# Dissipation induced non-stationary complex quantum dynamics

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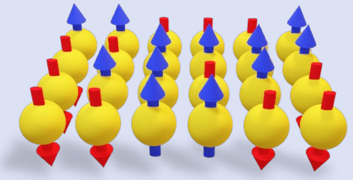
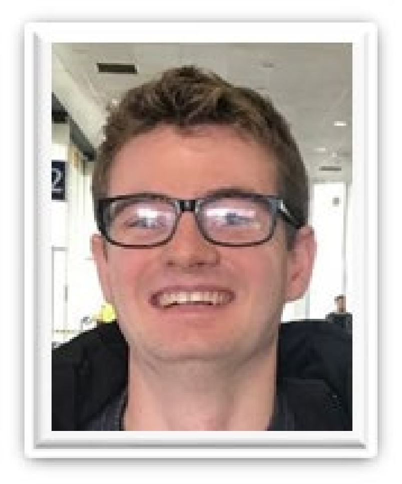
& University of Oxford



SPICE workshop, 27 June 2023

# Collaborators



Joey Tindall
PhD, PostDoc → Flatiron

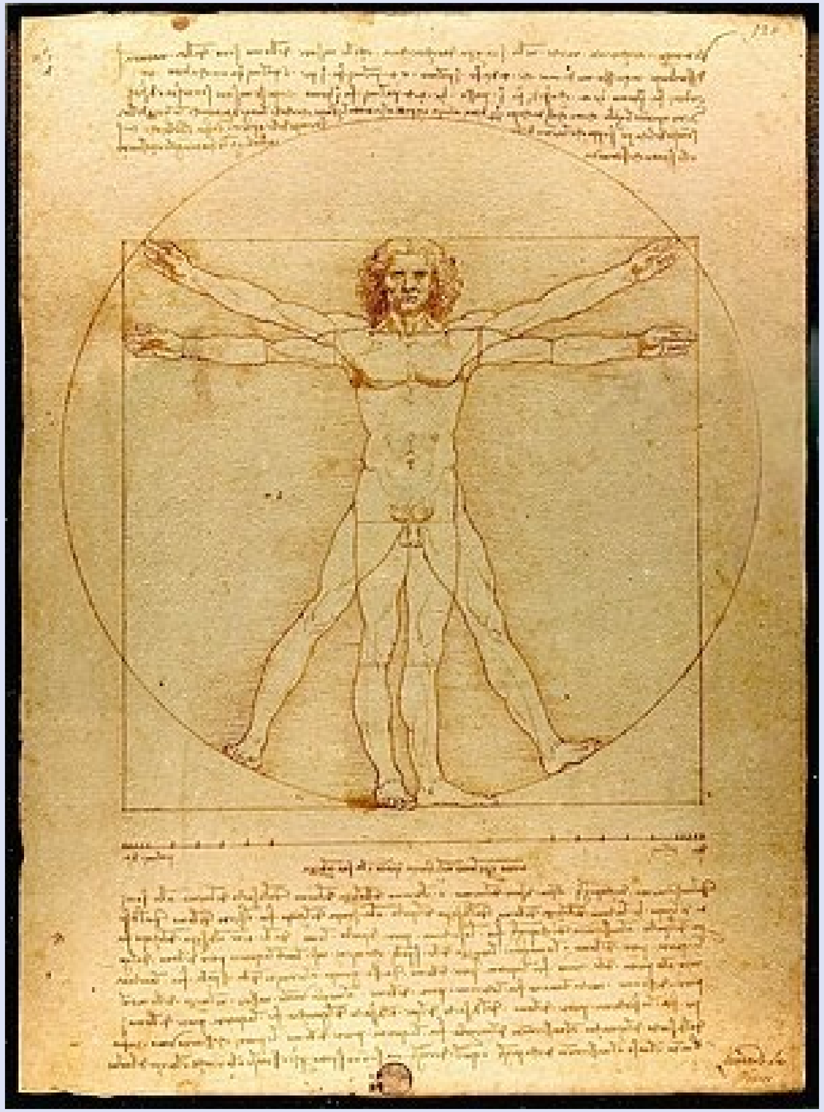
Berislav Buca
PostDoc → Copenhagen

Cameron Booker
PhD → industry

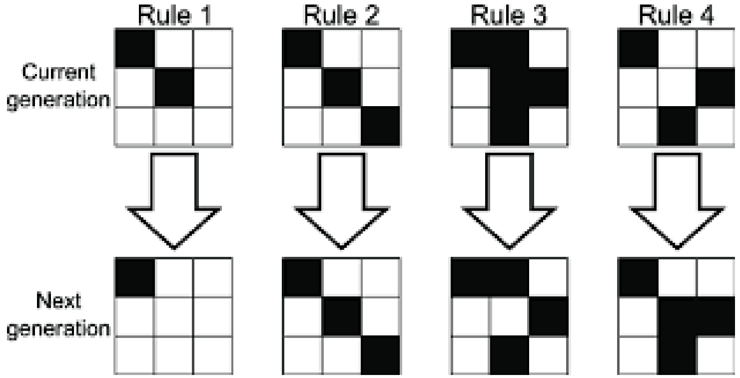
Carlos Sanchez-Munoz
PostDoc → Spain

- *Algebraic Theory of Quantum Synchronization and Limit Cycles under Dissipation,* B. Buca, C. Booker and D. Jaksch, SciPost Physics **12**, 097 (2022).
- *Dynamical order and superconductivity in a frustrated many-body system,* J Tindall, F Schlawin, M Buzzi, D Nicoletti, JR Coulthard, H Gao, A Cavalleri, MA Sentef, D Jaksch, Physical Review Letters **125**, 137001 (2020).
- *Complex coherent quantum many-body dynamics through dissipation,* B. Buca, J. Tindall, D. Jaksch, Nature Communications **10**, 1730 (2019).

# Complex systems – no stationarity



# Conway’s game of life




**Simple rules → complex dynamics**

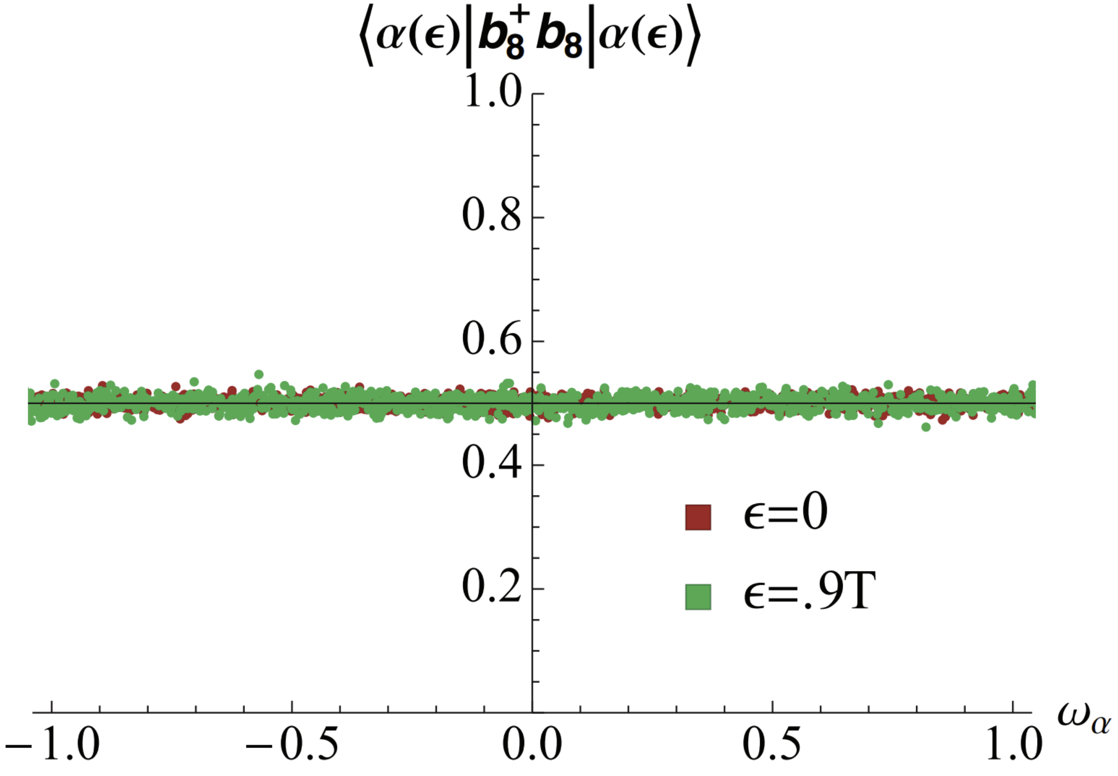
from: http://web.stanford.edu/~cdebs/GameOfLife/

# Quantum systems – driven

- Closed system driven by external field

- *"… here we show that for generic nonintegrable interacting systems, … Floquet eigenstates of the driven system at quasienergy $\omega_\alpha$ consist of a mixture of the exponentially many eigenstates of the undriven Hamiltonian, which are thus drawn from the entire extensive undriven spectrum."*

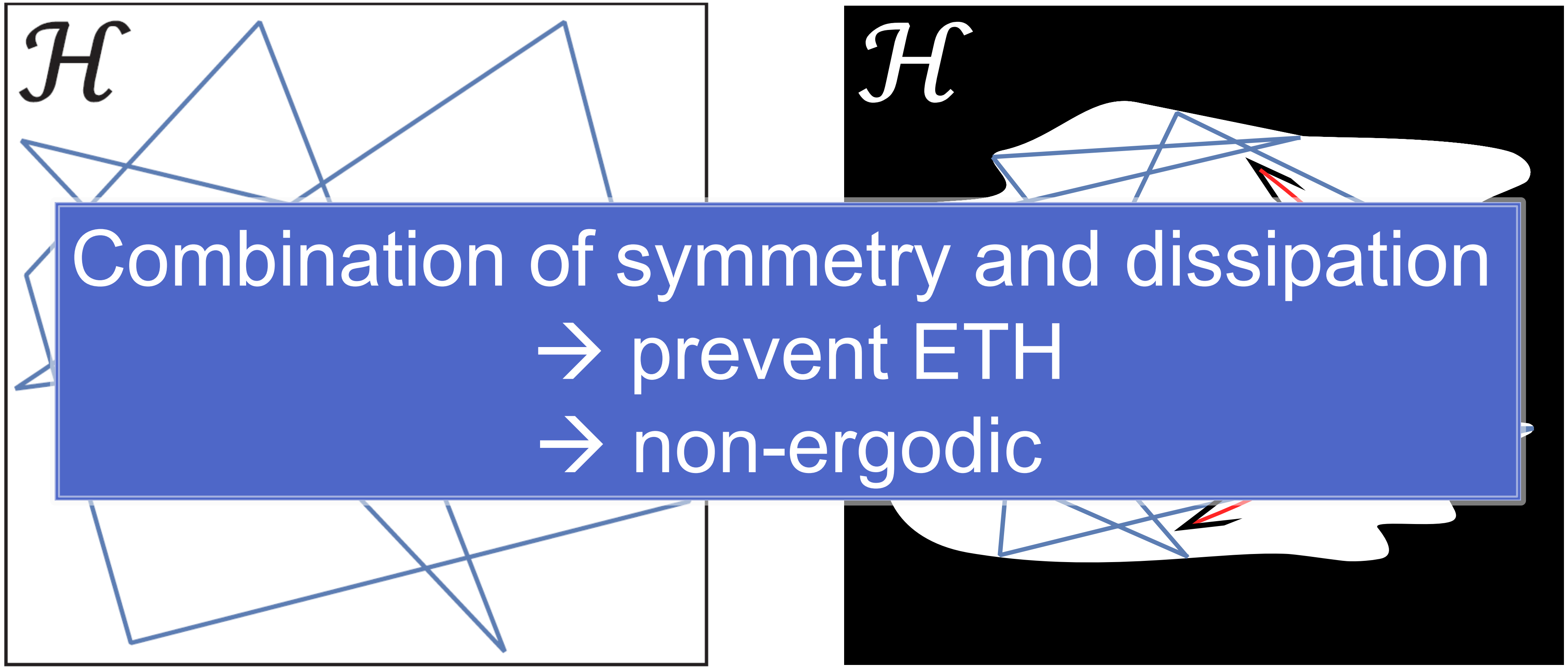
- A. Lazarides, A. Das, and R. Moessner, Phys. Rev. E **90**, 012110 (2014)




7 driven hardcore bosons on 14 lattice sites

Population in 8-th site Floquet eigenstate

# Non-stationary quantum matter




Combination of symmetry and dissipation
→ prevent ETH
→ non-ergodic

Can we dynamically engineer non-equilibrium stable many-body quantum states?

# Eigenvalues $\lambda_i$ and eigenstates $\rho_i$ of the dynamics

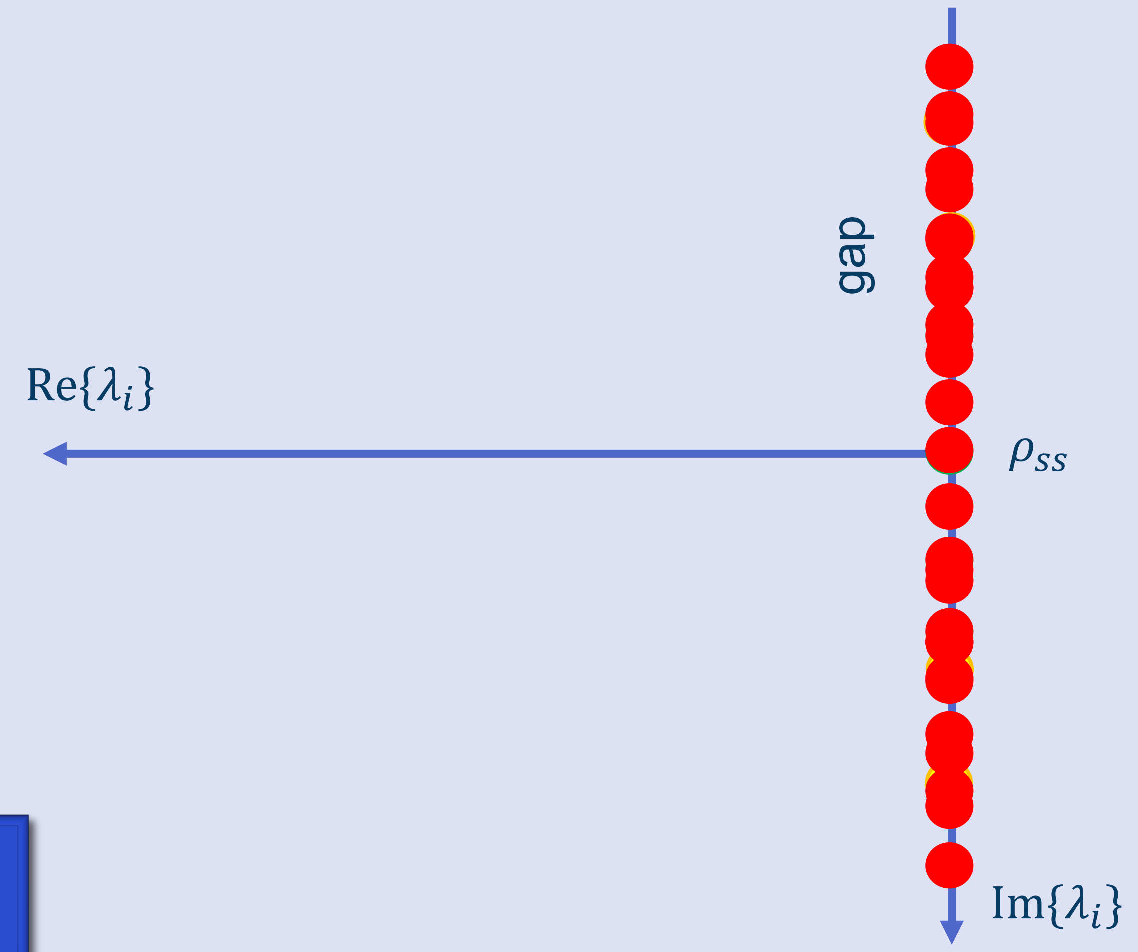
$$\dot{\rho} = \mathcal{L}\rho = -\mathrm{i}[H, \rho]$$
$$+\sum_\mu \left(2\, L_\mu\, \rho\, L_\mu^\dagger - L_\mu^\dagger L_\mu\, \rho - \rho L_\mu^\dagger L_\mu\right)$$

$$\rho_i(t) = e^{\lambda_i t} \rho_i$$

**Closed system**
- purely imaginary $\lambda_i$
- dephasing

**Open system**
- negative real parts
- dissipation

Generic condition for ●
$[H, A] = -\lambda A$ and $\left[A, L_\mu^\dagger\right] = \left[A, L_\mu\right] = 0 \quad \forall \mu$



# Generic condition

- If there exists a lowering operator $A$ with the following properties

$$[H, A] = -\lambda A \quad \text{and} \quad [A, L_\mu^\dagger] = [A, L_\mu] = 0 \quad \forall \mu$$

with $\lambda \neq 0$ then $\rho_{nm}$ of the form (with integer $m, n > 0$, $m \neq n$)

$$\rho_{nm} = \mathrm{A}^n \rho_{00} (A^\dagger)^m$$

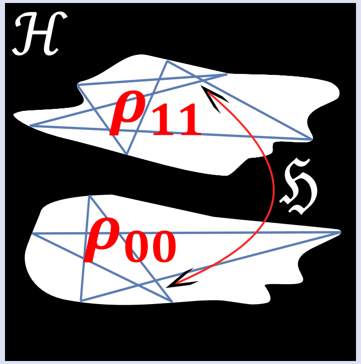
with $\rho_{00}$ a stationary state evolves according to

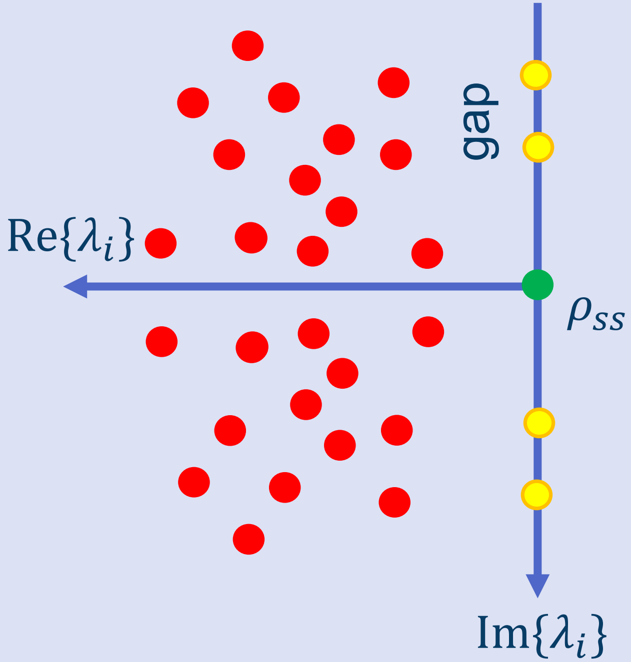
$$\mathcal{L}\rho_{nm} = \mathrm{i}(m-n)\lambda\rho_{nm}$$

- The “mixed coherences” are not necessarily decoupled from the environment

$$L_\mu \rho_{nm} L_\mu^\dagger \neq 0$$

- The combination of system and environment drives the long-term dynamics among the states on the imaginary axis.



# Generalized Gibbs ensemble

- In general we may have a set of dynamical symmetries $A_j$ with values $\lambda_j$.
- This includes conventional symmetries with a value of $\lambda_j = 0$.
- In the long-time limit we can assume that the system evolves to a maximum entropy state consistent with the presence of dynamical symmetries.
- This state is a generalized Gibbs ensemble

$$\rho(t \to \infty) = \frac{1}{Z} \exp\left(-\sum_j \mu_j \exp\left(-\mathrm{i}\lambda_j t\right) A_j + \mathrm{h.c.}\right)$$

- The constant amplitudes $\mu_j$ are determined by the initial state.
- Note that the operators $A_j$ are not necessarily extensive observables.

# Example: driven superconductivity

# Systems with $\eta$-pairing symmetry

- $\eta$-pairing symmetry

$$\eta^z = \frac{1}{2}\sum_j (n_j - 1)\,,$$

$$\eta^+ = \sum_j \tau(j)\eta_j^+, \quad \eta_j^+ = c_{j,\uparrow}^\dagger\, c_{j,\downarrow}^\dagger,$$

$$\eta^- = \sum_j \tau(j)\eta_j^-\,, \qquad \eta_j^- = c_{j,\downarrow}\, c_{j,\uparrow},$$

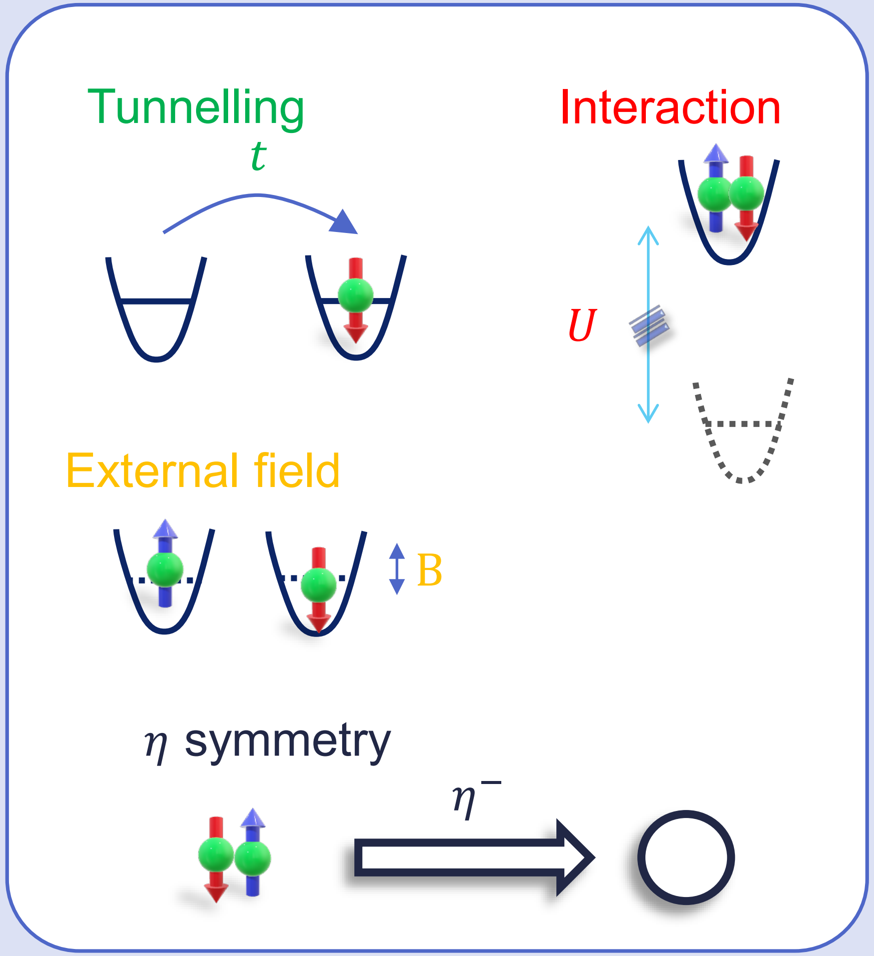
with $\tau(j)$ a chequerboard pattern.

- They fulfil

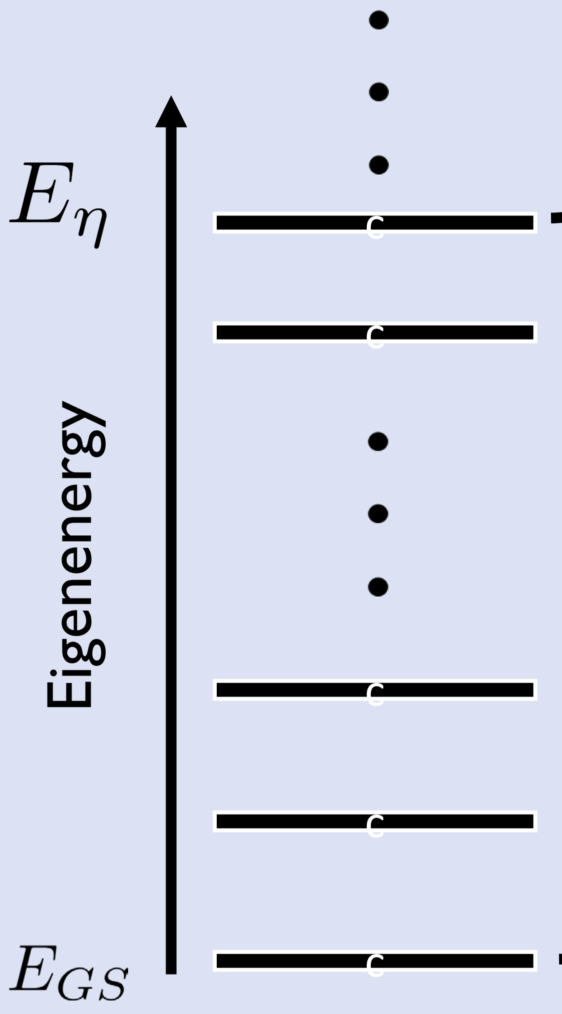
$$[H_{\text{Hub}}, \eta^z] = 0, \qquad [H_{\text{Hub}}, \eta^\pm] = \pm\, 2\mu\eta^\pm$$

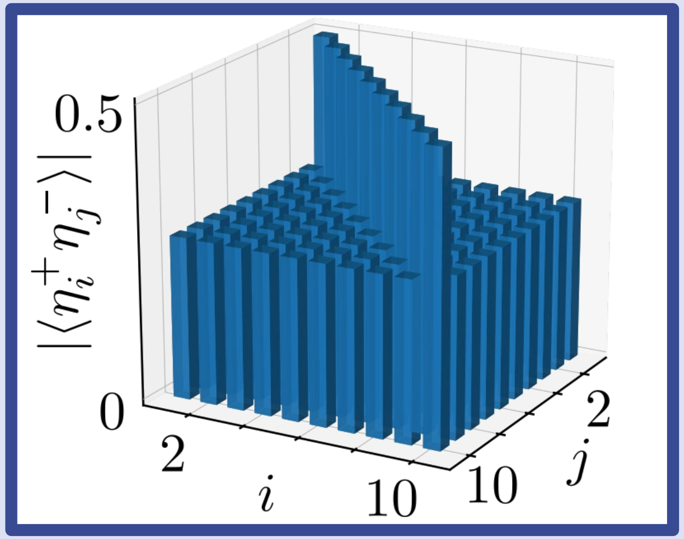
For instance

$$H_{\text{Hub}} = -t \sum_{\langle i,j\rangle,\sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow}n_{i\downarrow} + \frac{B}{2}\sum_i n_{i\uparrow} - n_{i\uparrow} - \mu \sum_i n_i$$



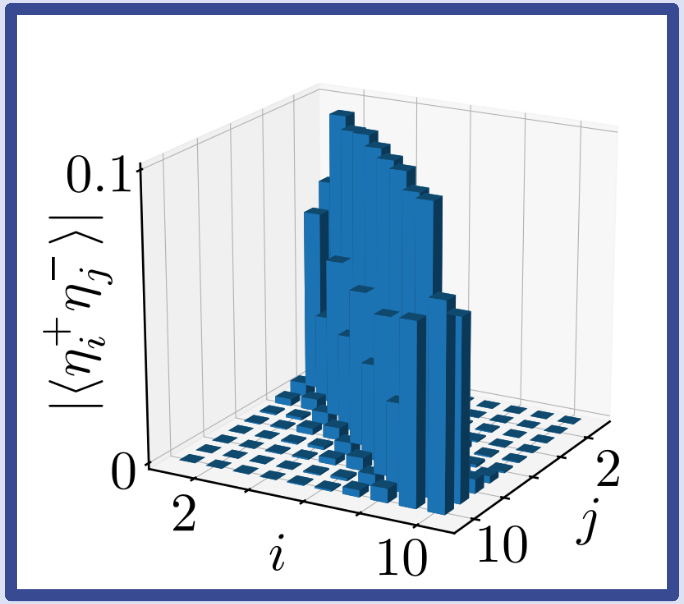
# $\eta$ – correlations in the Hubbard model



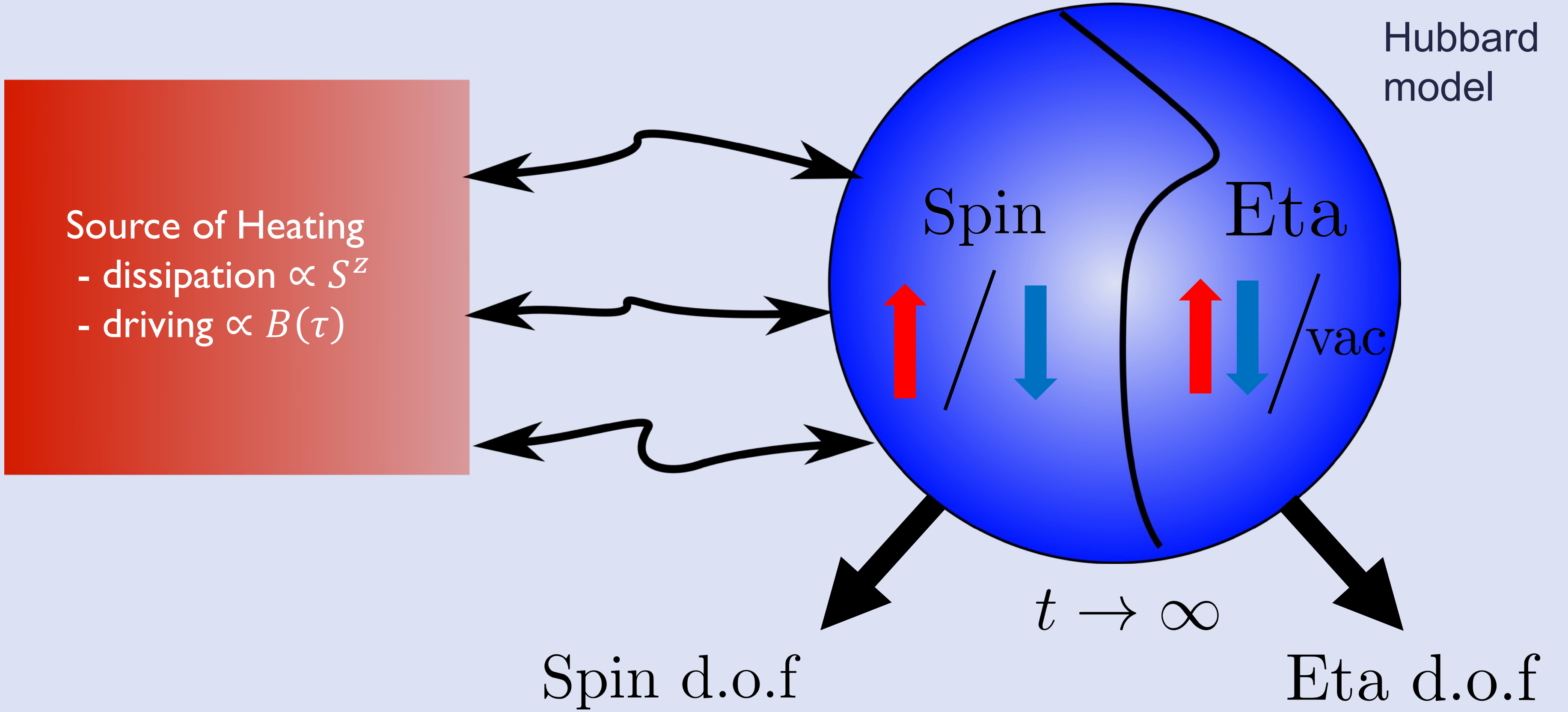

- High energy
- Off-diagonal long-range order
- Superconducting

*C. N. Yang, PRL **63**, 2144 (1989)*




- **Competing order**
- Decaying correlations

# Spin-heated Hubbard model

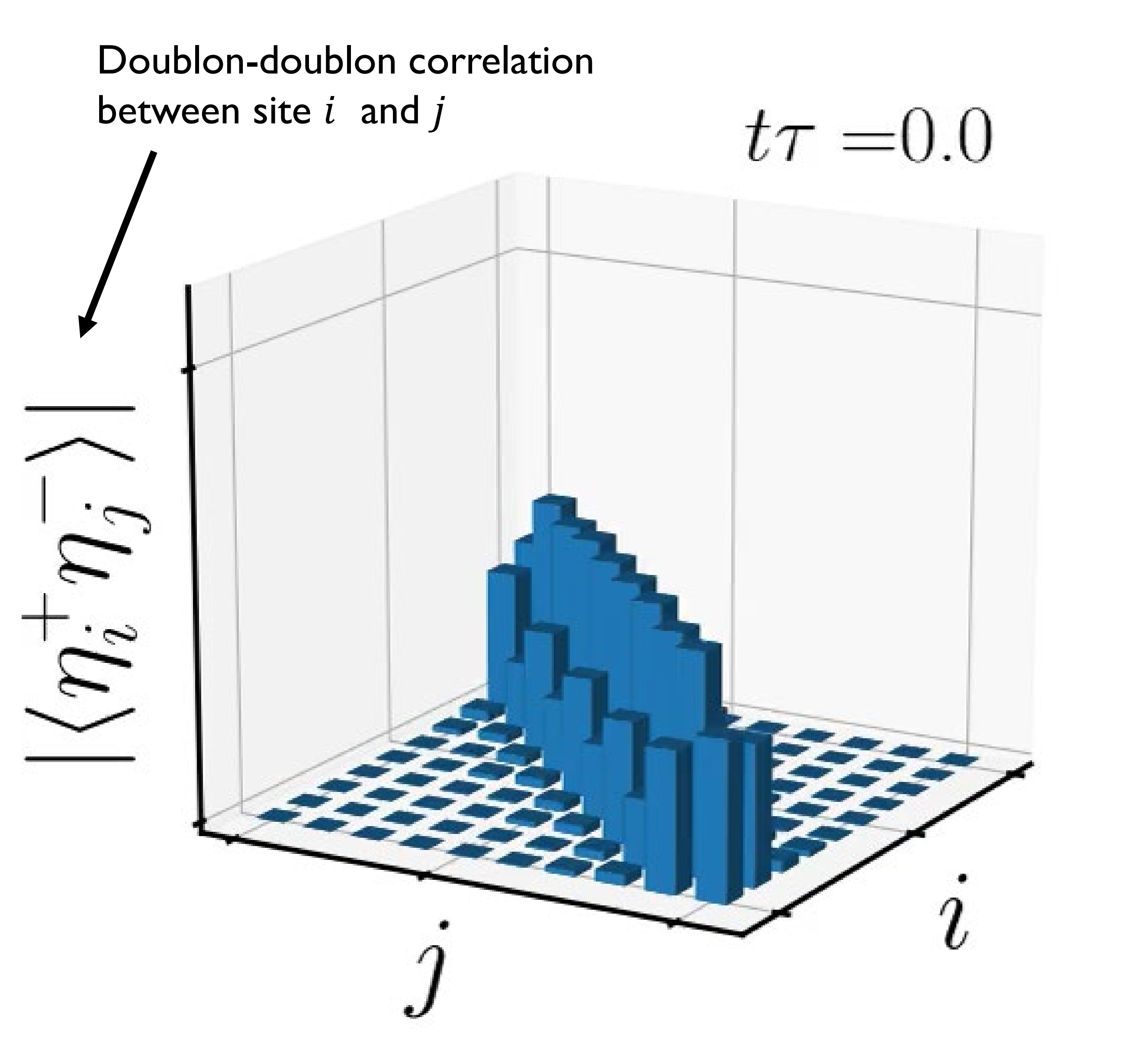



- High Temperature

- Off-diagonal long-range order

*Heating-Induced Long-Range $\eta$-Pairing in the Hubbard Model,* J. Tindall, B. Buca, J.R. Coulthard, DJ, PRL **123**, 030603 (2019)

# Spin-heating the Hubbard model from the ground state




- Ground state of the Hubbard model / Uniform off-diagonal correlations
- Superconducting correlations / Decaying order
- No off-diagonal order / Survives indefinitely

Parameters:
$U = t$

# Stationary State

- The quantum jump operators $L_\mu \propto n_\mu$ are Hermitian and thus the identity operator $\mathbb{I}$ is a stationary state of the dynamics

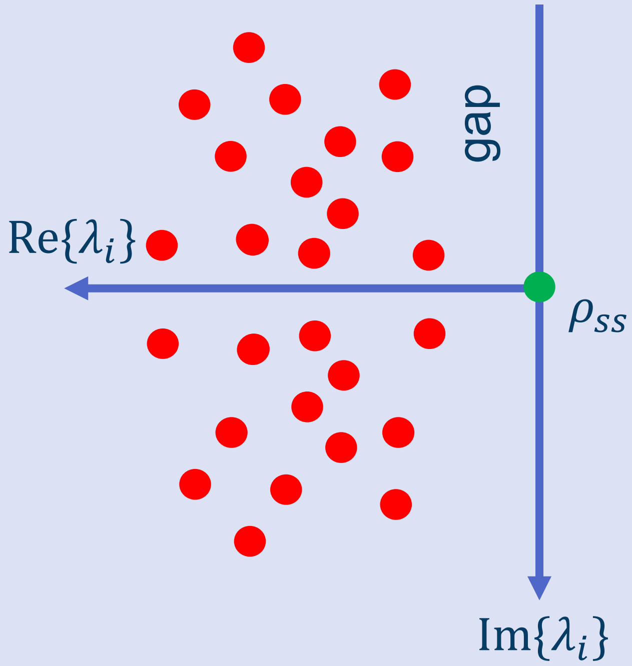
$$\mathcal{L}\mathbb{I} = -\mathrm{i}[H, \mathbb{I}] + \sum_\mu \left(2\, L_\mu \mathbb{I} L_\mu^\dagger - L_\mu^\dagger L_\mu \mathbb{I} - \mathbb{I} L_\mu^\dagger L_\mu\right) = 0$$

- Conserved quantities are $N_\downarrow$, $N_\uparrow$, and $\eta^+\eta^-$ and we assume a stationary state of the form (with Lagrange parameters $\beta_i$ fixing them)

$$\rho_{ss} \propto \exp(\beta_1 N_\downarrow + \beta_2 N_\uparrow + \beta_3 \eta^+\eta^-)$$

- Since in the stationary state $\rho_{ss}$ we have $[\rho_{ss}, P_{i,j}] = 0$, where $P_{i,j}$ swaps two lattice sites we can show that

$$Tr\{\rho_{ss}, \eta_i^+ \eta_{i+j}^-\} = \text{const.}$$




J. Tindall, F. Schlawin, M. Sentef and D. Jaksch, *Analytical Solution for the Steady States of the Driven Hubbard model*, PRB**103**, 035146 (2021).

# Lattice imbalance and $\eta$ condensates

## Lieb Theorem

$$\langle S^+ S^- \rangle \propto N^2$$

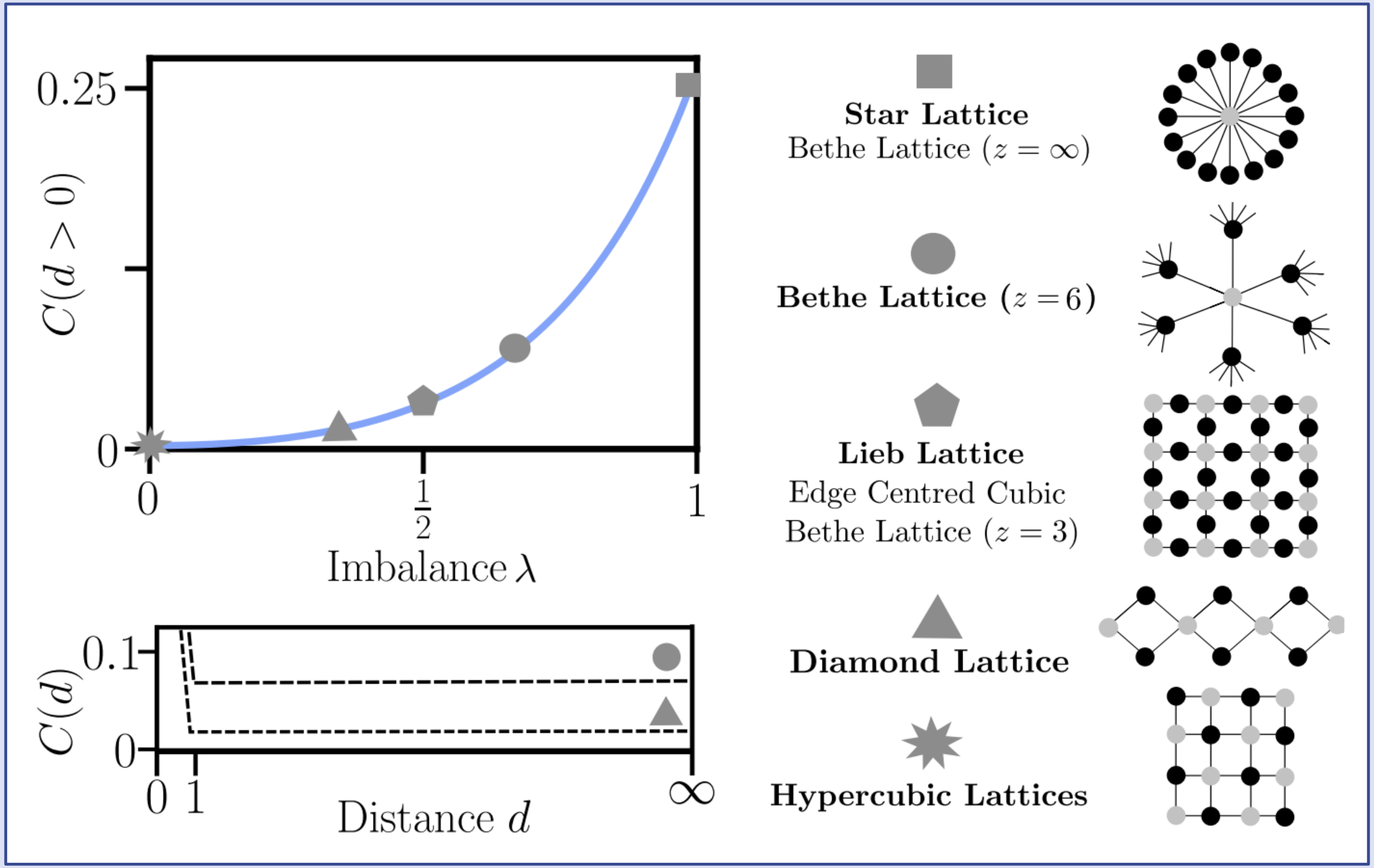
in unbalanced lattices with

$$N_a \neq N_b$$

and total number of lattice sites

$$N = N_a + N_b$$

Driving an unbalanced lattice thus leads to long-range $\eta$ correlations in the long-term

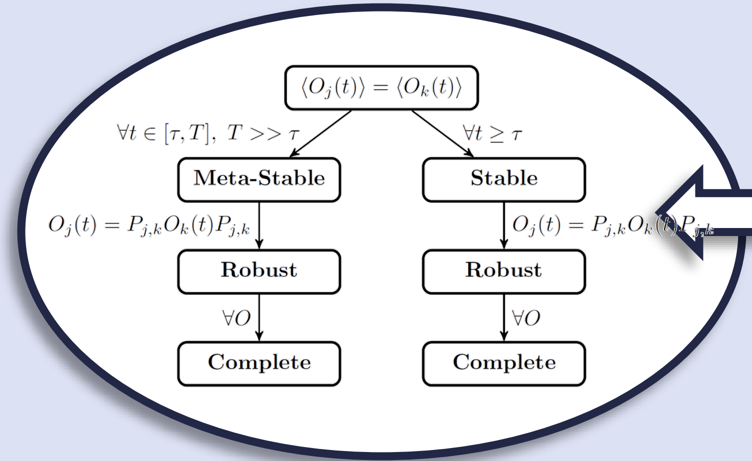



J. Tindall, F. Schlawin, M. Sentef and D. Jaksch, *Lieb Theorem and Maximum Entropy Condensates*, preprint arXiv:2103.04687.

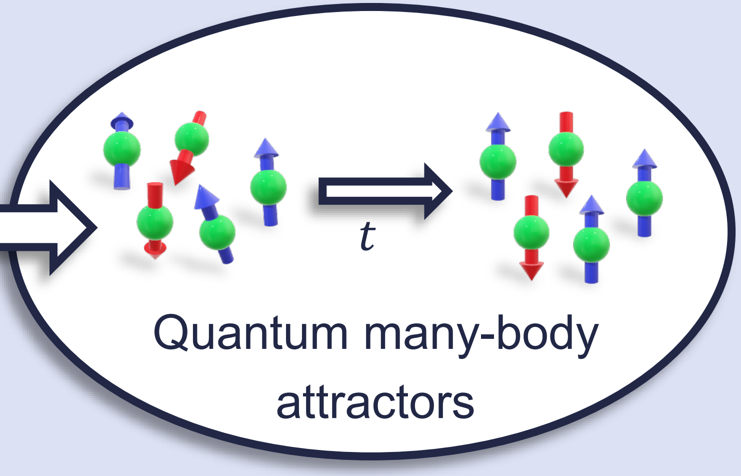
# Summary: Dynamical symmetries

$$[H, A] = -\lambda A$$

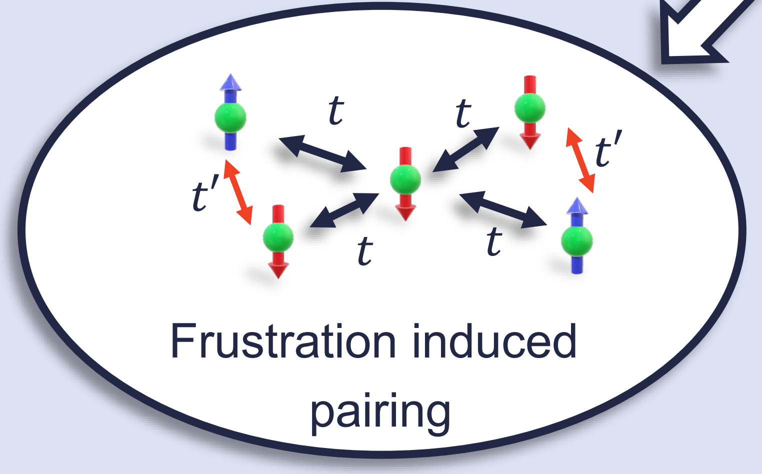
$$[A, L_\mu^\dagger] = [A, L_\mu] = 0 \quad \forall \mu$$



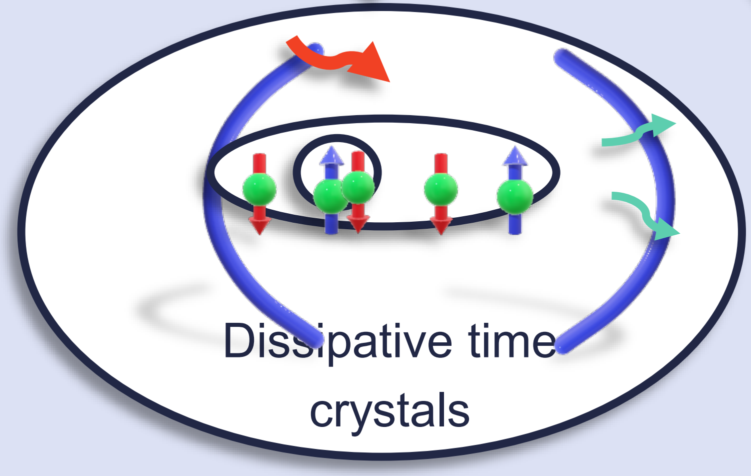

NJP **22**, 013026 (2020);
SciPost Physics **12**, 097 (2022)



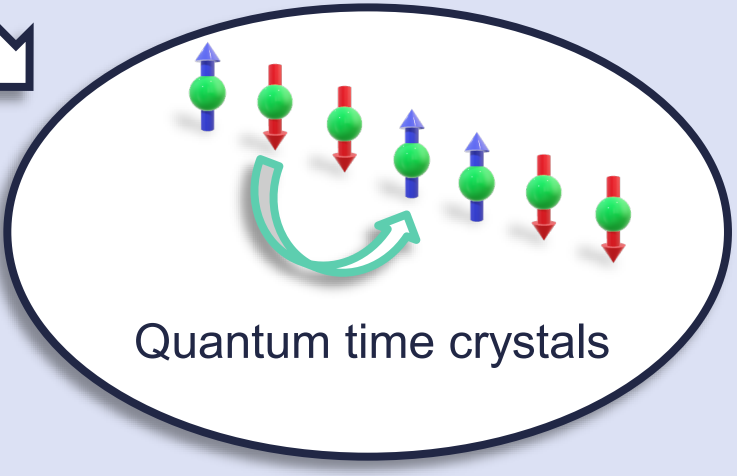

arXiv:2008.11166 (2020)




PRL **125**, 137001 (2020)




PRL **123**, 260401 (2019); NJP **22**, 085007 (2020)




PRL **123**, 030603 (2019);
Quantum **5**, 610 (2021)

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