Hot band sound

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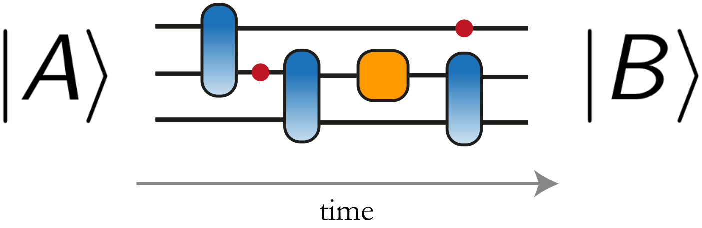
Why revisit quantum dynamics?

Unlike “real materials”, today’s quantum devices:



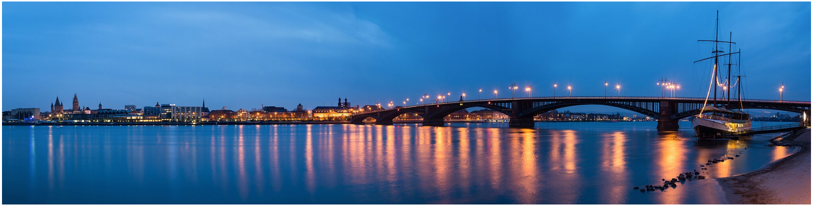
1. tend to start far from equilibrium
2. realize “model Hamiltonians” that may not be native to any material
3. transcend Hamiltonian time evolution (gates, measurements, feedback,...)

and thus probe dynamical regimes that are new to theorists.

How can we probe dynamics?

Old understanding: non-conserved degrees of freedom relax very quickly.
What's left over is **hydrodynamics**: splashing of a few slow modes.
e.g. the Rhine contains $\mathcal{O}(10^{23})$ water molecules in each drop...

Figure: The Rhine in Mainz, Photo: Arcalino / Wikimedia Commons / CC BY-SA 3.0

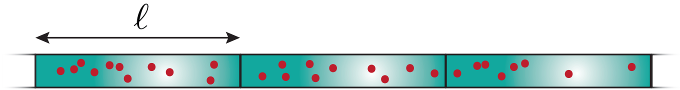


...but response to weak perturbations is just five equations.

When does hydrodynamics apply?

1. Require separation of scales:

$$\ell_{coll} \ll \ell \ll L.$$



(ℓ_{coll} = mean-free-path, ℓ=“coarse-graining length”, L=system size)

2. Typically want $T > 0$

That’s all!

Some textbook examples of hydrodynamic theory

Some textbook examples (that may not strike you as examples):

Classical physics:

- Fourier's law of heat conduction, $\partial_t n_E = \kappa \partial_x^2 n_E$ (1822)
- Fick's law of diffusion, $\partial_t n = D \partial_x^2 n$ (1855)

Quantum physics:

- Drude-Sommerfeld theory (1927)
- Landau's Fermi liquid theory (1956)

Hydrodynamics and beyond

For condensed matter systems, used to be thought of as "boring":

1. in lattice models, $T > 0$, only expect E, Q, $\mathbf{S}$ at best
2. no "interesting" hydrodynamics, just diffusion
3. more-or-less known for centuries

I'll argue that quantum devices can realize much richer regimes of hydrodynamics...

...some of which have unforeseen cond-mat counterparts.

Why do artificial quantum systems have interesting hydrodynamics?

Additional slow modes are generally a feature of "ideal" quantum dynamics.

- Disorder, phonons, substrates etc. in cond-mat systems often hinder observation there
- Simplest example: non-interacting systems
- Simplest *non-trivial* example: integrable systems[1]

Many artificial quantum systems are "close" to being integrable[2].

For concrete cond-mat examples, think Luttinger liquids in $d = 1$ or zero sound in $d > 1$.

[1] See e.g. VBB, Vasseur, Karrasch, Moore, PRL 2017 and VBB, PRB 2020

[2] For precise notions, see VBB, Huse, Gopalakrishnan, PRB 2022.

Sound is special too!

Unlike conventional heat ($x \sim (Dt)^{1/2}$), sound is ballistic ($x \sim ct$)

Ballistic modes are "special" and need extra symmetry:

- The extreme case: free or integrable particles, ∞ symmetries, ∞ conservation laws, ∞ ballistic modes[1].
- Ordinary sound comes from translation symmetry.

Can get emergent integrability in fermion lattices as $T \to 0$, or electron sound in ultrapure 2D metals[2].

But generically, we don't expect sound modes in a hot lattice.

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[1] Castro-Alvaredo, Doyon, Yoshimura, PRX 2016, Bertini, Collura, De Nardis, Fagotti, PRL 2016

[2] Bandurin et al., Science 2016, Moll et al., Science 2016

Introducing hot band sound

This talk will sketch how to get long-lived ballistic sound modes:

- On a lattice (no momentum conservation)
- In chaotic systems (no exact integrability)
- At high temperature (no emergent integrability)

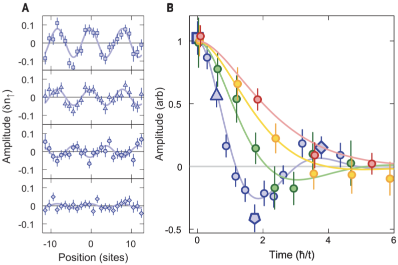
If I relax any of these conditions, easy.

With all three, no sound waves are expected.

Hot band sound

Underdamped sound in the Fermi-Hubbard model

Ultracold Li-6 realization of Fermi-Hubbard model looked at relaxation of charge density fluctuations[1]:



- Crossover from underdamped to overdamped ("bad metal") charge propagation with increasing wavelength
- This is a hot, chaotic, lattice model: expect normal diffusion

How does underdamped sound survive?

[1] Brown et al., Science 2019

Why shouldn't there be a sound mode?

Simpler to think about this effect in one dimension.

- Consider interacting spinless fermion chains in 1D:

$$H = \hat{H}_0 + \hat{V}, \quad \hat{H}_0 = -\sum_{x,x'=1}^{L} t_{|x-x'|} \hat{c}^\dagger_{x'} \hat{c}_x,$$

$$\hat{V} = \sum_{x,x'=1}^{L} U_{|x-x'|} (\hat{n}_{x'} - 1/2)(\hat{n}_x - 1/2). \tag{1}$$

- Local charge conservation holds as an operator equation,

$$\partial_t \hat{n}_x + \hat{j}_{x+1} - \hat{j}_x = 0. \tag{2}$$

- But the total charge current

$$\hat{J} = \sum_x \hat{j}_x = \sum_k v_k \, \hat{c}^\dagger_k \hat{c}_k \tag{3}$$

generically relaxes as $t \to \infty$. This means no sound mode.

Isolating hot band sound

To isolate this effect, we need to make the decay of $\hat{J}$ as slow as possible.

- We look for "hot band sound" by solving the following variational problem:

  Minimize $\langle \dot{\hat{J}}^2 \rangle_{\beta=0}$ **subject to** $\langle \hat{J}^2 \rangle_{\beta=0}$ *and* $\langle \hat{V}^2 \rangle_{\beta=0}$ *constant.*

- Forces slow decay of $\hat{J}$ in a strongly interacting regime.
- Constraints steer clear of trivial solutions (no hopping, no interactions).

Optimal models

We solved this for nearest-neighbour hopping. Yields an optimal model for any allowed interaction ranges $x \leq R$:

$$U_x^*(R) = \frac{2}{\sqrt{2R+1}} \cos \frac{\pi(x-1/2)}{(2R+1)}, \quad 1 \leq x \leq R, \qquad (4)$$

These “optimal models” are tabulated below:

R	U_1^*	U_2^*	U_3^*	U_4^*	U_5^*	U_6^*	U_7^*	$\langle \dot{\hat{J}}^2 \rangle$
1	1	0	0	0	0	0	0	L
2	0.851	0.526	0	0	0	0	0	$0.382L$
3	0.737	0.591	0.328	0	0	0	0	$0.198L$
4	0.657	0.577	0.429	0.228	0	0	0	$0.121L$
5	0.597	0.549	0.456	0.326	0.170	0	0	$0.081L$
6	0.551	0.519	0.457	0.368	0.258	0.133	0	$0.058L$
7	0.514	0.491	0.447	0.384	0.304	0.210	0.107	$0.044L$

Note that current decay gets **arbitrarily slow** as $R \to \infty$.

Do the optimal models deliver?

Remains to simulate dynamics and check for hot band sound. Our protocol:

- Start from weak density modulation:

$$\hat{\rho}(0) = \frac{1}{Z}\left(1 + \epsilon \sum_{x=1}^{L} \sin(qx)(\hat{n}_x - \langle \hat{n}_x \rangle_{\beta=0})\right), \qquad (5)$$

  with $\epsilon = 0.01$.

- Evolve numerically under Schrödinger evolution

$$\hat{\rho}(t) = e^{-i\hat{H}t}\hat{\rho}(0)e^{i\hat{H}t} \qquad (6)$$

- Look at lowest Fourier mode of the charge density

$$n_q(t) = \sqrt{\frac{2}{L}} \sum_{x=1}^{N} \sin(qx)\mathrm{tr}[\hat{\rho}(t)\hat{n}_x] \qquad (7)$$

  with $q = 2\pi/L$.

Apparently so...

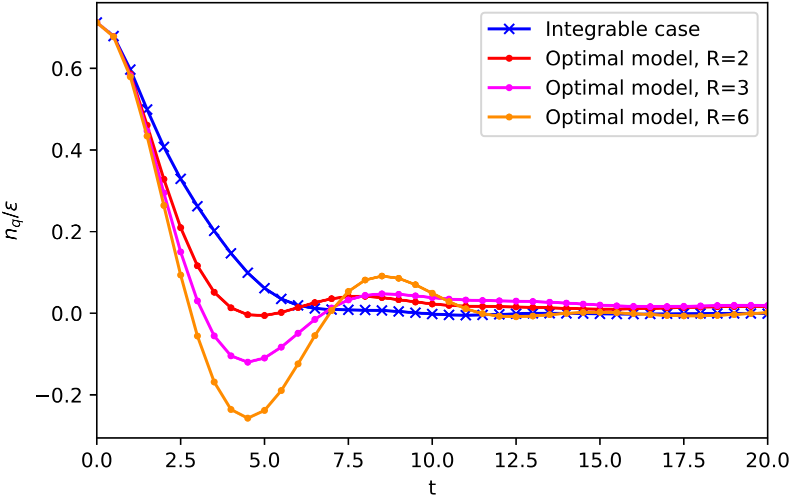


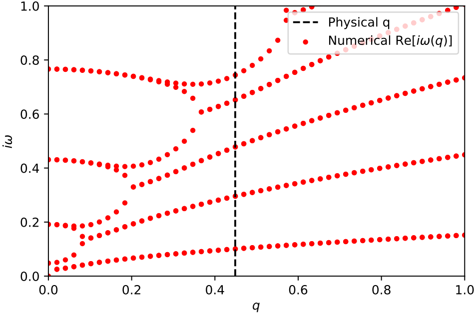
Figure: optimal models with interaction ranges $R \in \{1, 2, 3, 6\}$, half-filled chains, $L = 14$ sites, exact diagonalization.

Explaining underdamped sound from kinetic theory

This family of models has a simple limiting kinetic theory, "phase-space hydrodynamics":

$$\partial_t \delta\rho_k + v_k \partial_x \delta\rho_k = D\partial_k^2 \delta\rho_k.$$

Solving numerically over Brillouin zone, find an infinite "tower" of underdamped modes. "Sound" is just the bottom of the tower:



Integrable-like hydrodynamics in a chaotic system.

What's really going on here?

The underlying physics: slow momentum diffusion in phase space.

- Once diagnosed, connects to both integrable systems[1] and 2D metals[2]

We think the two most pressing questions are:

1. Sorting out a precise quantum-classical correspondence
2. Developing sharp experimental diagnostics

In 1D, trapped ions? In 2D, cold atoms or even ordinary metals?

[1] Bastianello, De Nardis, De Luca, PRB 2020

[2] See Ledwith, Guo, Shytov, Levitov, PRL 2019 and subsequent works by al. et Levitov

Thank you for listening!

Brought to you in collaboration with David A. Huse (Princeton)

See arXiv 2208.13767 for further details.

A Lagrangian in model space

Minimizing $\dot{\hat{J}}$ in model space generates this Lagrangian:

$$\begin{aligned} \mathcal{L}(t_r, U_r, \lambda_1, \lambda_2) &= \langle \dot{\hat{J}} \dot{\hat{J}} \rangle_{\beta=0} \\ &+ \lambda_1 \left(\langle \hat{V}^2 \rangle_{\beta=0} - \sigma_V^2 \right) + \lambda_2 \left(\langle \hat{J}^2 \rangle_{\beta=0} - \sigma_J^2 \right), \end{aligned} \tag{8}$$

which is a function of hopping and interaction strengths at each range r, i.e.

$$\langle \dot{\hat{J}} \dot{\hat{J}} \rangle_{\beta=0} = \frac{1}{2} \sum_{r>0} r^2 t_r^2 \sum_x \sum_{y \neq x, x-r} \left(U_{|y-x|} - U_{|y-x+r|} \right)^2 \tag{9}$$

and

$$\langle \hat{V}^2 \rangle_{\beta=0} = \frac{L}{4} \sum_{r>0} U_r^2, \quad \langle \hat{J}^2 \rangle_{\beta=0} = \frac{L}{2} \sum_{r>0} r^2 t_r^2. \tag{10}$$

Generally quartic and intractable. Not even clear that solutions exist! (in general they don't...)

Solving for optimal models

An exactly solvable special case occurs for nearest-neighbour hopping $t_1 = t$ and $t_r = 0$ for $r > 1$.
Then we optimize over interactions up to some range $\vec{U} = (U_1, U_2, \ldots, U_R)$.
This yields

$$\mathcal{L}(t, \vec{U}, \lambda_1, \lambda_2) = Lt^2 \left(\sum_{n=1}^{R-1} (U_{n+1} - U_n)^2 + U_R^2 \right) + \frac{L\lambda_1}{4} \left(\sum_{n=1}^{R} U_n^2 - 1 \right) + \frac{L\lambda_2}{2} \left(t^2 - 1 \right). \quad (11)$$

- Key idea: view as a quadratic form.
- Then the constraint commutes with the objective function.
- **So this is just matrix diagonalization!**

An embarassment of models

Optimality of $\dot{\hat{J}}$ then demands that $A\vec{U} = \alpha\vec{U}$, where the R-by-R matrix A

$$A = \begin{pmatrix} 1 & -1 & 0 & 0 & \dots & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & \dots & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & \dots & 0 & -1 & 2 \end{pmatrix}. \qquad (12)$$

- The matrix is simple enough that we can diagonalize by hand.
- We find R distinct solutions $\vec{U}^{(m)}$ with wavenumber

$$k_m = \frac{(2m+1)\pi}{2R+1}, \quad m = 0, 1, \dots, R-1. \qquad (13)$$

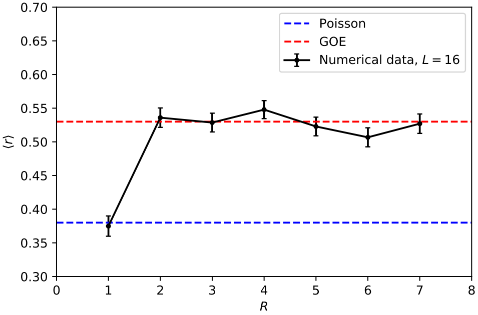
- Each solution has decay rate $\langle \dot{\hat{J}}^2 \rangle = 4\sin^2(k_m/2)L$.
- "Harmonics" of the interaction potential. $m = 0$ is best.

Sanity check: no integrability

To test for chaos, we looked at the $\langle r \rangle$ statistic (*Oganesyan, Huse, '07*),

$$\langle r \rangle = \langle r_n \rangle, \quad r_n = \frac{\min(\delta_n, \delta_{n+1})}{\max(\delta_n, \delta_{n+1})}, \tag{14}$$

where $\delta_n = E_{n+1} - E_n$ and adjacent energy levels $\ldots > E_{n+1} > E_n > \ldots$ (in a non-degenerate sector).



Clear evidence for quantum chaos.