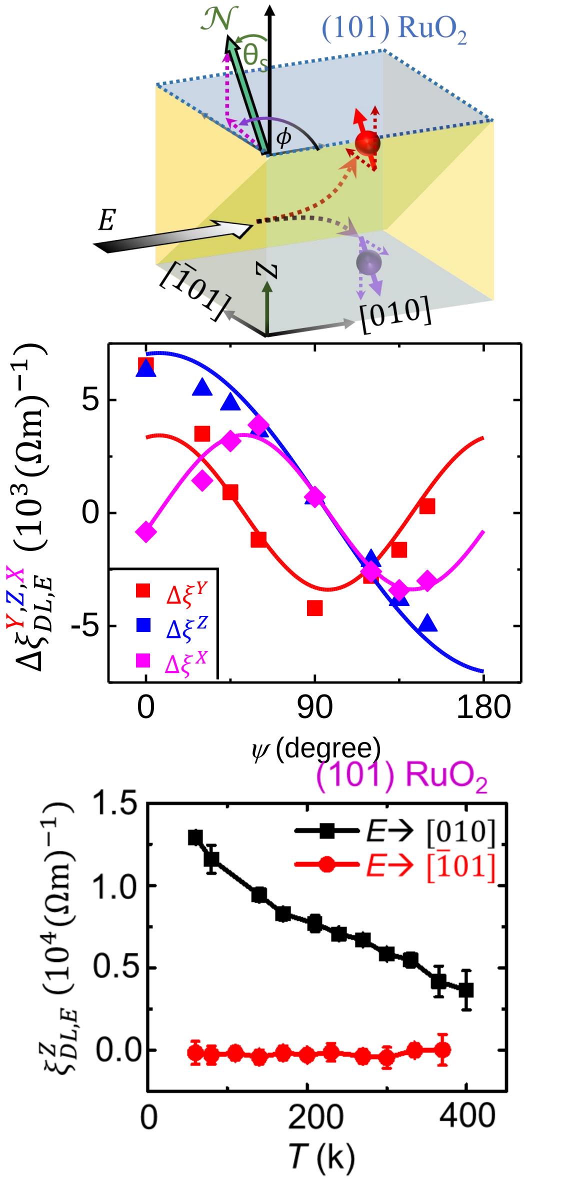
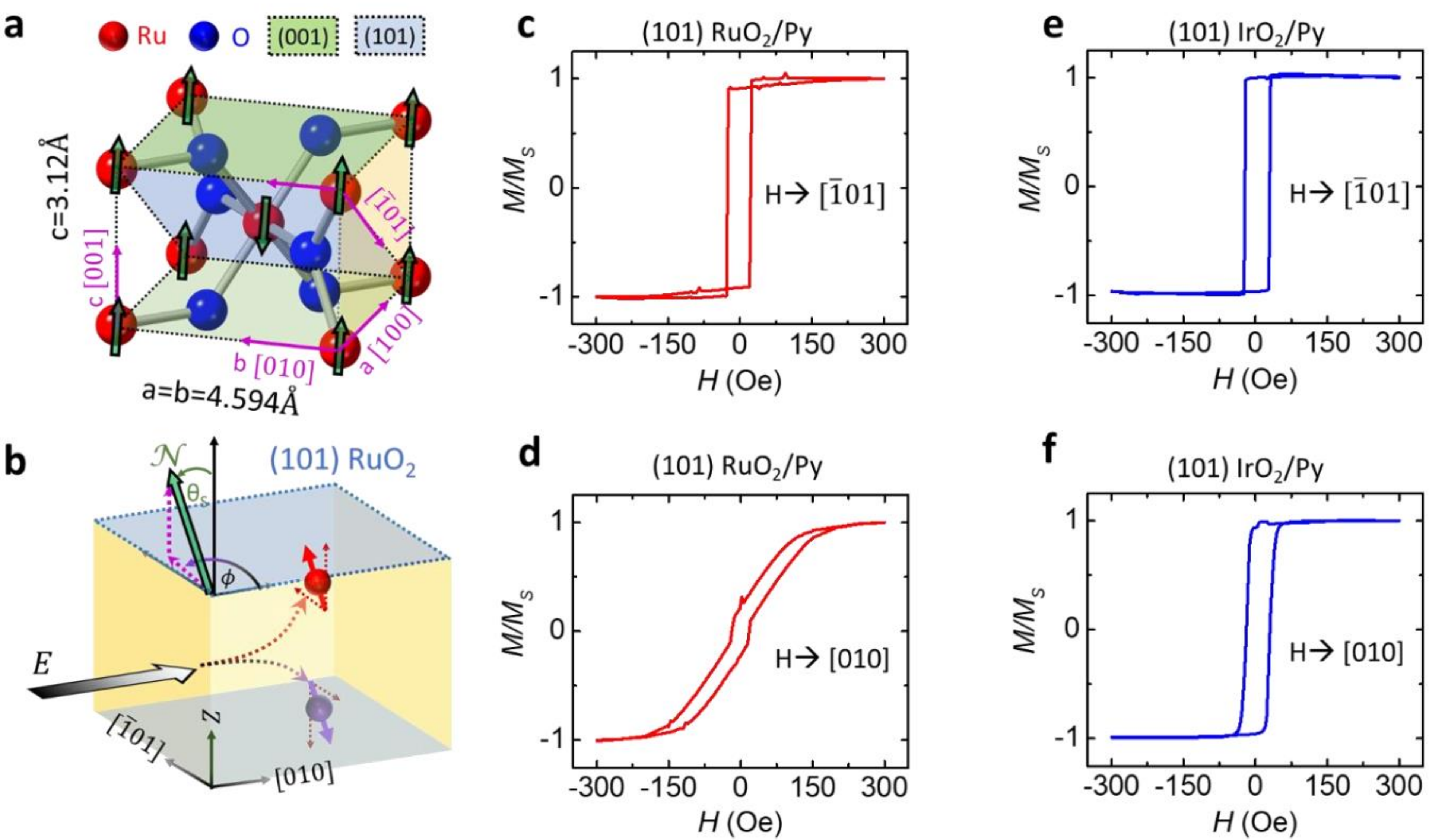
R. Jain, Cornell, Physics, AEP

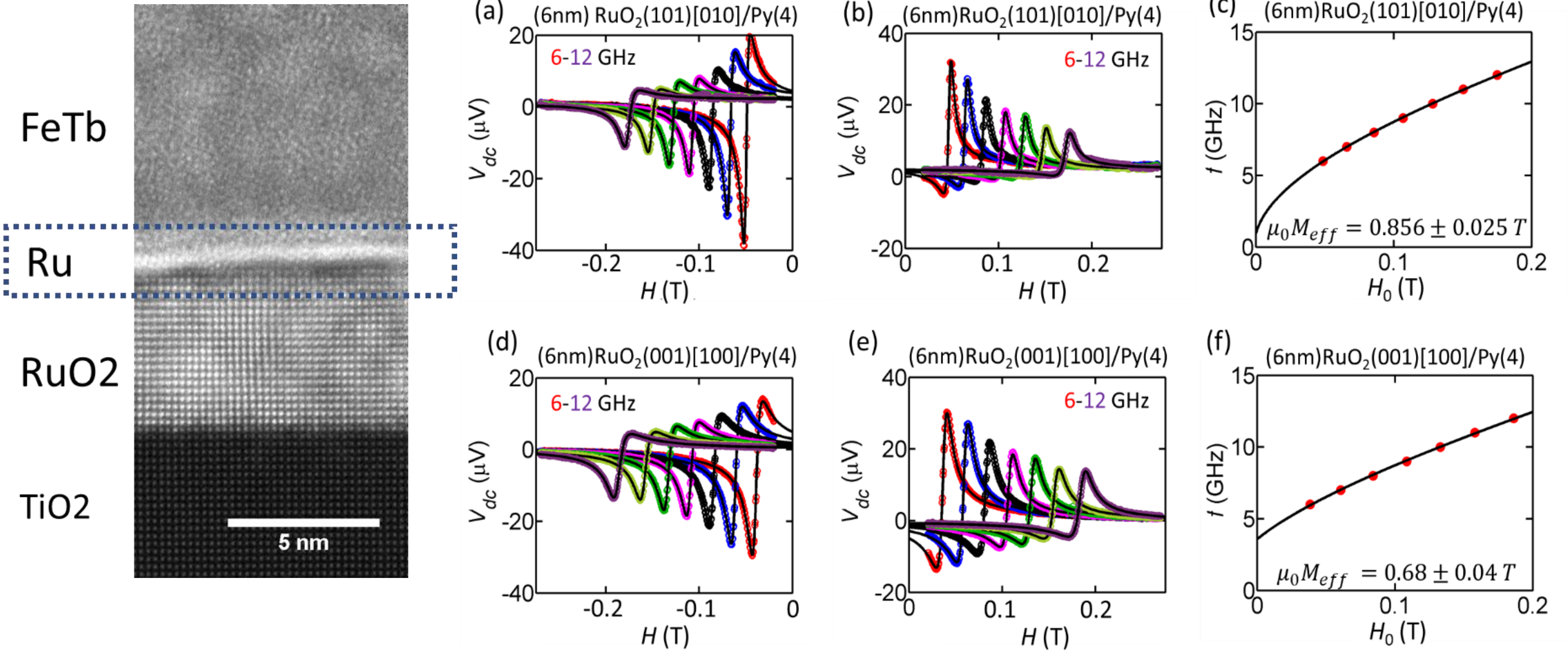
D. C. Ralph and R. A. Buhrman (Cornell)

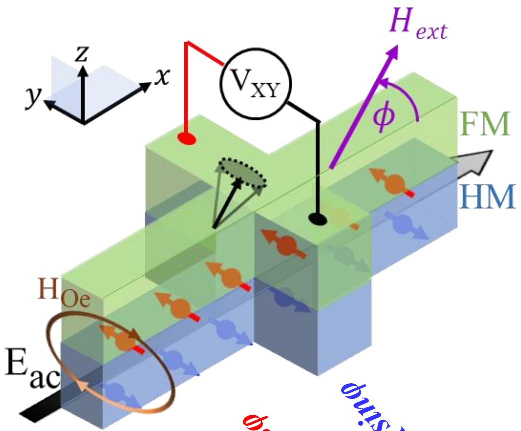
Exp (*Nature Elecl.5, 267 (2022)*)

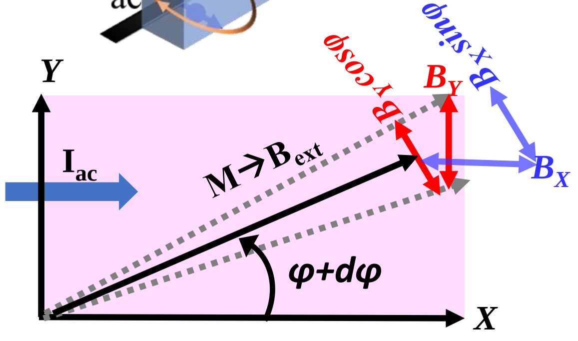


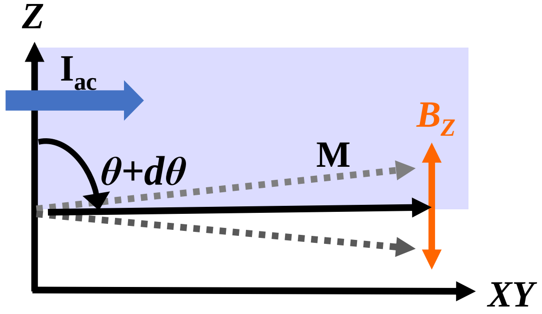


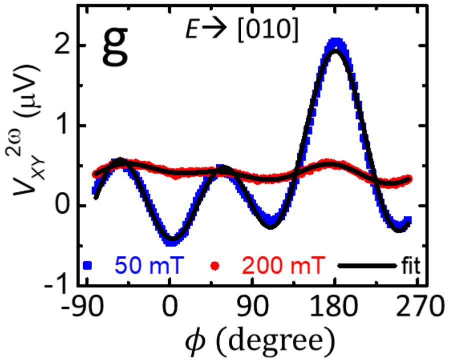
a
Ru
O
(001)
(101)
c=3.12Å
c [001]
[$\bar{1}$01]
b [010]
a [100]
a=b=4.594Å
b
(101) RuO_2
$\mathcal{N}$
θ_S
ϕ
E
Z
[$\bar{1}$01]
[010]
c
(101) RuO_2/Py
M/M_S
H→ [$\bar{1}$01]
H (Oe)
d
(101) RuO_2/Py
H→ [010]
e
(101) IrO_2/Py
H→ [$\bar{1}$01]
f
(101) IrO_2/Py
H→ [010]

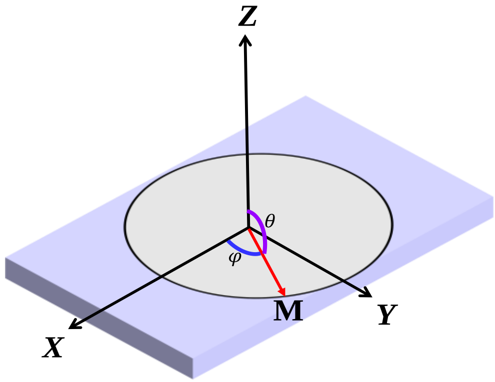

FeTb
Ru
RuO2
TiO2
5 nm
(a) (6nm) RuO2(101)[010]/Py(4)
6-12 GHz
Vdc (μV)
H (T)
(b) (6nm)RuO2(101)[010]/Py(4)
6-12 GHz
Vdc (μV)
H (T)
(c) (6nm)RuO2(101)[010]/Py(4)
f (GHz)
μ0Meff = 0.856 ± 0.025 T
H0 (T)
(d) (6nm)RuO2(001)[100]/Py(4)
6-12 GHz
Vdc (μV)
H (T)
(e) (6nm)RuO2(001)[100]/Py(4)
6-12 GHz
Vdc (μV)
H (T)
(f) (6nm)RuO2(001)[100]/Py(4)
f (GHz)
μ0Meff = 0.68 ± 0.04 T
H0 (T)











SHH

$$R_{xy}(\theta, \varphi) = R_0 + R_P \sin 2\varphi \sin^2\theta + R_A \cos\theta$$

$$R_{xy}(\theta + d\theta, \varphi + d\varphi) \approx R(\theta, \varphi) + \sum_i \frac{\partial R}{\partial \theta} d\theta + \sum_i \frac{\partial R}{\partial \varphi} d\varphi$$

Since in-plane magnet: $\theta = 90°$

$$\left(\frac{\partial R}{\partial \varphi}\right)_{\theta=90°,\varphi} = 2R_P \cos 2\varphi \qquad \left(\frac{\partial R}{\partial \theta}\right)_{\theta=90°,\varphi} = -R_A$$

$$d\varphi \sim \frac{B_y \cos\varphi}{B_{ext}} \sin\omega t \qquad d\theta \sim \frac{B_Z \cos\varphi}{B_{ext} + B_k} \sin\omega t$$

$$\Gamma_{DL} \rightarrow \left(m \times \left(\sigma^Y \times m\right)\right) \rightarrow B_{DL} = B_Z \cos\varphi$$

$$\Gamma_{FL} \rightarrow \left(m \times \sigma^Y\right) \rightarrow B_{FL} = B_y \cos\varphi$$

$$V_{xy}(\omega t) = R_{xy}\left(\theta + d\theta(t), \varphi + d\varphi(t)\right) I_0 \sin\omega t$$

$$V_{xy}(\omega t) = I_0/2(1 - \cos 2\omega t)\left[\frac{-R_A B_Z}{B_{ext} + B_k} \cos\varphi + \frac{2R_p B_y}{B_{ext}} \cos\varphi \cos 2\varphi\right] + I_0 \sin\omega t \, R(\theta, \varphi)$$

For: $\sigma = \sigma^Y \rightarrow V_{2\omega} \rightarrow D_{DL}^Y \cos\phi + F_{FL}^Y \cos\phi \cos 2\phi$

For: $\sigma = \sigma^X \rightarrow V_{2\omega} \rightarrow D_{DL}^X \sin\phi + F_{FL}^X \sin\phi \cos 2\phi$

For: $\sigma = \sigma^Z \rightarrow V_{2\omega} \rightarrow F_{FL}^Z + D_{DL}^Z \cos 2\phi$

$$\Gamma_{DL} \rightarrow \left(m \times \sigma_{z,x}^{X,Y,Z} \times m\right)$$

$$\Gamma_{FL} \rightarrow \left(m \times \sigma_{z,x}^{X,Y,Z}\right)$$

FMR

$$\frac{d\hat{m}}{dt} = -\gamma_0\left(\hat{m} \times \vec{H}_{eff}\right) + \alpha\left(\hat{m} \times \frac{d\hat{m}}{dt}\right)$$ LLG. (En. 1)

Precession Damping

Evaluate it by multiplying $\hat{m} \times$ (En 1)

$$\frac{d\hat{m}}{dt} = -\gamma\left(\hat{m} \times \vec{H}_{eff}\right) - \alpha\gamma\, \hat{m} \times \left(\hat{m} \times \vec{H}_{eff}\right)$$ LLG: (En. 2)

$$\vec{H}_{eff} = \vec{H}_{ext} + \frac{-1}{\mu_0 Vol.M_s}\frac{\delta E_{mag}}{\delta \hat{m}} = \vec{H}_{ext} + (H_{\parallel} m_x \hat{x} - H_{\perp} m_z \hat{z})$$

Action of ac magnetic field: $(h_y\hat{y}, h_z\hat{z})$ superimposed on a dc field: $H_{ext}\hat{x}$

$$\vec{H}_{eff} = H_{ext}\hat{x} + (H_{\parallel} m_x \hat{x} - H_{\perp} m_z \hat{z}) + h_y\hat{y} + h_z\hat{z}$$

$$\delta\dot{m}_{y'} = -\omega_1\,\delta m_{z'} - \alpha\omega_2\,\delta m_{y'} + \gamma h_{z'} + \alpha\gamma h_{y'} \qquad (3a)$$

$$\delta\dot{m}_{z'} = \omega_2\,\delta m_{y'} - \alpha\omega_1\,\delta m_{z'} - \gamma h_{y'} + \alpha\gamma h_{z'} \qquad (3a)$$

LLG linearized: small fluctuation: $m \approx m_x, m_y \to 0, m_z \to 0$

$h_{y'} = h_{y'}e^{-i\omega t}$, $h_{z'} = h_{z'}e^{-i\omega t}$, $\delta m_{y'} = Ae^{-i\omega t}$, $\delta m_{z'} = Be^{-i\omega t}$

$$\begin{pmatrix} A \\ B \end{pmatrix} = \gamma \begin{pmatrix} \frac{-i\alpha\omega + (1+\alpha^2)\omega_1}{-\omega^2 + (1+\alpha^2)\omega_1\omega_2 - i\alpha\omega(\omega_1+\omega_1)} & \frac{-i\omega}{-\omega^2 + (1+\alpha^2)\omega_1\omega_2 - i\alpha\omega(\omega_1+\omega_1)} \\ \frac{i\omega}{-\omega^2 + (1+\alpha^2)\omega_1\omega_2 - i\alpha\omega(\omega_1+\omega_1)} & \frac{-i\alpha\omega + (1+\alpha^2)\omega_2}{-\omega^2 + (1+\alpha^2)\omega_1\omega_2 - i\alpha\omega(\omega_1+\omega_1)} \end{pmatrix} \begin{pmatrix} h_{y'} \\ h_{z'} \end{pmatrix}$$

$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} \chi_{11}^H & \chi_{12}^H \\ \chi_{21}^H & \chi_{22}^H \end{pmatrix} \begin{pmatrix} h_{y'} \\ h_{z'} \end{pmatrix}$$

$$\omega_1 = \gamma(H_{\perp} + H_{\parallel} + H_0)$$

$$\omega_2 = \gamma(H_{\parallel} + H_0)$$

FMR → ST-FMR

$$\begin{pmatrix} A \\ B \end{pmatrix} = \gamma \begin{pmatrix} \frac{-i\alpha\omega + (1+\alpha^2)\omega_1}{-\omega^2 + (1+\alpha^2)\omega_1\omega_2 - i\alpha\omega(\omega_1+\omega_1)} & \frac{-i\omega}{-\omega^2 + (1+\alpha^2)\omega_1\omega_2 - i\alpha\omega(\omega_1+\omega_1)} \\ \frac{i\omega}{-\omega^2 + (1+\alpha^2)\omega_1\omega_2 - i\alpha\omega(\omega_1+\omega_1)} & \frac{-i\alpha\omega + (1+\alpha^2)\omega_2}{-\omega^2 + (1+\alpha^2)\omega_1\omega_2 - i\alpha\omega(\omega_1+\omega_1)} \end{pmatrix} \begin{pmatrix} h_{y'} \\ h_{z'} \end{pmatrix}$$

$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} \chi_{11}^H & \chi_{12}^H \\ \chi_{21}^H & \chi_{22}^H \end{pmatrix} \begin{pmatrix} h_{y'} \\ h_{z'} \end{pmatrix} \qquad \omega_1 = \gamma(H_\perp + H_\parallel + H_0) \,;\, \omega_2 = \gamma(H_\parallel + H_0)$$

$$\chi_{11}^H = \frac{-i\alpha\omega + \omega_1}{-\omega^2 + \omega_1\omega_2 - i\alpha\omega(\omega_1+\omega_1)} \qquad \chi_{11}^H = \frac{(i\alpha\omega - \omega_1)[(\omega^2 - \omega_1\omega_2) - i\alpha\omega(\omega_1+\omega_1)]}{(\omega^2 - \omega_1\omega_2)^2 + \alpha^2\omega^2(\omega_1+\omega_2)^2}$$

At resonance:

$(\omega^2 - \omega_1\omega_2)^2 = 0$

$\omega^2 = \omega_1\omega_2$

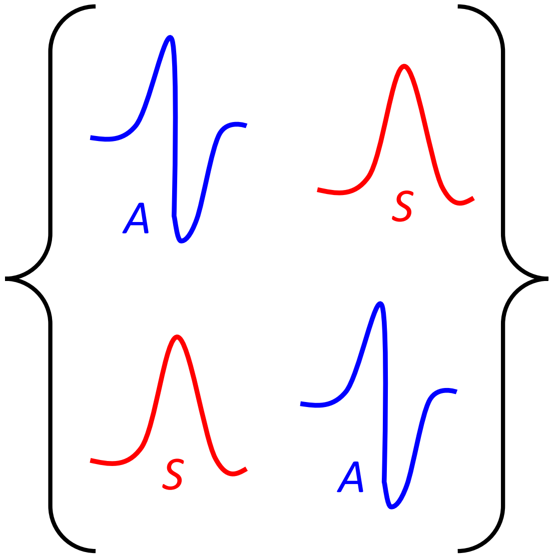
(Kittel's equation)

$$\chi_{11}^H(H_{ext}) \approx \sqrt{\frac{\omega_{1,0}}{\omega_{2,0}}} \frac{\gamma}{\alpha(\omega_{1,0}+\omega_{2,0})} \frac{\Delta H}{2} \left[\frac{(H_{ext}-H_r)}{(H_{ext}-H_r)^2 + \left(\frac{\Delta H}{2}\right)^2} + i \frac{\left(\frac{\Delta H}{2}\right)}{(H_{ext}-H_r)^2 + \left(\frac{\Delta H}{2}\right)^2} \right]$$

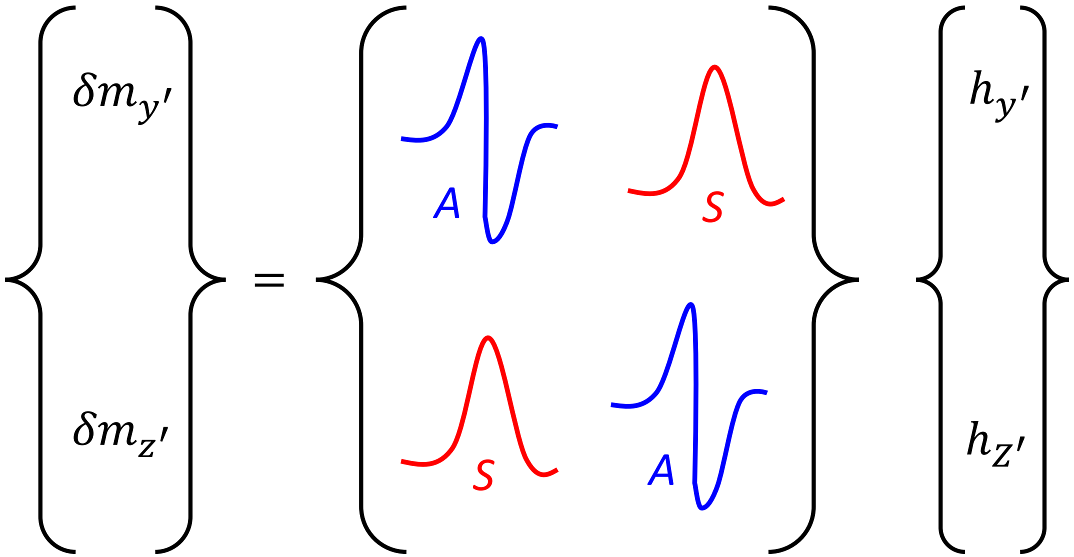
$$\chi_{12}^H(H_{ext}) \approx \frac{-\gamma}{\alpha(\omega_{1,0}+\omega_{2,0})} \frac{\Delta H}{2} \left[\frac{\frac{\Delta H}{2}}{(H_{ext}-H_r)^2 + \left(\frac{\Delta H}{2}\right)^2} - i \frac{(H_{ext}-H_r)}{(H_{ext}-H_r)^2 + \left(\frac{\Delta H}{2}\right)^2} \right] = -\chi_{21}^H(H_{ext})$$

$$\chi_{22}^H(H_{ext}) \approx \sqrt{\frac{\omega_{2,0}}{\omega_{1,0}}} \frac{\gamma}{\alpha(\omega_{1,0}+\omega_{2,0})} \frac{\Delta H}{2} \left[\frac{(H_{ext}-H_r)}{(H_{ext}-H_r)^2 + \left(\frac{\Delta H}{2}\right)^2} + i \frac{\left(\frac{\Delta H}{2}\right)}{(H_{ext}-H_r)^2 + \left(\frac{\Delta H}{2}\right)^2} \right]$$

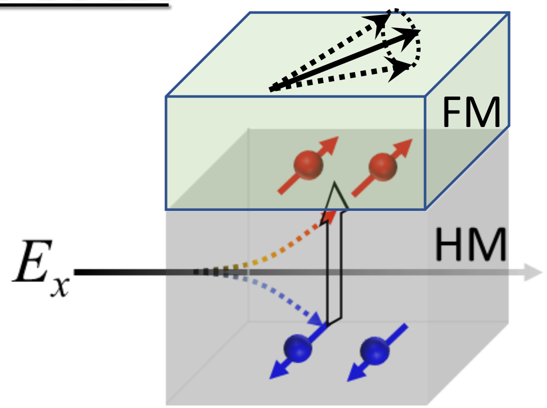
$Re(\chi^H) =$



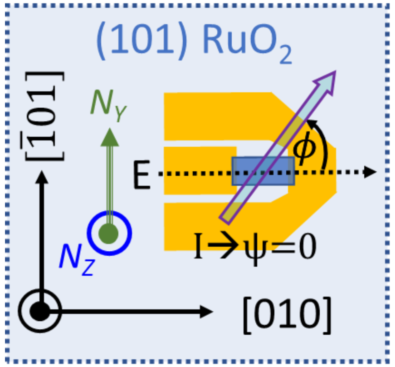
Susceptibility tensor for the field



ST-FMR



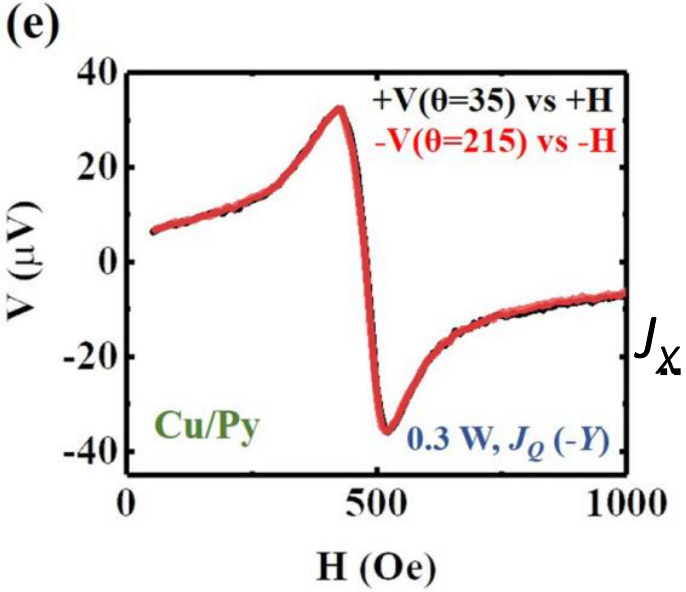
Magnet oscillates in 3d

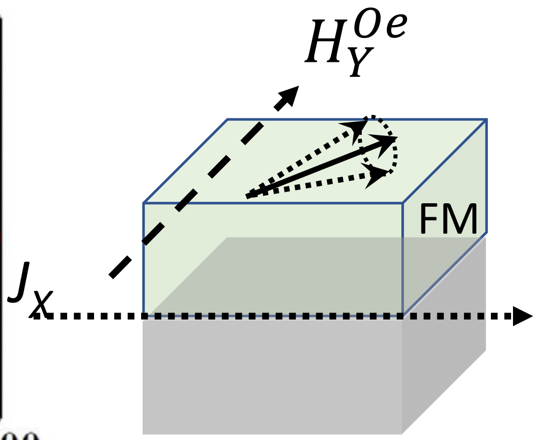


$$V_{xx}^{mix} = \frac{I_{rf}}{2} Re[m] \frac{dR}{dm}$$

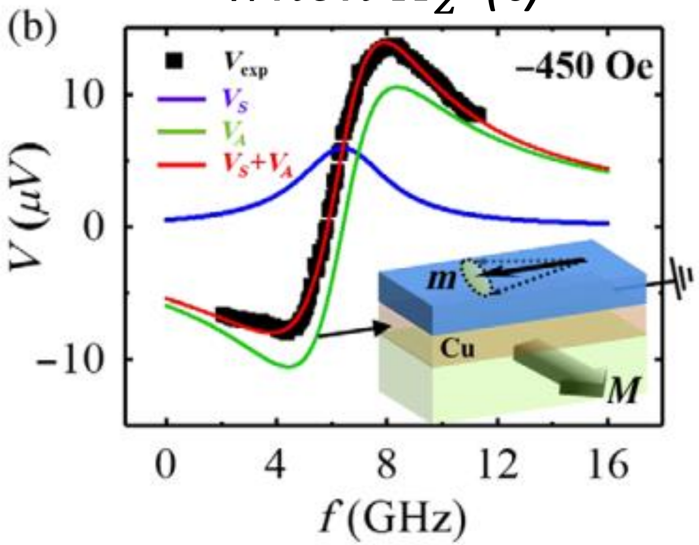
For in-plane magnet: $\left[\frac{dR}{dm_z}\right] = 0$

With AMR detection ($R_{xx} \propto \cos^2 \varphi$)
we are sensitive to the $\Delta\phi$ only





when H_z (t)



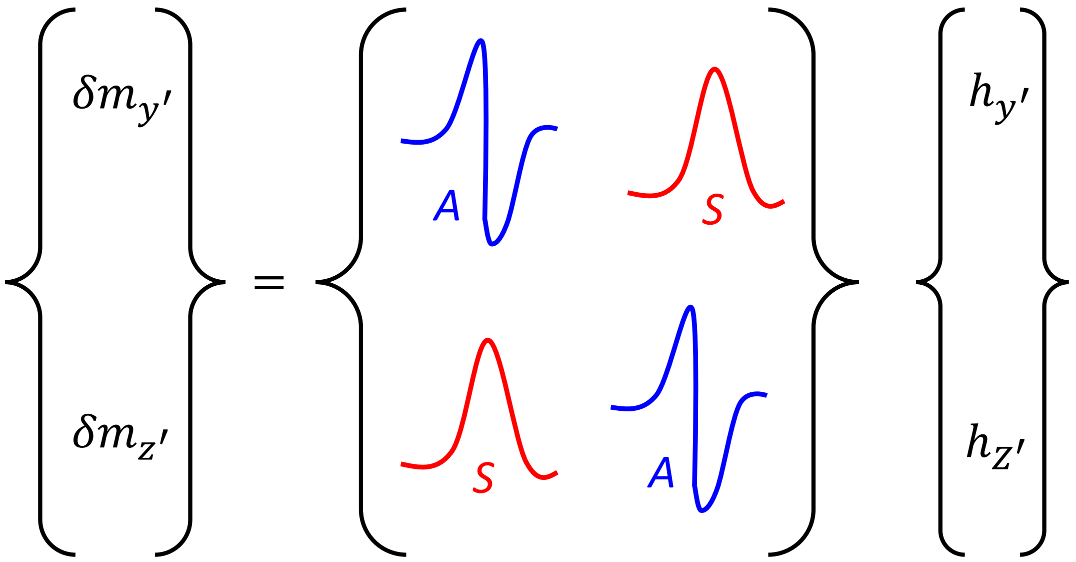
$$V_{XX}^{\text{mix}} = \frac{I_{\text{rf}}}{2} R_{\text{AMR}} \,\text{Re}\,[m_x] \sin 2\phi$$

$$\Gamma_{FL} \rightarrow \left(m \times \sigma_{z,x}^{X,Y,Z}\right)$$

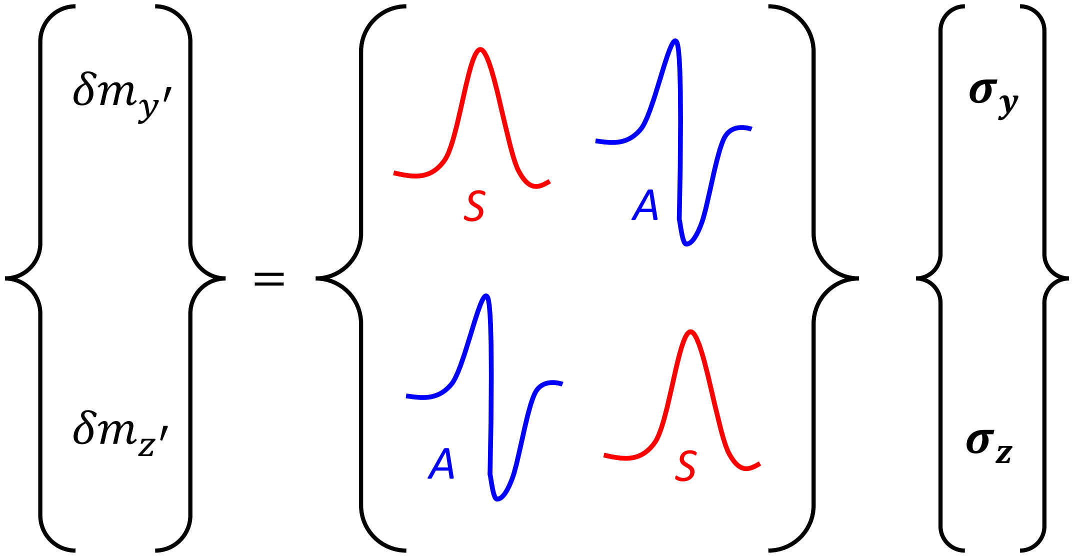
$$V_{xx}^{mix} = \begin{cases} F_A^y \sin 2\varphi \cos \varphi \\ F_A^x \sin 2\varphi \sin \varphi \\ F_S^z \sin 2\varphi . 1 \end{cases}$$

ST-FMR

Susceptibility tensor for the field



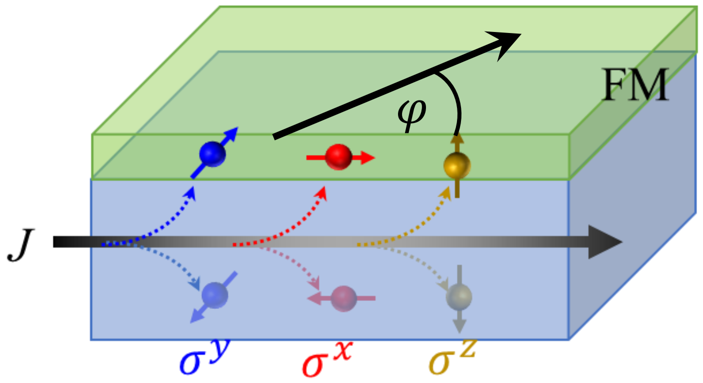
Susceptibility tensor for the spin-current



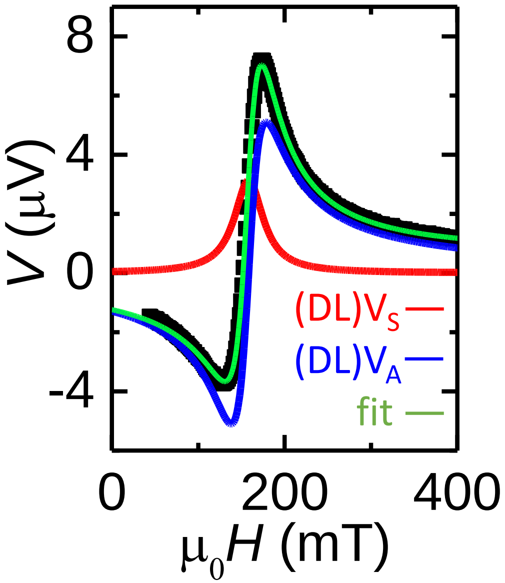
FMR from damping like torque (DLT)

$$\frac{d\hat{m}}{dt} = -\gamma(\hat{m} \times \vec{H}_{net}) + \alpha\gamma(\hat{m} \times \vec{H}_{net} \times \hat{m}) + \Gamma_{FL}(\hat{m} \times \sigma) + \Gamma_{DL}(\hat{m} \times (\boldsymbol{\sigma} \times \hat{\boldsymbol{m}}))$$

Effective field for DLT



	Effective field
$\hat{m} \times (\boldsymbol{\sigma_x} \times \hat{\boldsymbol{m}})$	$\sin\varphi\, h_z$
$\hat{m} \times (\boldsymbol{\sigma_y} \times \hat{\boldsymbol{m}})$	$\cos\varphi\, h_z$
$\hat{m} \times (\boldsymbol{\sigma_z} \times \hat{\boldsymbol{m}})$	*1 (In-plane)*

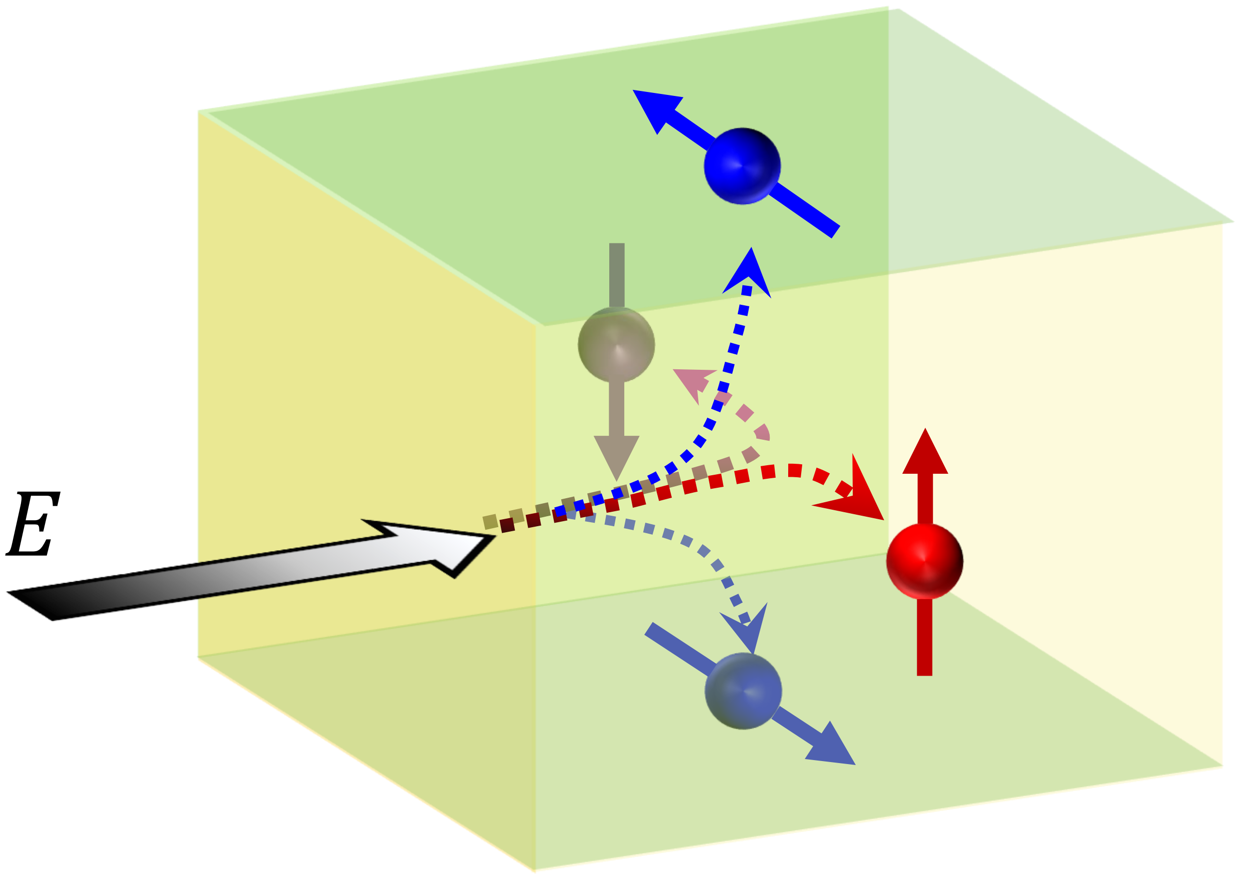


$$V_{XX}^{\text{mix}} = \frac{I_{\text{rf}}}{2} R_{\text{AMR}} \operatorname{Re}[m_x] \sin 2\phi$$

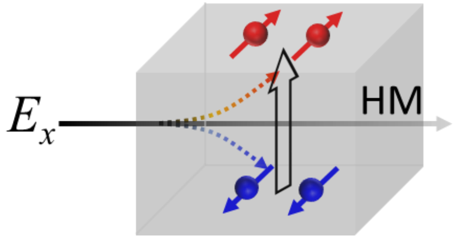
Angular ST-FMR ($\phi \rightarrow$ 0 to 360 degree)

$$S = (S_{DL}^{Y} \cos\phi + S_{DL}^{X} \sin\phi + S_{FL}^{Z}) \sin 2\phi$$

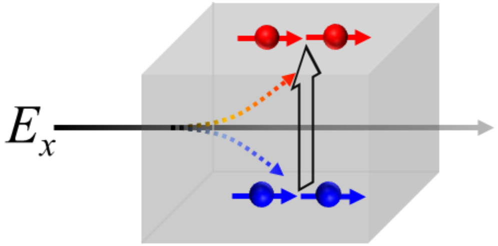
$$A = (A_{FL}^{Y} \cos\phi + A_{FL}^{X} \sin\phi + A_{DL}^{Z}) \sin 2\phi$$


E

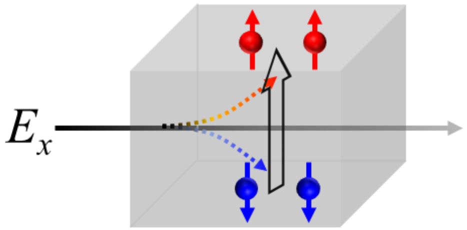
Types of SOT and measurement techniques



SHE, *REE* etc.



(Dresselhaus etc.)



(Unconventional, WTe_2, RuO_2)

$$\Gamma_{DL} \rightarrow (m \times \sigma_{z,x}^{X,Y,Z} \times m)$$
$$\Gamma_{FL} \rightarrow (m \times \sigma_{z,x}^{X,Y,Z})$$

In-plane 2nd Harmonic Hall

$$+D_{DL}^{Z} \cos 2\phi + F_{FL}^{Z}$$

IP-torque: $\overrightarrow{E_{AHE}} \rightarrow \vec{M} \times \vec{E_x}$

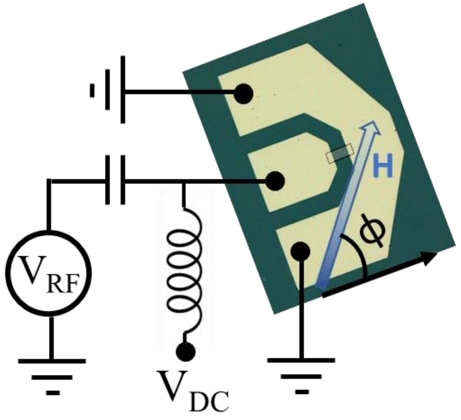
OP-torque: $\overrightarrow{E_{AMR,PHE}} \rightarrow \vec{M} \times \vec{E_x} \times \vec{M}$

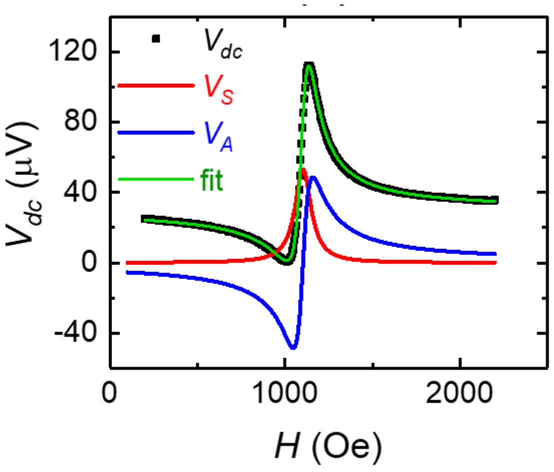
$$V_{XY}^{2\omega} \rightarrow \begin{cases} D_{DL}^{Y} \cos\phi + F_{FL}^{Y} \cos\phi \cos 2\phi \\ +D_{DL}^{X} \sin\phi + F_{FL}^{X} \sin\phi \cos 2\phi \\ +D_{DL}^{Z} \cos 2\phi + F_{FL}^{Z} \end{cases}$$

Constants D and F depend on H_{ext}

[PRB **89**, 144425 (2014), arXiv:2108.09150 (2021)]

ST-FMR





$$V_S = S\left(\frac{\Delta^2}{(H-H_0)^2+\Delta^2}\right) \qquad V_A = A\left(\frac{(H-H_0)\Delta}{(H-H_0)^2+\Delta^2}\right)$$

In-plane torques | Out-plane torques

$\{m \times (\sigma_{z,x}^{Y} \times m)\}$ $\{m \times (\sigma_{z,x}^{X} \times m)\}$ $\{m \times \sigma_{z,x}^{Z}\}$

$$S = S_{DL}^{Y} \cos\phi \sin 2\phi + S_{DL}^{X} \sin\phi \sin 2\phi + S_{FL}^{Z} \sin 2\phi$$
$$A = A_{FL}^{Y} \cos\phi \sin 2\phi + A_{FL}^{X} \sin\phi \sin 2\phi + A_{DL}^{Z} \sin 2\phi$$

$\{m \times \sigma_{z,x}^{Y}\}$ $\{m \times \sigma_{z,x}^{X}\}$ $\{m \times (\sigma_{z,x}^{Z} \times m)\}$

[PRL106, 036601 (2011)]