Young Research Leaders Group Workshop: Recent advances in non-equilibrium and magnetic phenomena

Non-linear Anomalous Hall effects detect topological phase transitions in moiré super lattices

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nature physics

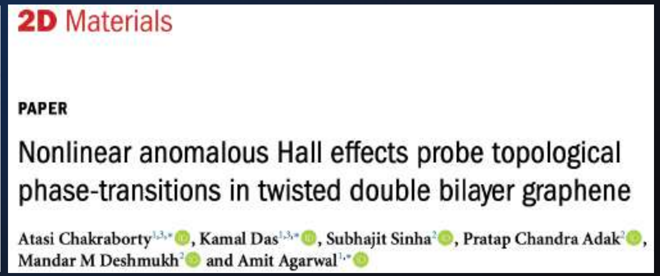
LETTERS

https://doi.org/10.1038/s41567-022-01606-y

Check for updates

Berry curvature dipole senses topological transition in a moiré superlattice

Subhajit Sinha[1,6]✉, Pratap Chandra Adak[1,6], Atasi Chakraborty[2], Kamal Das[2], Koyendrila Debnath[3], L. D. Varma Sangani[1], Kenji Watanabe[4], Takashi Taniguchi[5], Umesh V. Waghmare[3], Amit Agarwal[2]✉ and Mandar M. Deshmukh[1]✉



2D Materials

PAPER

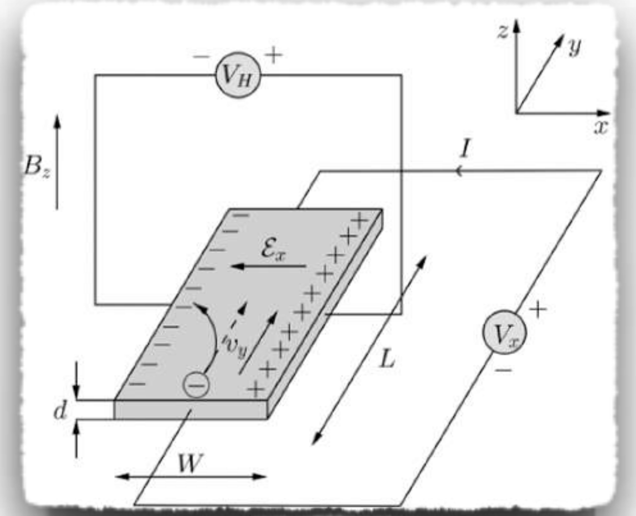
Nonlinear anomalous Hall effects probe topological phase-transitions in twisted double bilayer graphene

Atasi Chakraborty[1,3,*], Kamal Das[1,3,*], Subhajit Sinha[2], Pratap Chandra Adak[2], Mandar M Deshmukh[2] and Amit Agarwal[1,*]

July 27, 2023

Background

- **1879: Edwin Hall** made a revolution by the first observation of transverse potential difference in a conductor, fixed in position with respect to a steady magnetic field at right angles to the applied current.



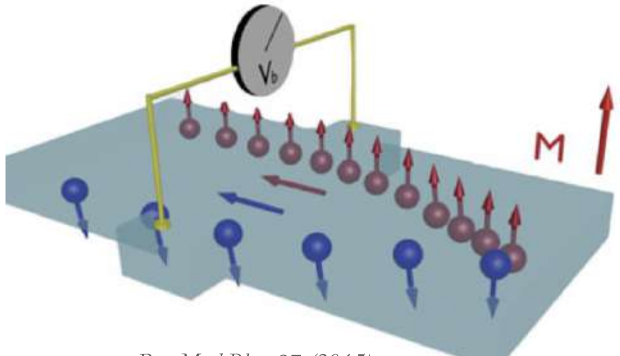
$$eE_x = ev_yB$$

$$\rho_H = \frac{E_x}{j_y} = \frac{B}{nq}$$

- **Transverse response**
- **Determine carrier density**
- **Determine carrier type**

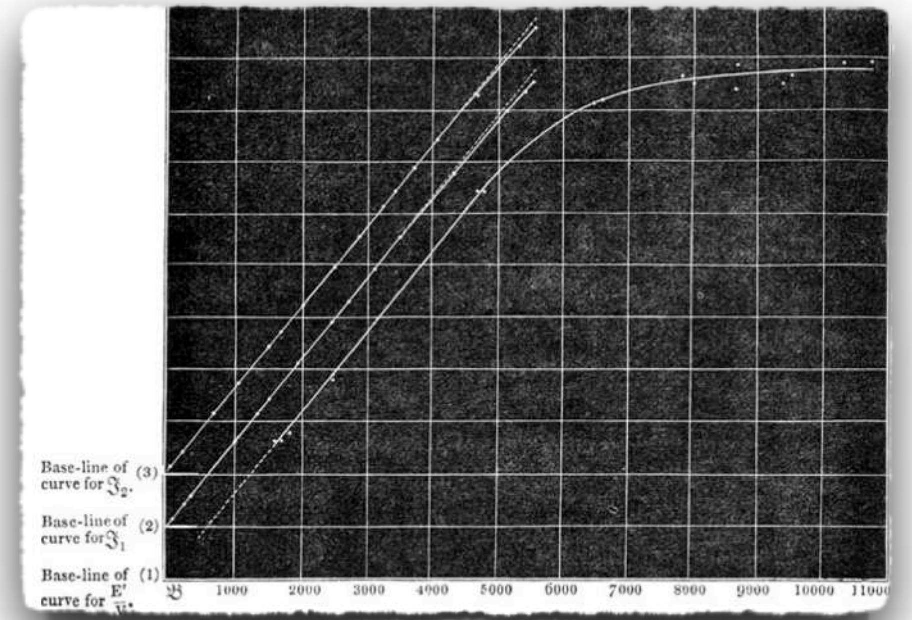
- **1881: Edwin Hall** observed the existence of Hall responses even in absence of external magnetic field but for ferromagnetic systems only.

Anomalous Hall !



Rev Mod Phys ***87*** *(2015)*

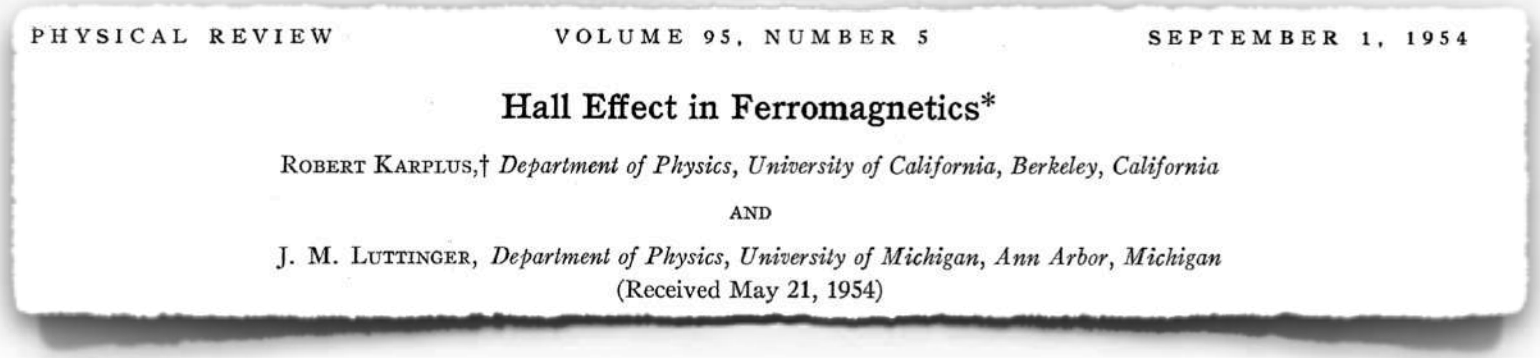
Transverse signal ∝ magnetisation of the system



Philosophical magazine and journal of science ***5*** *(1881)*

Theory of Anomalous Hall effect

In the early stages of research on the anomalous Hall conductivity, it was widely believed that its origins were solely related to **Extrinsic** contributions— scattering of charge carriers by impurities, defects, or other extraneous factors within the material.



PHYSICAL REVIEW VOLUME 95, NUMBER 5 SEPTEMBER 1, 1954

Hall Effect in Ferromagnetics*

ROBERT KARPLUS,† *Department of Physics, University of California, Berkeley, California*

AND

J. M. LUTTINGER, *Department of Physics, University of Michigan, Ann Arbor, Michigan*

(Received May 21, 1954)

Semi classical equations of motion

$$\dot{\mathbf{k}} = e(\mathbf{E} + \dot{\mathbf{x}} \times \mathbf{B})$$

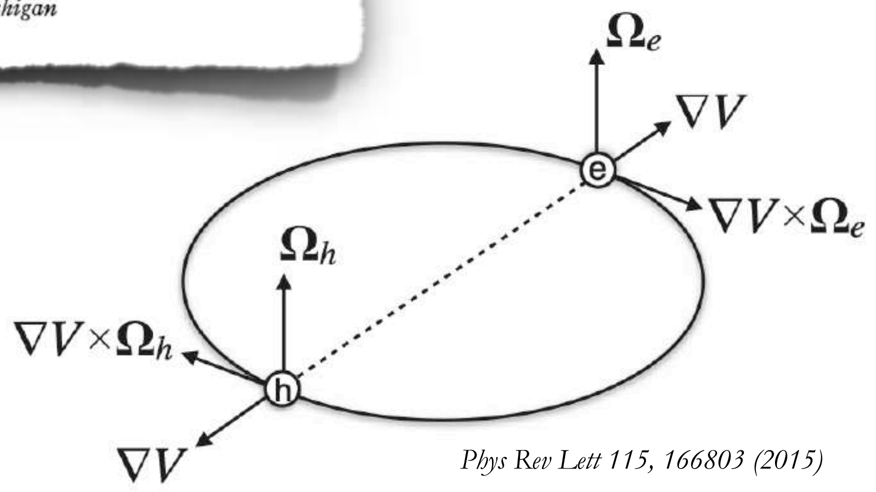
$$\dot{\mathbf{x}} = \frac{\partial \epsilon(\mathbf{k},\mathbf{x})}{\partial \mathbf{k}} \underbrace{-(\dot{\mathbf{k}} \times \boldsymbol{\Omega})}_{\text{anomalous velocity}}$$

Berry curvature

$$\boldsymbol{\Omega}_n(\mathbf{k}) = \nabla \times \mathbf{A}_n(\mathbf{k})$$

Berry connection

$$\mathbf{A}_n(\mathbf{k}) = -i < \psi_{\mathbf{k}} | \nabla_{\mathbf{k}} | \psi_{\mathbf{k}} >$$



Phys Rev Lett 115, 166803 (2015)

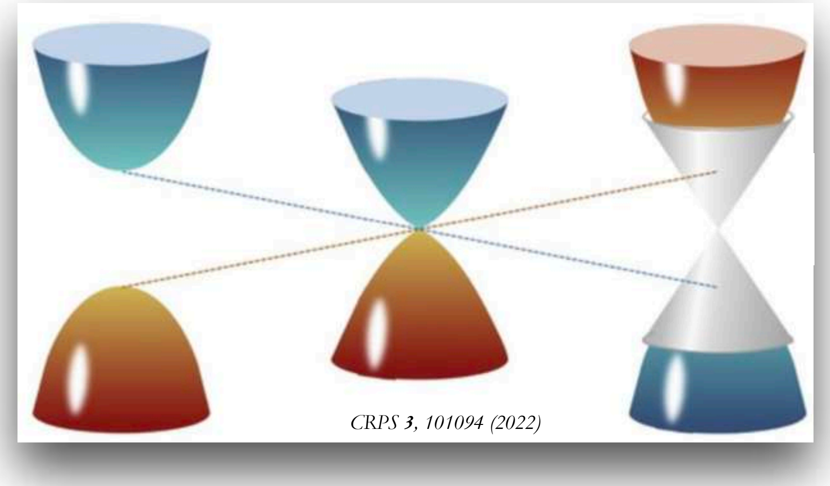
Anomalous Hall conductivity:

$$\sigma_{xy} = \frac{e^2}{h}\Sigma\int \Omega_n(\mathbf{k})d\mathbf{k}$$
$$= \frac{e^2}{h}C$$

Chern number
Topological Quantization !

- ***Broken Time Reversal symmetry***

  $\boldsymbol{\Omega}(\mathbf{k}) \neq -\boldsymbol{\Omega}(-\mathbf{k})$



CRPS 3, 101094 (2022)

Time Reversal Symmetry : Non-linear AHE

Can we obtain **Anomalous Hall Effect** in systems with **TRS**?

PRL **115,** 216806 (2015) PHYSICAL REVIEW LETTERS week ending 20 NOVEMBER 2015

Quantum Nonlinear Hall Effect Induced by Berry Curvature Dipole in Time-Reversal Invariant Materials

Inti Sodemann and Liang Fu

Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

Time Reversal symmetry: $\boldsymbol{\Omega}(\mathbf{k}) = -\boldsymbol{\Omega}(-\mathbf{k})$

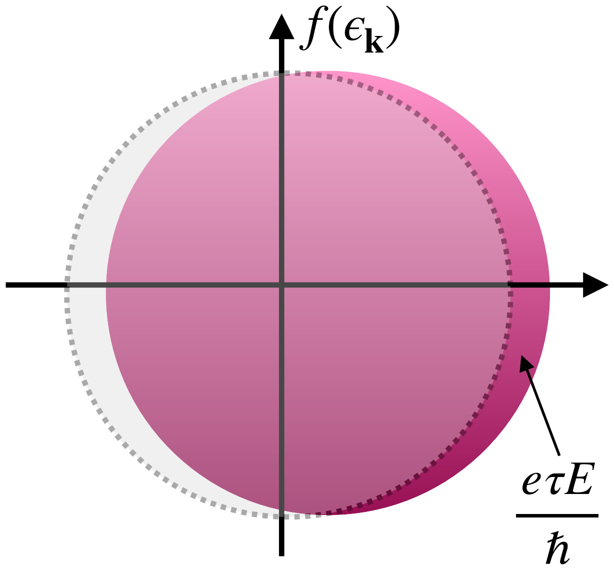
Space ~~Inversion~~ symmetry: $\boldsymbol{\Omega}(\mathbf{k}) = \boldsymbol{\Omega}(-\mathbf{k})$

In presence of both TRS and SIS, the Berry curvature is identically zero at every BZ points

Anomalous velocity: $v(\mathbf{k}) = \frac{1}{\hbar}\frac{\partial \epsilon(\mathbf{k})}{\partial \mathbf{k}} + \dot{\mathbf{k}} \times \boldsymbol{\Omega}_n(\mathbf{k})$

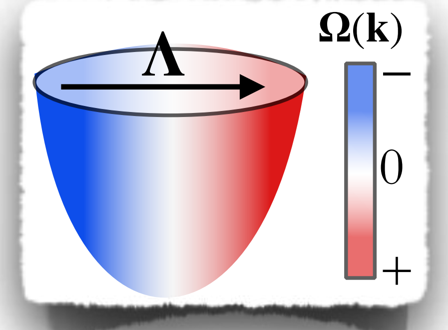
Out of equilibrium $\Rightarrow f(\epsilon_{\mathbf{k}}) \neq f(\epsilon_{-\mathbf{k}})$

$$j_H = -e \int \left[f_o(\epsilon_{\mathbf{k}}) + \underbrace{\delta f(\epsilon_{\mathbf{k}})}_{\propto E} \right] \underbrace{v(\mathbf{k})}_{\propto E} d\mathbf{k}$$



Berry curvature dipole

$$\Lambda_{ab} = \int f_0\, (\partial_a \Omega_b)\, [d\mathbf{k}] = -\int \Omega_b\, (\partial_a f_0)\, [d\mathbf{k}]$$



Feasibility of topology from tilted Dirac model

Do we have any **Topological characterisation** from **NL-AHE** in presence of **TRS?**

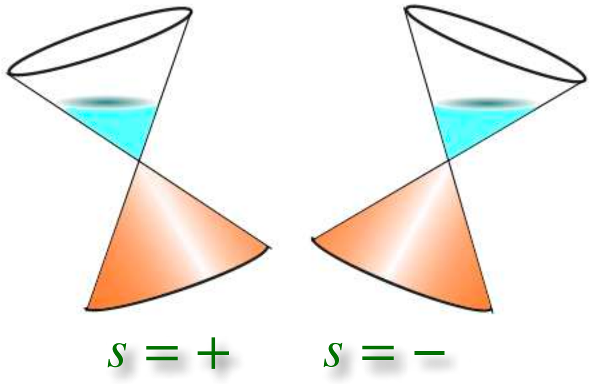
$$H_s = \hbar v_F(k_x\sigma_y - s k_y\sigma_x) + s\hbar v_t k_x\sigma_0 + \Delta\sigma_z$$

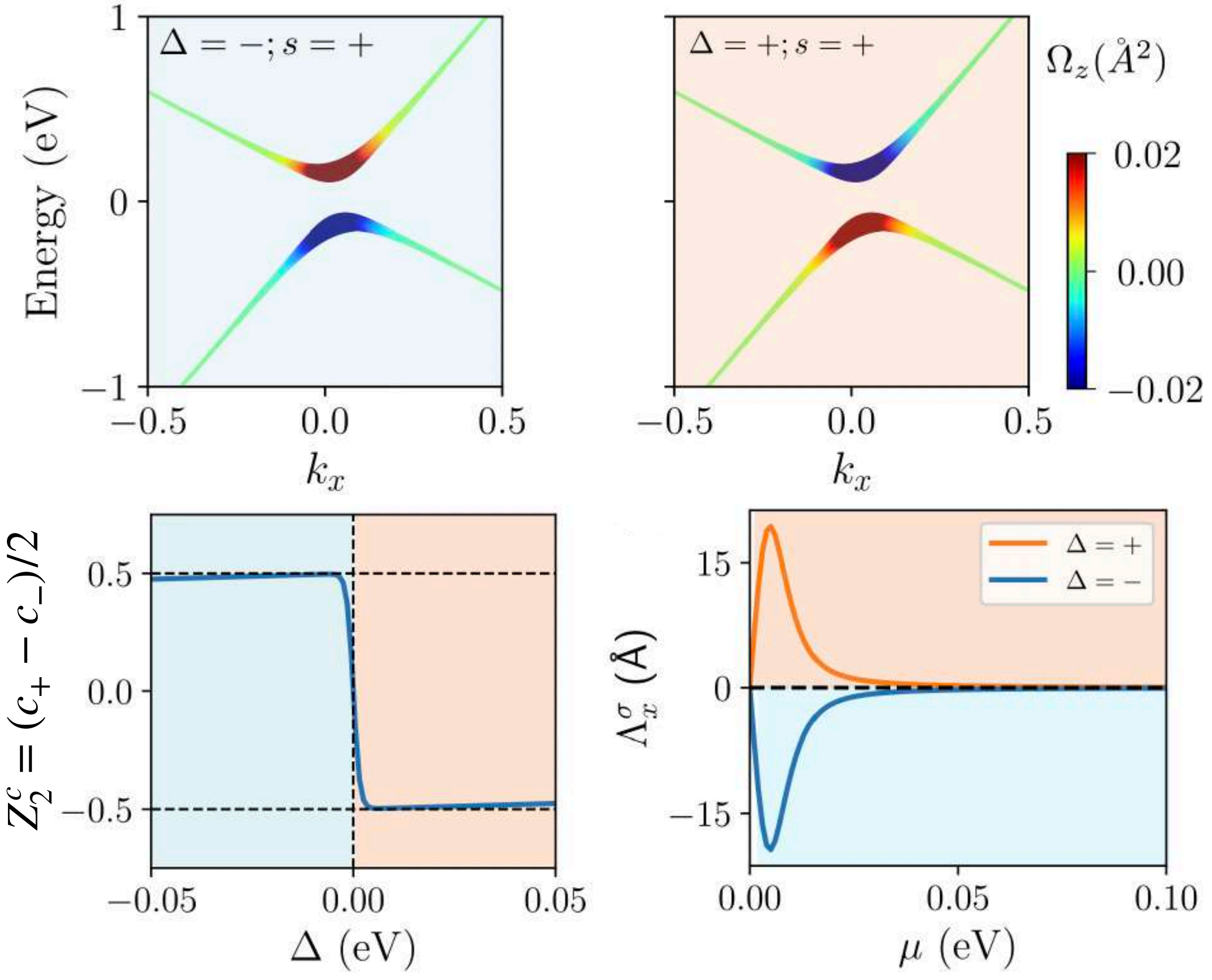
Berry curvature

$$\Omega_z = \mp s\frac{\hbar^2 v_F^2\Delta}{2(\hbar^2 v_F^2 k^2 + \Delta^2)^{3/2}}$$

Valley Chern number

$$c_v = \frac{1}{2\pi}\int \Omega_z d\mathbf{k}$$





- **Time Reversal symmetry preserved**
- **Space Inversion symmetry broken**
- **Tenability with external perturbations**

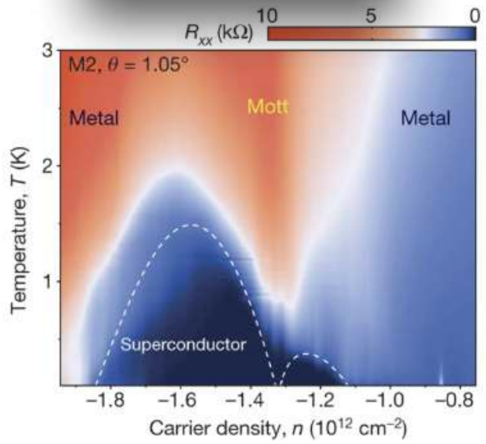
Moiré systems

2D Materials **9**, 045020 (2022)

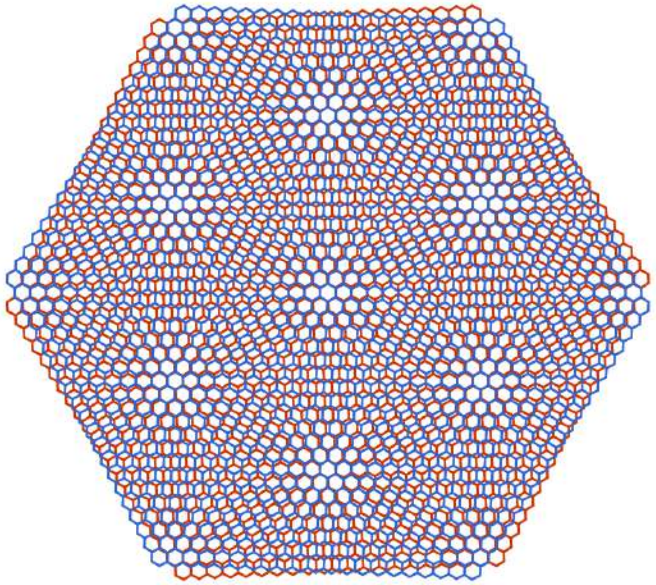
Twistronics

In 2018, the world break through has happened in the field of condensed matter physics with discovery of ***magic*** angle in twisted graphene superlattice namely ``**Moiré** ” systems

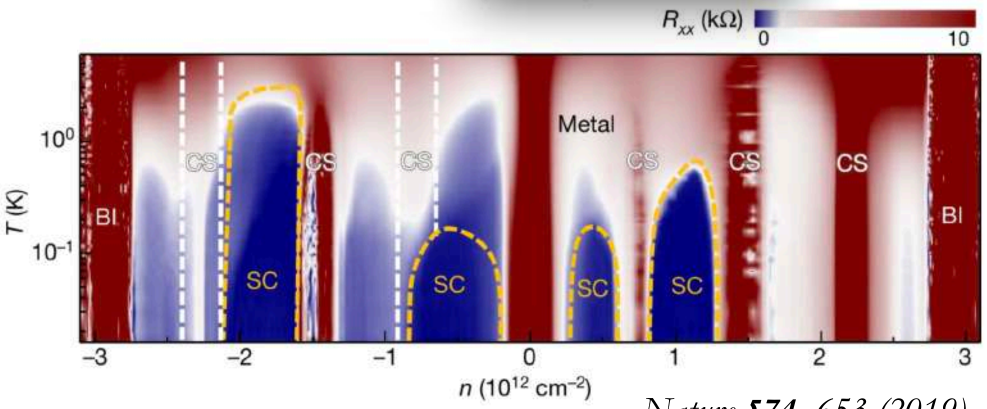
Pablo J Herrero
(2018)



Nature ***556***, *43 (2018)*



Dmitri Efetov
(2019)



Nature ***574***, *653 (2019)*

LETTER

doi:10.1038/nature26154

Correlated insulator behaviour at half-filling in magic-angle graphene superlattices

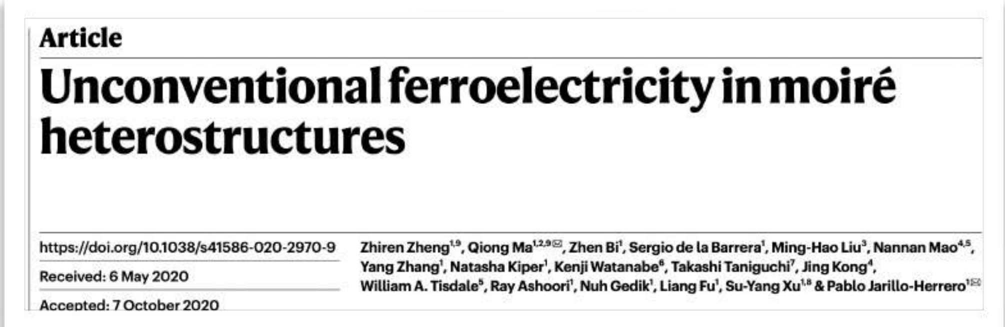
Yuan Cao[1], Valla Fatemi[1], Ahmet Demir[1], Shiang Fang[2], Spencer L. Tomarken[1], Jason Y. Luo[1], Javier D. Sanchez-Yamagishi[2], Kenji Watanabe[3], Takashi Taniguchi[3], Efthimios Kaxiras[2,4], Ray C. Ashoori[1] & Pablo Jarillo-Herrero[1]

ARTICLE

doi:10.1038/nature26160

Unconventional superconductivity in magic-angle graphene superlattices

Yuan Cao[1], Valla Fatemi[1], Shiang Fang[2], Kenji Watanabe[3], Takashi Taniguchi[3], Efthimios Kaxiras[2,4] & Pablo Jarillo-Herrero[1]



Article

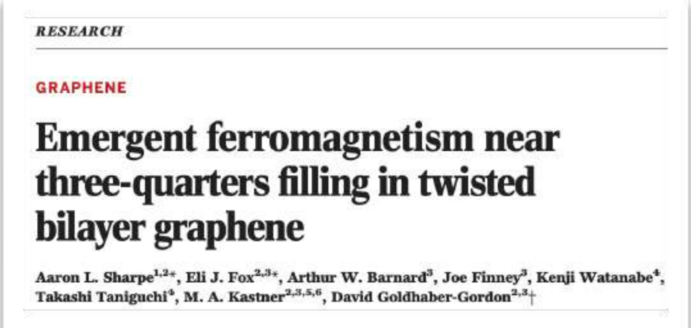
Unconventional ferroelectricity in moiré heterostructures

https://doi.org/10.1038/s41586-020-2970-9

Received: 6 May 2020

Accepted: 7 October 2020

Zhiren Zheng[1,9], Qiong Ma[1,2,9]✉, Zhen Bi[1], Sergio de la Barrera[1], Ming-Hao Liu[3], Nannan Mao[4,5], Yang Zhang[1], Natasha Kiper[1], Kenji Watanabe[6], Takashi Taniguchi[7], Jing Kong[4], William A. Tisdale[5], Ray Ashoori[1], Nuh Gedik[1], Liang Fu[1], Su-Yang Xu[1,8] & Pablo Jarillo-Herrero[1]✉



RESEARCH

GRAPHENE

Emergent ferromagnetism near three-quarters filling in twisted bilayer graphene

Aaron L. Sharpe[1,2]*, Eli J. Fox[2,3]*, Arthur W. Barnard[3], Joe Finney[3], Kenji Watanabe[4], Takashi Taniguchi[4], M. A. Kastner[2,3,5,6], David Goldhaber-Gordon[2,3]†

and it goes on …

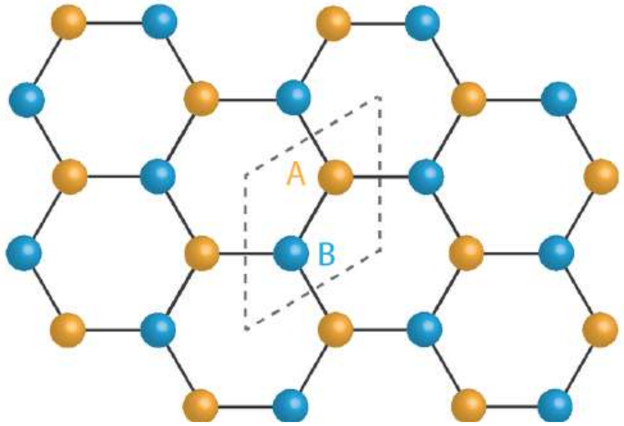
RESEARCH

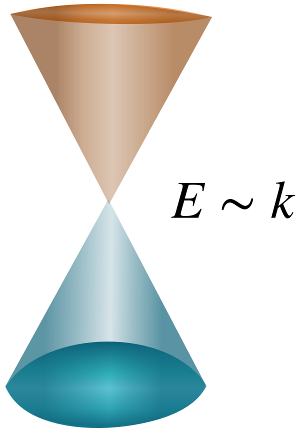
TOPOLOGICAL MATTER

Intrinsic quantized anomalous Hall effect in a moiré heterostructure

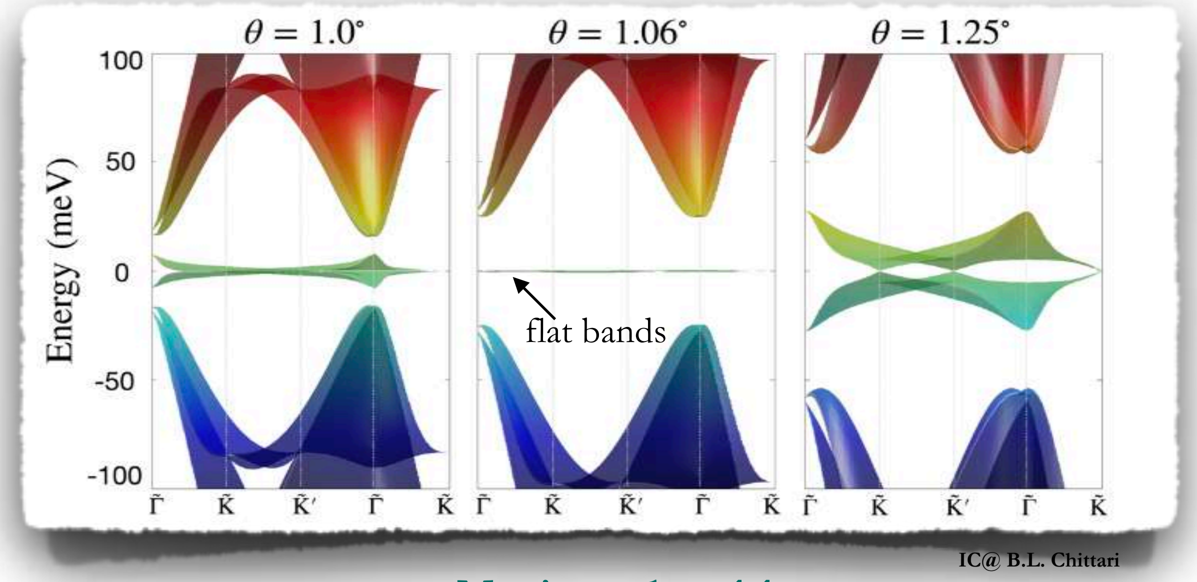
M. Serlin[1]*, C. L. Tschirhart[1]*, H. Polshyn[1]*, Y. Zhang[1], J. Zhu[1], K. Watanabe[2], T. Taniguchi[2], L. Balents[3], A. F. Young[1]†

Illustration of energy dispersion

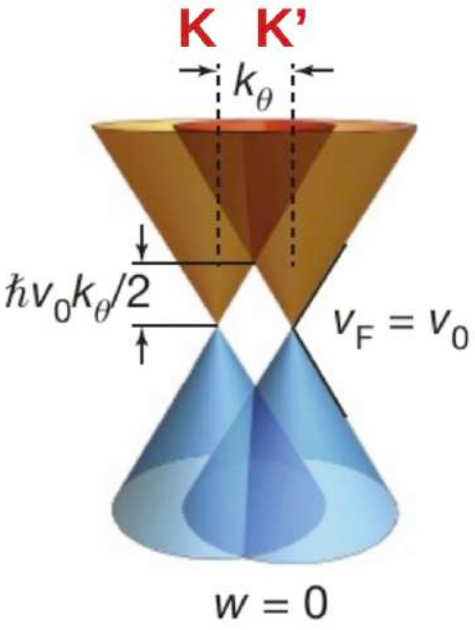




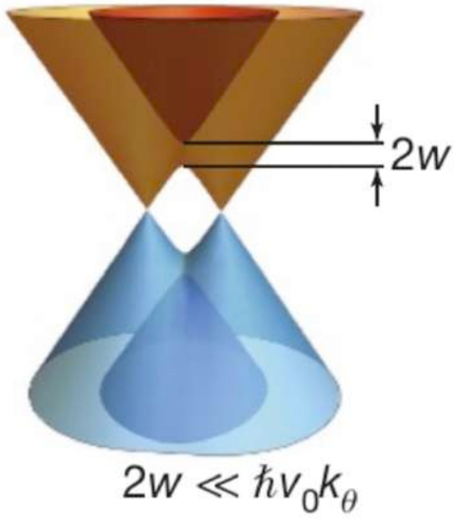
Low energy continuum model:



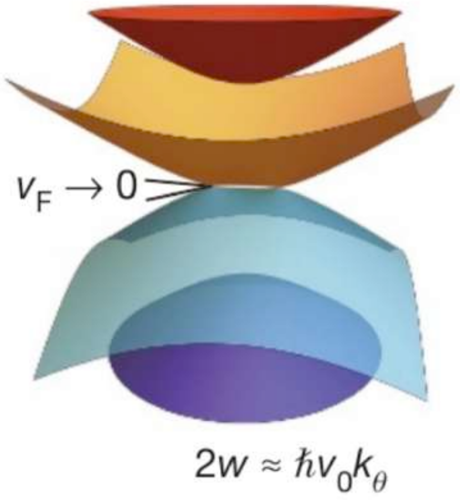
Magic angle ~ 1.1°



- *twist* -



- *Interlayer tunnelling* -

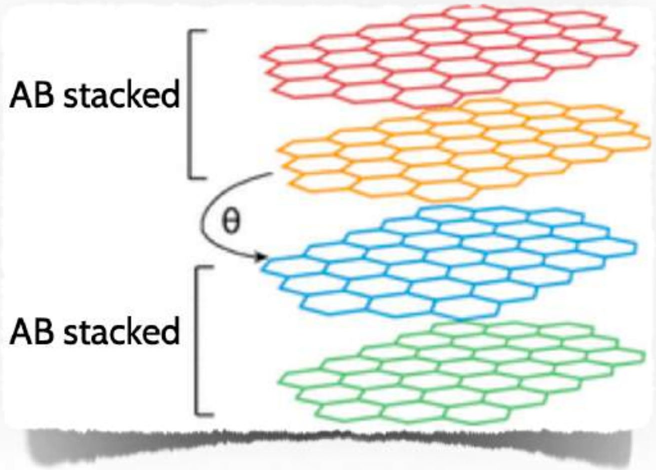


- *decreasing twist angle* -

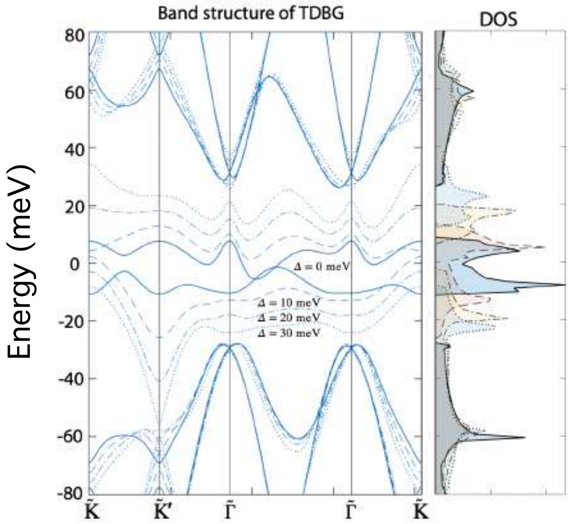
Phys Rev Lett ***99****, 256802 (2007)*
PNAS ***108****, 12233 (2011)*
Nature ***556****, 80 (2019)*

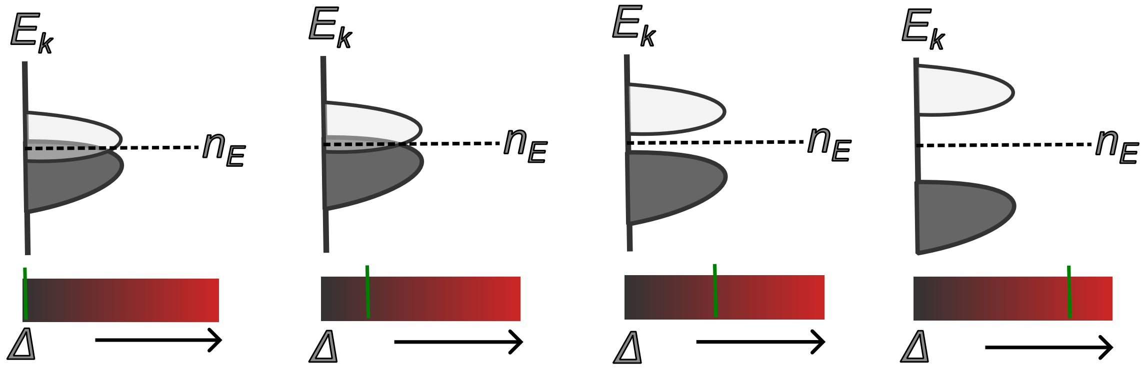
Key criteria for NL Anomalous Hall effect

- ✓ Time Reversal symmetry preserved
- ✓ Space Inversion symmetry broken
- ✓ Tenability with external perturbations

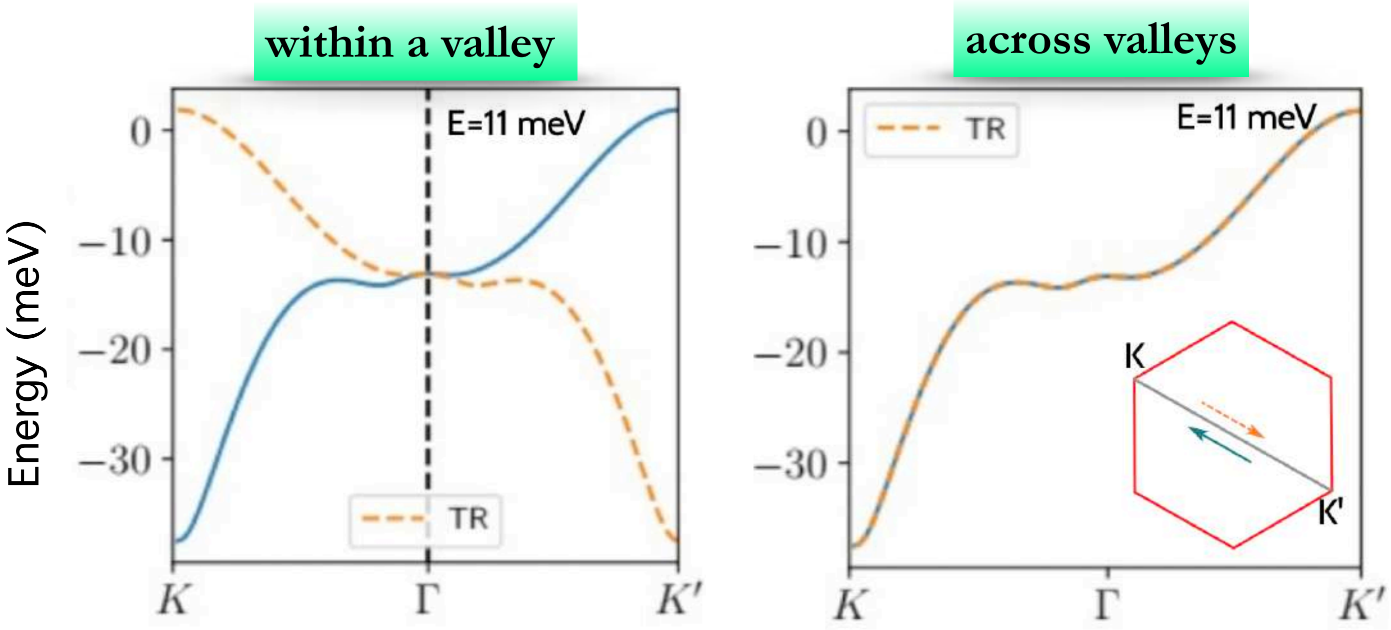


ABAB twisted double bilayer graphene (TDBG)



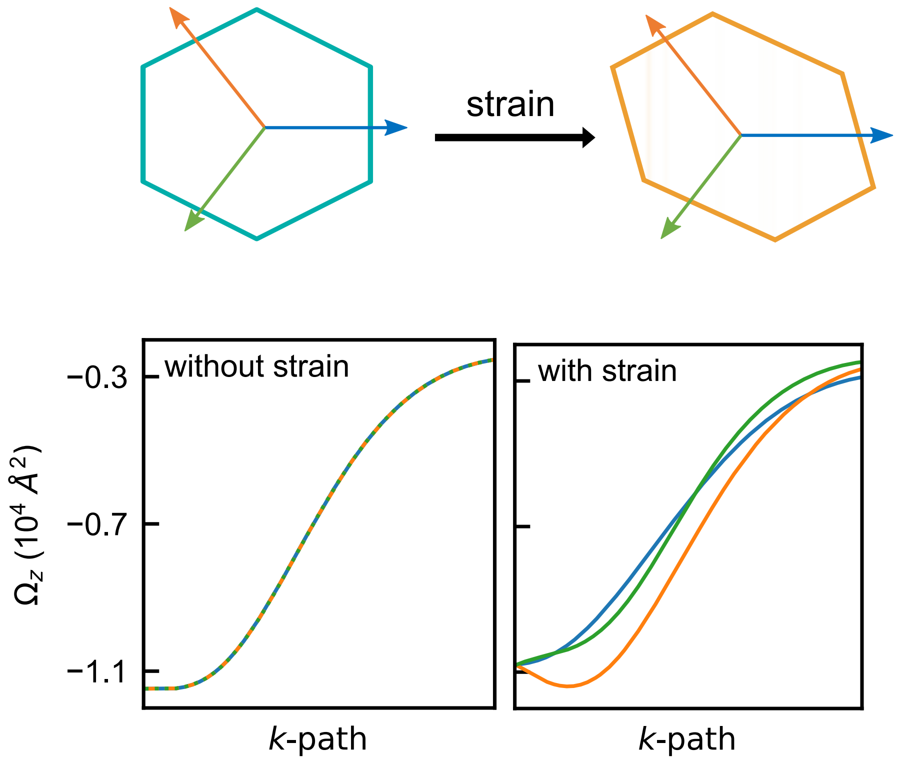


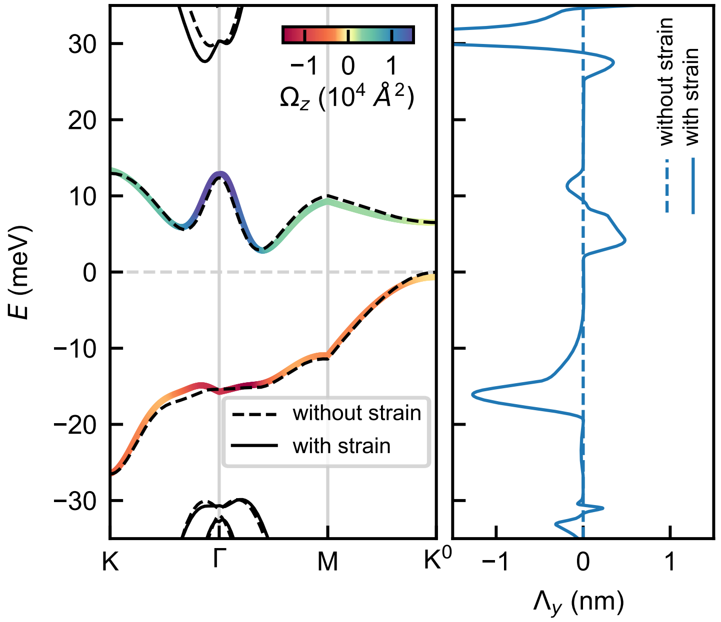
External vertical electric field induces metal insulator transition by opening a gap and charge neutrality point



The time reversal symmetry of the whole system is preserved

Second order Hall response

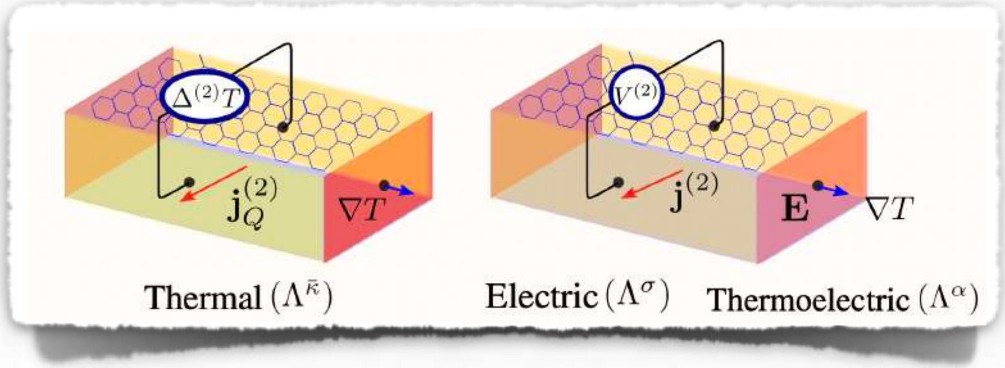




Breaking of C_3 symmetry is important to get finite BCD hence NLA Hall response

Non-linear Hall current:

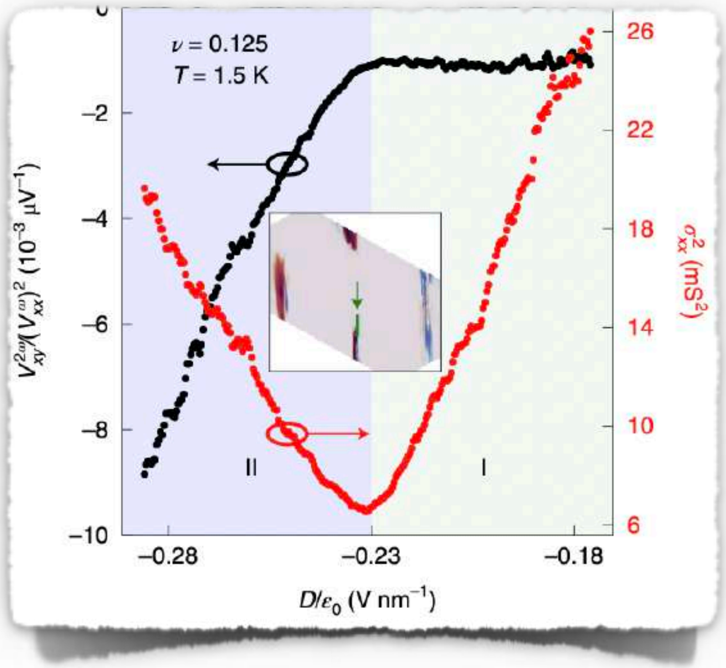
$$\begin{pmatrix} j_a \\ j_b^Q \end{pmatrix} = \begin{pmatrix} \sigma_{abc} & \alpha_{abc} \\ \alpha_{abc} & \kappa_{abc} \end{pmatrix} \begin{pmatrix} E_b E_c \\ \nabla T_b \nabla T_c \end{pmatrix}$$

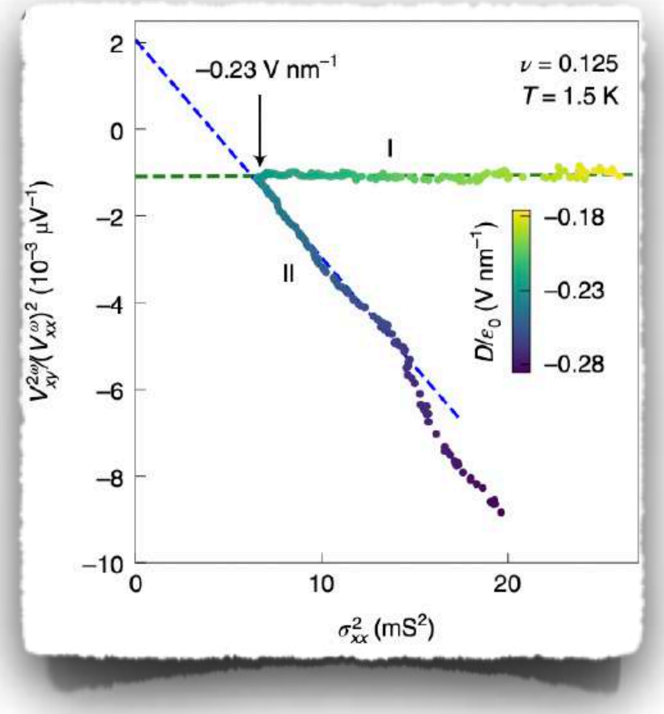


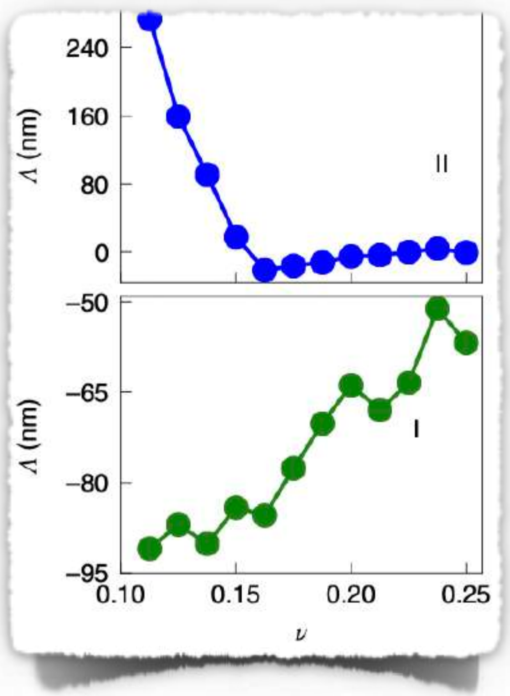
NL conductivity	**Berry curvature dipole**
$\sigma_{abc} = \epsilon_{abd}\dfrac{e^3\tau}{\hbar^2}\Lambda^{\sigma}_{dc}$	$\Lambda^{\sigma}_{dc} = -\Sigma_n \displaystyle\int [d\mathbf{k}]\Omega^n_d\,(\partial_c f^n_0)$
$\alpha_{abc} = \epsilon_{abd}\dfrac{e\tau k_B^2}{\hbar^2}\Lambda^{\alpha}_{dc}$	$\Lambda^{\alpha}_{dc} = -\Sigma_n \displaystyle\int \dfrac{(\epsilon-\mu)^2}{(k_B T)^2}[d\mathbf{k}]\Omega^n_d\,(\partial_c f^n_0)$
$\bar{\kappa}_{abc} = \epsilon_{abd}\dfrac{e\tau k_B^3 T}{\hbar^2}\Lambda^{\bar{\kappa}}_{dc}$	$\Lambda^{\bar{\kappa}}_{dc} = -\Sigma_n \displaystyle\int [d\mathbf{k}]\dfrac{(\epsilon-\mu)^3}{(k_B T)^3}\Omega^n_d\,(\partial_c f^n_0)$

Nat Phys ***18****, 765 (2022)*
2D Materials ***9****, 045020 (2022)*

Experimental observation





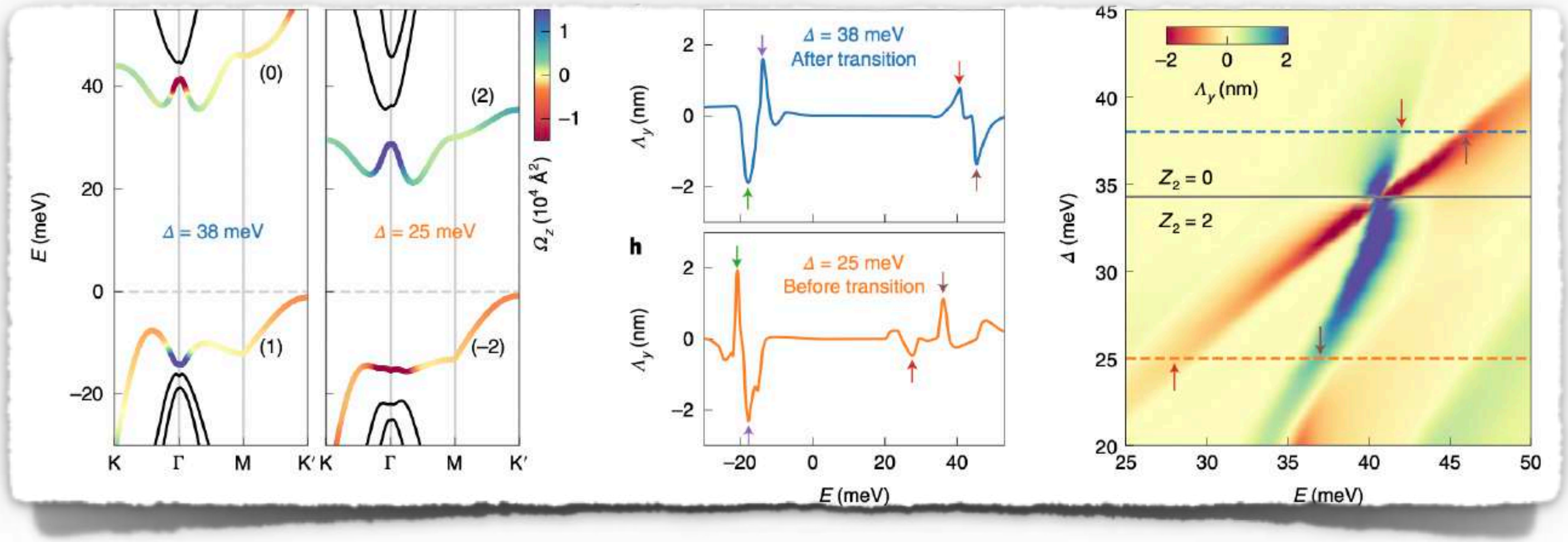


NLH voltage (at 2ω) and longitudinal voltage (at ω) scaling analysis:

$$\frac{V_y^N}{(V_x^L)^2} = A\sigma_{xx}^2 + B$$

Slope: Extrinsic contribution

Intercept: BCD



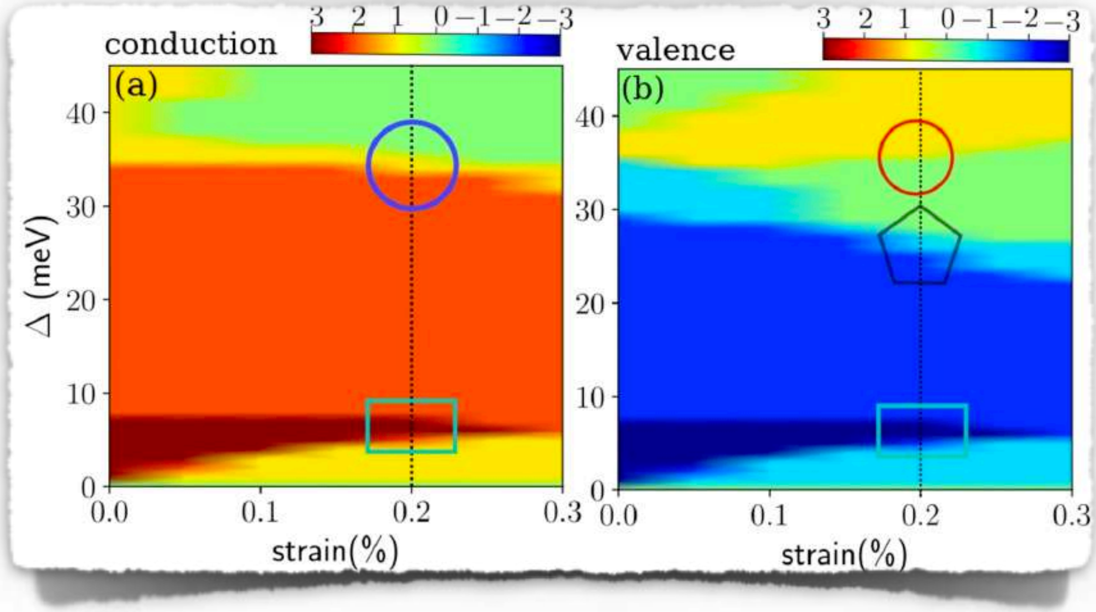
Topological index:

$$Z_2 = (C_k - C_k')/2$$

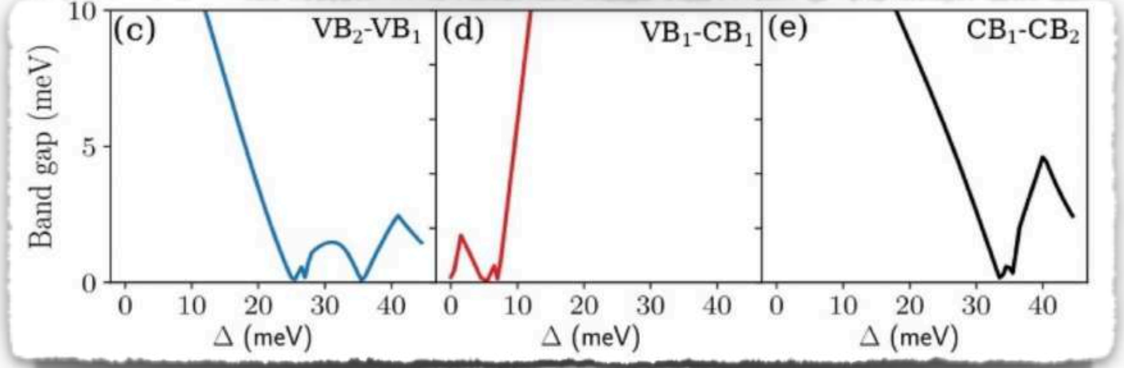
Nat Phys ***18****, 765 (2022)*
2D Materials ***9****, 045020 (2022)*
Nat commun ***10****, 3047 (2019)*

Non-linear AHE as indicator of topological phase transition

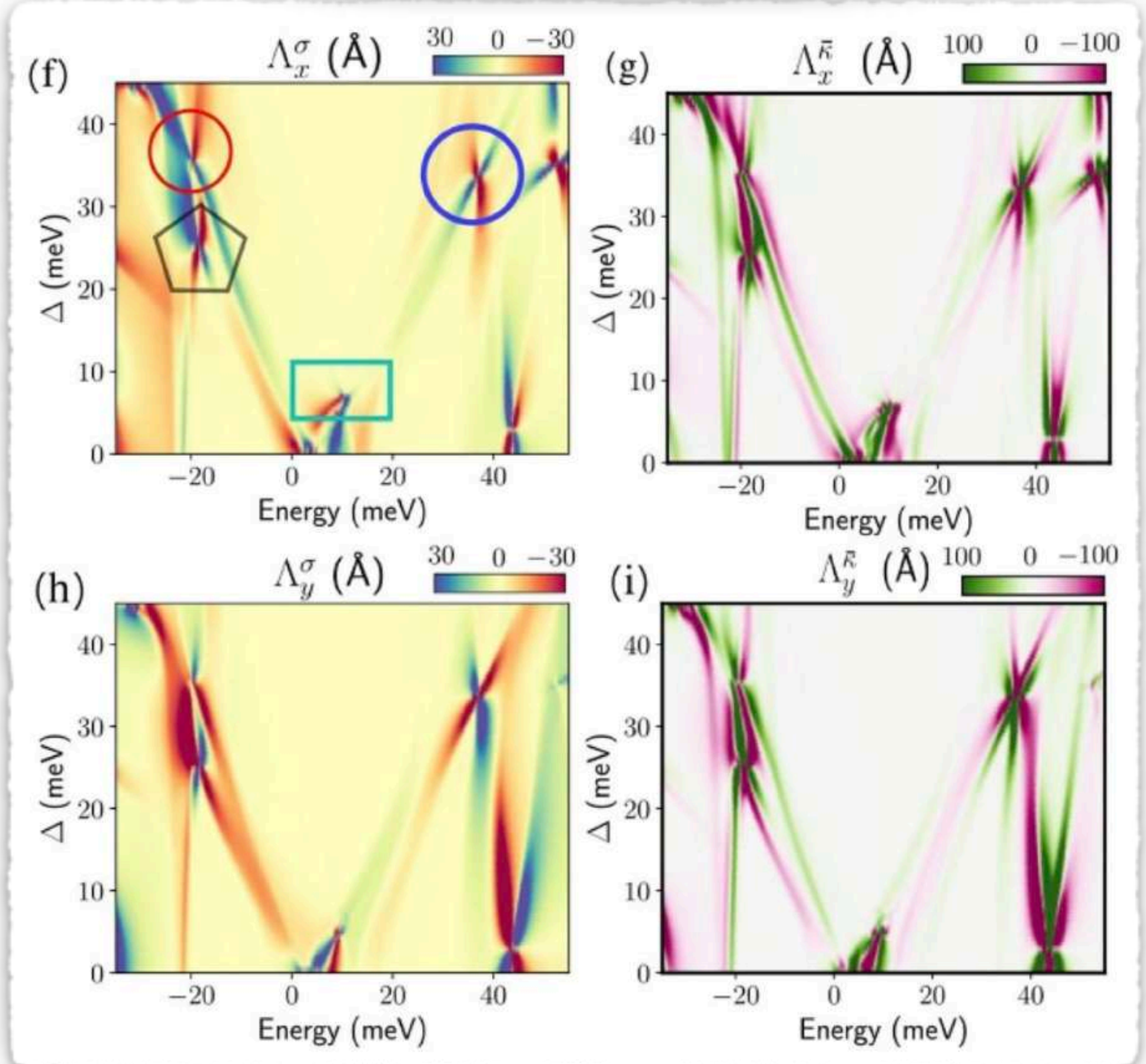
phase diagram of Z_2 index



direct band gap closing



different NLAH coefficients



Summary:

Detection of topological transition through NL anomalous Hall responses not restricted to TDBG only rather is very generic to Moiré as well as for three dimensional time reversal symmetric systems too.

Nat Phys ***18****, 765 (2022)*
2D Materials ***9****, 045020 (2022)*

Acknowledgement

Prof. Amit Agarwal @ IIT Kanpur, India

Dr. Kamal Das @ IIT Kanpur, India
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Dr. L.D. Varma Sangini @ Aalto Univ, Finland
Prof. Kenji Watanabe@ NIMS, Japan
Prof.Takashi Taniguchi @ NIMS, Japan

SERB INDIA

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Thank you