Floquet time-crystals as sensors of AC fields

Fernando Iemini

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NEUQUAM group– till 2024
(Henriette Herz programme)

F. Iemini, R. Fazio, A. Sanpera, **arXiv:2306.03927** (2023)

In collaboration with:

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  (ICTP / Università di Napoli)

The Abdus Salam International Centre for Theoretical Physics

Università degli Studi di Napoli Federico II

UAB Universitat Autònoma de Barcelona

- Anna Sanpera
  (ICREA/ Universitat Autonoma de Barcelona)

ICREA

Outline

- Introduction:

    - definition: what is a discrete time-crystal?

        . from theoretical proposals to experimental observation.

- Sensing with DTC’s

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- **Landau's symmetry breaking** is a cornerstone of modern physics:

. equilibrium ***spontaneuos symmetry breaking (SSB)*** occurs when the ground state or low-temperature states of a system fail to be invariant under symmetries of the Hamiltonian;

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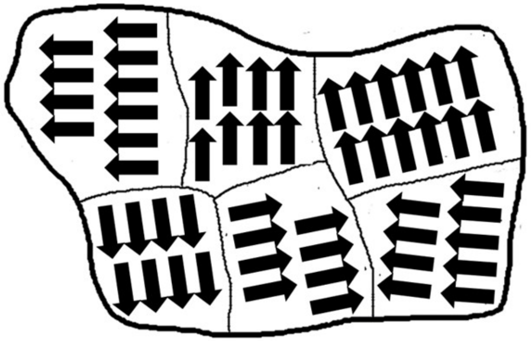
(spin rotation SB)



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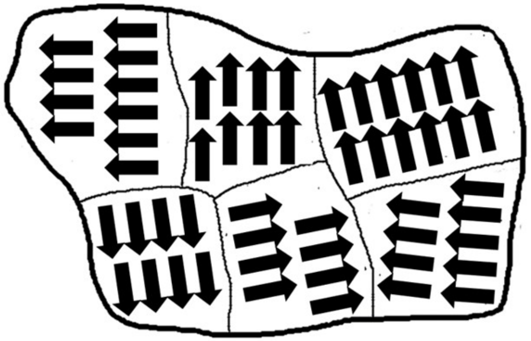

Crystal lattices
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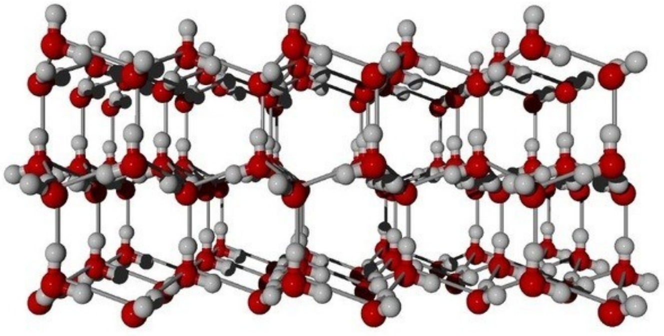

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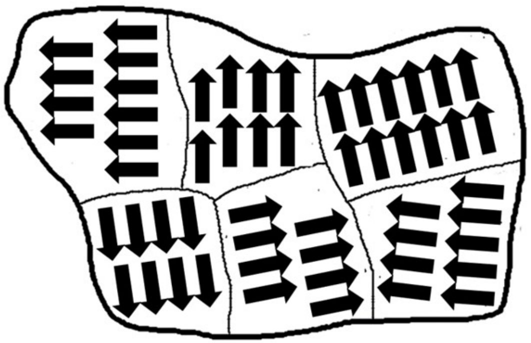

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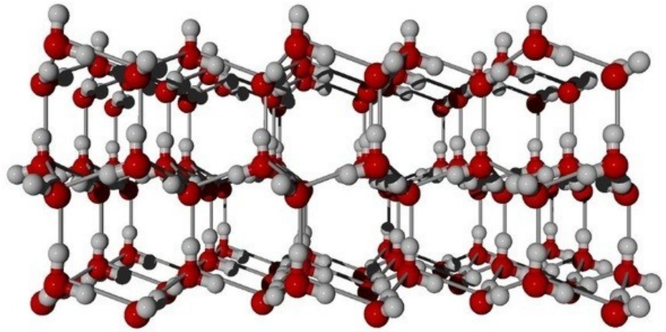
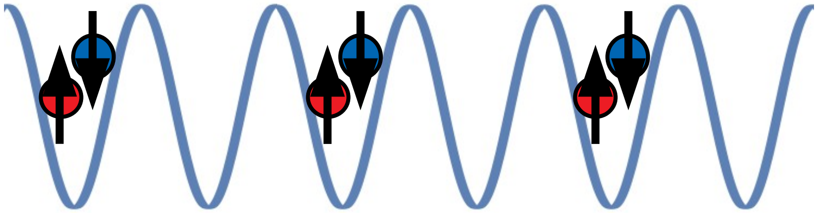


Charge-density-waves
(discrete SB)



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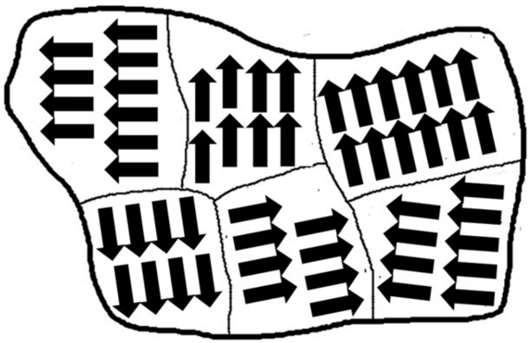

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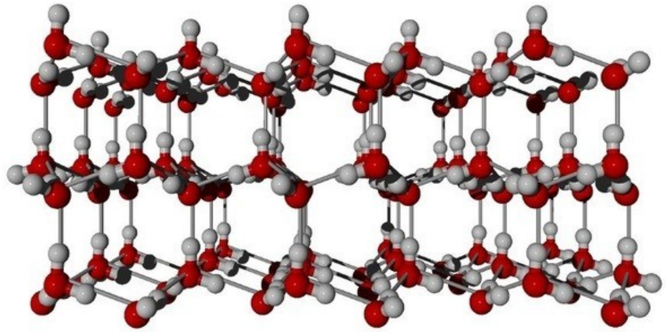

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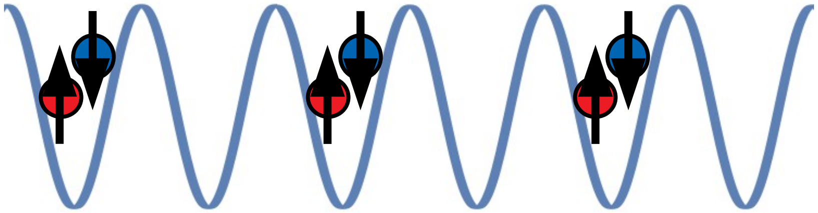

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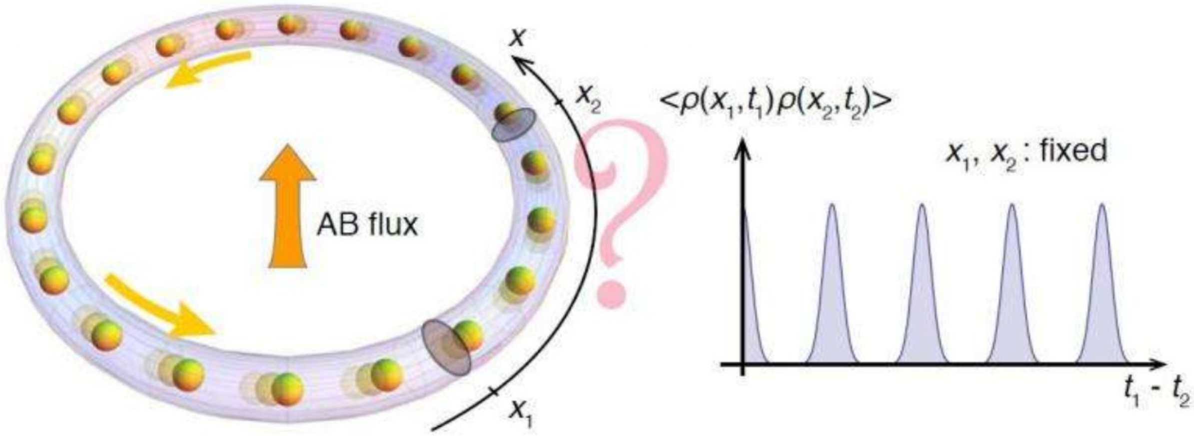
- Can **<u>time-translational invariance</u>** be spontaneously broken?

(F. Wilczek, PRL. 109, 160401 (2012))

Time translation symmetry breaking

ring particle model: **ground states** with moving "lumps"



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$$|x-x'|\to\infty$$

$$\rho(x,t) = \text{local order parameter}$$

Time translation symmetry breaking

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T. Li et al, PRL 109, 163001(2012);
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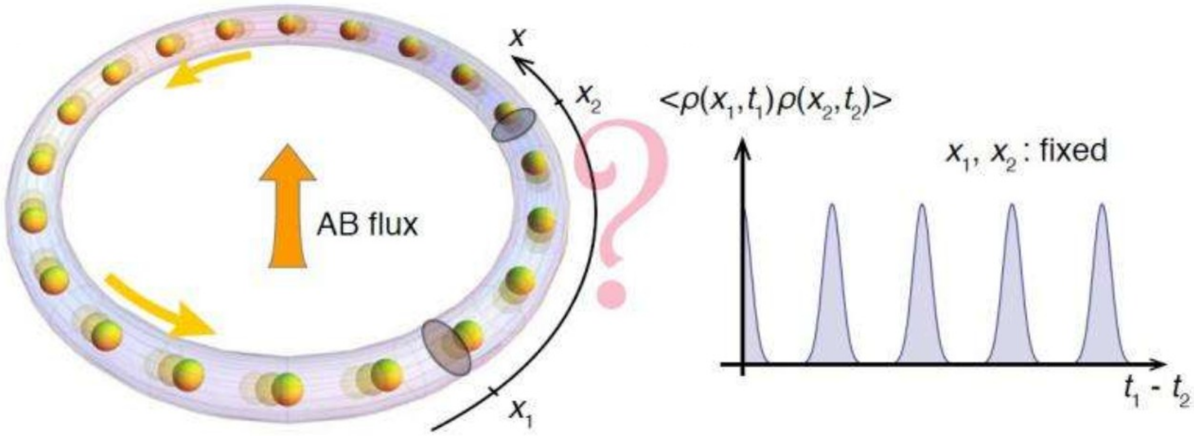

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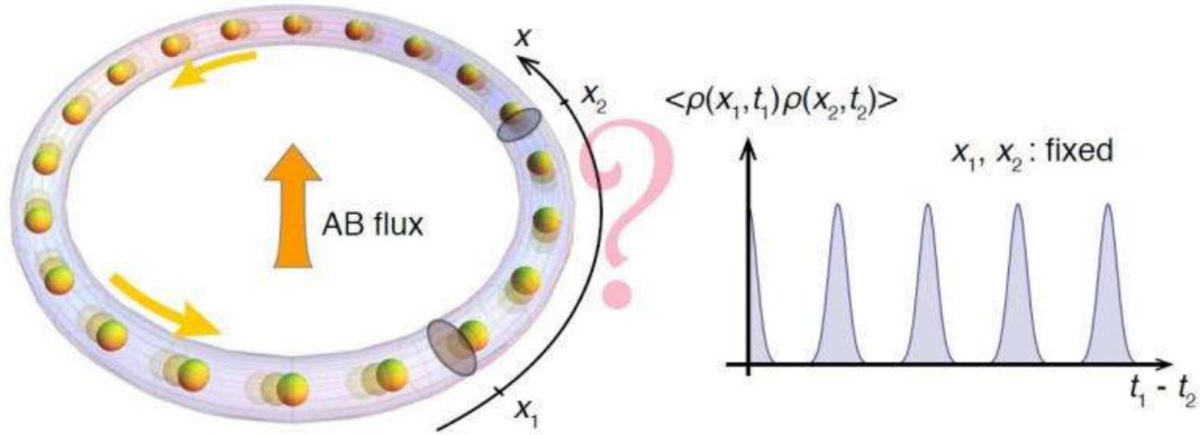

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- No-go theorem: systems in thermal equilibrium cannot manifest any time-crystalline behavior (*short-range Hamiltonians)

(H. Watanabe and M. Oshikawa, PRL 114, 251603 (2015))

- right context should be out of equilibrium: *e.g., preparing the system in an excited state, driven dynamics*, ...

- Important steps on periodically driven systems (**Floquet dynamics**):

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III) non-trivial oscillations persist for infinitely long times ***only*** the thermodynamic limit;

Floquet time-crystals

[D. V. Else, Bela Bauer, and Chetan Nayak, PRL 117, 090402 (2016)]

Combining **Floquet dynamics** + **MBL** (many-body localization) can support a **DTC**:

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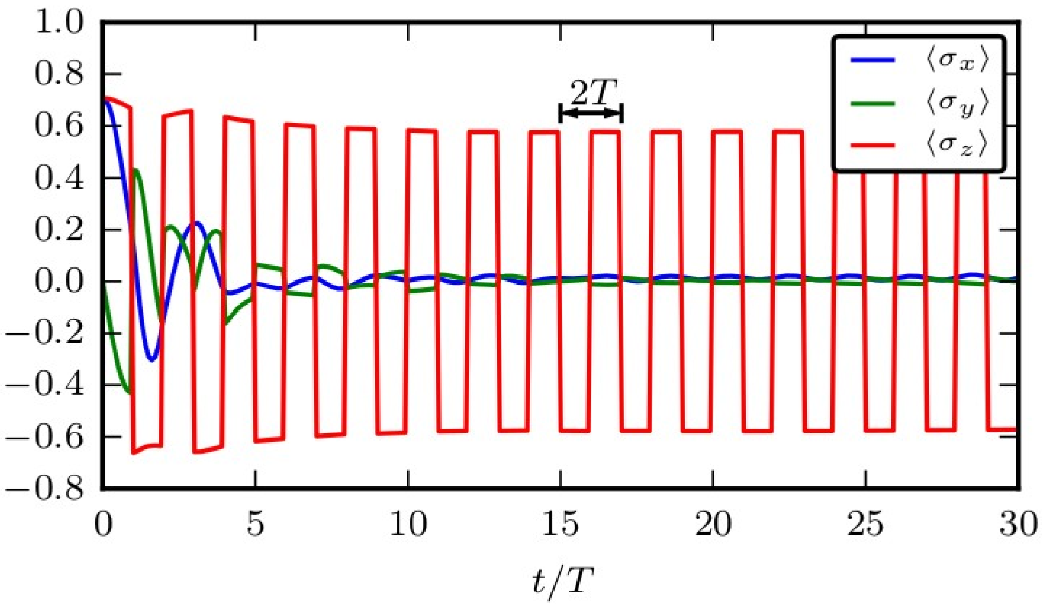
period doubling for magnetization



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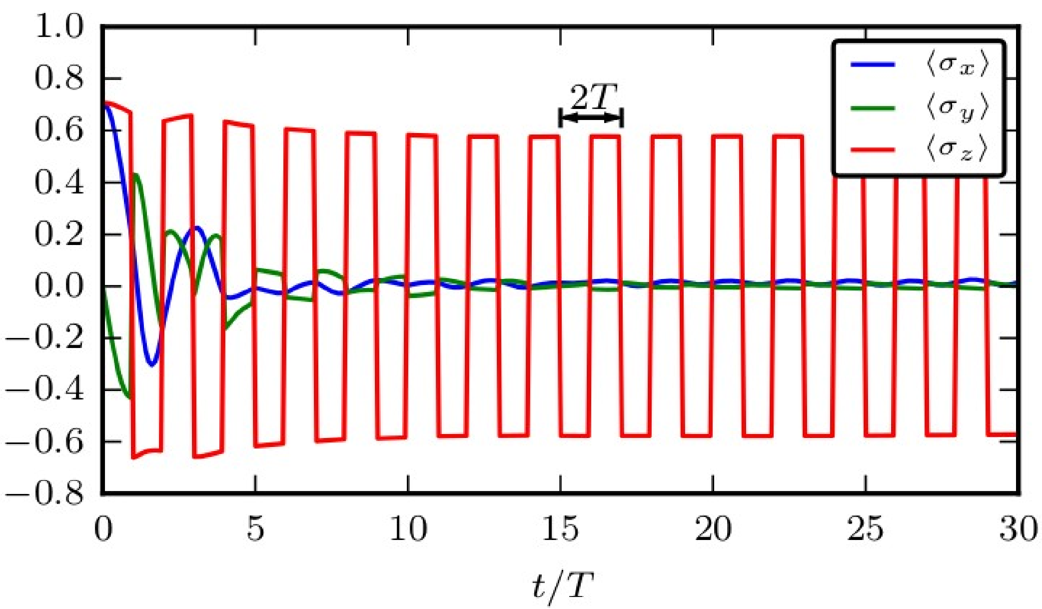
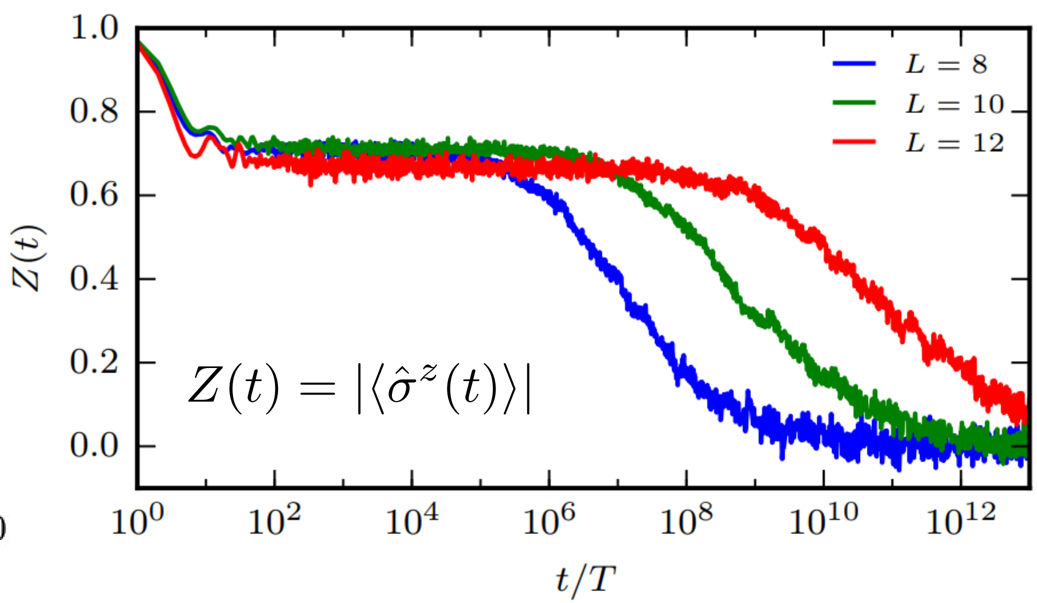


*persistence **only** in the thermodynamic limit*



trapped ytterbium ions

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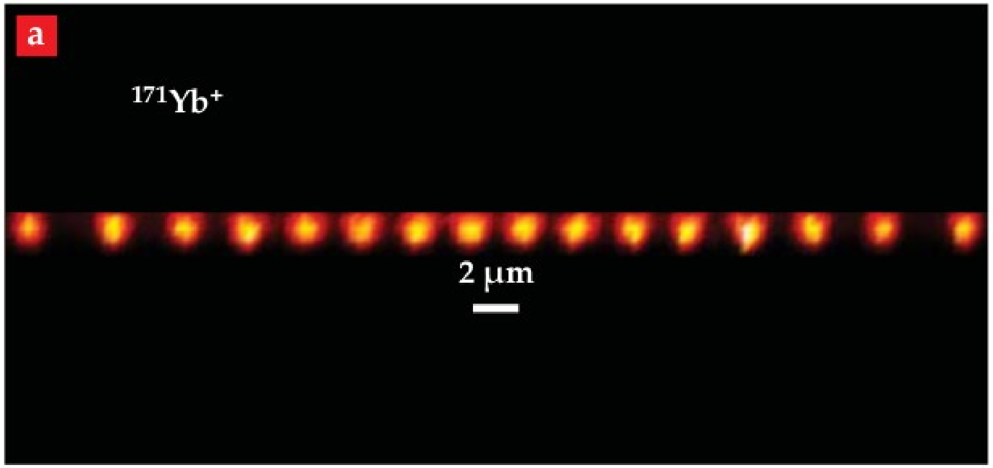

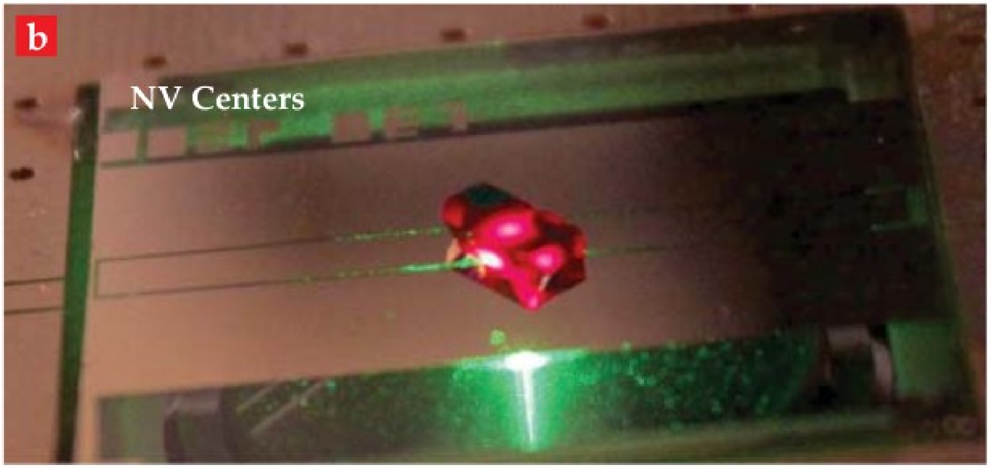

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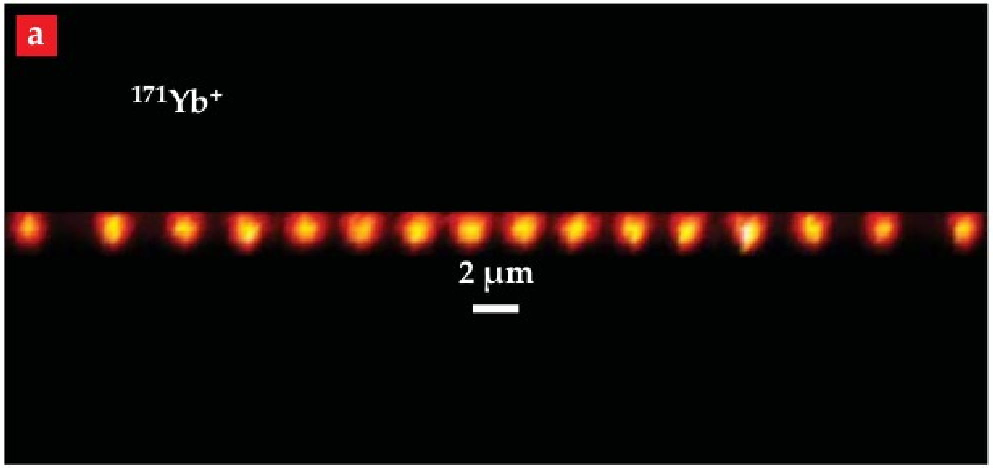

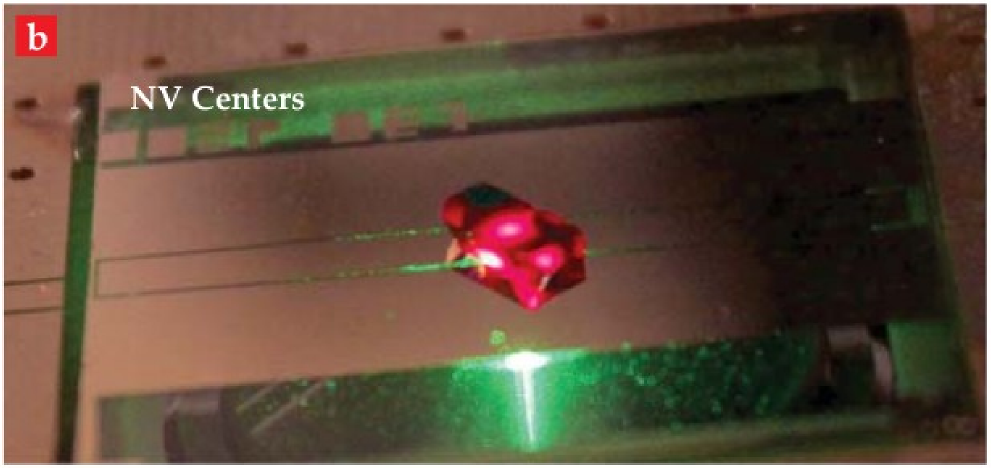

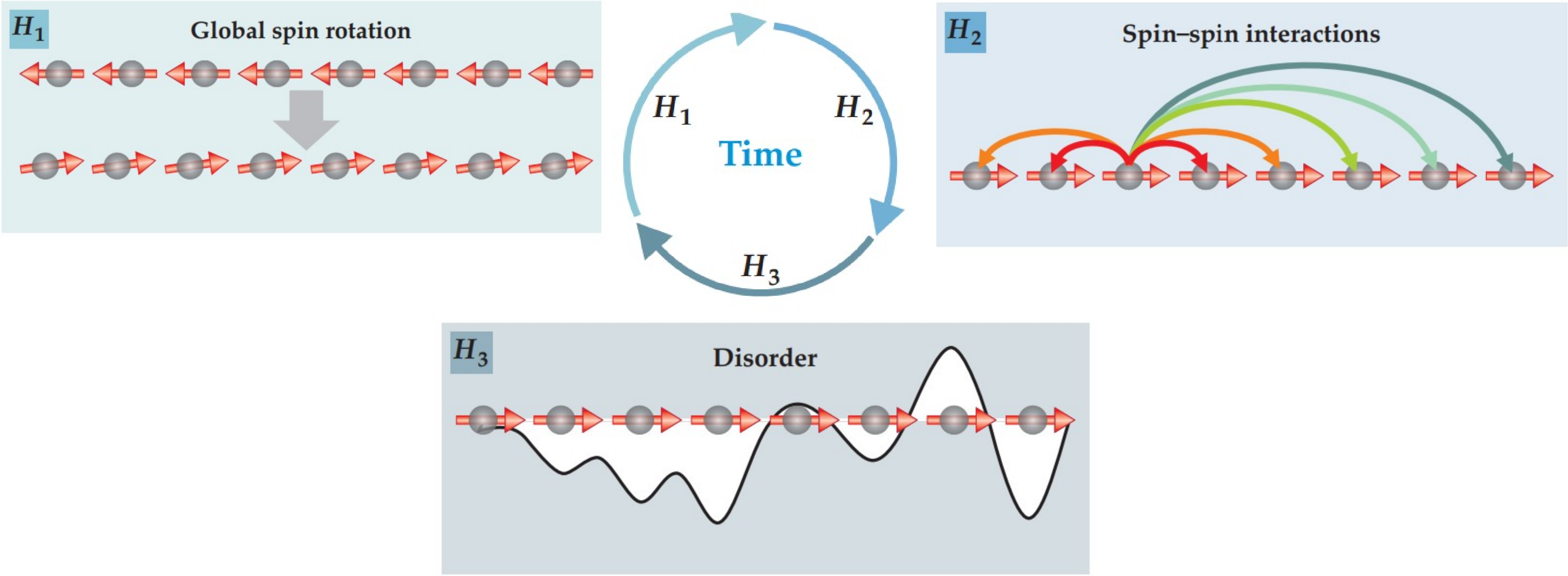

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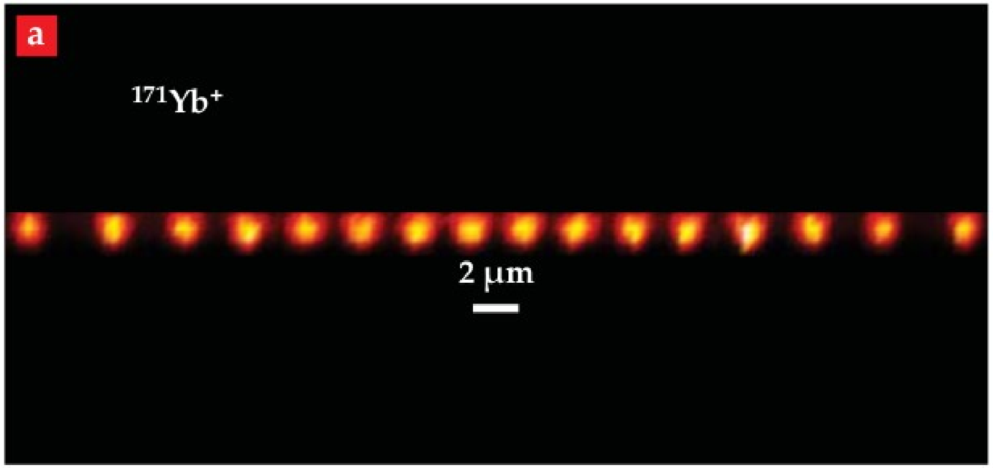
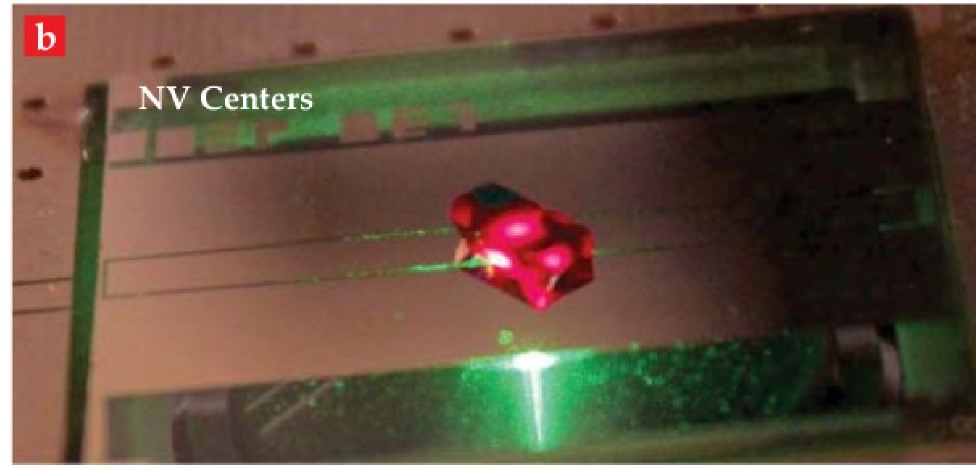
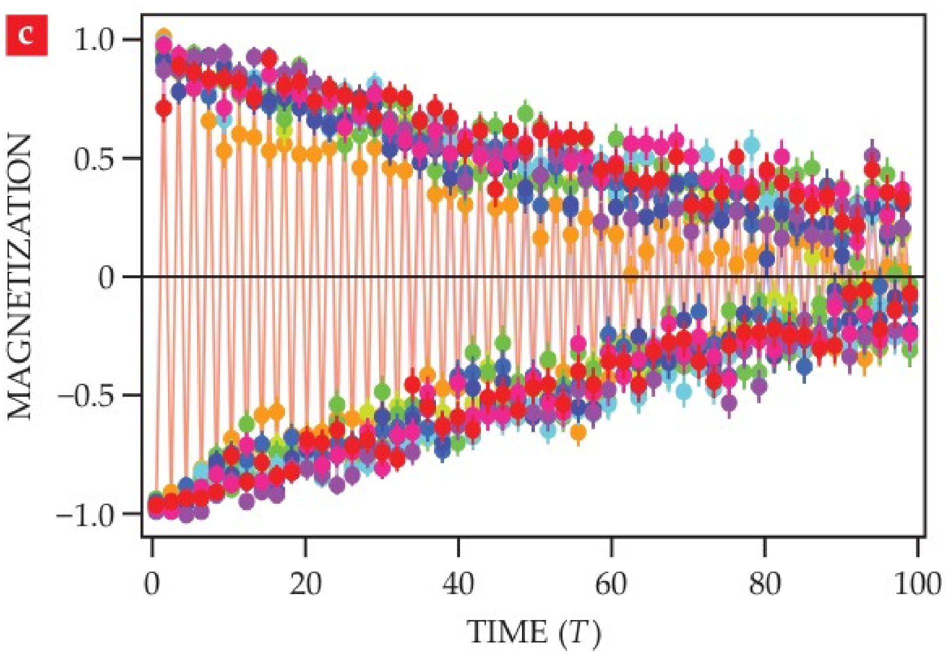
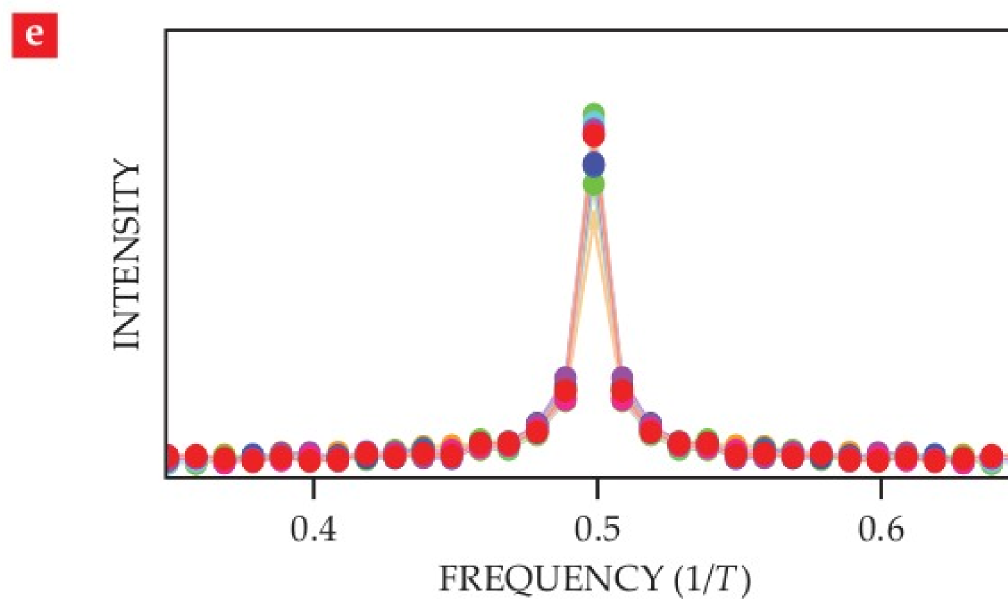
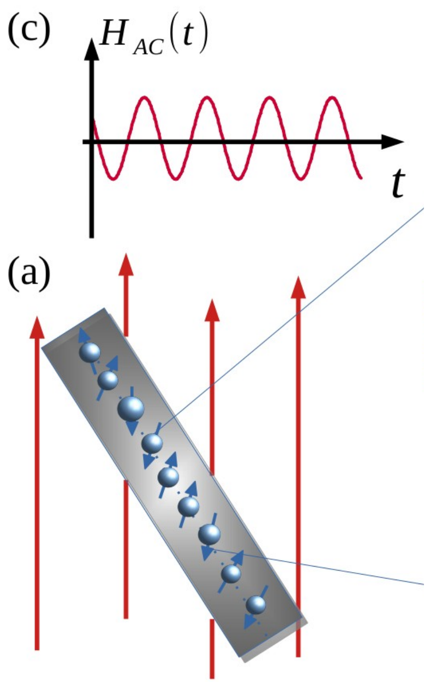








Sensing AC fields



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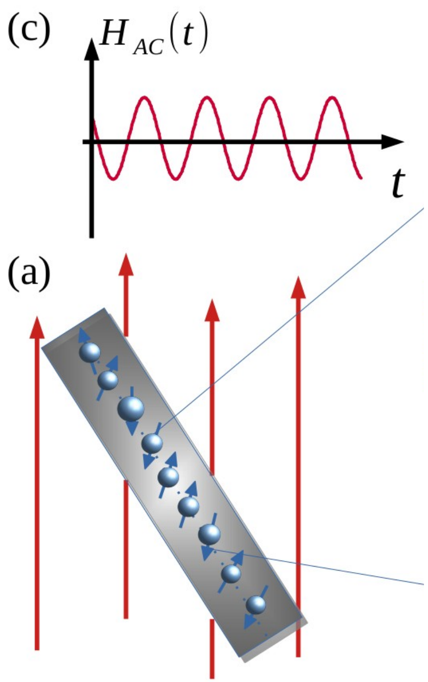

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improve performance

non-eq. interacting /
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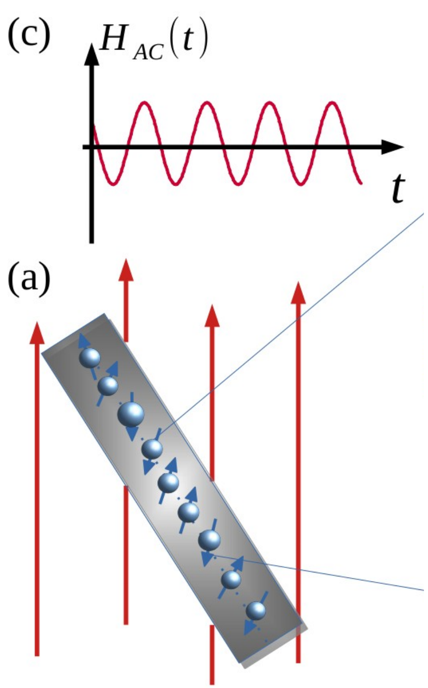

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Multitude of approaches

- dynamical decoupling spins by a series of fast pulses – mitigate decoherence/interactions:

. Dieter Suter et al. Rev. Mod. Phys. 88 (2016).
. Hengyun Zhou et al., PRX 10, 031003 (2020).

- strong interactions and fast drive: prepare and (prethermal) stabilize of GHZ-like states:

. Soonwon Choi et al. ArXiv:1801.00042 (2017)

- “inspirational work”: non-equilibrium dynamics of an integrable (Ising chain) many-body system

. Utkarsh Mishra and Abolfazl Bayat, Scientific Reports 12, 14760 (2022)

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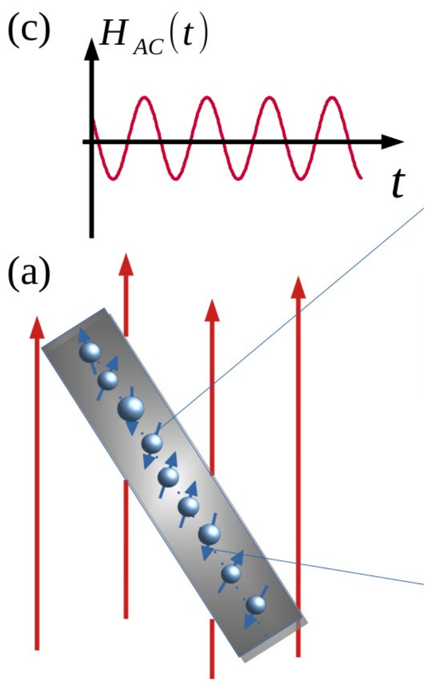

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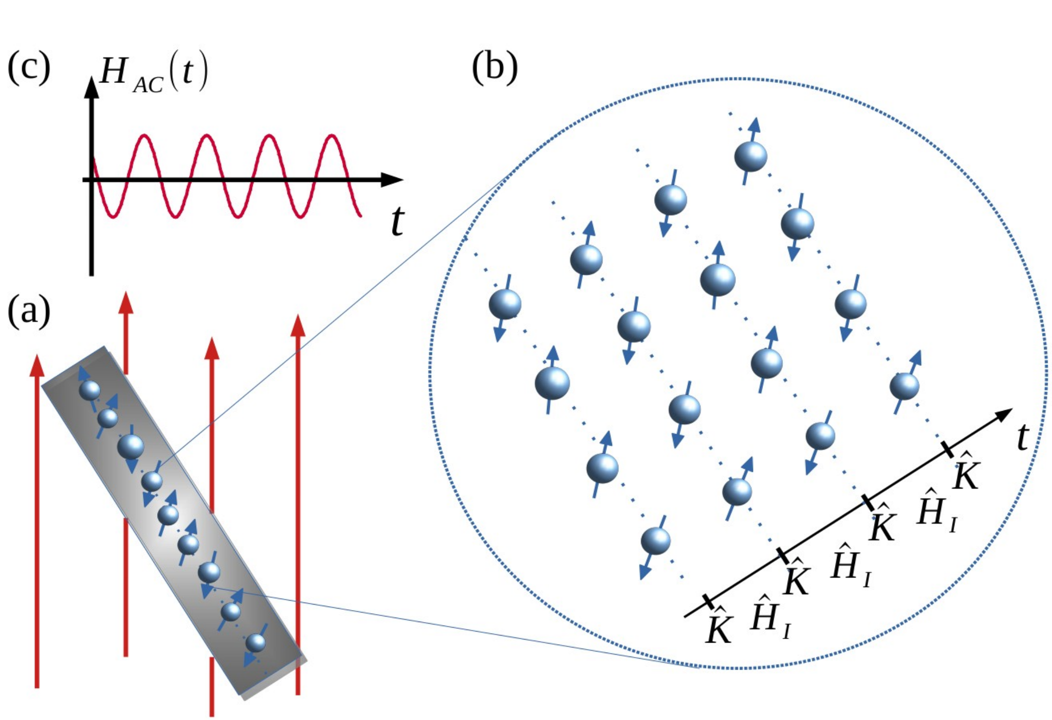
exploit long-range spatial and time ordering for enhanced metrological protocols.

Floquet time-crystals as sensors of AC fields



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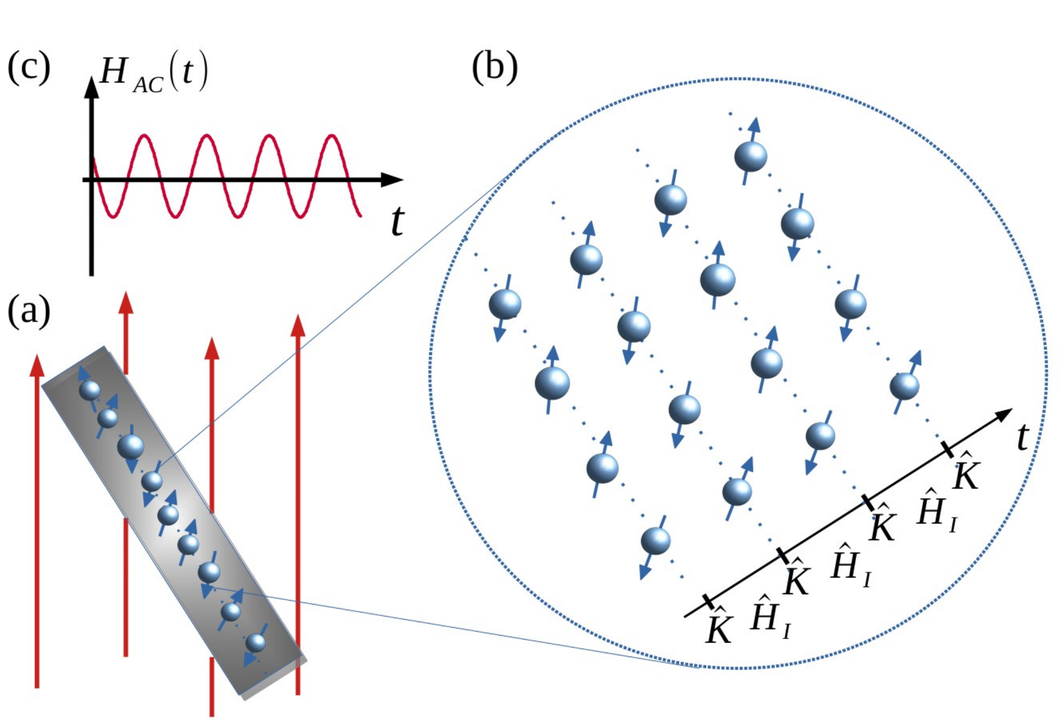

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Sensing steps:

i) initialize the state; $\hat{\rho}_o$

$\downarrow$

ii) interact with signal; $\hat{\rho}(t, h_{\mathrm{ac}})$

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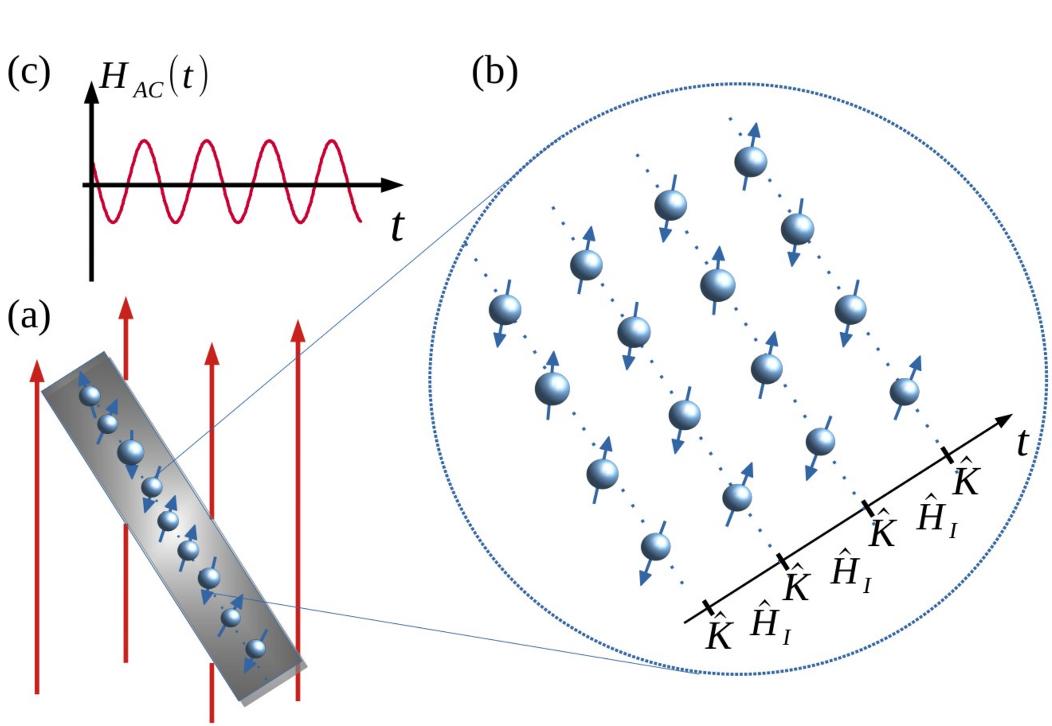

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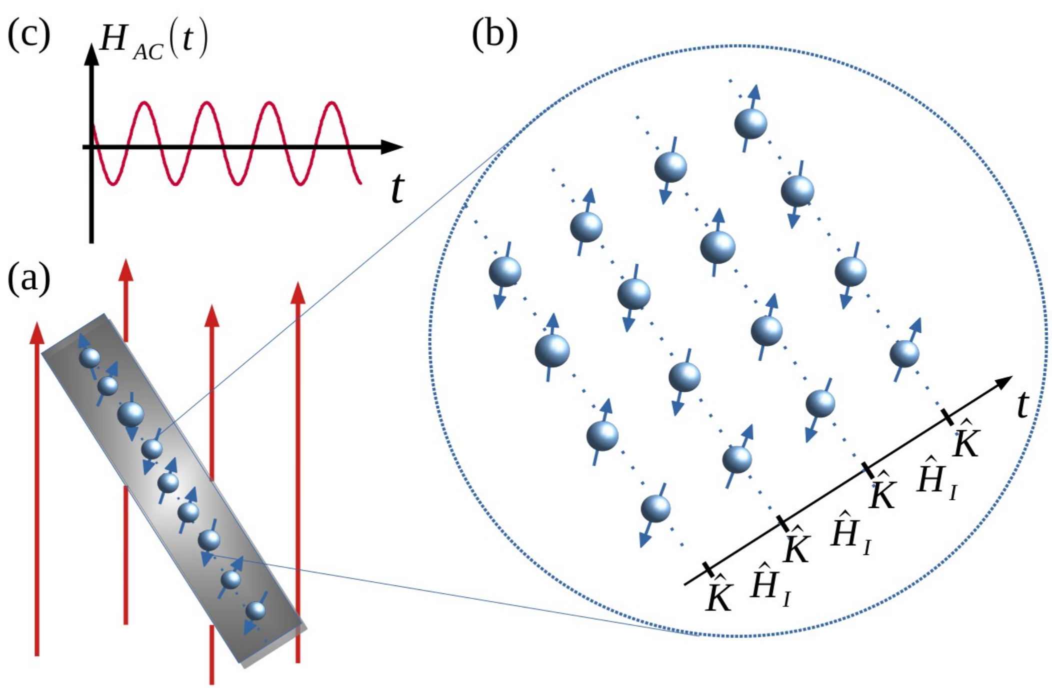

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(quantum Fisher information)

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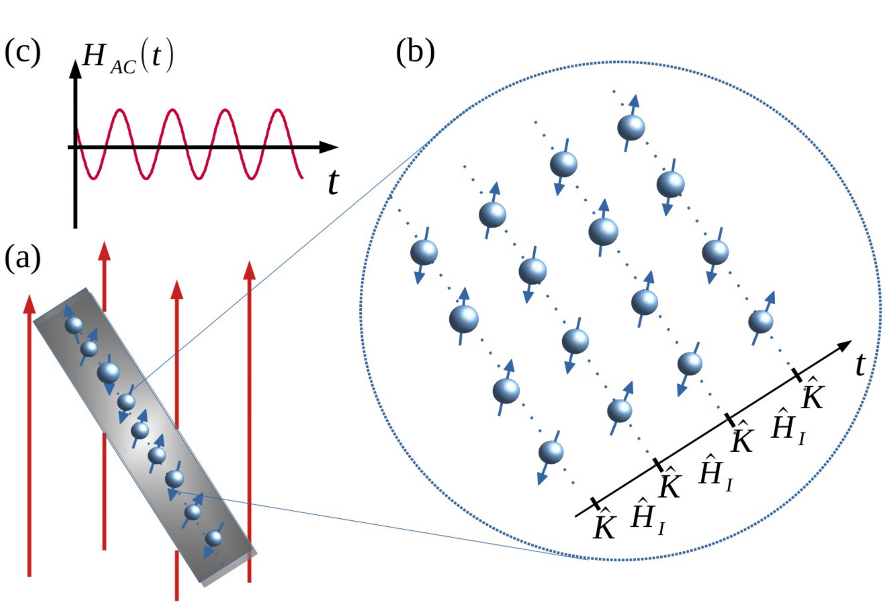

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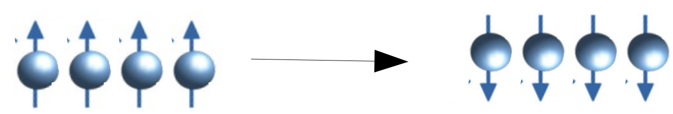
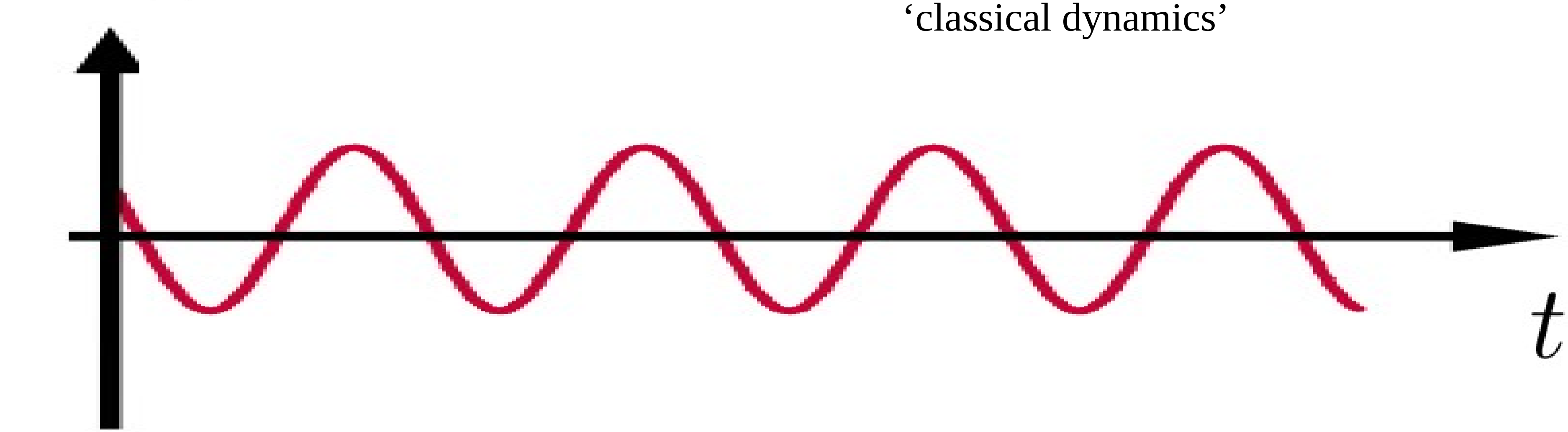
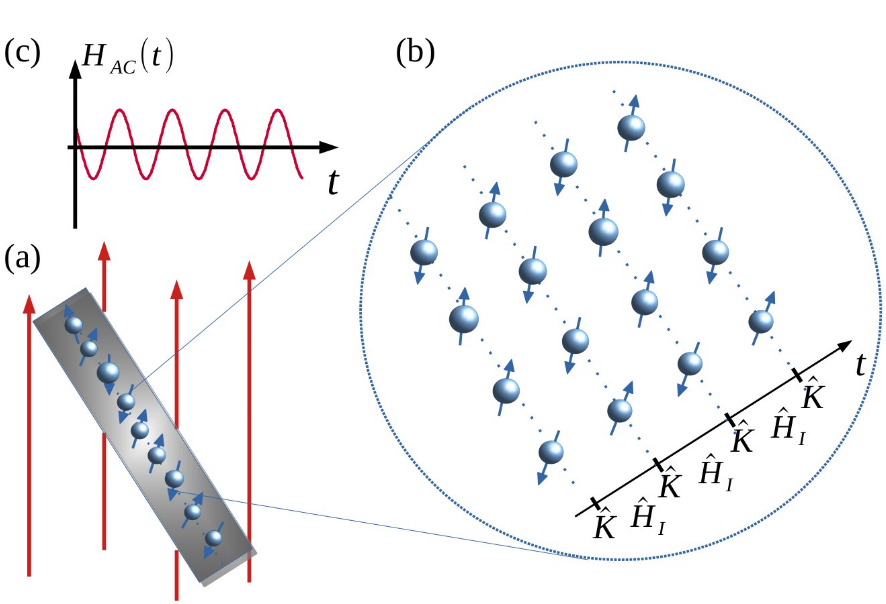
Case with $\phi = \pi,\ h_i^x = 0,\ h_{\mathrm{ac}} \to 0$



'classical dynamics'



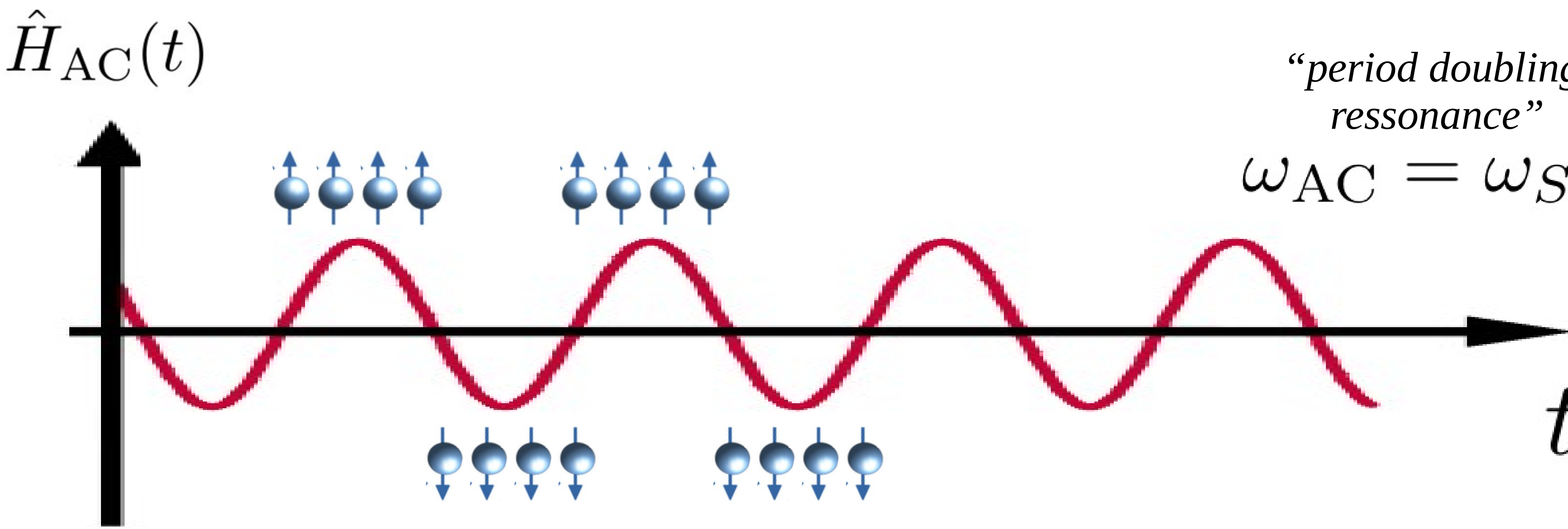
Floquet time-crystals as sensors of AC fields



$$\hat{H}_0(t) = \sum_i \left(J_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z + \sum_{\alpha=x,z} h_i^\alpha \hat{\sigma}_i^\alpha \right) + \\ + \sum_{n=1}^{\infty} \delta(t - nT_S)\phi \hat{S}^x,$$

$$\hat{H}_{\mathrm{AC}}(t) = h_{\mathrm{AC}} \sin(\omega_{\mathrm{AC}} t + \theta_{\mathrm{AC}}) \hat{S}_z$$

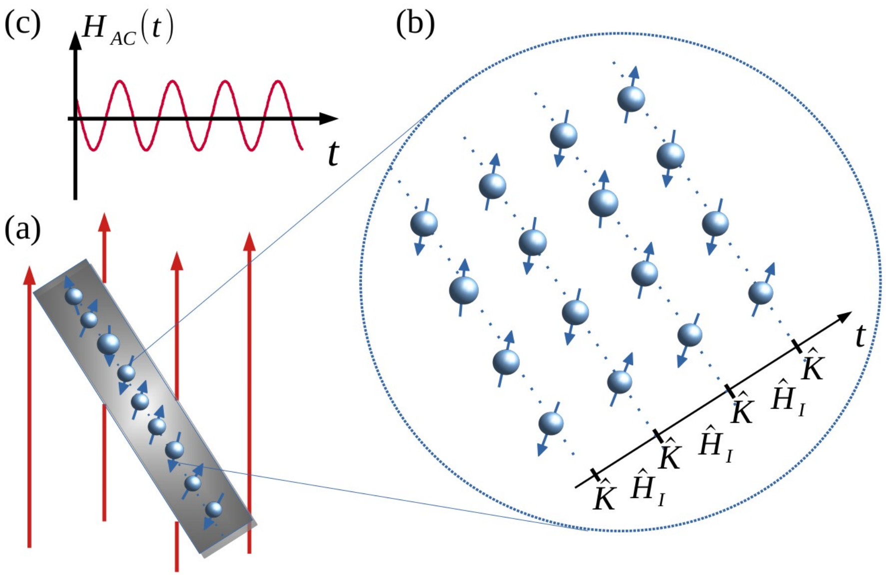
$\boldsymbol{Case\ with}\ \phi = \pi,\ h_i^x = 0,\ h_{\mathrm{ac}} \to 0$



"period doubling ressonance"

$$\omega_{\mathrm{AC}} = \omega_S / 2$$

Floquet time-crystals as sensors of AC fields



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$\boldsymbol{Case\ with}\ \phi = \pi,\ h_i^x = 0,\ h_{ac} \to 0$

. Fisher information: $F_{h_{ac} \to 0}(t) \sim \varphi(t)^2 \mathrm{var}(\hat{\mathrm{S}}_{\mathrm{z}}(0))$

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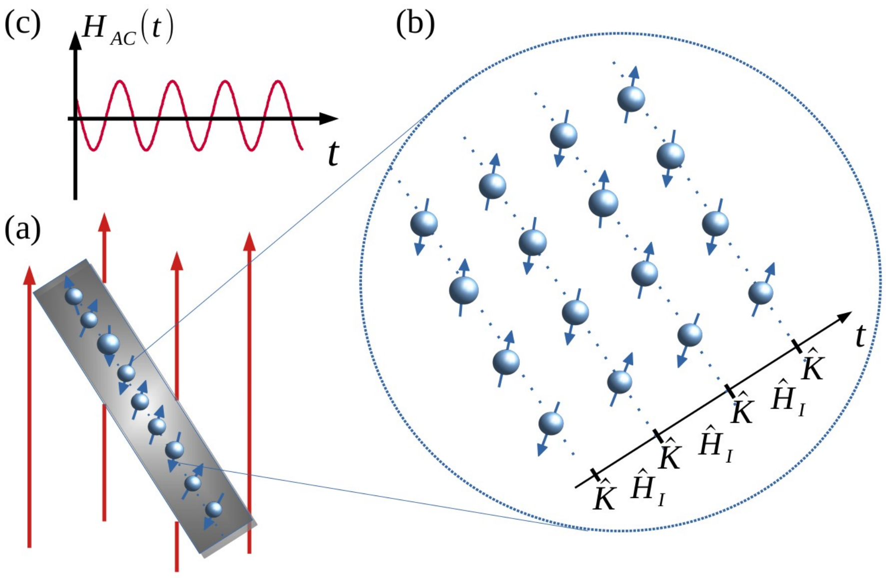

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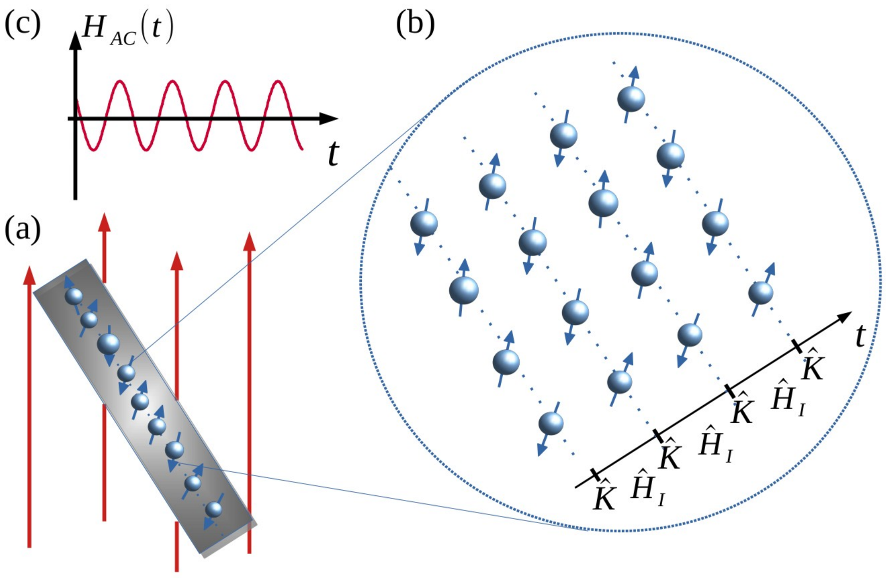
(i) sensor can be enhanced due to the correlations among its spins,

Floquet time-crystals as sensors of AC fields



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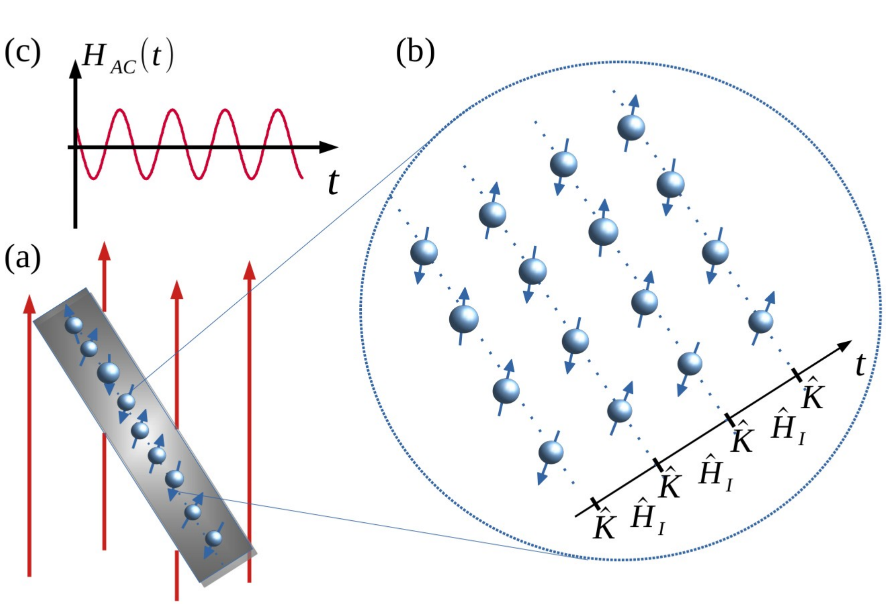
$$\varphi|_{\omega_S = 2\omega_{\mathrm{AC}}} \sim t \cos(\theta_{\mathrm{AC}})$$

(ii) optimal performance occurs for period-doubling resonance,

Floquet time-crystals as sensors of AC fields



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(i) sensor can be enhanced due to the correlations among its spins,

$$\varphi|_{\omega_S = 2\omega_{\mathrm{AC}}} \sim t \cos(\theta_{\mathrm{AC}})$$

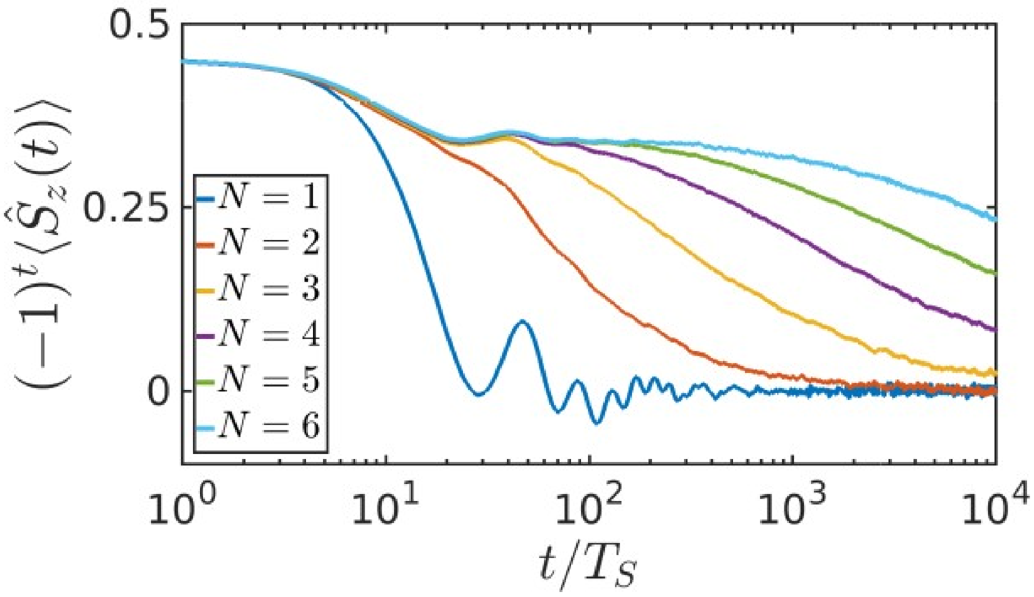
(ii) optimal performance occurs for period-doubling resonance,

$$\varphi|_{\omega_S = 2\omega_{\mathrm{AC}}+\epsilon} : \quad t^* \sim \theta_{AC}/\epsilon$$

(iii) off-resonance time scale for the optimal growth of Fisher information.

General case: $\phi \neq \pi \;\; h_i^x \neq 0$

(linear response $h_{\mathrm{AC}} \to 0$)



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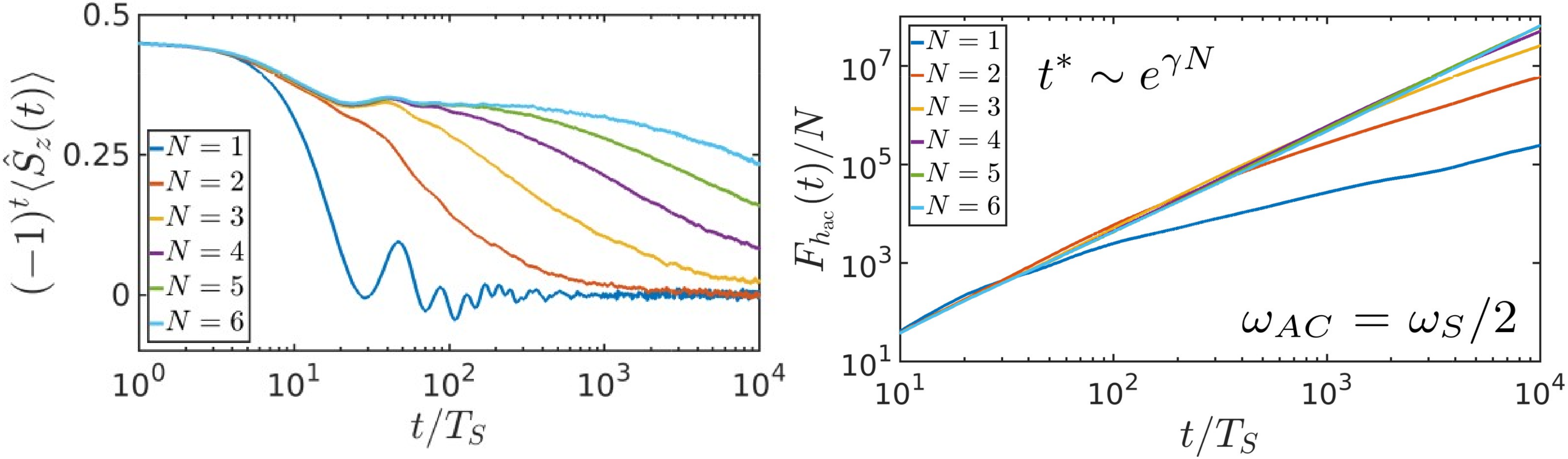

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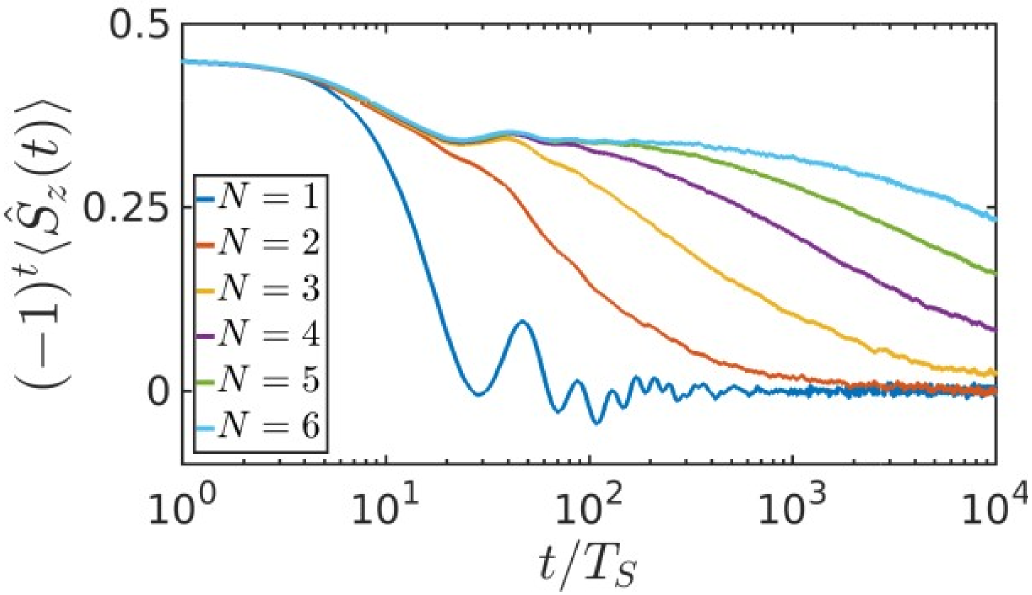

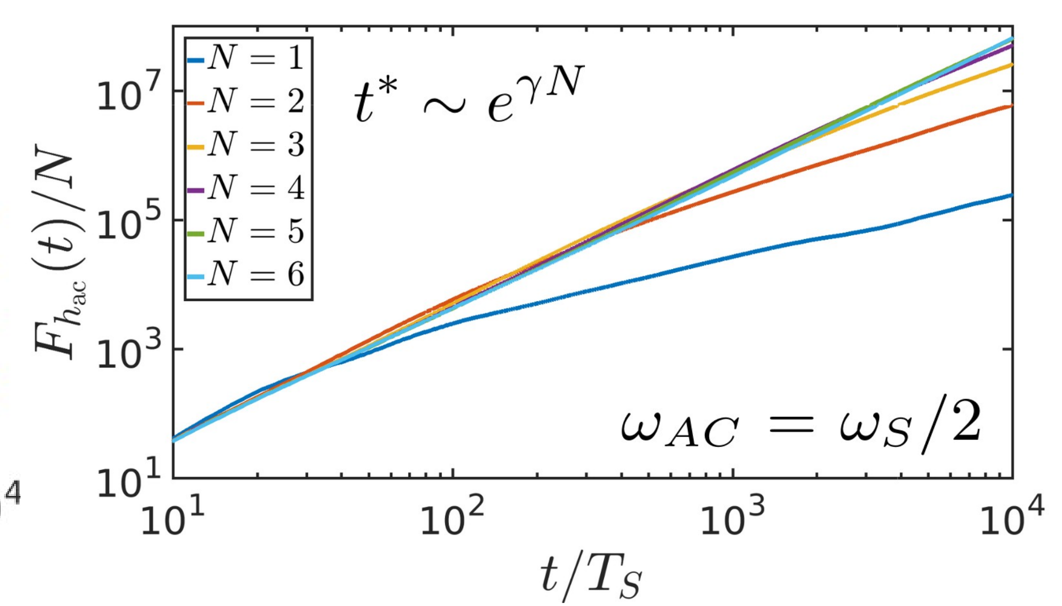

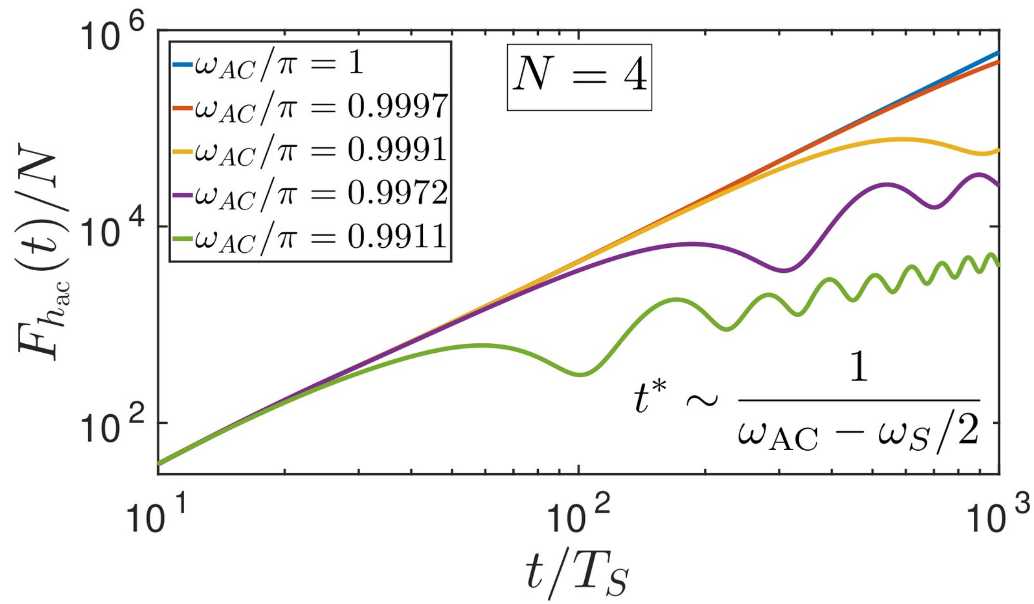

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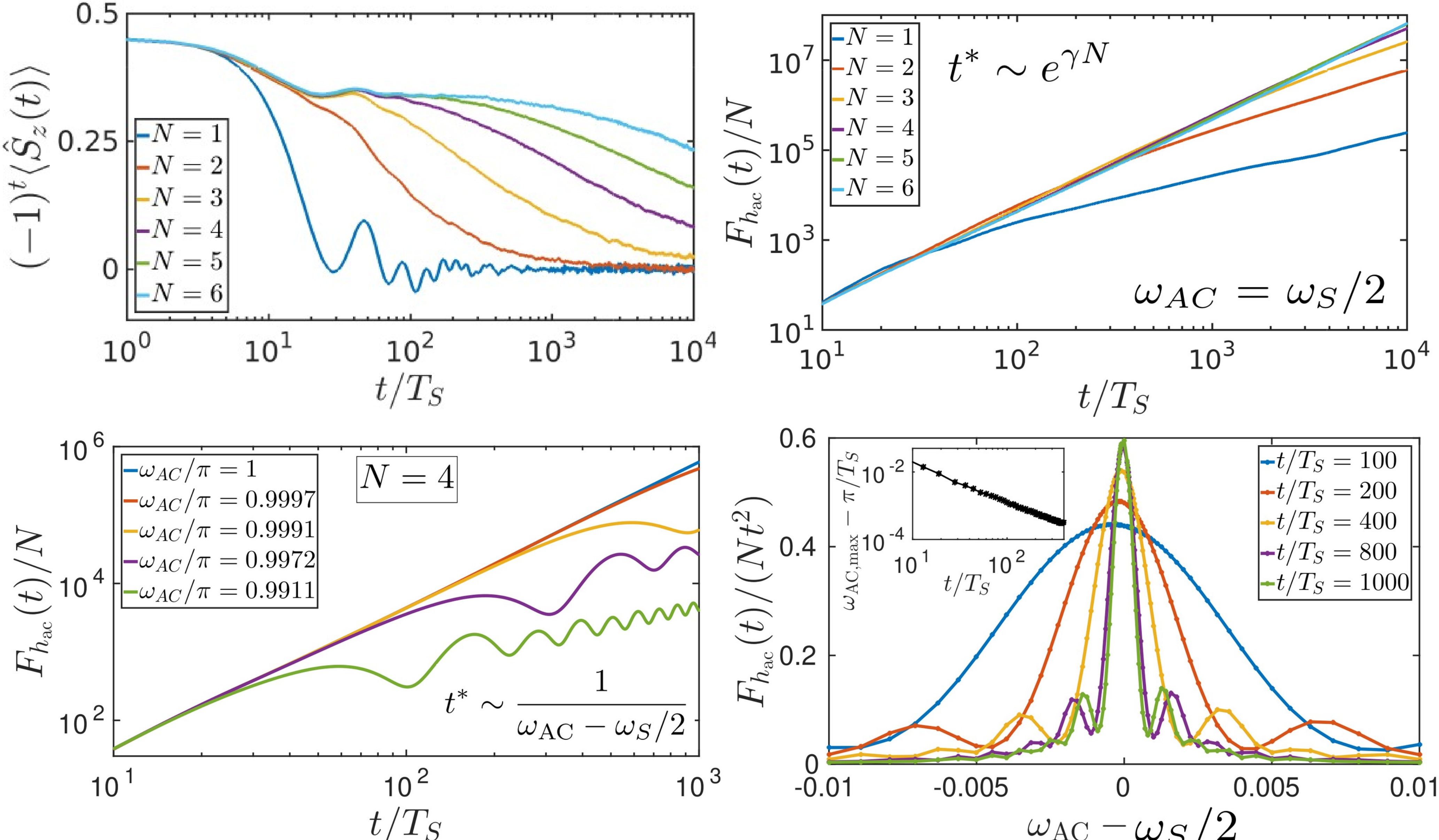
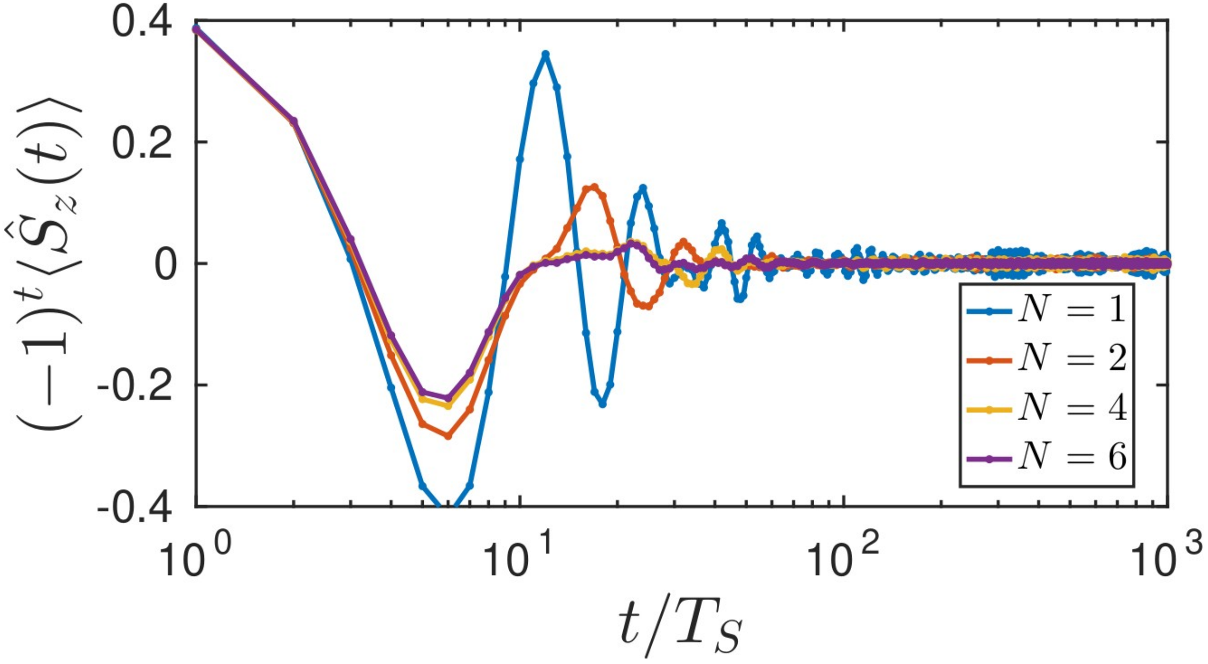


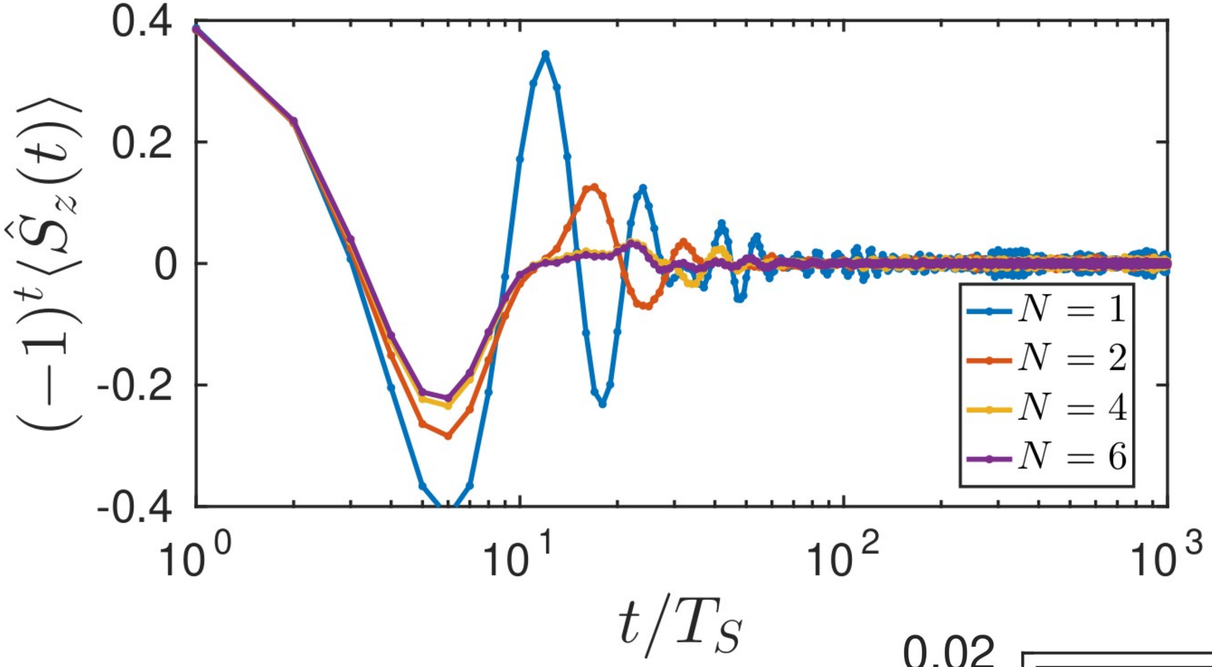
$$\max_{\omega} F_{h_{\mathrm{ac}}}(t) \sim t^{\beta},\; \beta > 2$$

Ergodic Sensor



- fast thermalization time ~ O(T);

Ergodic Sensor



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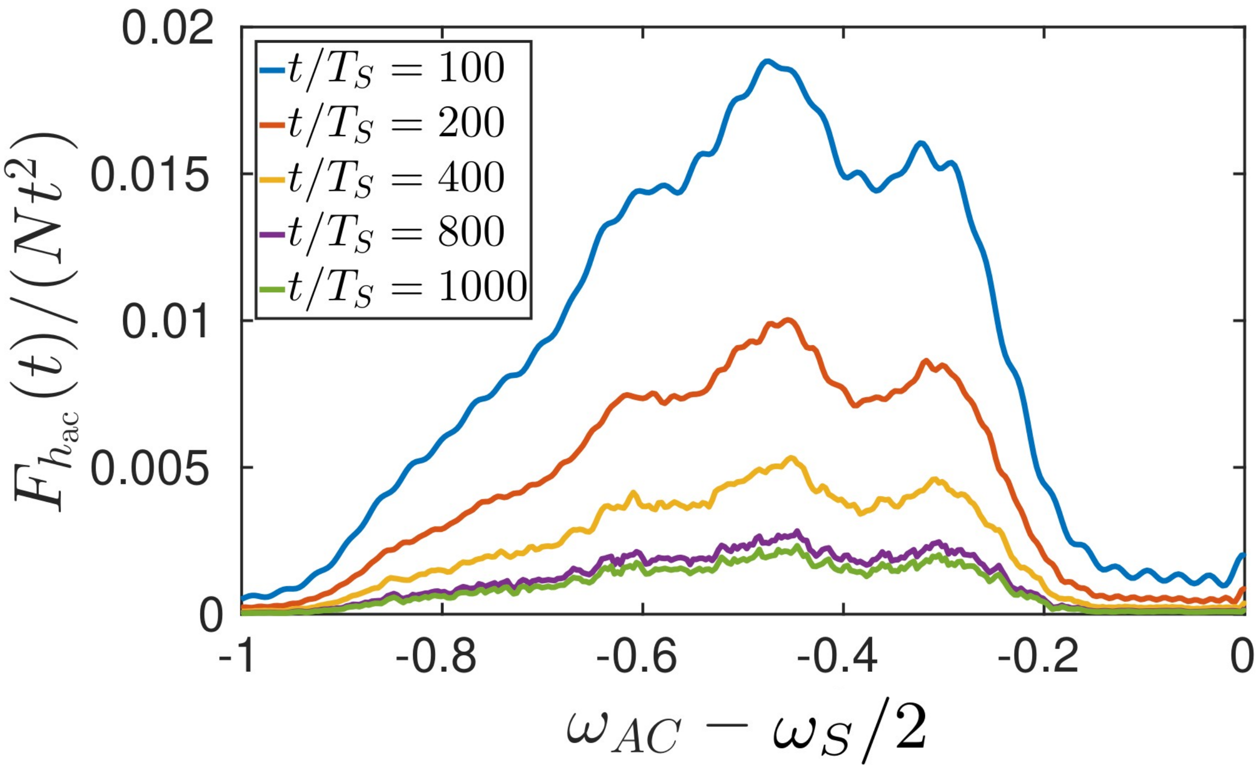
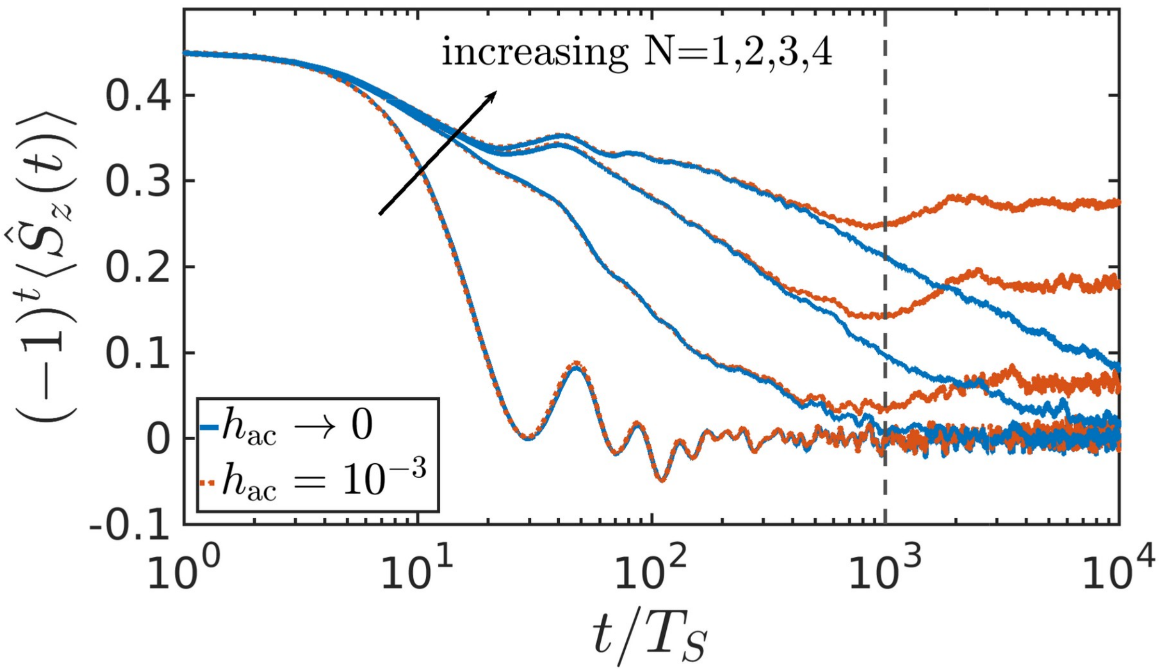


- noisy response;

- below standart (classical) limit for longer times;

beyond linear response, $h_{\mathrm{AC}} \neq 0,\ \omega_{AC} = \omega_S/2$

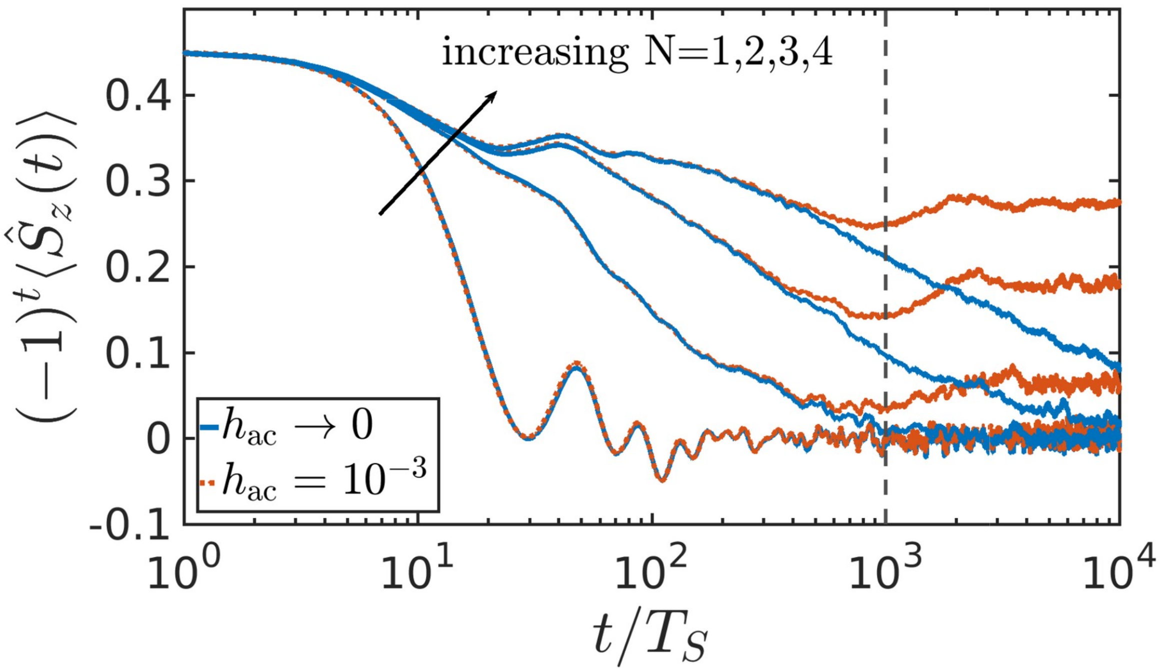


- sets a characteristic time scale,

$$t_{AC} \sim h_{\mathrm{ac}}^{-1}$$

(dominant AC fields over dynamics)

beyond linear response, $h_{\rm AC} \neq 0,\ \ \omega_{AC} = \omega_S/2$



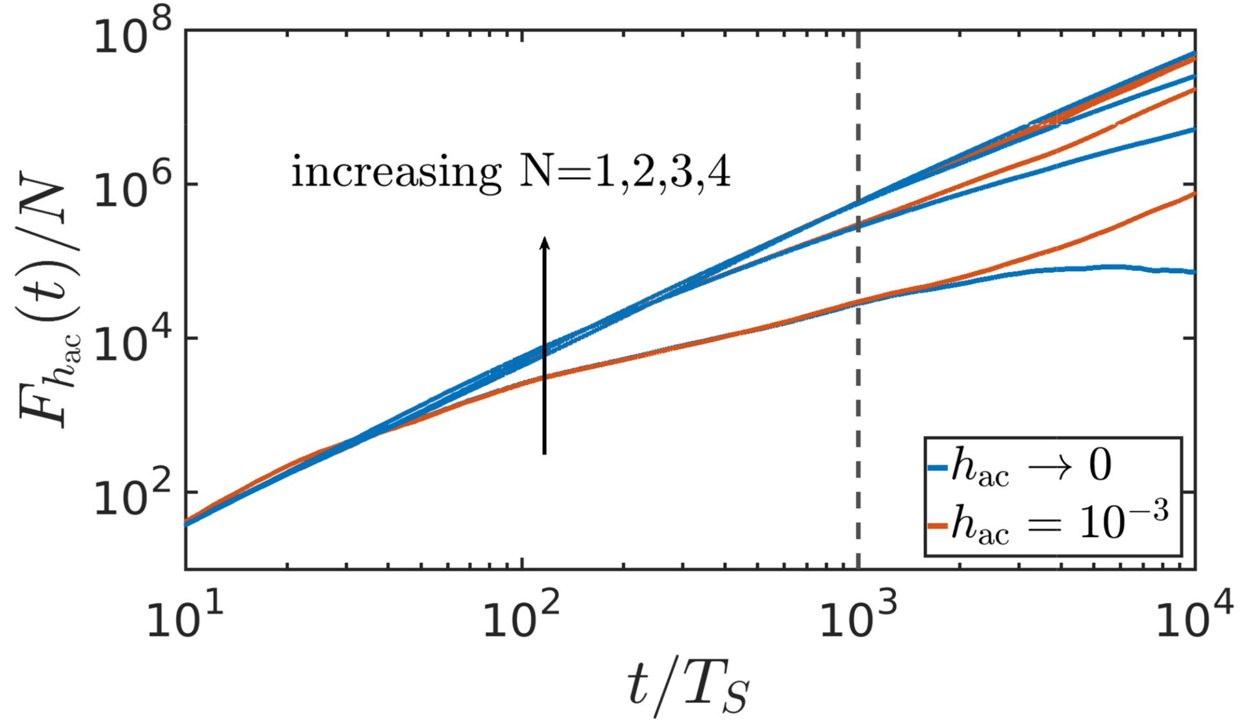
- sets a characteristic time scale,

$$t_{AC} \sim h_{\rm ac}^{-1}$$

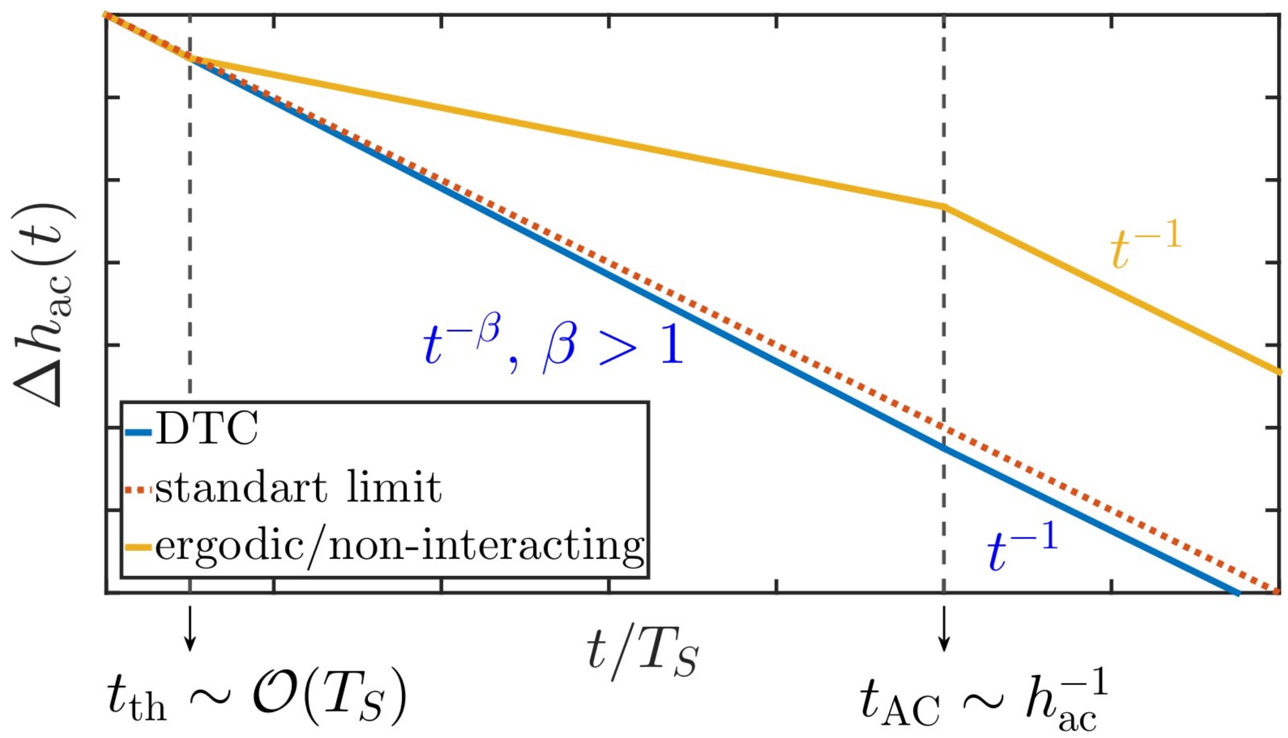
(dominant AC fields over dynamics)

- recovers Fisher quadratic growth,

$$F_{h_{\rm ac}}(t) \sim t^2$$



Floquet time-crystals as sensors of AC fields



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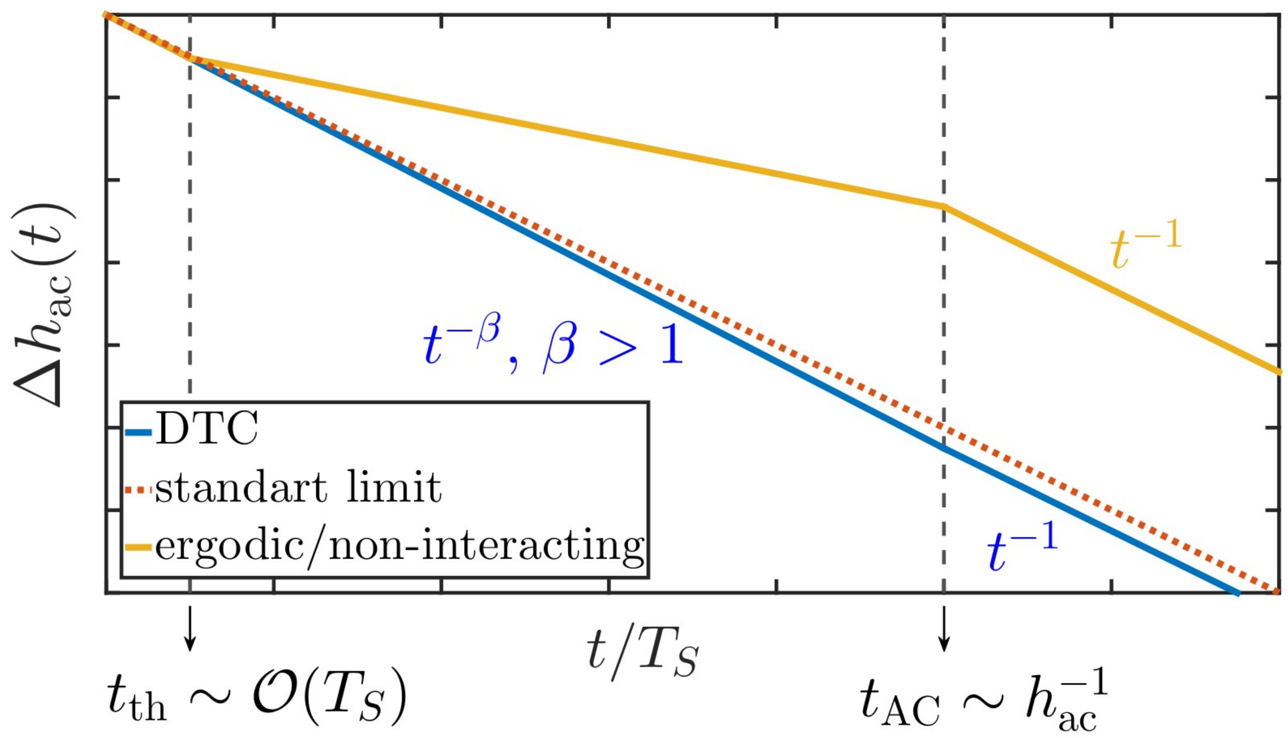


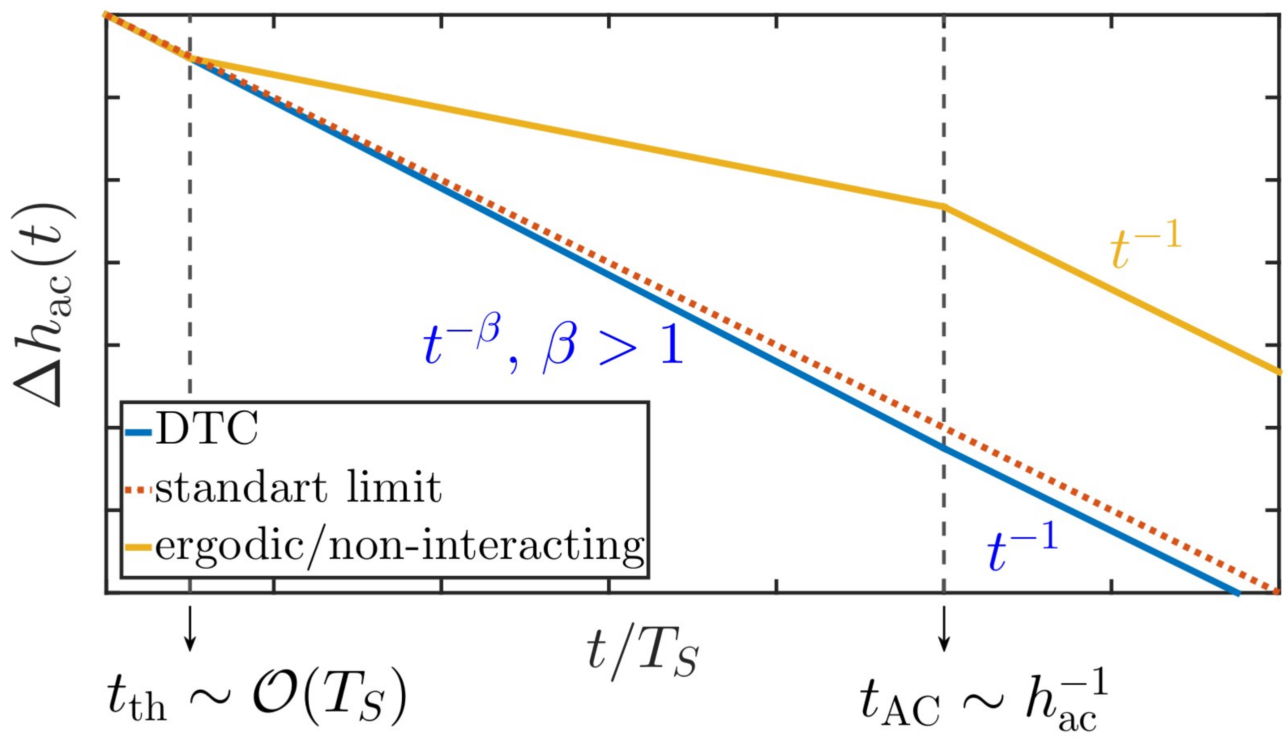
In summary, Discrete Time Crystal sensors offer several advantages:

- **slow heating** – exponentially slow with system size N

- useful quantum correlations - **overcome classical limit;**

- **robust** protocol.

Floquet time-crystals as sensors of AC fields



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