California Institute of Technology

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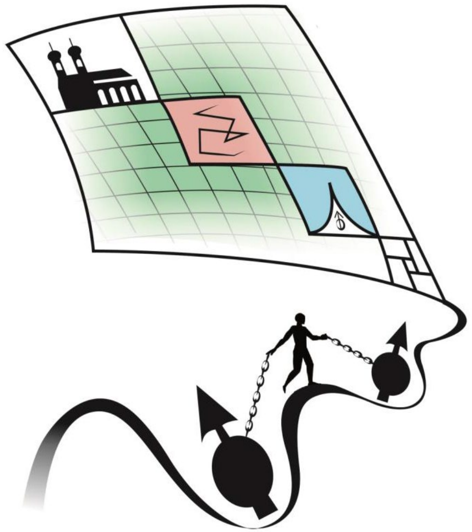
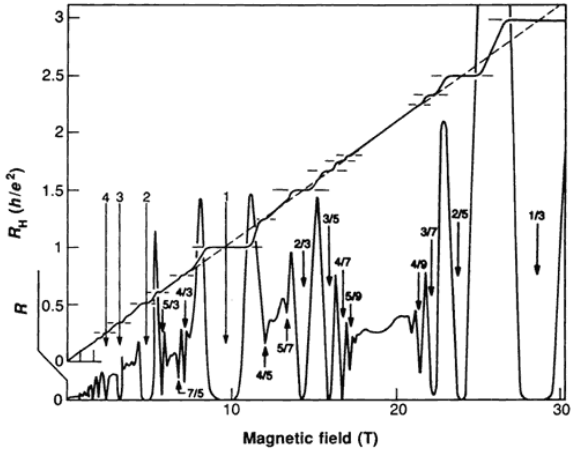
The role of unconventional symmetries in the dynamics of many-body systems

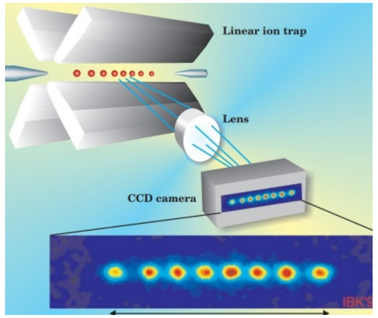


Pablo Sala de Torres-Solanot

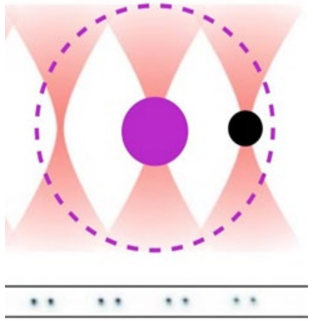
Young SPICE, 25/07/2023



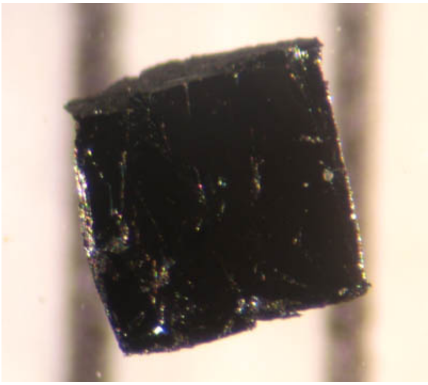
Science (1990)



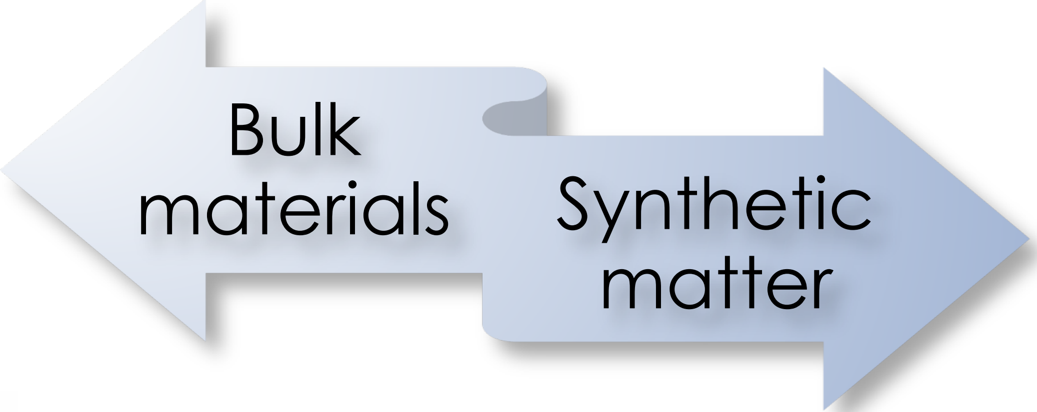
Physics Today 57, 3 (2004)

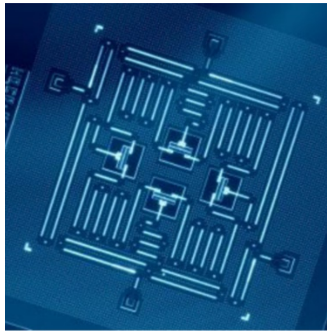


Nat. Phys. 16, 857 (2020)

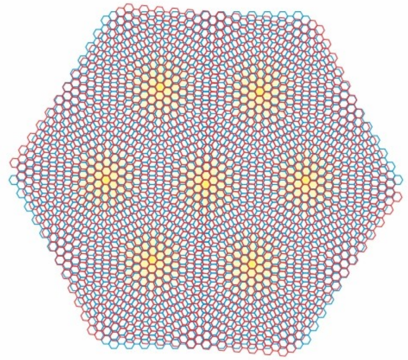


BSSCO (Wikipedia)

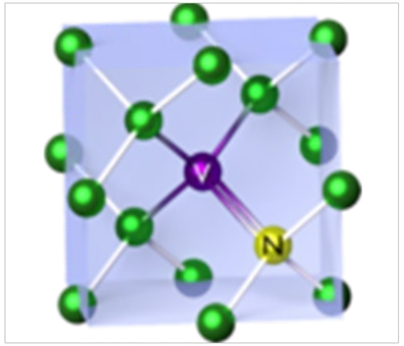




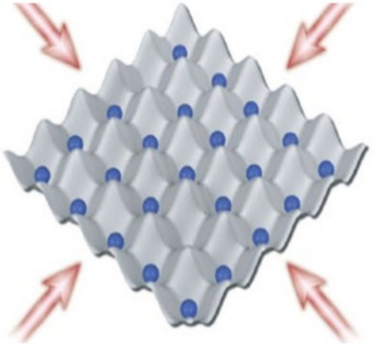
Npj Quantum Inf 3, 2 (2017)



Physics Today 71, 5, 15 (2018)



NIST



Science 352, 1547 (2016)

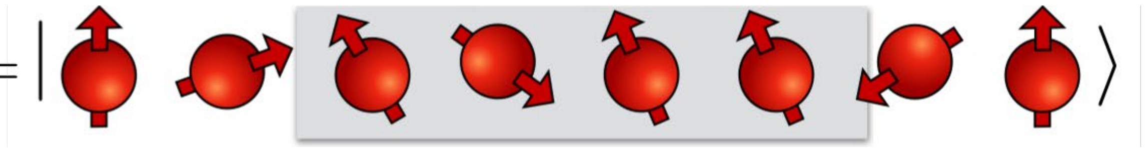
and more

Quantum thermalization

When does an isolated quantum many-body system thermalize?

$$i\frac{d}{dt}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$$

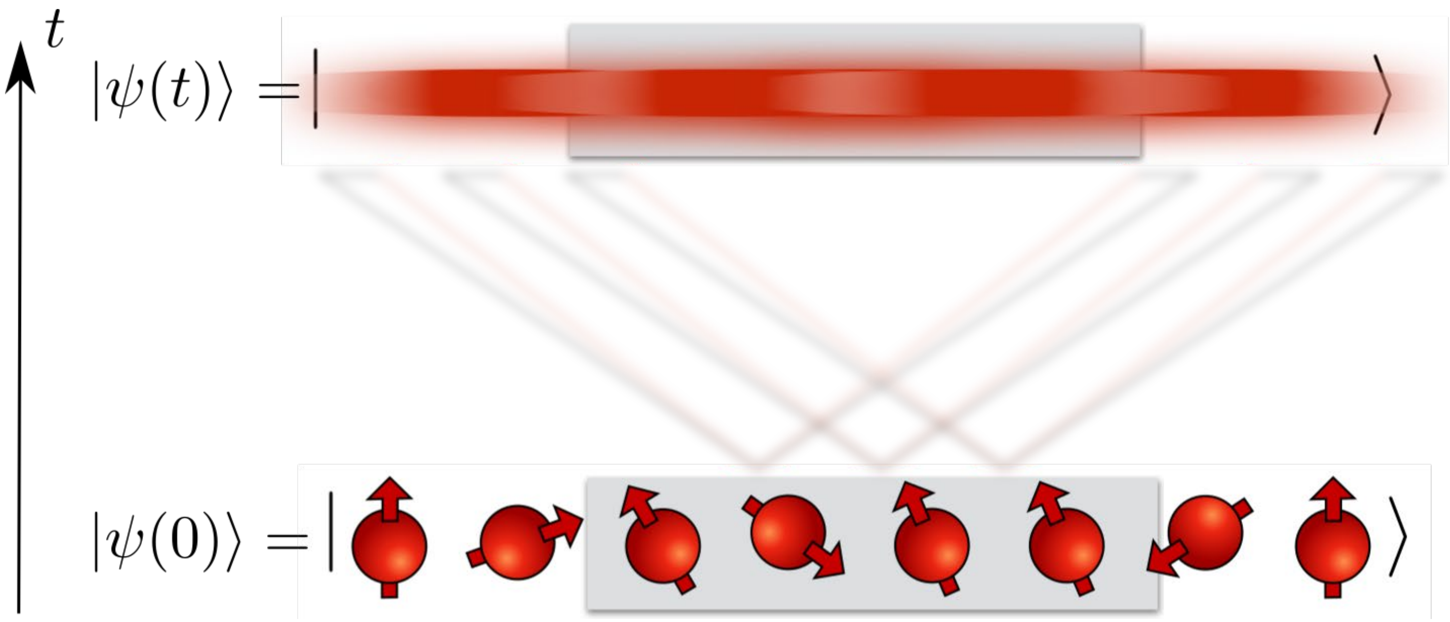
$$|\psi(0)\rangle =$$



Quantum thermalization

When does an isolated quantum many-body system thermalize?

$$\langle\psi(t)|\hat{O}|\psi(t)\rangle \xrightarrow{t\to\infty} \mathrm{tr}\left(\rho_{\mathrm{th}}(\beta,\mu,\ldots)\hat{O}\right)$$

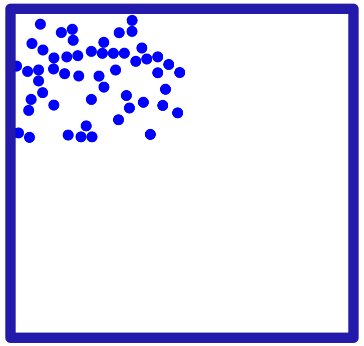


Different answers

Different answers

Thermalizing

- ✓ Eigenstates are thermal.
- ✓ Observables relax to thermal values.



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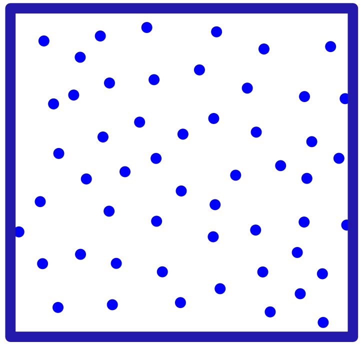

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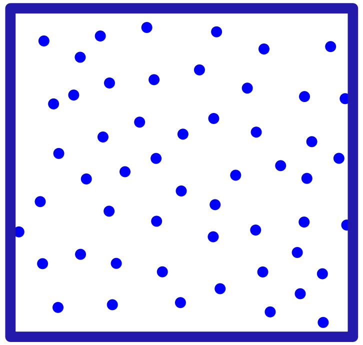
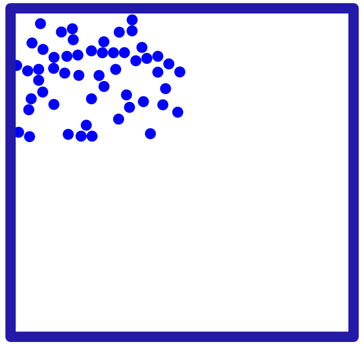


Many-body localization (MBL)

- ✓ Eigenstates are not thermal.
- ✓ Memory of the initial state.



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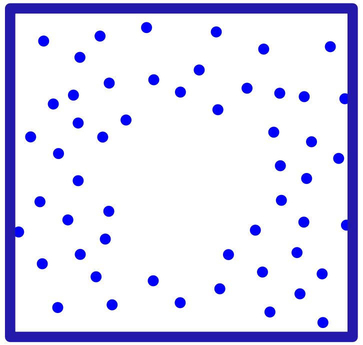
Intermediate regimes



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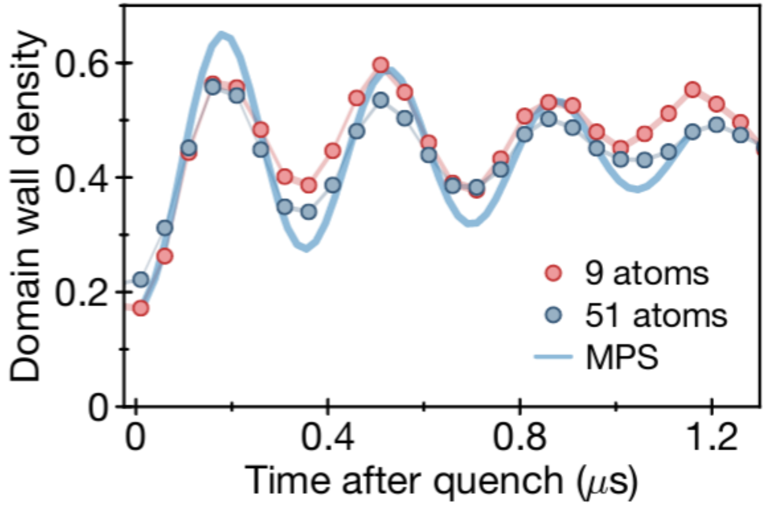
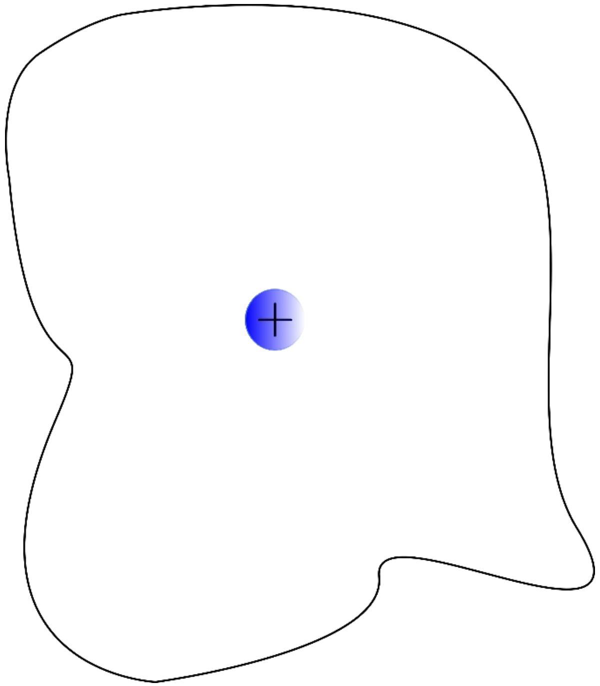


- ✓ Some initial states fail to thermalize.
- ✓ Quantum many-body scars.

Bernien et al., Nature 551, 597 '17

Fractonic dynamics

Charge: $Q = \sum_{i=1}^{N} q_i$ Dipole moment: $\vec{P} = \sum_{i=1}^{N} \vec{r}_i q_i$



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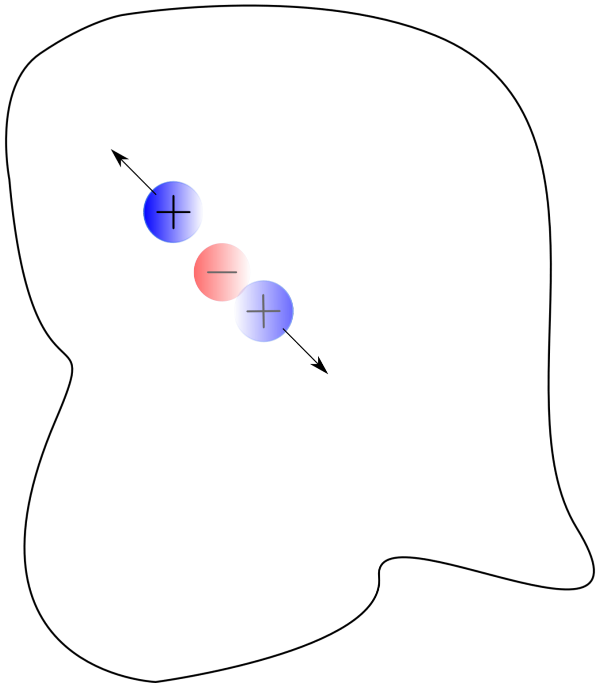

Fractonic dynamics

Key feature: Excitations with restricted mobility.

Fractonic dynamics

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- Fracton phases: generalized topological order.

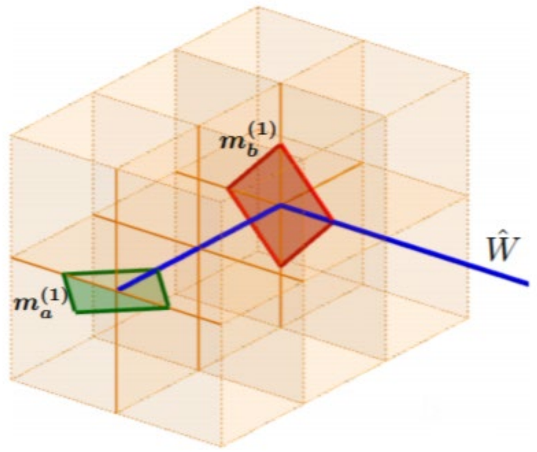
  Haah PRA 83, 042330 '11; Vijay, Haah ,Fu PRB 94, 235157 '16; Pretko PRB 95, 115139 '17,...

- Glassy behavior in strongly-correlated systems.

  Newman, Moore, PRE 60, 5068 '99; Chamon PRL 94, 040402 '05

- Localization with dipole conservation?

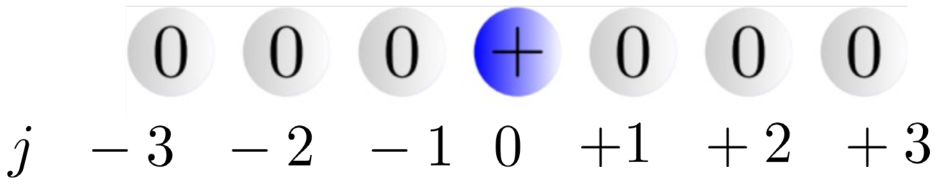
  Pai, Pretko, Nandkinshore PRX 9, 021003 '19



Simple model

Charge: $Q = \sum_{j=1}^{L} \hat{S}_j^z$ Dipole moment: $P = \sum_{j=1}^{L} j\hat{S}_j^z$

'Charges'= Spin-1: $|+\rangle = |\uparrow\rangle,\ |0\rangle = |0\rangle,\ |-\rangle = |\downarrow\rangle.\ \ |\pm\rangle = \hat{S}^{\pm}|0\rangle$



PS, Rakovszky, Verresen, Knap, Pollmann Phys. Rev. X 10, 011047 '20

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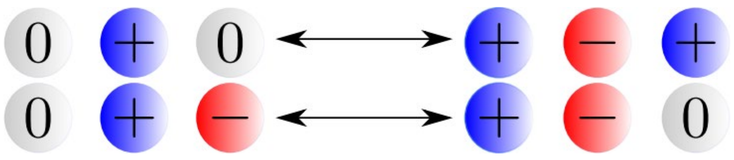
$$\color{red}{\hat{H}_3} = \sum_j \hat{S}^+_j \left(\hat{S}^-_{j+1}\right)^2 \hat{S}^+_{j+2} + \text{H.c.}$$



PS, Rakovszky, Verresen, Knap, Pollmann Phys. Rev. X 10, 011047 '20

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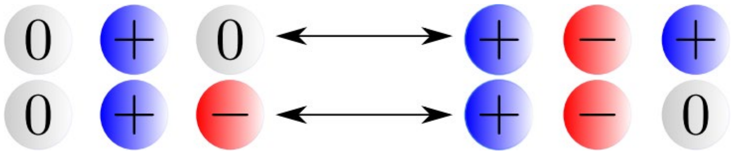
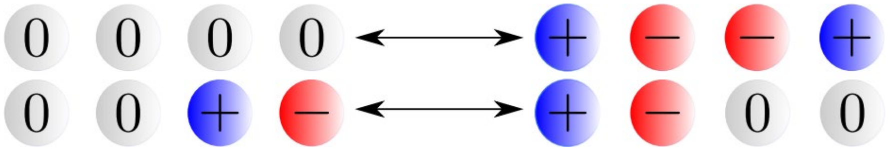


$$\hat{H}_4 = \sum_j \hat{S}_j^+ \hat{S}_{j+1}^- \hat{S}_{j+2}^- \hat{S}_{j+3}^+ + \text{H.c.}$$



PS, Rakovszky, Verresen, Knap, Pollmann Phys. Rev. X 10, 011047 '20

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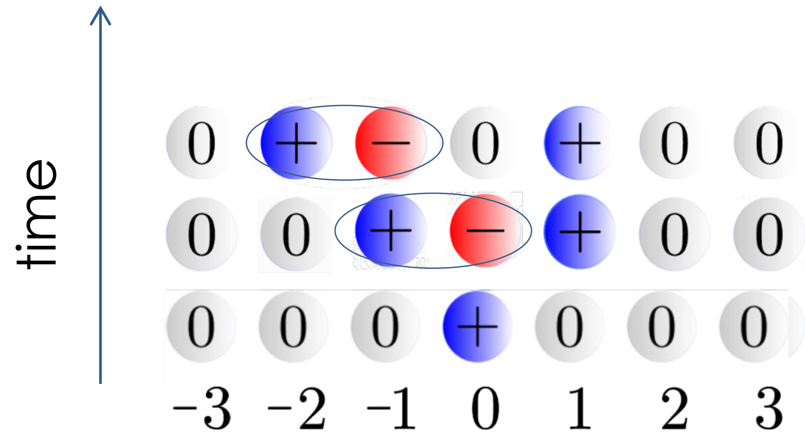
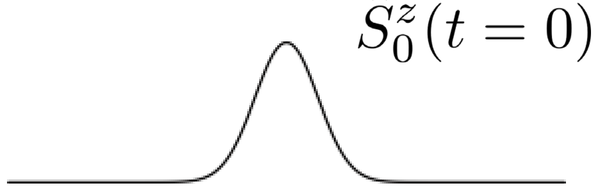


PS, Rakovszky, Verresen, Knap, Pollmann Phys. Rev. X 10, 011047 '20

Does the system thermalize?

$$C_j(t) = \langle \hat{S}^z_j(t)\hat{S}^z_0(0)\rangle_{\beta=0}$$



PS, Rakovszky, Verresen, Knap, Pollmann Phys. Rev. X 10, 011047 '20

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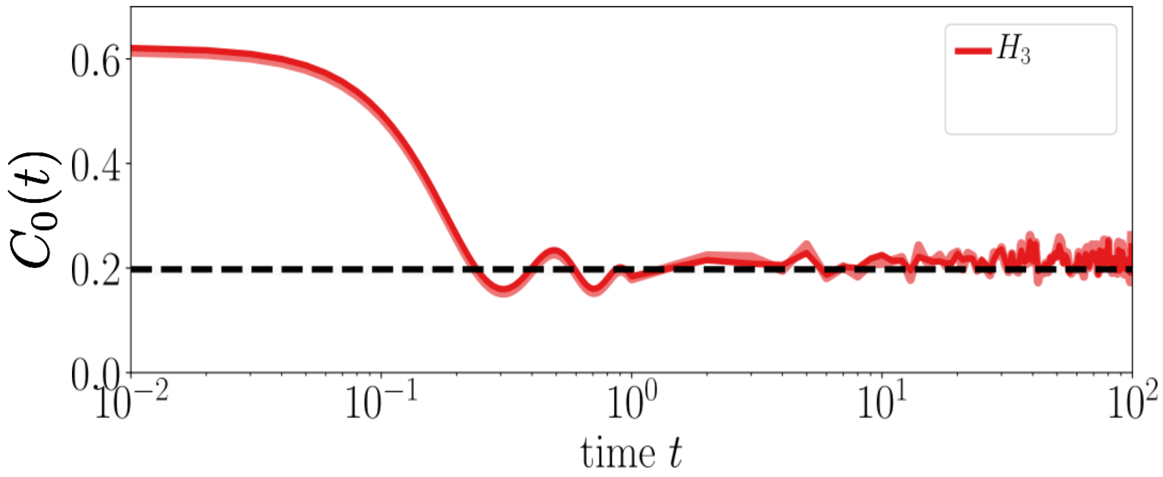
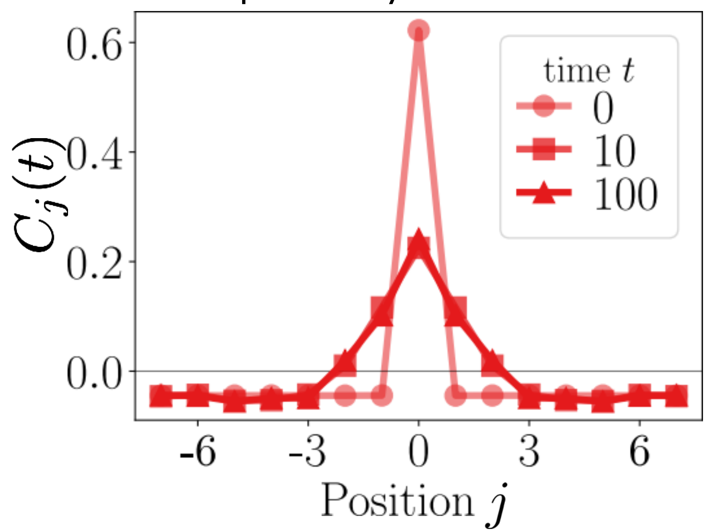


Spatially localized



PS, Rakovszky, Verresen, Knap, Pollmann Phys. Rev. X 10, 011047 '20

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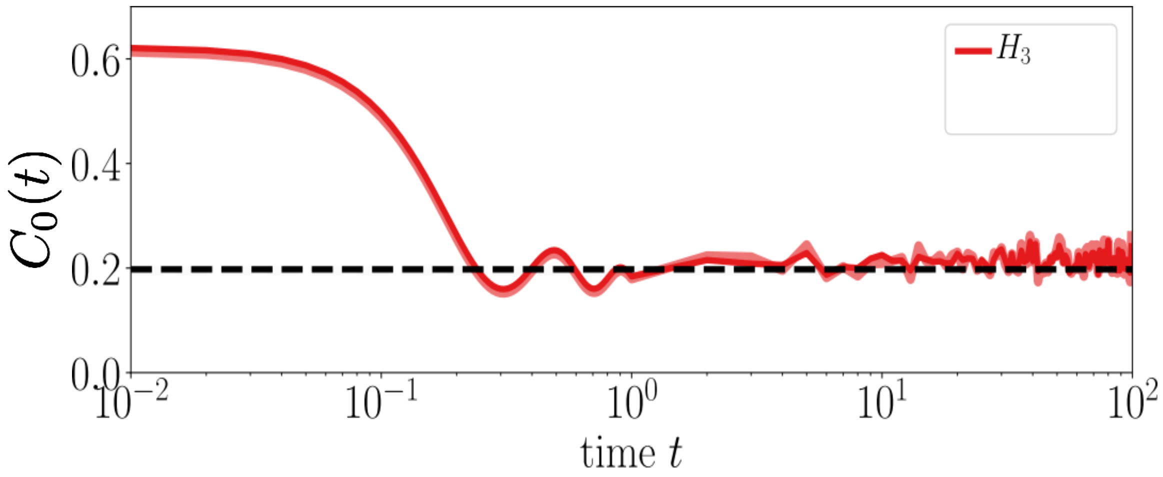

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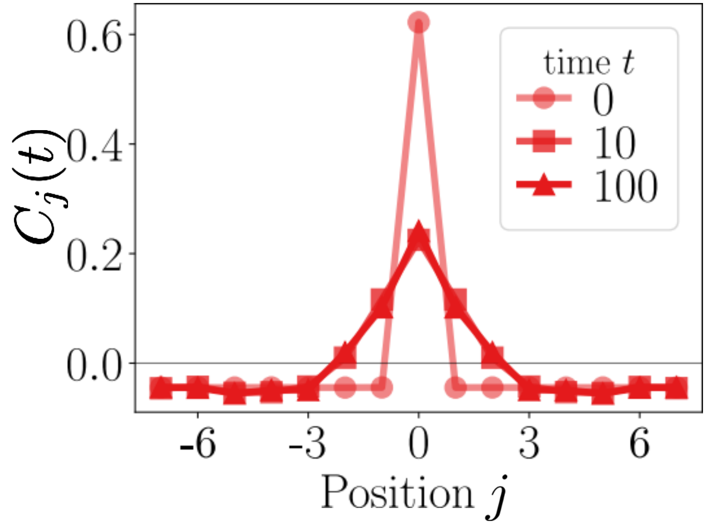

$\hat{H}_3$ Hilbert space fragmentation

$$\lim_{t\to\infty} \lim_{N\to\infty} C_0(t) > 0$$

PS, Rakovszky, Verresen, Knap, Pollmann Phys. Rev. X 10, 011047 '20

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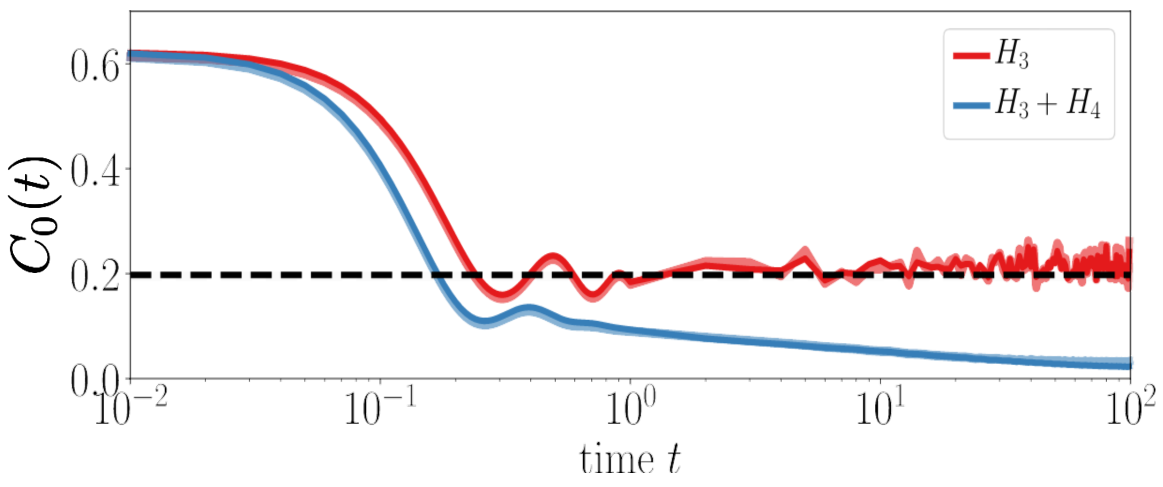

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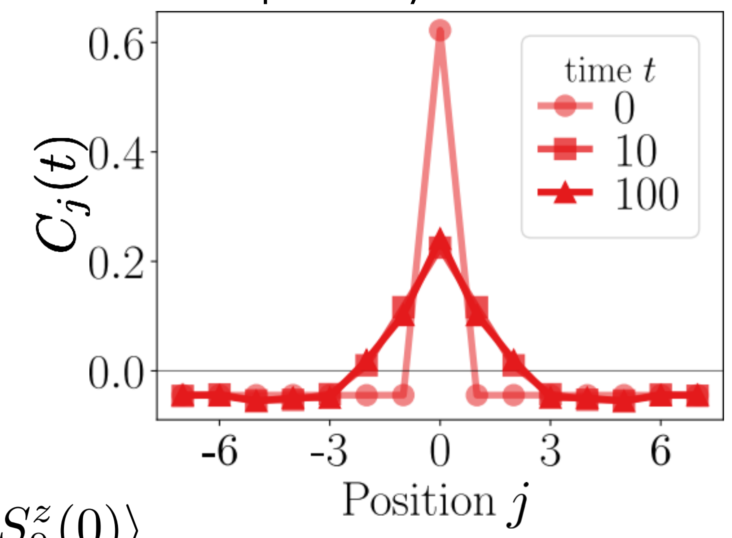
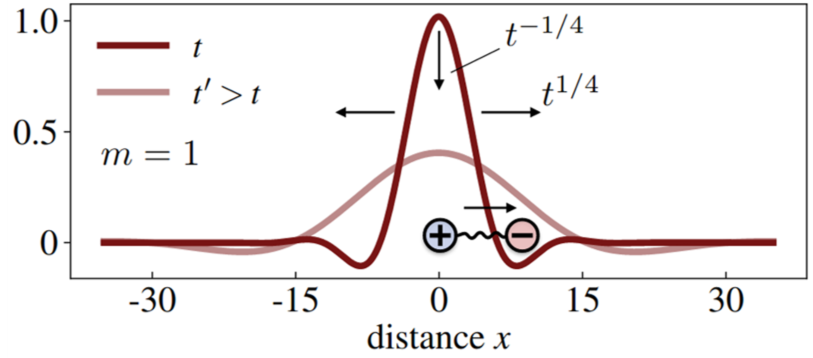


$\hat{H}_3 + \hat{H}_4$ Hydrodynamics

$\langle S^z_x(t)S^z_0(0)\rangle$



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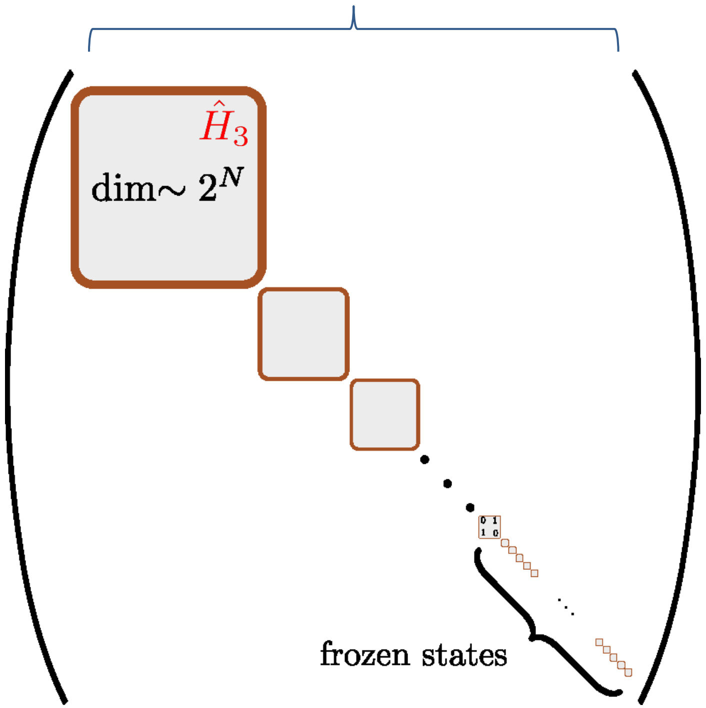
Hilbert space fragmentation

Each (Q,P) global symmetry sector fragments

Hilbert space dimension is 3^N



$$|\psi_1\rangle = |+\rangle_1|-\rangle_2|-\rangle_3|+\rangle_4$$

$$|\psi_2\rangle = |-\rangle_1|+\rangle_2|+\rangle_3|-\rangle_4$$

Local z-basis

PS, Rakovszky Verresen, Knap, Pollmann PRX 10, 011047 & PRB 101, 174204 '20 [see also PRB 101, 174204 '20]

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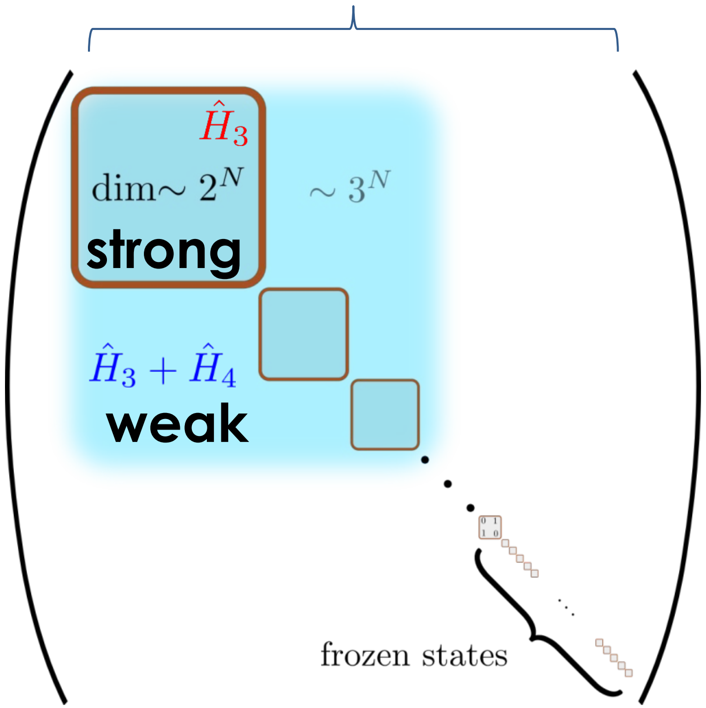

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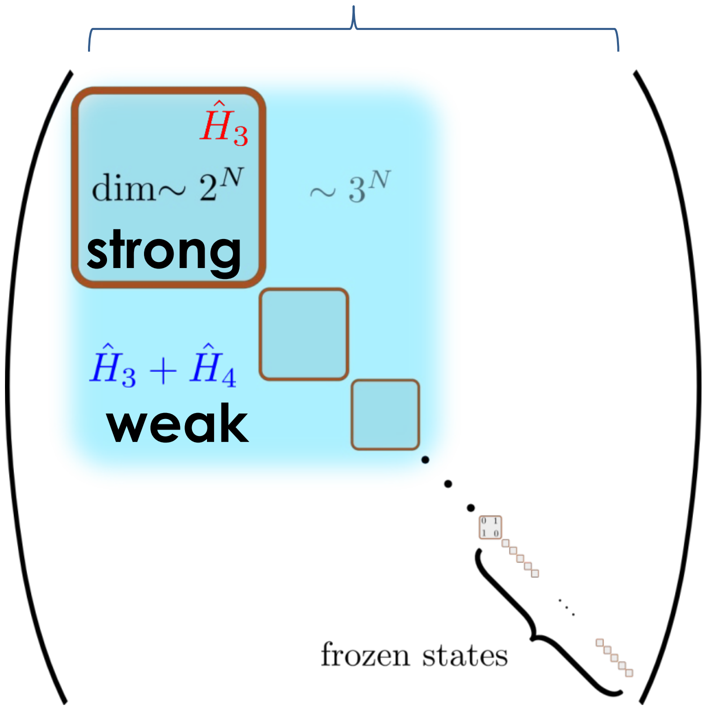
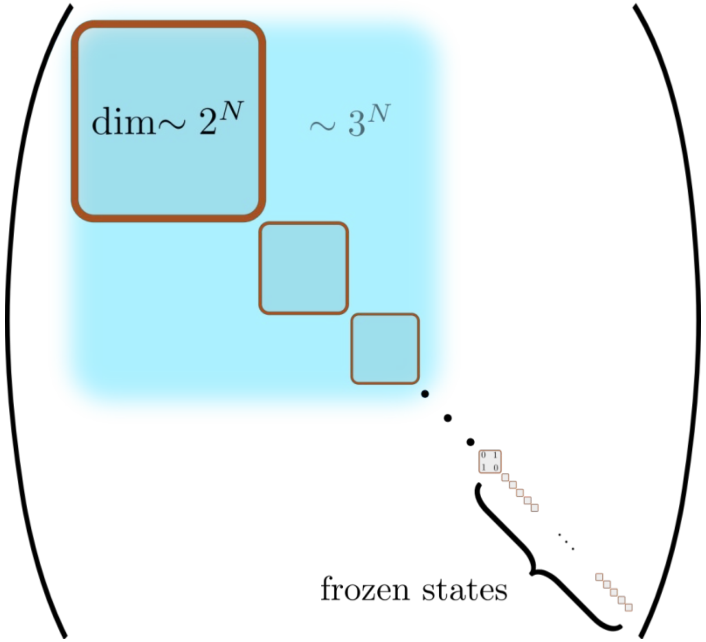


✓ Strong initial state dependence.

✓ Quantum scars and frozen states.

PS, Rakovszky Verresen, Knap, Pollmann PRX 10, 011047 & PRB 101, 174204 '20 [see also PRB 101, 174204 '20]

Fragmentation	Strong	Weak
# of sectors	~exp[N]	~exp[N]
Size of largest sector	$\sim 2^N \times 2^N$	$\sim 3^N \times 3^N$



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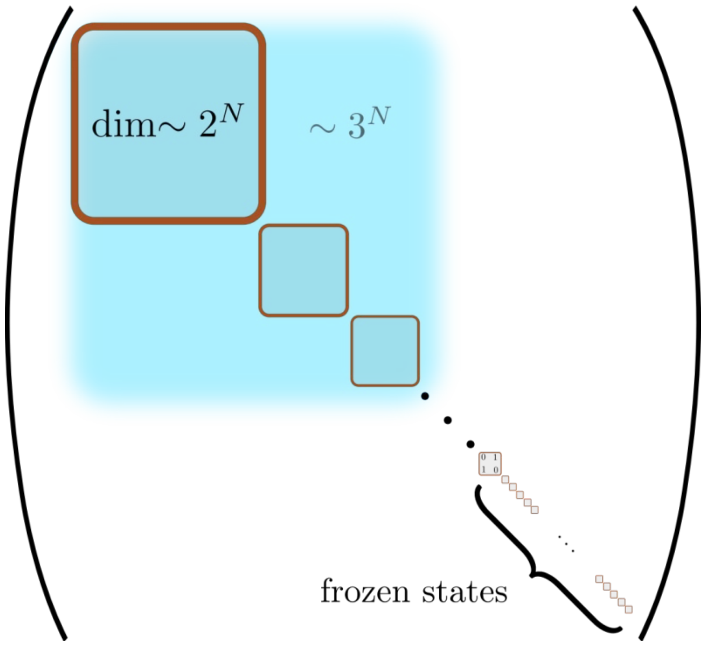


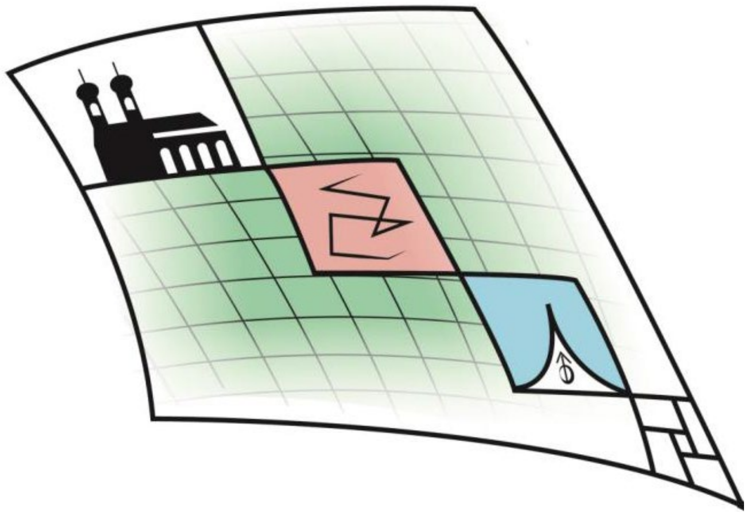
✓ Non-local conserved quantities and the ETH.

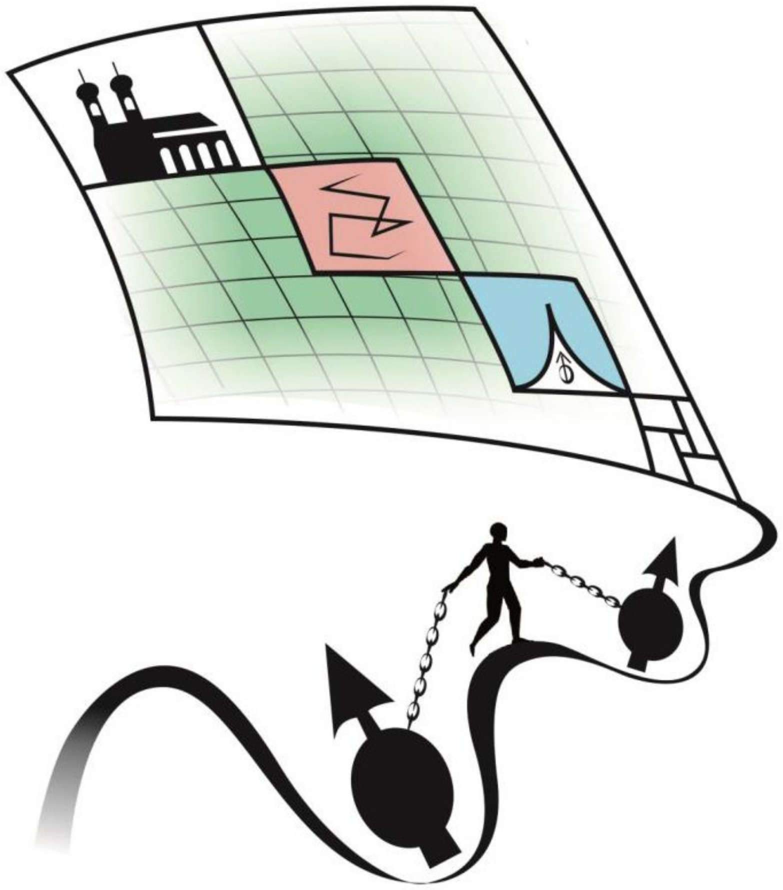
✓ Strong zero modes and SPT physics.

Thorngren, Vishwanath, Verresen PRB 104, 075132 ‘21

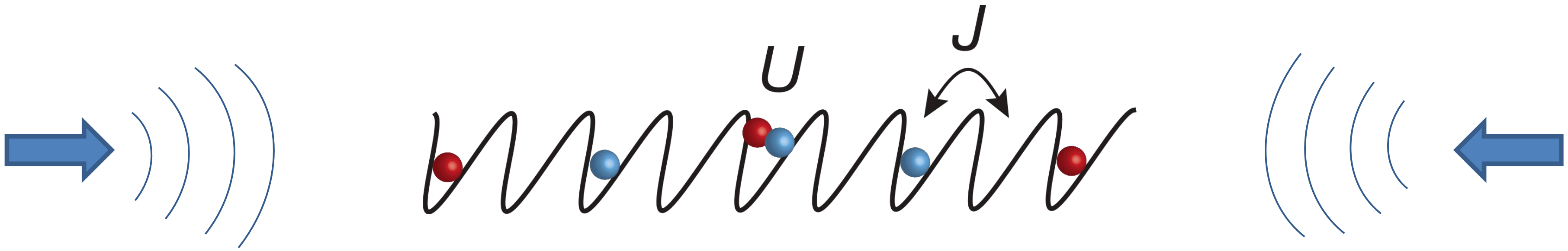
✓ Appearing in many other systems.

PS, Rakovszky Verresen, Knap, Pollmann PRX 10, 011047 & PRB 101, 174204 ’20 [see also PRB 101, 174204 ‘20]





Relation to experiments

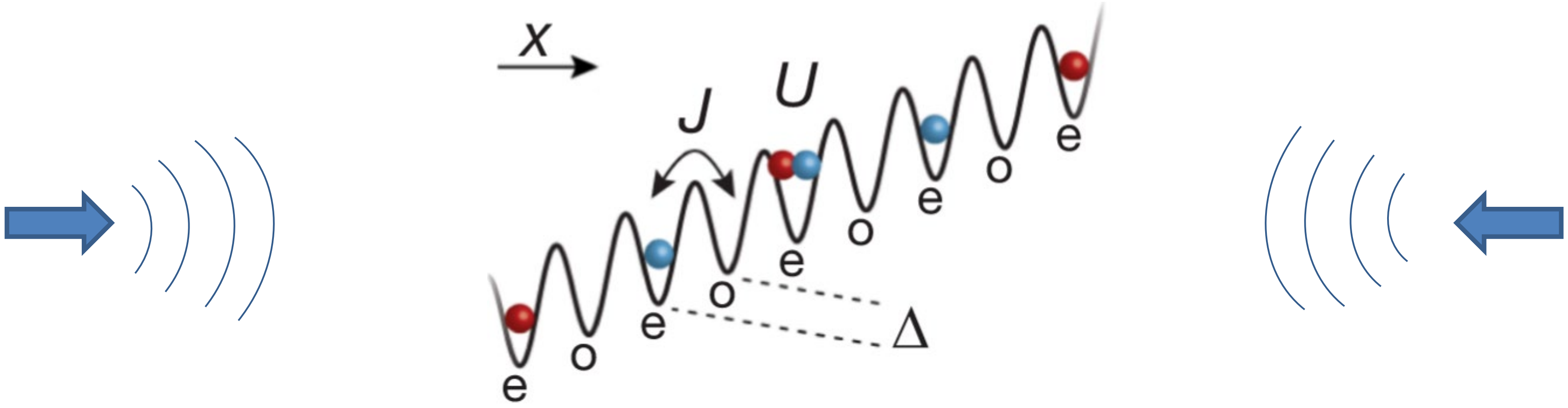


Fermi Hubbard model

$$\hat{H} = -J \sum_{j,\sigma=\uparrow,\downarrow} (\hat{c}_{j,\sigma}\hat{c}^{\dagger}_{j+1,\sigma} + \mathrm{H.c.}) + U \sum_{j} \hat{n}_{j,\uparrow}\hat{n}_{j,\downarrow}$$

Scherg, Köhlert, PS, Pollmann, Madhusudhana, Bloch, Aidelsburger Nat Comm 12, 4490 ‘21 & PRL 130, 010201 ‘23

Relation to experiments



Tilted Fermi Hubbard model

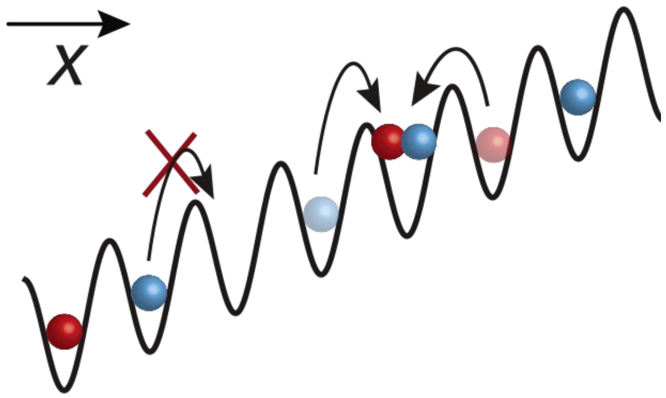
$$\hat{H} = -J \sum_{j,\sigma=\uparrow,\downarrow} (\hat{c}_{j,\sigma}\hat{c}^{\dagger}_{j+1,\sigma} + \text{H.c.}) + U \sum_j \hat{n}_{j,\uparrow}\hat{n}_{j,\downarrow} + \Delta \sum_j j\hat{n}_j$$

Scherg, Köhlert, PS, Pollmann, Madhusudhana, Bloch, Aidelsburger Nat Comm 12, 4490 '21 & PRL 130, 010201 '23

Approximate dipole conservation

$$\hat{H} = -J \sum_{j,\sigma=\uparrow,\downarrow} (\hat{c}_{j,\sigma}\hat{c}^{\dagger}_{j+1,\sigma} + \mathrm{H.c.}) + U\sum_j \hat{n}_{j,\uparrow}\hat{n}_{j,\downarrow} + \Delta \sum_j j\hat{n}_j$$

Take $\Delta \gg U, J$



$$\hat{H}_{\mathrm{eff}} = \hat{H}_0 + J_3\hat{H}_3 + J_4\hat{H}_4 + \cdots$$

Dipole moment is conserved!

$$\hat{P} = \sum_j j\hat{n}_j$$

Scherg, Köhlert, PS, Pollmann, Madhusudhana, Bloch, Aidelsburger Nat Comm 12, 4490 '21 & PRL 130, 010201 '23

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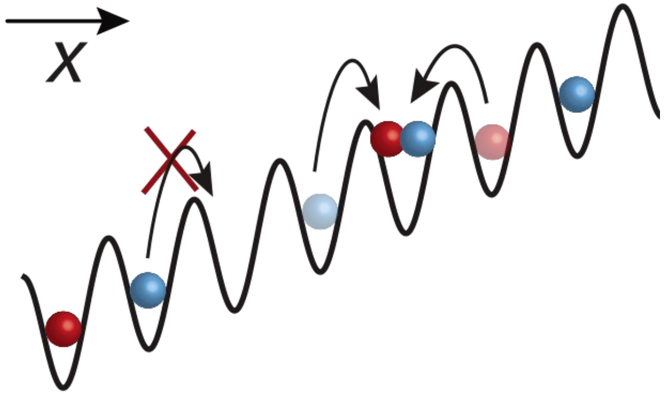
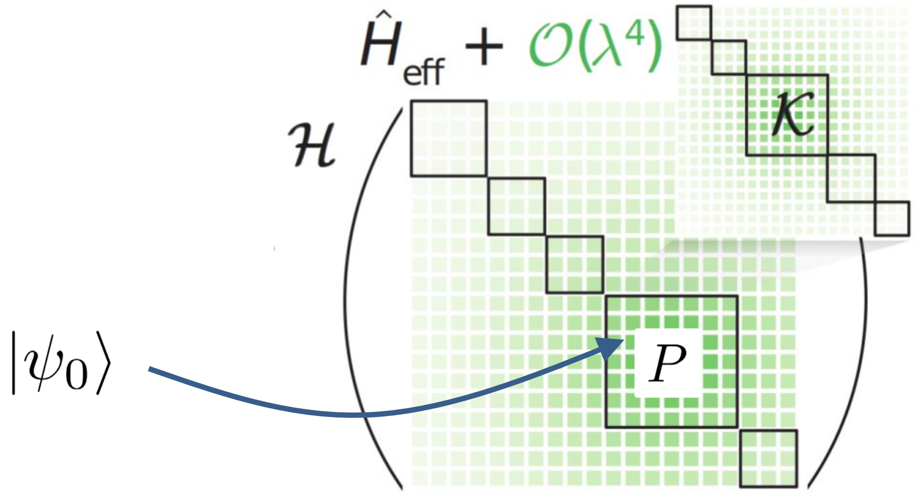


Hilbert space fragmentation

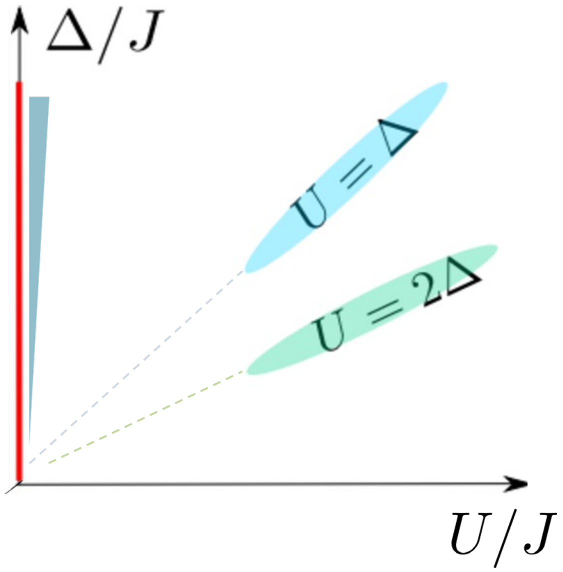


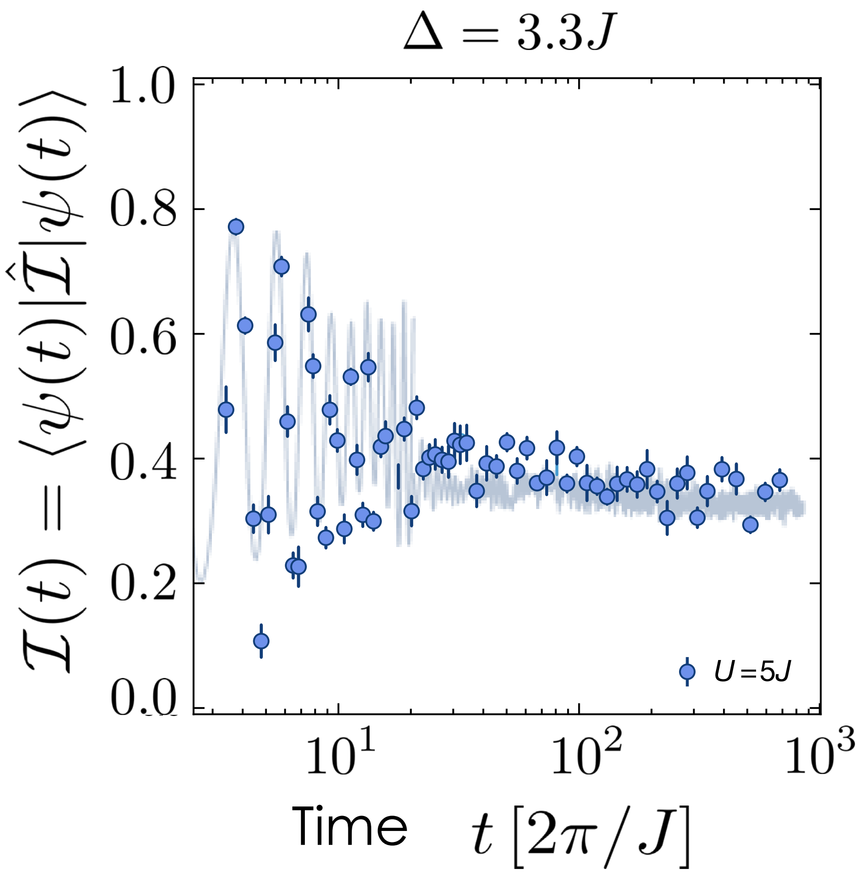
Scherg, Köhlert, PS, Pollmann, Madhusudhana, Bloch, Aidelsburger Nat Comm 12, 4490 '21 & PRL 130, 010201 '23

Experimental results

Different initial states

$$|\psi_0\rangle = |0 \uparrow 0 \downarrow 0 \downarrow 0 \uparrow 0 \downarrow 0 \uparrow 0\rangle$$

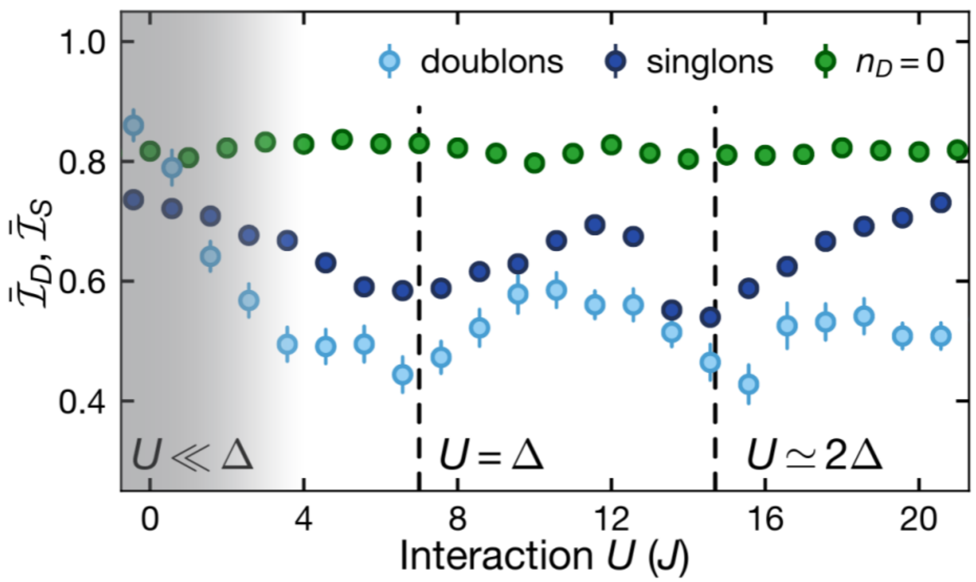




Scherg, Köhlert, PS, Pollmann, Madhusudhana, Bloch, Aidelsburger Nat Comm 12, 4490 '21 & PRL 130, 010201 '23

Experimental results

Different constrained dynamics



✓ Observed non-ergodic behavior.

✓ Description using fragmentation.

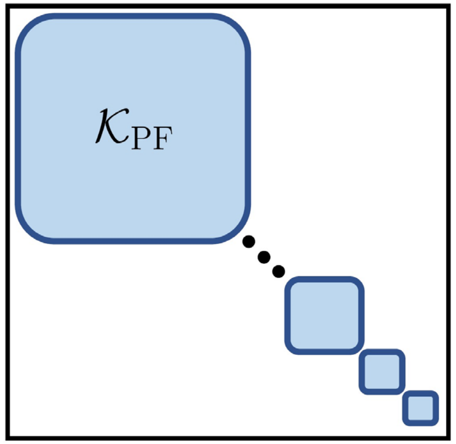
✓ Relevant for Stark Many-body Localization.

PRL 122 040606 '19 & PNAS 116, 9269 '19

Scherg, Köhlert, PS, Pollmann, Madhusudhana, Bloch, Aidelsburger Nat Comm 12, 4490 '21 & PRL 130, 010201 '23

Back to theory world

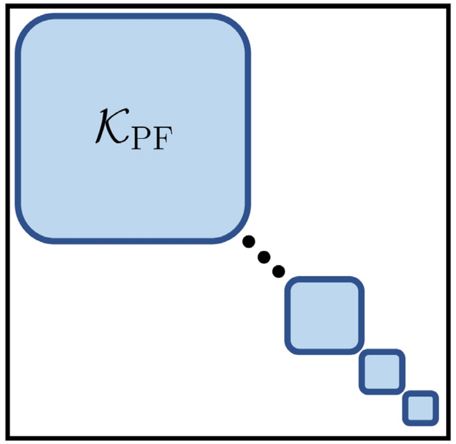
Beyond classical fragmentation



Family of Hamiltonians

$$H = \sum_j J_j \hat{h}_j$$

Beyond classical fragmentation



Family of Hamiltonians

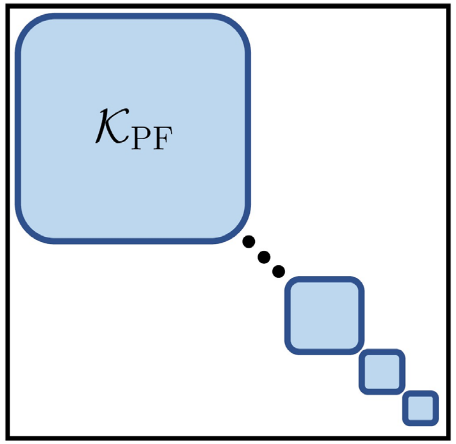
$$H = \sum_j J_j \hat{h}_j$$

$$\mathcal{A} = \langle\langle \{\hat{h}_j\} \rangle\rangle$$

$$\mathcal{C} = \{O : [O, \hat{h}_j] = 0 \; \forall j\}$$

Moudgalya, Motrunich PRX 12, 011050 ‘22

Beyond classical fragmentation



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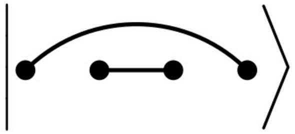
Moudgalya, Motrunich PRX 12, 011050 '22

$$|\psi_1\rangle = |+\rangle_1|-\rangle_2|-\rangle_3|+\rangle_4$$

$$|\psi_2\rangle = |-\rangle_1|+\rangle_2|+\rangle_3|-\rangle_4$$

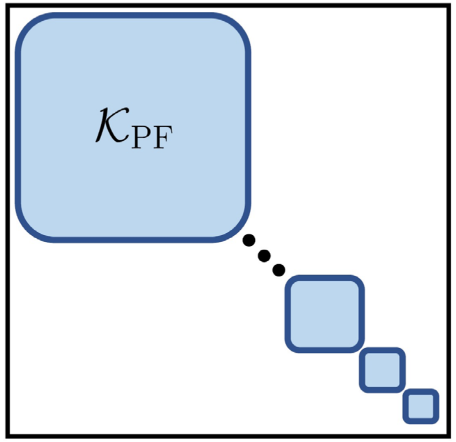
Entangled basis?

$|\psi_1\rangle =$

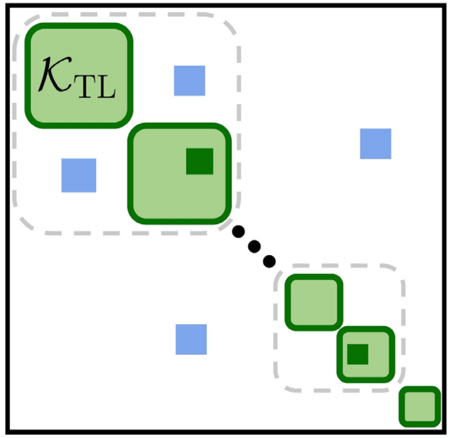


Beyond classical fragmentation

Classical



Quantum

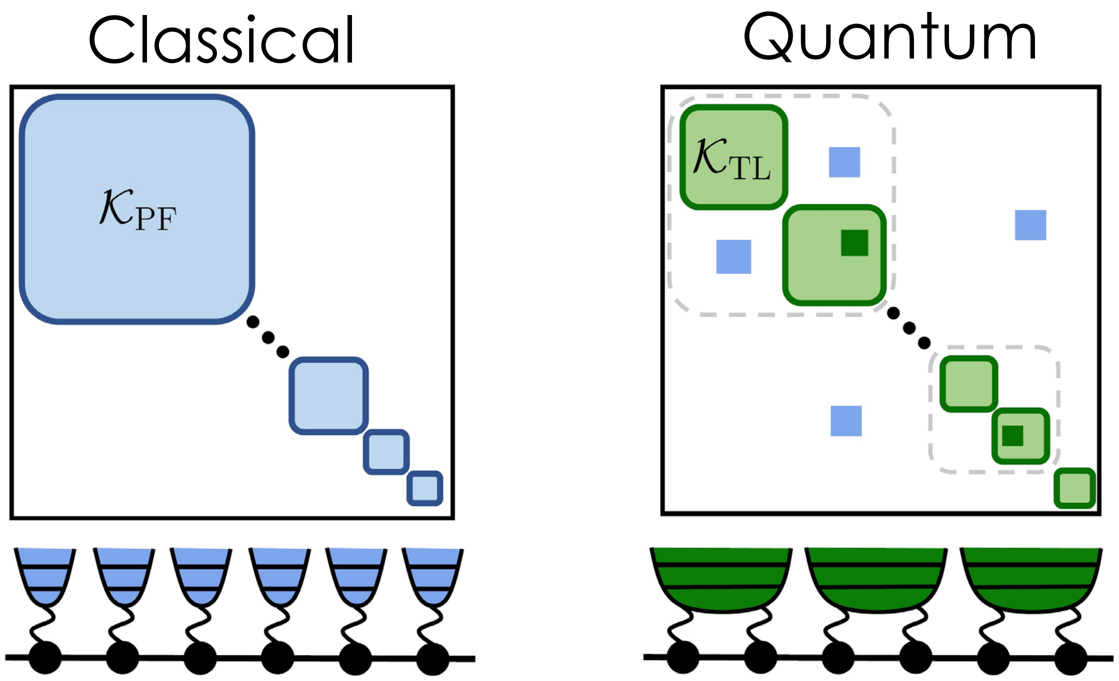


How to distinguish them in the dynamics?

$$|\psi_0\rangle \longrightarrow |\psi(t)\rangle$$

Li, PS, Pollmann arXiv:2305.06918

Beyond classical fragmentation



How to distinguish them in the dynamics?

$$|\psi_0\rangle \longrightarrow \rho(t)$$

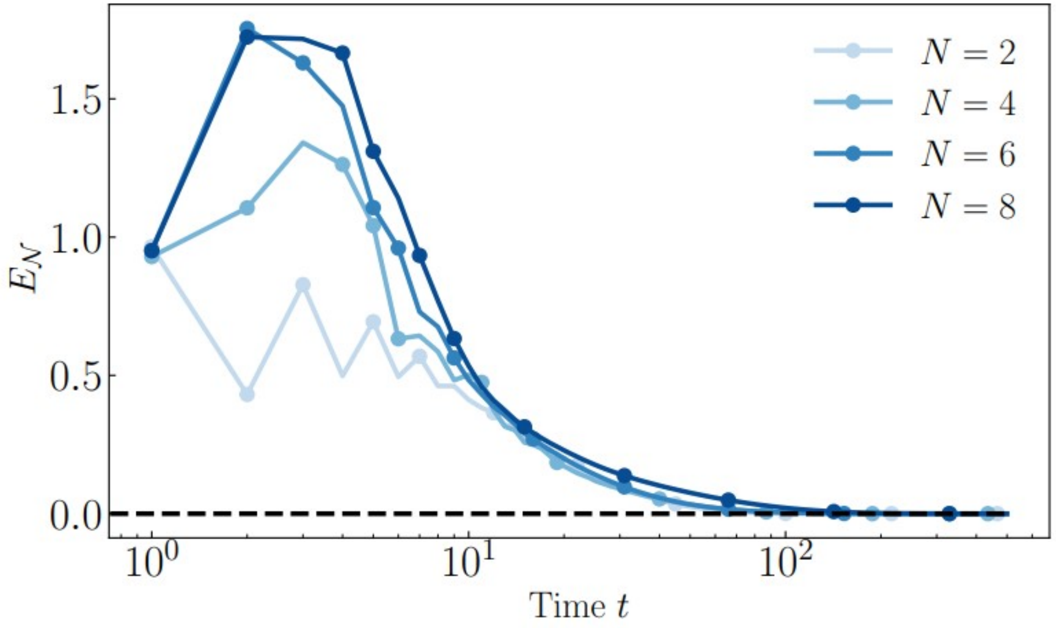
Entanglement negativity

$$E_N = \log ||\rho^{T_A}||_1$$

Li, PS, Pollmann arXiv:2305.06918

Beyond classical fragmentation

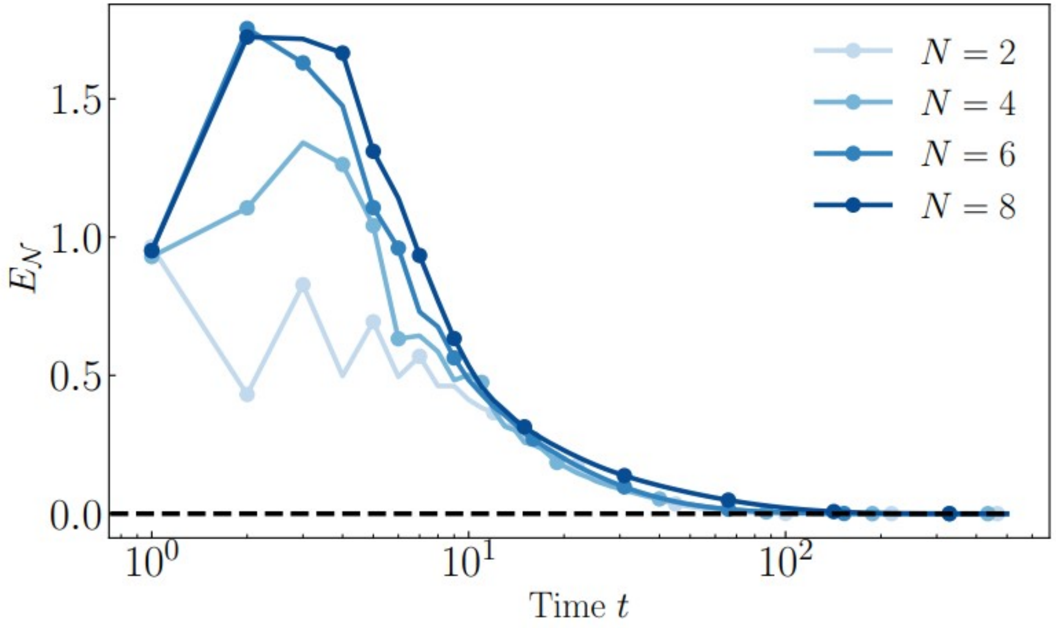
Classical



Li, PS, Pollmann arXiv:2305.06918

Beyond classical fragmentation

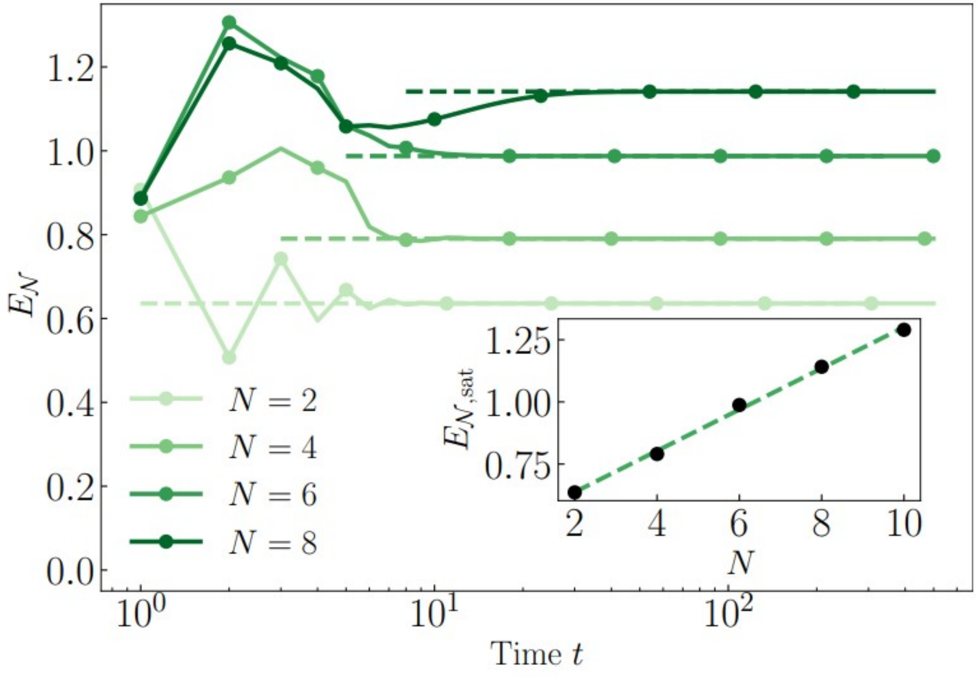
Classical



Rich structure in stationary state

$$\rho_{ss} = \sum_{\lambda,\alpha,\alpha'} (M_\lambda)_{\alpha\alpha'} \frac{\Pi^\lambda_{\alpha\alpha'} \mathbb{1}}{D_\lambda}$$

Quantum



Volume-law entanglement!

Li, PS, Pollmann arXiv:2305.06918

Conclusions

- Symmetries can constrain the dynamics.
- Experimental realization in tilted systems.
- Extended notion of symmetry in many-body systems.
- Quantum versus classical fragmentation.

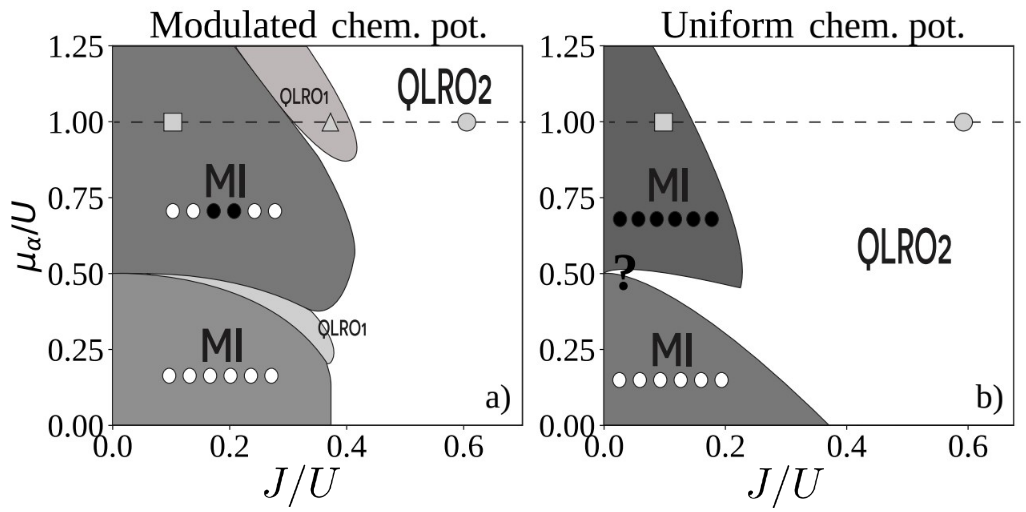
Thank you

Theory: F. Pollmann, M. Knap, T. Rakovszky, R. Verresen, G. De Tomasi, D. Hetterich, J. Feldmeier, Y. Li.

Experiments: B. Hebbe, S. Scherg, T. Kohlert, M. Aidelsburger, I. Bloch.

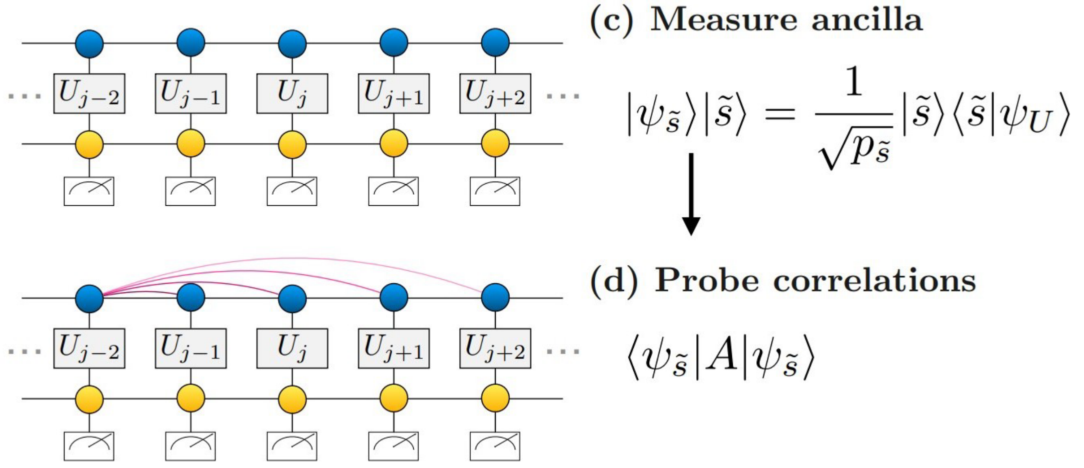
In case you are also interested

Exotic quantum liquids in Bose-Hubbard models with spatially-modulated symmetries



PS, Y. You, J. Hausschild, O. Motrunich
arXiv:2307,08761

Measurement-altered Ising quantum criticality



S. Murciano, PS, Y. Liu, R. Mong, J. Alicea
arXiv:2302,04325