Efficient method for quantum impurity problems out of equilibrium

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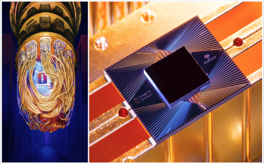
Quantum systems out of equilibrium

Thermalization

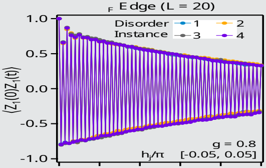
ETH or MBL?

Quantum computing



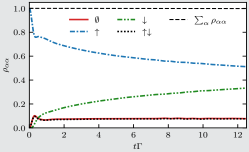
Quantum advantage?

Dynamical phases



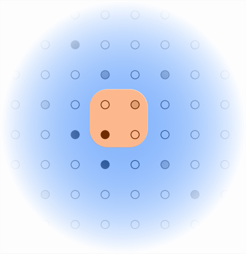
Time crystals, ...

Impurity models



Kondo physics, DMFT, NMR

Real time evolution of local observables



$$\langle O(t) \rangle_{\rho(t=0)=\rho_0} \qquad \text{or} \qquad \langle O(0)O(t) \rangle_{\rho_0}$$

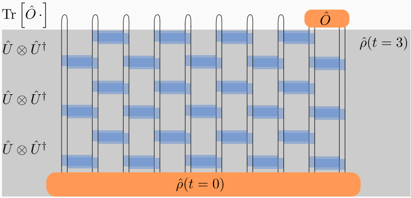
- Measurable quantity
- Transport properties: (Sub)diffusion vs ballistic
- Localization, Edge modes

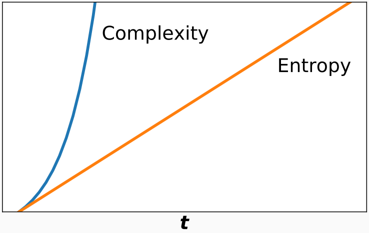
Wavefunction based approaches

- Represent the density matrix at different time $\rho(t)$
- Obtain observables by computing $\mathrm{Tr}[\rho(t)O]$

Problem:

Complexity grows exponentially with time!

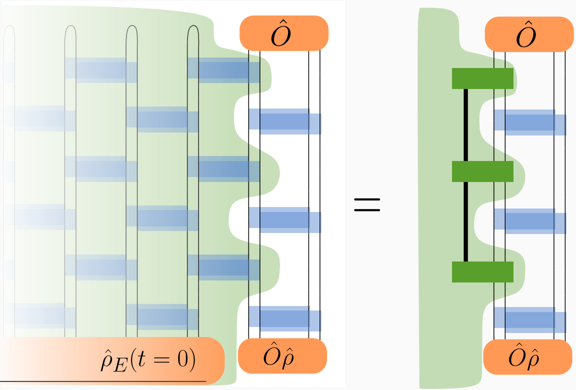




Influence functional approach[1]

Idea:

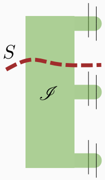
Describe the local subsystem as a quantum system coupled to a bath.



[1] Feynman and Vernon Jr 1963.

Temporal Entanglement

The numerical power of the Influence matrix approach is limited by **temporal entanglement** instead of spatial entanglement



- Represents the "Memory" of the quantum bath
- Low temporal entanglement $\Rightarrow$ efficient MPS representation
- Examples for low temporal entanglement:
  - Very chaotic systems $U \approx e^{i\frac{\pi}{4}Z_iZ_{i+1}+hZ_i}e^{i\frac{\pi}{4}X_i}$[1]
  - Close to integrability $U \approx e^{iJ_xX_iX_{i+1}+iJ_yY_iY_{i+1}}$[2]
  - Dissipative system[3]
  - Many body localized systems - Exact disorder averaging![4]

[1] Lerose, Sonner, and Abanin 2021a.

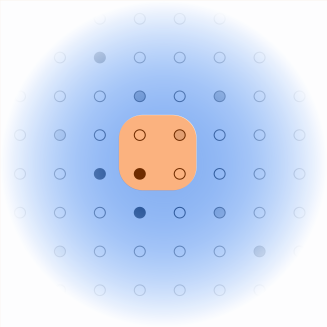
[2] Lerose, Sonner, and Abanin 2021b; Giudice et al. 2021; Thoenniss, Lerose, and Abanin 2022.

[3] Sonner, Lerose, and Abanin 2021; Mi et al. 2022.

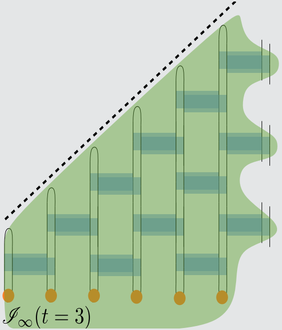
[4] Sonner, Lerose, and Abanin 2022.

Obtaining the Influence matrix

Free fermions



One dimensional models [1]



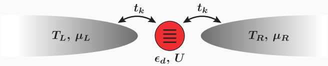
[1] Lerose, Sonner, and Abanin 2022; Frías-Pérez and Bañuls 2022; Banuls et al. 2009

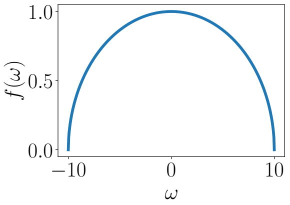
Model

Single Impurity Anderson model

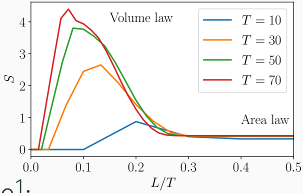
$$H = \sum_{k,\sigma} \left(\epsilon_{k,\sigma} c^\dagger_{k,\sigma} c_{k,\sigma} + V_{k,\sigma} d^\dagger_\sigma c_{k,\sigma} + h.c. \right) + U(n_\uparrow - \frac{1}{2})(n_\downarrow - \frac{1}{2})$$

- Impurity with on-site interaction coupled to free fermions
- Bath defined by spectral density $f(\omega) = 2\pi \sum_k |V_k|^2 \delta(\epsilon_k - \omega)$
- Kondo effect: Effective impurity spin forms singlet with the fermions in the bath
- Quantum embedding approaches

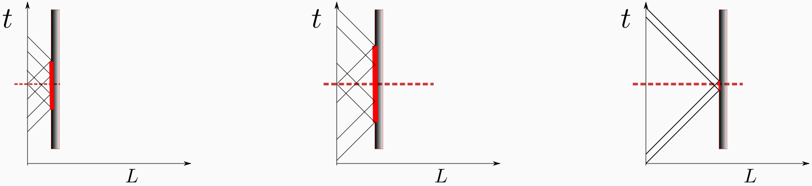




Entanglement barrier[2]



Quasiparticle picture[1]:



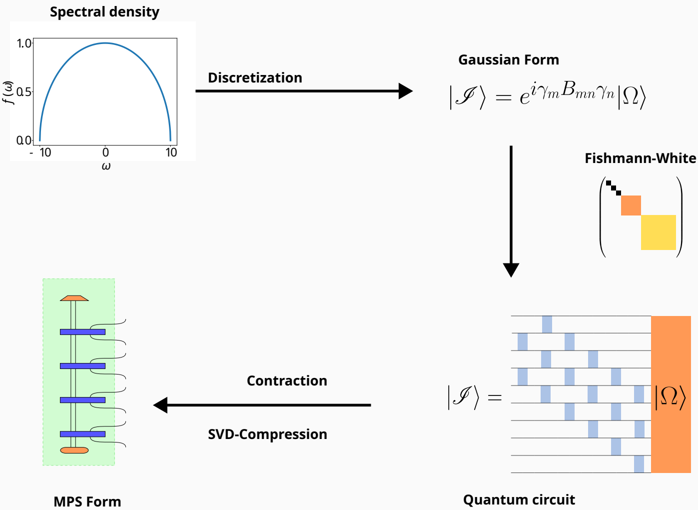
Reflection at the boundary cause high temporal entanglement in finite system IM!

But: thermodynamic IM has low temporal entanglement

[1]Calabrese and Cardy 2005.

[2]Lerose, Sonner, and Abanin 2022.

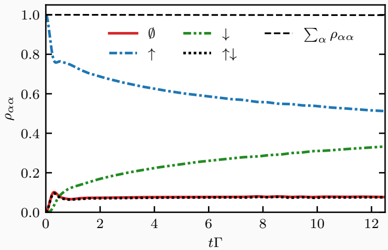
Algorithm[1]



[1]Thoenniss, Lerose, and Abanin 2022; Thoenniss et al. 2023.

Spin relaxation[2]

- Initialize impurity in the $|\uparrow\rangle$ state
- Slow spin relaxation in the Kondo regime
- Competitive with state of the art QMC [1]

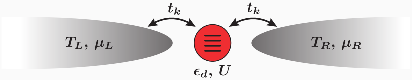


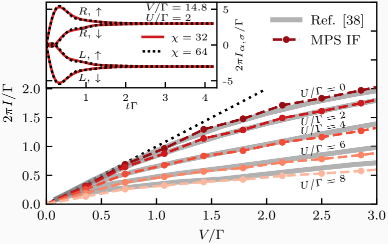
[1] Cohen et al. 2015

[2] Thoenniss et al. 2023.

Transport[2]

- Left and right reservoir at different chemical potential
- Small bond dimension $\chi = 32, 64$ (!)
- Steady state reachable
- Comparable with QMC [1]





[1]Bertrand et al. 2019

[2]Thoenniss et al. 2023.

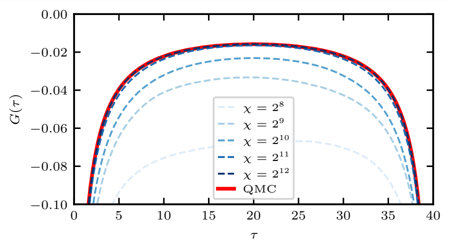
Imaginary time - RAMPS[1]

Imaginary time Green's function

$$G(\tau) = \mathrm{Tr}(e^{-(\beta-\tau)H} c^\dagger e^{-\tau H} c)$$

Benedikt Kloss,
Matthew T. Fishman,
E. M. Stoudenmire,
Olivier Parcollet,
Antoine Georges

- Central step for Dynamic mean field theory (DMFT)
- Requires relatively high bond dimension
- Multi-orbital possible, no sign problem!



[1] Kloss et al. 2023.

Summary and Outlook

Compute local observables by finding a compressed representation of the large quantum system as a bath (IM)

- Complexity of the IM can be low even in traditionally difficult regimes
- For free fermions there is an efficient algorithm to compute the IM directly
- Natural for impurity problems, DMFT!
- For one dimensional interactive models, we can be efficiently obtain the IM through sequencial contraction[1]

[1] Lerose, Sonner, and Abanin 2022; Frías-Pérez and Bañuls 2022.