

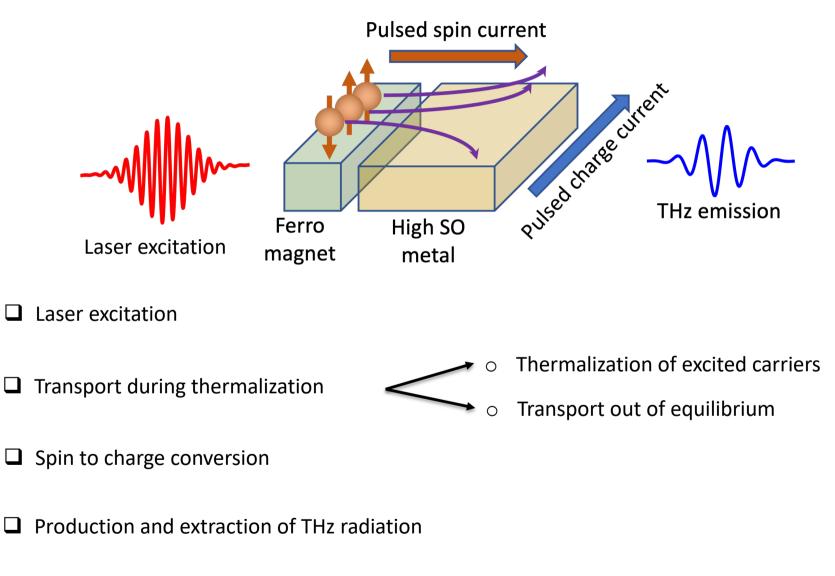
#### Fundamentals of spin transport for THz spintronic

Marco BATTIATO

Nanyang Technological University Singapore



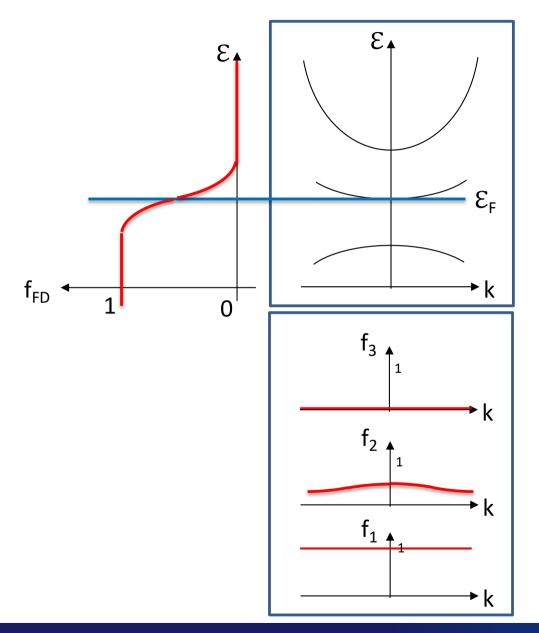
# Spintronics THz emitters: step by step



Kampfrath, Battiato et al. Nature Nanotech. 8, 256 (2013)



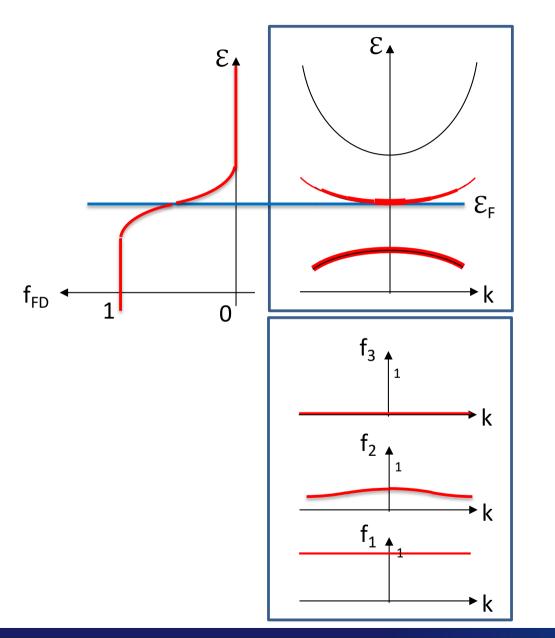
## **Population in excited material**



**9** 

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## **Population in excited material**

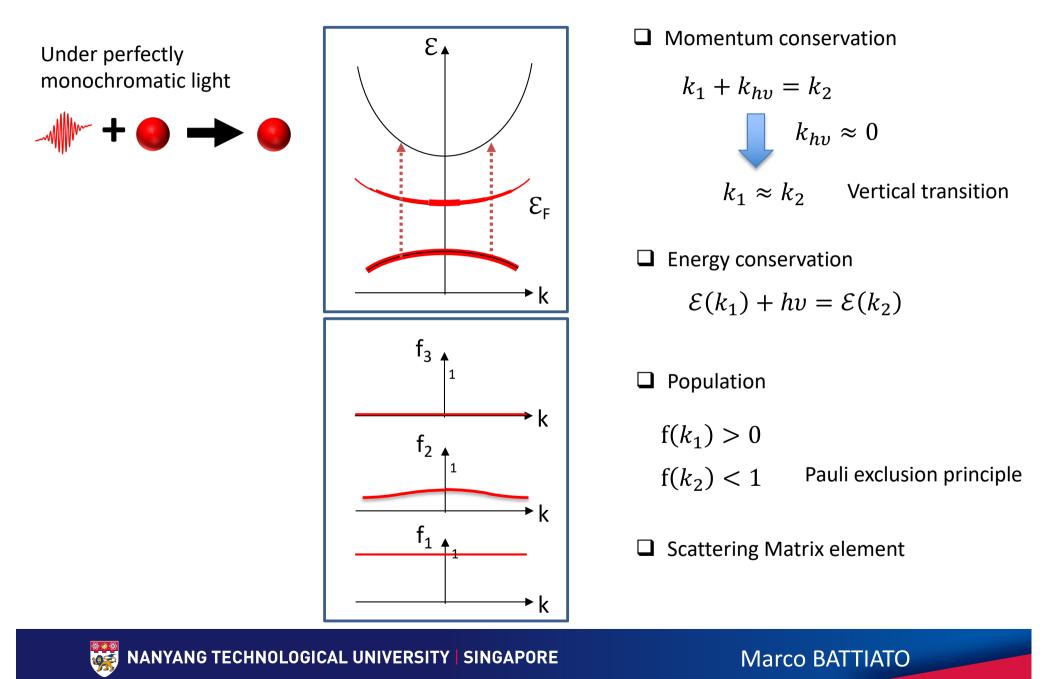


... We can do the same with phonon bands

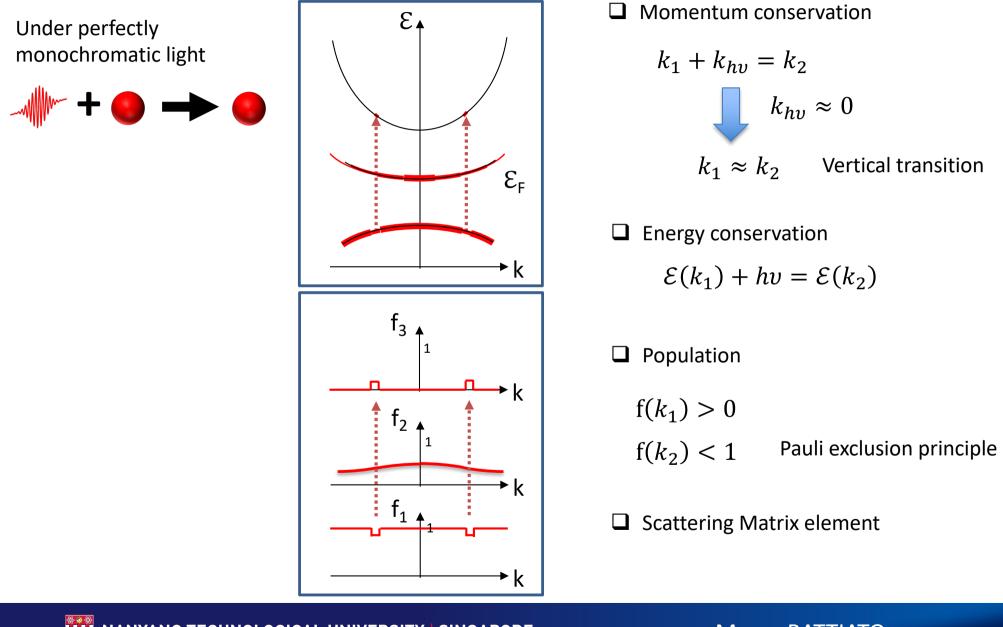


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# **Basics of laser excitation**



# **Basics of laser excitation**

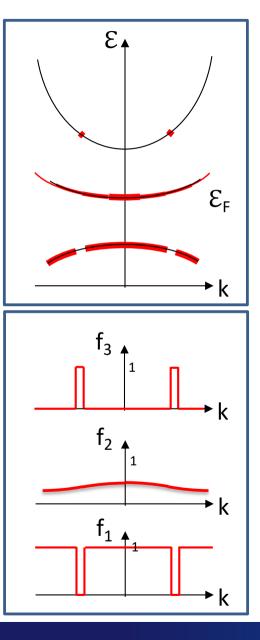


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# **Basics of laser excitation**

Under perfectly monochromatic light

If nothing else happens photons keep being absorbed and electrons being excited (we will see later what happens in reality)



Momentum conservation

$$k_{1} + k_{hv} = k_{2}$$

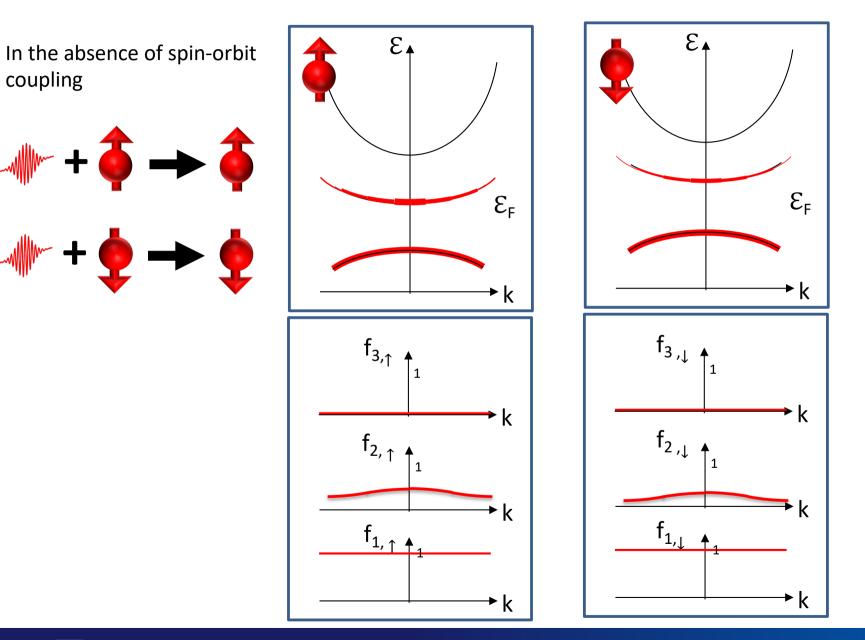
$$k_{hv} \approx 0$$

$$k_{1} \approx k_{2}$$

- □ Energy conservation
  - $\mathcal{E}(k_1) + hv = \mathcal{E}(k_2)$
- Population
  - $f(k_1) > 0$  $f(k_2) < 1$
- □ Scattering Matrix element

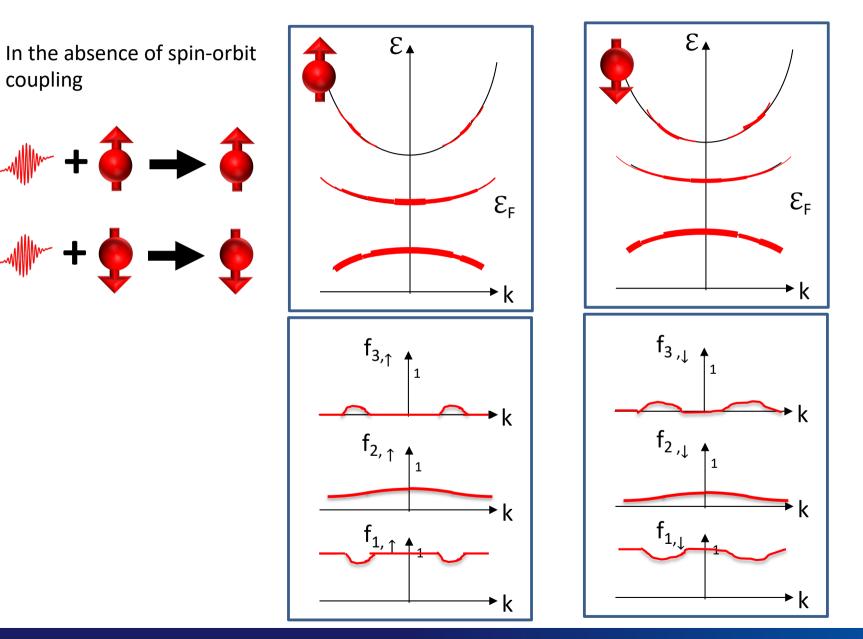


# Laser excitation (with a spin)



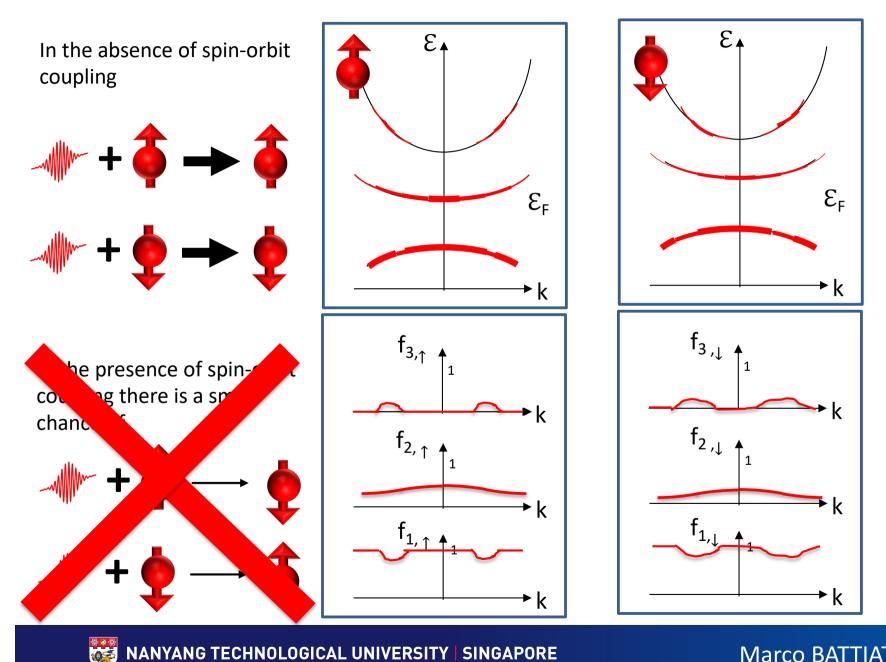
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# Laser excitation (with a spin)



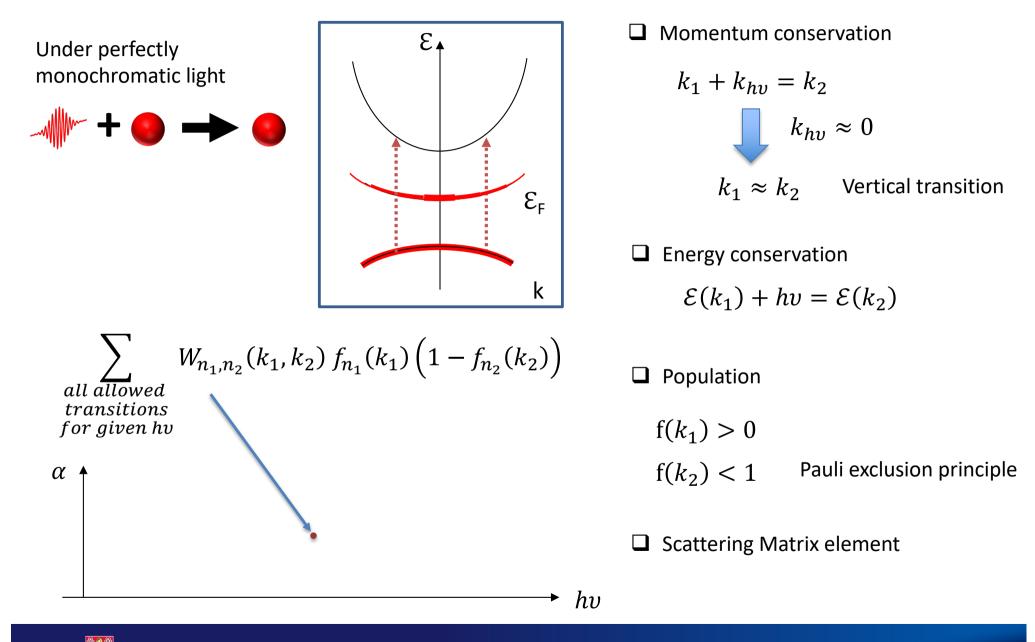
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## **Laser excitation (with a spin)**



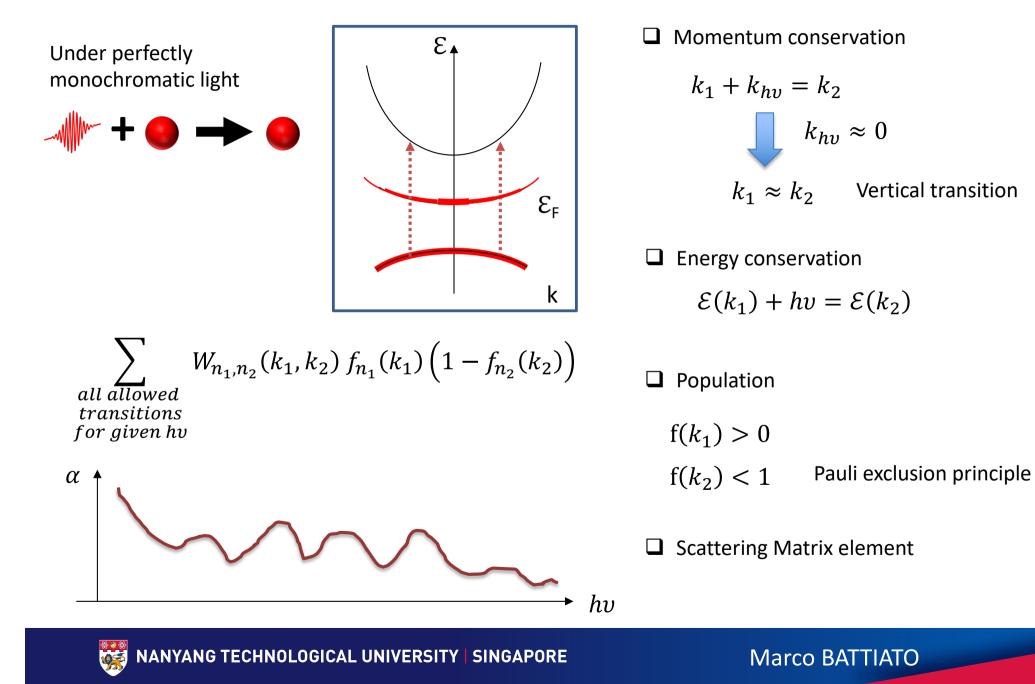
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# Laser excitation: absorption spectrum



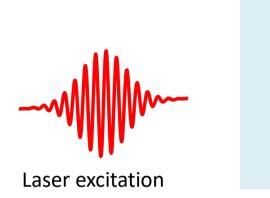


# Laser excitation: absorption spectrum

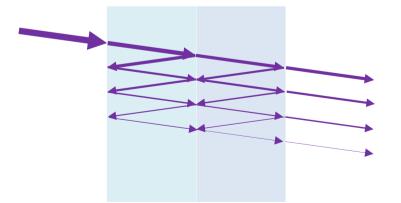


# Laser excitation: multilayer

What we said above is in the case of spatially uniform photon density in a single material. What happens when there is a multilayer



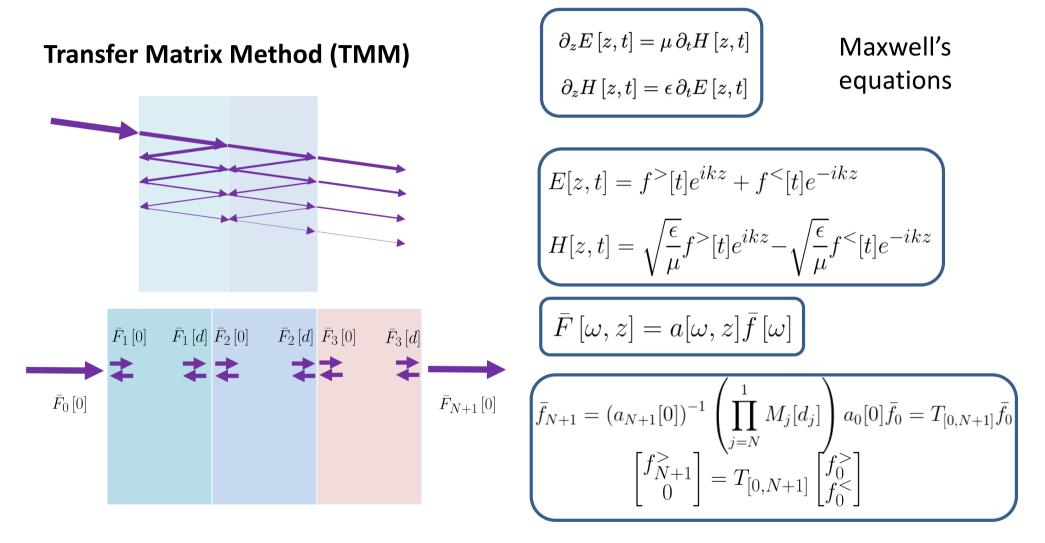
The photons, while being absorbed, will move throughout the sample, following Maxwell's equations





# Laser excitation: multilayer

To treat this we can use the well-known Transfer Matrix Method

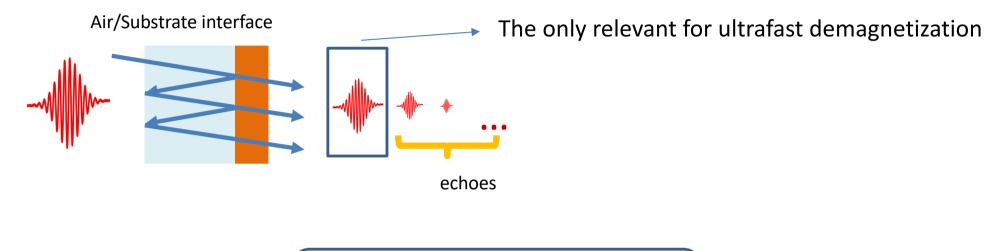


It is then possible to reconstruct the photon density and therefore the spatial profile of electron excitations

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## Laser excitation: echo

Using TMM naively for the laser excitation leads to an overestimation of the excitation



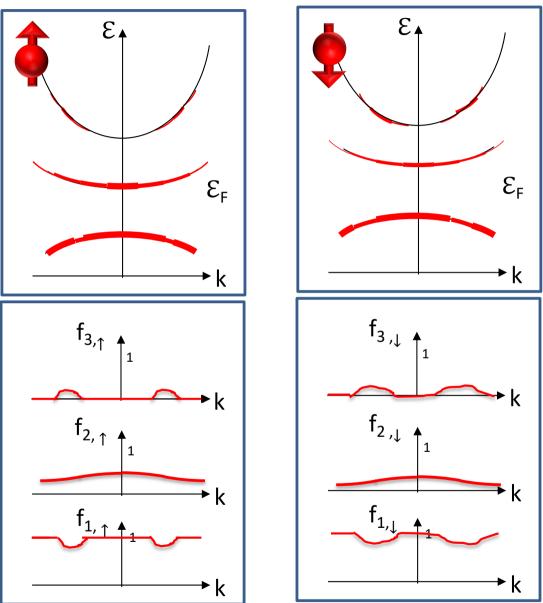
$$\begin{bmatrix} f_{N+1}^{>\text{no echo}} \\ 0 \end{bmatrix} = T_{[S,N+1]} \begin{bmatrix} f_S^{>*} \\ f_S^{<*} \end{bmatrix} = T_{[S,N+1]} \begin{bmatrix} t_{[0,S]} f_0^{>} \\ f_S^{<*} \end{bmatrix}$$
$$f_S^{>*} = t_{[0,S]} f_0^{>} \qquad f_{N+1}^{>\text{no echo}} = t_{[0,S]} t_{[S,N+1]} f_0^{>}.$$

We will see how echo has a way more serious effect on the THz frequency range

Y Yang, S Dal Forno, M Battiato, J Infr Millim, 42 1142 (2021)



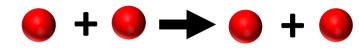
#### Laser excitation

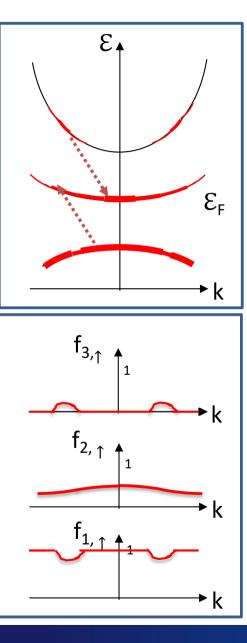


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# **Thermalisation**

Excited electrons can scatter





Momentum conservation

$$k_1 + k_2 = k_3 + k_4$$

In reality there is the possibility of umklapp and the correct equation is  $k_1 + k_2 = k_3 + k_4 + G$ 

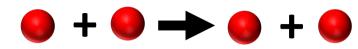
□ Energy conservation  $\mathcal{E}(k_1) + \mathcal{E}(k_2) = \mathcal{E}(k_3) + \mathcal{E}(k_4)$ 

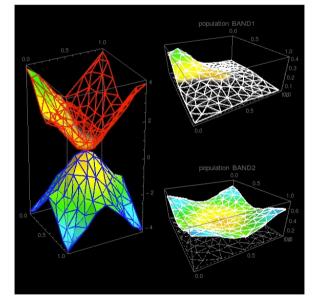
- Population
- $f(k_1) > 0$   $f(k_2) > 0$
- $\mathbf{f}(k_3) < 1 \quad \mathbf{f}(k_4) < 1$
- □ Scattering Matrix element

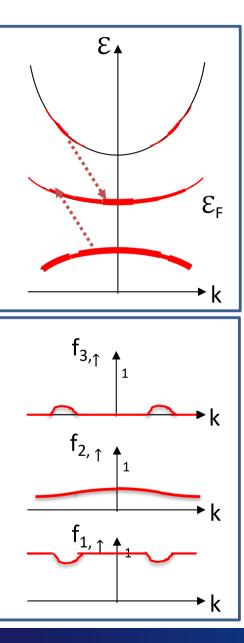


# **Thermalisation**

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Momentum conservation

$$k_1 + k_2 = k_3 + k_4$$

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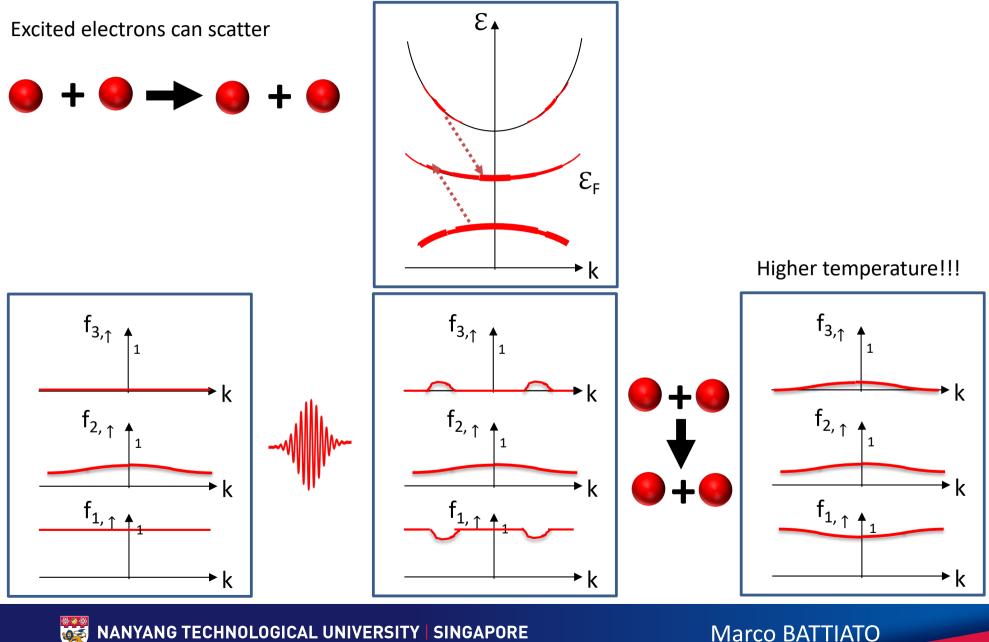
Energy conservation
$\mathcal{E}(k_1) + \mathcal{E}(k_2) = \mathcal{E}(k_3) + \mathcal{E}(k_4)$

- Population  $f(k_1) > 0$   $f(k_2) > 0$   $f(k_3) < 1$   $f(k_4) < 1$
- □ Scattering Matrix element



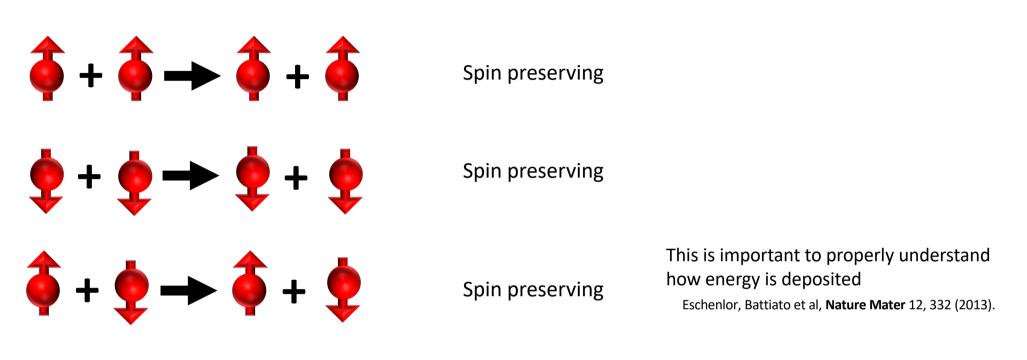
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# **Thermalisation**

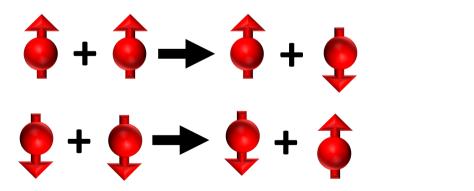


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## Thermalisation with spin



In the presence of spin orbit coupling, with lower probability there can be

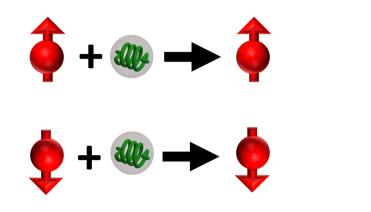


Spin flipped

Spin flipped



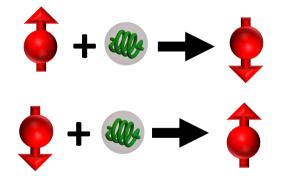
# **Cooling down with phonons**



Spin preserving

Spin preserving

In the presence of spin orbit coupling, with lower probability there can be



Spin flipped (Elliot-Yafet)

Spin flipped (Elliot-Yafet)

Usually slower than electron-electron scatterings (... well it's a bit more complicated than that)

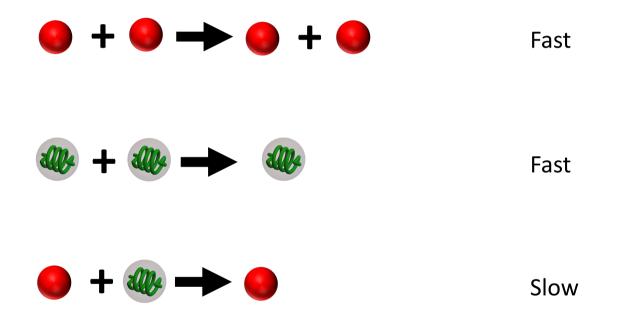


# **Cooling down with phonons**

What is the effect?

In general, it can be complicated and complex phenomena can arise, like phonon bottlenecks

Let us make some approximations just to get an intuitive understanding



Notice that this is not always a good approximation

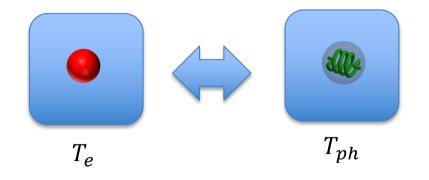


# **Cooling down with phonons**

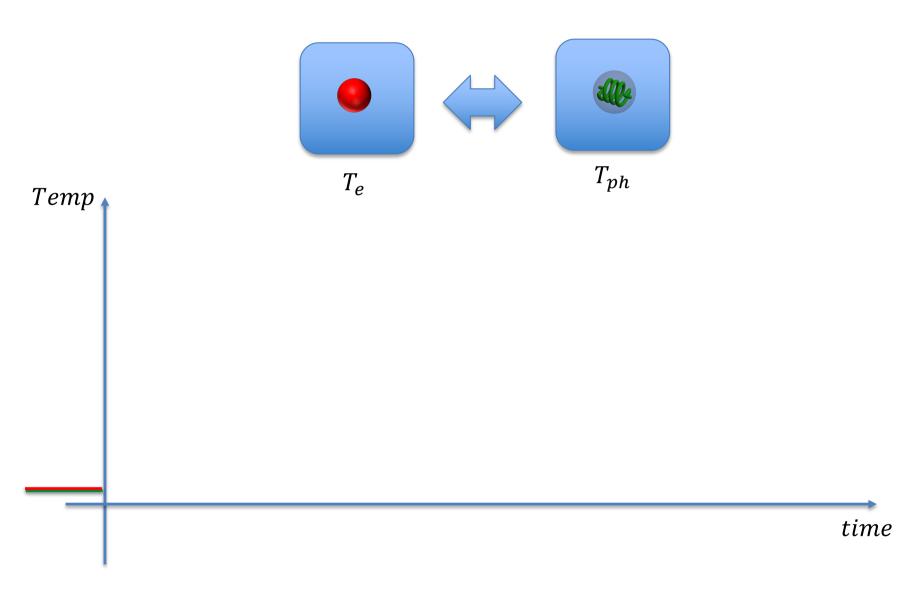
In that case, the electrons and the phonons can be described as two systems

- 1. Always very close to the thermalized state
- 2. Exchanging energy

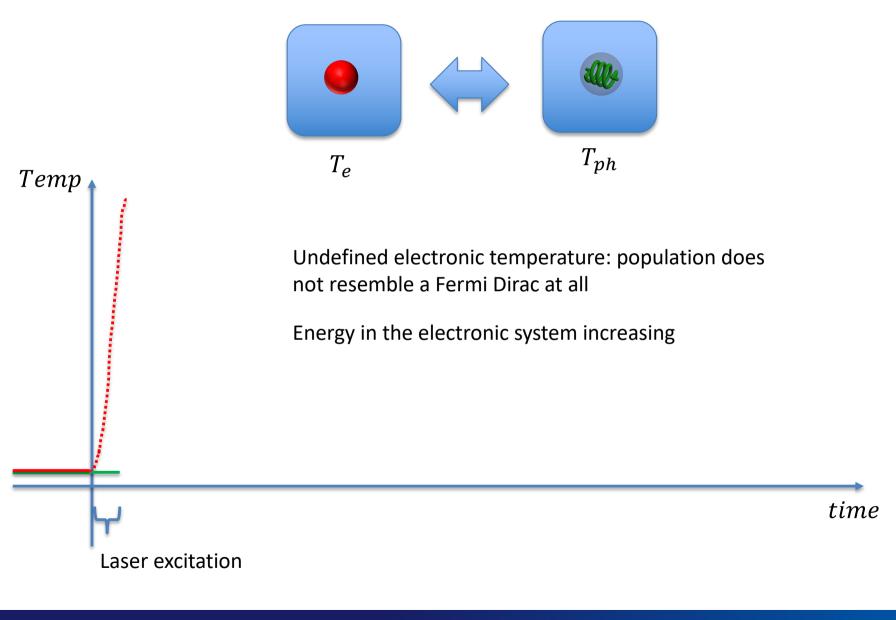
Thermodynamics tells us that they will equilibrate to the same temperature



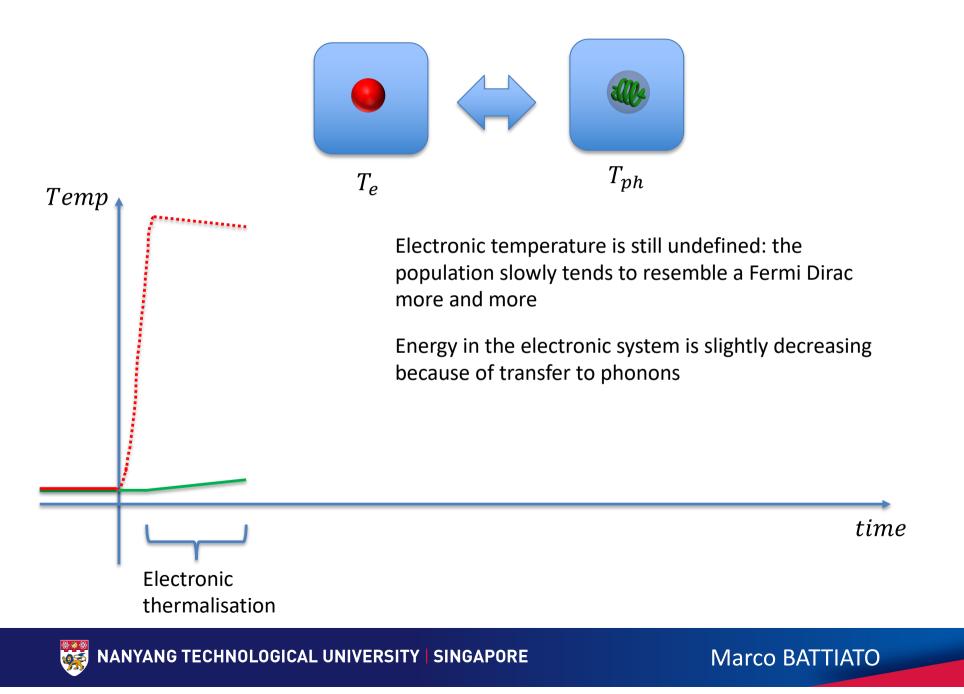


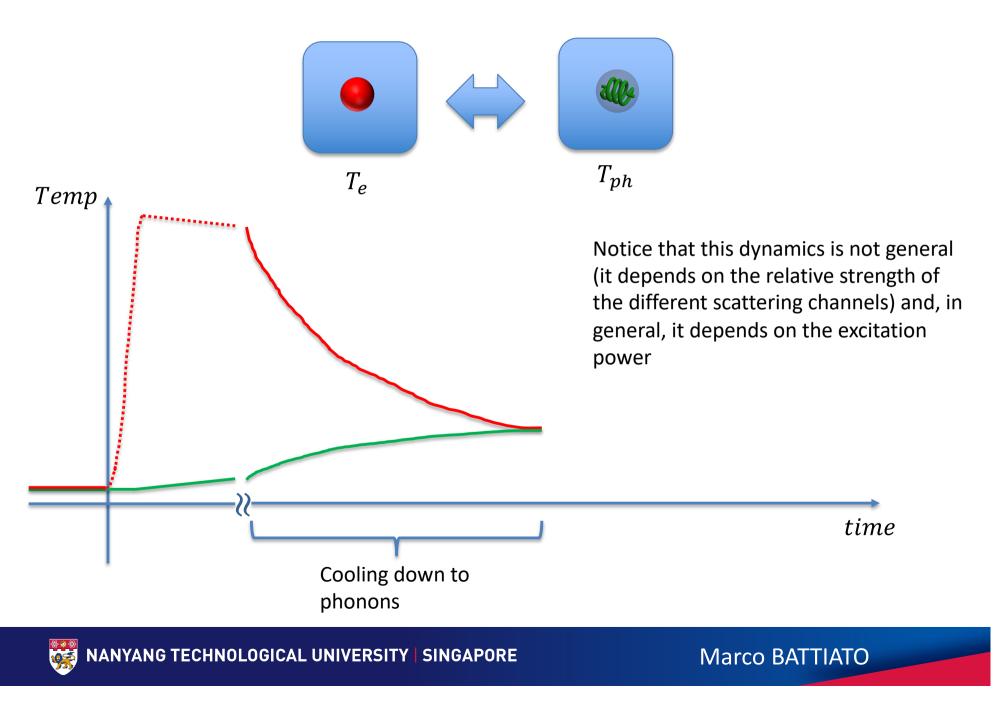


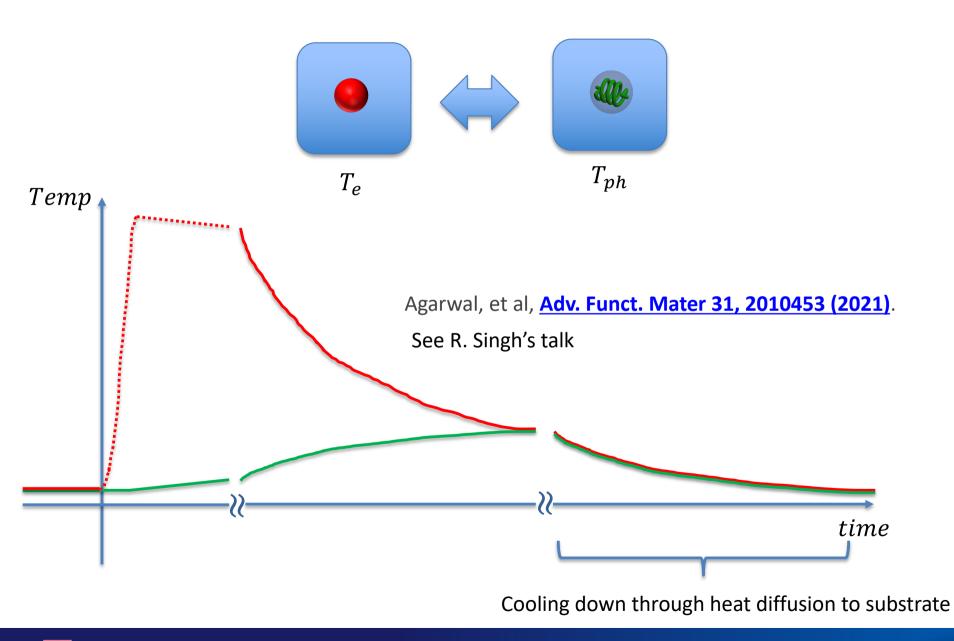












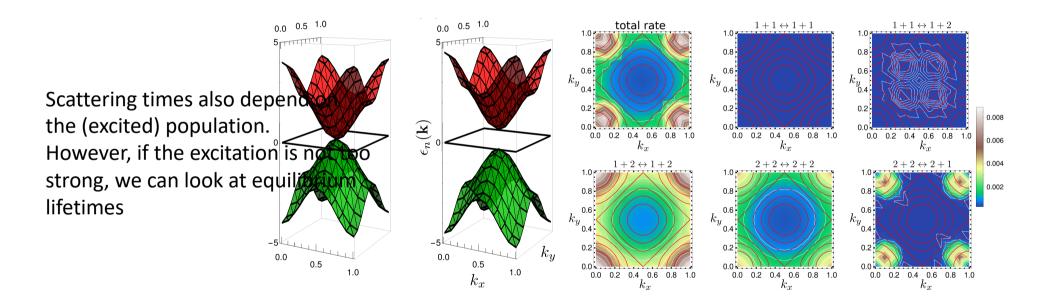


#### **Timescales**

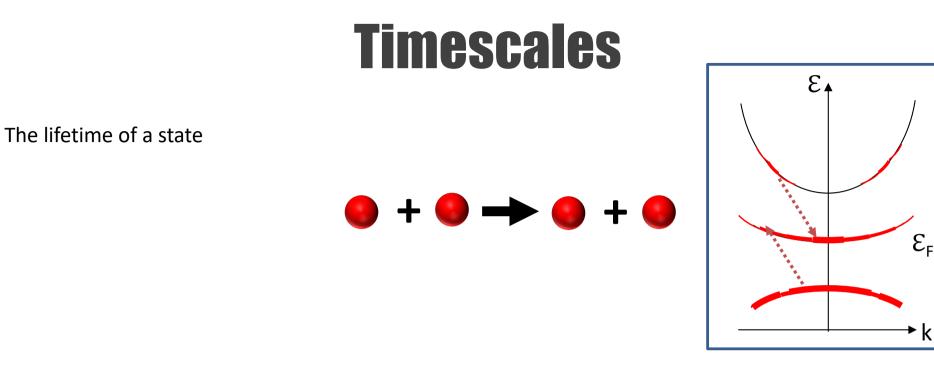
Despite what seemed from the previous slides, pinning down timescales is far from trivial

The decay time of excided states is a complex interplay of the decay timescales of higher (and to a certain extent, also lower) energy excitations and scattering lifetimes of the states.

The decay time of excided states is a complex interplay of the decay timescales of higher (and to a certain extent, also lower) energy excitations and scattering lifetimes of the states.



Wais, Held, Battiato, Comput. Phys. Commun. 264, 107877 (2021) Wadgaonkar, Jain, Battiato, Comput. Phys. Commun 263, 107863 (2021) Wadgaonkar, Wais, Battiato, Comput. Phys. Commun 271, 108207 (2021)



 $W_{n_1,n_2}(k_1,k_2,k_3,k_4) \int_{n_1} (k_1) f_{n_2}(k_2) \left(1 - f_{n_3}(k_3)\right) \left(1 - f_{n_4}(k_4)\right)$  $\overline{\tau(k_1)}$  $-\infty$ transitions for given hv

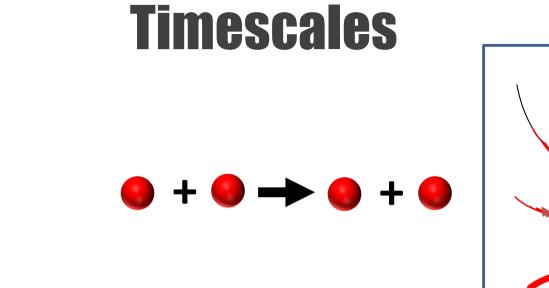
NOTE: Please notice that the one above is not the correct equation, but I did not want to spend time on technical details which are not really relevant for intuition. Check the refs below for the correct treatment

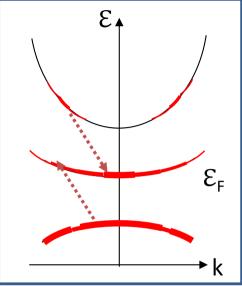
Wais, Held, Battiato, Comput. Phys. Commun. 264, 107877 (2021) Wadgaonkar, Jain, Battiato, Comput. Phys. Commun 263, 107863 (2021) Wadgaonkar, Wais, Battiato, Comput. Phys. Commun 271, 108207 (2021)



Marco BATTIATO

k





 $W_{n_1,n_2}(k_1,k_2,k_3,k_4) f_{n_1}(k_1) f_{n_2}(k_2) \left(1 - f_{n_3}(k_3)\right) \left(1 - f_{n_4}(k_4)\right)$  $\frac{1}{\tau(k_1)} \propto$ transitions

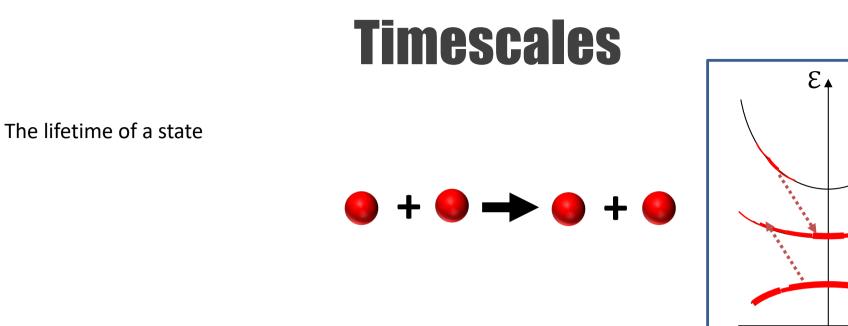
for given hv

NOTE: Please notice that the one above is not the correct equation, but I did not want to spend time on technical details which are not really relevant for intuition. Check the refs below for the correct treatment

Wais, Held, Battiato, Comput. Phys. Commun. 264, 107877 (2021) Wadgaonkar, Jain, Battiato, Comput. Phys. Commun 263, 107863 (2021) Wadgaonkar, Wais, Battiato, Comput. Phys. Commun 271, 108207 (2021)

The lifetime of a state

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$$\frac{1}{\tau(k_1)} \propto \sum_{\substack{\text{all allowed} \\ \text{transitions} \\ \text{for given } hv}} W_r$$

$$f_{n_1,n_2}(k_1,k_2,k_3,k_4) f_{n_1}(k_1) f_{n_2}(k_2) \left(1 - f_{n_3}(k_3)\right) \left(1 - f_{n_4}(k_4)\right)$$

NOTE: Please notice that the one above is not the correct equation, but I did not want to spend time on technical details which are not really relevant for intuition. Check the refs below for the correct treatment

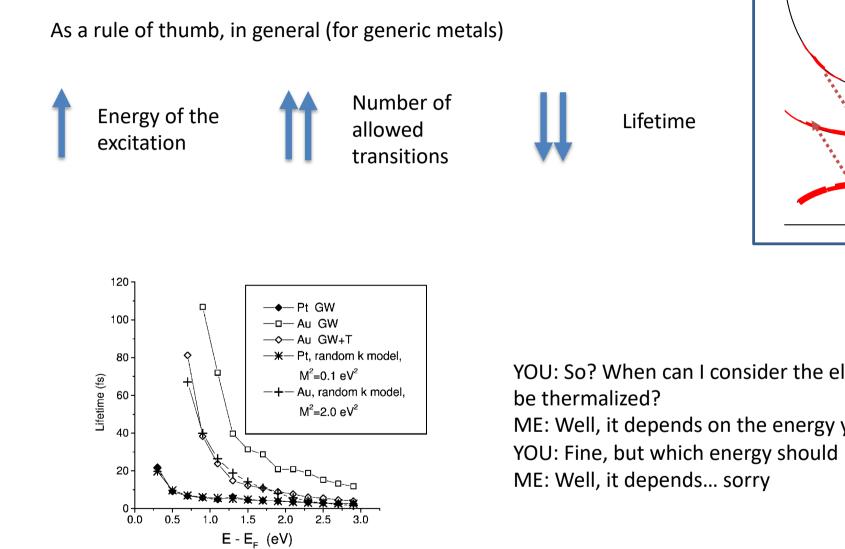
Wais, Held, Battiato, Comput. Phys. Commun. 264, 107877 (2021) Wadgaonkar, Jain, Battiato, Comput. Phys. Commun 263, 107863 (2021) Wadgaonkar, Wais, Battiato, Comput. Phys. Commun 271, 108207 (2021)

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 $\mathcal{E}_{F}$ 

k

## **Timescales**



V. Zhukov et al., Phys. Rev. B 73, 125105 (2006)

YOU: So? When can I consider the electronic system to

ME: Well, it depends on the energy you are looking at. YOU: Fine, but which energy should I look at?

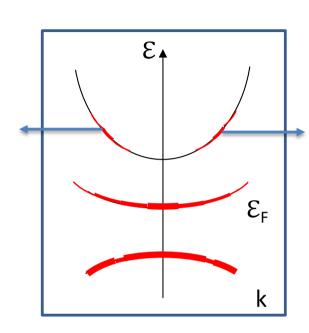
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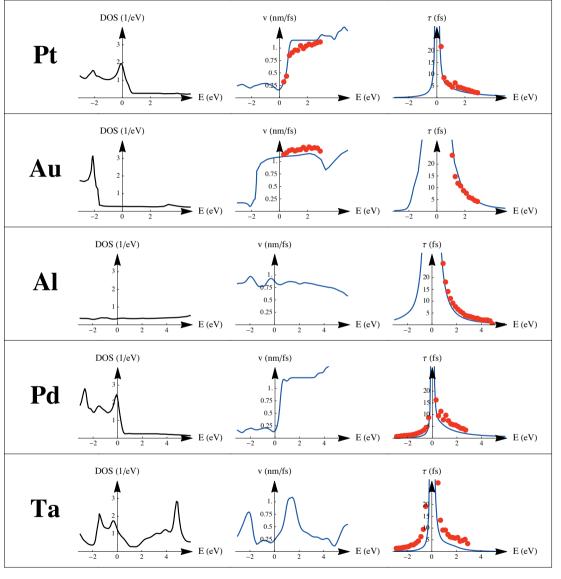
8

 $\mathcal{E}_{F}$ 

k

### **Transport out-of-equilibrium**



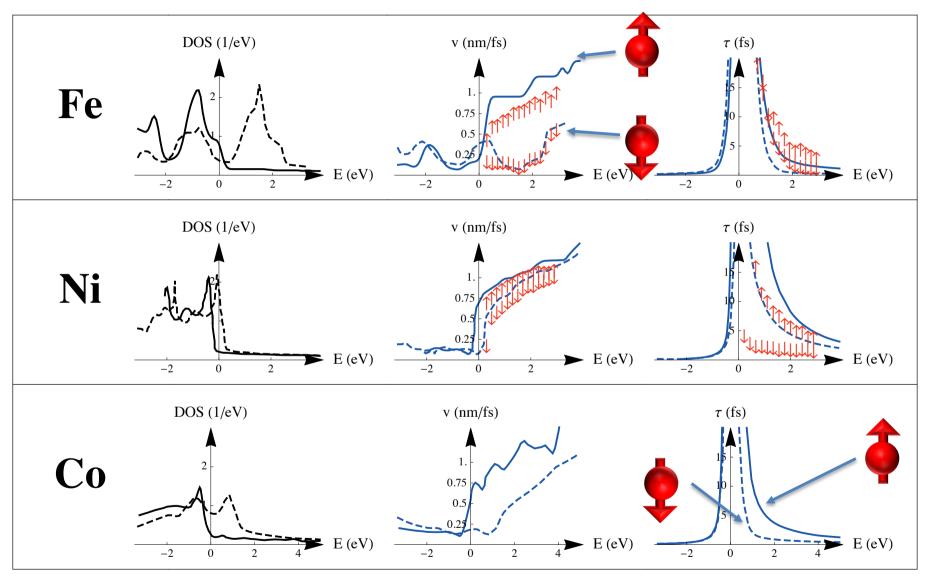


 $v(k) = \frac{1}{\hbar} \frac{\partial \mathcal{E}(k)}{\partial k}$ 

M. Battiato, PhD Thesis

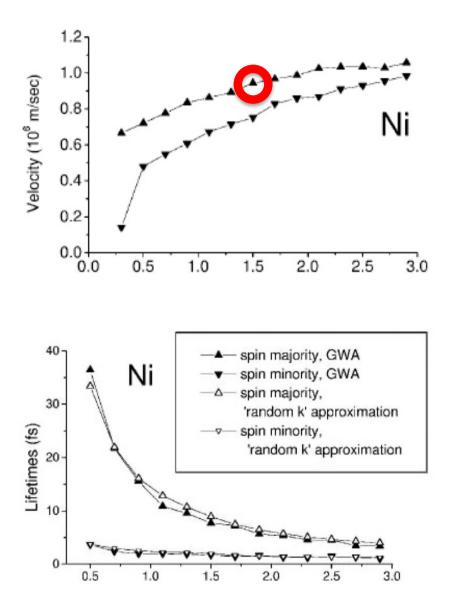


# Transport out-of-equilibrium with spin



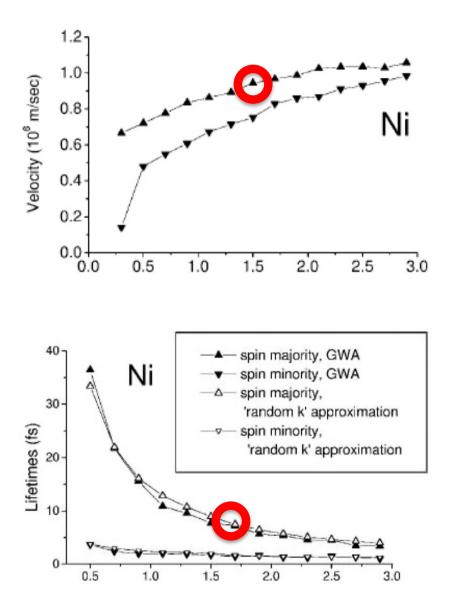
M. Battiato, PhD Thesis

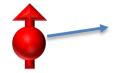
## Transport out of equilibrium with spin



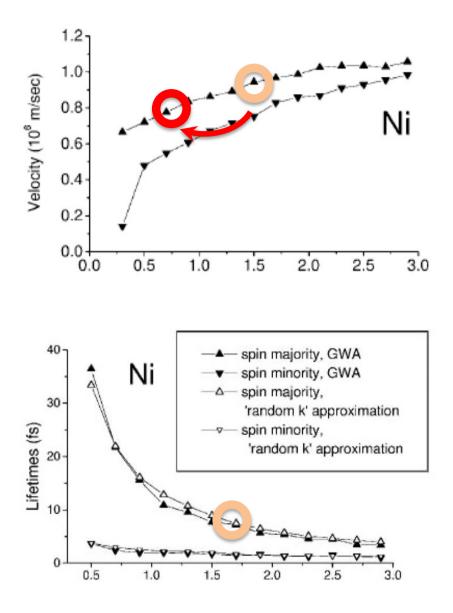


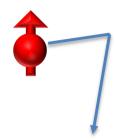
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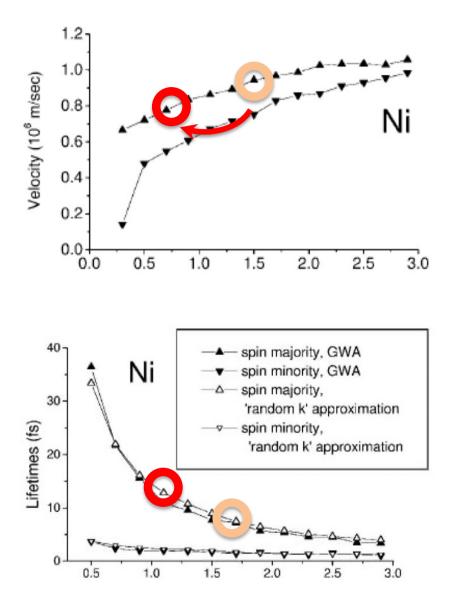


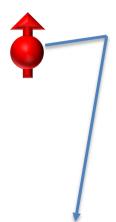
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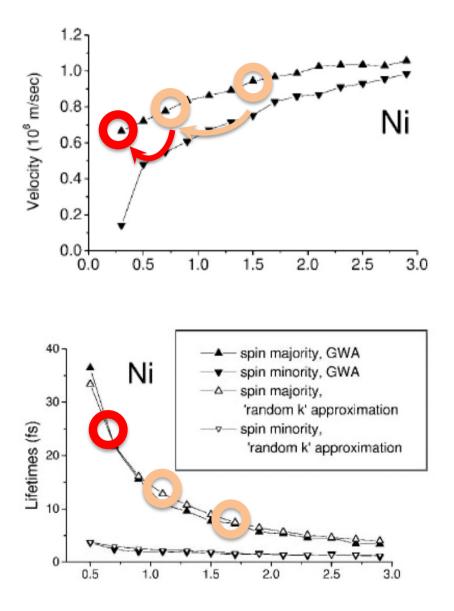


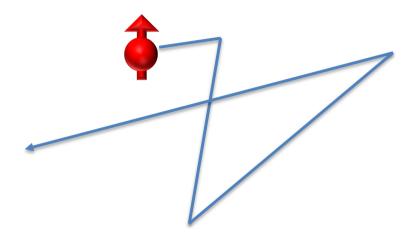




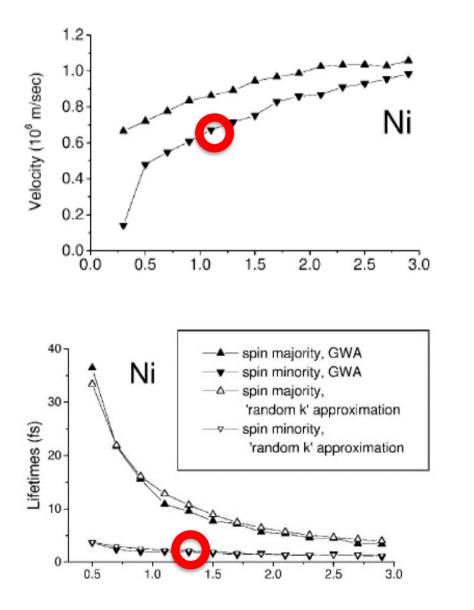


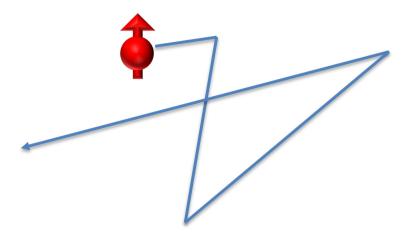




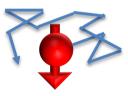








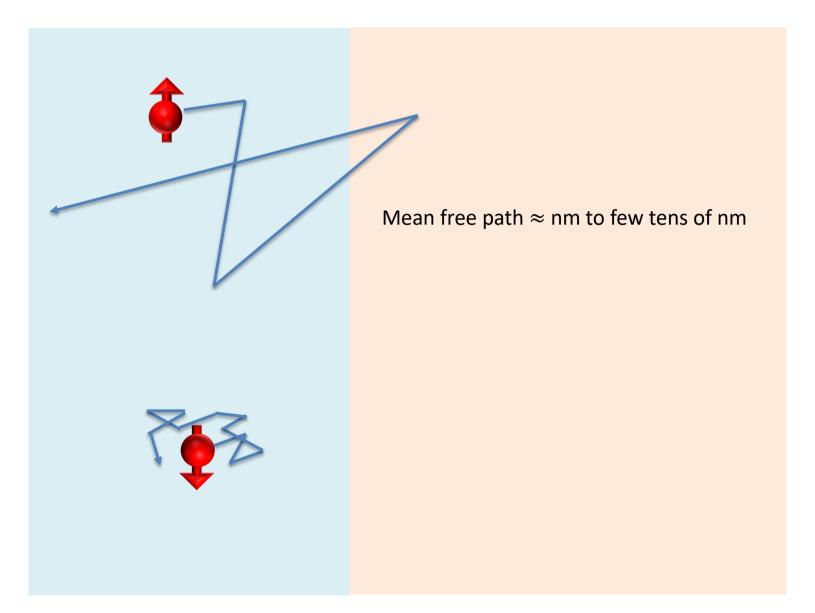
Superdiffusive regime -> efficient



Close to the diffusive regime -> inefficient

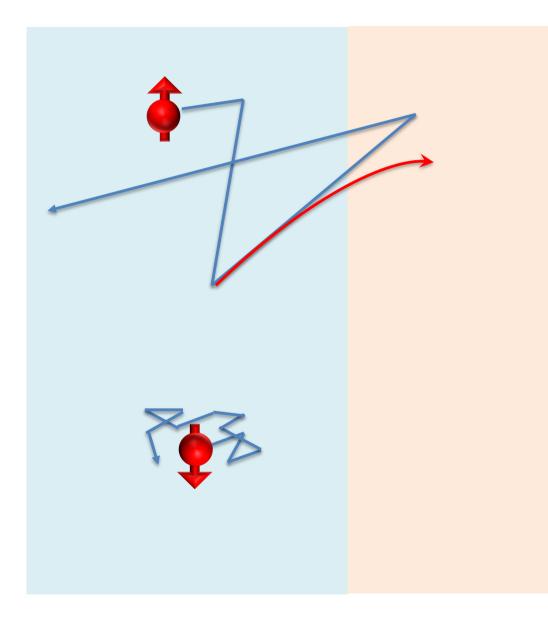


#### **Diffusion regimes**





#### What about drift?



Group velocity  $\approx 0.1-1$  nm/fs  $\approx 0.1-1 \ 10^6$  m/s

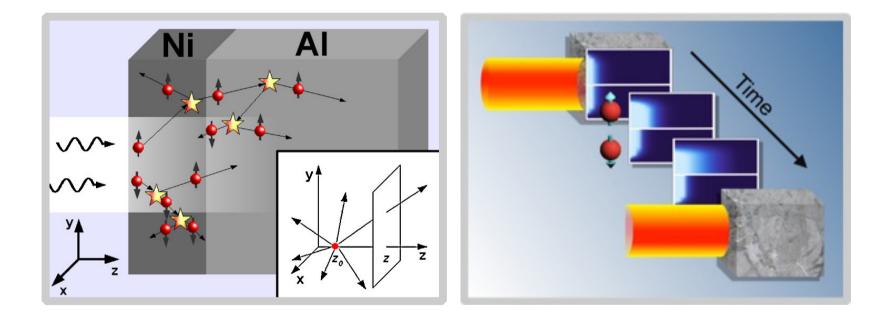
Typical mobility  $\approx 10^{-3} \text{ m}^2/\text{V s}$ 

To have drift velocities comparable to group velocities we need electric fields of  $\approx$  0.1-1 V/nm



#### Superdiffusive spin transport

Simplistically, majority carriers diffuse more efficiently than minority carriers and therefore there is a nest spin diffusion away from the excited area.

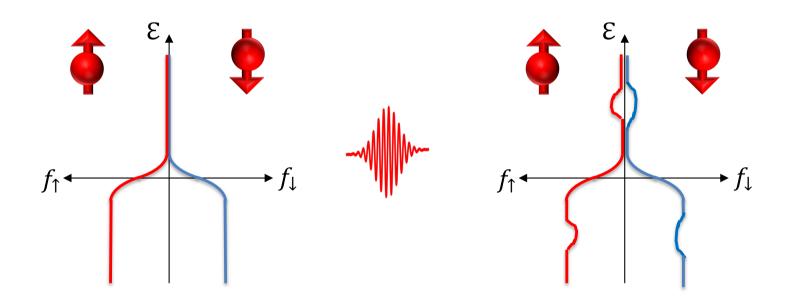


Battiato, Carva, Oppeneer, **Phys. Rev. Lett.** 105, 027203 (2010) Battiato, Carva, Oppeneer, **Phys. Rev. B** 86, 024404 (2012) Battiato, Maldonado, Oppeneer, **J. Appl. Phys.** 115, 172611 (2014).



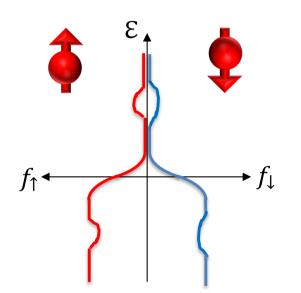
Carriers are excited at several energies (and their energy changes during thermalization). The question is: what are the most effective energies contributing to the spin transport?

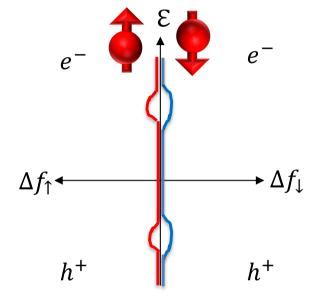
After the laser excitation, carriers in both spin channels are excited





Let us display only the excitation's population (since we know that the transport of the equilibrium population integrates to zero)



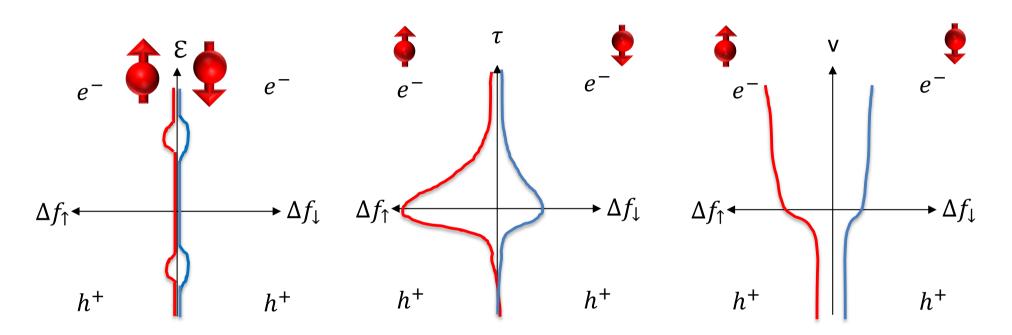


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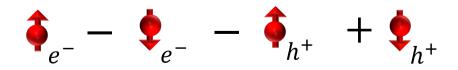
Both excited electrons and holes undergo diffusion Careful since spin up holes carry spin down



How do they behave?



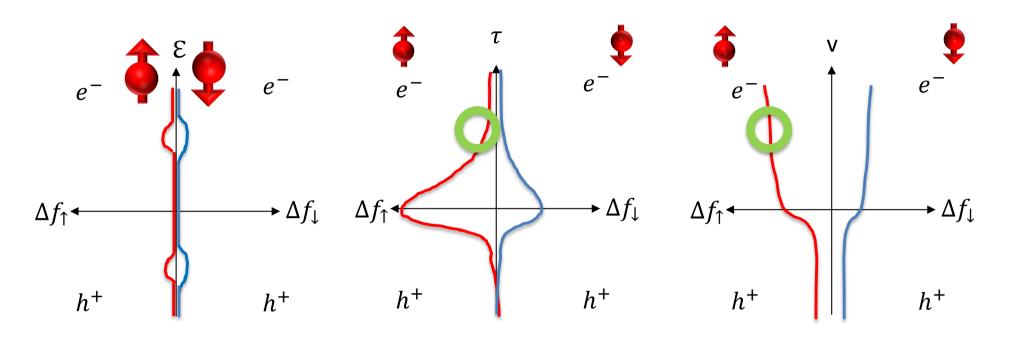
The net spin transport is



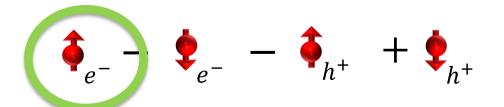


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How do they behave?



The net spin transport is



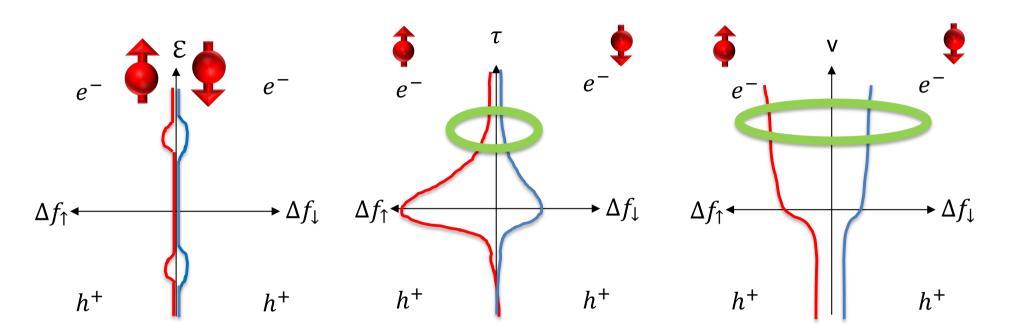
At high energy, the lifetimes are short,

- → the mean free paths are short
- → transport closer to standard diffusion
- → inefficient transport



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How do they behave?



The net spin transport is

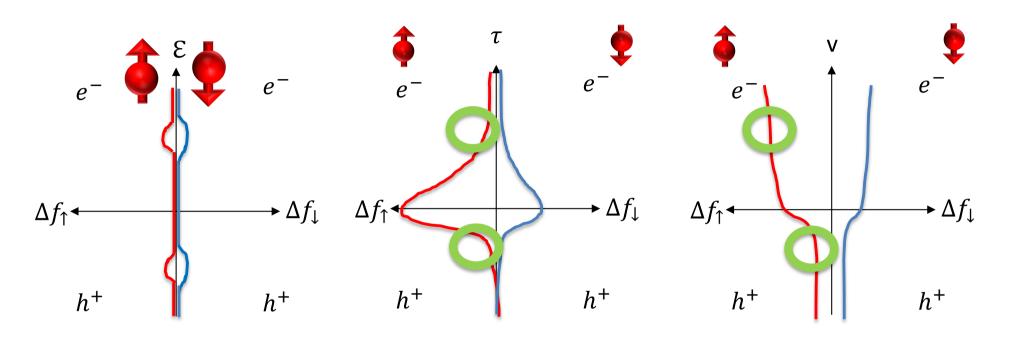


Where the maximal asymmetry is located usually depends on the ferromagnet

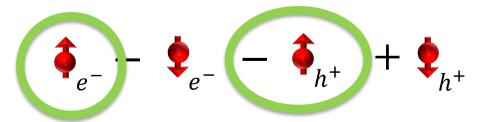


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How do they behave?



The net spin transport is

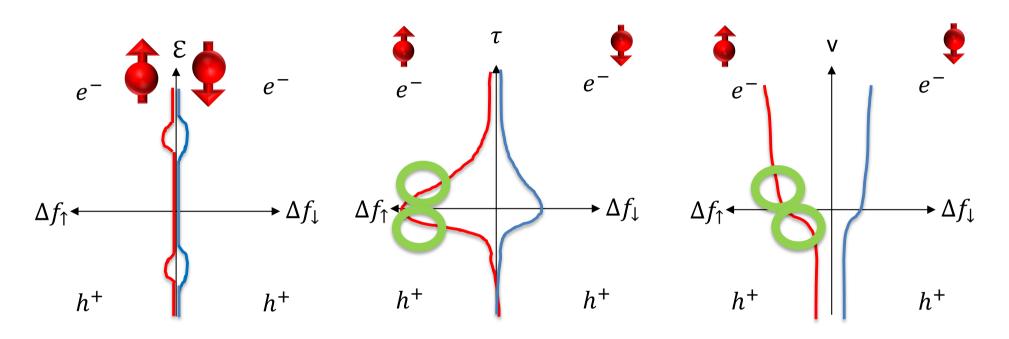


Usually holes considerably slower and/or much shorter lifetimes, except...

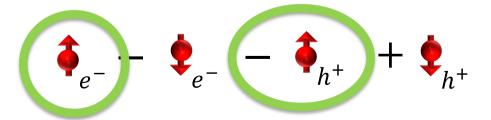


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How do they behave?



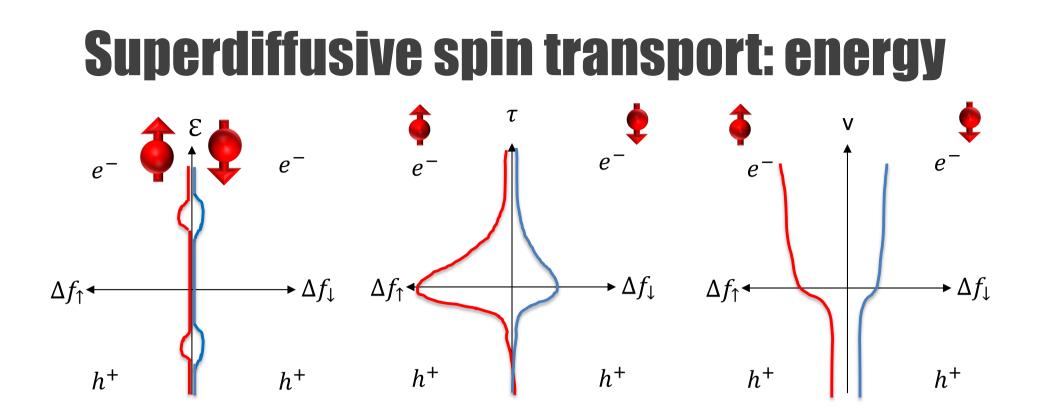
The net spin transport is



Usually holes considerably slower and/or much shorter lifetimes, except... when both electrons and holes are close to the Fermi level, then their transport cancels



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The states that contribute the most to the spin diffusion are at relatively low energies: high energies have too short lifetimes and their transport is inefficient (Notice that only a theoretical treatment that can describe superdiffusion can account for this

difference)

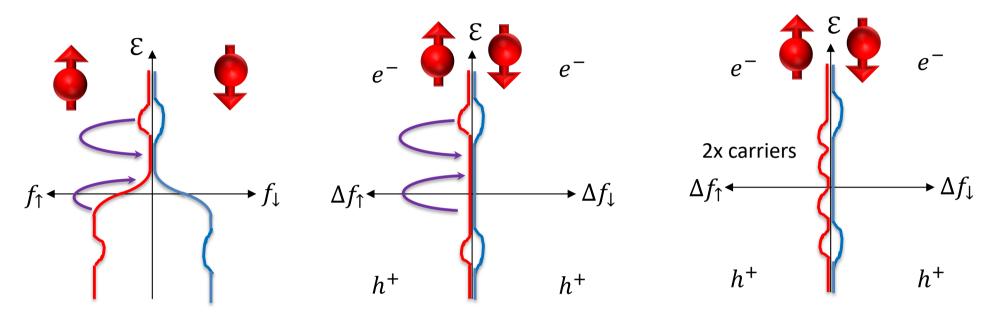
However, the energies should not be too low because in that case electron and hole transport cancel each other

**Marco BATTIATO** 

But there is another even more important effect that gives low energies an advantage...



When an excitation scatters with other electrons



Carrier multiplication

During thermalization to each photon corresponds a single high energy excitation and many low energy excitations

Meaning that relatively low energy excitations (but not too low), not only are more efficient in producing spin diffusion, but they are also much more numerous



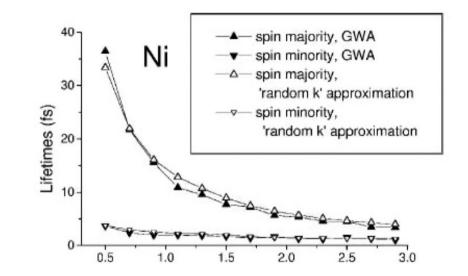
# Superdiffusive spin transport: timescale

We are now in a slightly better position to estimate the timescales that are relevant to the superdiffusive spin transport.

For thermalization typically several scattering events are needed.

Therefore, different energies can be thought to thermalize with different speeds.

Since the most important energy scale is the low energy one, we expect the population within that energy to be different from a Fermi–Dirac for few hundreds of fs.





### Superdiffusive spin transport: common misconceptions

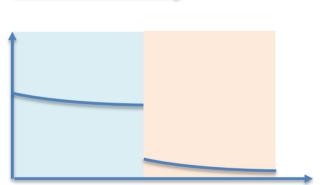
What drives the directionality?

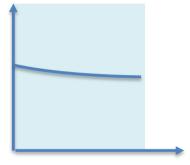
Except for very thick samples, the laser excitation profile is usually rather flat within a given layer (because of multiple reflections)

On the other hand, if there are different layers (with very different refractive indices) the absorption profile can be strongly non uniform...

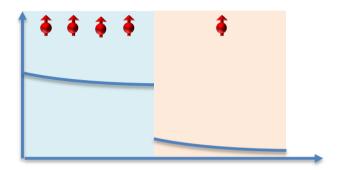
However, that is way less important than what one might think at first. Let's understand why





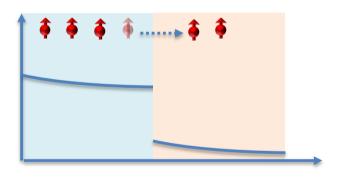


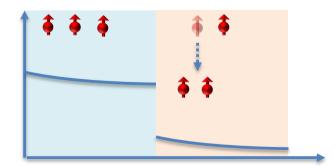
### Superdiffusive spin transport: common misconceptions



The initial high energy excitation is surely very non uniform, however...

As the excitations diffuse and enter the other layer...





When they scatter, they act as source of lower energy carriers (which more importantly produce transport) and so on.

Secondary excitations make the excitation profile more uniform, compared to what one might think at first



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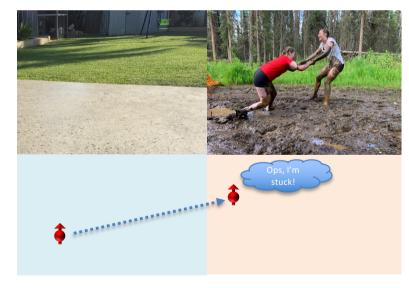
#### Superdiffusive spin transport: directionality

Where does the directionality come from?

Interfaces with vacuum

Different transport properties in different materials (differently for different spin channels)



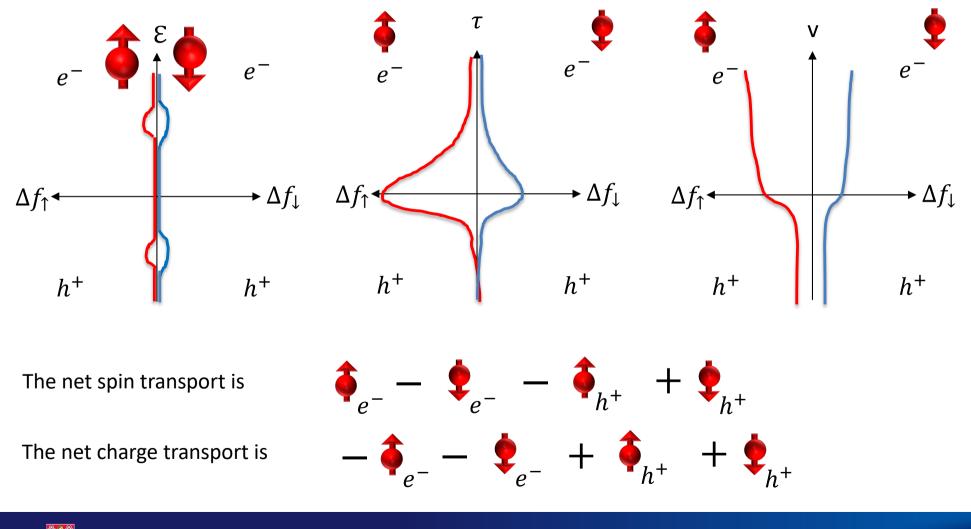


Battiato, Carva, Oppeneer, Phys. Rev. Lett. 105, 027203 (2010)
Battiato, Carva, Oppeneer, Phys. Rev. B 86, 024404 (2012)
Battiato, Maldonado, Oppeneer, J. Appl. Phys. 115, 172611 (2014).



#### Superdiffusive spin transport: Charge current

One last important point: charge current



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# Superdiffusive spin transport: Charge current

 $\mathbf{\Phi}_{e^-} - \mathbf{\Phi}_{e^-} - \mathbf{\Phi}_{h^+}$ 

 $-\phi_{\rho^-} - \phi_{\rho^-} + \phi_{h^+}$ 

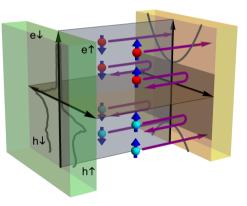
This means that there is indeed an associated charge current

The net spin transport is

The net charge transport is

It must be taken into account, since it generates electric fields that impact the transport. In metals, the carriers will try and screen the generated electric fields (and carry their spin while they move!!!)

To a first approximation, for low excitations, one can anyway expect that the majority of the screening is done by carriers around the Fermi energy. In most cases, this will lead to an enhancement of the spin current compared to what one might get neglecting the effect of the charge currents.



See E. Chia's talk

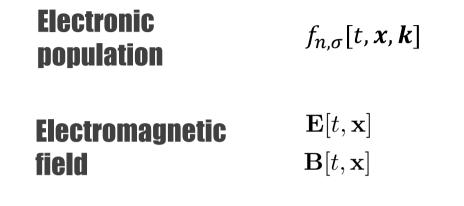
The effect becomes even more important when semiconductors are involved

Battiato, Held, **Phys. Rev. Lett.** 116, 196601 (2016). M. Battiato, **J. Phys. Condens. Matter** 29, 174001 (2017). Cheng, Wang, Yang, Chai, Yang, Chen, Wu, Chen, Chi, Goh, Zhu, Sun, Wang, Song, Battiato, Yang, Chia, **Nature Physics** (2019).



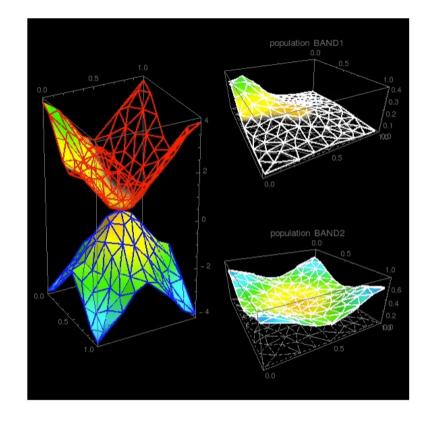


#### Boltzmann + Maxwell system



$$\frac{\partial f}{\partial t} + \frac{1}{\hbar} \nabla_{\mathbf{k}} \mathcal{E} \cdot \nabla_{\mathbf{k}} f + \frac{e}{\hbar} \mathbf{E} \cdot \nabla_{\mathbf{k}} f - \left(\frac{\partial f}{\partial t}\right)_{\mathrm{sc}} - \left(\frac{\partial f}{\partial t}\right)_{\mathrm{ex}} = 0$$

$$\begin{split} \nabla \cdot \boldsymbol{\epsilon} \mathbf{E} &= \rho \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \qquad \rho[t, \mathbf{x}] &= -\frac{e}{(2\pi)^3} \int f[t, \mathbf{x}, \mathbf{k}] d\mathbf{k} \\ \nabla \times \frac{\mathbf{B}}{\mu} &= \mathbf{j} + \frac{\partial \boldsymbol{\epsilon} \mathbf{E}}{\partial t} \qquad \mathbf{j}[t, \mathbf{x}] &= -\frac{e}{(2\pi)^3 \hbar} \int \frac{\partial \mathcal{E}}{\partial \mathbf{k}} f[t, \mathbf{x}, \mathbf{k}] d\mathbf{k} \end{split}$$



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#### **Boltzmann equation**

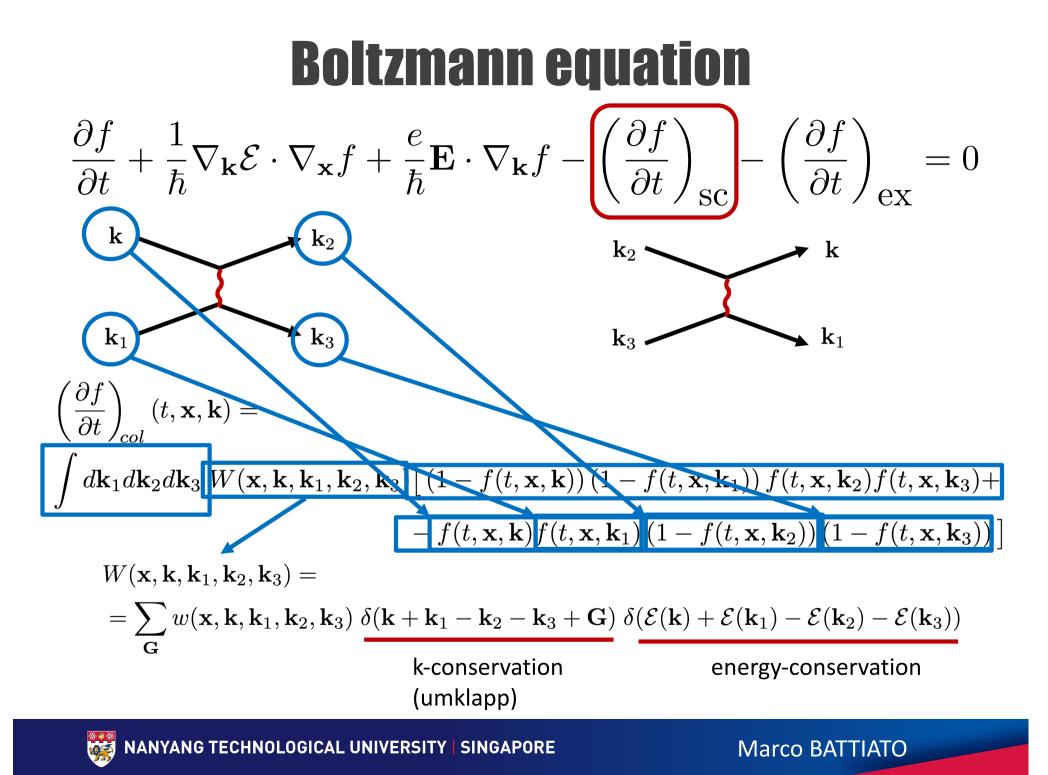
$$\frac{\partial f}{\partial t} + \left(\frac{1}{\hbar} \nabla_{\mathbf{k}} \mathcal{E} \cdot \nabla_{\mathbf{x}} f + \frac{e}{\hbar} \mathbf{E} \cdot \nabla_{\mathbf{k}} f\right) - \left(\frac{\partial f}{\partial t}\right)_{\mathrm{SC}} - \left(\frac{\partial f}{\partial t}\right)_{\mathrm{ex}} = 0$$

#### 1+1+3 dimensional problem: not cheap, but can be handled

#### Runge Kutta Discontinuous Galerkin

- Unstructured meshes
- Arbitrarily high order of convergence (depending on max polynomial power)
- Mass, energy and momentum conserving
- High stability in presence of shocks





$$\begin{split} S_{ijklm} = \sum_{\mathbf{G}} \int d\mathbf{k} d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3 \\ & \left( w(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \cdot \\ & \delta(\mathbf{k} + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3 + \mathbf{G}) \ \delta(\mathcal{E}(\mathbf{k}) + \mathcal{E}(\mathbf{k}_1) - \mathcal{E}(\mathbf{k}_2) - \mathcal{E}(\mathbf{k}_3)) \cdot \\ & b_i(\mathbf{k}) b_j(\mathbf{k}) b_k(\mathbf{k}_1) b_l(\mathbf{k}_2) b_m(\mathbf{k}_3) \right) \end{split}$$



$$\begin{split} \mathbf{S}_{ijklm} &= \sum_{\mathbf{G}} \int d\mathbf{k} d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3 \\ & \left( w(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \cdot \\ & \delta(\mathbf{k} + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3 + \mathbf{G}) \ \delta(\mathcal{E}(\mathbf{k}) + \mathcal{E}(\mathbf{k}_1) - \mathcal{E}(\mathbf{k}_2) - \mathcal{E}(\mathbf{k}_3)) \cdot \\ & b_i(\mathbf{k}) b_j(\mathbf{k}) b_k(\mathbf{k}_1) b_l(\mathbf{k}_2) b_m(\mathbf{k}_3) \right) \end{split}$$

5 dimensional tensor: number of entries is N<sup>5</sup>
 (400 basis functions -> 10<sup>13</sup> integrals to calculate and 10TB to load in memory)



$$\begin{split} S_{ijklm} &= \sum_{\mathbf{G}} \int \!\!\!\! d\mathbf{k} d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3 \\ & \left( w(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \cdot \\ & \delta(\mathbf{k} + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3 + \mathbf{G}) \ \delta(\mathcal{E}(\mathbf{k}) + \mathcal{E}(\mathbf{k}_1) - \mathcal{E}(\mathbf{k}_2) - \mathcal{E}(\mathbf{k}_3)) \cdot \\ & b_i(\mathbf{k}) b_j(\mathbf{k}) b_k(\mathbf{k}_1) b_l(\mathbf{k}_2) b_m(\mathbf{k}_3) \right) \end{split}$$

- 5 dimensional tensor: number of entries is N<sup>5</sup>
   (400 basis functions -> 10<sup>13</sup> integrals to calculate and 10TB to load in memory)
- 12 dimensional integral



$$\begin{split} S_{ijklm} = \sum_{\mathbf{G}} \int d\mathbf{k} d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3 \\ & \begin{pmatrix} w(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \cdot \\ \delta(\mathbf{k} + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3 + \mathbf{G}) \ \delta(\mathcal{E}(\mathbf{k}) + \mathcal{E}(\mathbf{k}_1) - \mathcal{E}(\mathbf{k}_2) - \mathcal{E}(\mathbf{k}_3)) \\ \delta_i(\mathbf{k}) b_j(\mathbf{k}) b_k(\mathbf{k}_1) b_l(\mathbf{k}_2) b_m(\mathbf{k}_3) \end{pmatrix} \end{split}$$

- 5 dimensional tensor: number of entries is N<sup>5</sup>
   (400 basis functions -> 10<sup>13</sup> integrals to calculate and 10TB to load in memory)
- 12 dimensional integral
- Dirac deltas inside the integral



$$\begin{split} S_{ijklm} &= \sum_{\mathbf{G}} \int d\mathbf{k} d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3 \\ & \left( w(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \cdot \\ & \delta(\mathbf{k} + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3 + \mathbf{G}) \ \delta(\mathcal{E}(\mathbf{k}) + \mathcal{E}(\mathbf{k}_1) - \mathcal{E}(\mathbf{k}_2) - \mathcal{E}(\mathbf{k}_3)) \cdot \\ & b_i(\mathbf{k}) b_j(\mathbf{k}) b_k(\mathbf{k}_1) b_l(\mathbf{k}_2) b_m(\mathbf{k}_3) \right) \end{split}$$

- 5 dimensional tensor: number of entries is N<sup>5</sup>
   (400 basis functions -> 10<sup>13</sup> integrals to calculate and 10TB to load in memory)
- 12 dimensional integral
- Dirac deltas inside the integral
- Failure to integrate to extreme precision leads to breaking of particle, energy and momentum conservation



#### Usually used approaches to bypass the problem

$$\begin{pmatrix} \frac{\partial f}{\partial t} \end{pmatrix}_{col} (t, \mathbf{x}, \mathbf{k}) = \int d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3 \ W(\mathbf{x}, \mathbf{k}, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \Big[ (1 - f(t, \mathbf{x}, \mathbf{k})) (1 - f(t, \mathbf{x}, \mathbf{k}_1)) f(t, \mathbf{x}, \mathbf{k}_2) f(t, \mathbf{x}, \mathbf{k}_3) + - f(t, \mathbf{x}, \mathbf{k}) f(t, \mathbf{x}, \mathbf{k}_1) (1 - f(t, \mathbf{x}, \mathbf{k}_2)) (1 - f(t, \mathbf{x}, \mathbf{k}_3)) \Big]$$

• Only electron-phonon scatterings included (phonons treated as bath)





#### Usually used approaches to bypass the problem

$$\begin{pmatrix} \frac{\partial f}{\partial t} \end{pmatrix}_{col} (t, \mathbf{x}, \mathbf{k}) = \\ \int d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3 \ W(\mathbf{x}, \mathbf{k}, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \Big[ (1 - f(t, \mathbf{x}, \mathbf{k})) (1 - f(t, \mathbf{x}, \mathbf{k}_1)) f(t, \mathbf{x}, \mathbf{k}_2) f(t, \mathbf{x}, \mathbf{k}_3) + \\ - f(t, \mathbf{x}, \mathbf{k}) f(t, \mathbf{x}, \mathbf{k}_1) (1 - f(t, \mathbf{x}, \mathbf{k}_2)) (1 - f(t, \mathbf{x}, \mathbf{k}_3)) \Big]$$

- Only electron-phonon scatterings included (phonons treated as bath)
- Close to equilibrium approximation



#### Usually used approaches to bypass the problem

$$\begin{pmatrix} \frac{\partial f}{\partial t} \end{pmatrix}_{col} (t, \mathbf{x}, \mathbf{k}) = \\ \int d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3 \ W(\mathbf{x}, \mathbf{k}, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \left[ (1 - f(t, \mathbf{x}, \mathbf{k})) (1 - f(t, \mathbf{x}, \mathbf{k}_1)) f(t, \mathbf{x}, \mathbf{k}_2) f(t, \mathbf{x}, \mathbf{k}_3) + \\ - f(t, \mathbf{x}, \mathbf{k}) f(t, \mathbf{x}, \mathbf{k}_1) (1 - f(t, \mathbf{x}, \mathbf{k}_2)) (1 - f(t, \mathbf{x}, \mathbf{k}_3)) \right]$$

- Only electron-phonon scatterings included (phonons treated as bath)
- Close to equilibrium approximation
- Low (electron or hole) population



#### Usually used approaches to bypass the problem

 $\left(\frac{\partial f}{\partial t}\right)_{col}(t,\mathbf{x},\mathbf{k}) =$ 

 $d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3 \ W(\mathbf{x}, \mathbf{k}, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \big[ \left(1 - f(t, \mathbf{x}, \mathbf{k})\right) \left(1 - f(t, \mathbf{x}, \mathbf{k}_1)\right) f(t, \mathbf{x}, \mathbf{k}_2) f(t, \mathbf{x}, \mathbf{k}_3) + \\ - f(t, \mathbf{x}, \mathbf{k}) f(t, \mathbf{x}, \mathbf{k}_1) \left(1 - f(t, \mathbf{x}, \mathbf{k}_2)\right) \left(1 - f(t, \mathbf{x}, \mathbf{k}_3)\right) \big]$ 

- Only clostron-phonon scatterings included (phonons treated as path)
- Close to equilibrium approximation
- Low (electron or hole) population
- Relavation time approximation

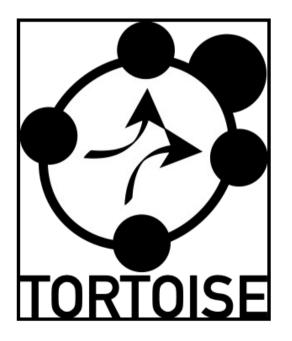




# Strongly out-of-equilibrium thermalisation and transport

Wais, Held, Battiato, Comput. Phys. Commun. 264, 107877 (2021) Wadgaonkar, Jain, Battiato, Comput. Phys. Commun 263, 107863 (2021) Wadgaonkar, Wais, Battiato, Comput. Phys. Commun 271, 108207 (2021)

Boltzmann equation

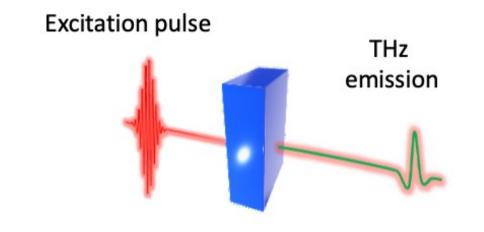


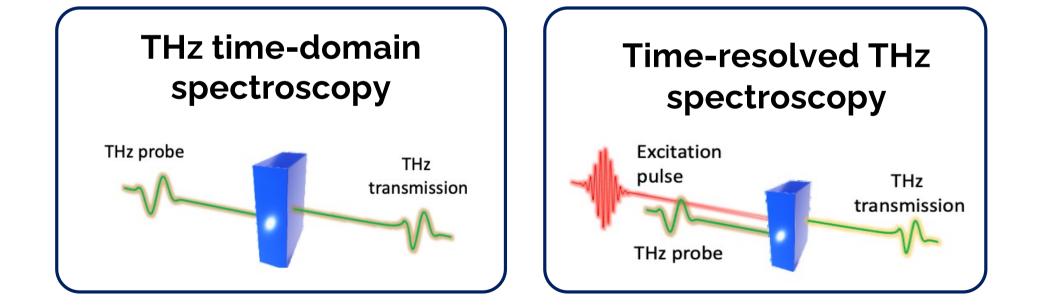
$$\frac{\partial f}{\partial t} = -\frac{1}{\hbar} \nabla_k \mathcal{E} \cdot \nabla_x f - \frac{e}{\hbar} \mathcal{E} \cdot \nabla_k f + \left(\frac{df}{dt}\right)_{scatt}$$

- No close to equilibrium assumptions
- No fixed population assumptions
- Any number of quasiparticles and bands
- Arbitrary dispersion (dispersion as input)
- Any number of scattering channels
- Scattering channels with 2, 3, and 4 legs (soon >4)
- Arbitrary scattering amplitude (required as input)
- Excellent scaling with precision
- Exact conservation of particle, energy and momentum

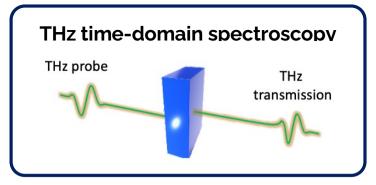


# THz from and through excited materials



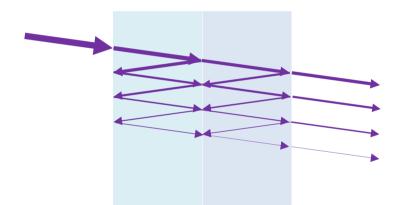






# THz through non exited multilayers

Transfer Matrix Method (TMM)



 $\bar{F}_{1}[d] \ \bar{F}_{2}[0]$ 

ささ

 $\bar{F}_{1}[0]$ 

 $\bar{F}_{0}[0]$ 

 $\partial_{z}E\left[z,t
ight] = \mu \,\partial_{t}H\left[z,t
ight]$   $\partial_{z}H\left[z,t
ight] = \epsilon \,\partial_{t}E\left[z,t
ight]$ 

Maxwell's equations

$$E[z,t] = f^{>}[t]e^{ikz} + f^{<}[t]e^{-ikz}$$
$$H[z,t] = \sqrt{\frac{\epsilon}{\mu}}f^{>}[t]e^{ikz} - \sqrt{\frac{\epsilon}{\mu}}f^{<}[t]e^{-ikz}$$

$$\bar{F}\left[\omega,z
ight] = a[\omega,z]\bar{f}\left[\omega
ight]$$

$$\bar{f}_{N+1} = (a_{N+1}[0])^{-1} \left(\prod_{j=N}^{1} M_j[d_j]\right) a_0[0]\bar{f}_0 = T_{[0,N+1]}\bar{f}_0$$
$$\begin{bmatrix} f_{N+1}^{>} \\ 0 \end{bmatrix} = T_{[0,N+1]} \begin{bmatrix} f_0^{>} \\ f_0^{<} \end{bmatrix}$$

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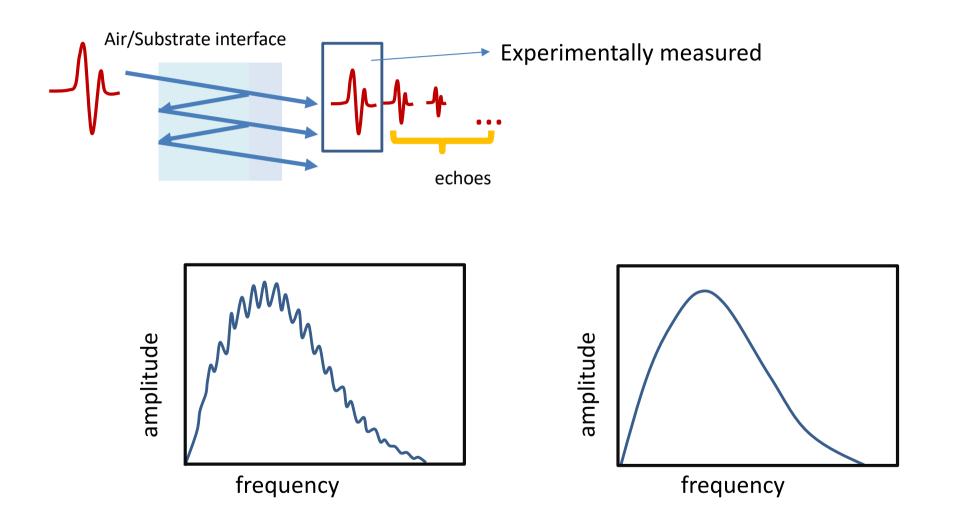
 $\bar{F}_{2}[d] \ \bar{F}_{3}[0]$ 

 $\bar{F}_3[d]$ 

 $\bar{F}_{N+1}[0]$ 

## An annoying issue: echo

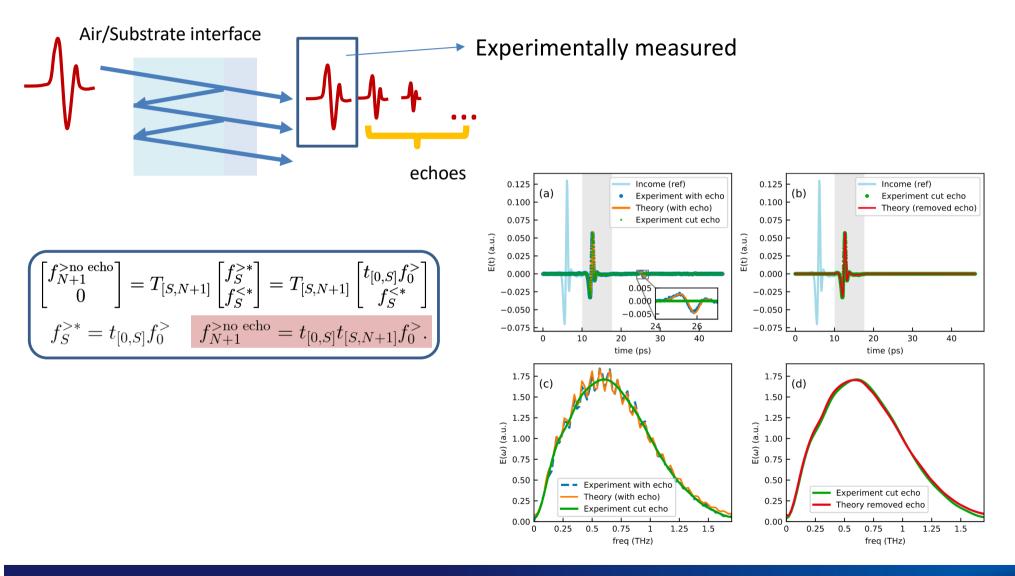
Y Yang, S Dal Forno, M Battiato, J Infr Millim, 42 1142 (2021)



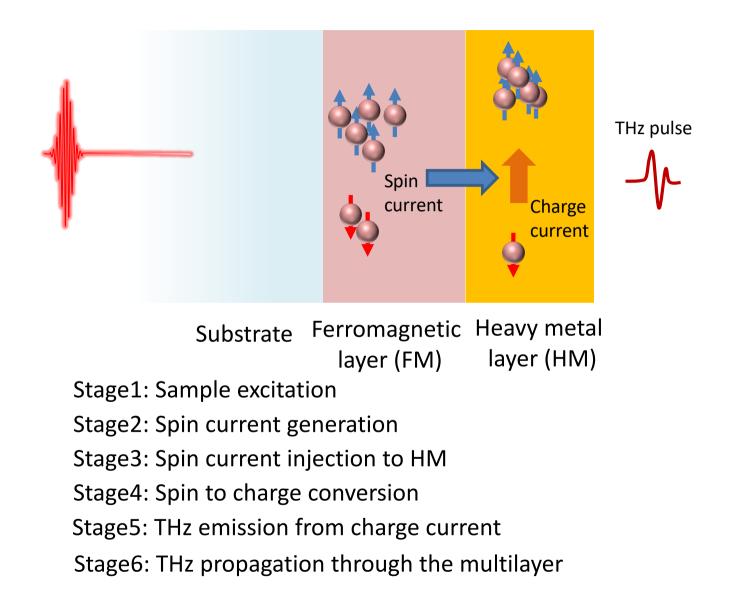


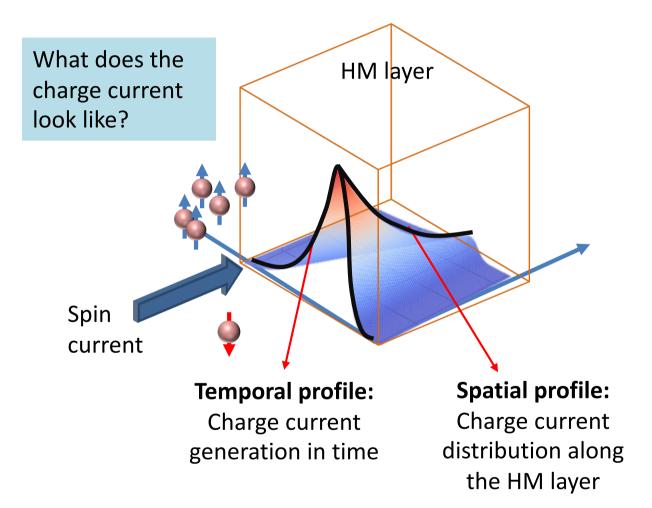
## An annoying issue: echo

Y Yang, S Dal Forno, M Battiato, J Infr Millim, 42 1142 (2021)



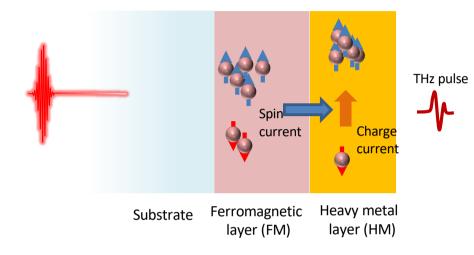








Y Yang, S Dal Forno, M Battiato, Phys Rev B 104 (15), 155437



Stage1: Sample excitation

Stage2: Spin current generation Stage3: Spin current injection to HM

- Stage4: Spin to charge conversion
- Stage5: THz emission from charge current

Stage6: THz propagation through the multilayer

A. Pump laser absorption profile  

$$Q_{loss} = -\int_{-\infty}^{+\infty} dt \oint_{S} (\mathbf{E} \times \mathbf{H}) \cdot d\mathbf{S}.$$

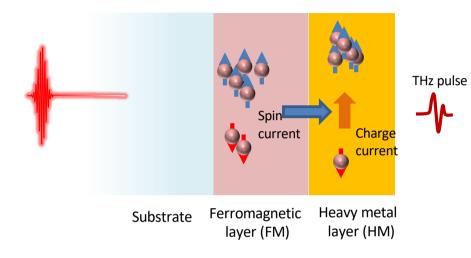
$$\Phi(z) = -\int_{-\infty}^{+\infty} E[t, z]H[t, z]dt,$$

$$A_{layer} = \frac{Q_{loss}}{Q_{in}} = \frac{\Phi(z_0) - \Phi(z_0 + d)}{Q_{in}},$$

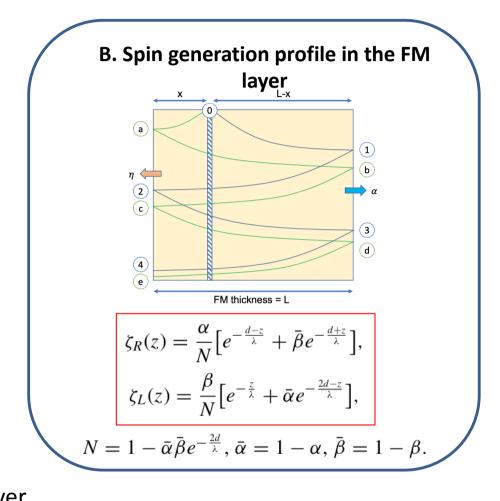
$$D(z) = -\frac{d(\Phi(z)/Q_{in})}{dz}.$$



Y Yang, S Dal Forno, M Battiato, Phys Rev B 104 (15), 155437

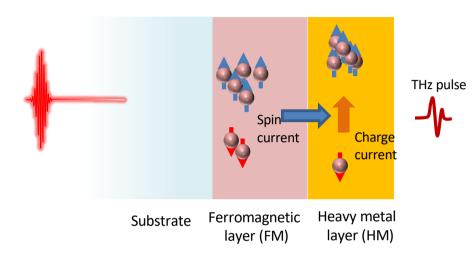


Stage1: Sample excitation
Stage2: Spin current generation
Stage3: Spin current injection to HM
Stage4: Spin to charge conversion
Stage5: THz emission from charge current
Stage6: THz propagation through the multilayer

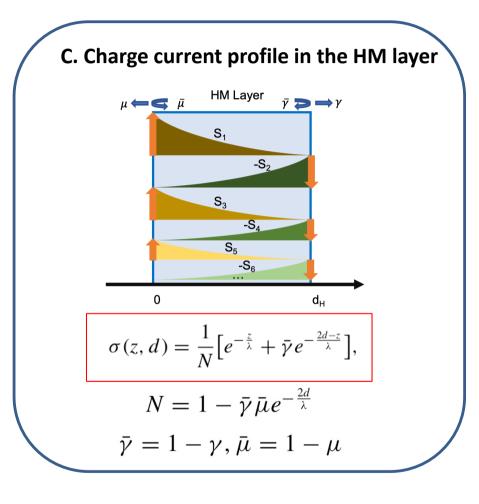




Y Yang, S Dal Forno, M Battiato, Phys Rev B 104 (15), 155437



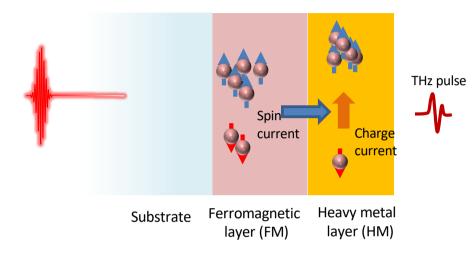
Stage1: Sample excitation
Stage2: Spin current generation
Stage3: Spin current injection to HM
Stage4: Spin to charge conversion
Stage5: THz emission from charge current
Stage6: THz propagation through the multilayer



#### λ Spin diffusion length



Y Yang, S Dal Forno, M Battiato, Phys Rev B 104 (15), 155437



Stage1: Sample excitation

Stage2: Spin current generation

Stage3: Spin current injection to HM

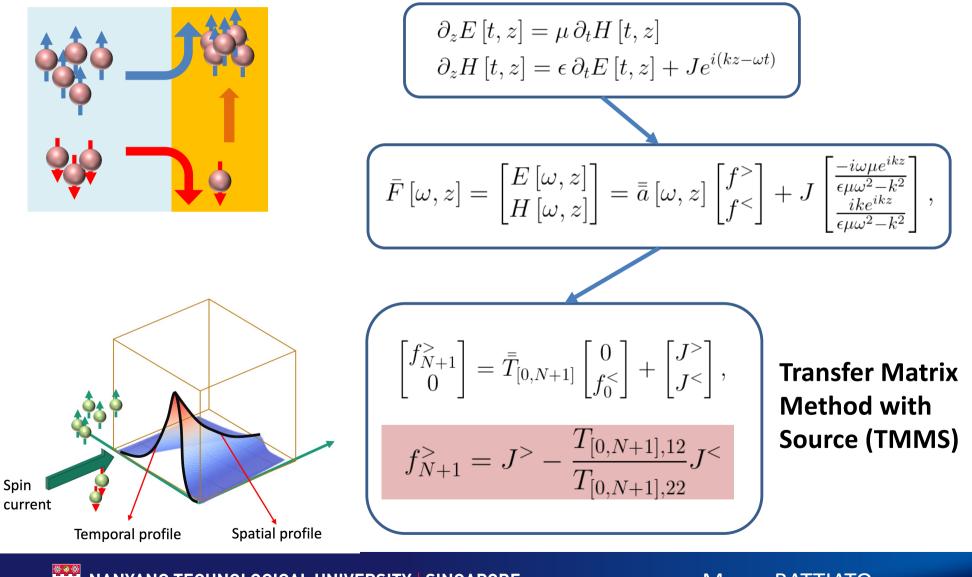
Stage4: Spin to charge conversion

Stage5: THz emission from charge current

Stage6: THz propagation through the multilayer

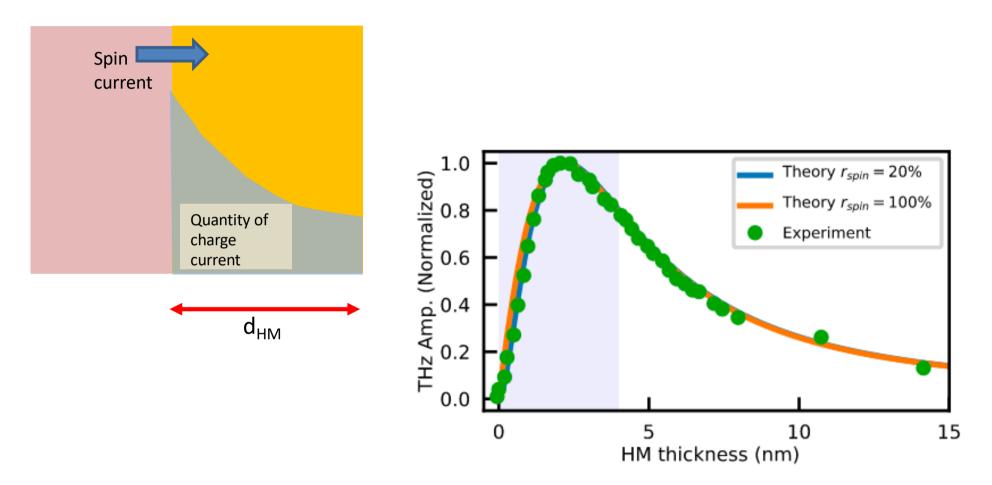


Y Yang, S Dal Forno, M Battiato, Phys Rev B 104 (15), 155437



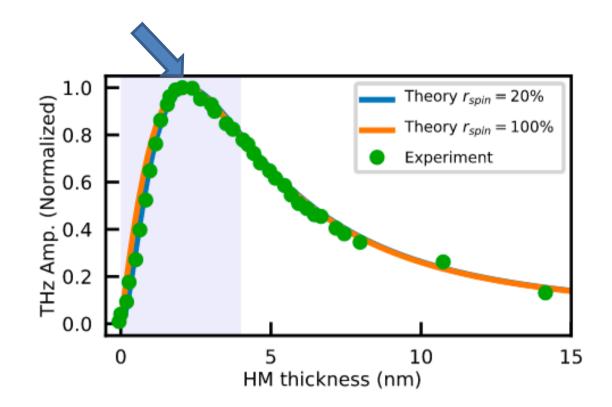
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Y Yang, S Dal Forno, M Battiato, Phys Rev B 104 (15), 155437



The large-thickness decay is controlled by the absorption of the THz emission within the metallic multilayer itself

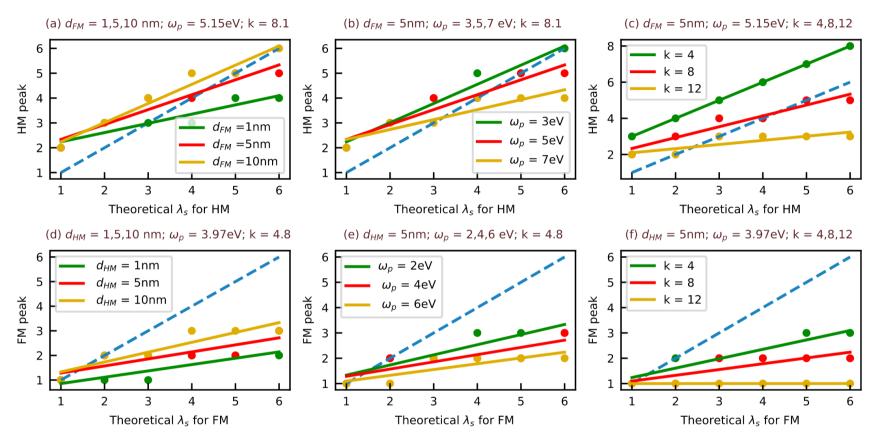
Y Yang, S Dal Forno, M Battiato, Phys Rev B 104 (15), 155437



Is the thickness corresponding to the maximal emission really indicative of the spin diffusion length?



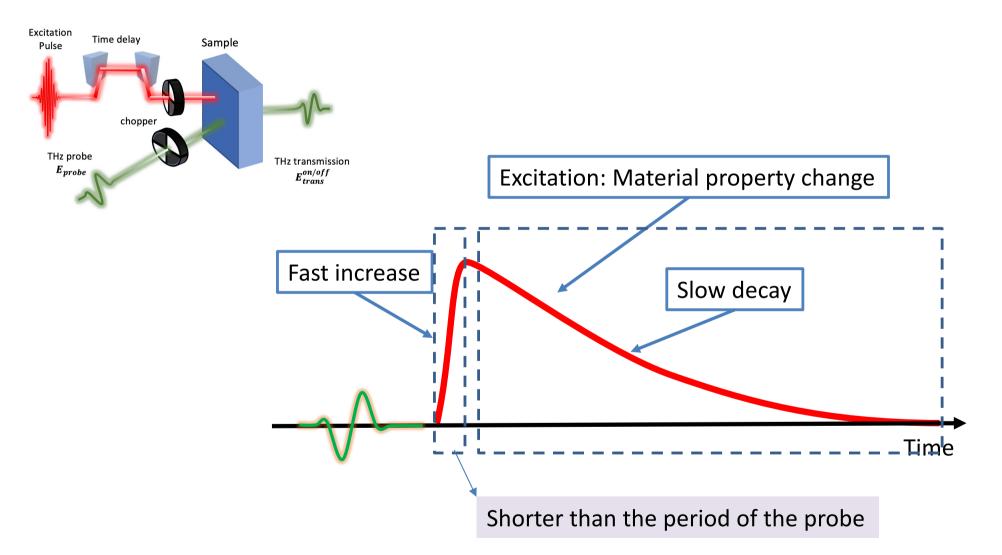
Y Yang, S Dal Forno, M Battiato, Phys Rev B 104 (15), 155437



Blue dashed line represents when Spin diffusion length = peak position

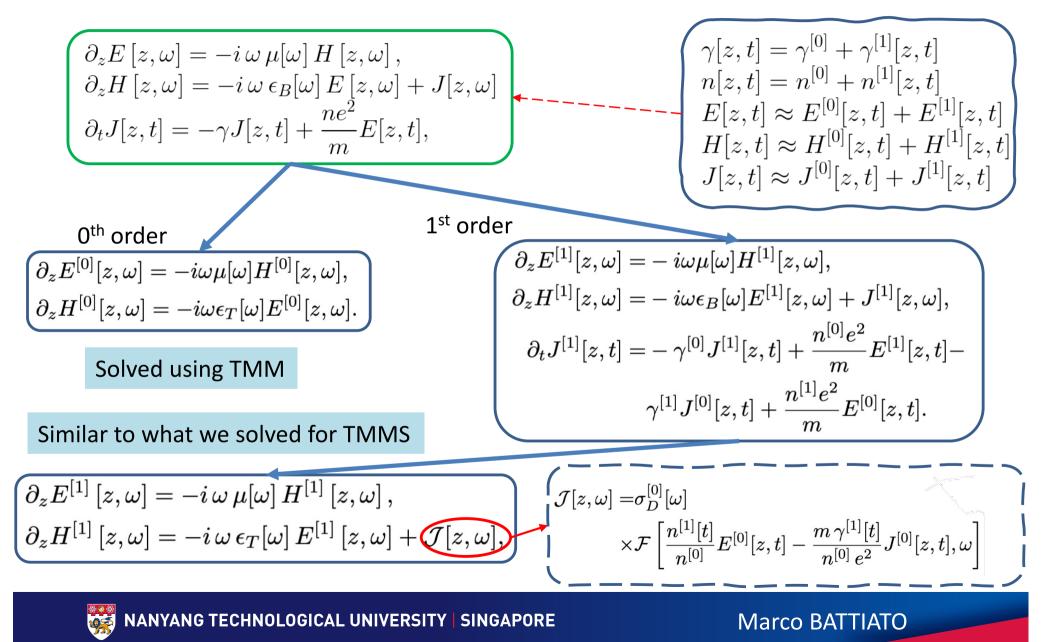
The thickness corresponding to the maximal emission is **not indicative at all** of the spin diffusion length

Y Yang, SD Forno, M Battiato, arXiv preprint arXiv:2206.00895

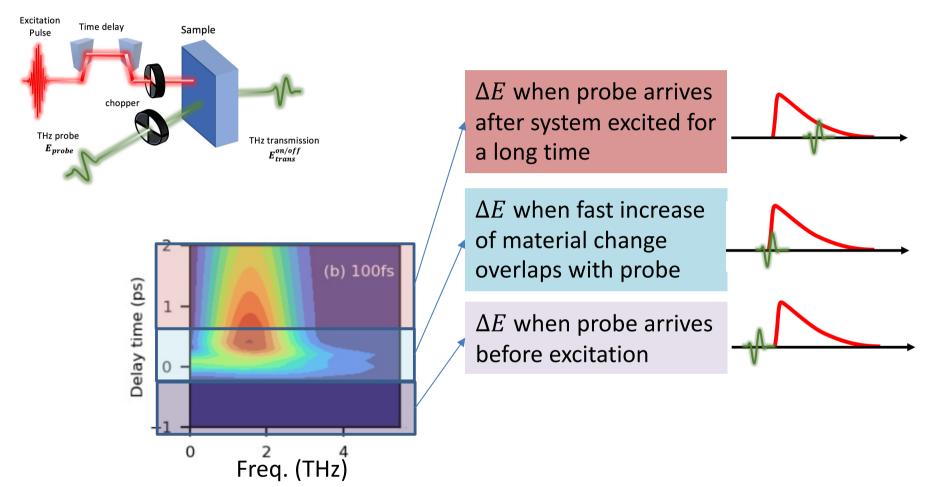


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Y Yang, SD Forno, M Battiato, arXiv preprint arXiv:2206.00895

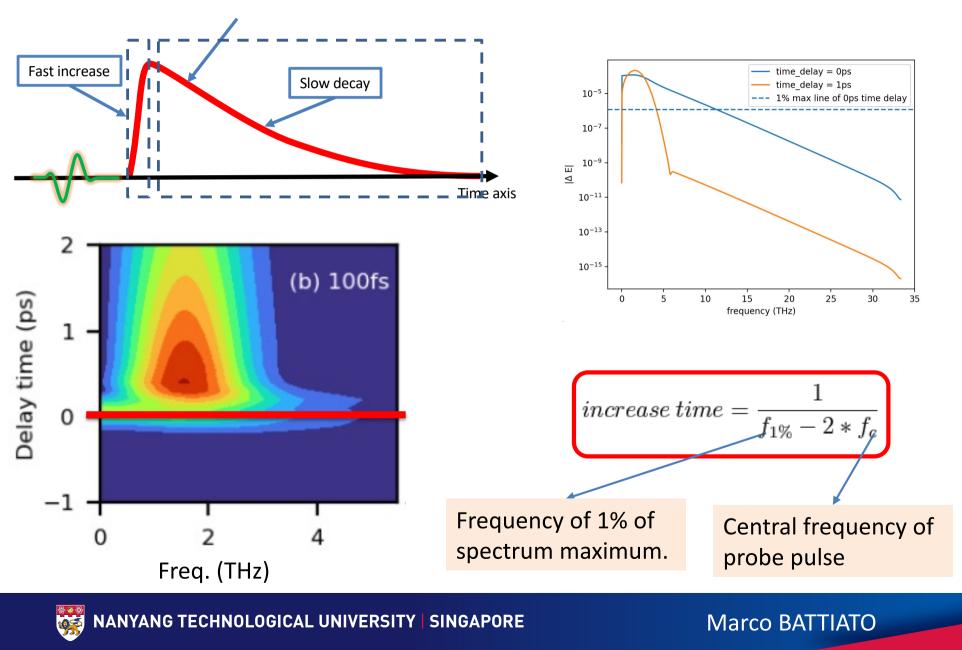


Y Yang, SD Forno, M Battiato, arXiv preprint arXiv:2206.00895

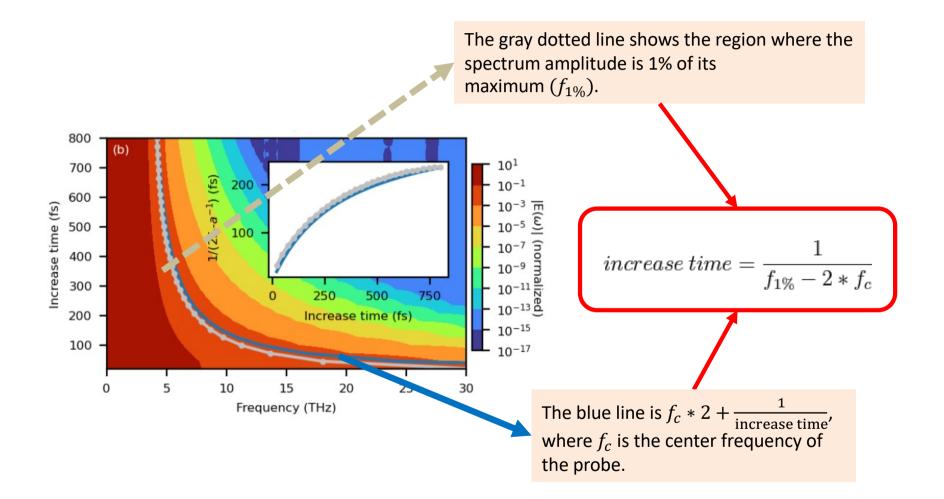




Y Yang, SD Forno, M Battiato, arXiv preprint arXiv:2206.00895



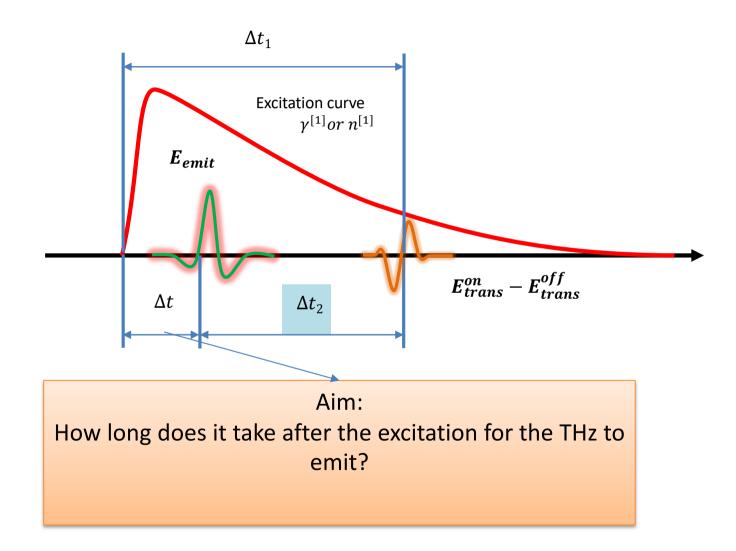
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# **How about spintronics THz emitters?**

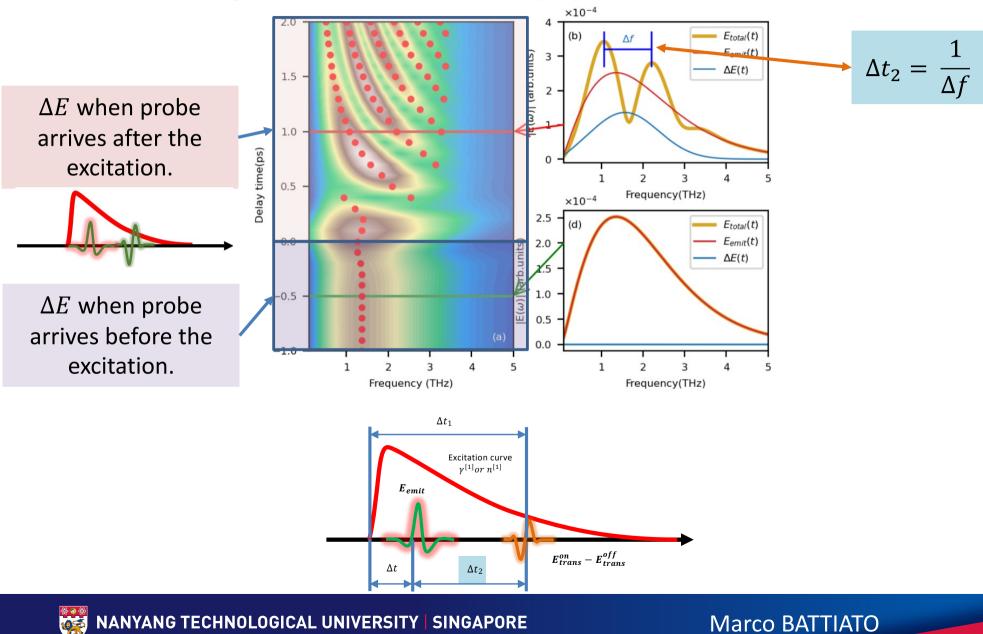
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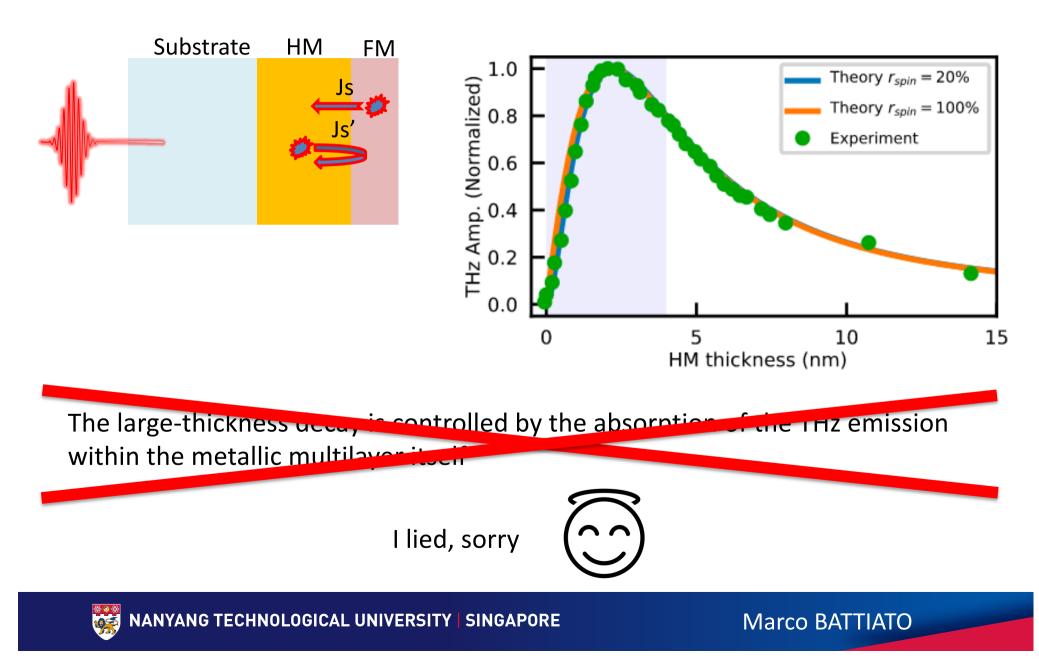
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# How about spintronics THz emitters?

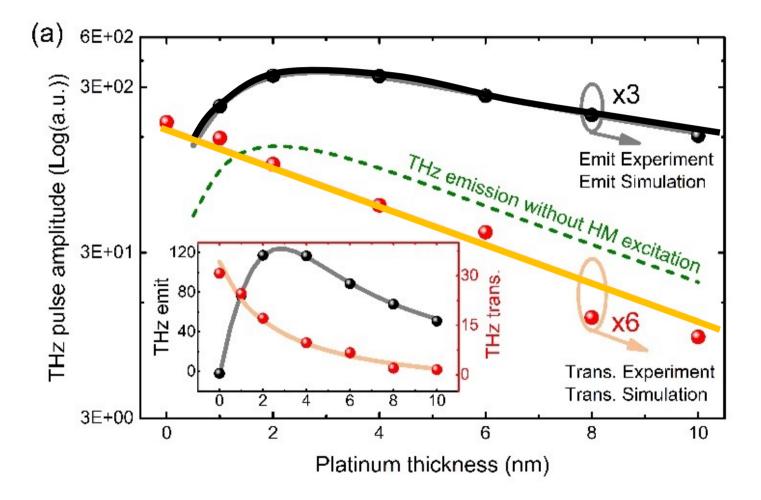
Y Yang, SD Forno, M Battiato, arXiv preprint arXiv:2206.00895



P Agarwal\*, Y Yang\*, R Medwal, H Asada, Y Fukuma, M Battiato, R Singh arXiv:2211.15135

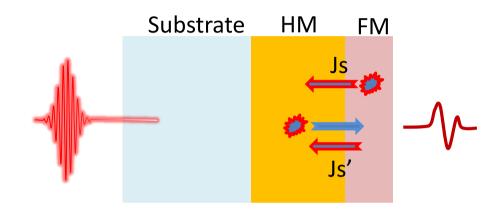


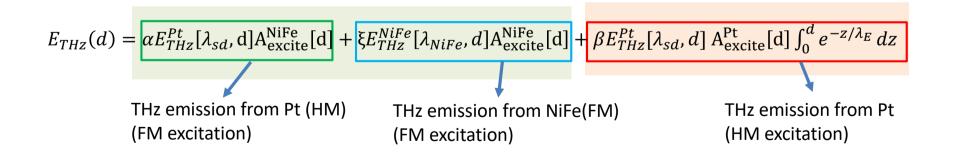
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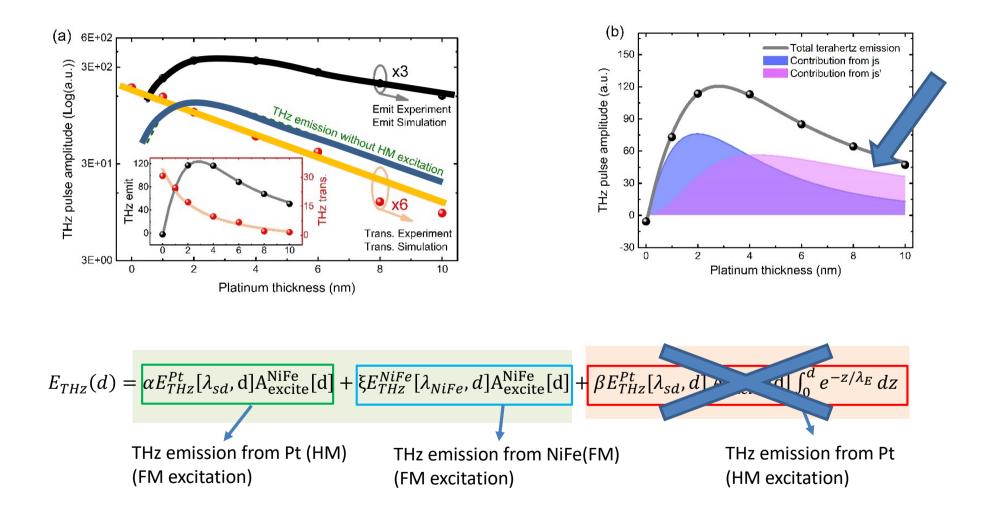
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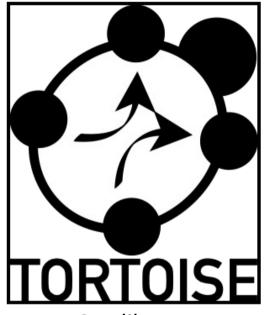
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### TORTOISE



C++ library

- No close to equilibrium assumptions
- No fixed population assumptions
- Any number of quasiparticles and bands
- Arbitrary dispersion (dispersion as input)
- Any number of scattering channels
- Scattering channels with 2, 3, and 4 legs (soon >4)
- Arbitrary scattering amplitude (required as input)
- Excellent scaling with precision
- Exact conservation of particle, energy and momentum

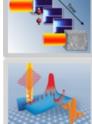
Wais, Held, Battiato, Comput. Phys. Commun. 264, 107877 (2021) Wadgaonkar, Jain, Battiato, Comput. Phys. Commun 263, 107863 (2021) Wadgaonkar, Wais, Battiato, Comput. Phys. Commun 271, 108207 (2021) https://sites.google.com/view/battiatomarco/tortoise

https://github.com/MarcoBattiato/TORTOISE





#### Thanks to



#### P. Maldonado, K. Carva, P. M. Oppeneer



T. Kampfrath G. Eilers J. Nötzold

Liang Cheng

Xinbo Wang

Jianwei Chai

Weifeng Yang

K. Held

S. Mährlein V. Zbarsky F. Freimuth

Ming Yang

Yang Wu

Mengji Chen

Xiaoxuan Chen

Y. Mokrousov S. Blügel M. Wolf

I. Radu M. Münzenberg

Dongzhi Chi Kuan Eng Johnson Goh Jian-Xin Zhu Handong Sun Shijie Wang Justin C. W. Song Hyunsoo Yang Elbert E. M. Chia



M. Wais, I. Wadgaonkar, Jain, K. Held



Y Yang, S Dal Forno

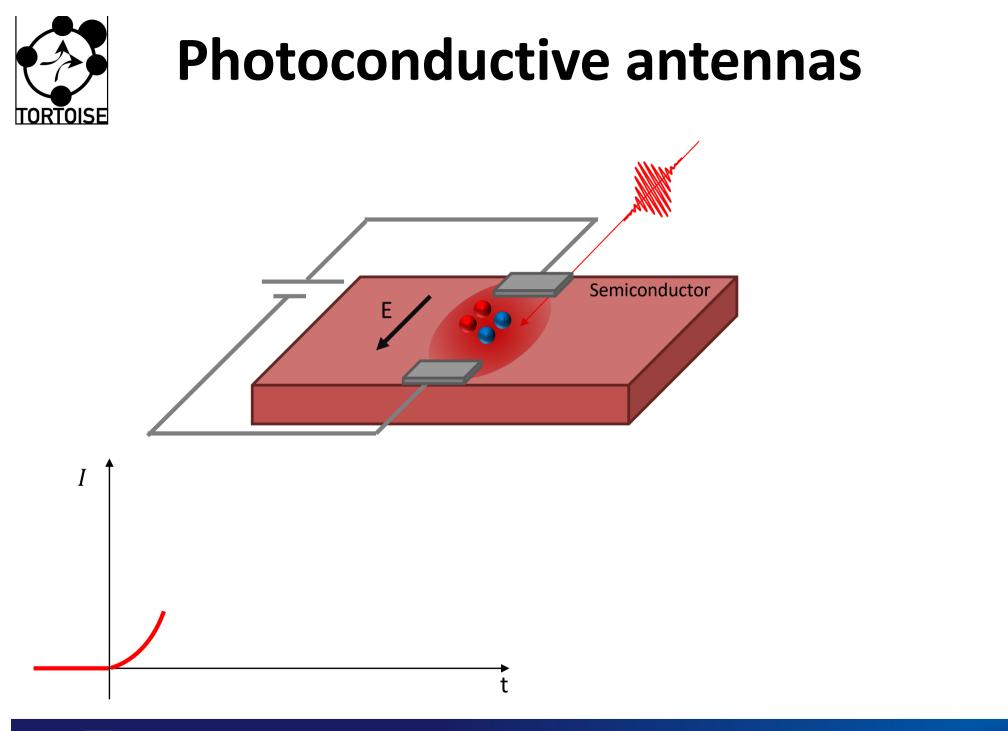


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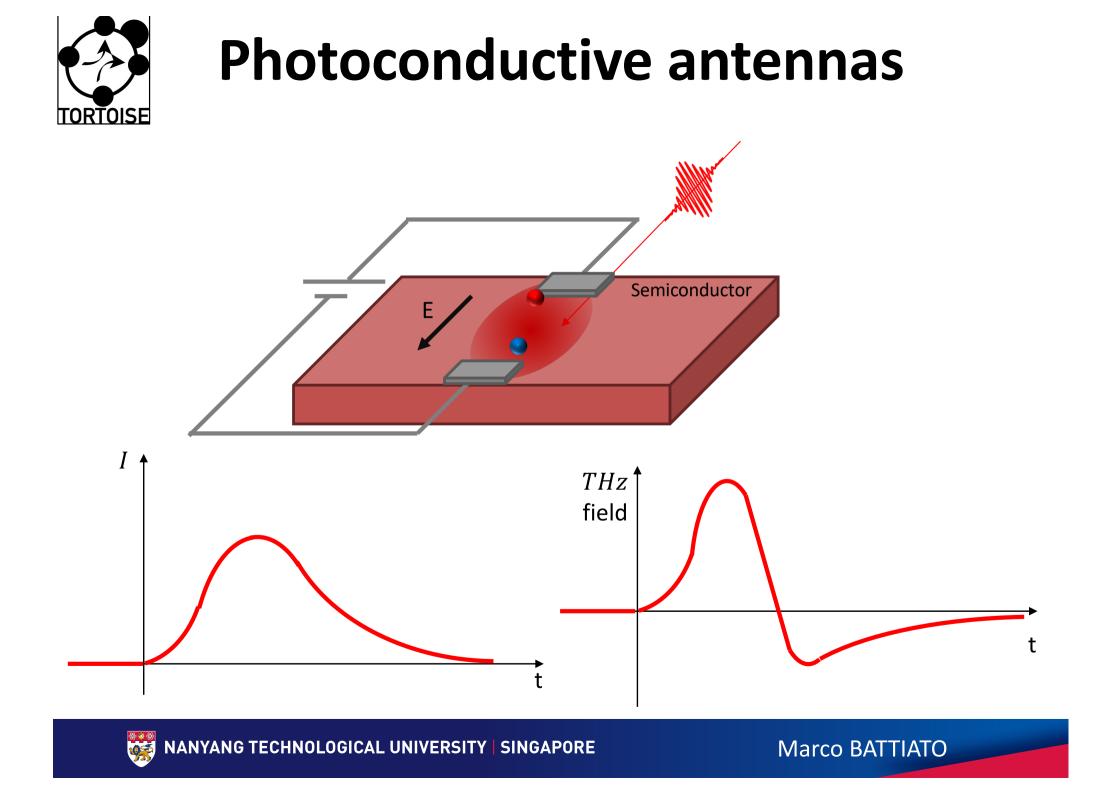


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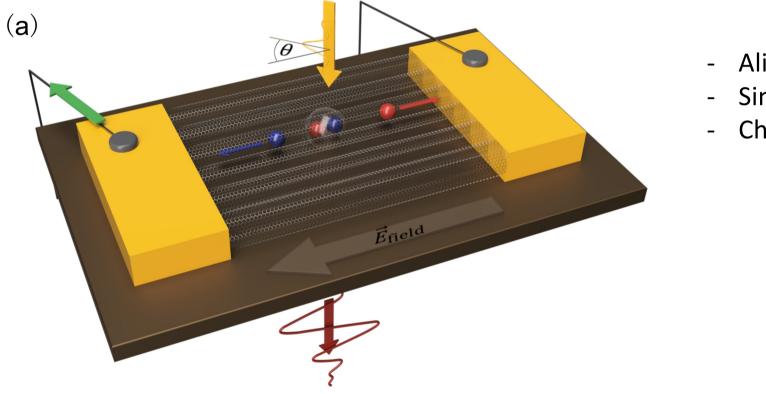


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Bagsican, Wais, Komatsu, Gao, Weber, Serita, Murakami, Held, Hegmann, Tonouchi, Kono, Kawayama, Battiato, **Nano Letters** 20, 3098 (2020).



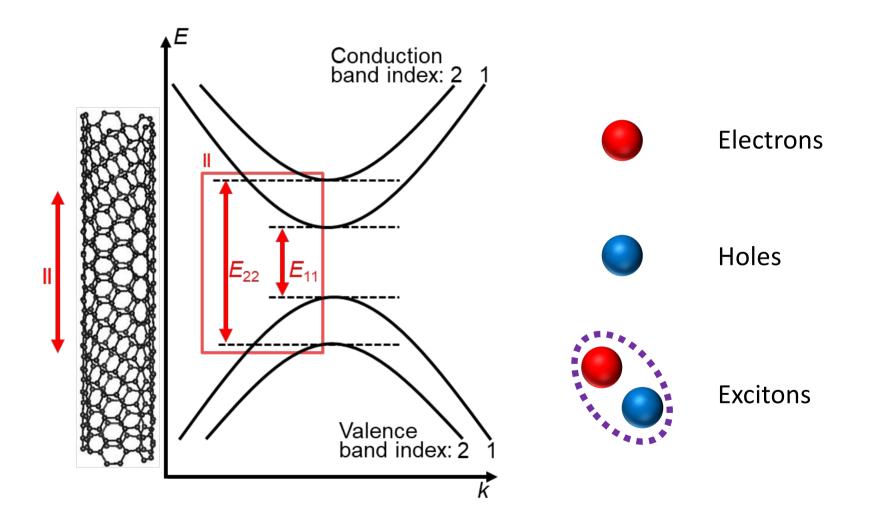
- Aligned
- Single walled
- Chirality enriched





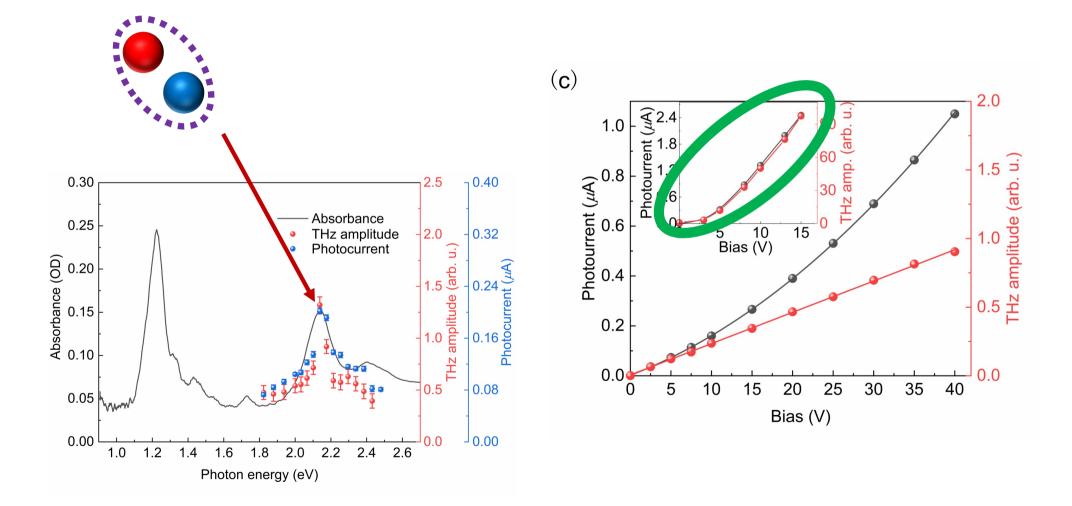


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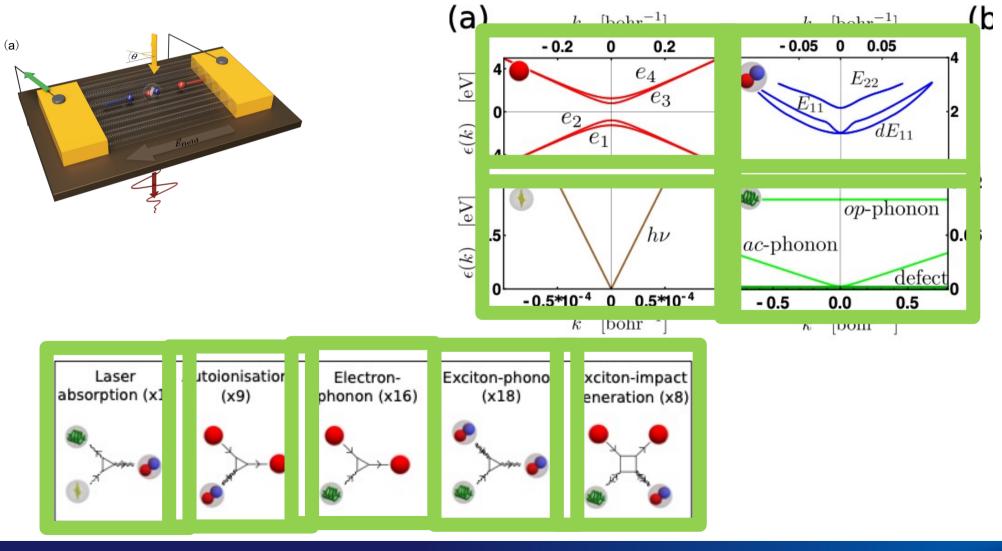
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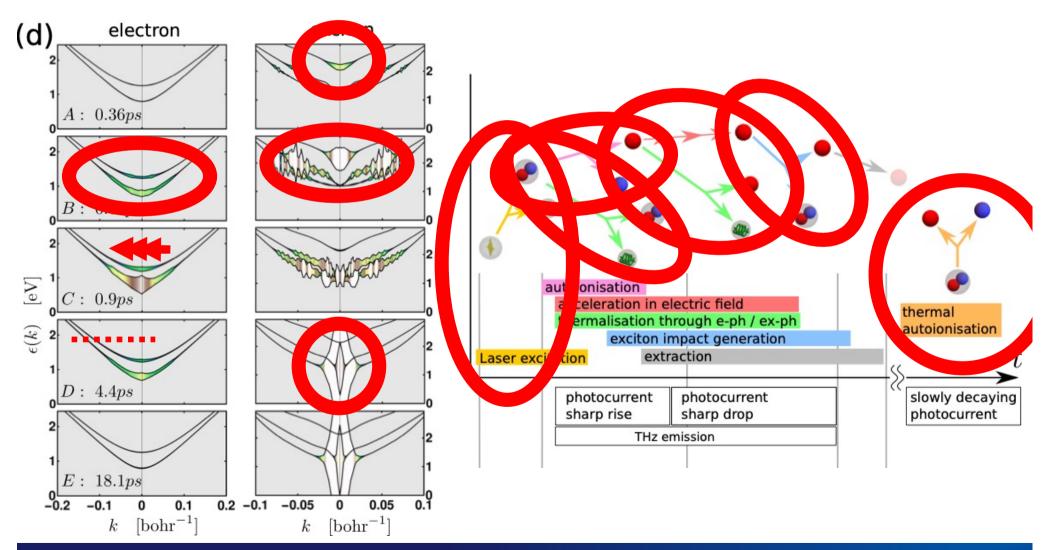
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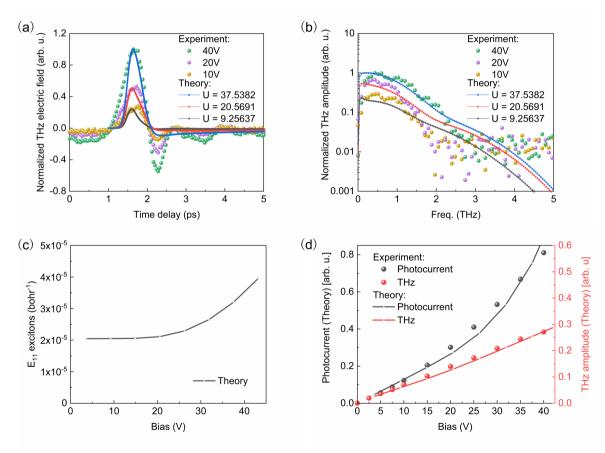
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Bagsican, Wais, Komatsu, Gao, Weber, Serita, Murakami, Held, Hegmann, Tonouchi, Kono, Kawayama, Battiato, **Nano Letters** 20, 3098 (2020).



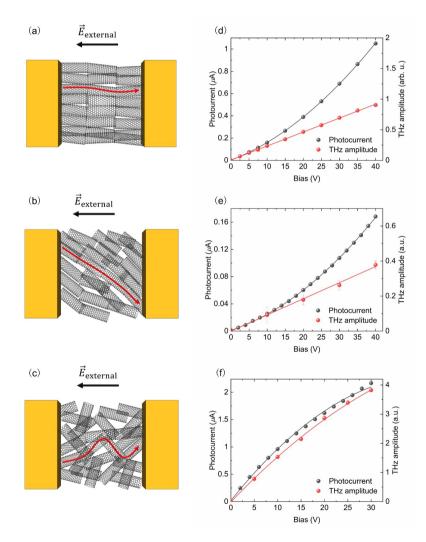
- Understood the unusual THz emission process in CNTs
- Uncovered the role of the "exciton impact generation" scattering
- Produced a very promising CNTbased photoconductive antenna





#### From ballistic to diffusive in CNTs

Wais<sup>\*</sup>, Bagsican<sup>\*</sup>, Komatsu, Gao, Serita, Murakami, Held, Kawayama, Kono, Battiato and Tonouchi, **Nano Letters** (2023).



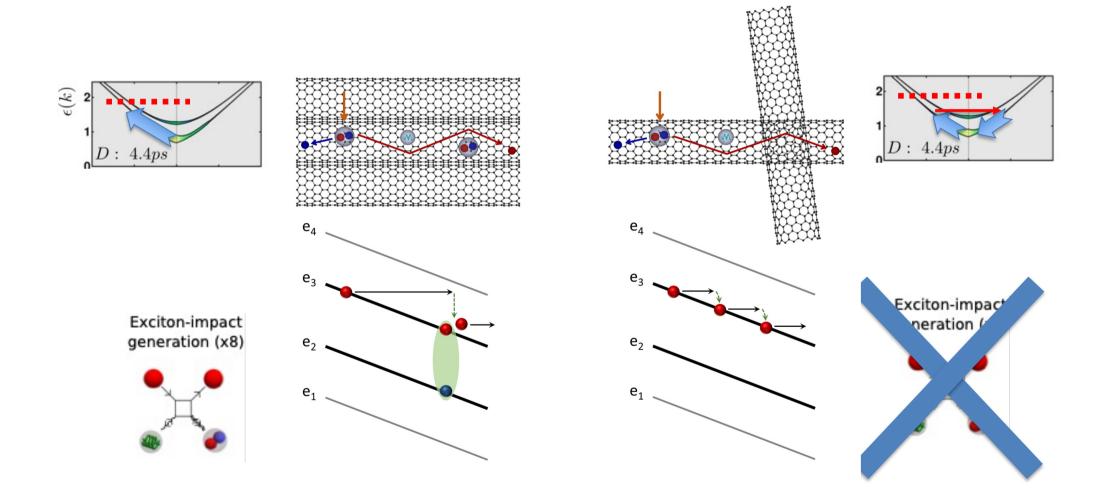


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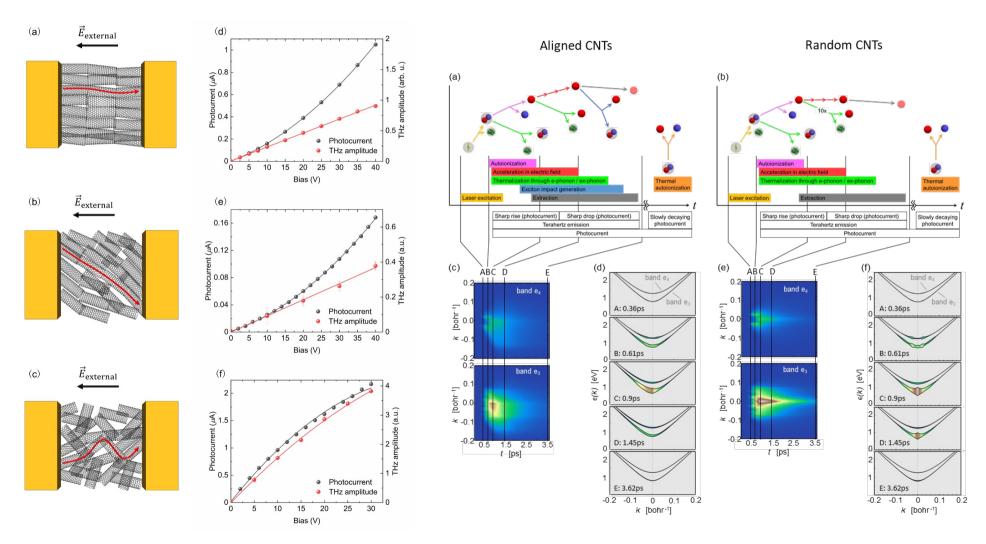






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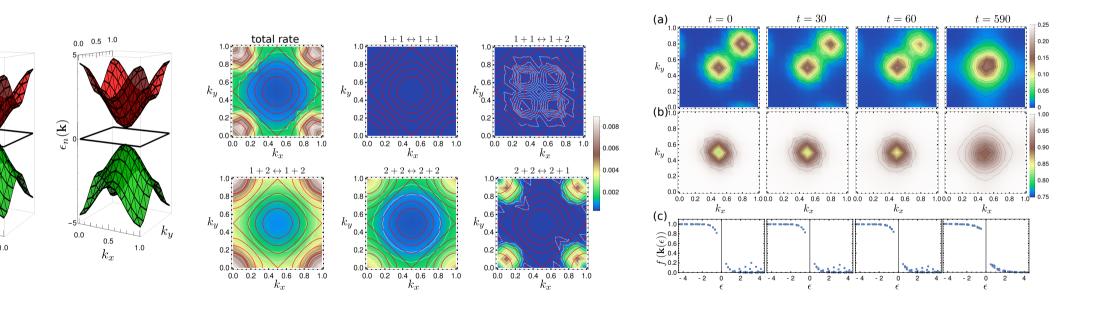


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# Strongly out-of-equilibrium thermalisation and transport

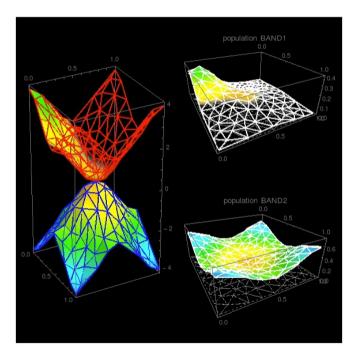
Wais, Held, Battiato, Comput. Phys. Commun. 264, 107877 (2021) Wadgaonkar, Jain, Battiato, Comput. Phys. Commun 263, 107863 (2021) Wadgaonkar, Wais, Battiato, Comput. Phys. Commun 271, 108207 (2021)

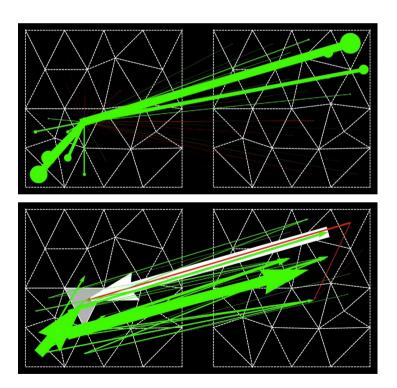




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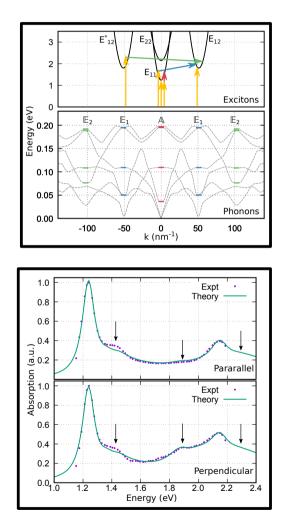


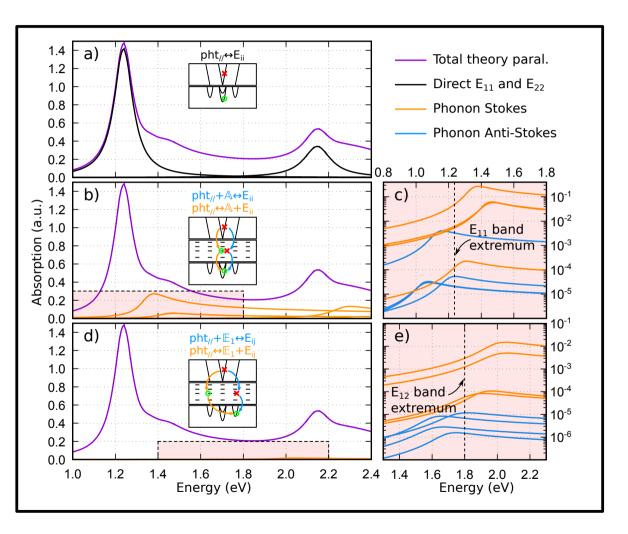




# **Time-resolved absorption spectra**

Dal Forno, Komatsu, Wais, Mojibpour, Wadgaonkar, Ghosh, Yomogida, Yanagi, Held, Kono, Battiato, **Carbon** 186, 465 (2021)

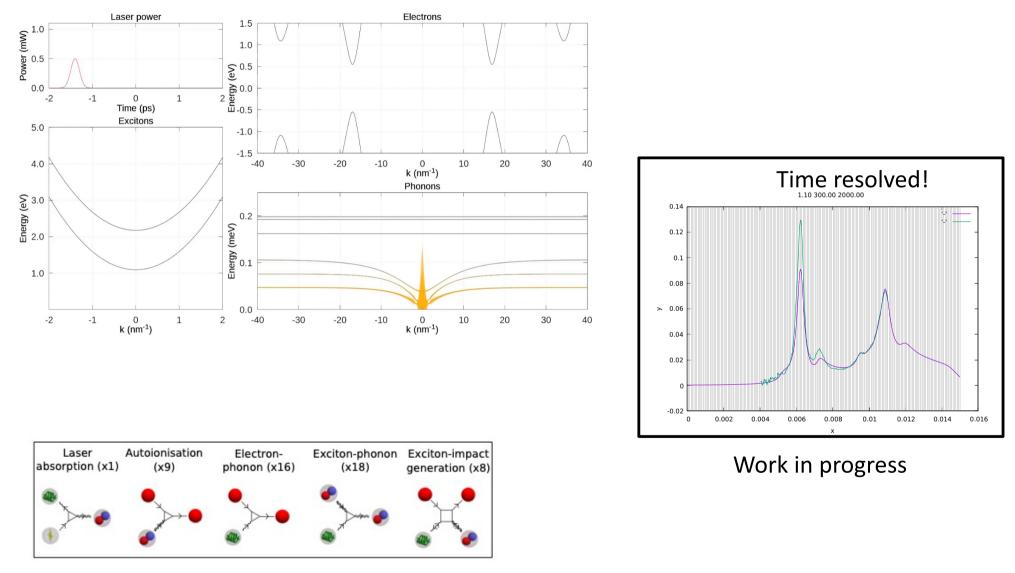






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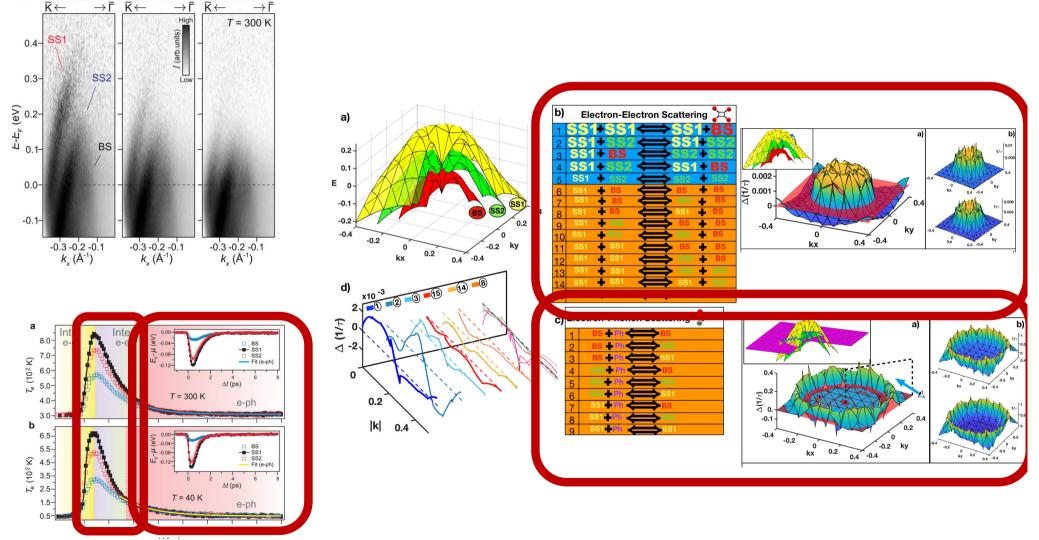
Dal Forno, Komatsu, Wais, Wadgaonkar, Held, Kono, Battiato, manuscript.





### Ultrafast thermalisation pathways in GeTe

TOISEClark, Wadgaonkar, Freyse, Springholz, Battiato, Sanchez-BarrigaAdvanced Materials34, 2200323 (2022).





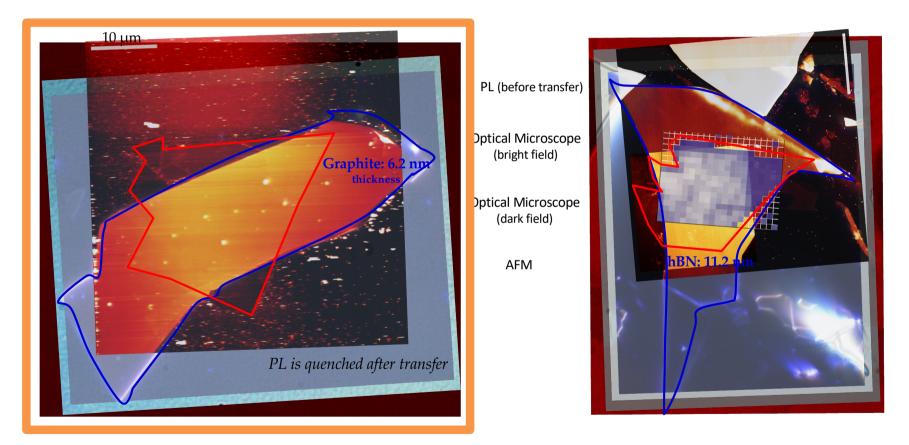
TORTOISE

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Woo, Wadgaonkar, Xiang, Battiato, Loh, work in progress

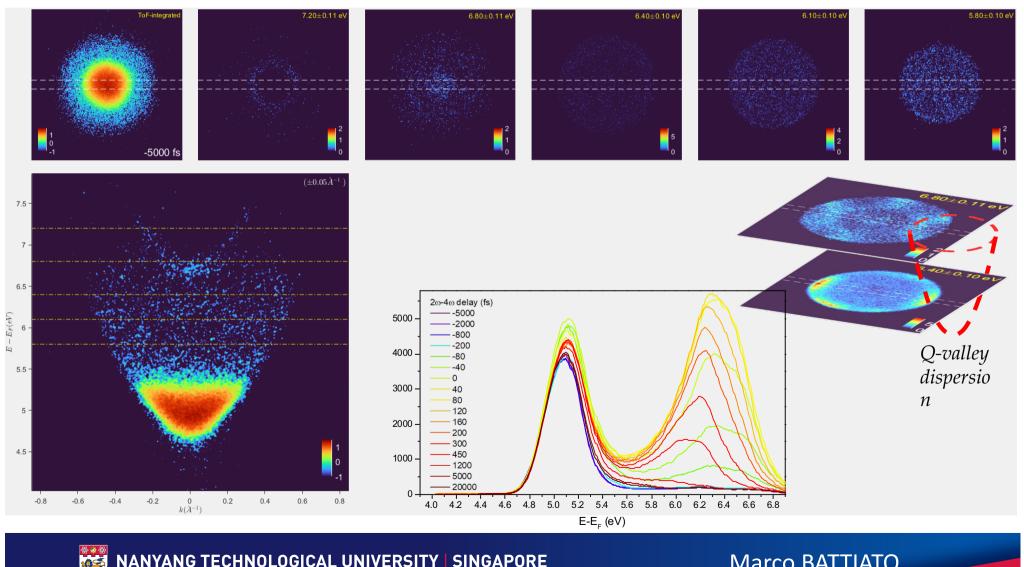
Samples we have: 1L- $WSe_2$  / Si with different supports







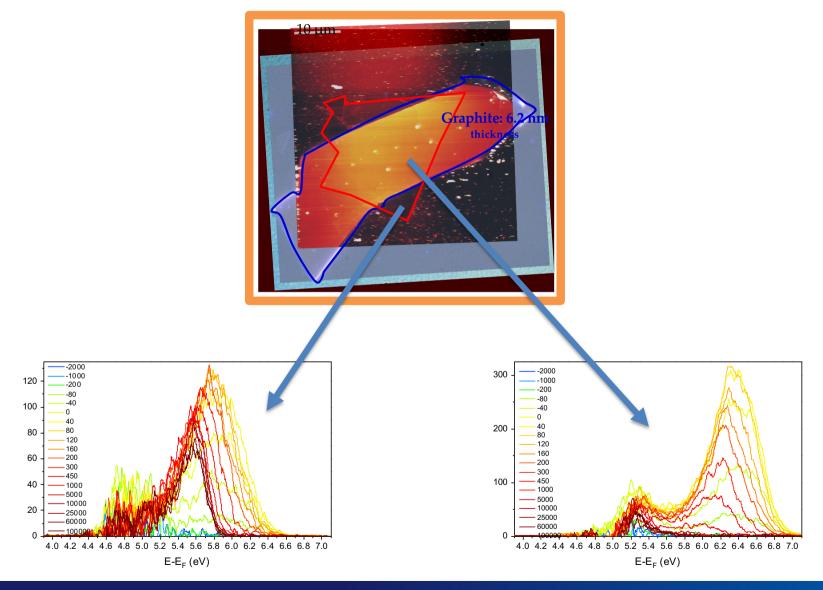
Woo, Wadgaonkar, Xiang, Battiato, Loh, work in progress



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Woo, Wadgaonkar, Xiang, Battiato, Loh, work in progress

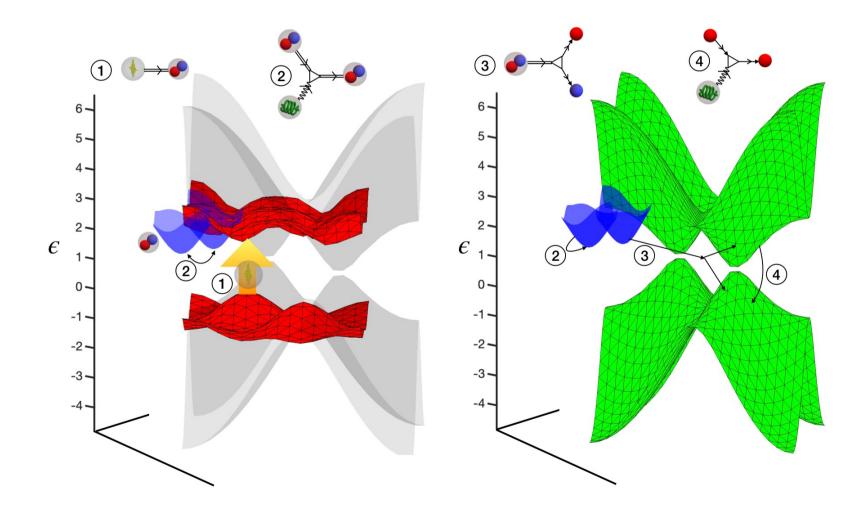




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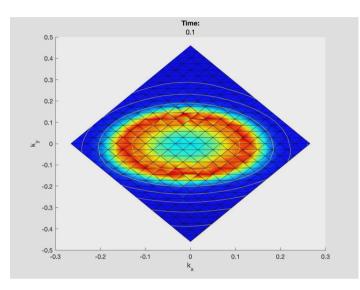
Woo, Wadgaonkar, Xiang, Battiato, Loh, work in progress

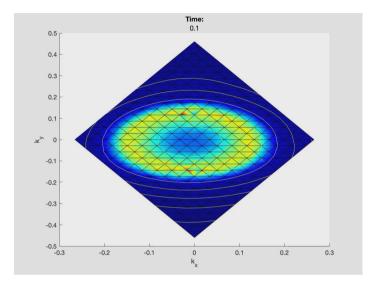


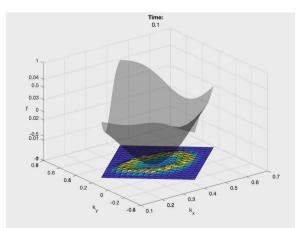




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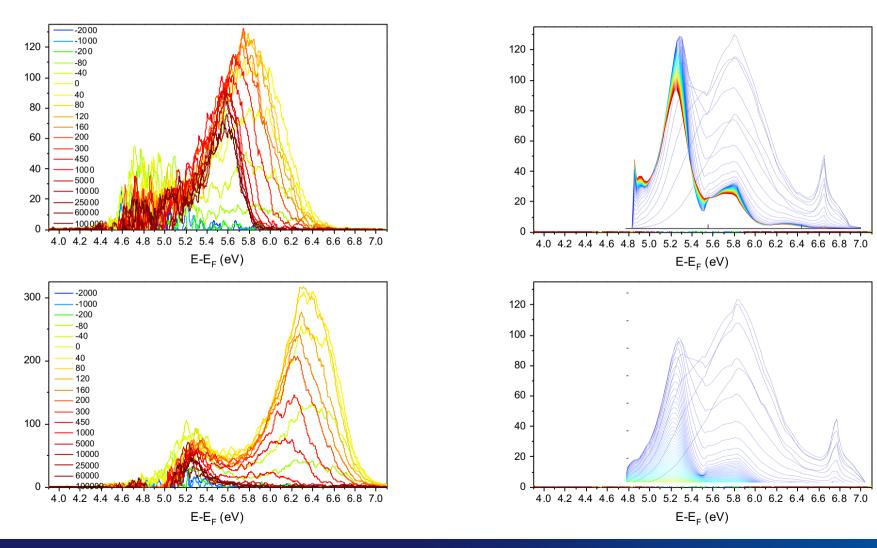








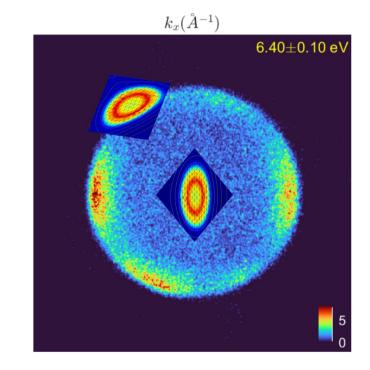
Woo, Wadgaonkar, Xiang, Battiato, Loh, work in progress







Woo, Wadgaonkar, Xiang, Battiato, Loh, work in progress



 $k_y(\mathring{A}^{-1})$ 

 $k_x(\mathring{A}^{-1})$ 

#### Movie: work in progress

