LUKAS KÖRBER // YRLGW 07/2023

Spin waves
in *curved* magnetic shells

TECHNISCHE UNIVERSITÄT DRESDEN

HZDR
HELMHOLTZ ZENTRUM DRESDEN ROSSENDORF

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

DRESDEN concept
SCIENCE AND INNOVATION CAMPUS

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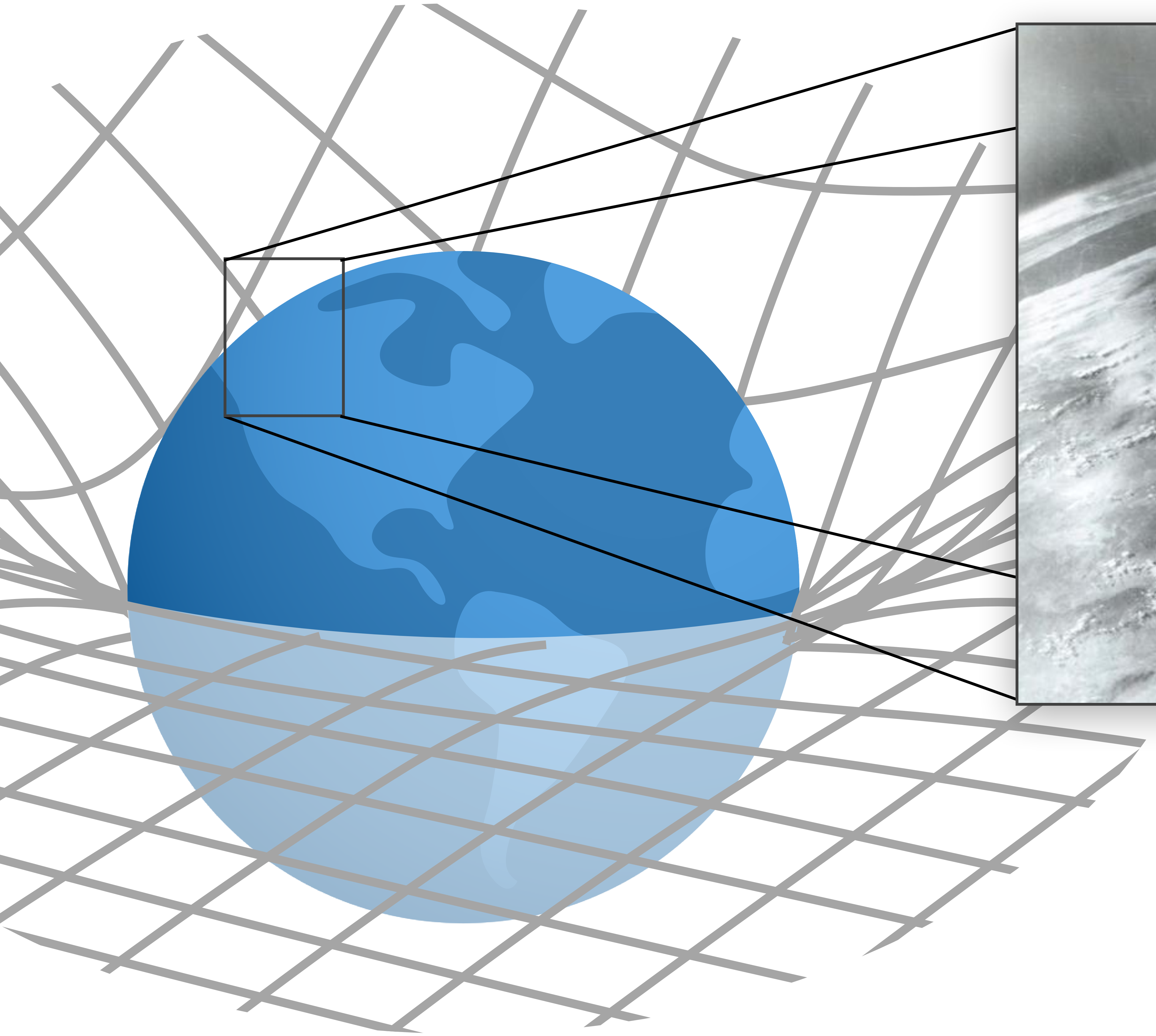
Curvature is a fundamental aspect of our physical world

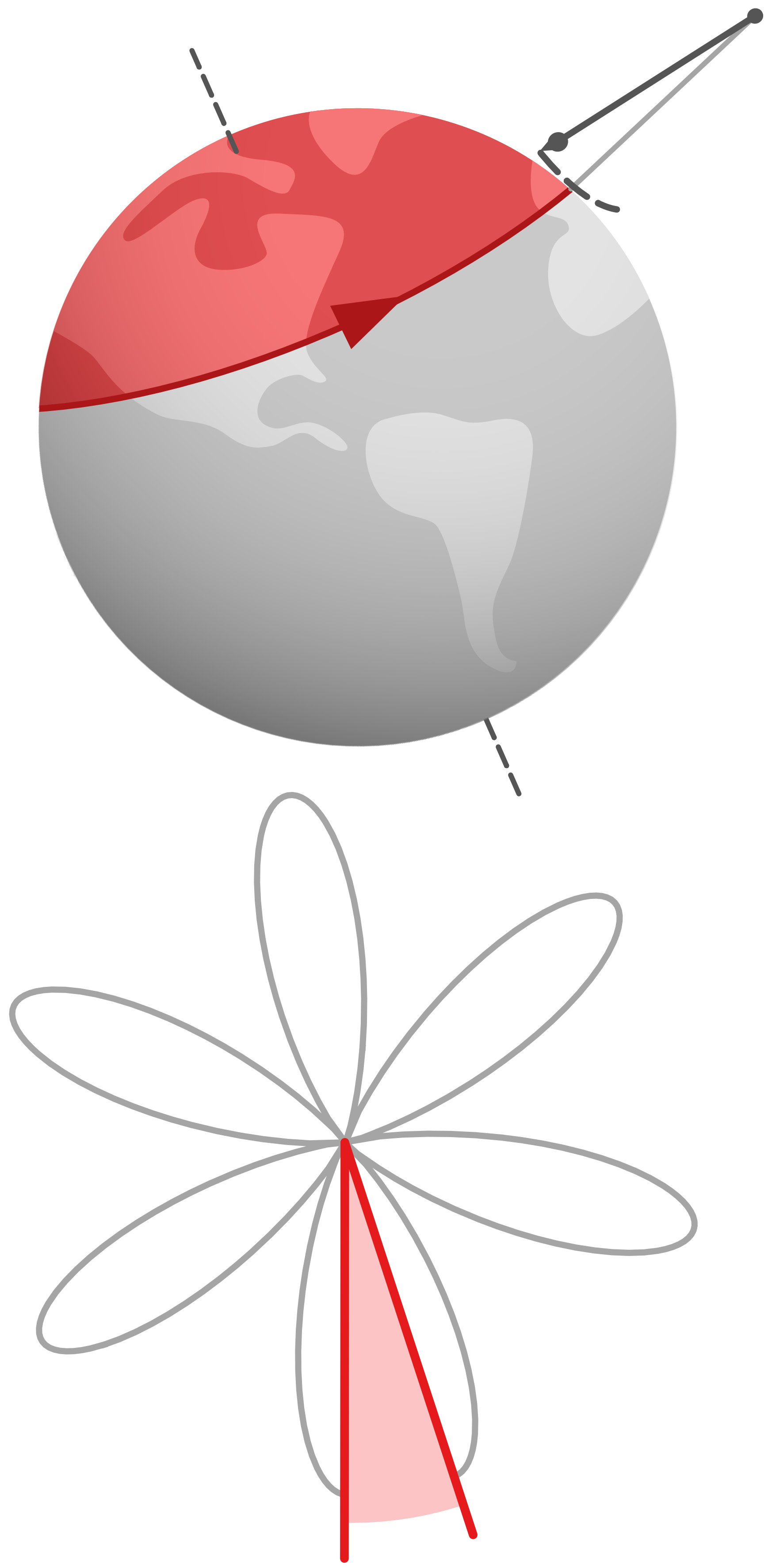


Curvature of spacetime

(public domain)

Photo of earth's curvature

taken in 1947 (using a V-2 rocket)

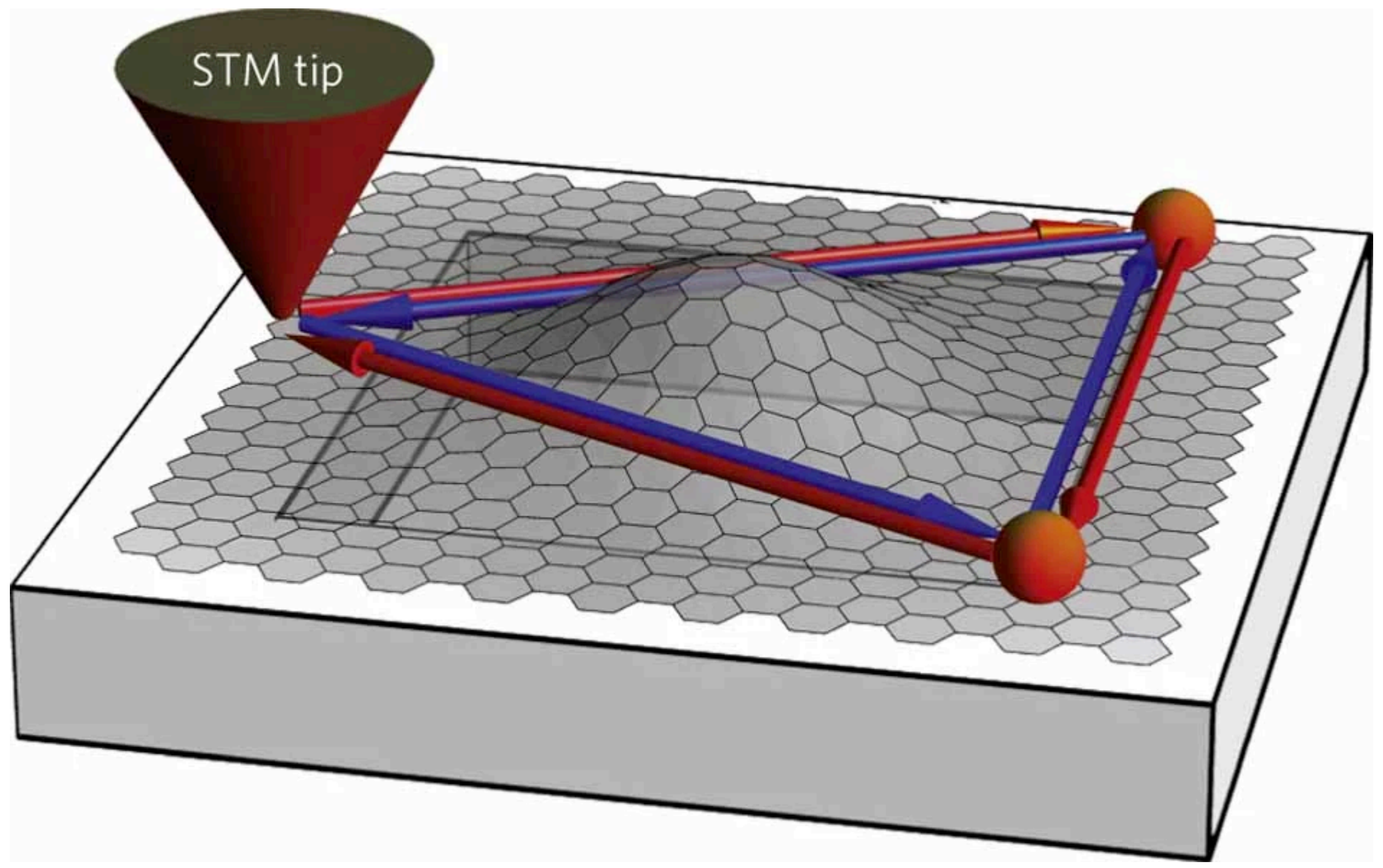


Hannay angle
from curvature

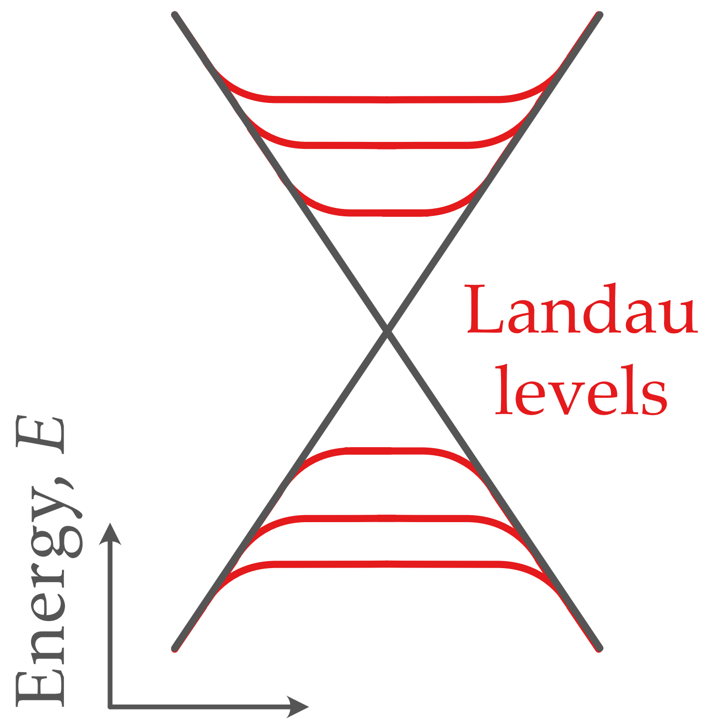
Microscopic properties changed by bending, deforming, rolling up

Strain-induced Berry phase in graphene

Hannay angle of pendulum → Berry phase of electrons



de Juan et al., *Nature Physics* **7**, 810–815 (2011)

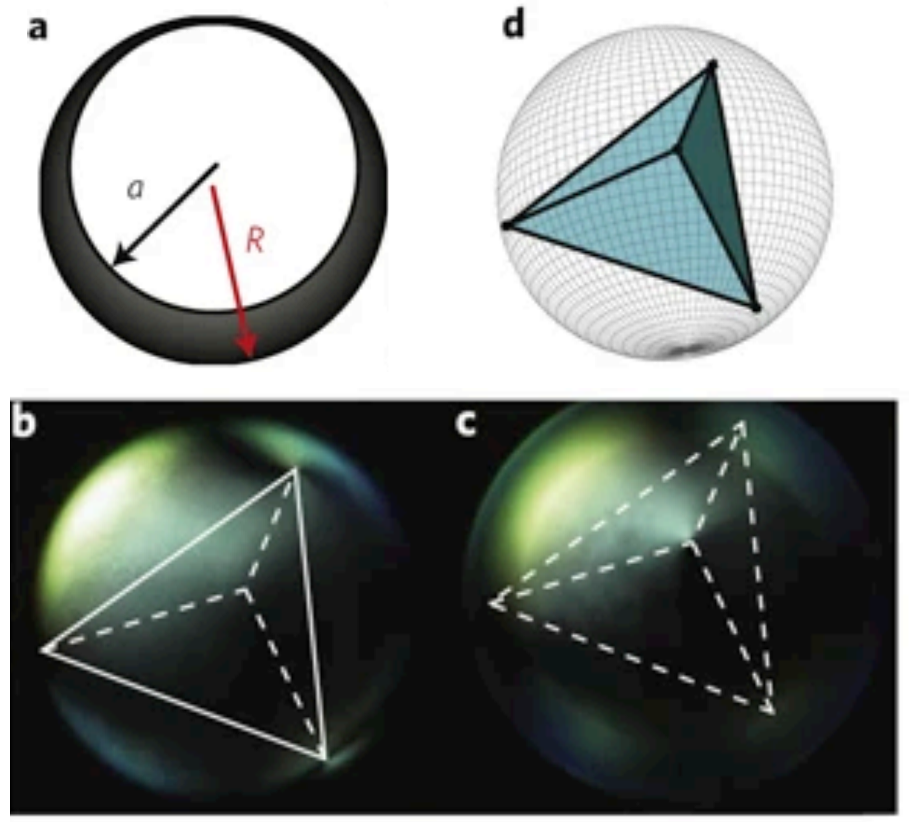


Nigge et al. *Sci. Adv.* **5**, eaaw5593 (2019)

Aharonov-Bohm interference

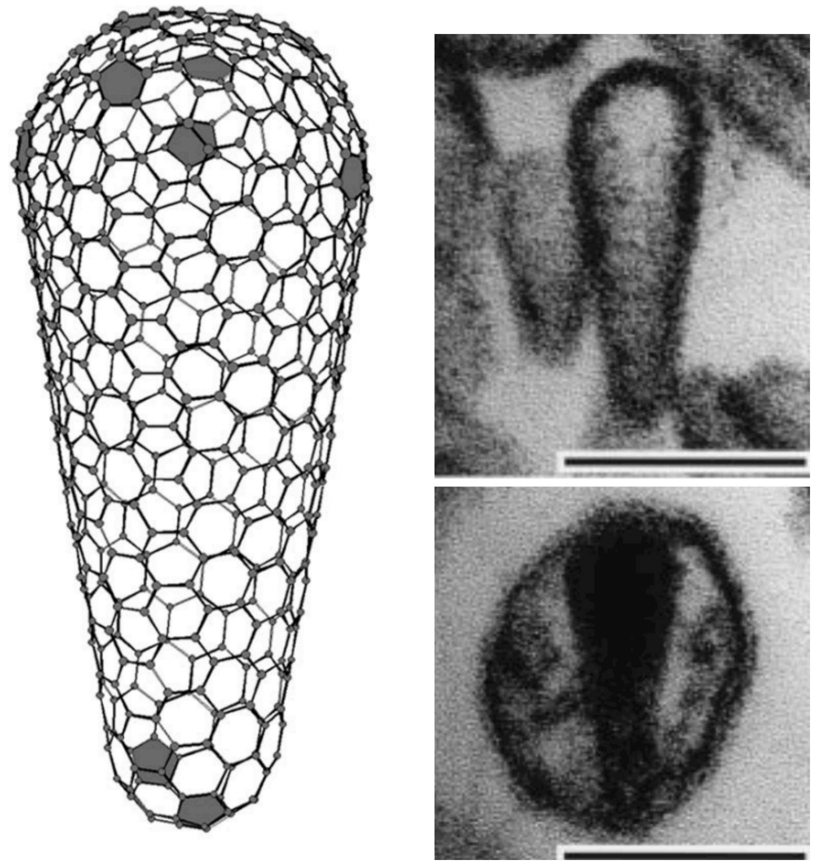
Other examples

geometric frustration in **liquid crystals**



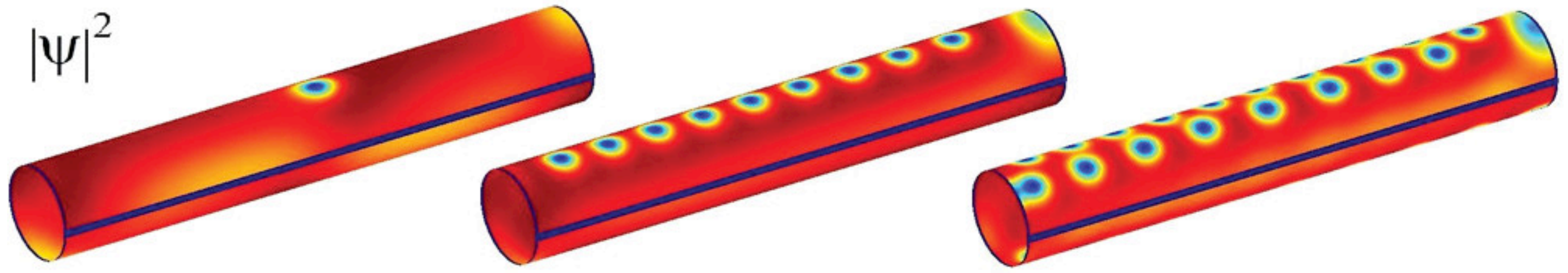
Lopez-Leon *et al. Nature Phys* **7**, 391–394 (2011)

assembly of **viral shells**



Ganser et al., *Science* **283**, 80-83 (1999)

vortex generation in **superconductor tubes**



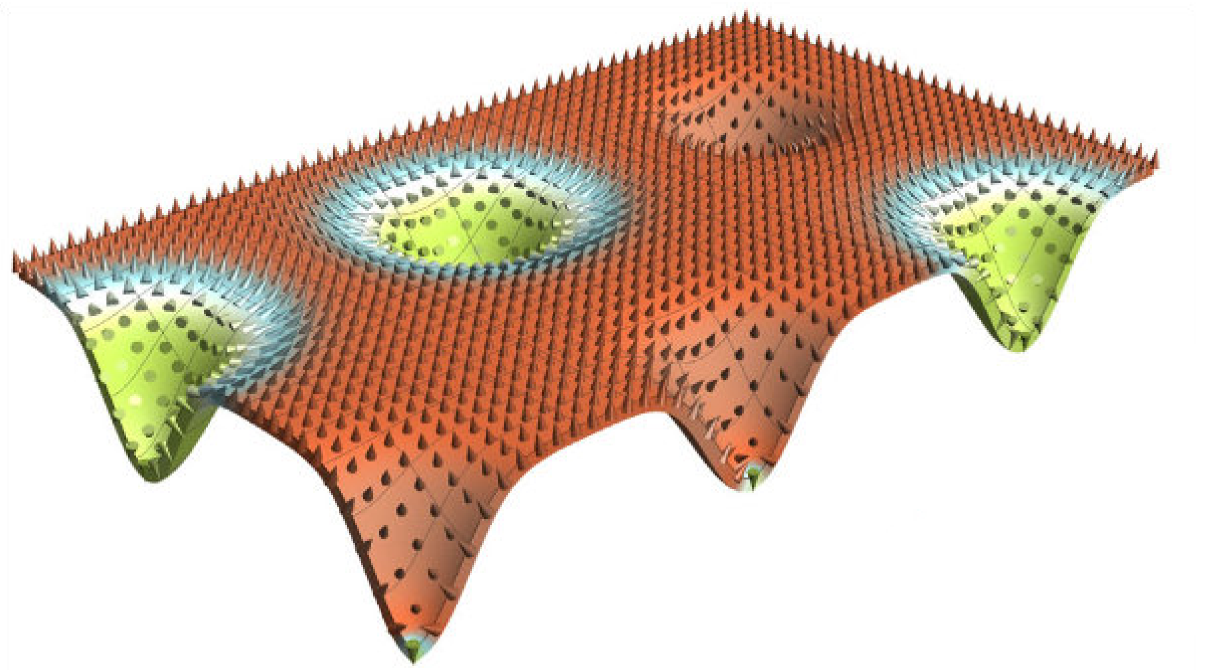
Fomin *et al., Nano Lett.* **12.3**, 1282-1287 (2012)

Magnetic materials are sensitive to curvature, topology etc.

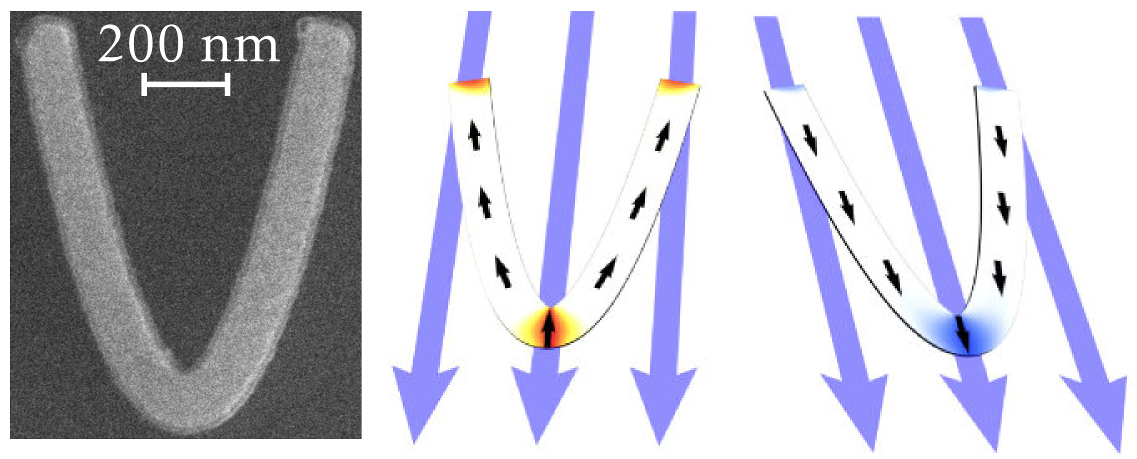
magnetization $\boldsymbol{M}$ = vector order parameter

Symmetry (e.g. parity)

emergent chiral interactions

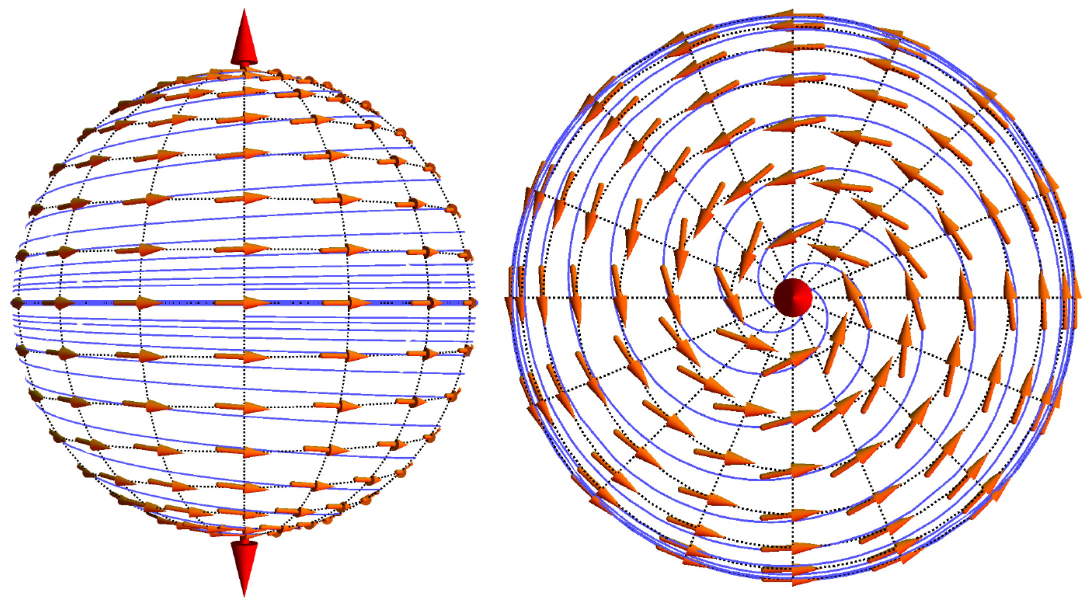


stabilization of skyrmions
Kravchuk *et al.*, PRL **120**, 067201 (2018)

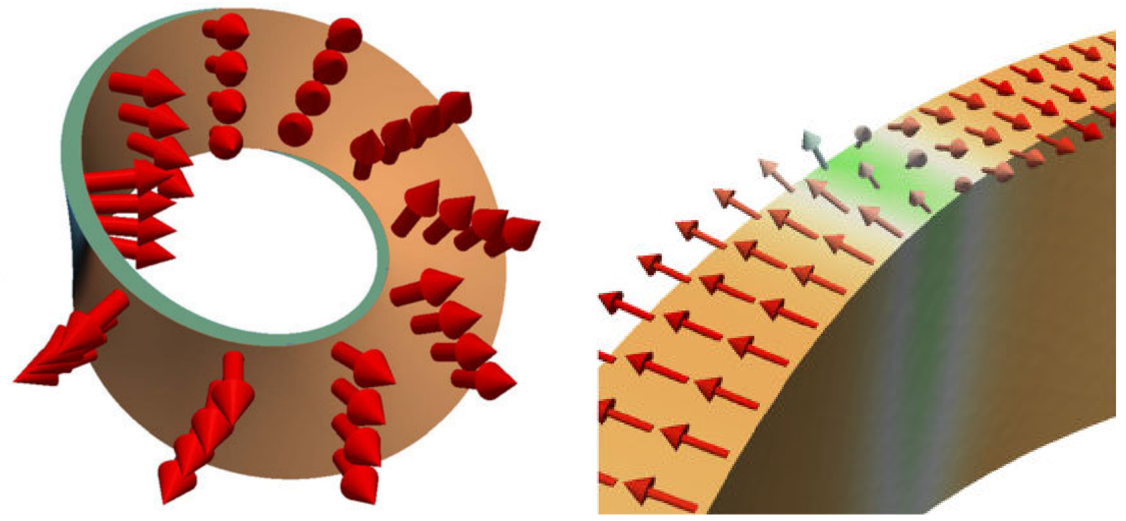


asymmetric domain wall pinning
Volkov *et al.*, PRL **123**, 077201 (2019)

Topology

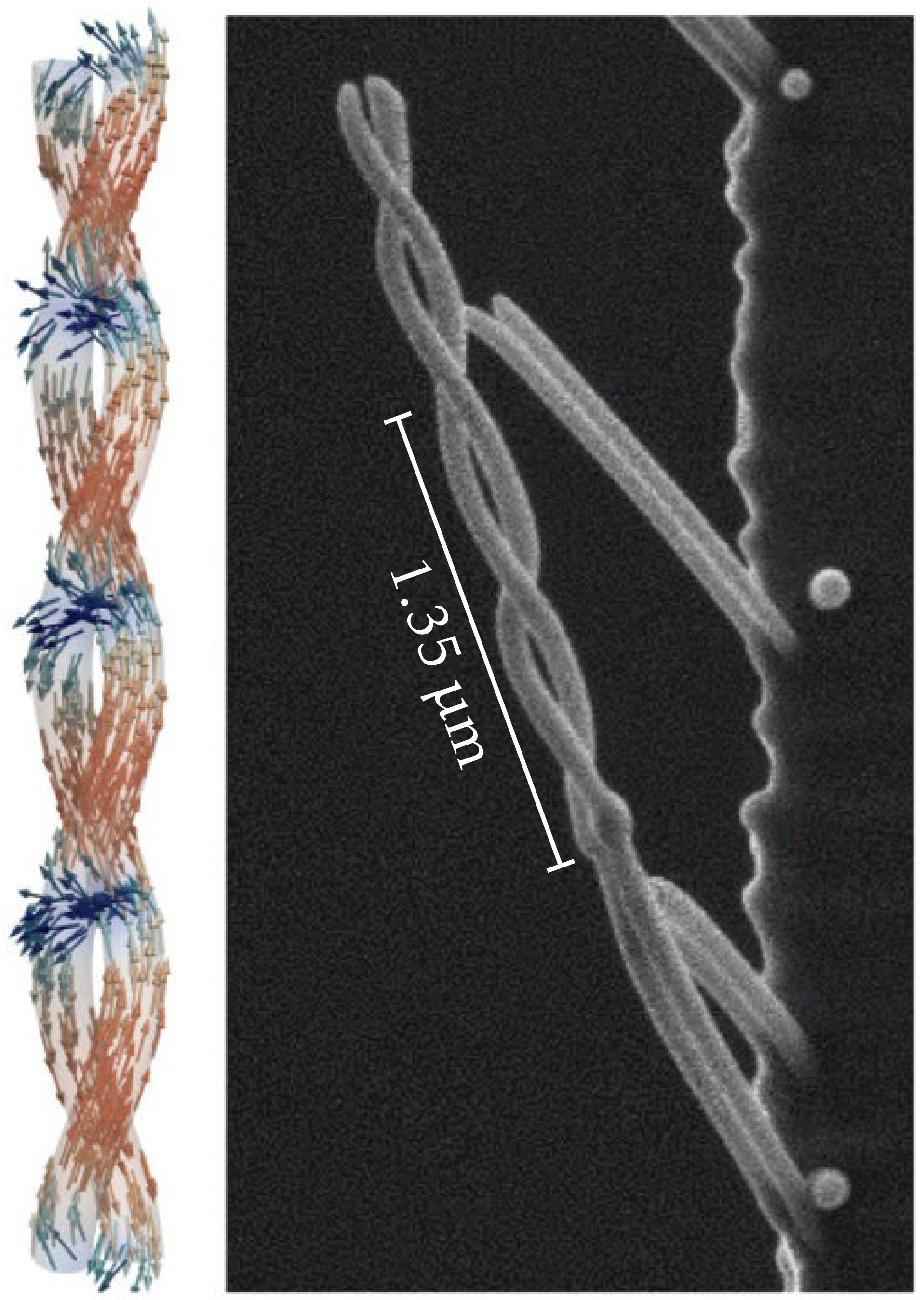


vortex pairs on spherical shells
Kravchuk et al., PRB **85**, 144433 (2012)



topologically-induced domain walls
on Möbius ribbons
Pylypovskyi *et al.*, PRL **114**, 197204 (2015)

other geometrical effects

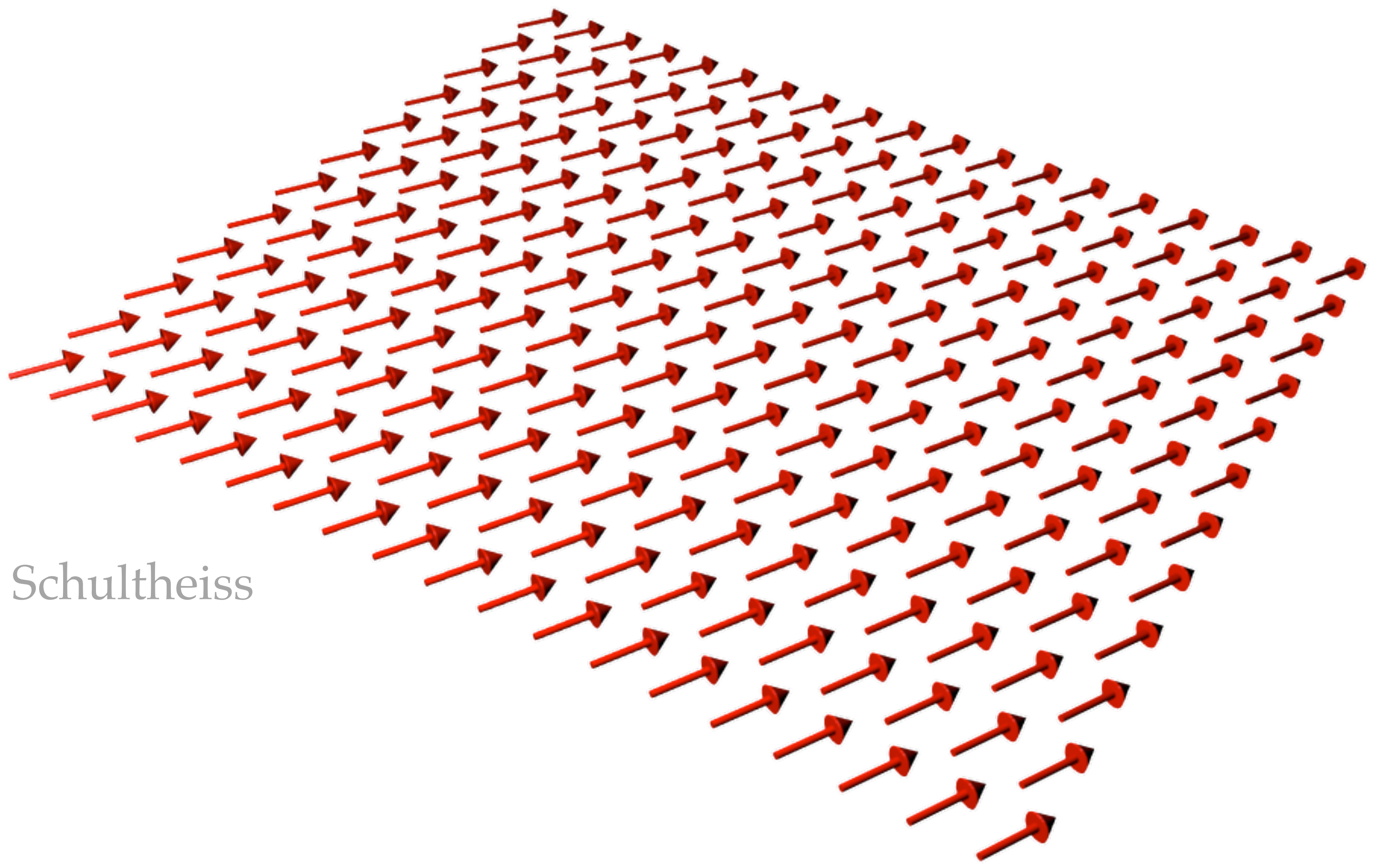


locked domain walls
in double helices
Donnelly *et al.*, Nat. Nano
17, 136–142 (2022)

effects relevant on nm to μm scale → technologically relevant

Statics is nice… but what about dynamics?

What is a spin wave?



© H. Schultheiss

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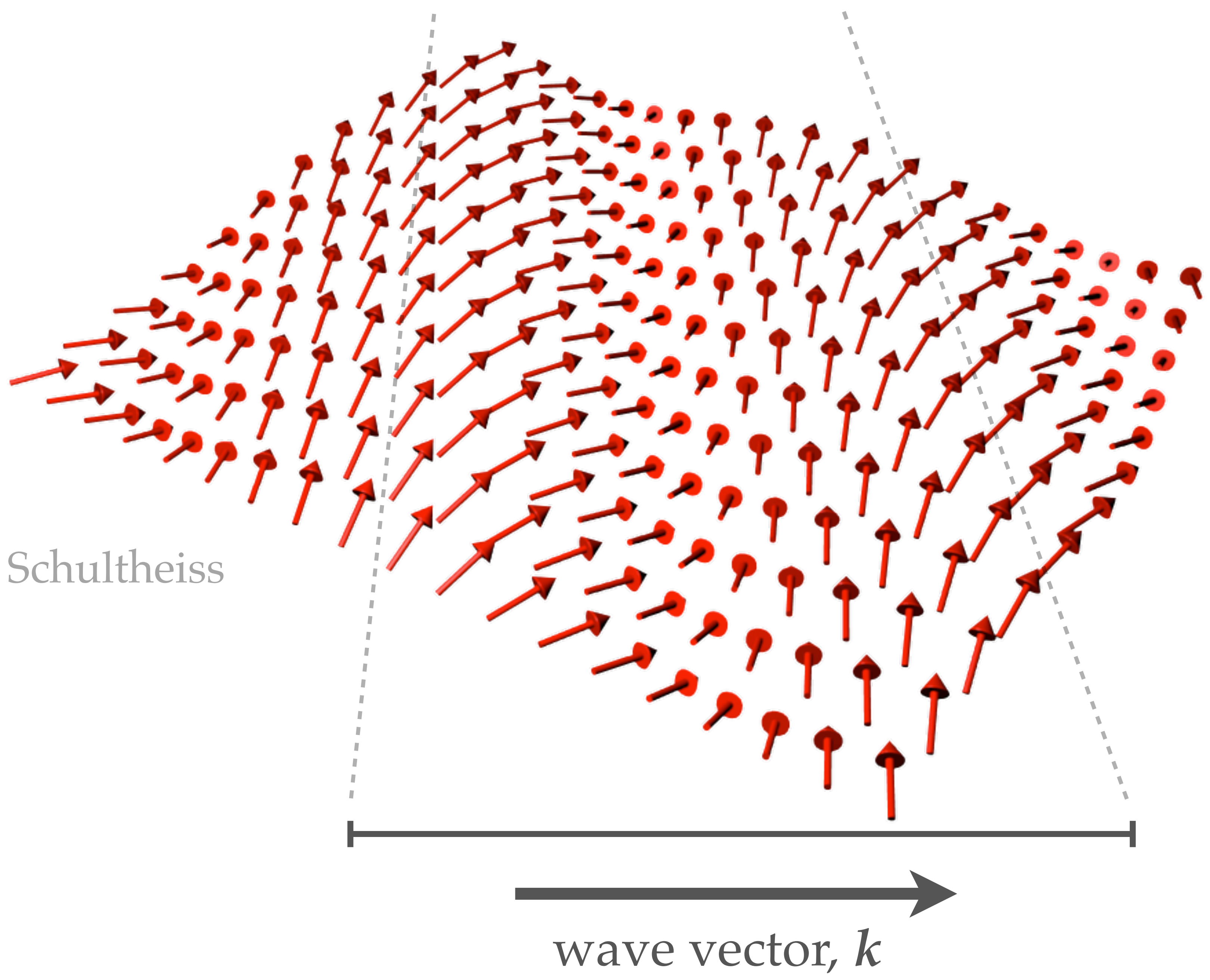
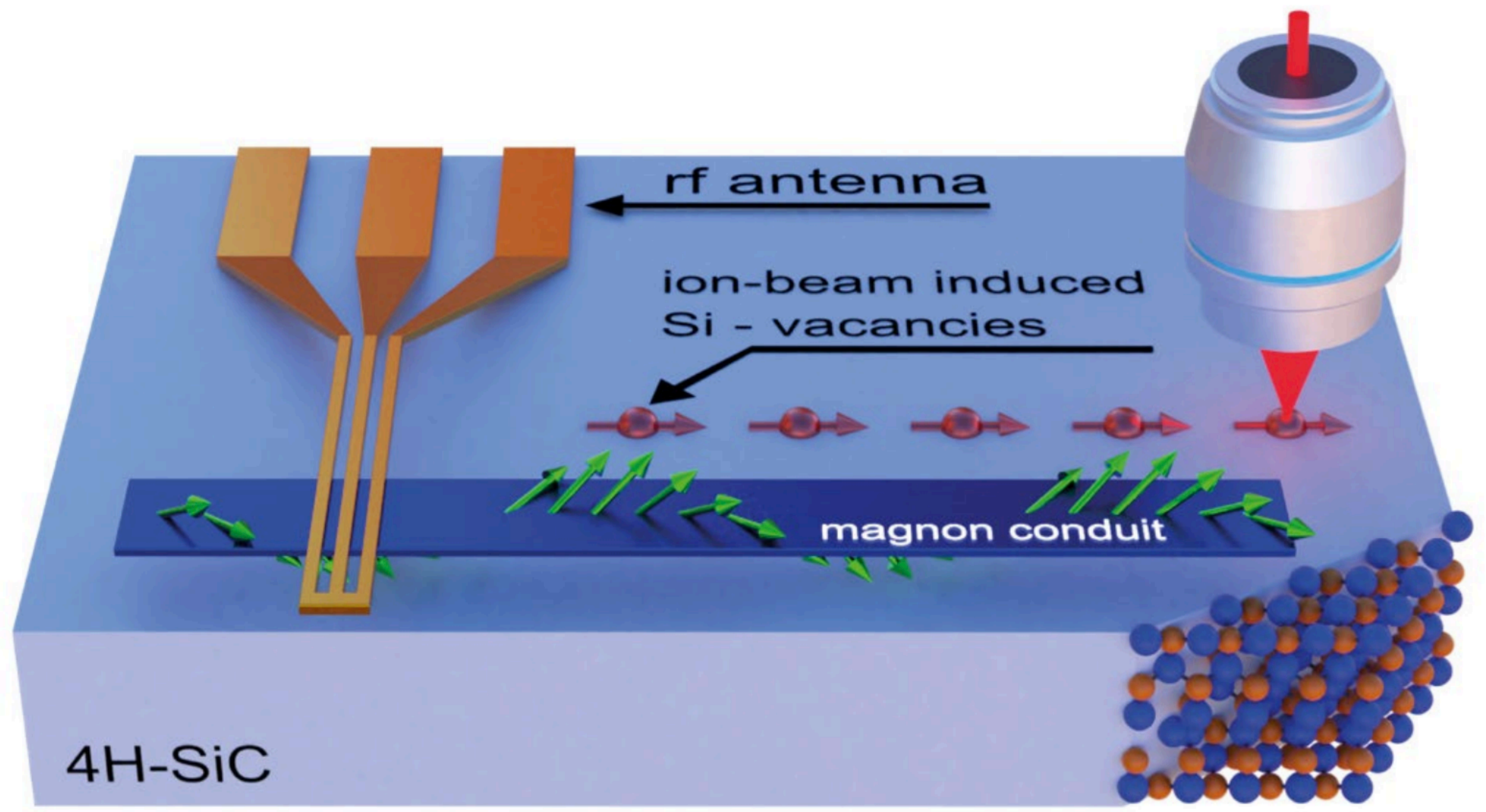
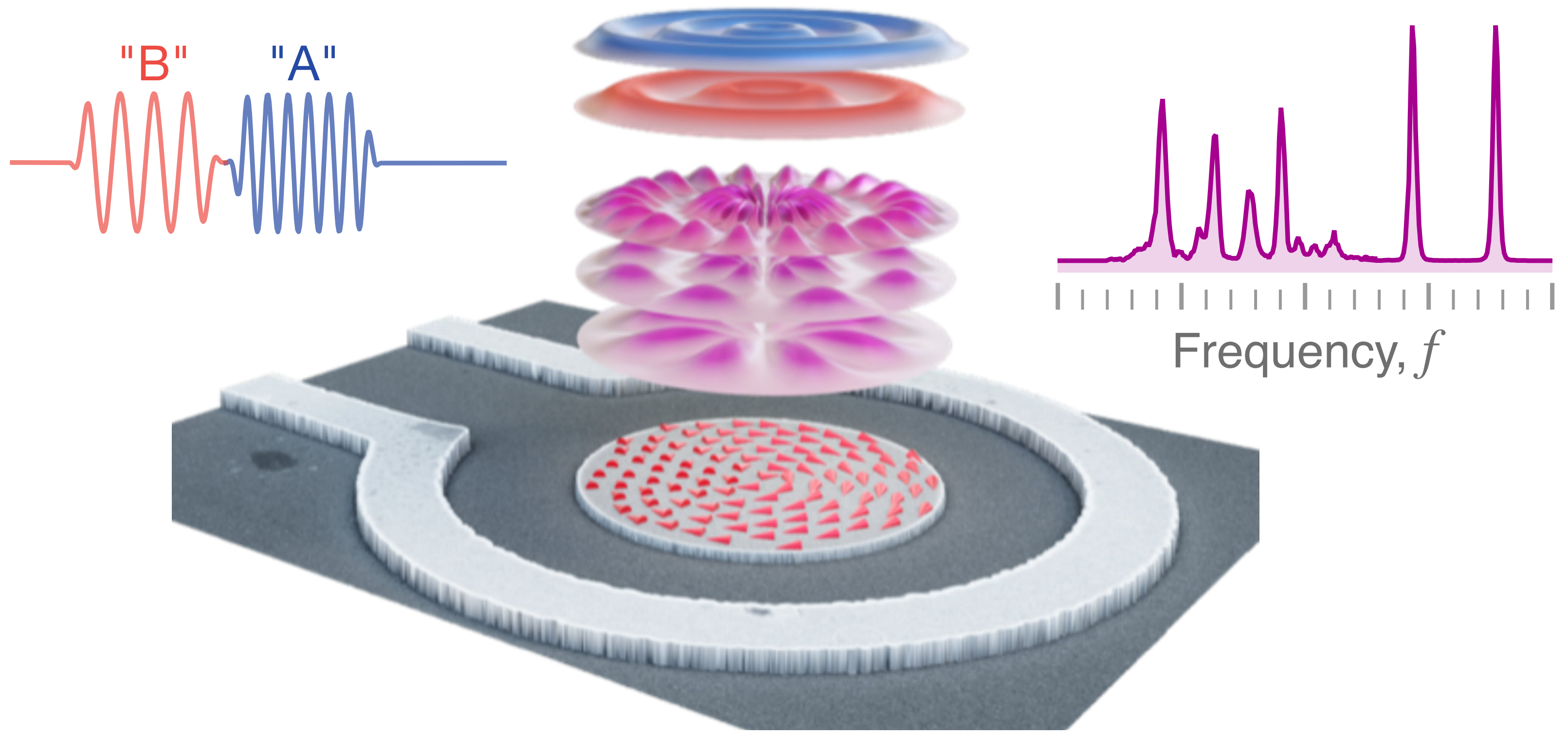


How they might be useful (examples)



…as interconnects to spin qubits

Bejarano *et al.*, arXiv:2208.09036v1 (2023)



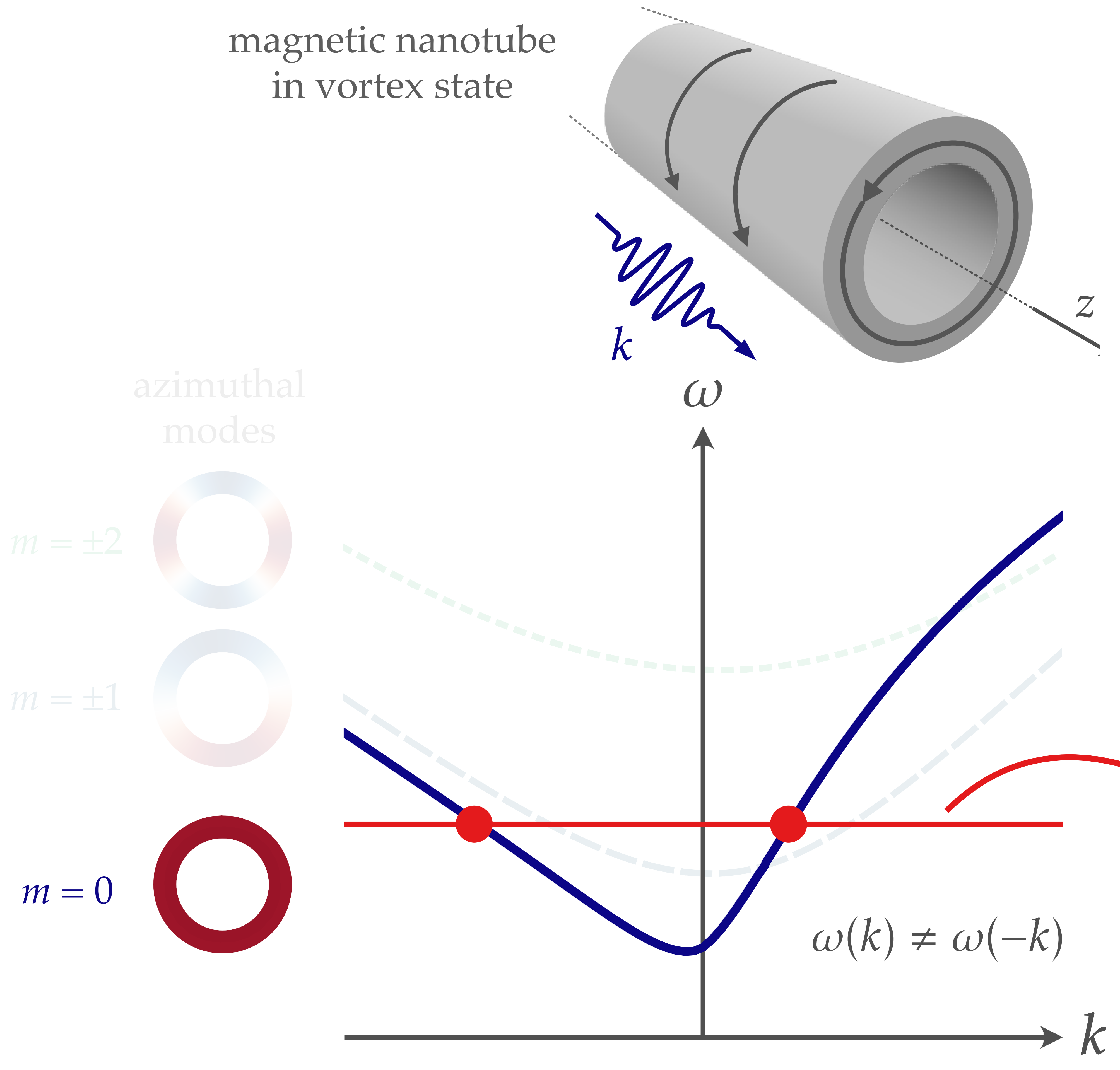
…for reservoir computing

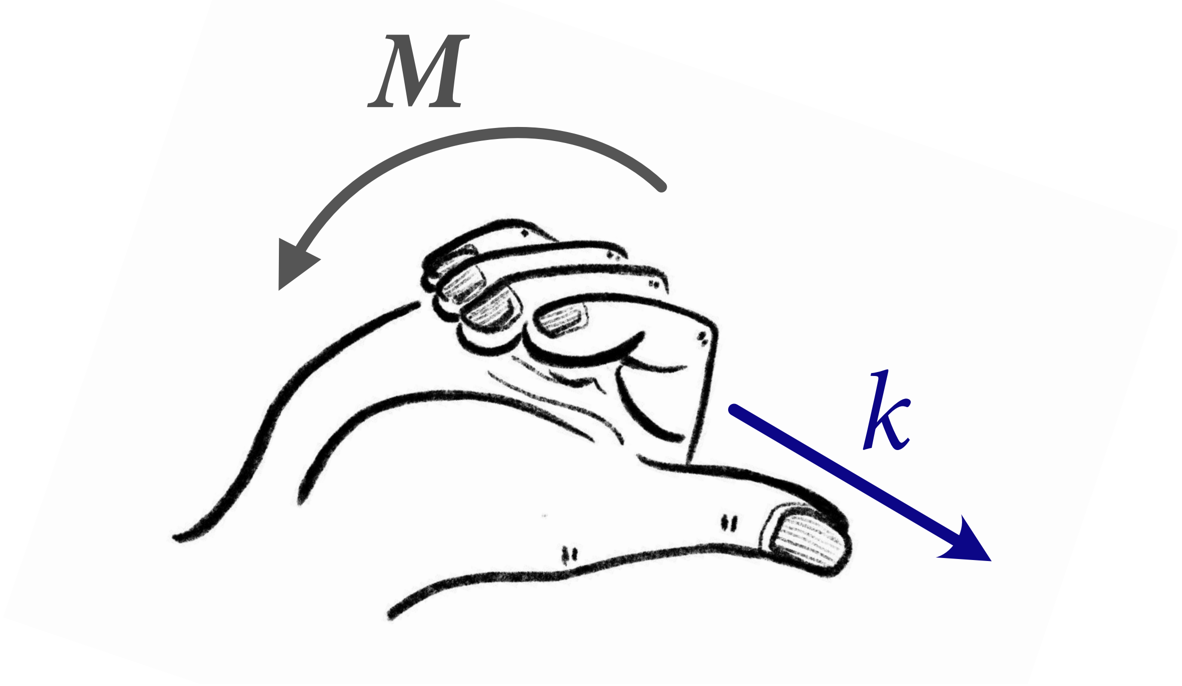
Körber, Heins *et al.*, accepted at Nat. Comm (2023)

Spin waves are also sensitive to geometrical effects

Example: Curvature-induced dispersion asymmetry

Otálora *et al.*, PRL **117**, 227203 (2016)

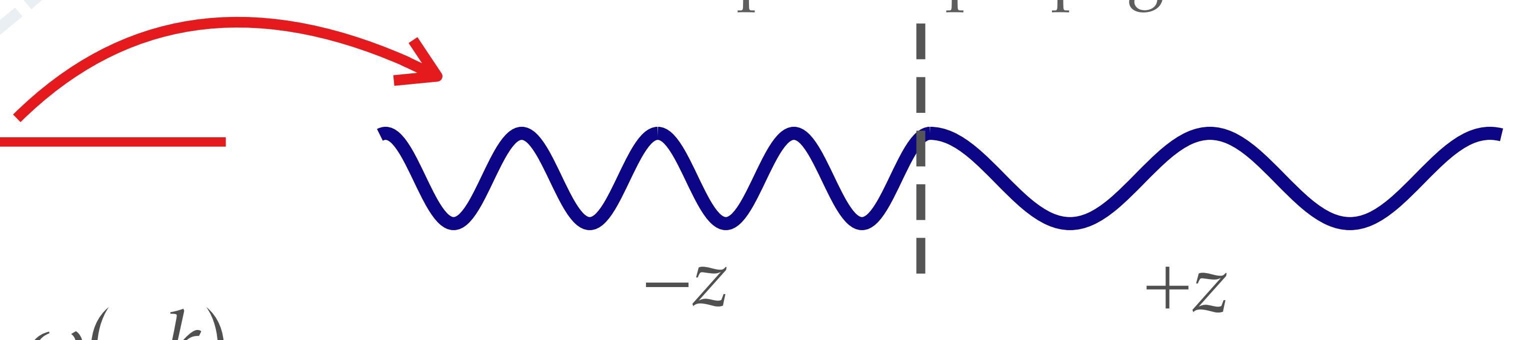




Curvature leads to

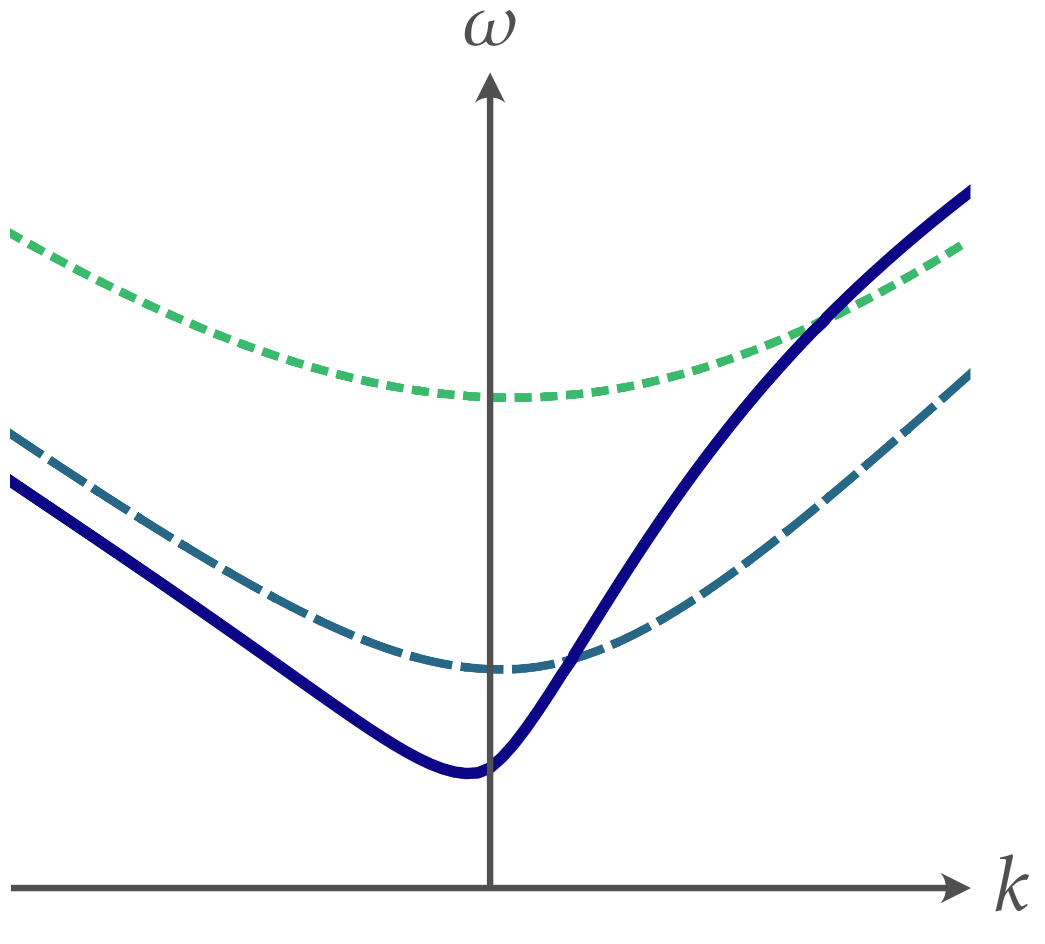
Nonlocal chiral symmetry breaking

(related to dipole-dipole interactions).



Magnetization dynamics in micromagnetic continuum theory

What governs spin-wave dispersion?



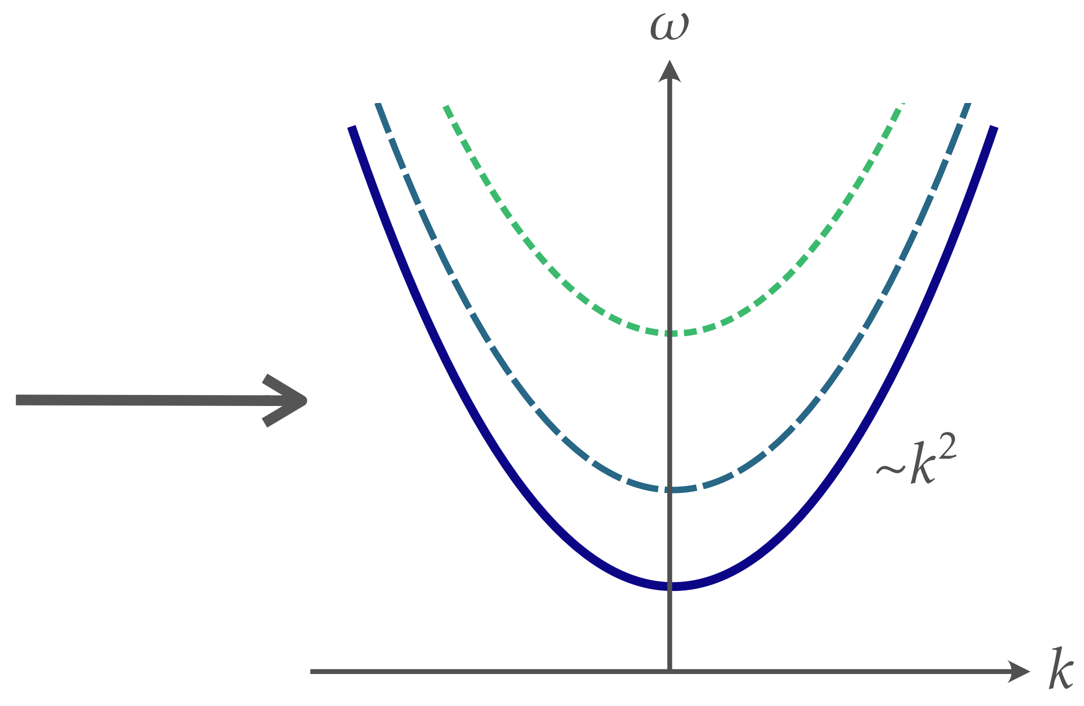
Continuum limit: Landau-Lifshitz-Gilbert equation (LLG)

$$\frac{\mathrm{d}\boldsymbol{M}}{\mathrm{d}t} = -\gamma \boldsymbol{M} \times \boldsymbol{H}_{\text{eff}}(\boldsymbol{M})$$

$$\boldsymbol{H}_{\text{eff}} = \boldsymbol{H}_{\text{exc}} + \boldsymbol{H}_{\text{dip}} + \boldsymbol{H}_{\text{ani}} + \boldsymbol{H}_{\text{ext}} + \ldots$$

Exchange interaction
(from Pauli exclusion principle)

strong & short-range (local)



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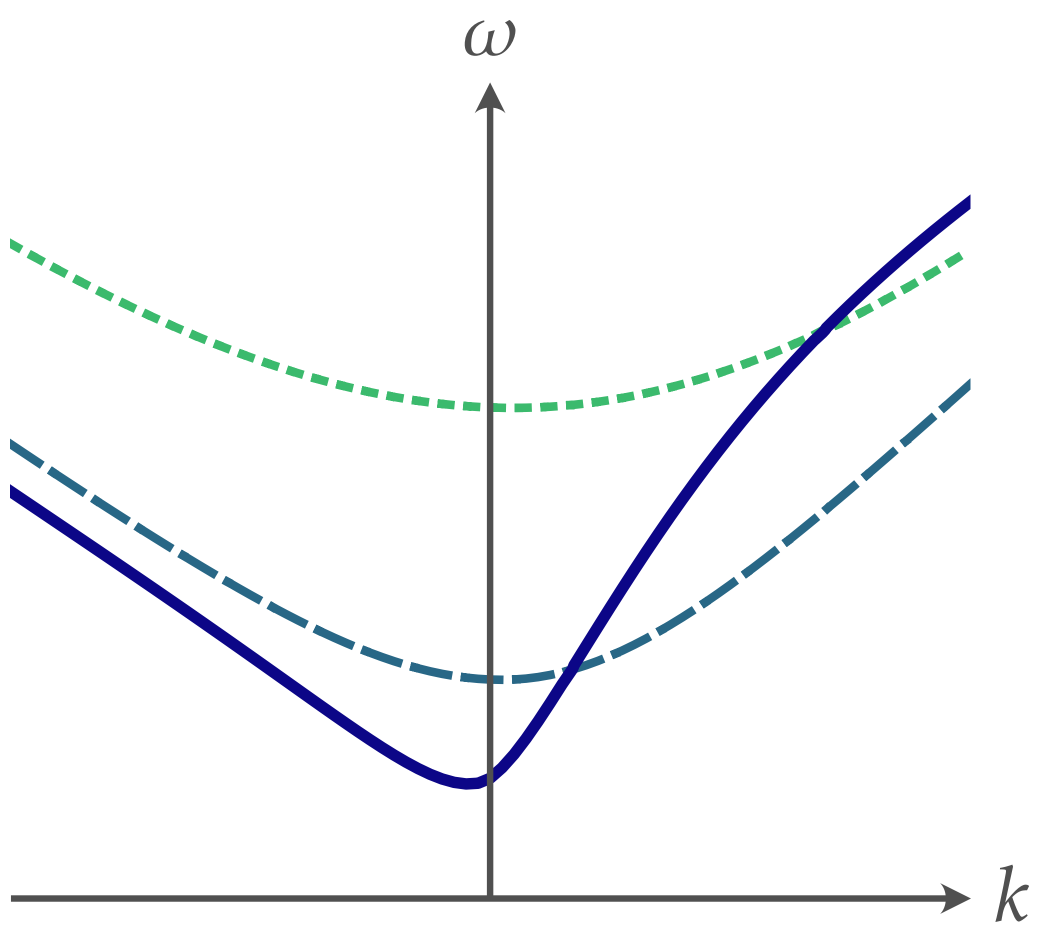

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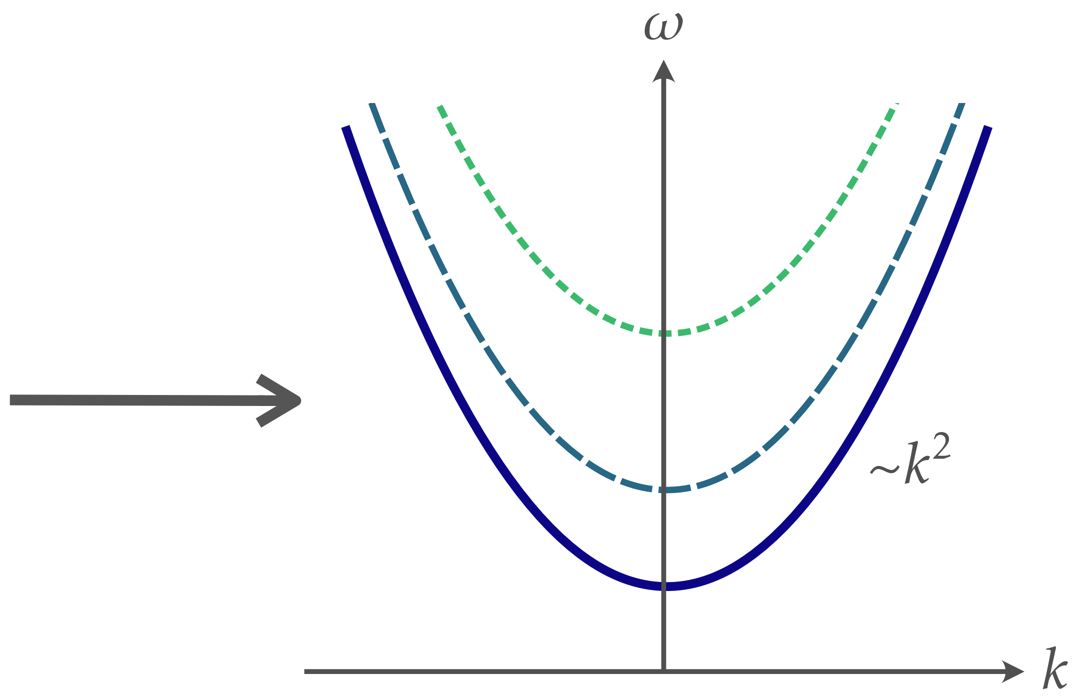
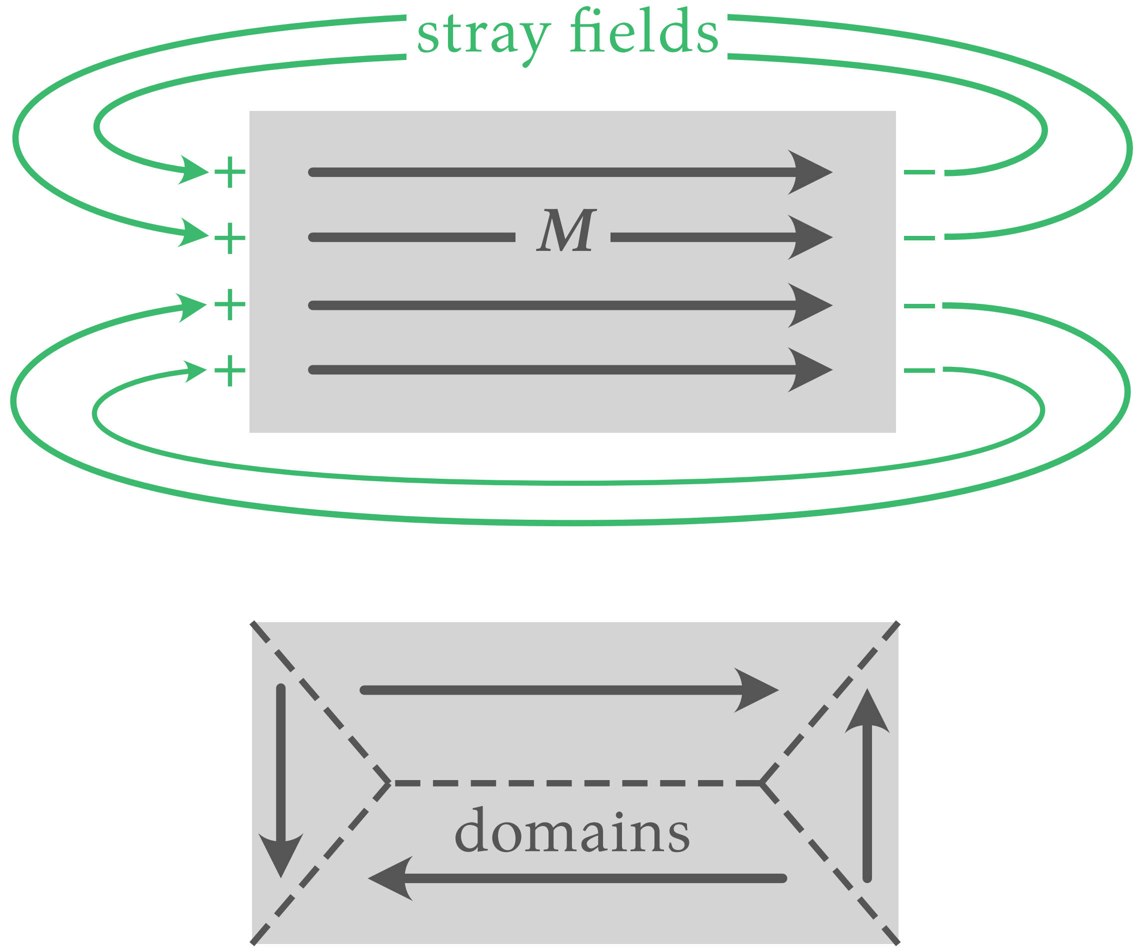


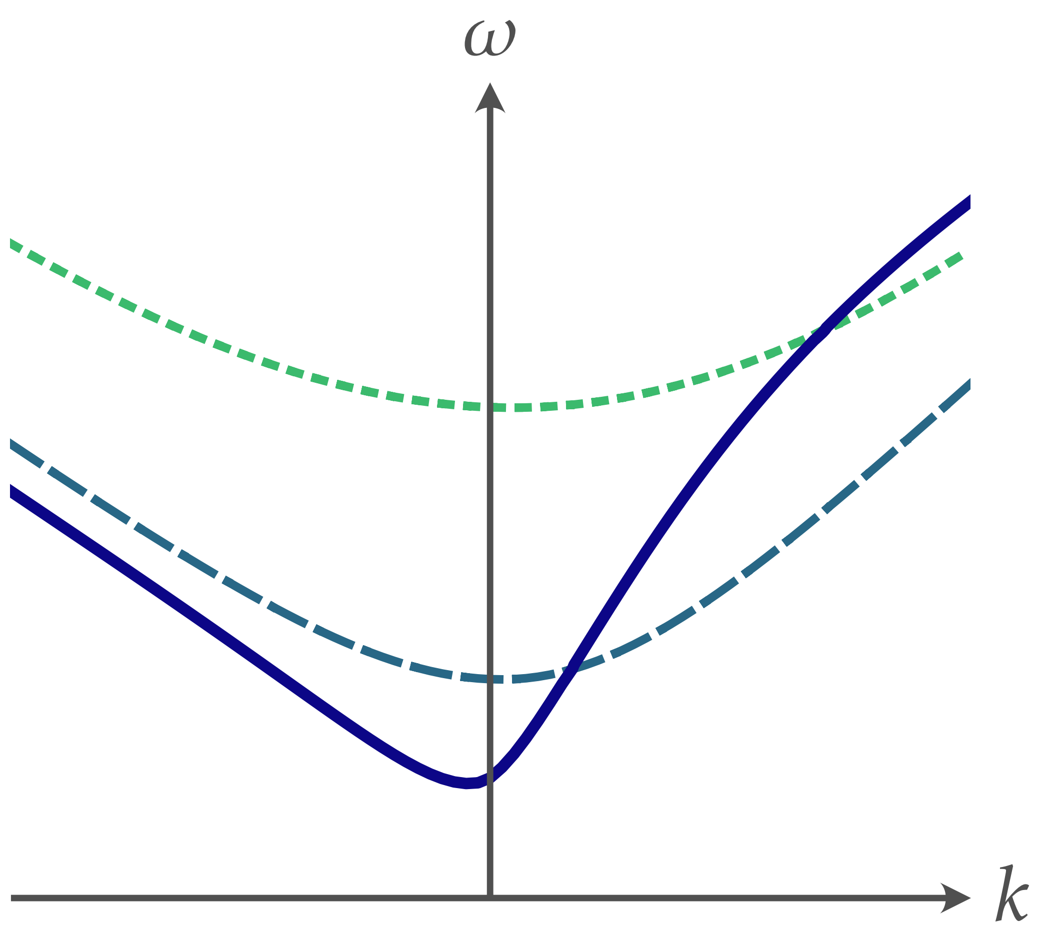
Dipole-dipole interaction



weak & long-range (non-local)

Magnetization dynamics in micromagnetic continuum theory

What governs spin-wave dispersion?



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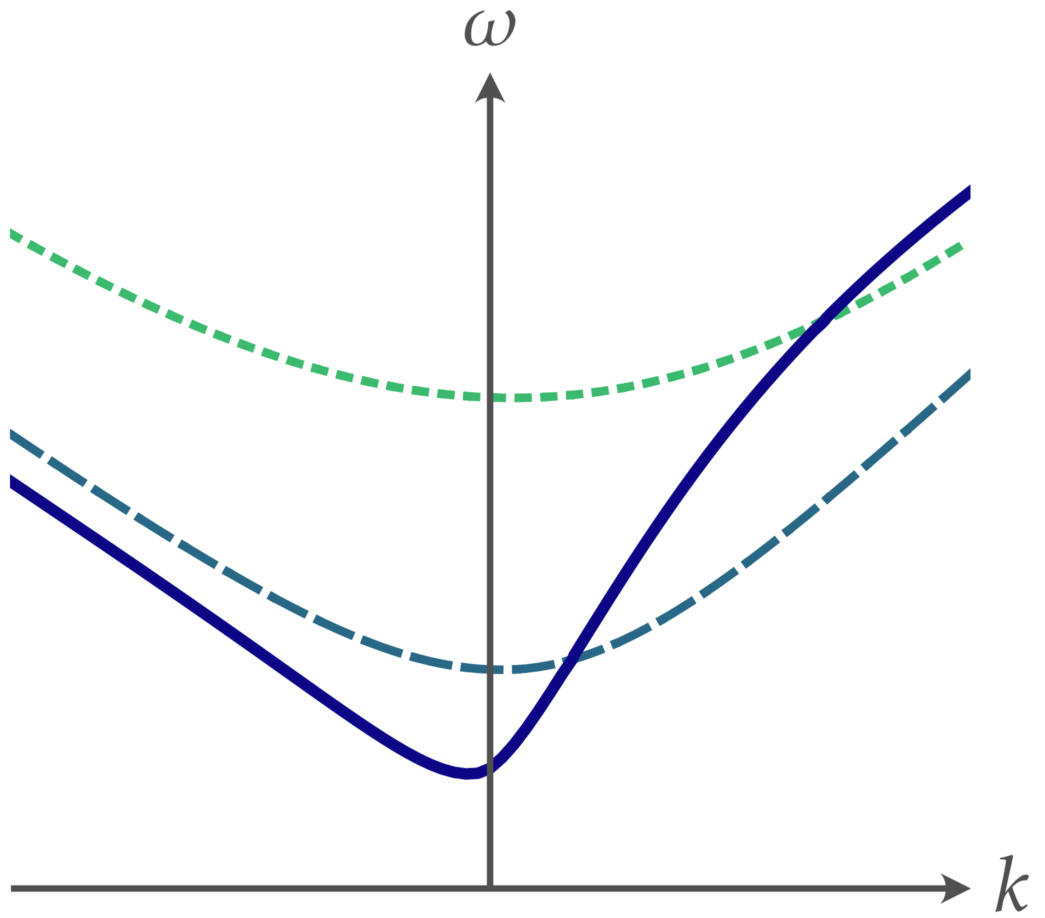
Magnetic pseudocharges

Magnetization dynamics in micromagnetic continuum theory

What governs spin-wave dispersion?



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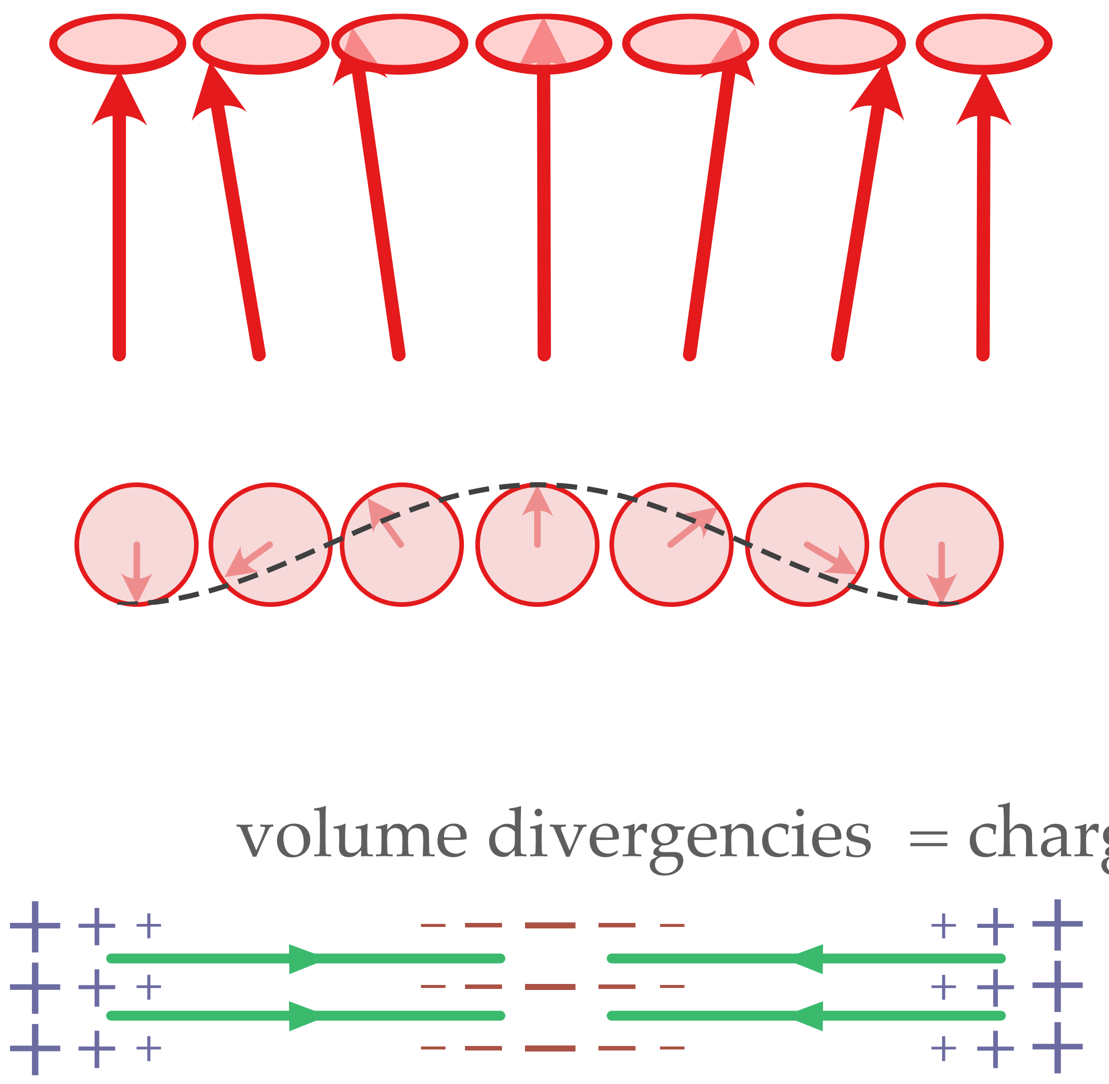
Magnetic pseudocharges



volume divergencies = charges

$\nabla \boldsymbol{M} \neq \nabla \boldsymbol{B}$

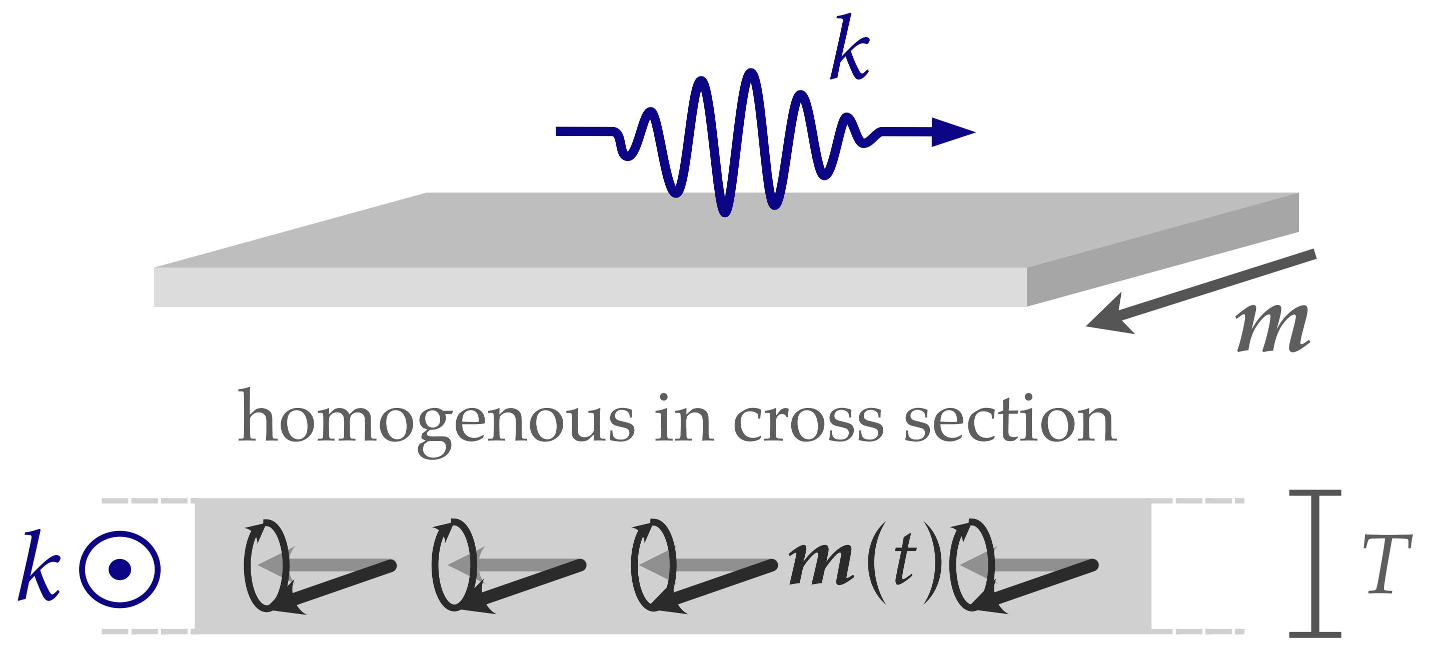
$\boldsymbol{H}_{\text{dip}} = -\nabla\Phi$

internal dipolar field

(magnetostatic Maxwell/Poisson)

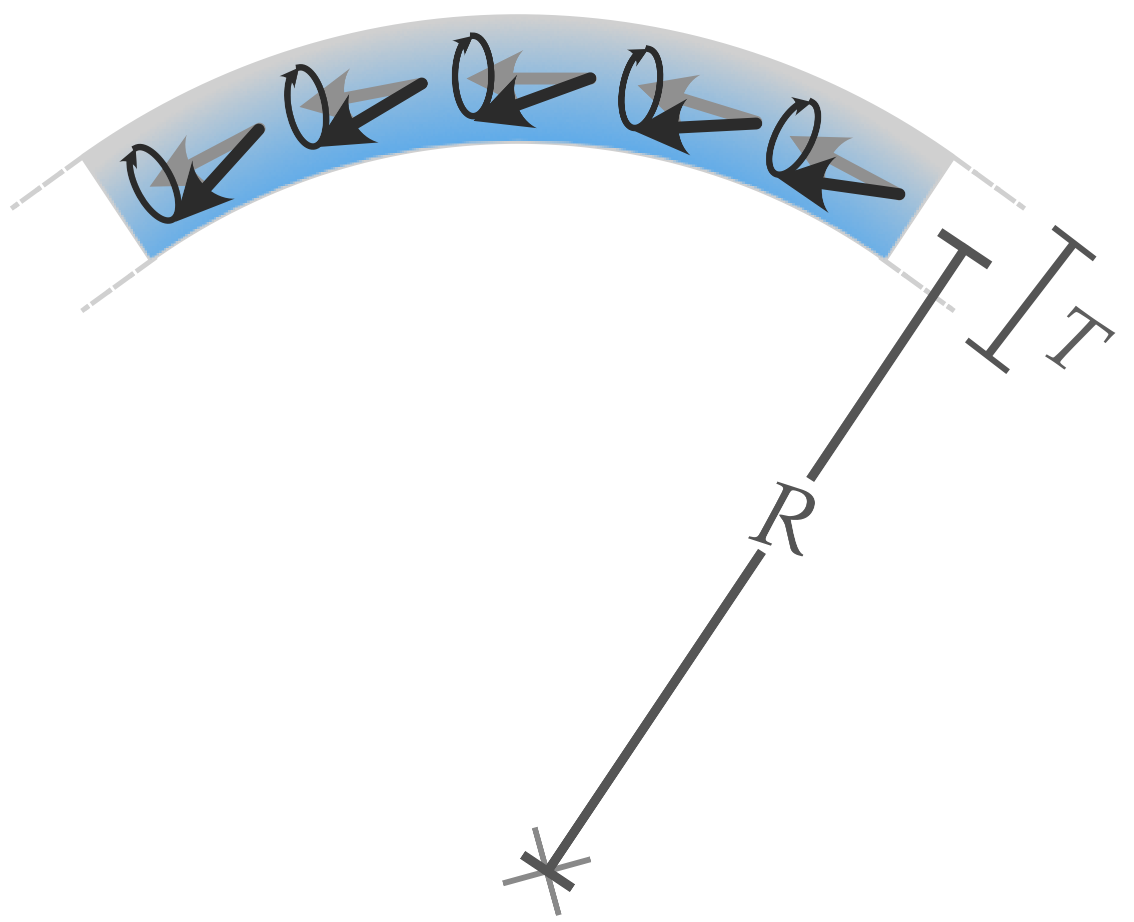
$$\Delta\Phi = \begin{cases} \nabla \boldsymbol{M} & \text{inside} \\ 0 & \text{outside} \end{cases}$$

Nonlocal chiral symmetry breaking due to geometric charges



Homogeneous along normal direction!
(otherwise not analytically solvable)

dynamic volume charges



Volume charges

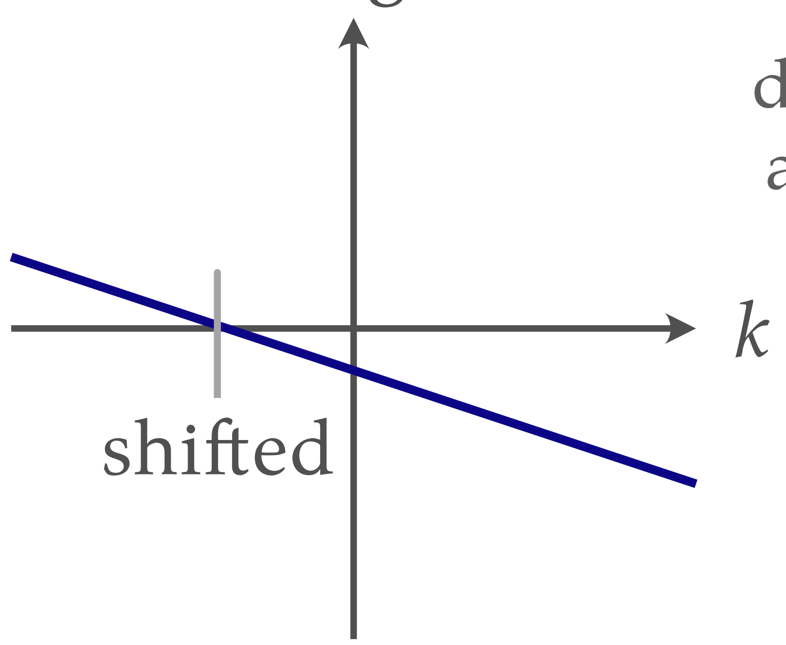
(in curvilinear frame of reference)

Mean curvature
(of central surface)
tube: $\mathcal{H}_0 = \frac{1}{R}$

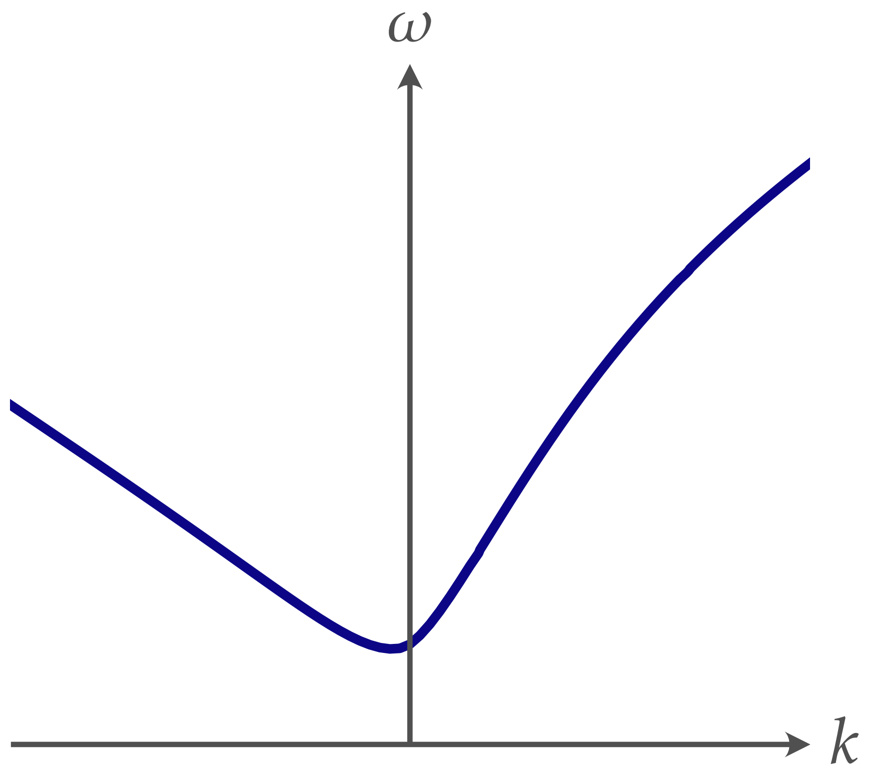
$$\nabla \boldsymbol{m} = \underbrace{(\partial_1 m^1 + \partial_2 m^2)}_{\text{spatial variation}} + \underbrace{\mathcal{H}_0 m_n}_{\text{geometric charges}}$$

Sheka *et al.*, Comm. Phys. 3, 128 (2020)

$$\nabla \boldsymbol{m} \sim k + \frac{1}{R} \quad \text{for thin-shell tube}$$



developed analytical model



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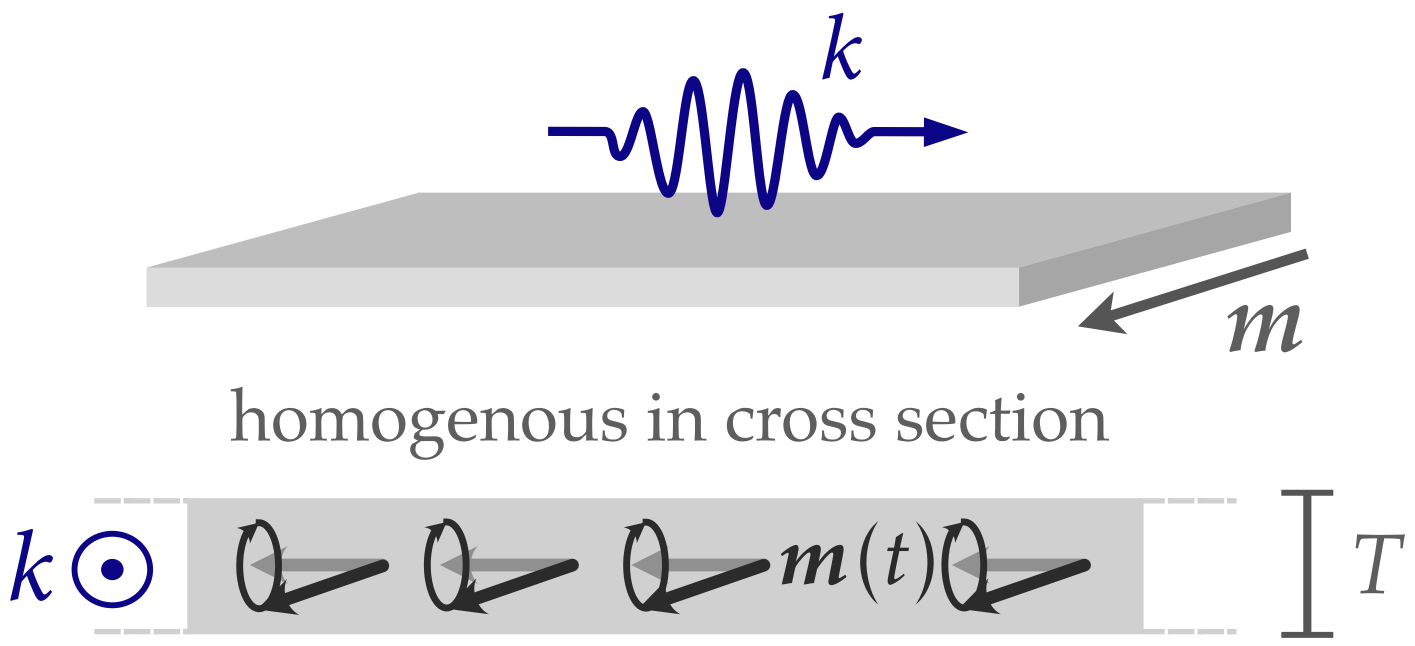

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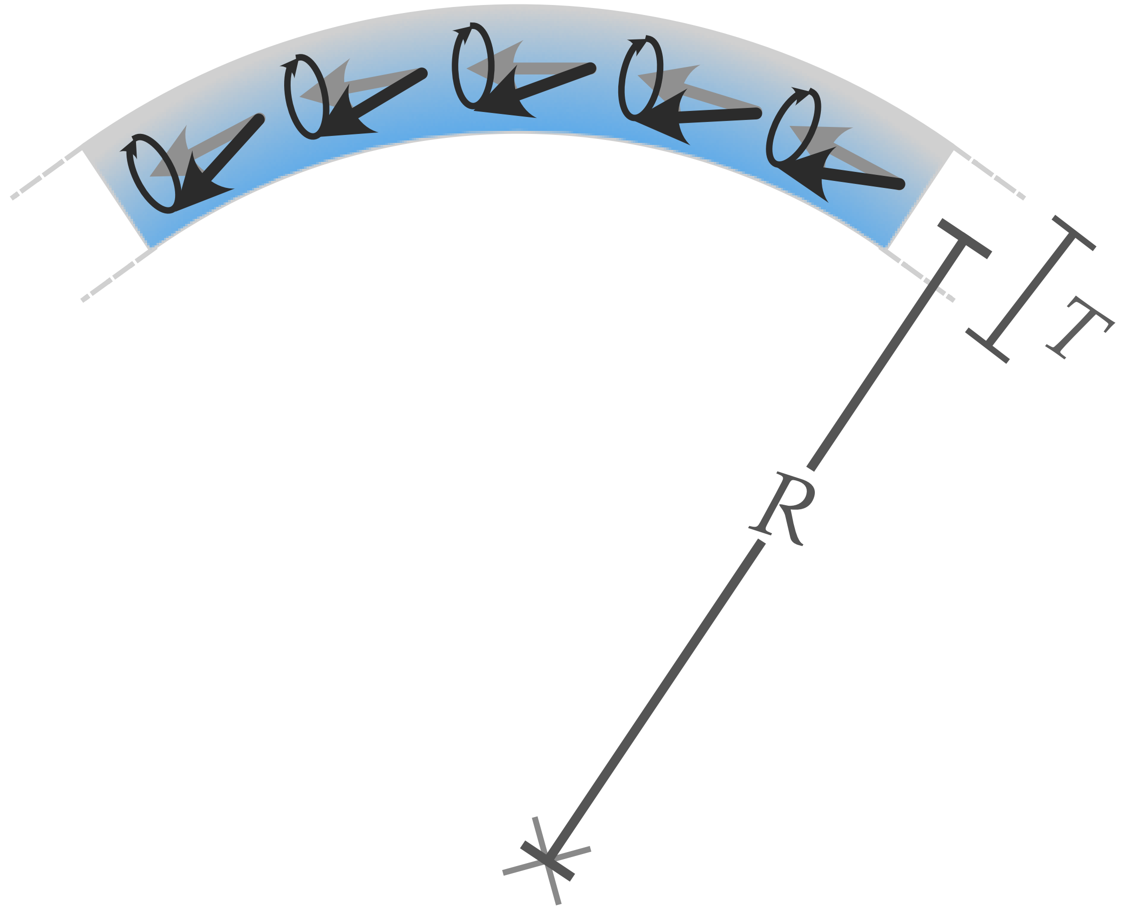

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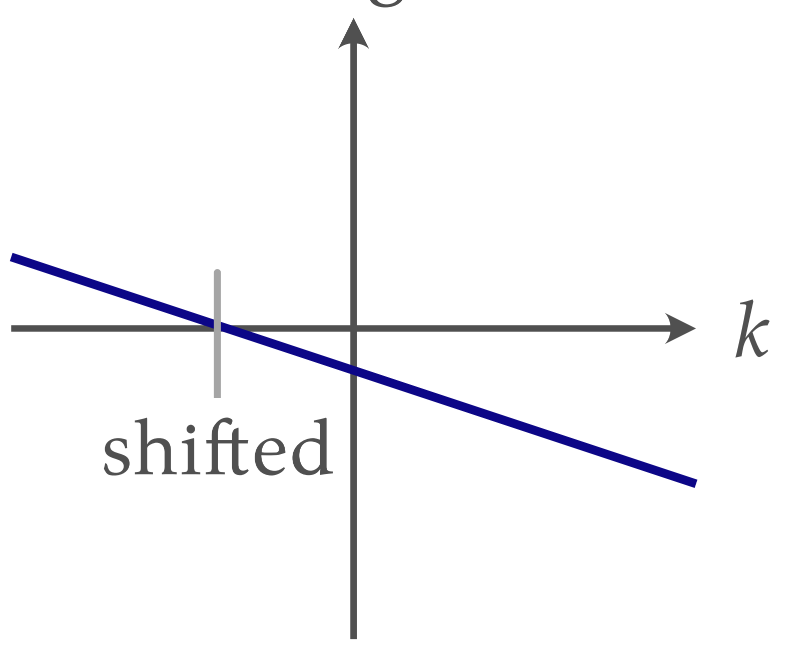
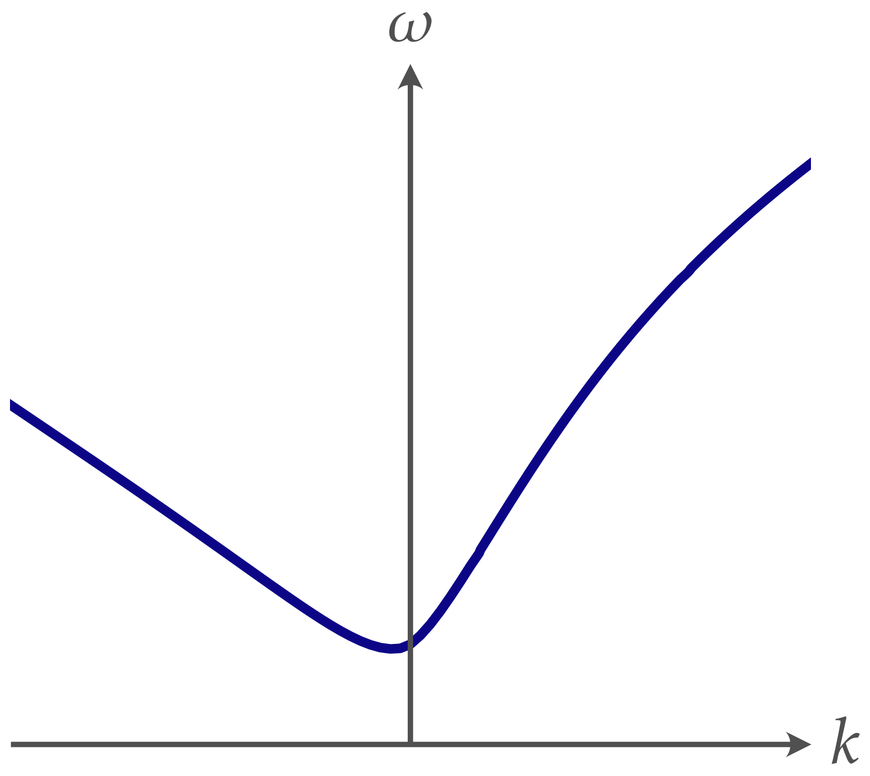
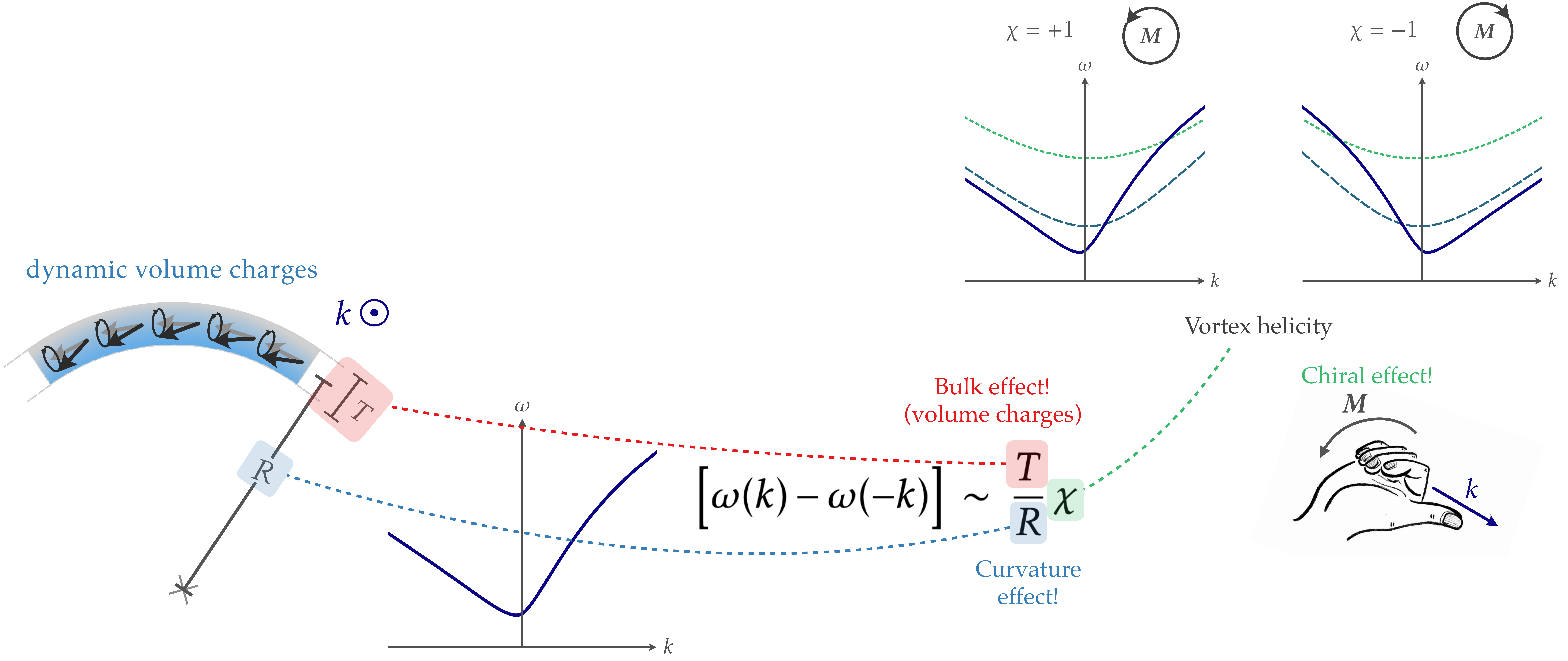




In limit $T \ll R$:

$$\left[\omega(k) - \omega(-k)\right] \sim \frac{T}{R}\chi$$

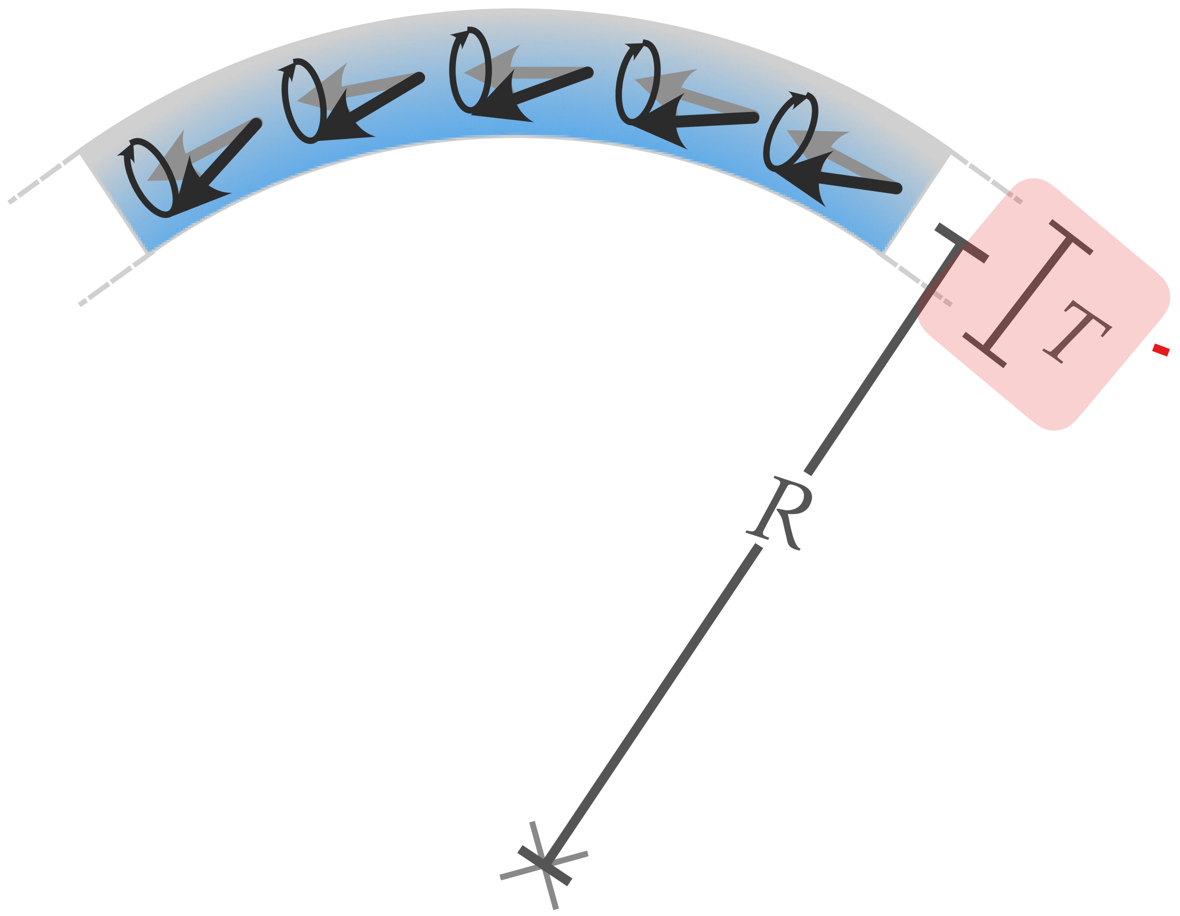
Nonlocal chiral symmetry breaking due to geometric charges

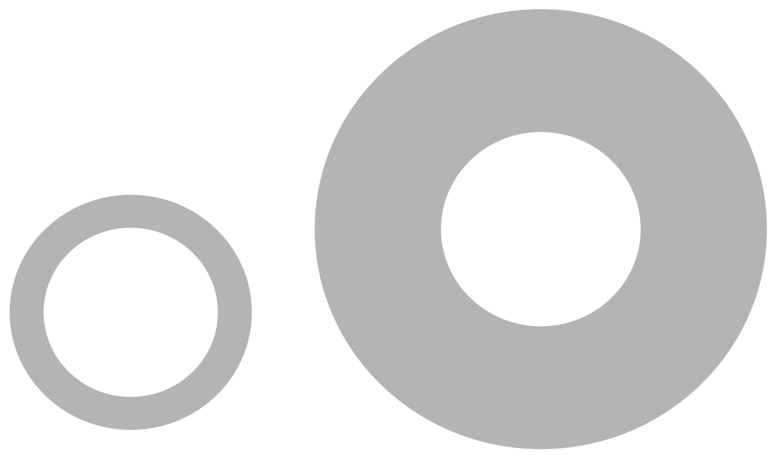


If is a bulk effect… thin-shell approximation is boring

Homogeneous along normal direction!
(otherwise not analytically solvable)

dynamic volume charges





How to calculate dispersion for thick shells?

Numerics!

Bulk effect!
(volume charges)

$$\left[\omega(k) - \omega(-k)\right] \sim \frac{T}{R}\chi$$

Development of finite-element dynamic-matrix method

Landau-Lifshitz-Gilbert equation (LLG)

$$\frac{\mathrm{d}\boldsymbol{M}}{\mathrm{d}t} = -\gamma \boldsymbol{M} \times \boldsymbol{H}_{\mathrm{eff}}(\boldsymbol{M}) + \text{damping}$$

linearize around
some equilibrium

$$\boldsymbol{M}(t) = \boldsymbol{M}_0 + \boldsymbol{m}(t)$$

$$\frac{\mathrm{d}\boldsymbol{m}}{\mathrm{d}t} = -\gamma \left[\boldsymbol{M}_0 \times \boldsymbol{h}_{\mathrm{eff}}(\boldsymbol{m}) + \boldsymbol{m} \times \boldsymbol{H}_{\mathrm{eff}}(\boldsymbol{M}_0) \right]$$

mumax^3
GPU-accelerated micromagnetism

OOMMF

TetraMag

Time integration + Post processing can take extremely long (several days to weeks)

Development of finite-element dynamic-matrix method

Landau-Lifshitz-Gilbert equation (LLG)

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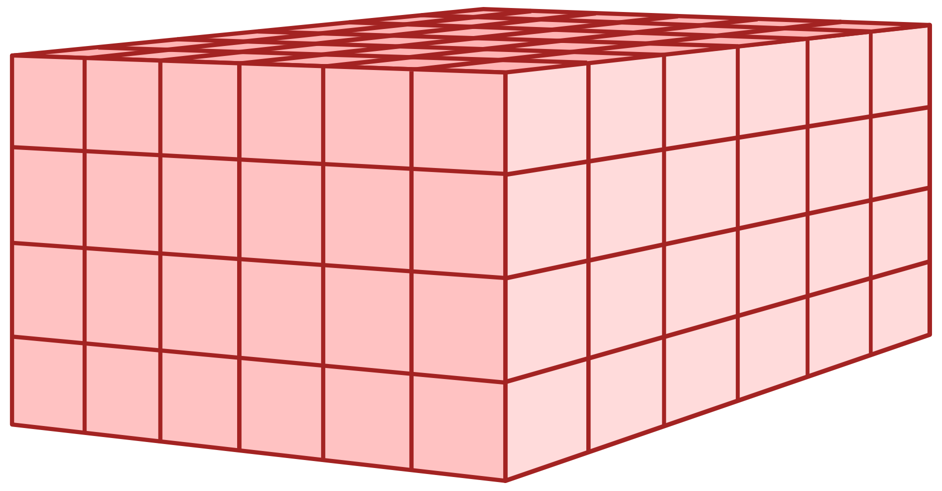
discretized eigenvalue problem

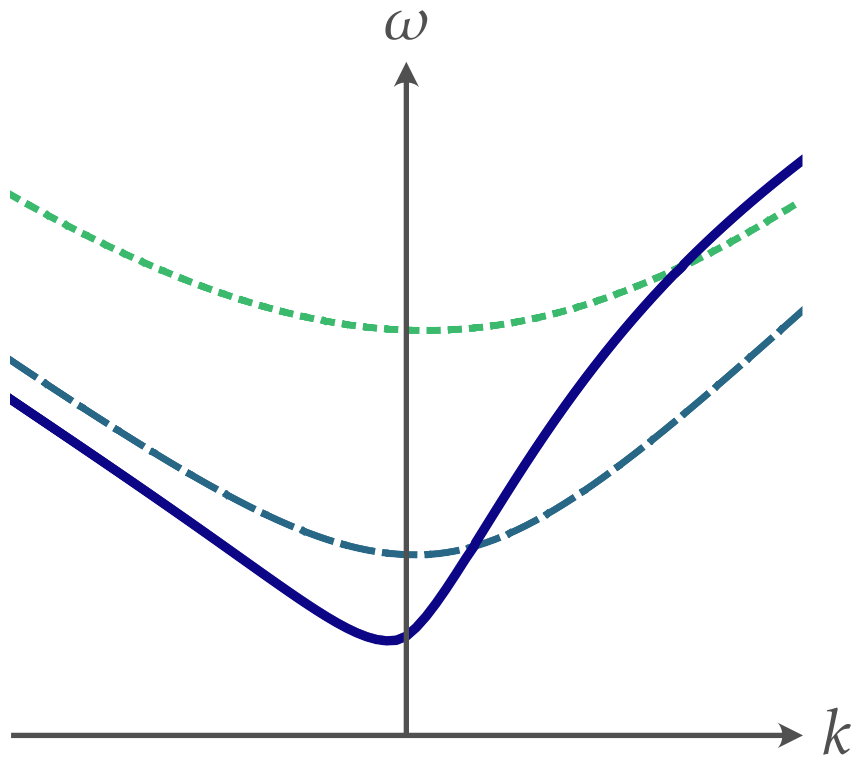
dynamic matrix

$$\omega \begin{pmatrix} m_x^{(1)} \\ \vdots \\ m_x^{(N)} \\ m_y^{(1)} \\ \vdots \\ m_y^{(N)} \end{pmatrix} = \begin{pmatrix} \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\ \vdots & \mathbf{D}_{xx} & \vdots & \vdots & \mathbf{D}_{xy} & \vdots \\ \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\ \vdots & \mathbf{D}_{yx} & \vdots & \vdots & \mathbf{D}_{yy} & \vdots \\ \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \end{pmatrix} \begin{pmatrix} m_x^{(1)} \\ \vdots \\ m_x^{(N)} \\ m_y^{(1)} \\ \vdots \\ m_y^{(N)} \end{pmatrix}$$

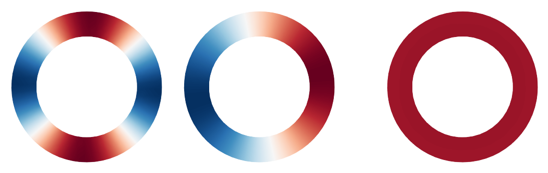


`scipy.sparse.eigs()`

eigenvalues



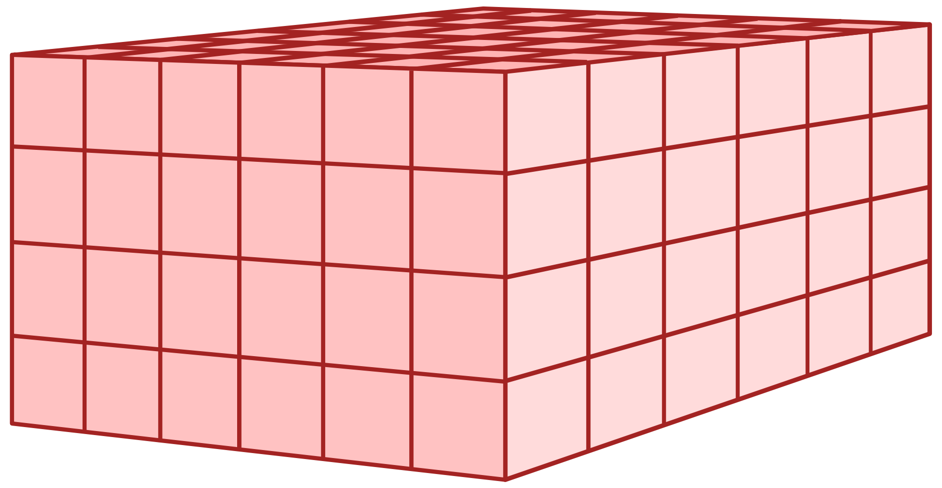
eigenvectors



Development of finite-element dynamic-matrix method

dynamic matrix

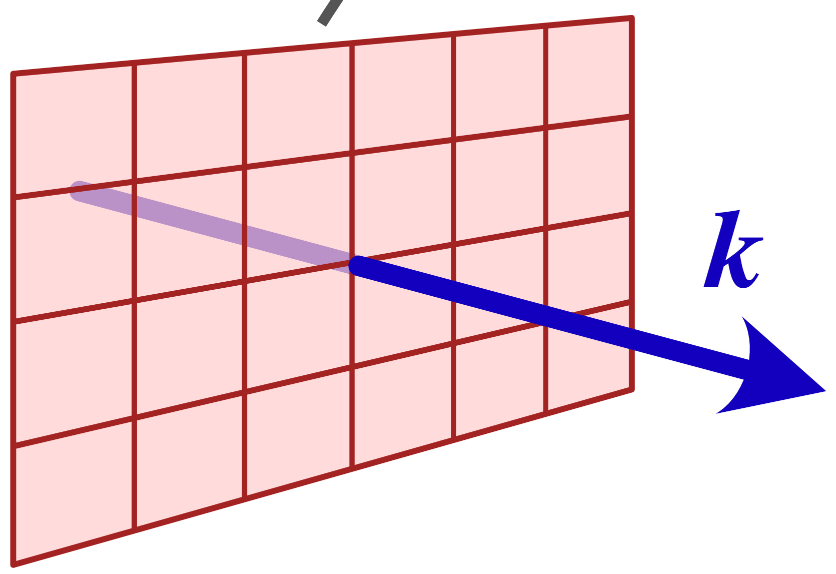
$$\omega \begin{pmatrix} m_x^{(1)} \\ \vdots \\ m_x^{(N)} \\ m_y^{(1)} \\ \vdots \\ m_y^{(N)} \end{pmatrix} = \begin{pmatrix} \mathbf{D}_{xx} & \mathbf{D}_{xy} \\ \mathbf{D}_{yx} & \mathbf{D}_{yy} \end{pmatrix} \begin{pmatrix} m_x^{(1)} \\ \vdots \\ m_x^{(N)} \\ m_y^{(1)} \\ \vdots \\ m_y^{(N)} \end{pmatrix}$$



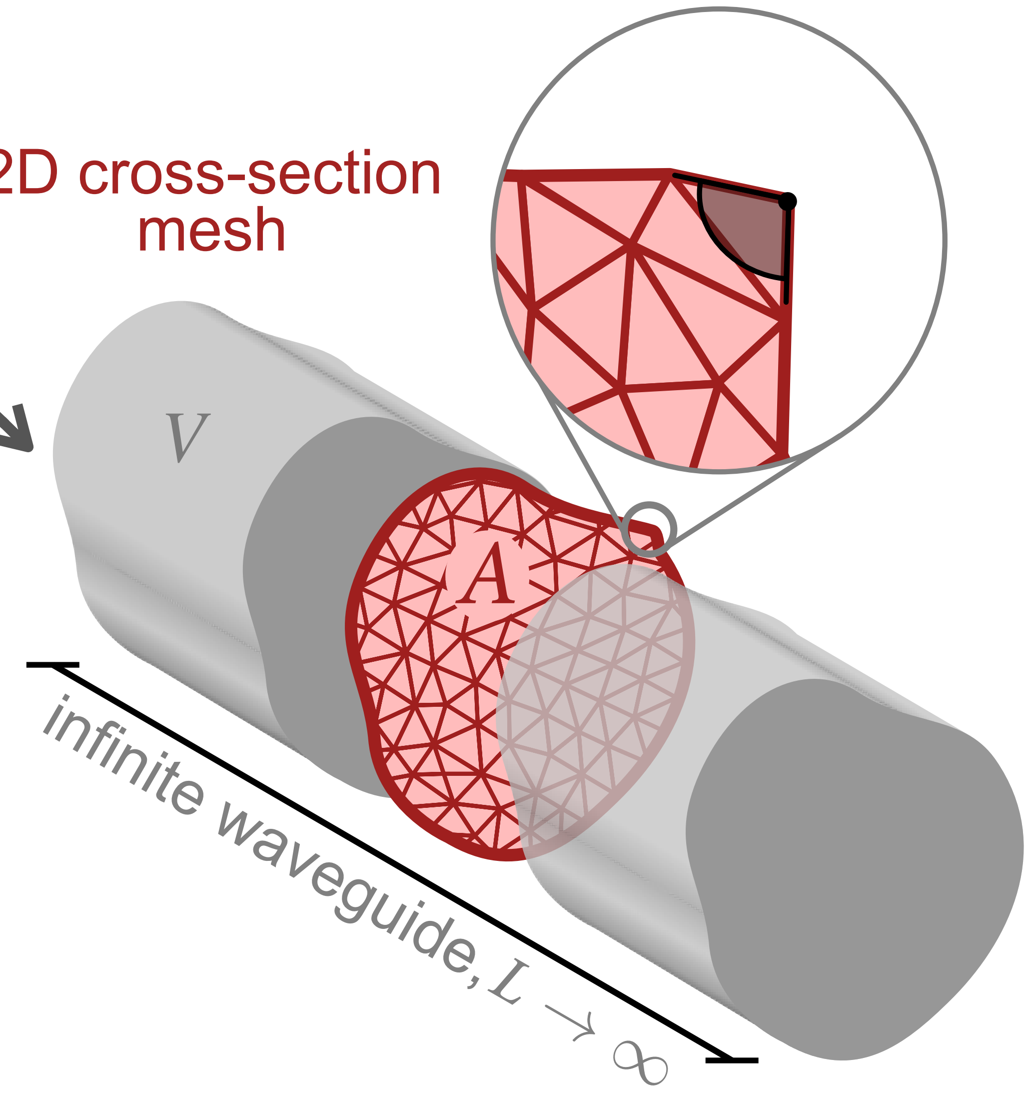
project to single cross section

$$\boldsymbol{m}_\nu \to \boldsymbol{\eta}_{\nu k} = \boldsymbol{m}_\nu e^{-ikz}$$

$$\hat{\mathbf{D}} \to \hat{\mathbf{D}}_k = e^{-ikz}\hat{\mathbf{D}}e^{ikz}$$



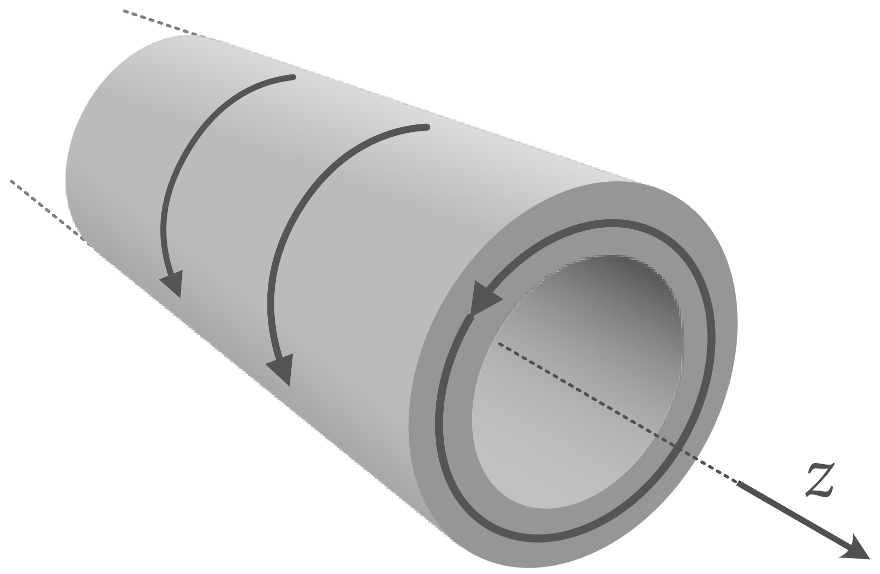
finite-element method (FEM)

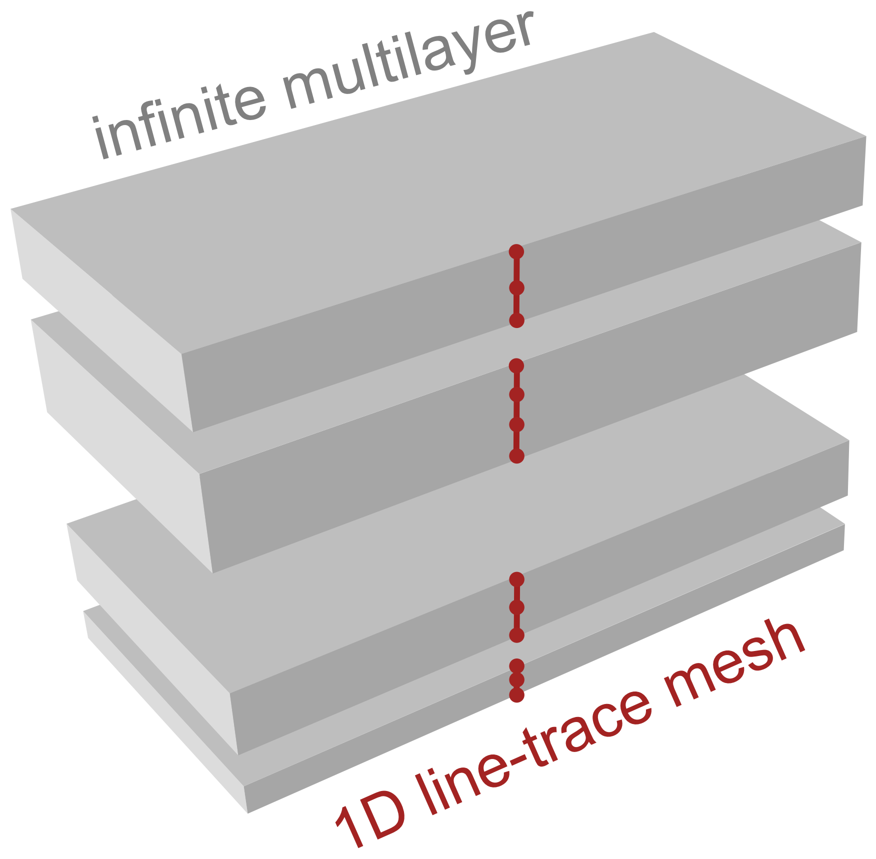


Körber *et al.*,
AIP Advances **11**, 095006 (2021)

not good for (infinity) long systems
(FFT in space still required)

not good for
curved systems





Körber, Hempel *et al.*,
AIP Advances **12**, 115206 (2022)

Notoriously tricky: Calculation of dynamic dipolar fields

Linearized equation of motion

$$\frac{\mathrm{d}\boldsymbol{m}}{\mathrm{d}t} = -\gamma\left[\boldsymbol{M}_0 \times \boldsymbol{h}_{\mathrm{eff}}(\boldsymbol{m}) + \boldsymbol{m} \times \boldsymbol{H}_{\mathrm{eff}}(\boldsymbol{M}_0)\right]$$

contains

Dipolar fields of propagating waves?

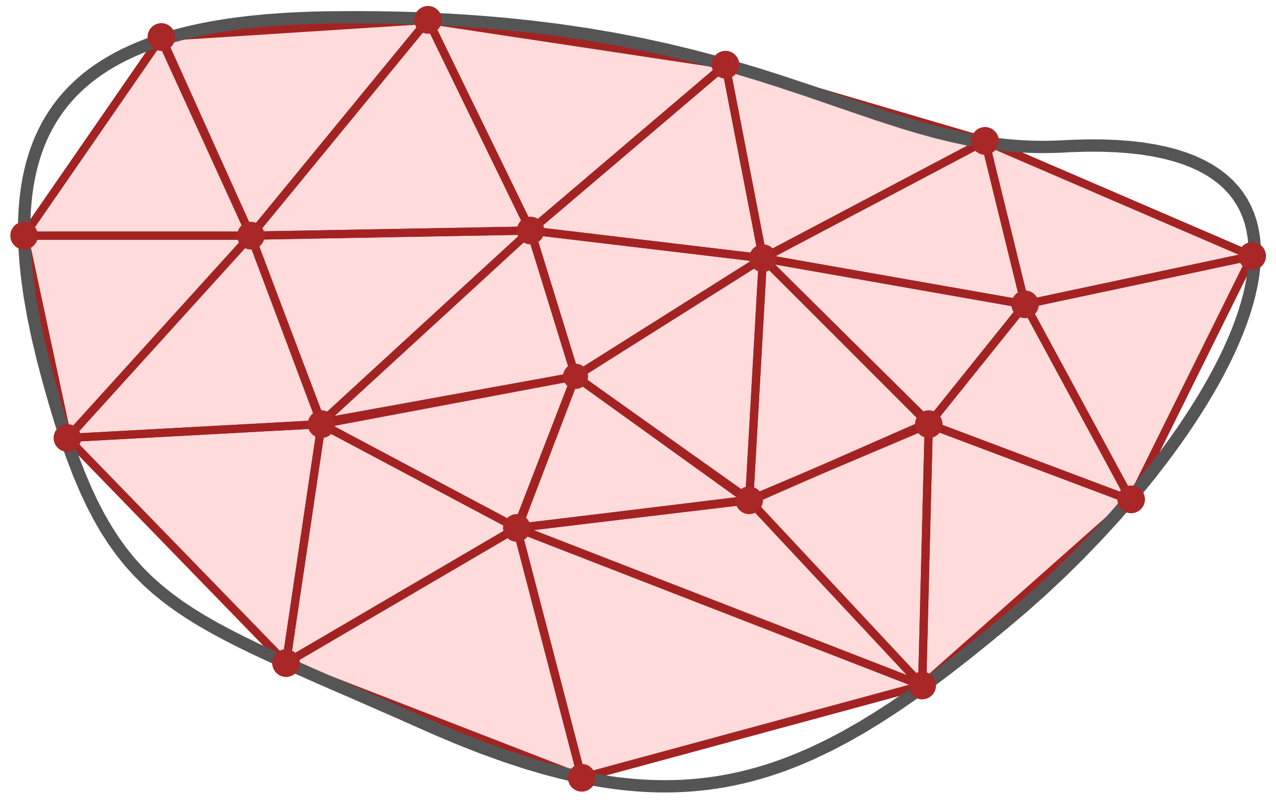
$$\boldsymbol{h}_{\mathrm{dip}} = \nabla\, \psi\, e^{i(kz-\omega t)}$$

obeys

**Screened Poisson equation/
Yukawa equation**

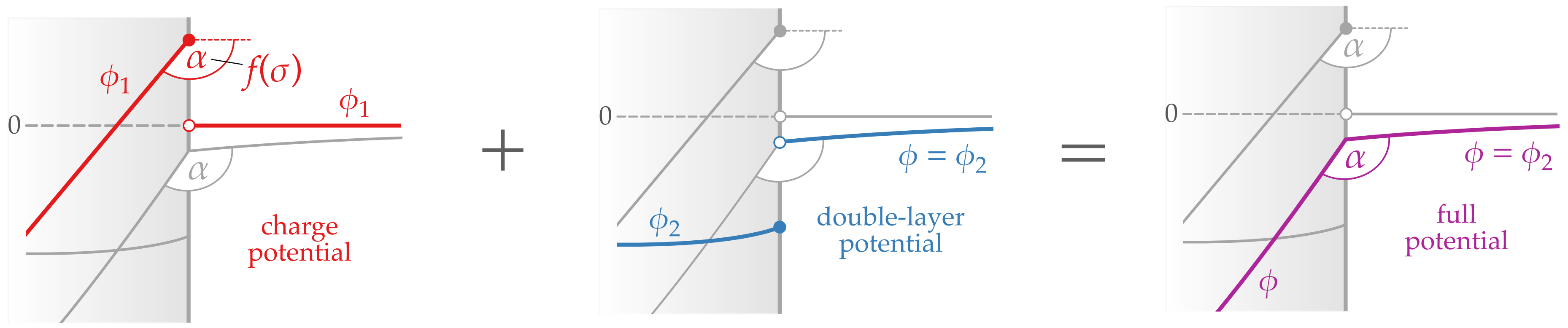
$$(\Delta - k^2)\psi = \begin{cases} (\nabla + ik\boldsymbol{e}_z)\boldsymbol{m} & \text{inside} \\ 0 & \text{outside} \end{cases} \quad + \text{interface conditions}$$

How to solve this (efficiently) on a FEM mesh?



Hybrid FEM/BEM method for screened Poisson equation

(largest part of FEM dynamic-matrix method)

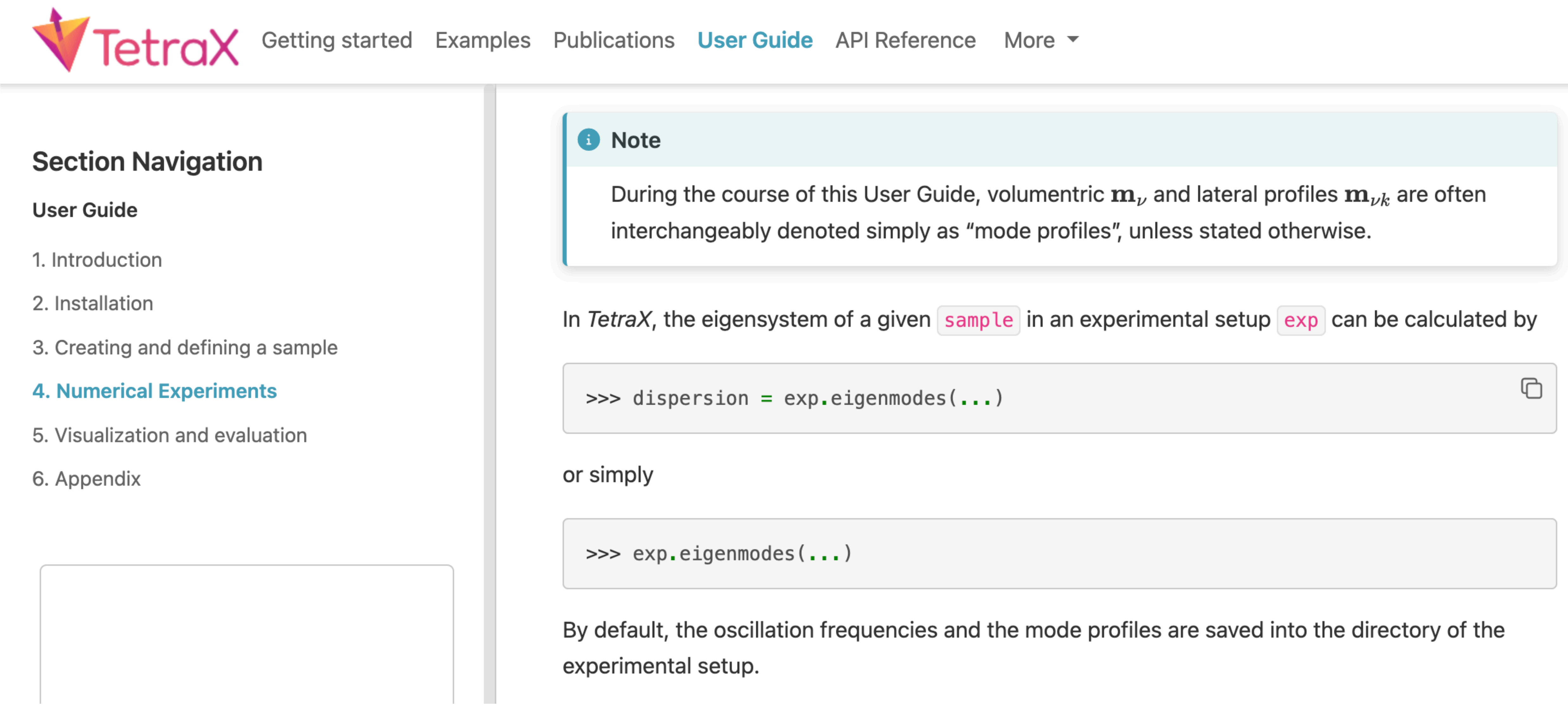


Körber *et al.*, AIP Advances **11**, 095006 (2021)
Körber, Hempel *et al.*, AIP Advances **12**, 115206 (2022)

Method published as open-source package TetraX

Map of *confirmed* users

Documentation at tetrax.readthedocs.io with many examples



TetraX Getting started Examples Publications **User Guide** API Reference More

Section Navigation

User Guide

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2. Installation
3. Creating and defining a sample
4. **Numerical Experiments**
5. Visualization and evaluation
6. Appendix

Note

During the course of this User Guide, volumentric $\mathbf{m}_\nu$ and lateral profiles $\mathbf{m}_{\nu k}$ are often interchangeably denoted simply as “mode profiles”, unless stated otherwise.

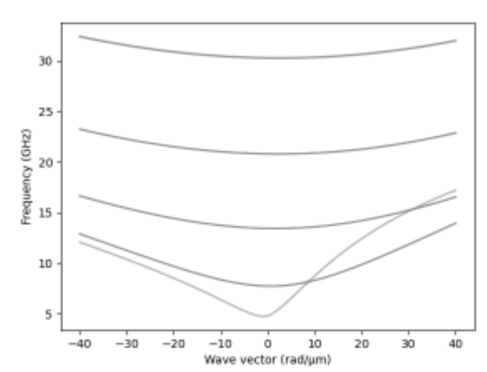
In *TetraX*, the eigensystem of a given `sample` in an experimental setup `exp` can be calculated by

```
>>> dispersion = exp.eigenmodes(...)
```

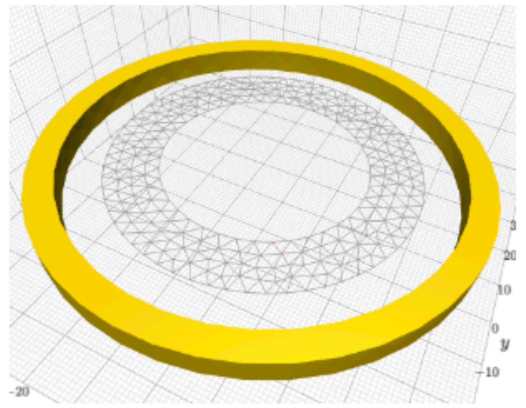
or simply

```
>>> exp.eigenmodes(...)
```

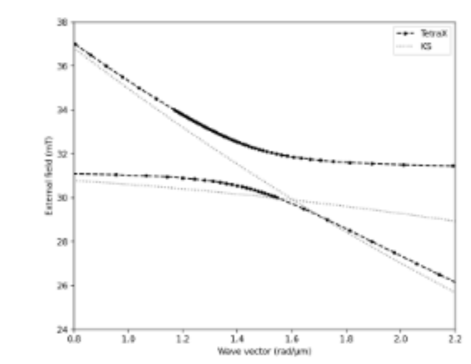
By default, the oscillation frequencies and the mode profiles are saved into the directory of the experimental setup.



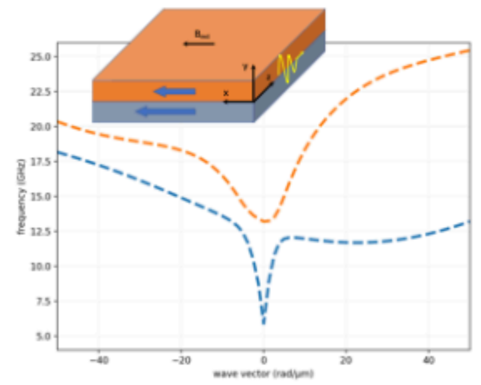
Dispersion of a nanotube in vortex state



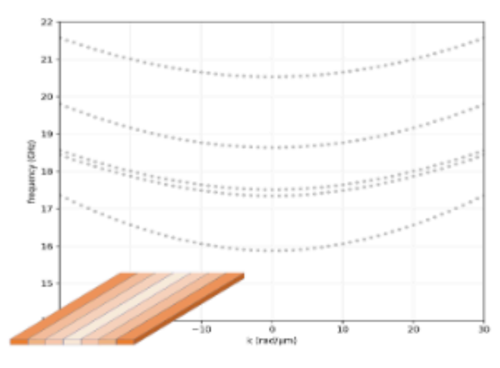
Absorption of a nanotube with antennae

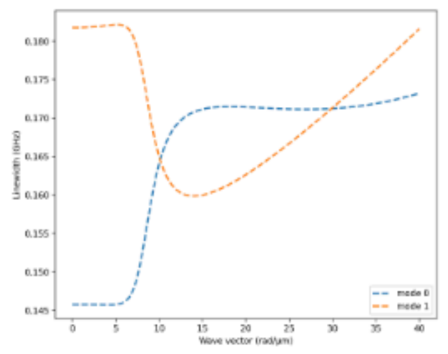


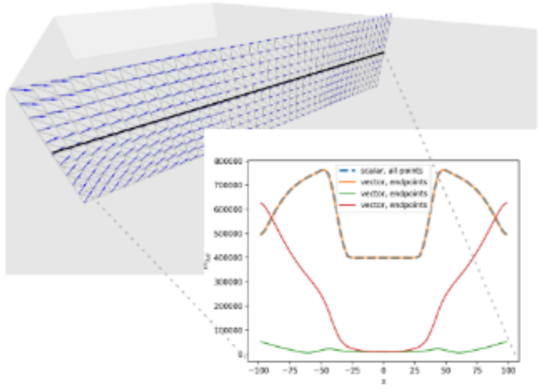
Thick YIG film - spin-wave hybridization

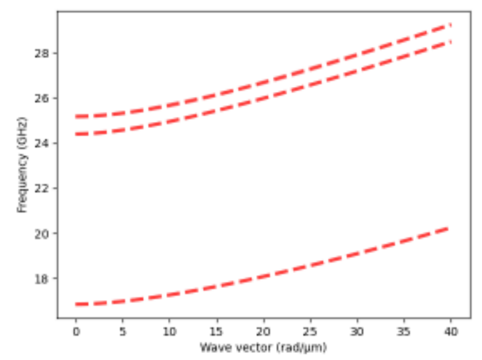


Double layers of Py / CoFeB

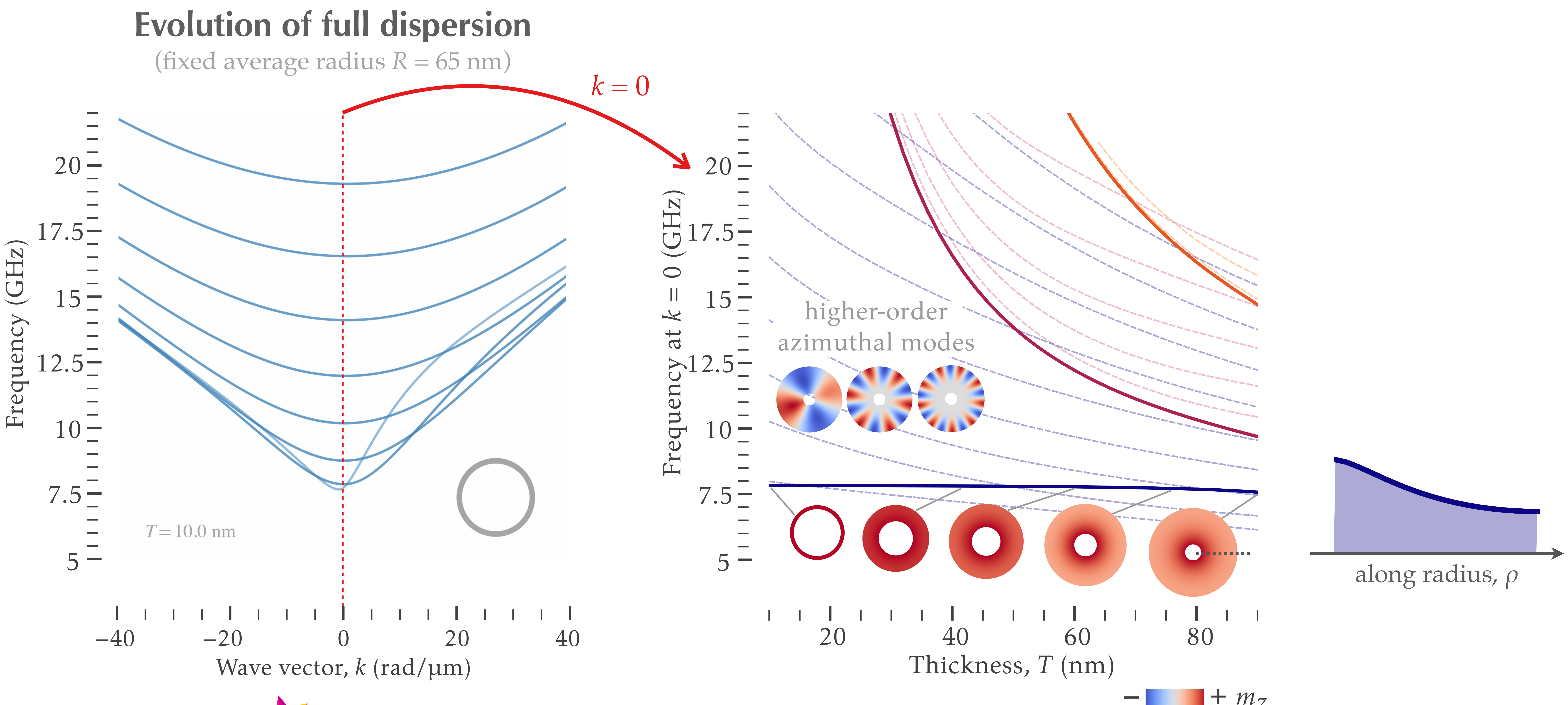




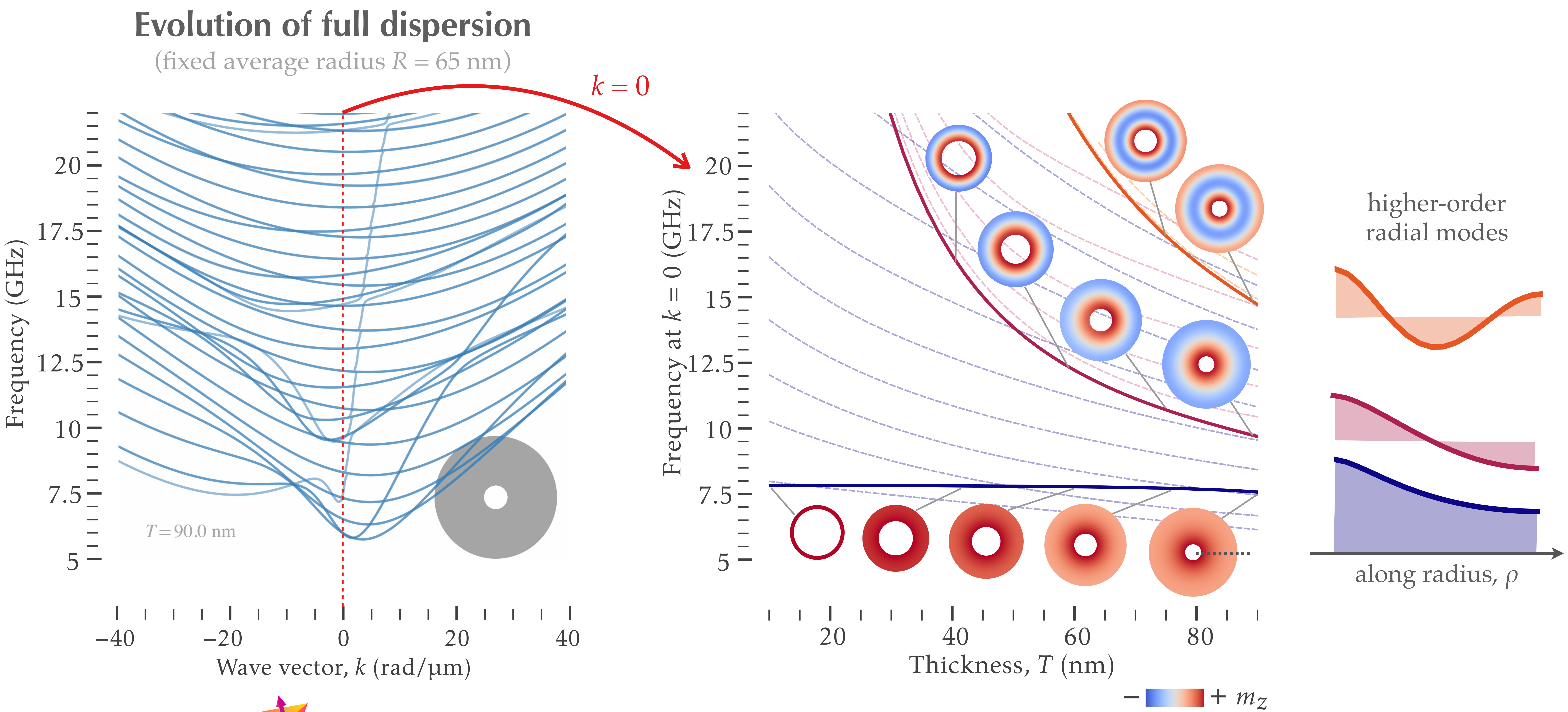




Thickness evolution of spin-wave spectrum

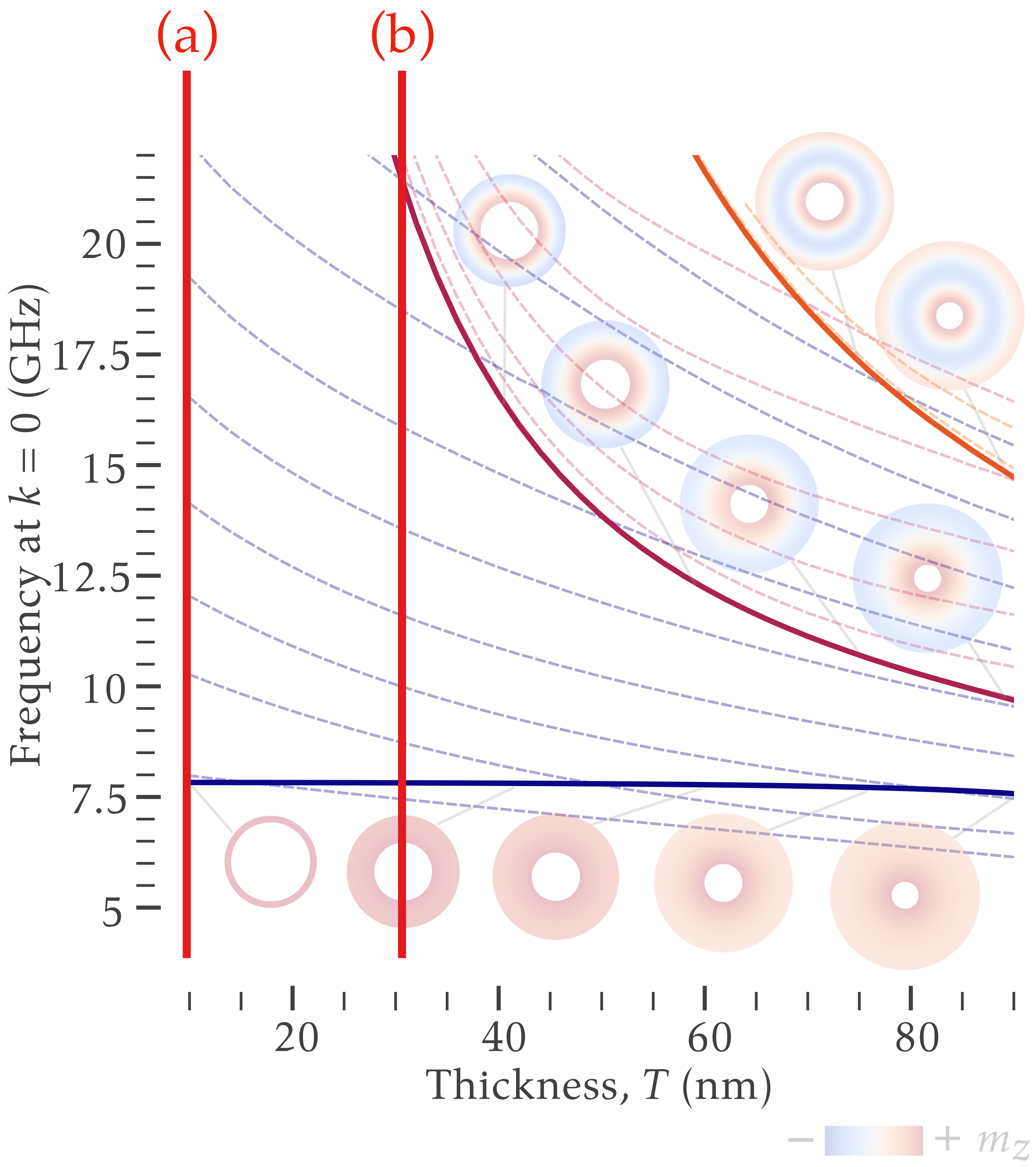


Thickness evolution of spin-wave spectrum

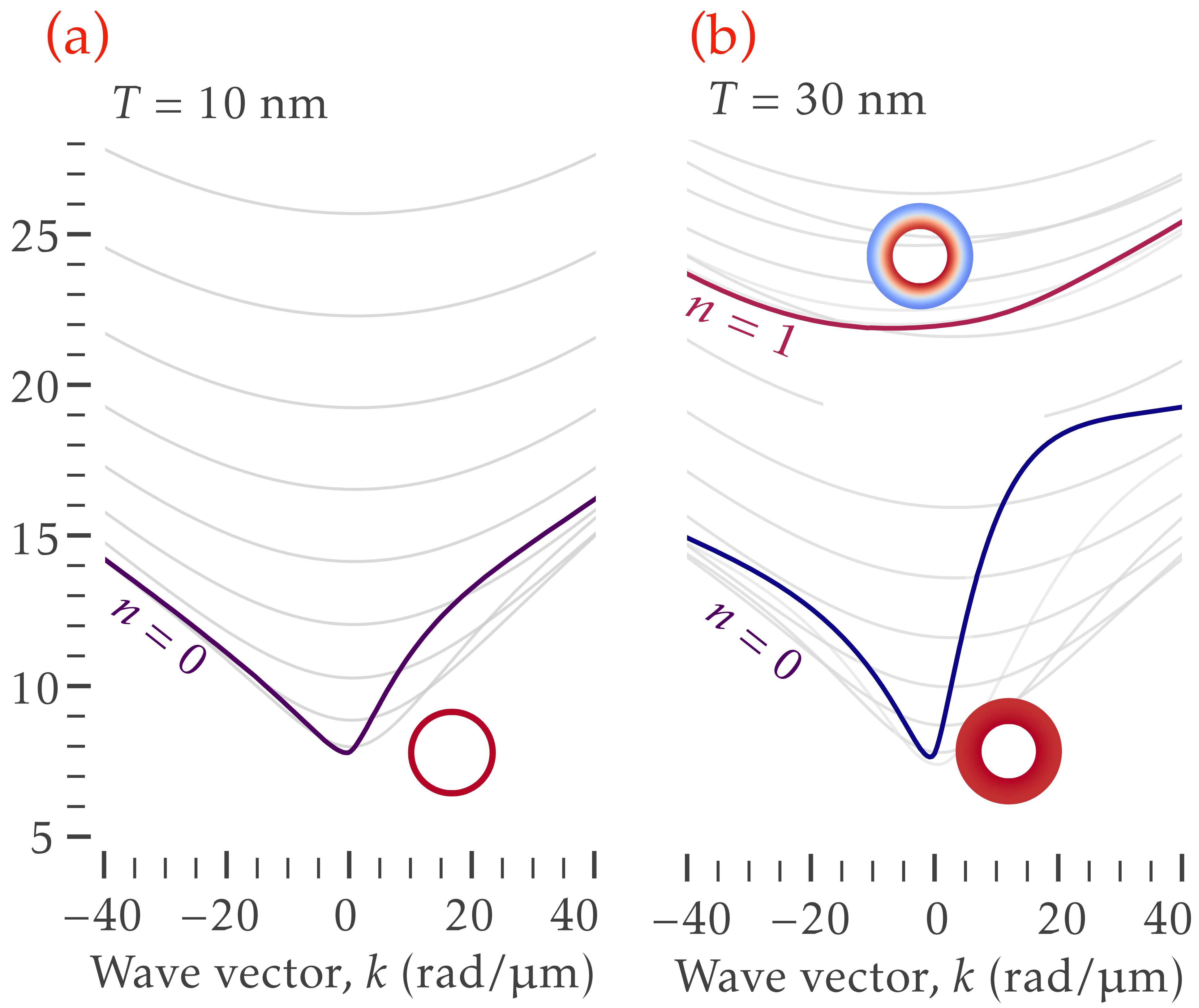


TetraX

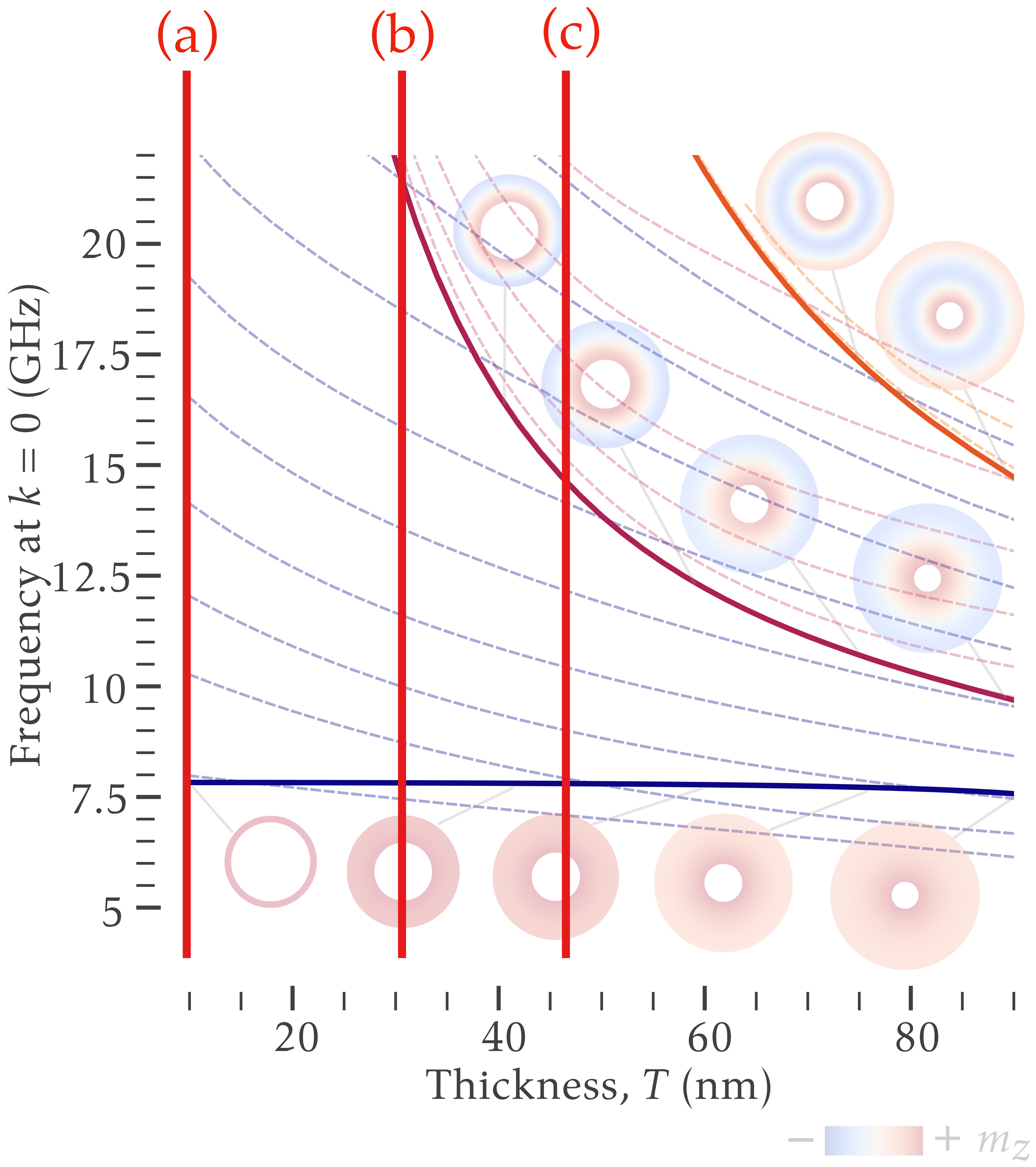
Thickness evolution of spin-wave spectrum



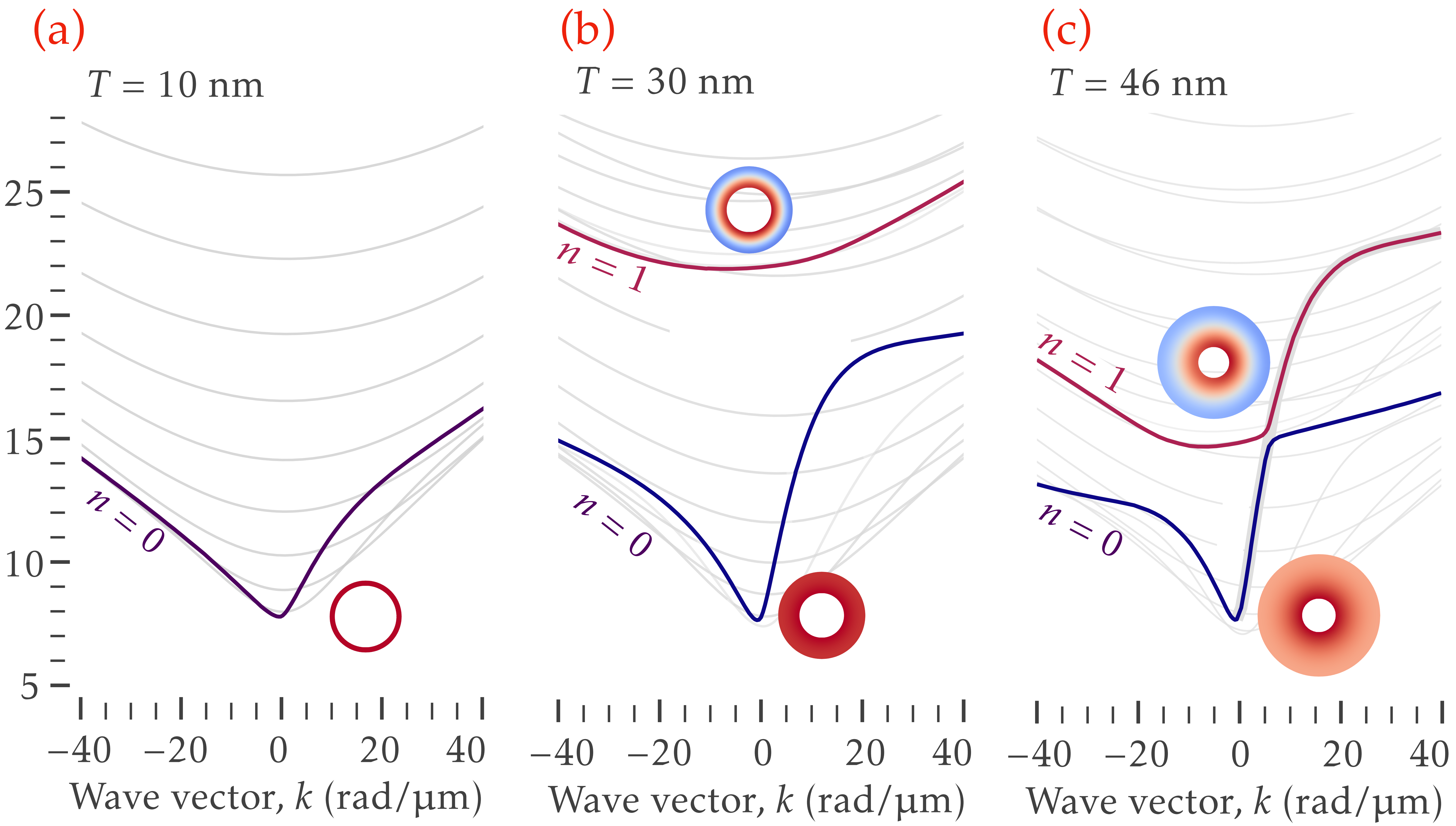
Evolution of full dispersion (snapshots)



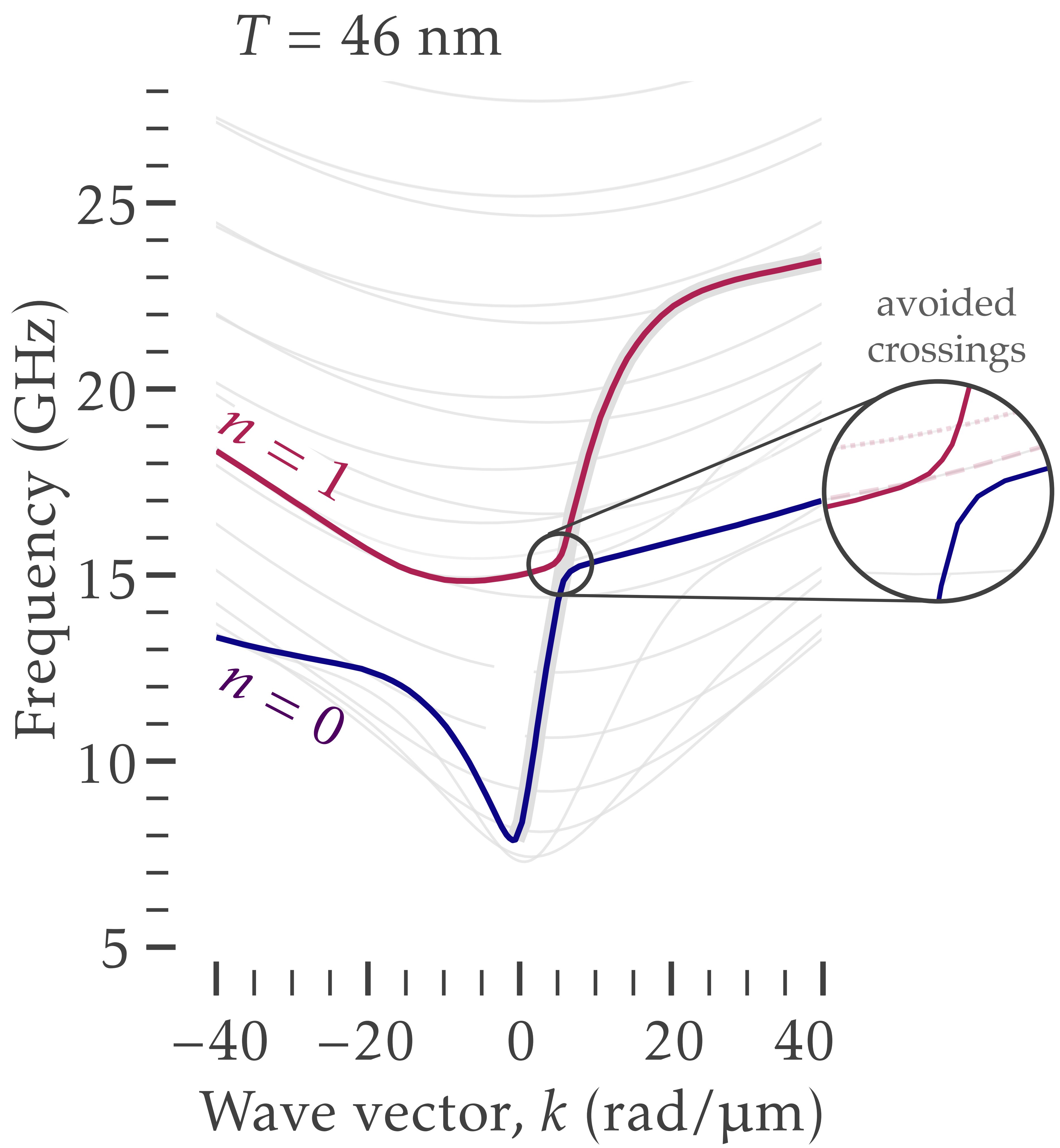
Thickness evolution of spin-wave spectrum

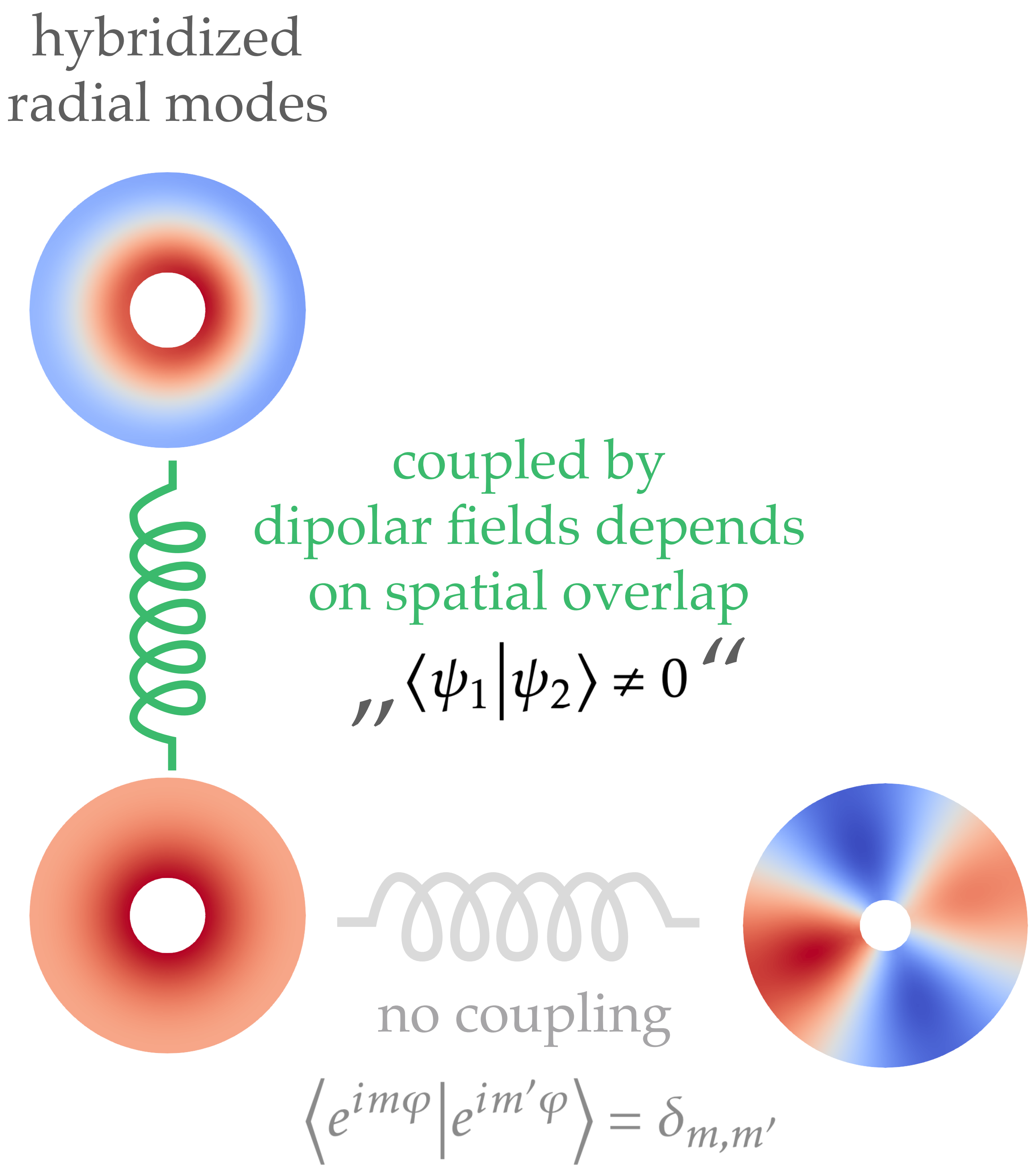


Evolution of full dispersion (snapshots)

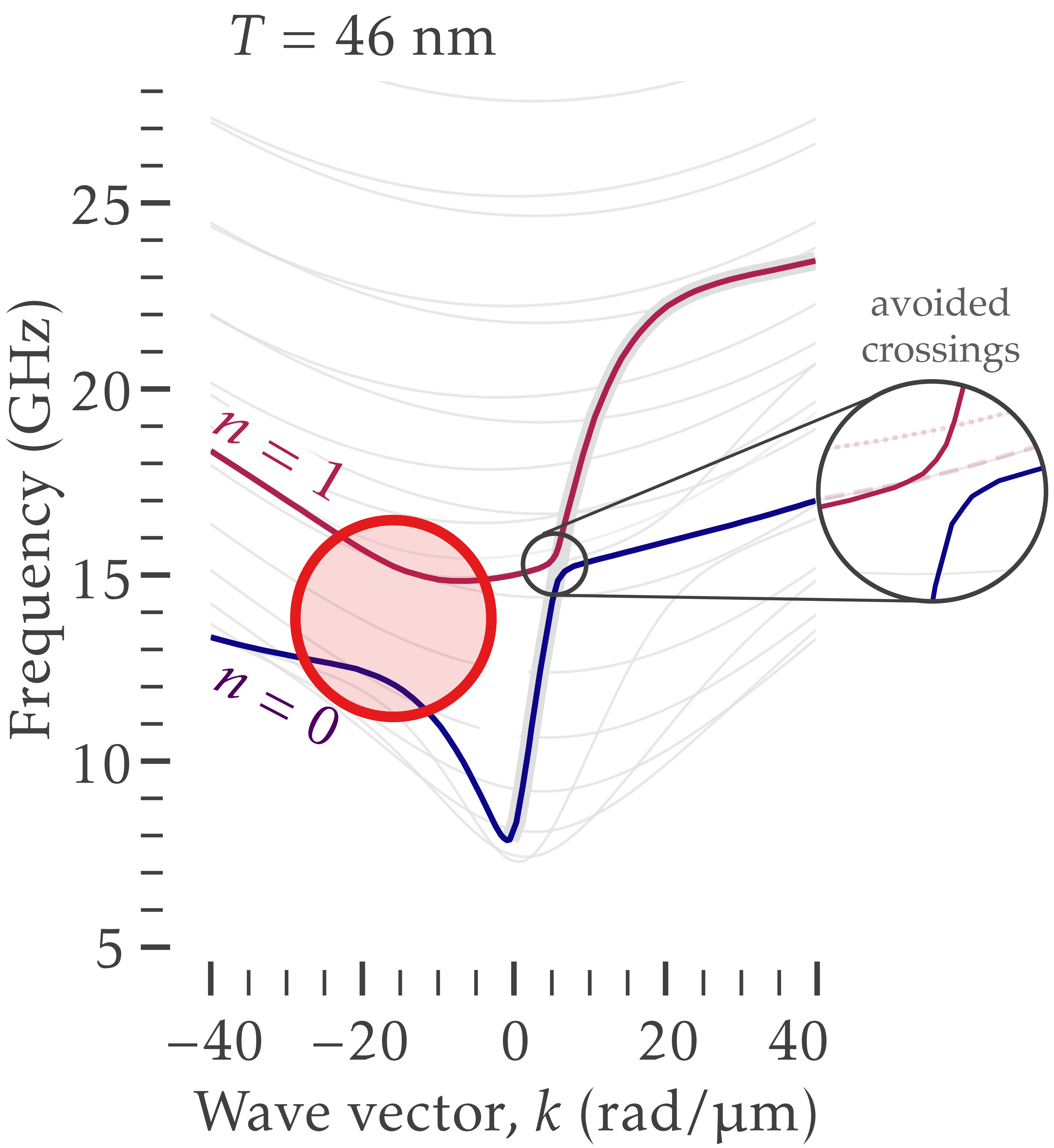


Different radial modes are hybridized/mixed





Hybridization between radial modes is nonreciprocal!



asymmetric crossings

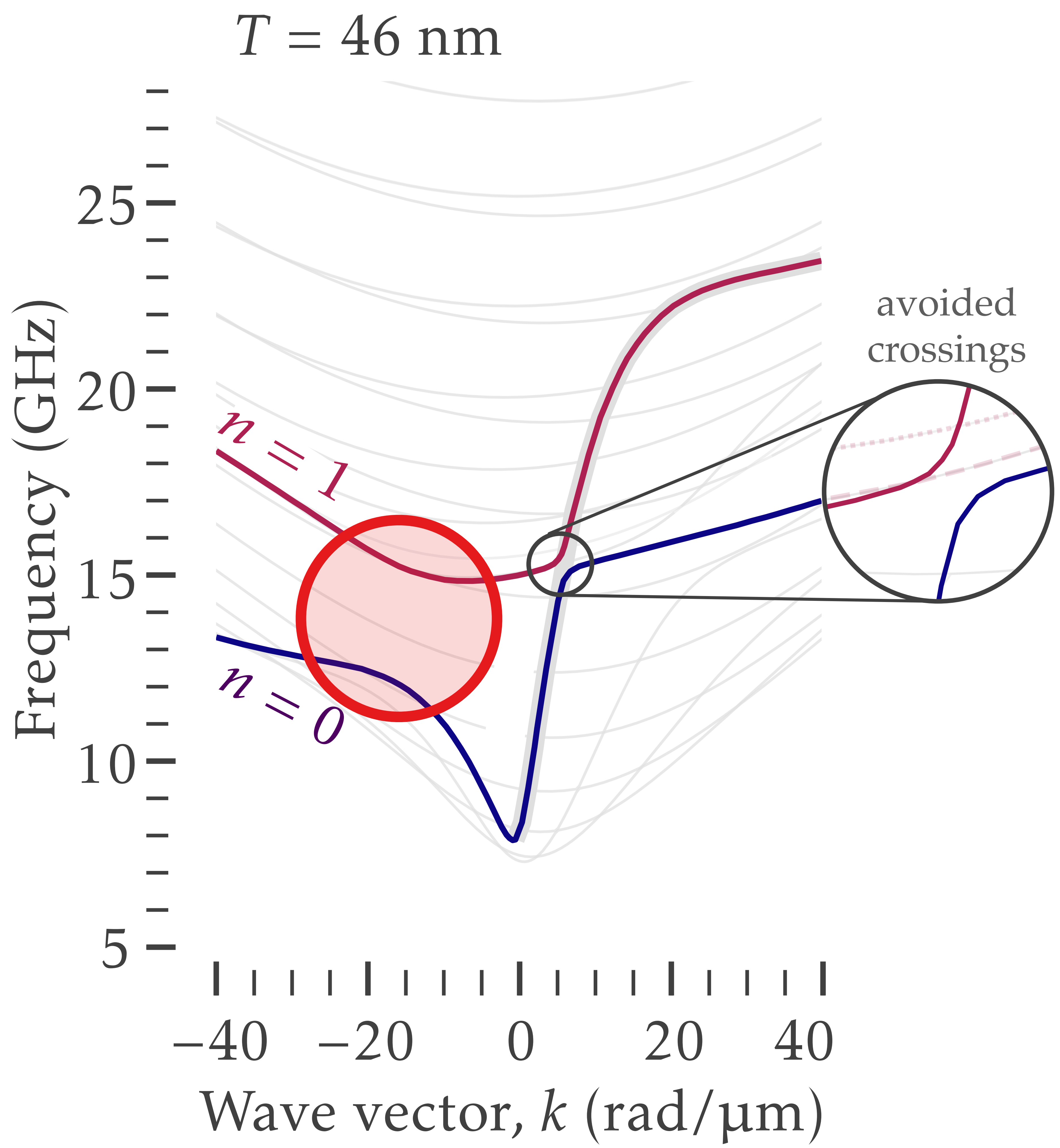
„ $\langle \psi_1 | \psi_2 \rangle$ "

↓

from nonreciprocal radial profiles?

„ $|\psi_1(k)\rangle \neq |\psi_1(-k)\rangle$ "

Hybridization between radial modes is nonreciprocal!



asymmetric crossings

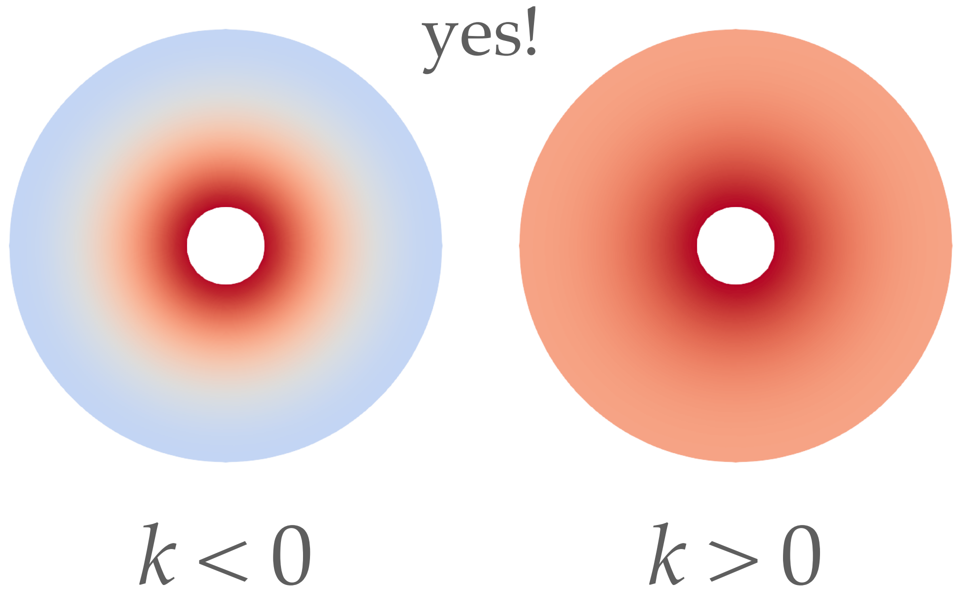
„ $\langle \psi_1 | \psi_2 \rangle$ "

↓

from nonreciprocal radial profiles?

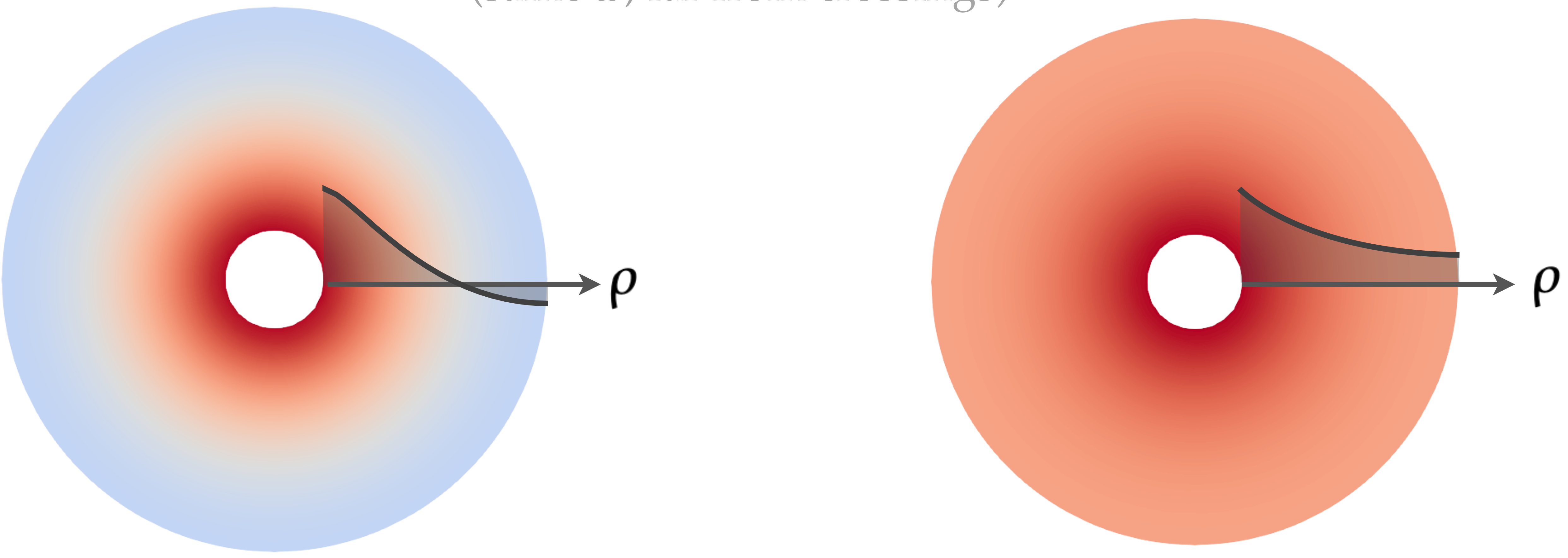
„ $|\psi_1(k)\rangle \neq |\psi_1(-k)\rangle$ "

yes!



Nonreciprocity of profiles explained by inhomogeneous geometric charges

(same ω, far from crossings)



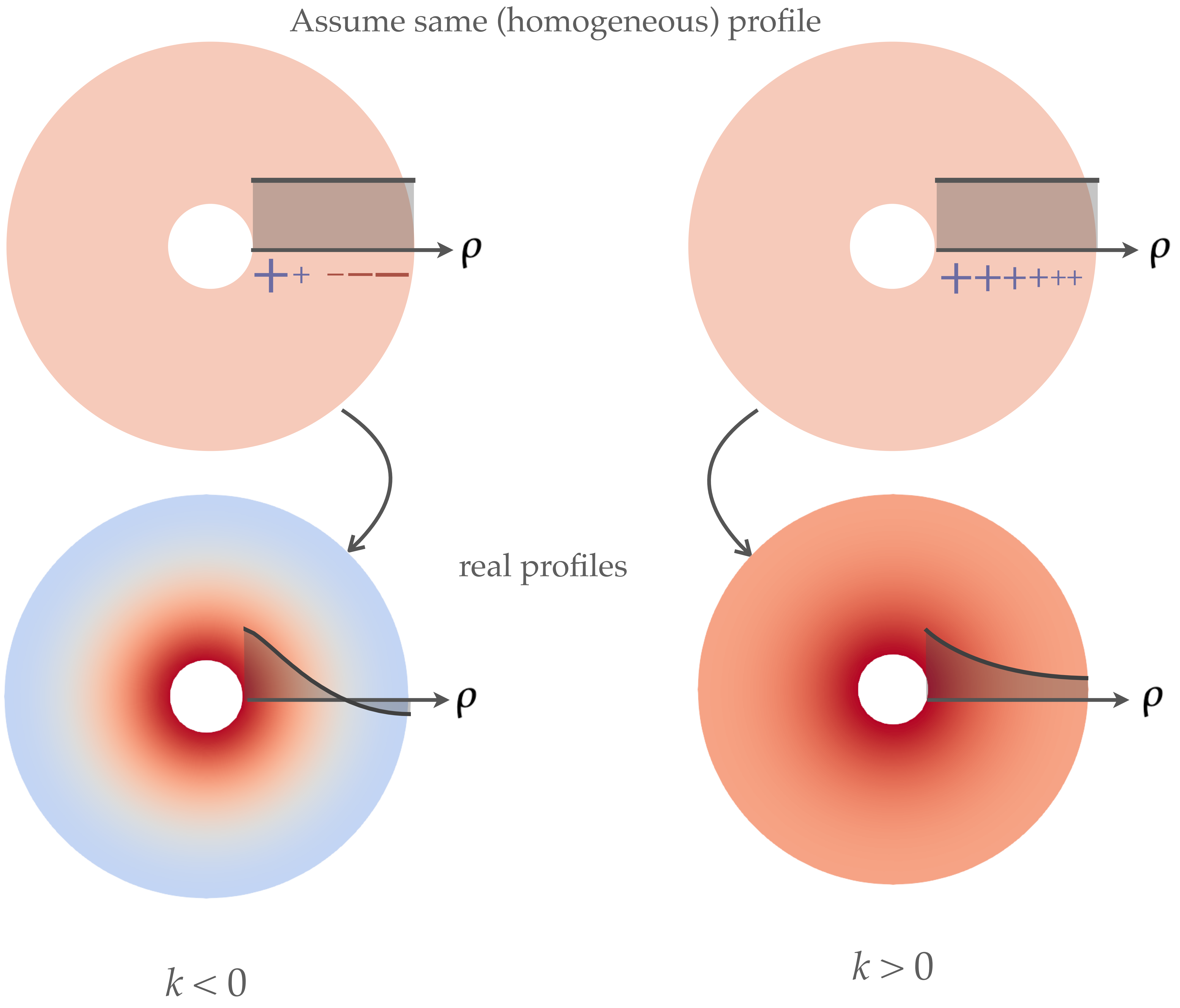
$k < 0$ $\qquad$ $k > 0$

Charges extended to thick shells:

$$\nabla \boldsymbol{m} = \left(\frac{1}{1 + \zeta \mathcal{H}_0} \partial_1 m^1 + \partial_2 m^2 + \partial_\zeta m_n \right) + \mathcal{H}(\boldsymbol{r}) m_n$$

Körber *et al.*, PRB 105, 014405 (2022)

Nonreciprocity of profiles explained by inhomogeneous geometric charges



Charges extended to thick shells:

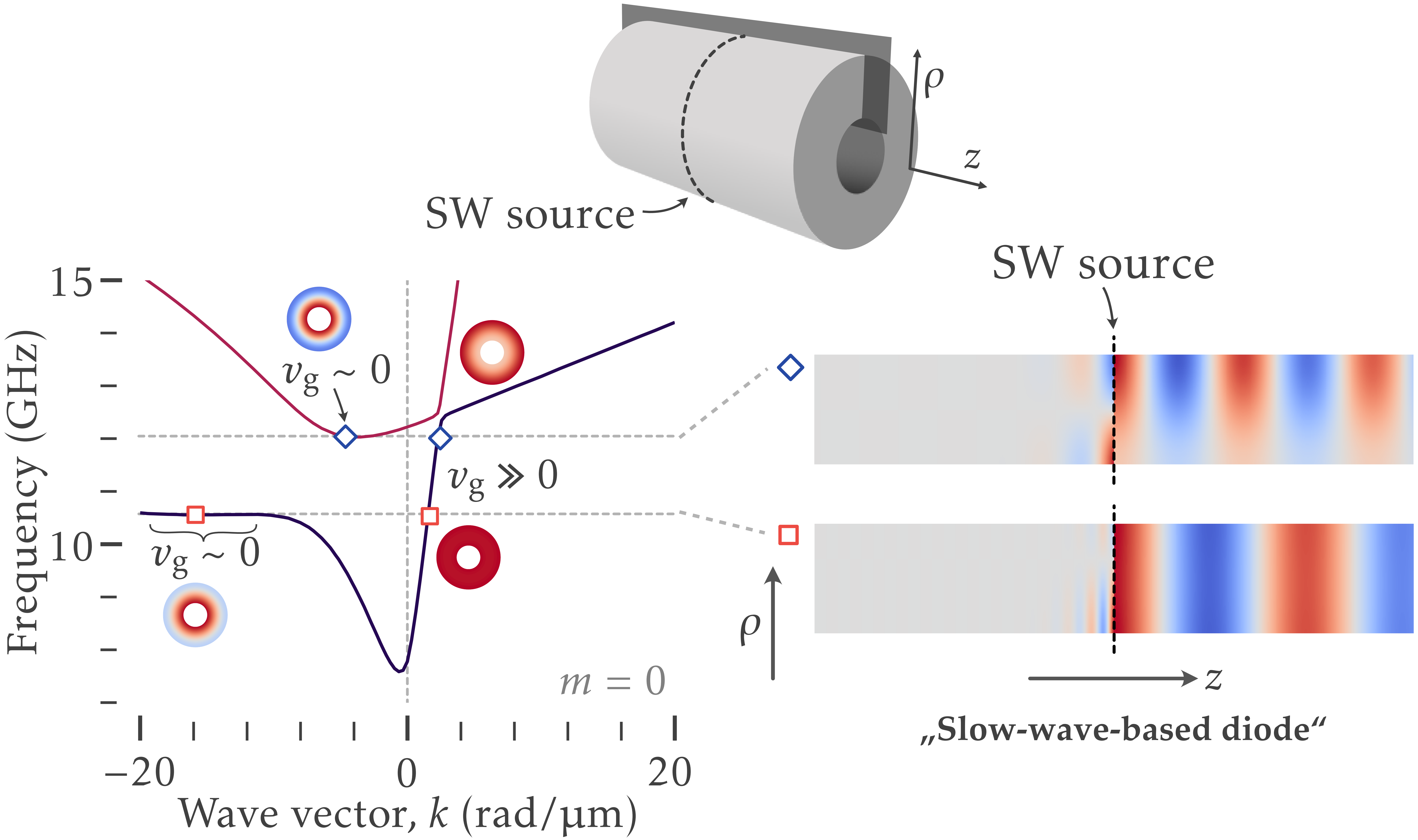
$$\nabla \boldsymbol{m} = \left(\frac{1}{1 + \zeta \mathcal{H}_0} \partial_1 m^1 + \partial_2 m^2 + \partial_\zeta m_n \right) + \mathcal{H}(\mathbf{r}) m_n$$

$$\sim k + \frac{1}{\rho}$$

Körber *et al.*, PRB 105, 014405 (2022)

Depends on normal direction.

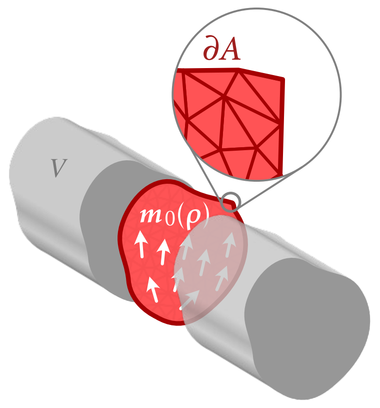
Nonreciprocal hybridization allows unidirectional propagation



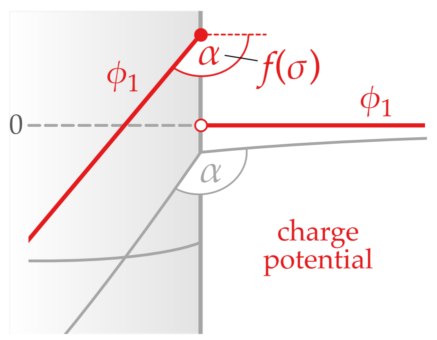
„Slow-wave-based diode"

Summary: Spin waves in curved magnetic shells

Development of numerical methods



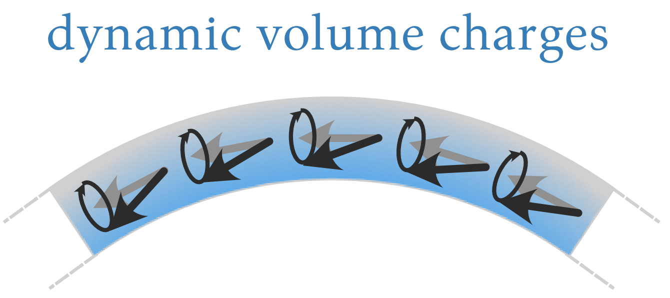
FEM dynamic-matrix method to study spin waves



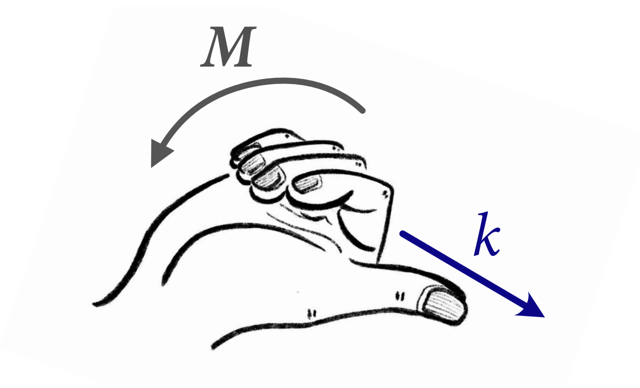
FEM/BEM method for dipolar fields of propagating waves

open-source package TetraX

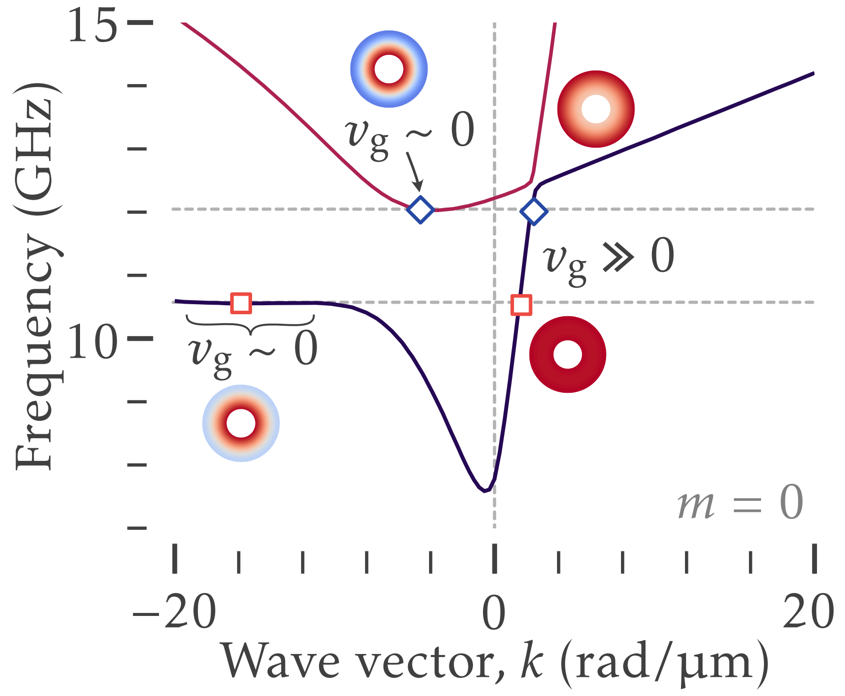
Nonlocal chiral symmetry breaking



geometric charges

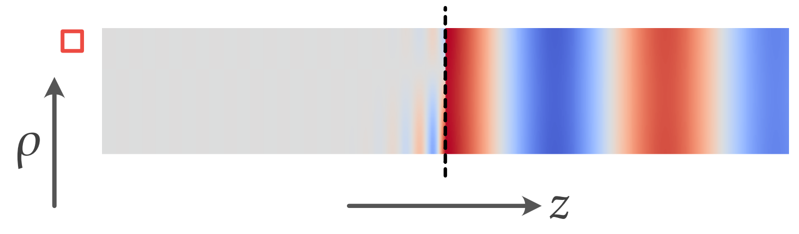


chiral symmetry breaking (bulk)



dispersion & mode profile asymmetry

nonreciprocal hybridization



unidirectional propagation

Thank you for your attention!