

innovating nanoscience

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Machine learning as a tool to accelerate magnetic materials discovery

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COMPUTATIONAL SPINTRONICS

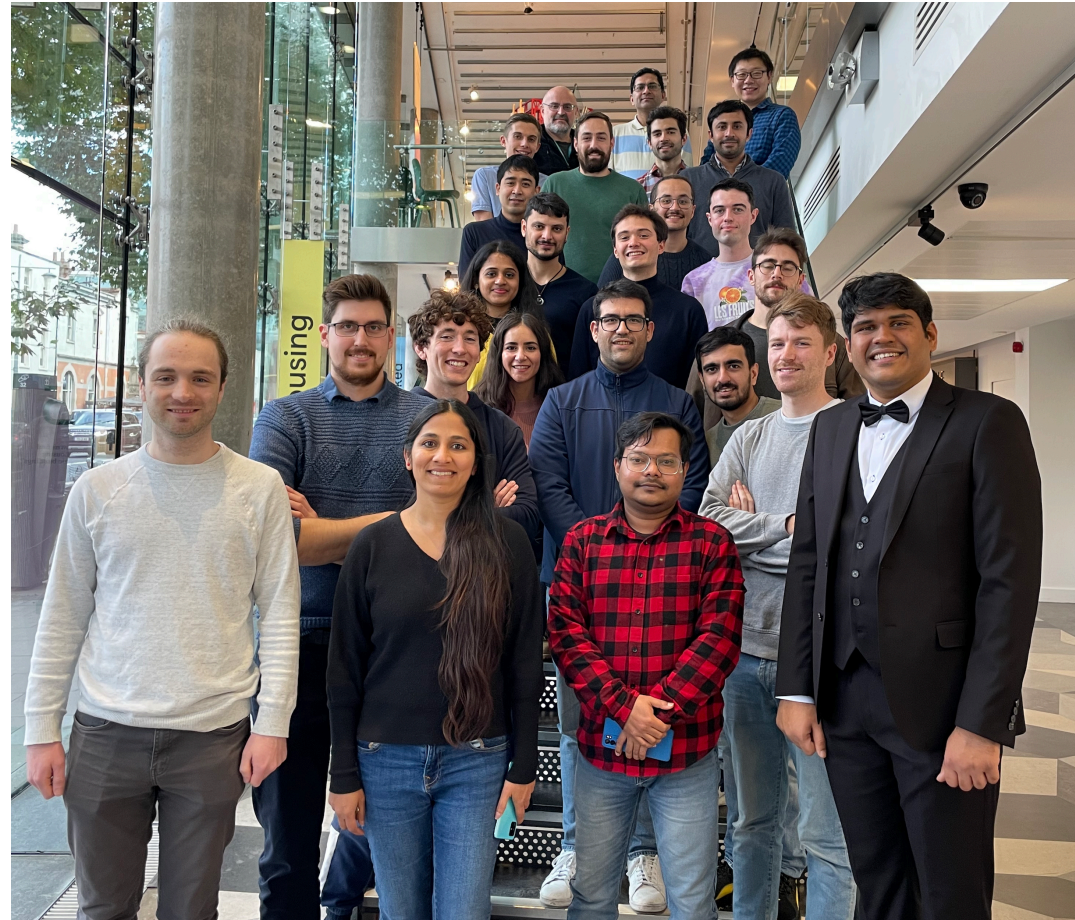
SANVITO RESEARCH GROUP
TRINITY COLLEGE, DUBLIN



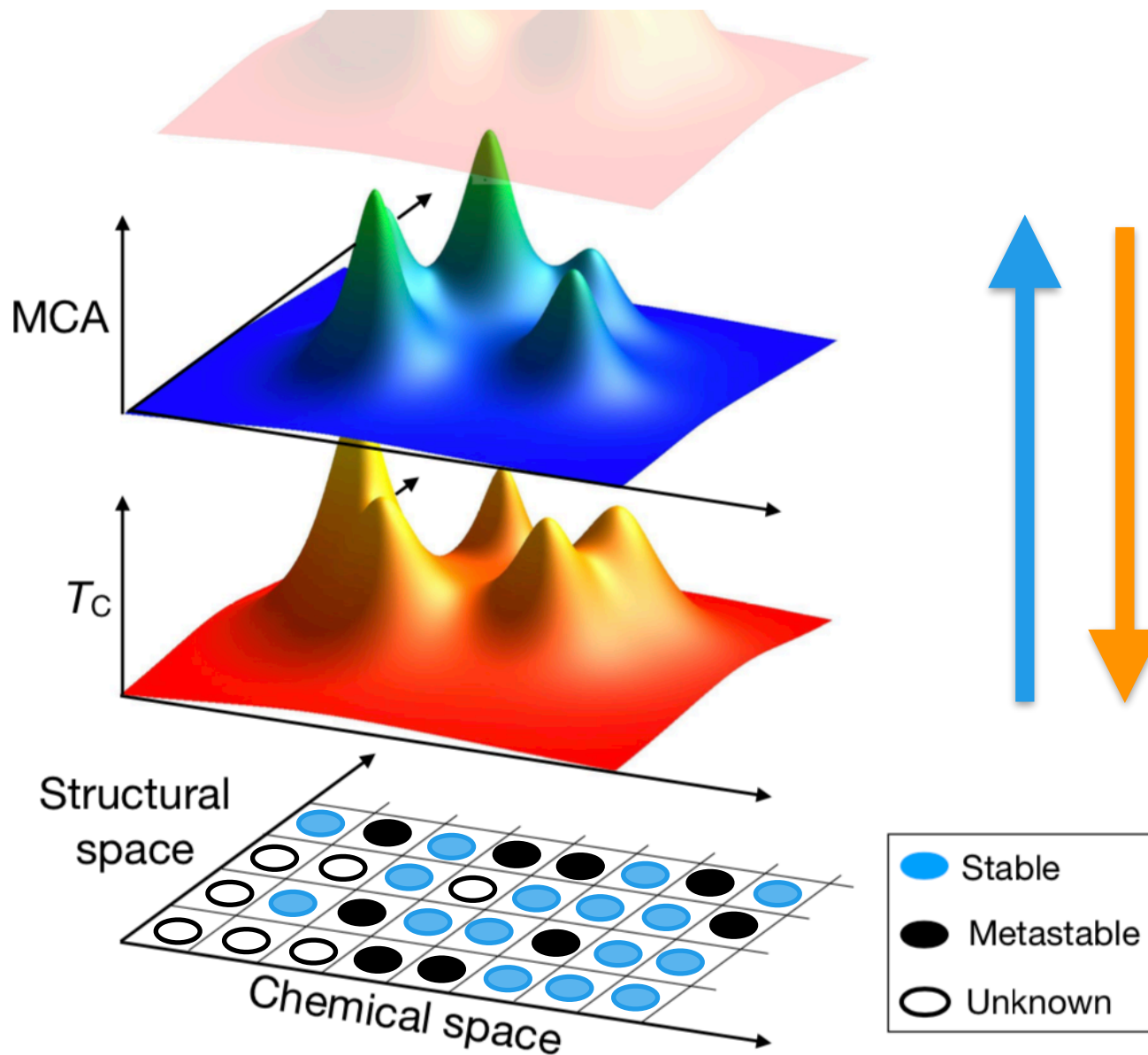
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Direct vs Inverse problem



The problems of making a roadmap



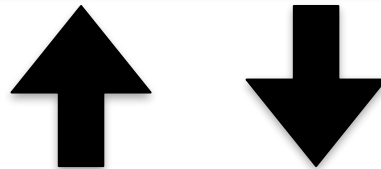
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Accuracy

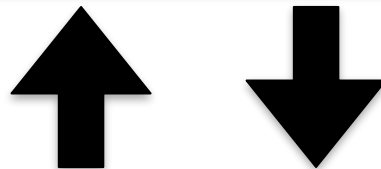
LAB

Environment

Materials



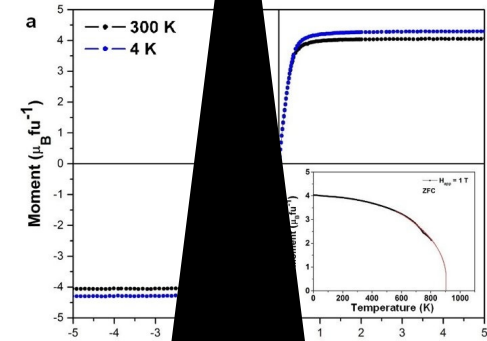
Properties



Explore structural space

and

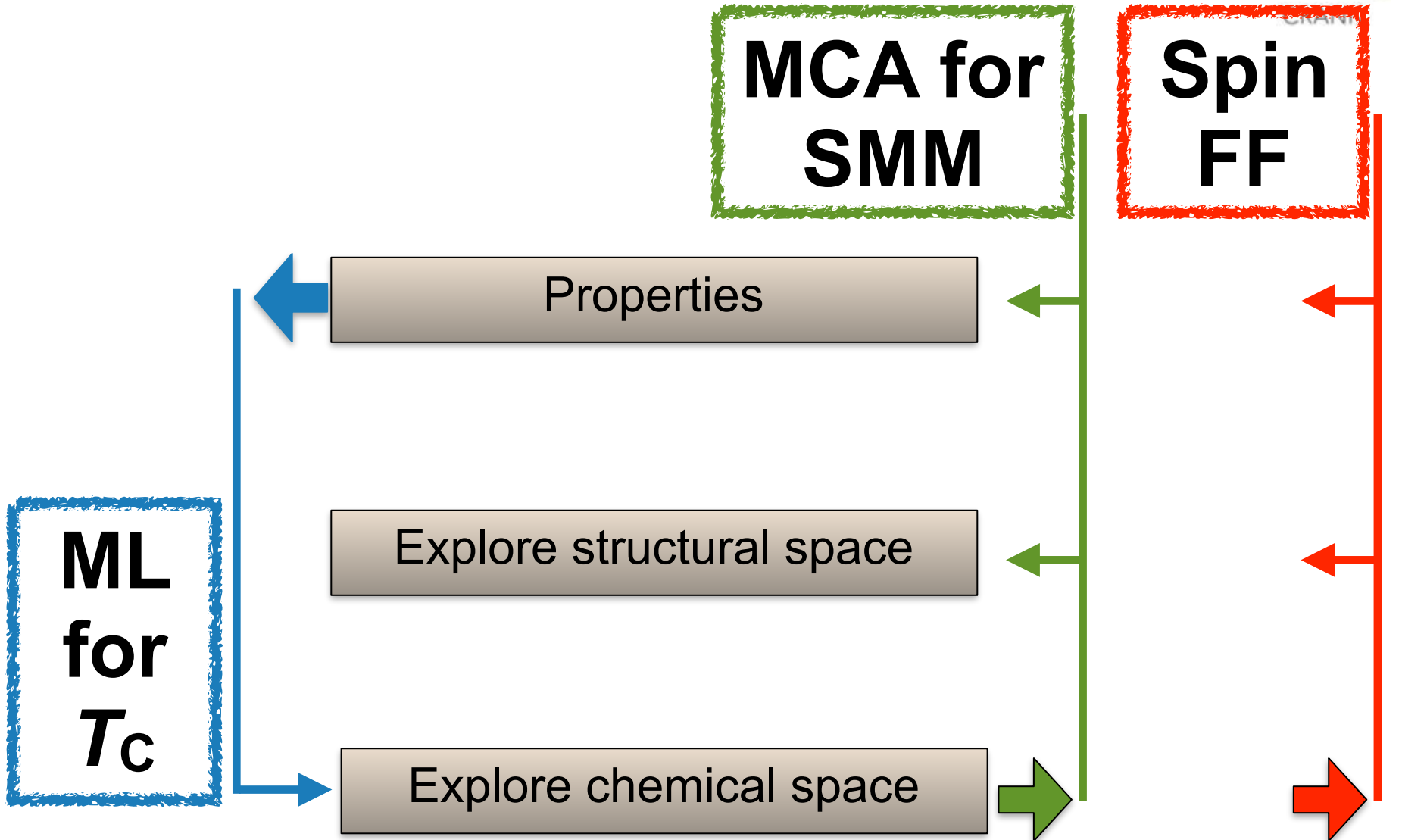
Explore chemical space



PERIODIC TABLE OF ELEMENTS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
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Outline

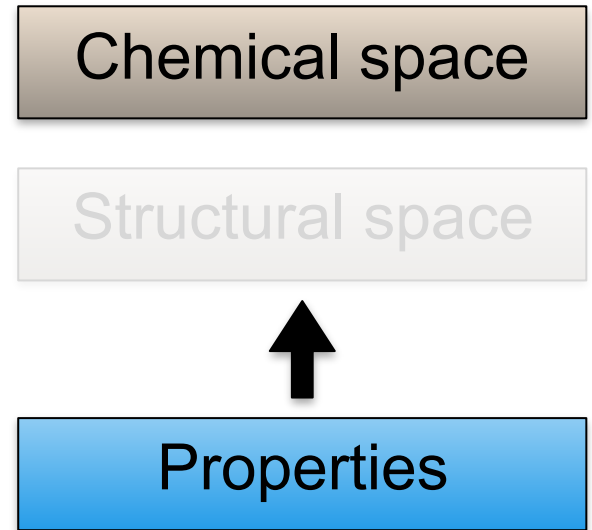
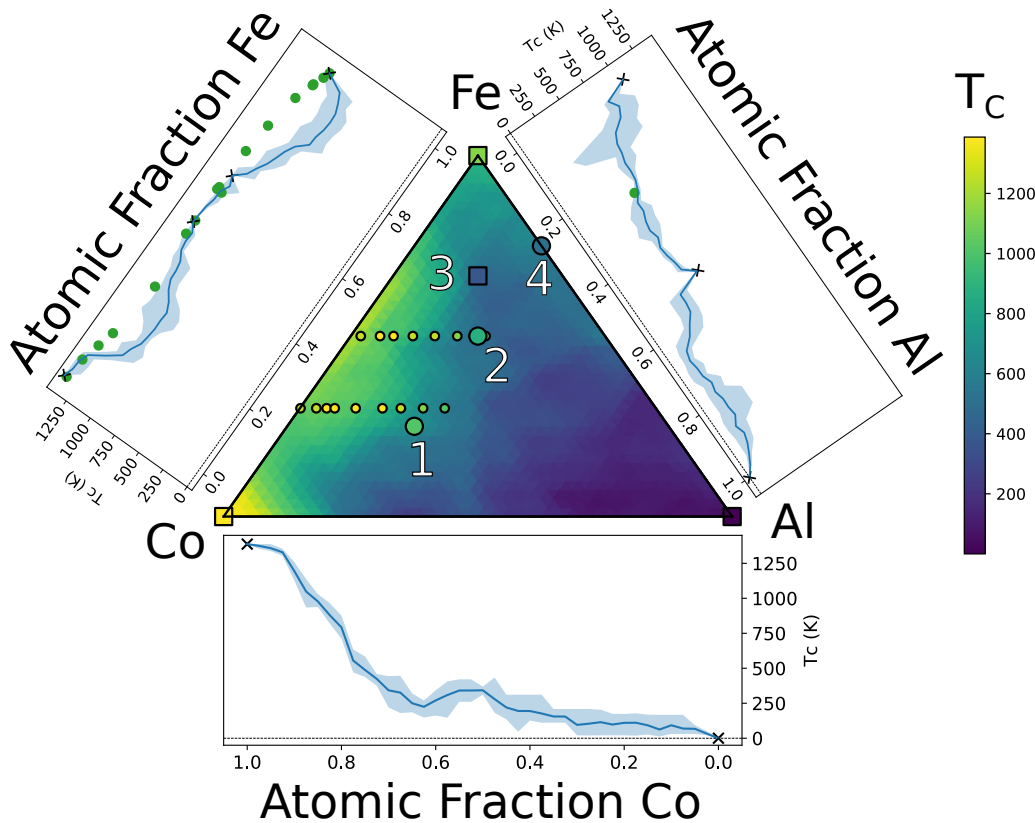




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James Nelson

Data to predictions: T_C of ferromagnets

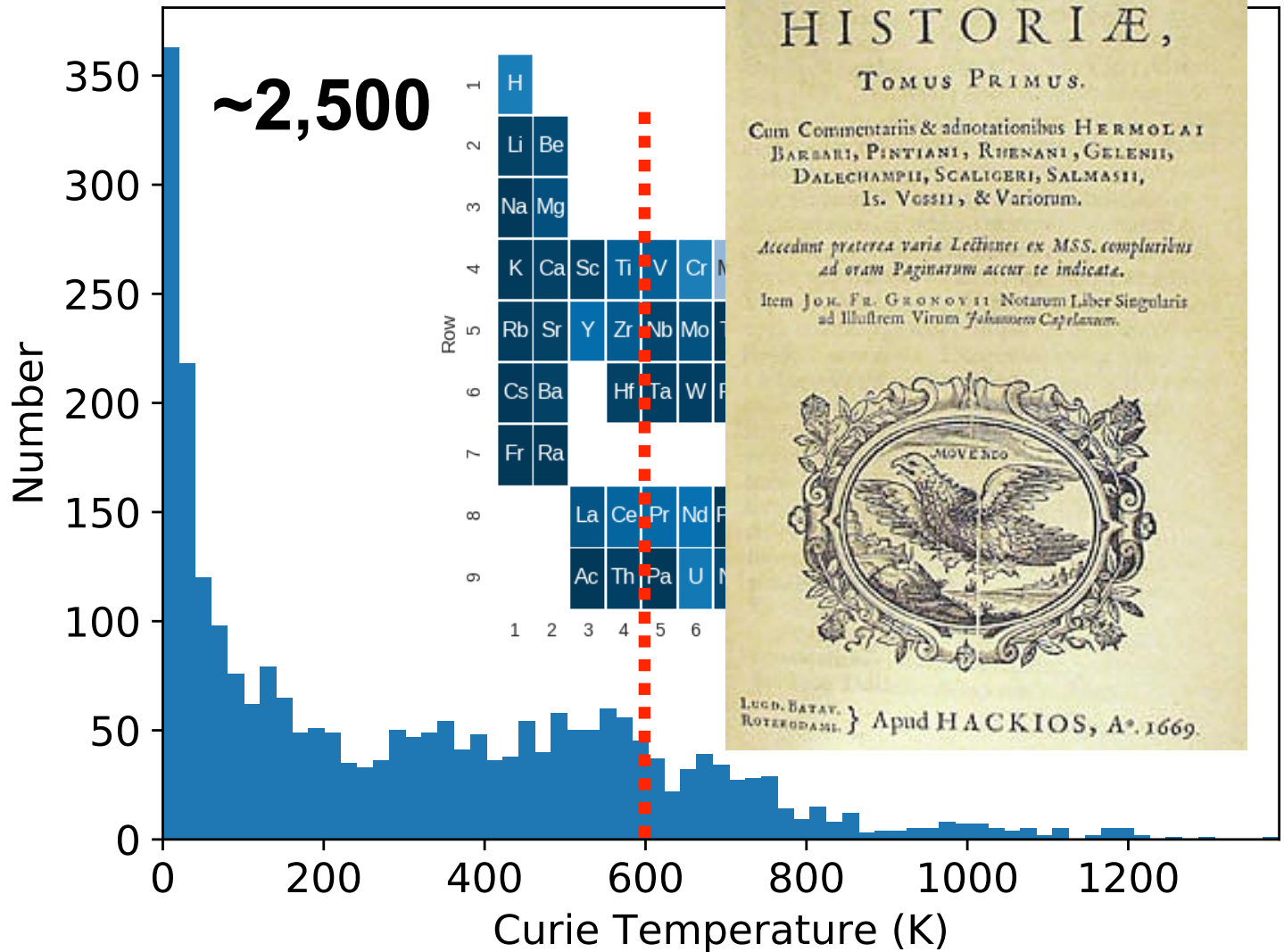


Example: T_C of ferromagnets

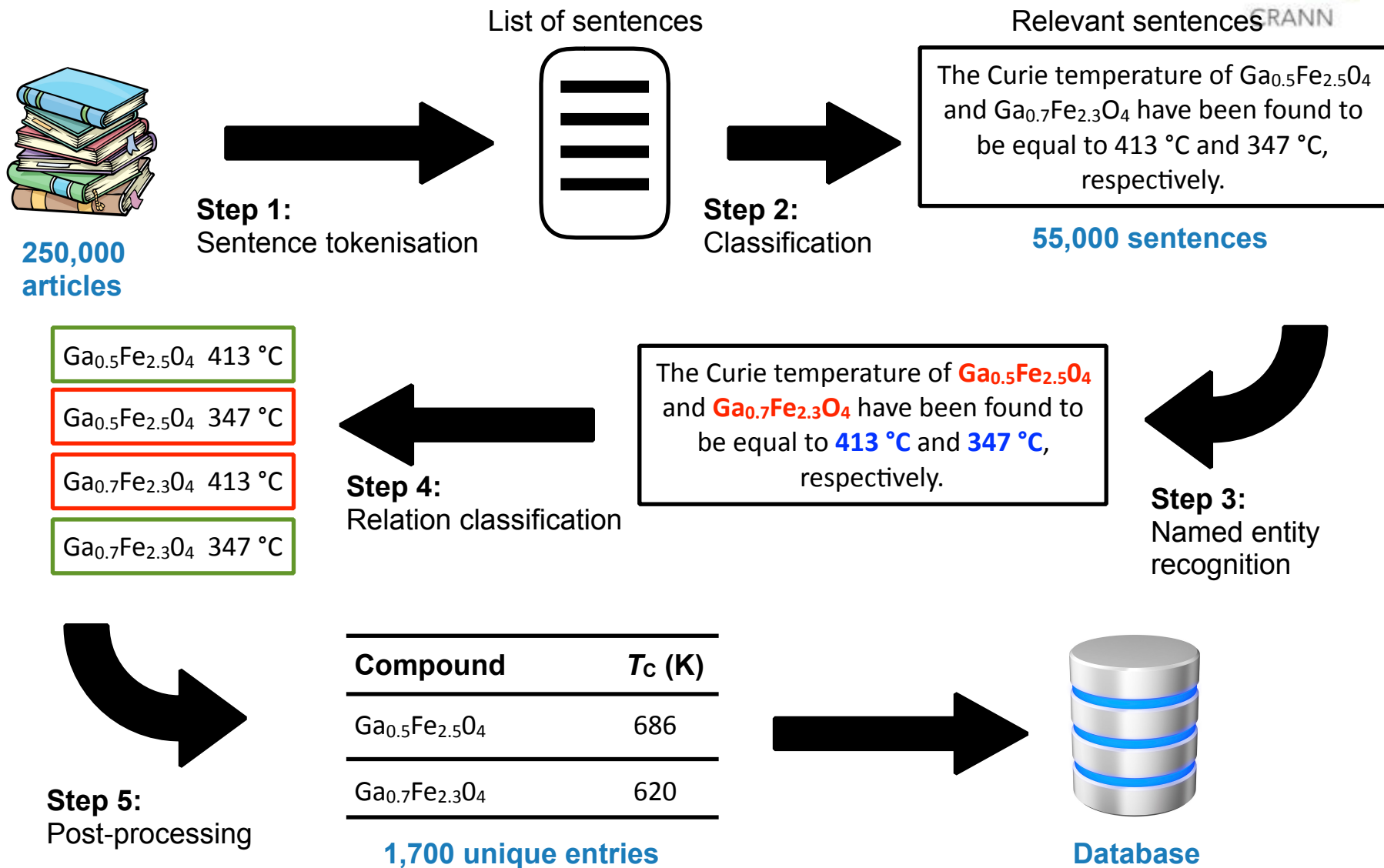
Goal: predict T_C of ferromagnets



Start from
experimental
data



Automated Extraction

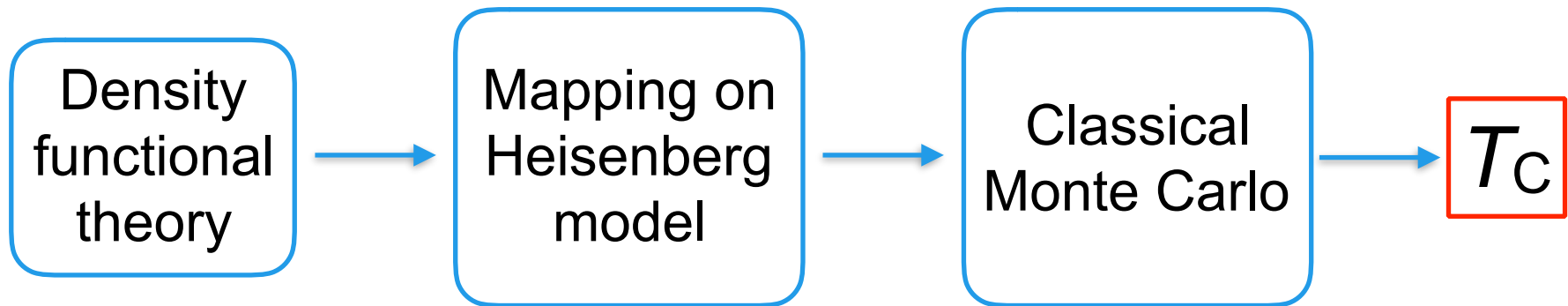


Example: T_C of ferromagnets



Goal: predict T_C of ferromagnets

“Standard way”



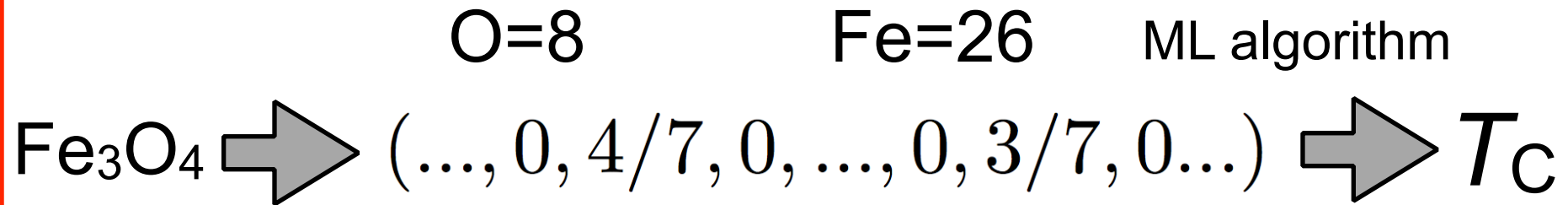
Approximation
Cell size
Broken symmetry
....

Non-magnons
Interaction range
....

Quantum effects
....

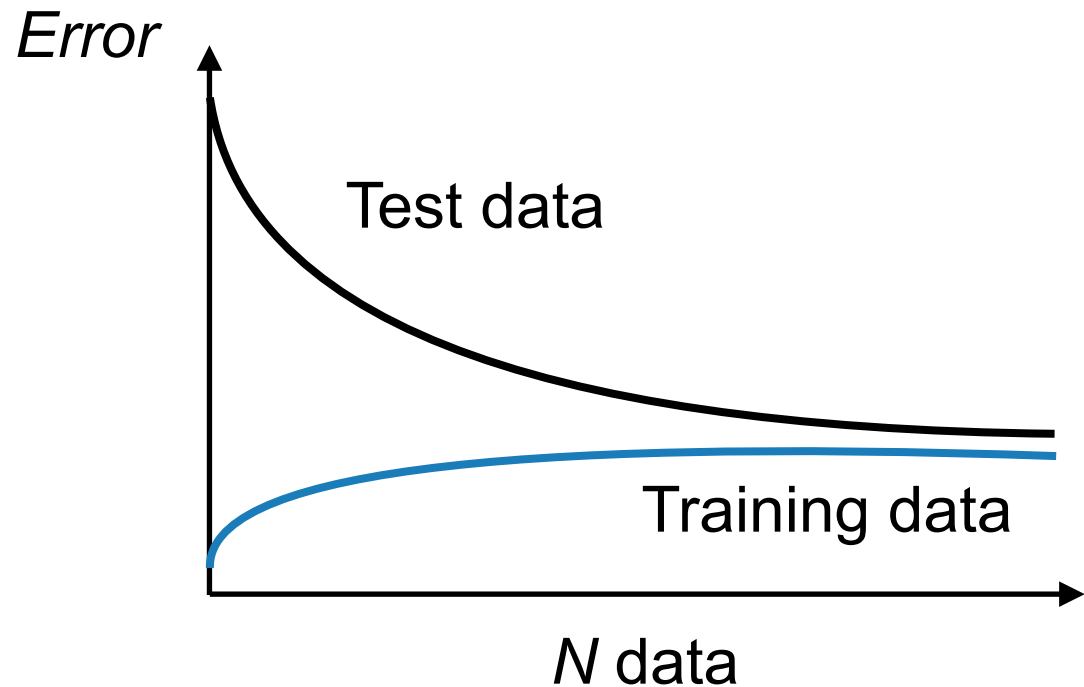
Often if we do not know the answer we cannot ‘predict’

Example: T_C of ferromagnets



1818 Training

746 Test



Example: T_C of ferromagnets

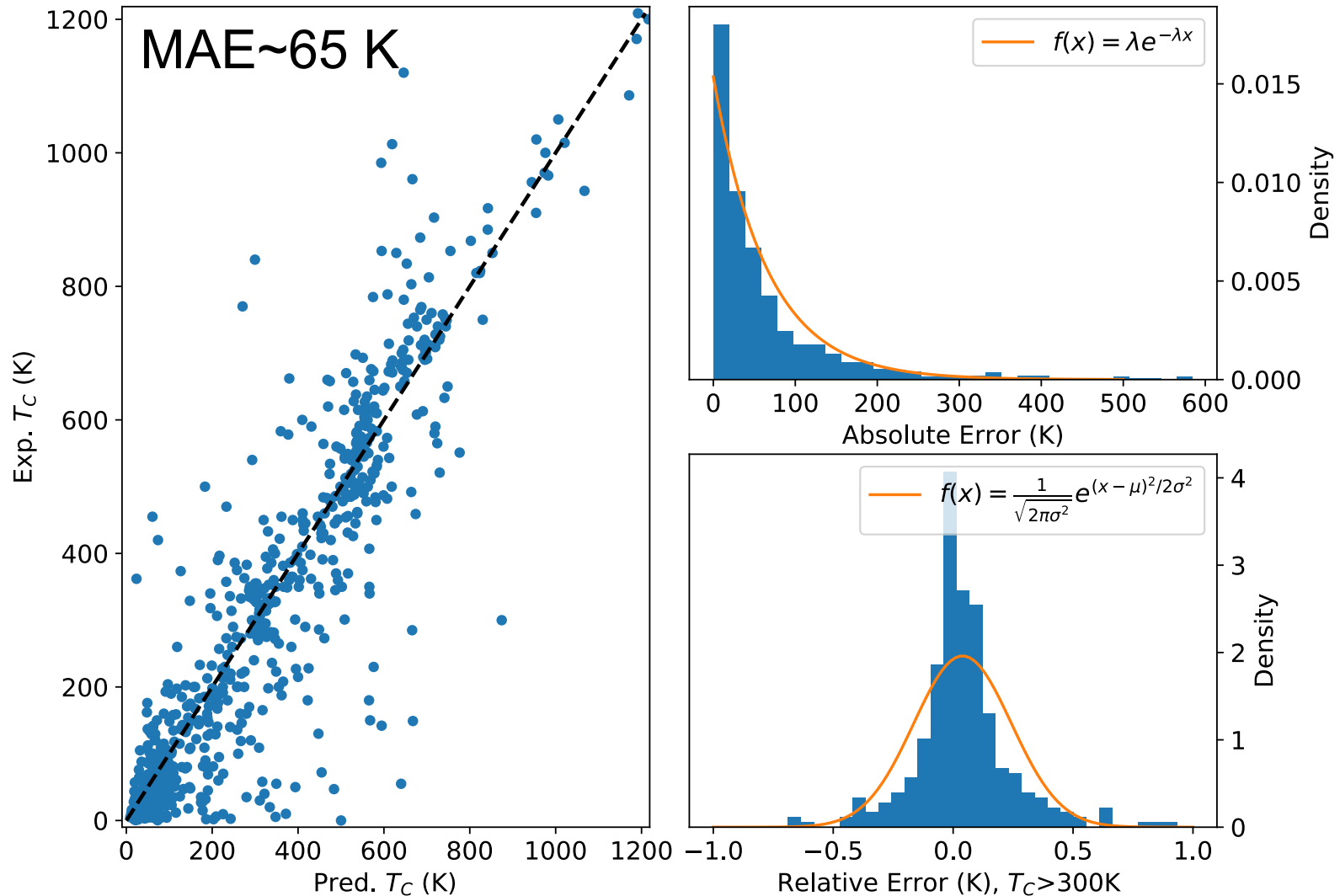


R^2	All	C10	C20	C40	C80	P10	P20	P40	P80
Ridge	0.50	0.48	0.50	0.51	0.50	0.27	0.30	0.32	0.37
KRR	0.70	0.72	0.76	0.74	0.71	0.70	0.71	0.70	0.69
NN	0.75	0.74	0.77	0.77	0.75	0.71	0.74	0.75	0.73
RF	0.83	0.78	0.80	0.80	0.81	0.73	0.75	0.74	0.73

Ridge = Ridge Regression
KRR = Kernel Ridge Reg.
NN = Neural Network
RF = Random Forest

$$R^2 = 1 - \frac{\sum_i [y^{(i)} - f(\mathbf{x}^{(i)})]^2}{\sum_i [y^{(i)} - \mu]^2}$$

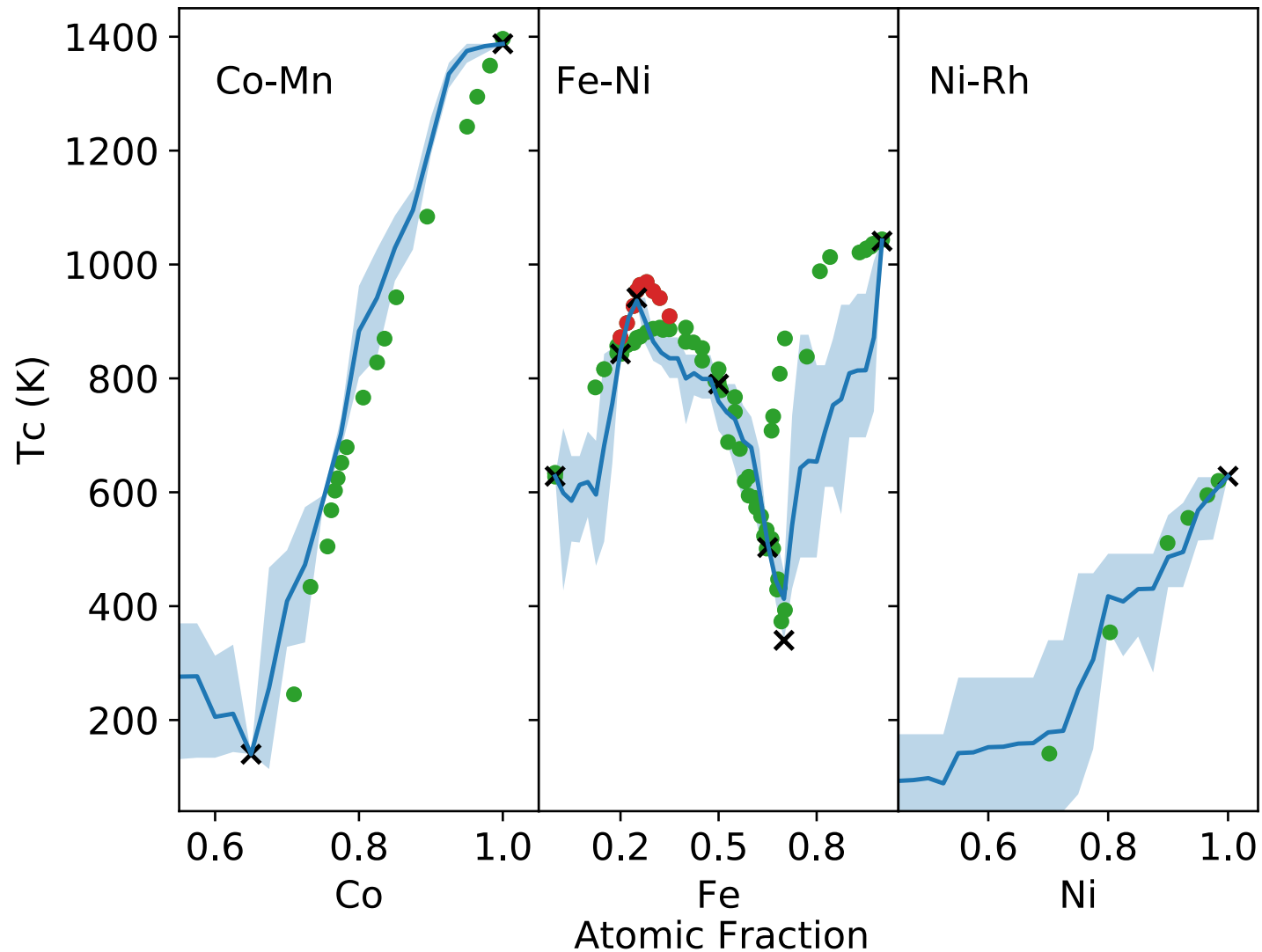
Example: T_C of ferromagnets



Example: T_C of ferromagnets



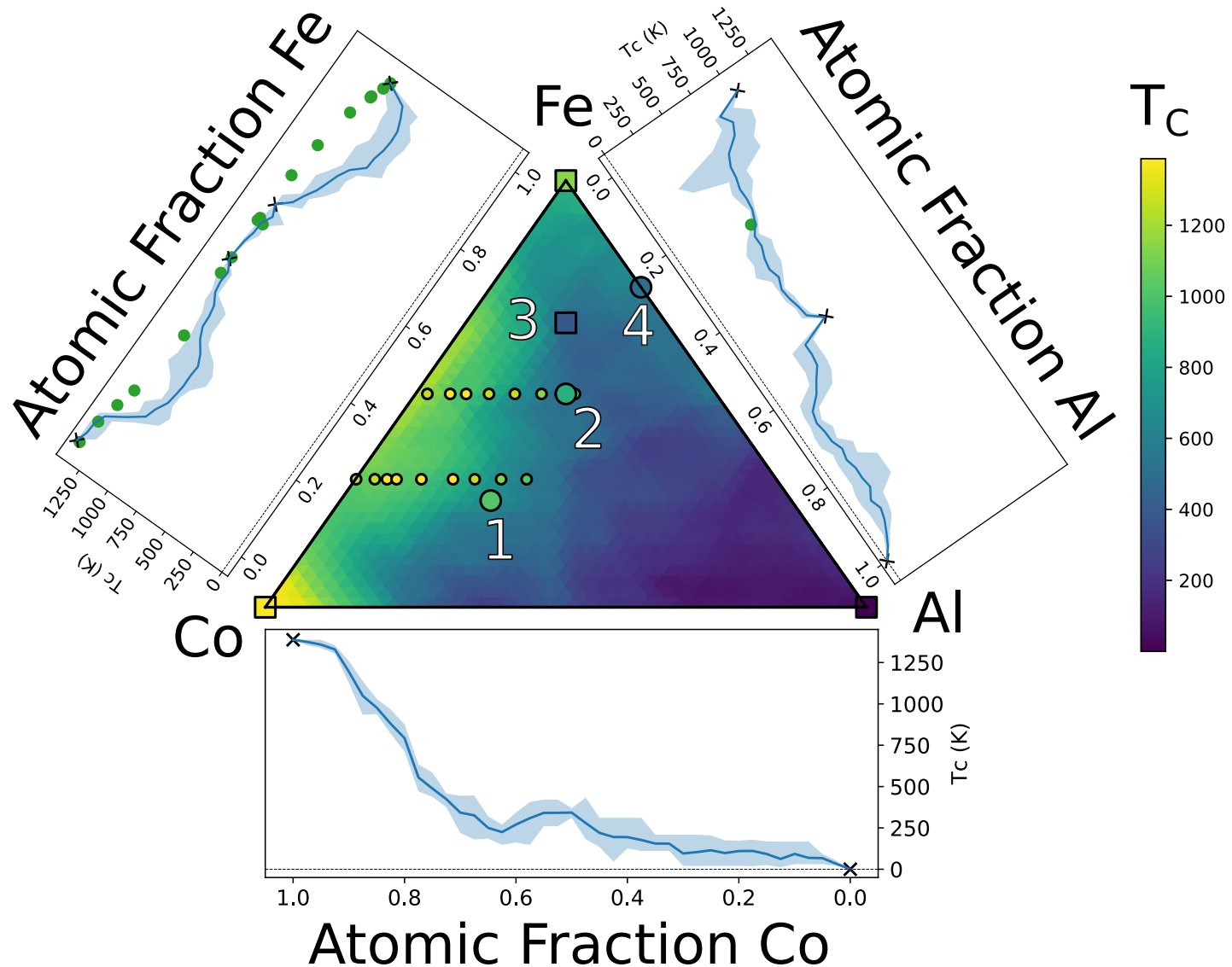
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Example: T_C of ferromagnets



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Data can be used to construct models !

Useful for an initial exploration

Extremely high throughput

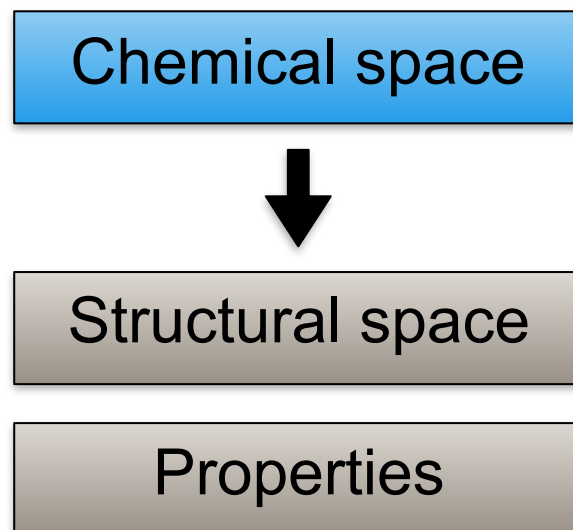
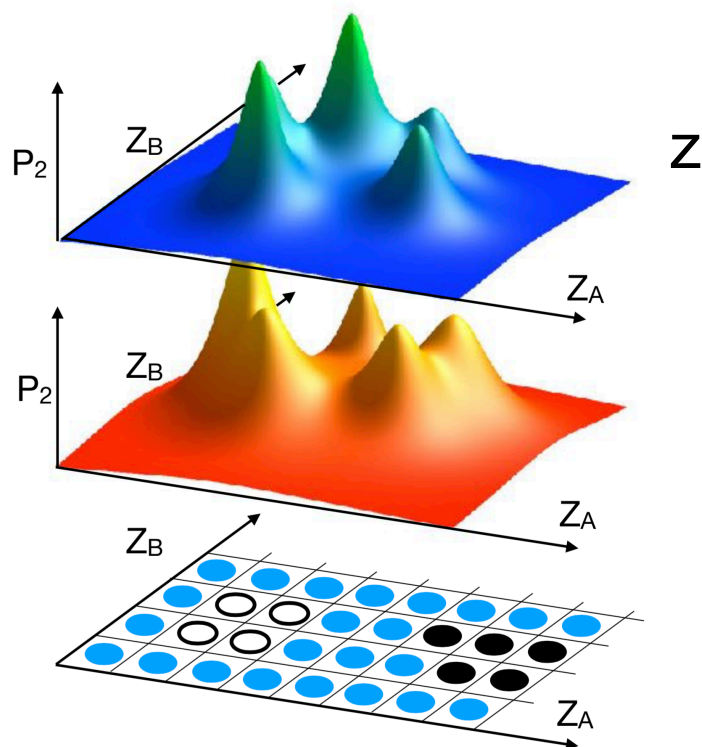
Only experimental data are needed

Need to harvest more data: automation possible now!!

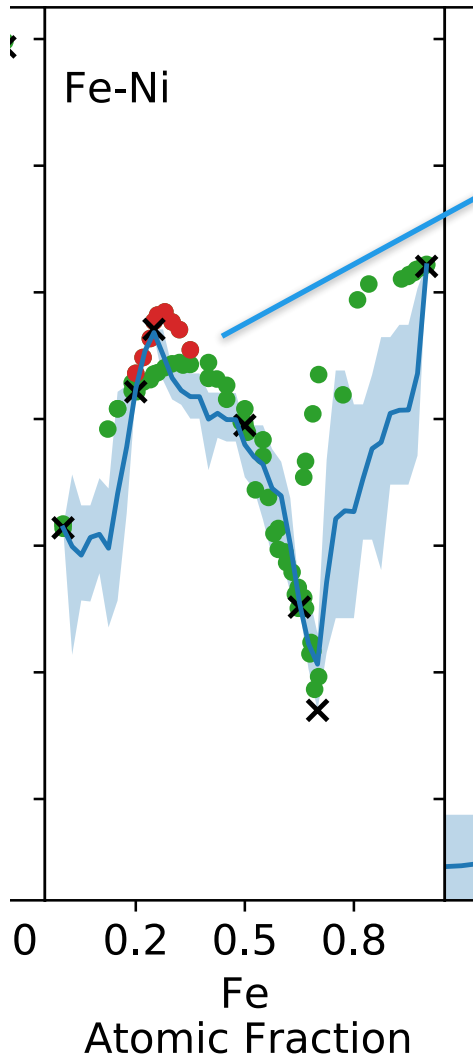
Structure difficult to introduce

Alessandro Lunghi, Yanhui Zhang, Michelangelo Domina

Continuous properties representation



Representing atomic distributions

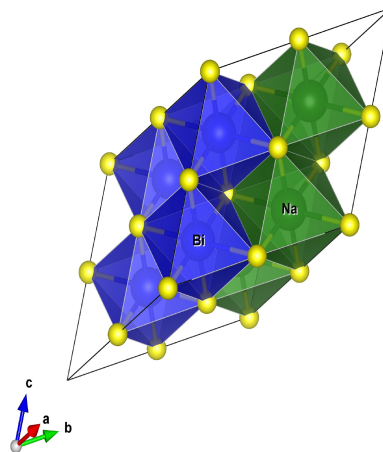


We need to distinguish different structures

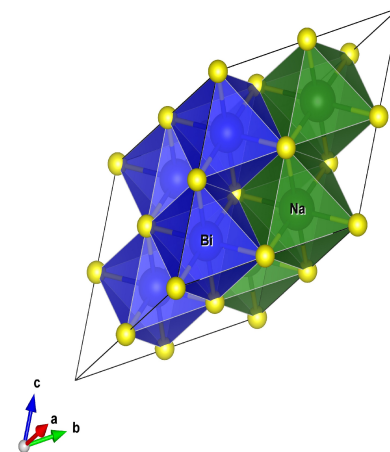
This is not trivial:

Space groups ?

No. 225



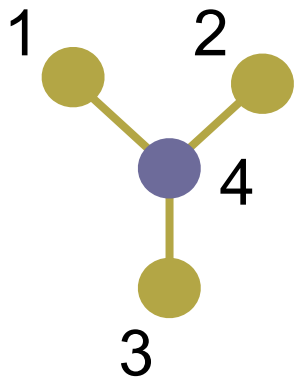
No. 166



Representing atomic distributions



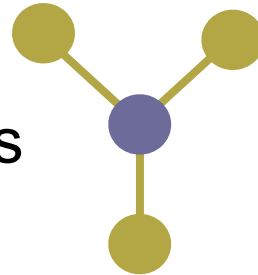
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Cartesian coordinates

$$\vec{v} = (\vec{x}_1, \vec{x}_2, \vec{x}_3, \vec{x}_4)$$

$$\vec{v} + \vec{w}$$

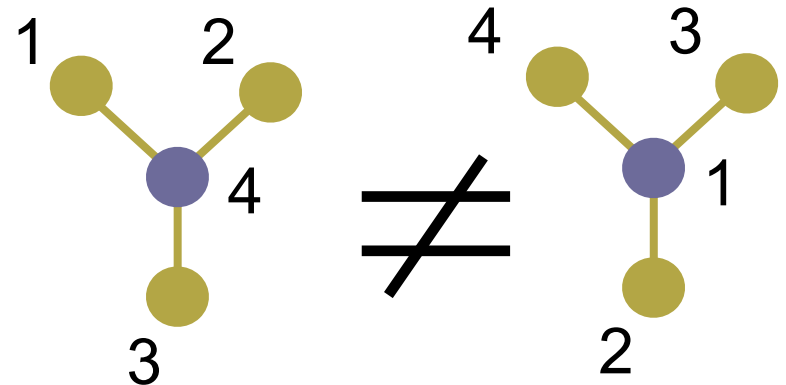


Invariant by translations

Locality fixes the problem

$$Q_{\text{local}}(\mathbf{r}^N) = \sum_{i=1}^N Q_i(\mathbf{r}'_i)$$

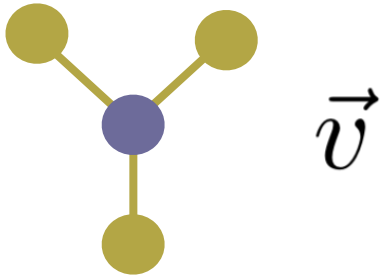
Locality fixes also invariance by permutation



Representing atomic distributions

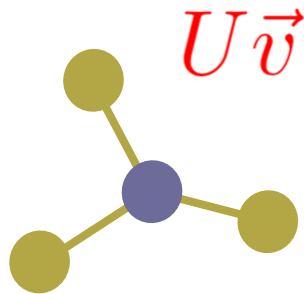


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\vec{v}

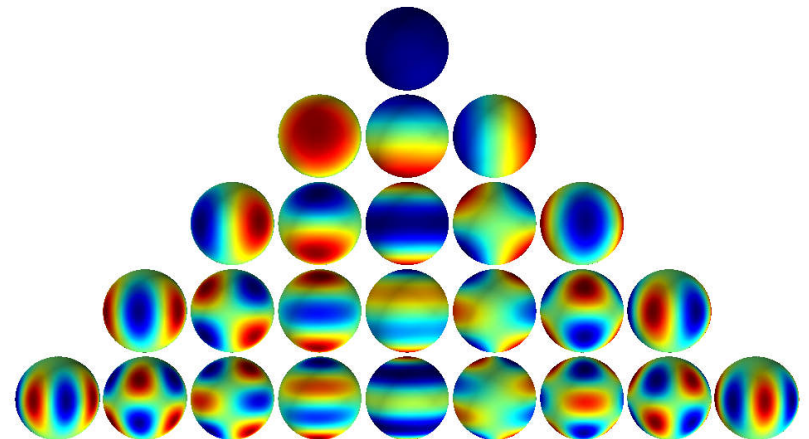
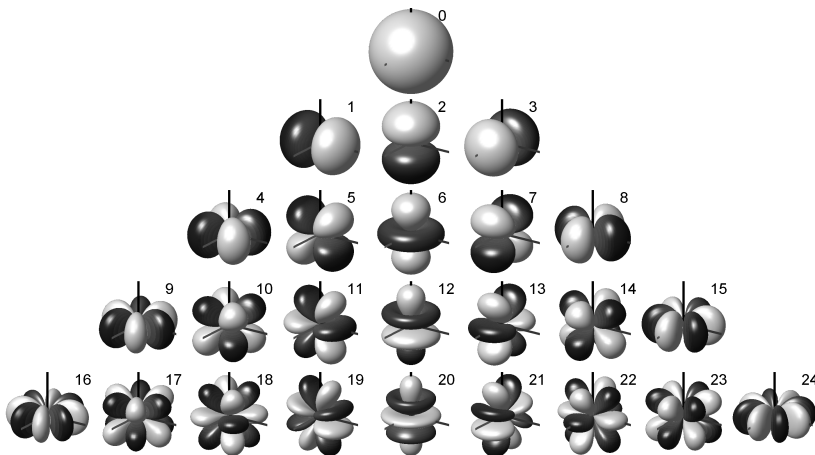
Invariant by rotations



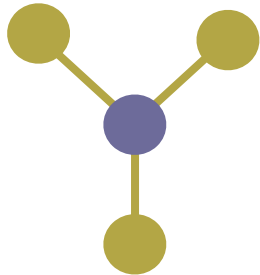
$U\vec{v}$

Needs to be built in

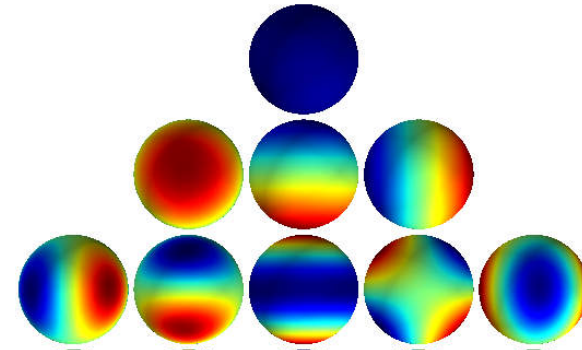
Usually spherical harmonics expansion



Representing atomic distributions



$$\rho(\mathbf{r}) = \sum_{\mathbf{i}} \alpha_{\mathbf{i}} \delta(\mathbf{r} - \mathbf{r}_{\mathbf{i}})$$



$$|\rho\rangle = \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} \sum_{m=-l}^l c_{nlm} |nlm\rangle$$

This is a rotational invariant
(*power spectrum*)

$$p_{nl} = \sum_{m=-l}^l |c_{nlm}|^2$$

Need continuous representation



Spectral Neighbour Analysis Potential (SNAP)

Clever representation of the local chemical environment

$$\rho_i(\mathbf{r}) = \delta(\mathbf{r}) + \sum_{r_{ii'} < R_{cut}} f_c(r_{ii'}) w_{i'} \delta(\mathbf{r} - \mathbf{r}_{ii'})$$

$$\rho(\mathbf{r}) = \sum_{j=0, \frac{1}{2}, \dots}^{\infty} \sum_{m=-j}^j \sum_{m'=-j}^j u_{m,m'}^j U_{m,m'}^j(\theta_0, \theta, \phi)$$

Rotationally invariant *descriptors*

$$B_{j_1, j_2, j} = \sum_{m_1, m'_1 = -j_1}^{j_1} \sum_{m_2, m'_2 = -j_2}^{j_2} \sum_{m, m' = -j}^j (u_{m, m'}^j)^* H_{j_1 m_1 m'_1}^{j m m'} H_{j_2 m_2 m'_2}^{j m m'} u_{m_1, m'_1}^{j_1} u_{m_2, m'_2}^{j_2}$$

Total energy linear in the *descriptors*

$$E(\mathbf{r}^N) = E_{ref}(\mathbf{r}^N) + E_{local}(\mathbf{r}^N)$$

$$E_{local}(\mathbf{r}^N) = \sum_{i=1}^N E_i(\mathbf{q}_i)$$

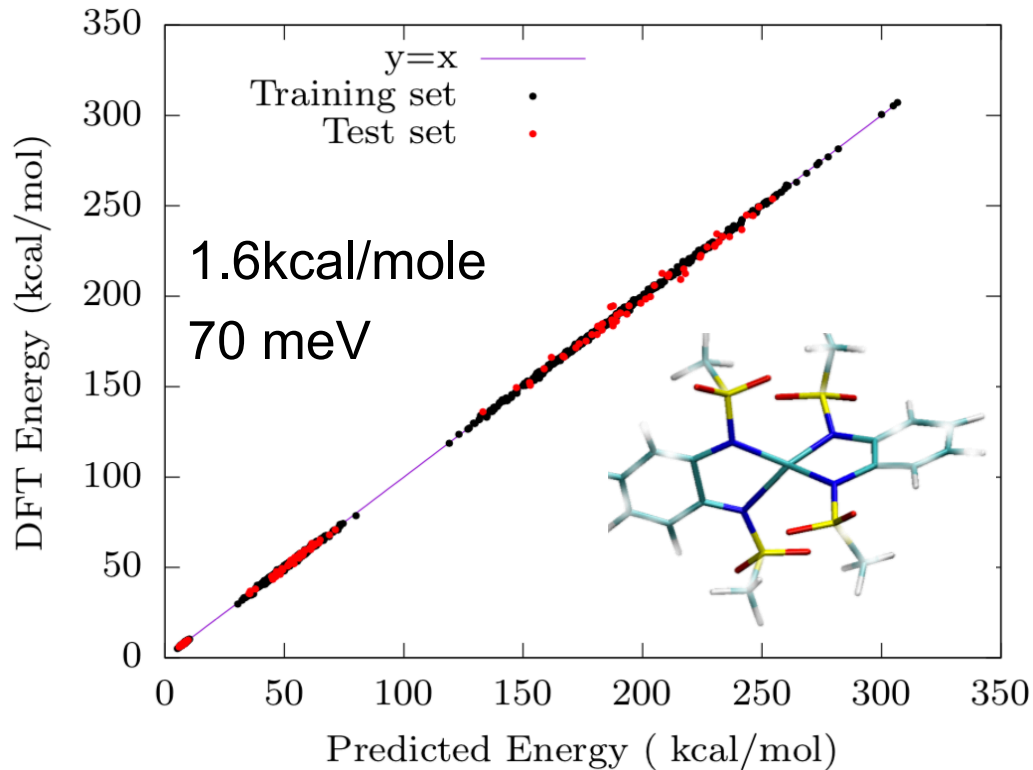
}

$$E_{SNAP}^i(\mathbf{B}^i) = \beta_0^{\alpha_i} + \sum_{k=1}^K \beta_k^{\alpha_i} B_k^i = \beta_0^{\alpha_i} + \boldsymbol{\beta}^{\alpha_i} \cdot \mathbf{B}^i$$

Total energy: $[\text{Co}(\text{pdms})_2]^{2-}$



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“Useful” configurations:
phonon, stress tensor,

First force field

New force field

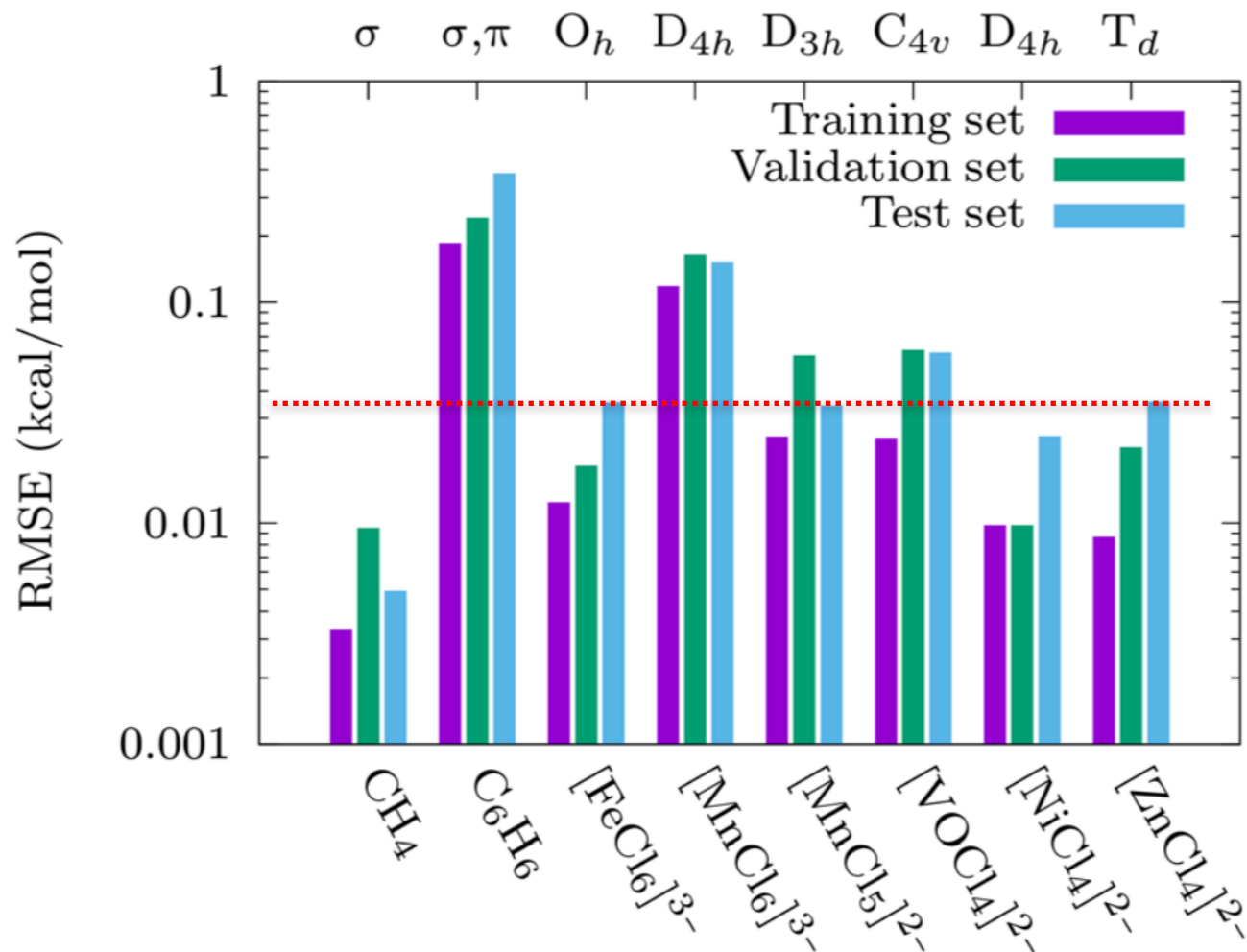
Monitor

MD at finite T:
new configurations

Phonon
spectrum

Phonon-
phonon
coupling

Total energy: agnostic of the bond nature

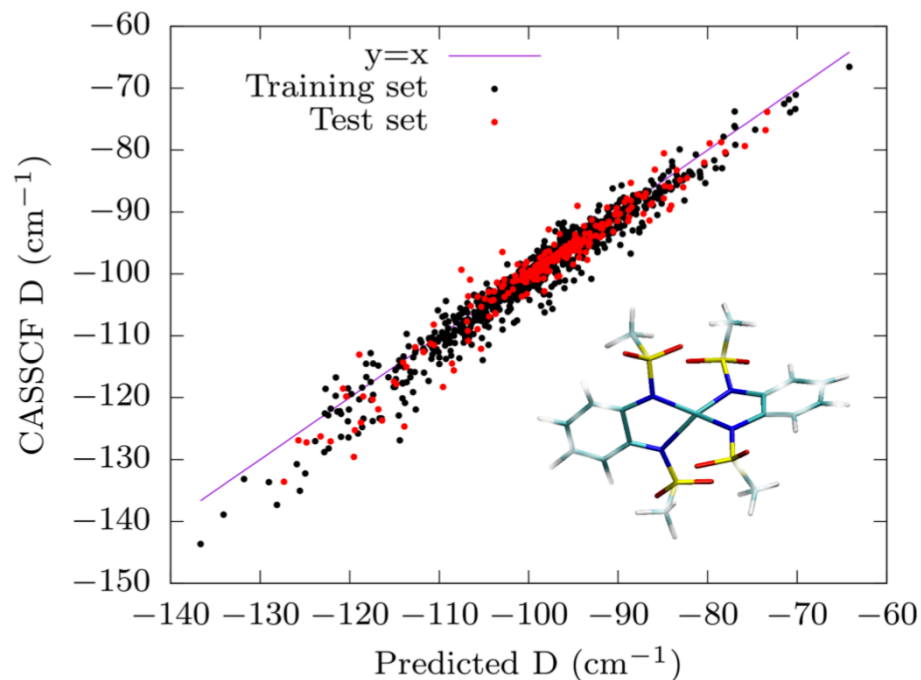


1kcal/mol = 40meV

Not just for total energy



$$\hat{H}_S = D\hat{S}_z$$

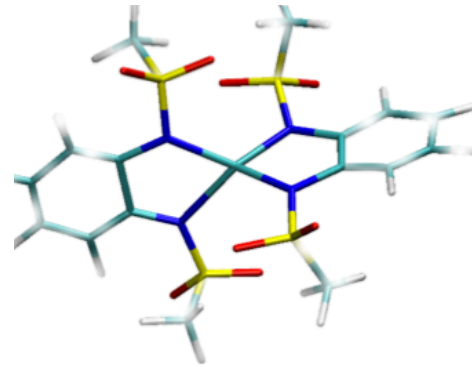
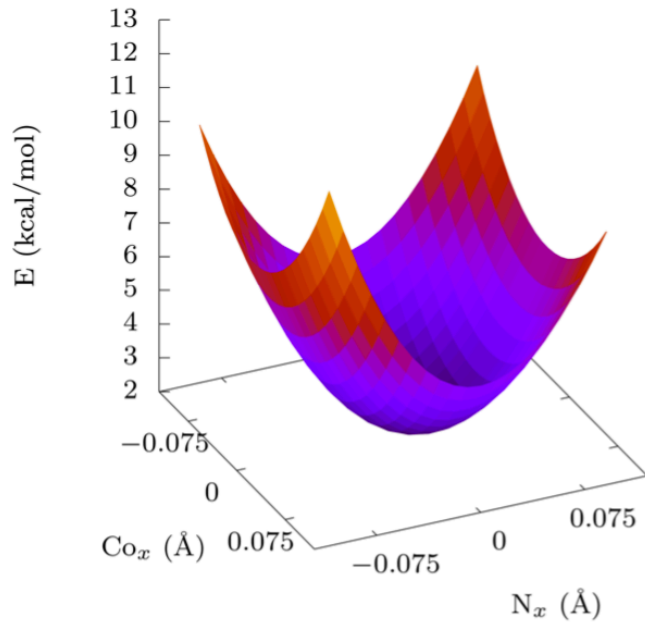


2.2 cm^{-1}

You can search in continuous surfaces!

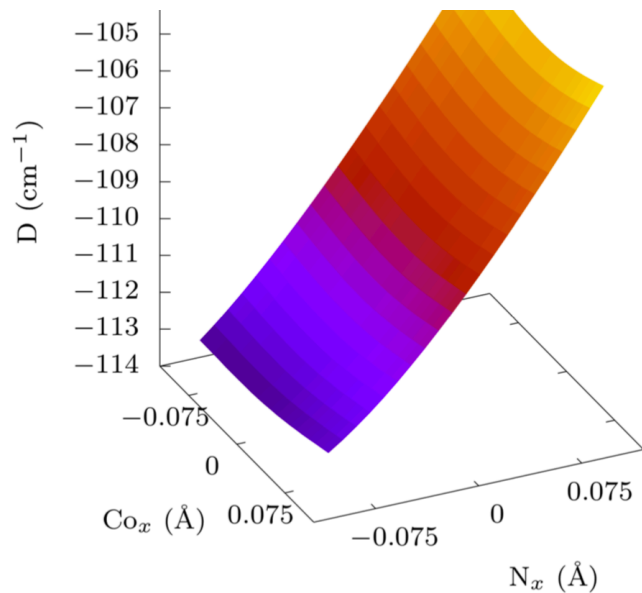


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5% reduction of: d_{Co-N}
 $N-Co-N$

50% increase of D



$$H(q_\alpha) = E(q_\alpha) + \lambda D(q_\alpha)$$

What is this useful for?



Very accurate atomic models possible

Useful for finite temperature

Mainly energy but other quantities possible

Only moderately large training set needed

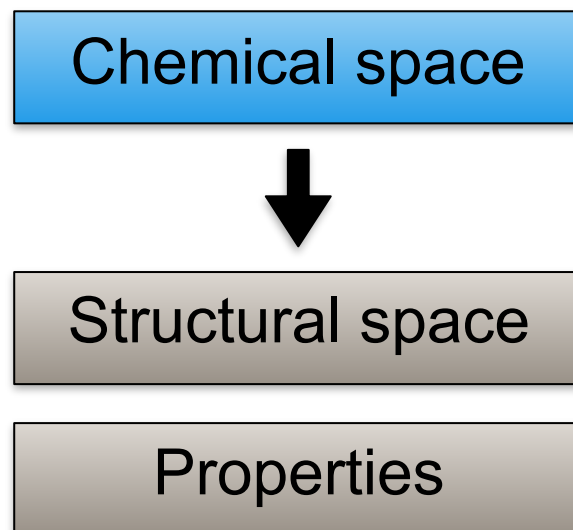
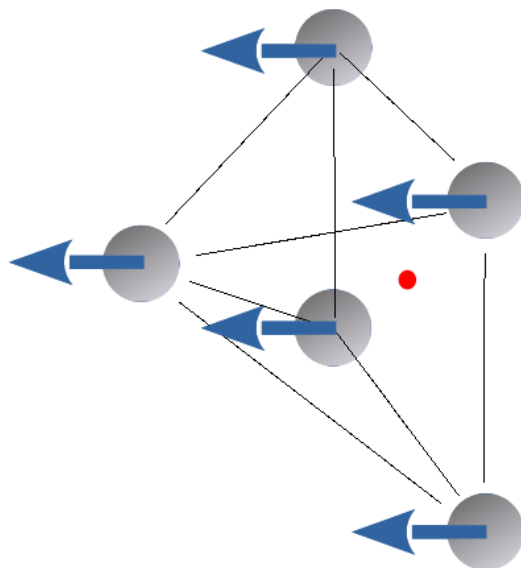
Much better for molecules than solids

High risk of overfitting

Not really interpretable

Michelangelo Domina, Matteo Cobelli

ML potential for vector fields



Extension to spin



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The idea is to construct a force-field, which depends on a vector field

$$\boldsymbol{\rho}(\mathbf{r}) = \sum_i \alpha_i \delta(\mathbf{r} - \mathbf{r}_i) \mathbf{s}_i$$

Then expand

$$|\boldsymbol{\rho}\rangle = \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} \sum_{m=-l}^l \sum_{q=-1}^1 c_{nlmq} |nlmq\rangle$$

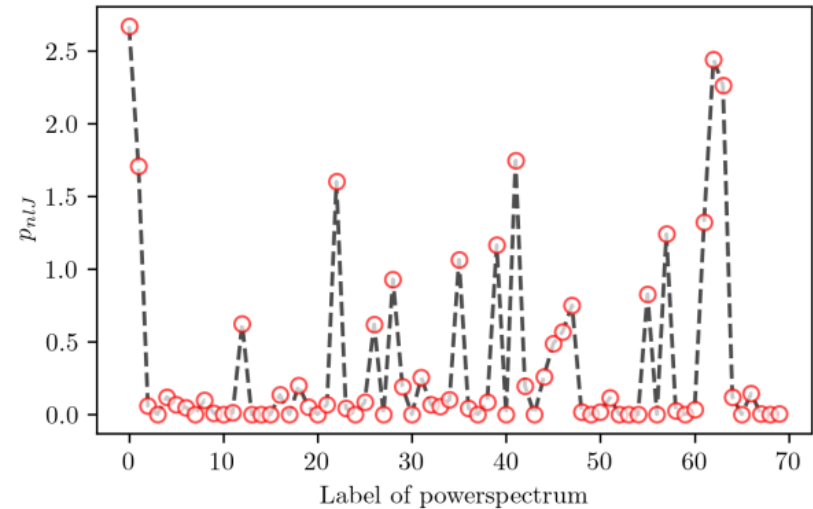
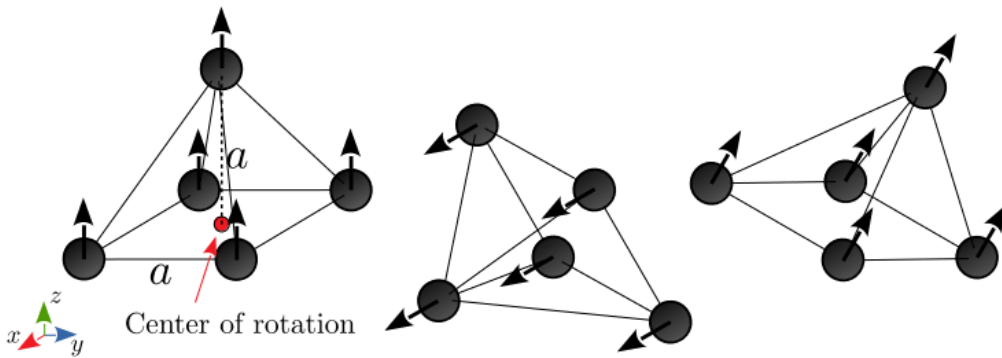
In position representation: $g_n(r)$ $Y_l^m(\hat{\mathbf{r}})$ $\hat{\mathbf{e}}_q$

$$p_{nlJ} = \sum_M |u_{nlJM}|^2$$

Extension to spin



$$p_{nlJ} = \sum_M |u_{nlJM}|^2 \text{ is fully rotational invariant}$$



The $l=0$ term is the Heisenberg model

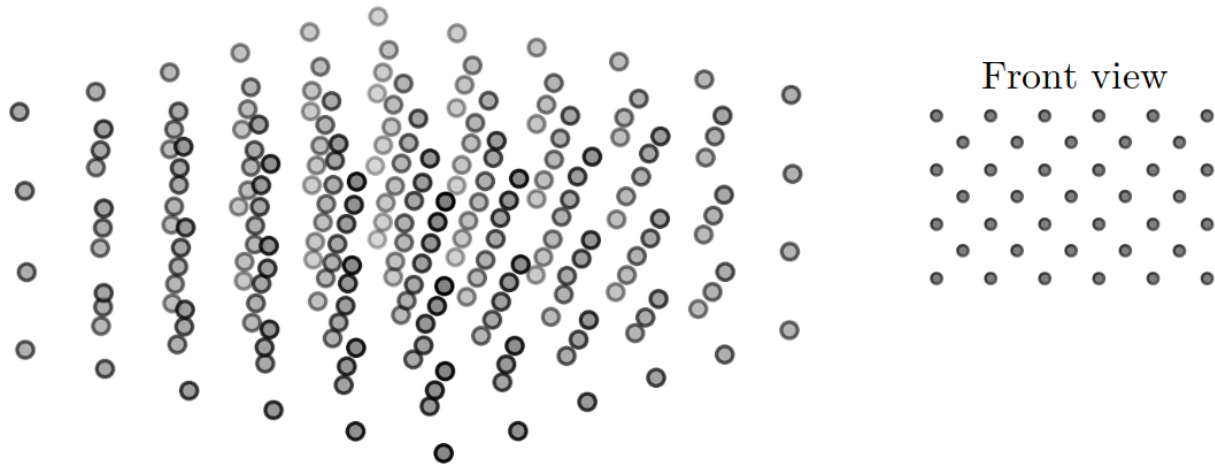
$$p_{n0J} \propto \sum_{ab} \alpha_a \alpha_b g_{n0}(r_a) g_{n0}(r_b) \mathbf{S}_a \cdot \mathbf{S}_b$$

Test against simple models



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$$E \simeq \boldsymbol{\theta} \cdot \sum_i \mathbf{p}^{(i)} = \sum_{nlJ} \theta_{nlJ} \sum_i p_{nlJ}^{(i)}$$



$$H_H = -\frac{1}{2} \sum_{\langle i,j \rangle} J_{ij}(r_{ij}) \mathbf{S}_i \cdot \mathbf{S}_j \quad \longrightarrow \quad \text{Transverse excitations}$$

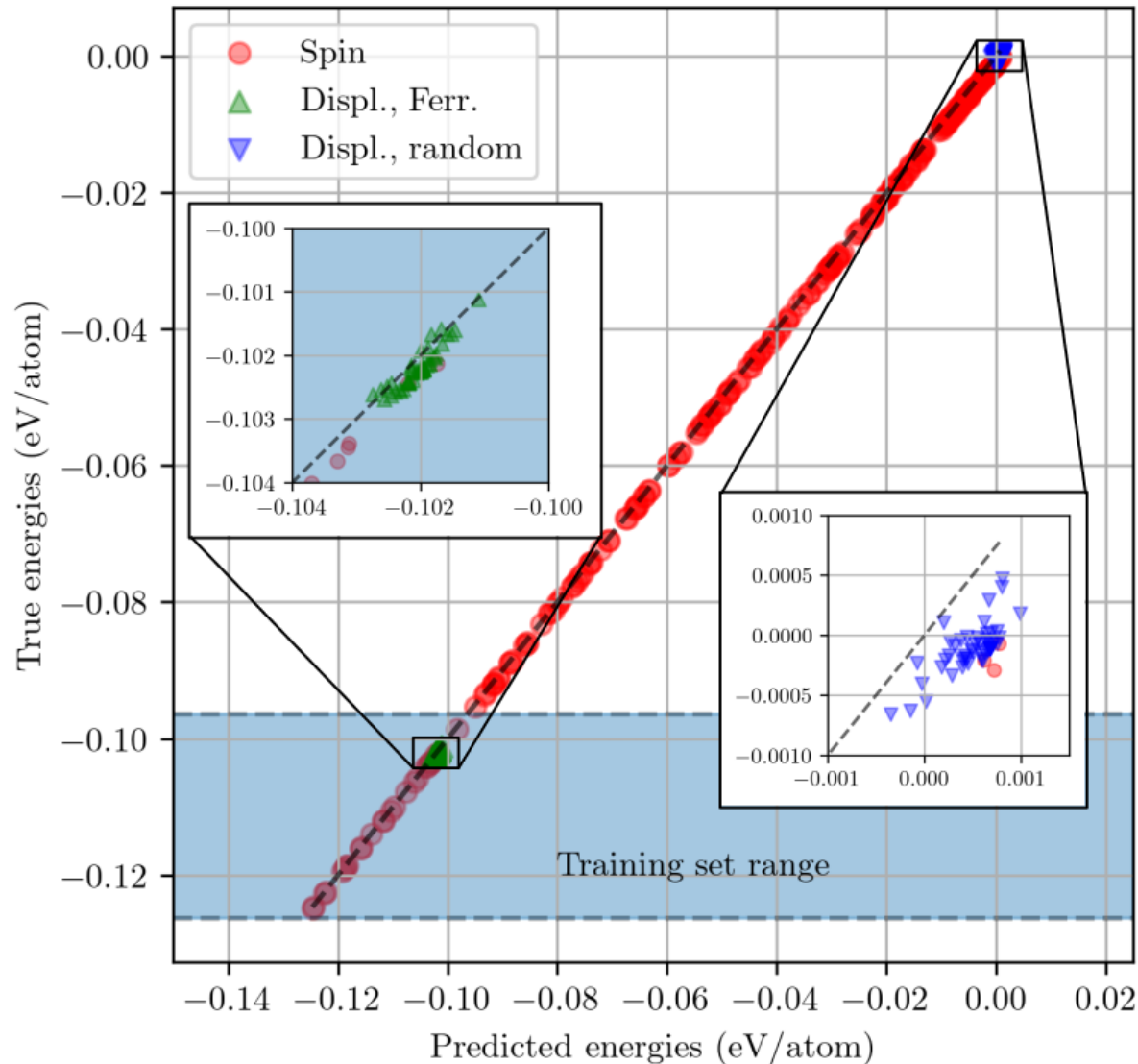
$$H_L = \sum_i (AS_i^2 + BS_i^4 + CS_i^6) \quad \longrightarrow \quad \text{Longitudinal excitations}$$

Test against simple models: H_H



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Predictions (H_H), Ferromagnetic training, 10%



E error
~0.1%

35 features

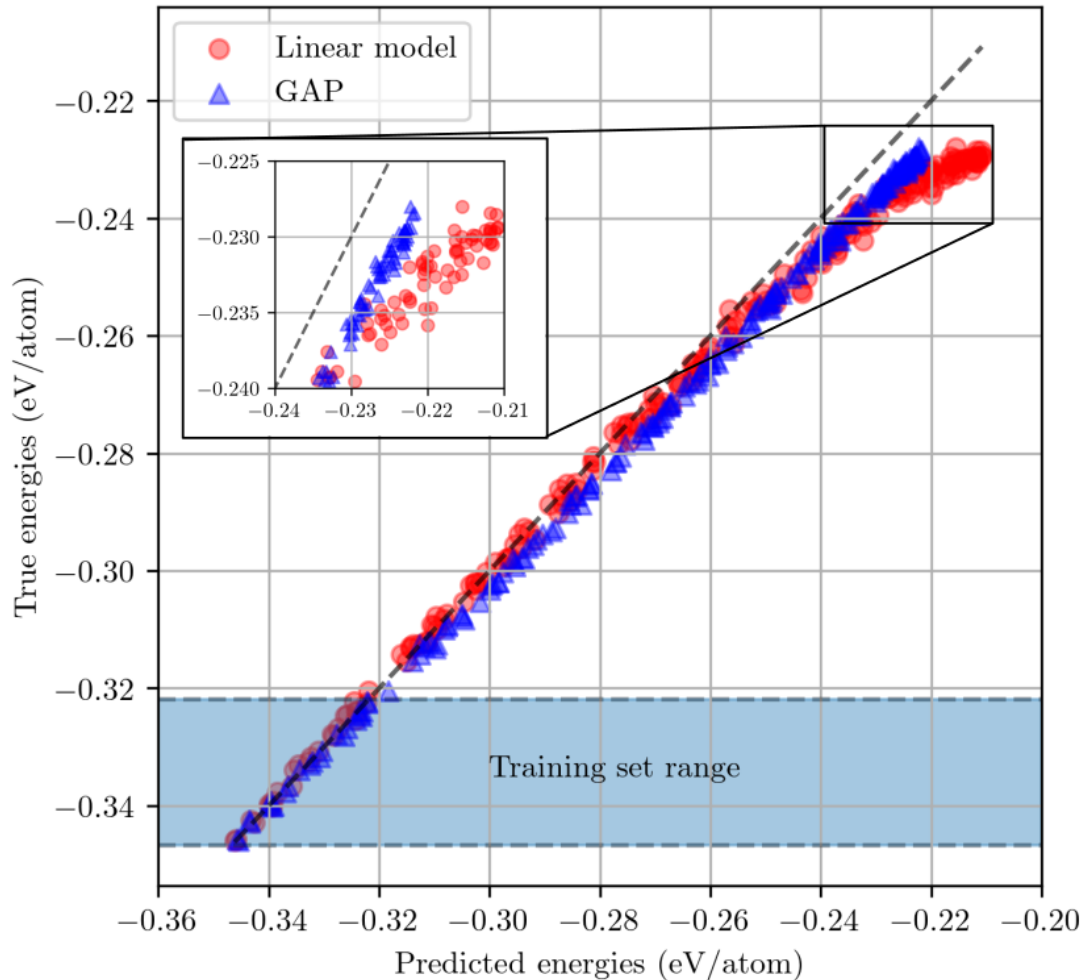
~200 training
data

Test against simple models: $H_H + H_L$



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Predictions, Full Hamiltonian



GAP ML model

$$\varepsilon_i = \sum_{t=0}^{N_{\text{train}}} \theta_t S(\mathbf{p}^i, \mathbf{p}^t) = \sum_t \theta_t e^{-\frac{1}{2\sigma}(\mathbf{p}^i - \mathbf{p}^t)^2}$$

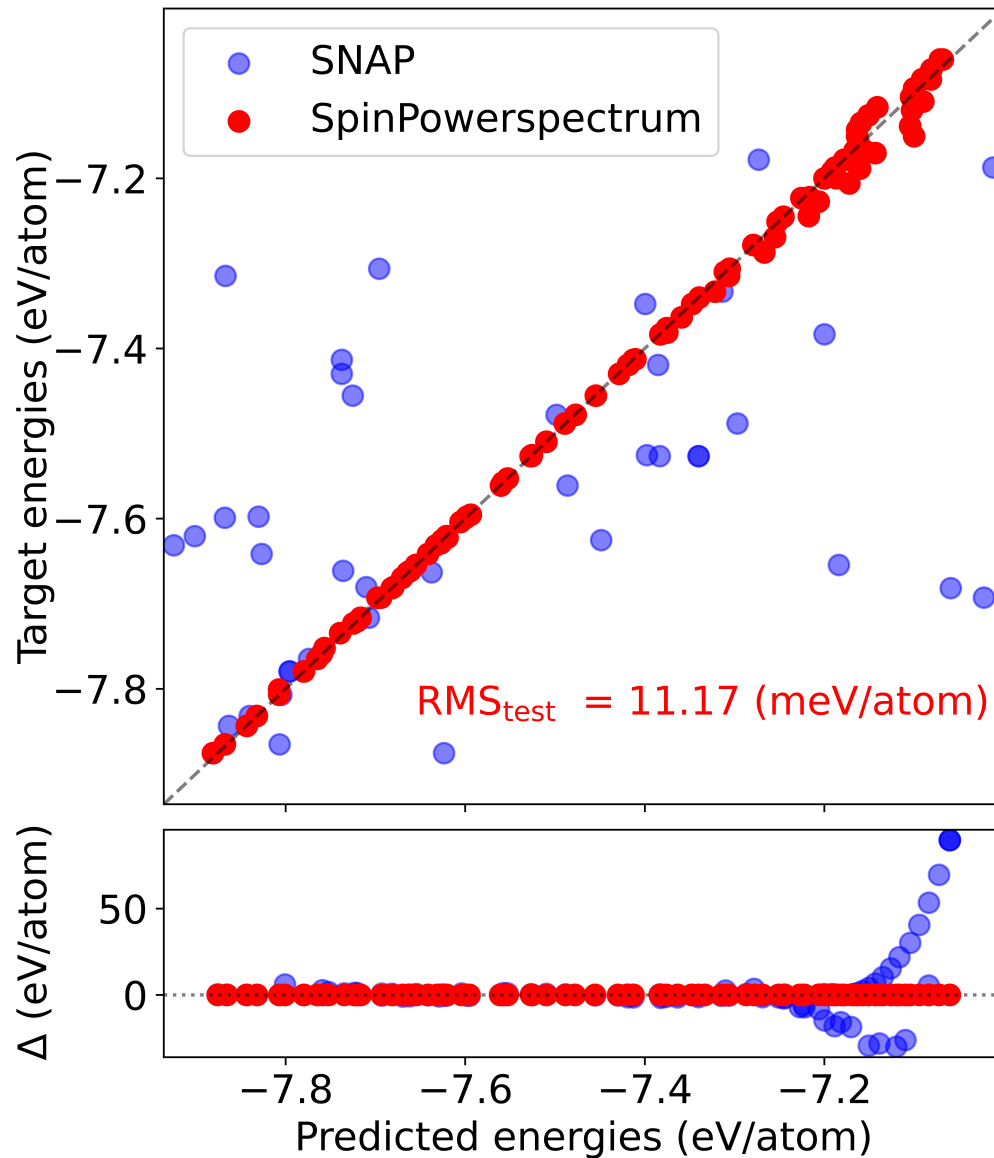
Test against simple models



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Now fitting ab initio data

α -(bcc)- ϵ (hcp)





Extension of FF to vector fields possible

Very general and agnostic of the PES

Formalism extendible to bi-spectrum

Small training sets needed (so far)

Concept extendable to other FF classes

Need non-linear model to go beyond Heisenberg

Constructing dataset is complicated

