

Geometry-governed effects in curvilinear magnetism

Novel physical effects caused by tailoring geometry

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Nanomagnetism in 3D

SPICE Workshop

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WASEM Monastery, Ingelheim, Germany



Overview of curvilinear magnetism

Curvilinear magnetism

[Streubel, Fischer, Kronast, Kravchuk, Sheka, Gaididei, Schmidt, Makarov, J Phys D (2016)]

Reviews on curvilinear magnetism

[Fernández-Pacheco, Streubel, Fruchart, Hertel, Fischer, Cowburn, Nature Comm (2017)]

[Streubel, Tsymbal, Fischer, JAP (2021)]
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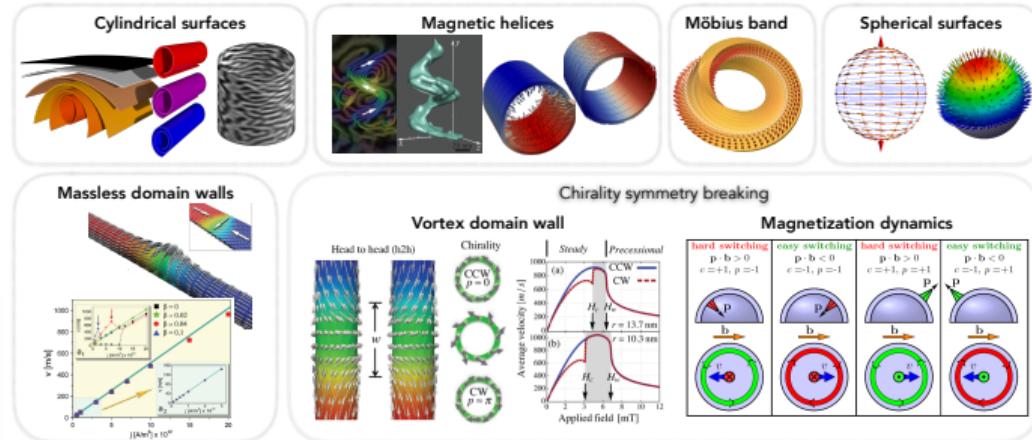
[Sheka, Pylypovskiy, Volkov, Yershov, Kravchuk, Makarov, Small (2022)]

[Makarov, Volkov, Kákay, Pylypovskiy, Budinská, Dobrovolskiy, Advanced Materials (2022)]

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[Makarov, Sheka, Curvilinear Micromagnetism: From Fundamentals to Applications, Springer (2022)]

<https://doi.org/10.1007/978-3-031-09086-8>



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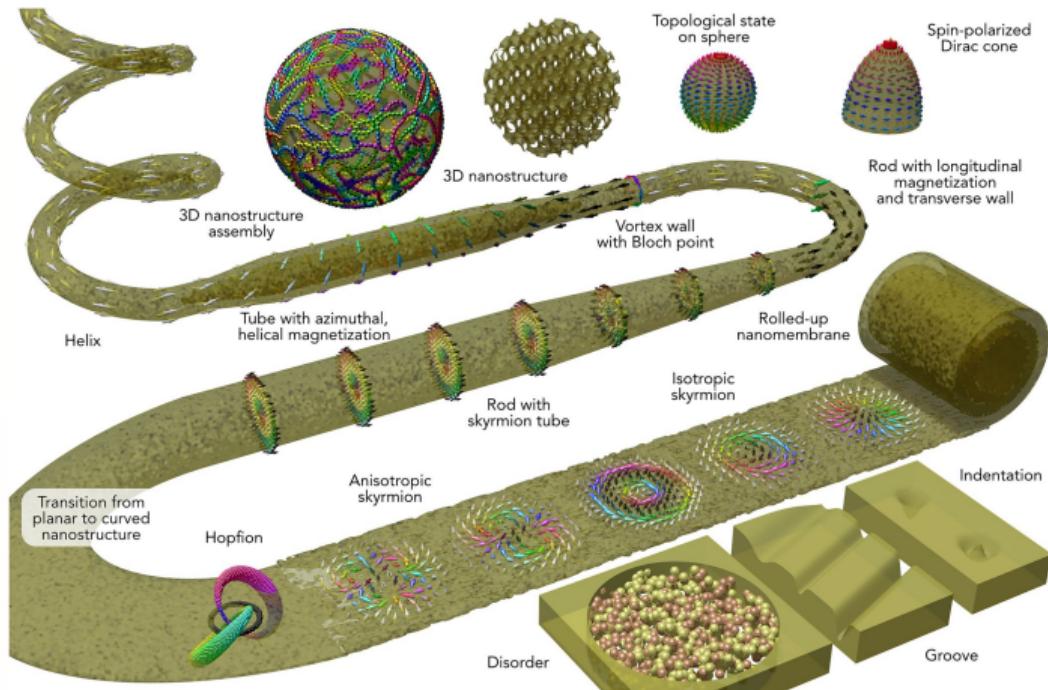
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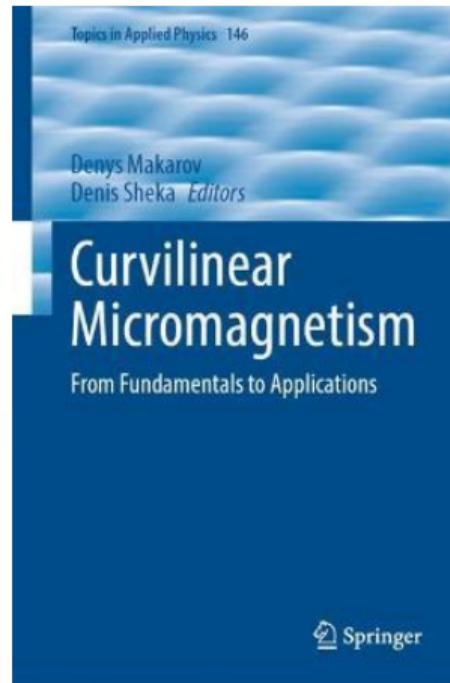
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- **Flat magnet**

⇒ spatial inversion symmetry

- **Curved magnet**

✓ Local geometrical properties

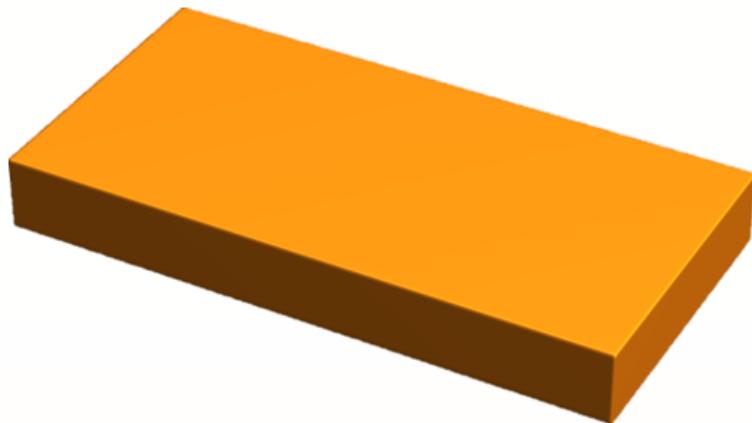
⇒ geometry-governed interactions

⇒⇒ geometry-governed anisotropy

⇒⇒ geometry-governed chiral interactions

⇒ Geometrical magnetochiral effects

Flat film



✓ Global geometrical properties

⇒ Topology of curved magnets

⇒⇒ Topological patterning

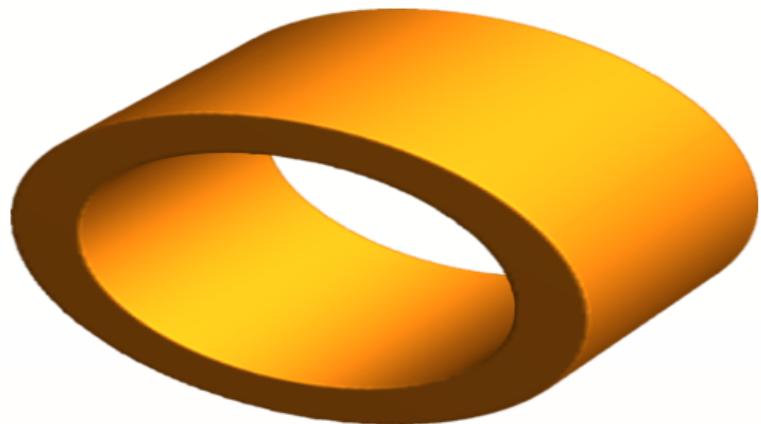
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Curved film: **geometry**



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Curved film: topology



Impact of curvature on nanomagnets

Typical magnetic interactions: minimal model

$$E = \int d^3x \left[\underbrace{\mathcal{E}^X}_{\text{exchange}} + \underbrace{\mathcal{E}^A}_{\text{anisotropy}} \right] + \underbrace{\mathcal{E}^{\text{DMI}}}_{\text{Dzyaloshinskii-Moriya interaction}} + \underbrace{\mathcal{E}^{\text{MS}}}_{\text{magnetostatics}} \right]$$

- Ginzburg-Landau functional for the vector order-parameter

⇒ General approach for various condensed matter systems:

- Magnets
- Liquid crystals
- Superconductors

- The curvature gets into the game due to the geometry-dependent anisotropy

$$\vec{\epsilon}_{\text{an}} = \vec{\epsilon}_{\text{an}}(x, y, z)$$

- Strong anisotropy ⇒ Curvilinear reference frame

Impact of curvature on nanomagnets

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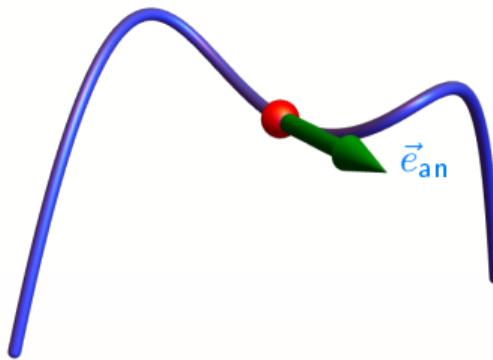
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\mathcal{E}^{DMI} Dzyaloshinskii–Moriya interaction + \mathcal{E}^{MS} magnetostatics

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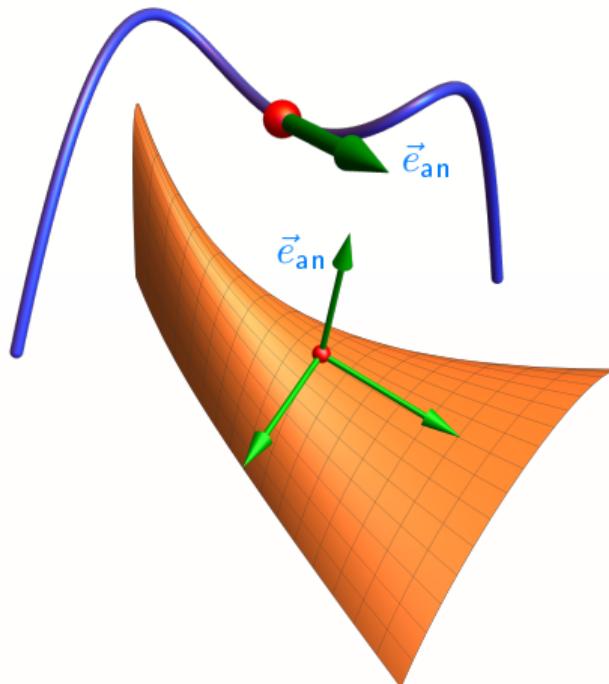
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Minimal model of a curved magnet: exchange driven effects

Exchange energy:

$$\mathcal{E}^x = \underbrace{\mathcal{A} (\vec{\nabla} m_i) (\vec{\nabla} m_i)}_{\text{Cartesian frame}} = \underbrace{\mathcal{E}_0^x + \mathcal{E}_A^x + \mathcal{E}_D^x}_{\text{Curvilinear frame}}$$

$\mathcal{E}_0^x = \mathcal{A} (\partial_\alpha m_\nu) (\partial_\alpha m_\nu)$ is a 'common' exchange
 $\partial_\alpha m_\nu$ is covariant derivative

$\nu, \mu = 1, 2, 3$
 $\alpha, \beta = 1, 2$

Geometry-governed Anisotropy

$\mathcal{E}_A^x = K_{\mu\nu} m_\mu m_\nu$ κ_1, κ_2 are principle curvatures
 $K_{\mu\nu} = \mathcal{A} \text{ diag } (\kappa_1^2, \kappa_2^2, \kappa_1^2 + \kappa_2^2)$ are coefficients of effective biaxial anisotropy

Geometry-governed Dzyaloshinskii–Morita Interaction (DMI)

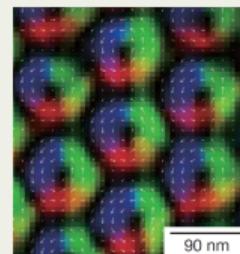
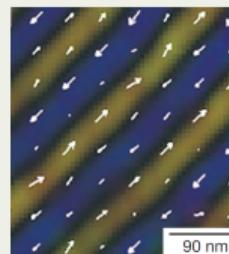
$$\mathcal{E}_D^x = \mathcal{A} \kappa_1 L_{13}^{(1)} + \mathcal{A} \kappa_2 L_{23}^{(2)}$$

$L_{\mu\nu}^{(\alpha)} = m_\mu \partial_\alpha m_\nu - m_\nu \partial_\alpha m_\mu$ is a curvilinear-geometry analogue of Lifshitz invariants

Lifshitz invariants come from spin-orbit coupling, leading to helical structures and skyrmion crystals

[Bogdanov and Röbler, PhysRevLett (2001)]

[Yu, Onose, Kanazawa, Park, Han, Matsui, Nagaosa, Tokura, Nature (2010)]



[Gaididei, Kravchuk, Sheka, PhysRevLett (2014); Sheka, Kravchuk, Gaididei, JPhysA (2015)]

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 is a curvilinear-geometry analogue of Lifshitz invariants

Curvilinear geometry as a **source** of emergent
Dzyaloshinskii–Moriya interaction

[Gaididei, Kravchuk, Sheka, PhysRevLett (2014); Sheka, Kravchuk, Gaididei, JPhysA (2015)]

Analogy with mechanics

Magnetic system in a curved frame

$$\mathcal{E} = \mathcal{E}_0 + \underbrace{K_{\alpha\beta} m_\alpha m_\beta}_{\text{Anisotropy}} + \underbrace{\mathcal{A} \kappa_\alpha (m_\alpha \vec{\partial}_\alpha m_3 - m_3 \vec{\partial}_\alpha m_\alpha)}_{\text{Dzyaloshinskii-Moriya interaction}}$$

Mechanical particle in a non inertial frame

$$E = E_0 + \underbrace{\frac{m}{2} [\vec{\Omega} \times \vec{r}]^2}_{\text{Centrifugal}} + \underbrace{m \dot{\vec{r}} \cdot [\vec{\Omega} \times \vec{r}]}_{\text{Coriolis}}$$

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Centrifugal force in action



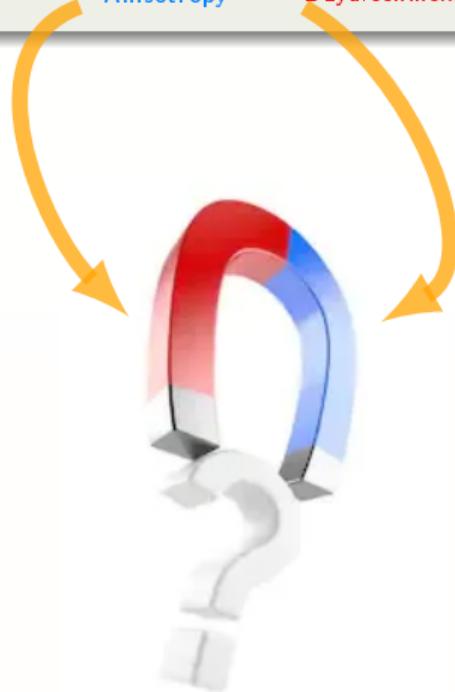
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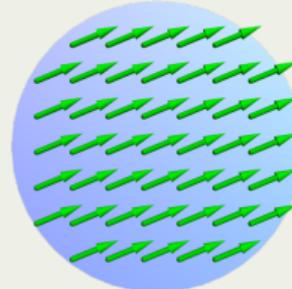
Coriolis force in action



Topological patterning

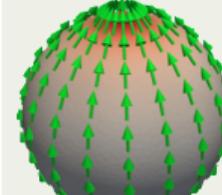
Formation of **topologically protected textures** in curved geometries

Flat Film

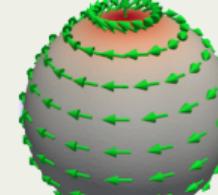


Spherical shell

3D onion



Whirligig



Nanotube



Toroidal shell

Hyperboloid shell

Möbius ring

Topological patterning

Formation of **topologically protected textures** in curved geometries

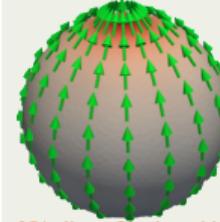
Hairy Ball Theorem

You cannot comb a hairy ball flat without creating a cowlick [Poincaré (1885)]

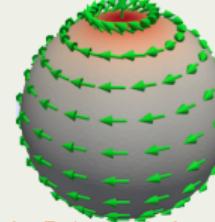


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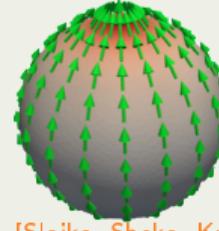
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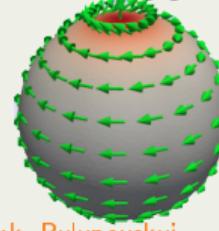


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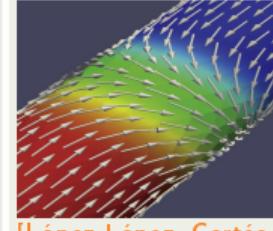


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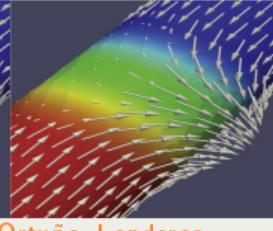


Nanotube

Vortex wall



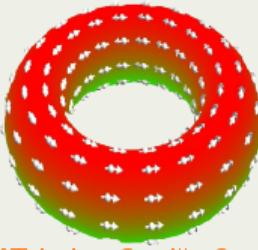
Transversal wall



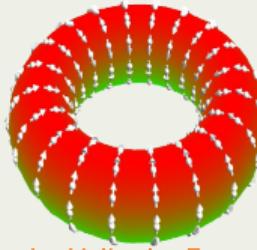
[López-López, Cortés-Ortuño, Landeros, JMMM (2012)]

Toroidal shell

Toroidal vortex

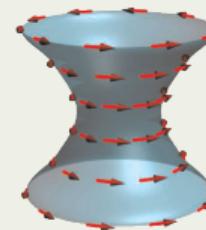


Poloidal vortex

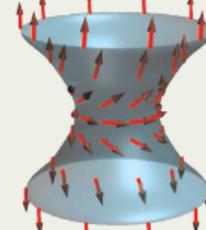


Hyperboloid shell

Vortex

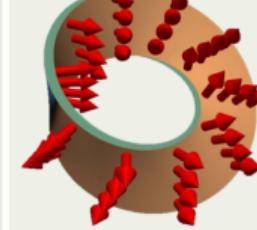


Skyrmion



Möbius ring

t -wall



ℓ -wall



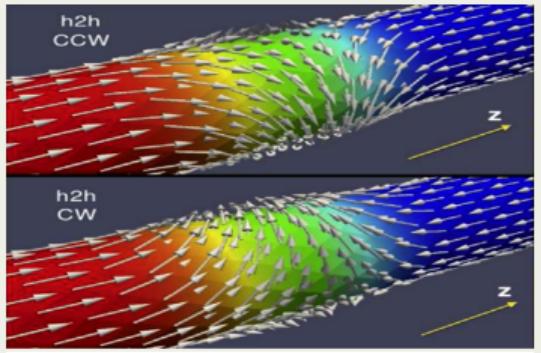
[Teixeira, Castillo-Sepúlveda, Vojkovic, Fonseca, Altbir, Núñez, Carvalho-Santos, JMMM (2019)]

[Carvalho-Santos, Elias, Altbir, Fonseca, JMMM (2015)]

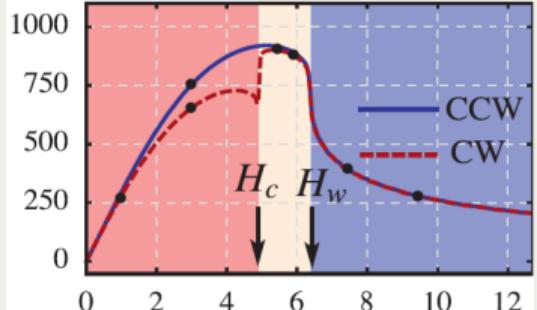
[Pylypovskiy, Kravchuk, Sheka, Makarov, Schmidt, Gaididei, PRL (2015)]

Geometrical magnetochiral effects: pattern-induced chirality breaking

Chiral symmetry breaking in moving vortex domain wall



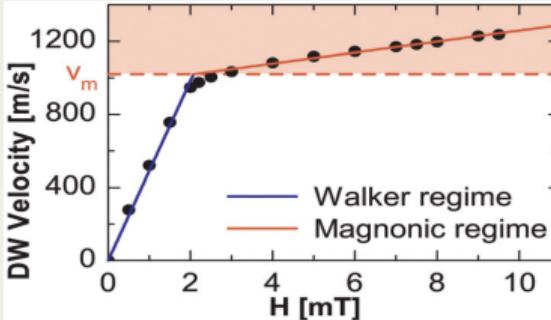
$$R = 17.3 \text{ nm}, r = 13.7 \text{ nm}$$



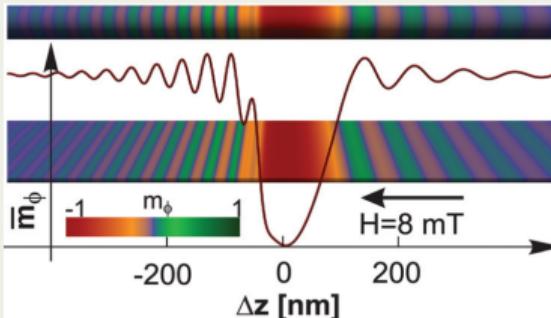
[Landeros, Núñez, JAP (2010)]

Ultrafast domain walls in nanotubes

Suppression of Walker breakdown



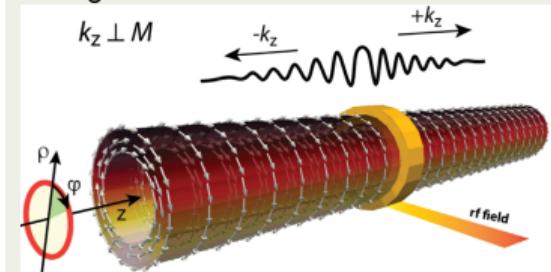
Spin-Cherenkov effect



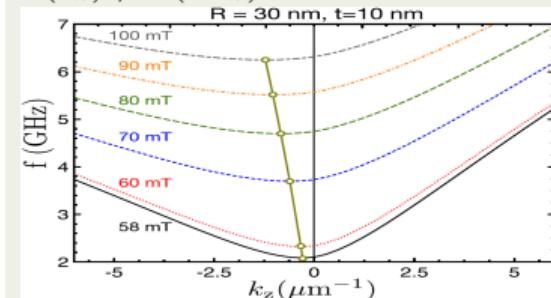
[Yan, Andreas, Kákay, García-Sánchez, Hertel, APL (2011)]

Chiral spin-wave dynamics

Spin waves on a vortex state background



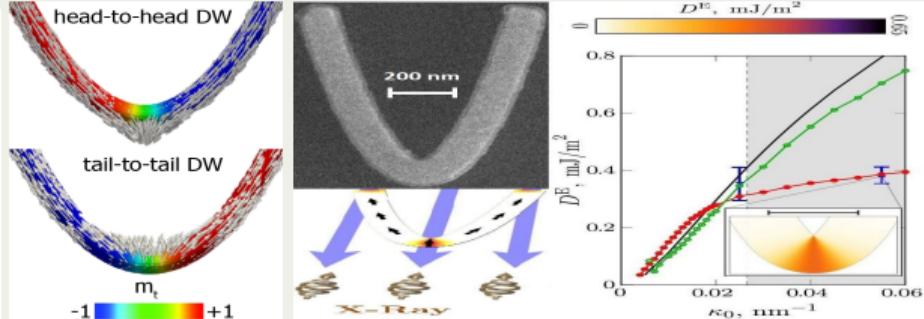
Asymmetric dispersion law
 $\omega(k_z) \neq \omega(-k_z)$



[Otálora, Yan, Schultheiss, Hertel, Kákay, PRL (2016); PRB (2017)]

Geometrical magnetochiral effects: geometry-induced chirality breaking

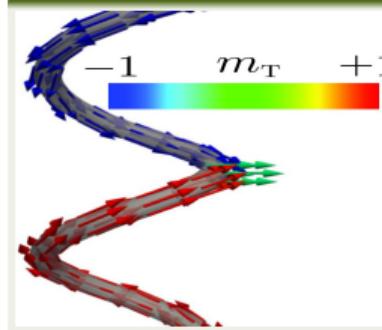
Flat curved stripes: domain wall pinning



[Yershov, Kravchuk, Sheka, Gaididei, PRB (2015)]

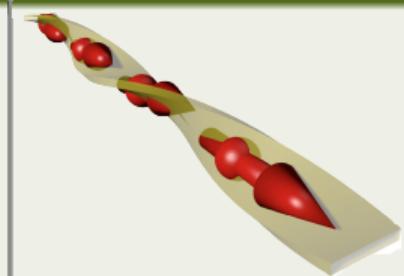
[Volkov, Kákay, Kronast, Mönch, Mawass, Fassbender, Makarov, PRL (2019)]

Helix wire



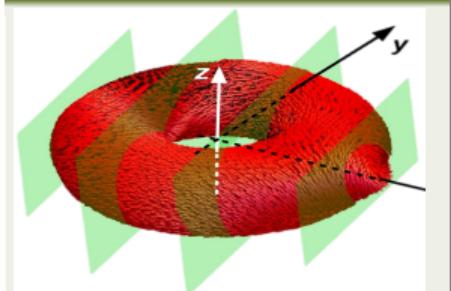
[Yershov, Kravchuk, Sheka, Gaididei, PRB (2016)]

Helicoid ribbon



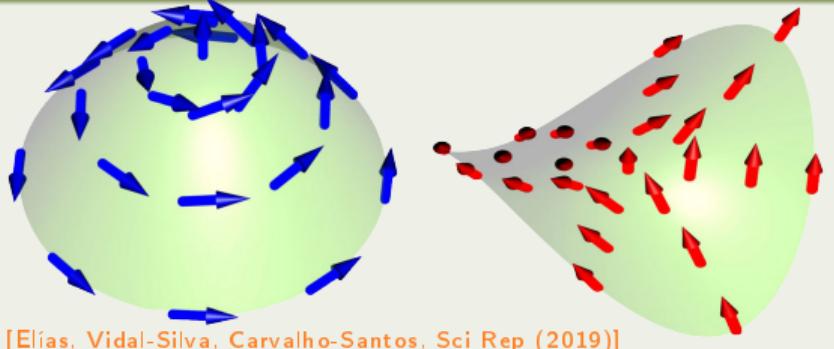
[Gaididei, Goussev, Kravchuk, Pylypovskiy, Robbins, Sheka, Slastikov, Vasylkevych, JPA (2017)]

Toroidal shell



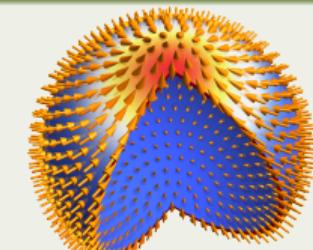
[Vojkovic, Carvalho-Santos, Fonseca, Nunez, JAP (2017)]

Parabolic and Hyperbolic shells



[Elías, Vidal-Silva, Carvalho-Santos, Sci Rep (2019)]

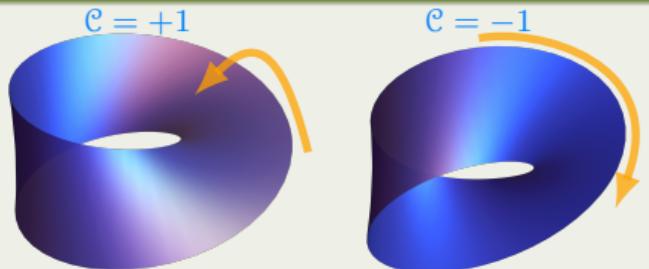
Skyrmion on a spherical shell



[Kravchuk, Rößler, Volkov, Sheka, van den Brink, Makarov, Fuchs, Fangohr, Gaididei, PRB (2016)]

Coupling between geometrical chirality \mathcal{C} and the magnetochirality \mathfrak{C}

Möbius geometrical chirality



Topologically protected domain wall

Chiral symmetry breaking

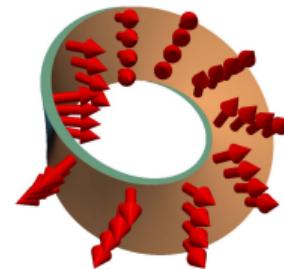
t -wall energy gain $\Delta E \propto \mathcal{C} \cdot \mathfrak{C}$

$m_{\text{thickness}} = \mathfrak{C} = \pm 1$ (inward or outward)

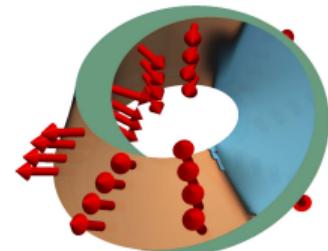
● Vortex state



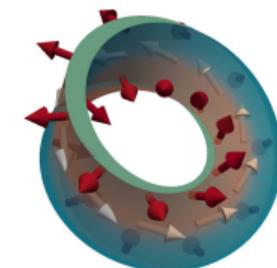
□ t -wall



▼ Three t -walls



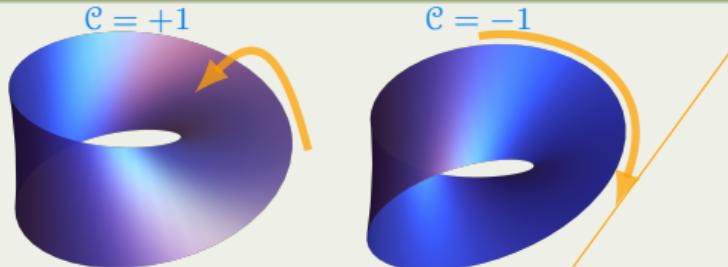
■ ℓ -wall



[Pylypovskiy, Kravchuk, Sheka, Makarov, Schmidt, Gaididei, PhysRevLett (2015)]

Coupling between geometrical chirality \mathcal{C} and the magnetochirality \mathfrak{C}

Möbius geometrical chirality



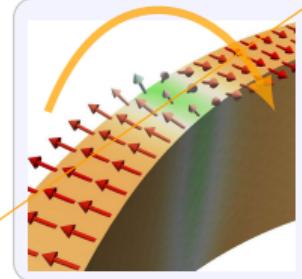
Topologically protected domain wall

Chiral symmetry breaking

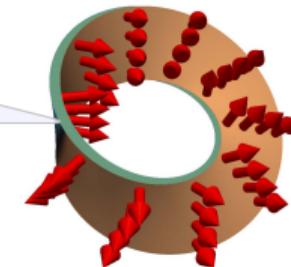
$t\text{-wall energy gain } \Delta E \propto \mathcal{C} \mathfrak{C}$

$m_{\text{thickness}} = \mathfrak{C} = \pm 1$ (inward or outward)

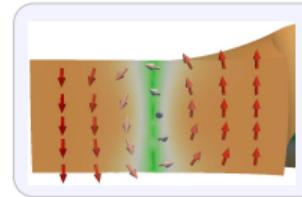
$t\text{-wall (details)}$



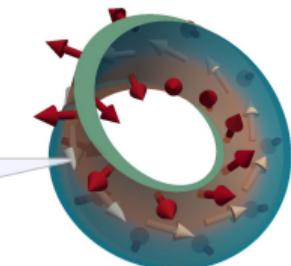
$t\text{-wall}$



$\ell\text{-wall (details)}$



$\ell\text{-wall}$



[Pylypovskiy, Kravchuk, Sheka, Makarov, Schmidt, Gaididei, PhysRevLett (2015)]

Geometrical magnetochiral effects: geometry-induced chirality breaking

Coupling between geometrical chirality \mathcal{C} and the magnetochirality \mathcal{E}

Co Möbius (FEBID)



[Skoric, Sanz-Hernández, Meng, Donnelly, Merino, Aceituno, Fernández-Pacheco, Nano Lett (2020)]

Topologically protected domain wall

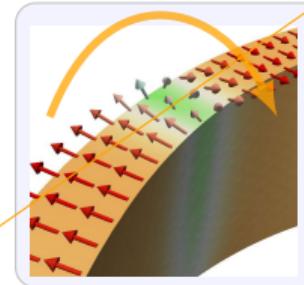
Chiral symmetry breaking

$$t\text{-wall energy gain } \Delta E \propto \mathcal{C} \mathcal{E}$$

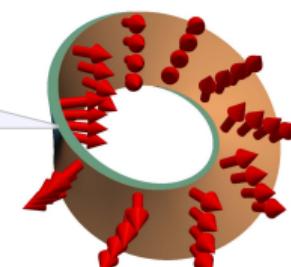
$$m_{\text{thickness}} = \mathcal{E} = \pm 1 \text{ (inward or outward)}$$

[Pylypovskiy, Kravchuk, Sheka, Makarov, Schmidt, Gaididei, PhysRevLett (2015)]

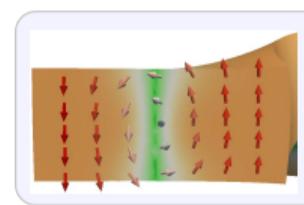
t -wall (details)



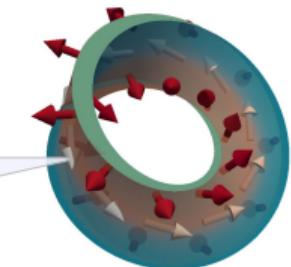
t -wall



ℓ -wall (details)



ℓ -wall



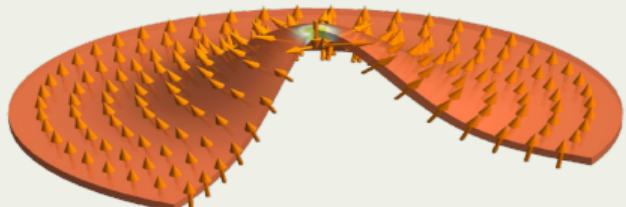
Geometrical magnetochiral effects: exchange-driven DMI

$$\text{Geometry-governed DMI: } \mathcal{E}_D^X = \mathcal{A}\kappa_1 L_{13}^{(1)} + \mathcal{A}\kappa_2 L_{23}^{(2)}$$

Small radius skyrmions: theory

Radius R is determined by \mathcal{H}

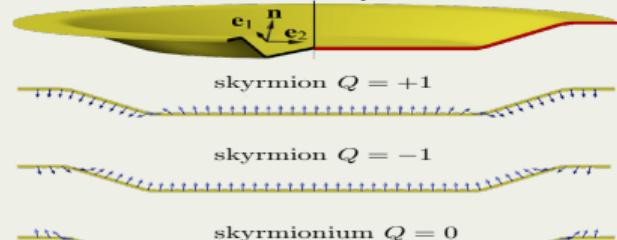
$\mathcal{H} = \kappa_1 + \kappa_2$ is a mean curvature



[Kravchuk, Sheka, Kákay, Volkov, Röbler, van den Brink, Makarov, Gaididei, PRL (2018)]

Tunable radius skyrmion: theory

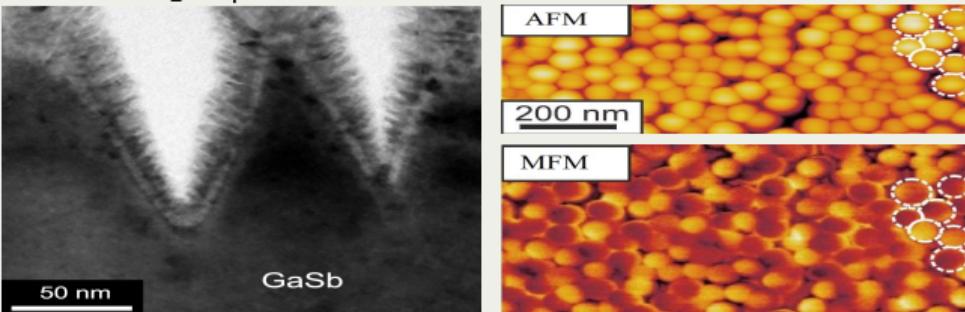
Radius R is determined by $\nabla \mathcal{H}$



[Pylypovskiy, Makarov, Kravchuk, Gaididei, Saxena, Sheka, PR Applied (2018)]

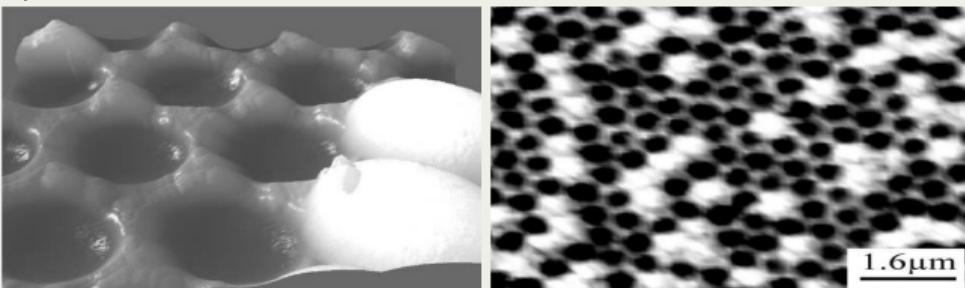
Geometrically stabilized magnetic states: experiment

CoCrPt:SiO₂ deposited on nanocones



[Ball, Lenz, Fritzsche, Varvaro, Günther, Krone, Makarov, Mücklich, Facsko, Fassbender, Albrecht, Nanotechnology (2014)]

Spherical nanoindentations



[Makarov, Baraban, Guhr, Boneberg, Schift, Gobrecht, Schatz, Leiderer, Albrecht, APL (2007)]

Geometrical magnetochiral effects: mesoscale DMI

Vector of Mesoscale DMI: geometrical tailoring of the magnetochirality

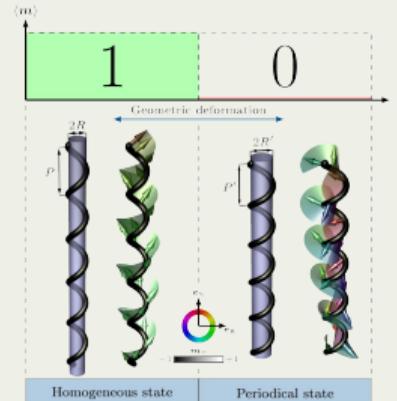
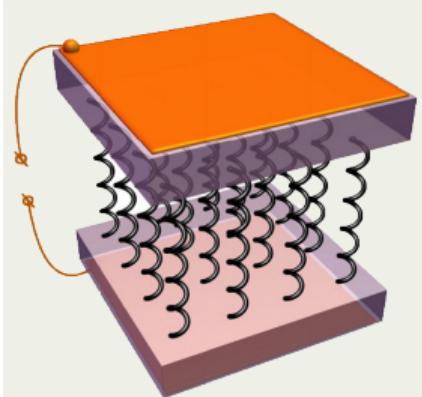
$$\vec{D} = \underbrace{\vec{D}^i}_{\text{intrinsic}} + \underbrace{\vec{D}^e}_{\text{extrinsic}}$$

Two types of DMI are characterized by:

- different scales
- different directions
- different symmetries

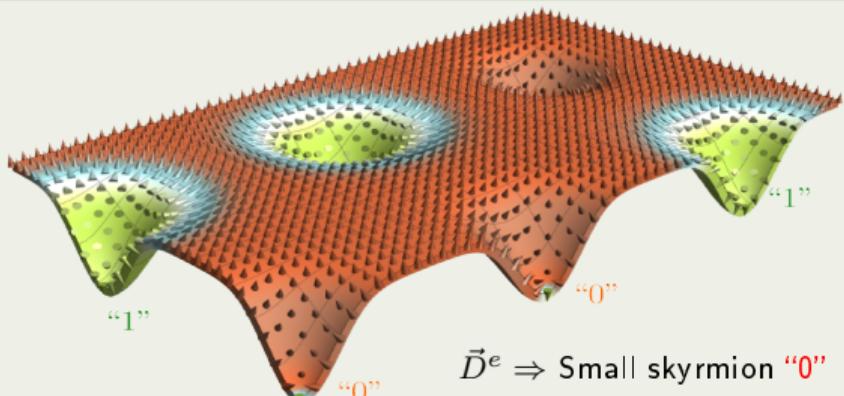
[Volkov, Sheka, Gaididei, Kravchuk, Röbler, Fassbender, Makarov, Sci. Rep. (2018)]

Concept of artificial magnetoelectric materials



[Volkov, Röbler, Fassbender, Makarov, JPD (2019)]

Reconfigurable skyrmion lattices



$\vec{D}^e \Rightarrow$ Small skyrmion "0"

$\vec{D} \Rightarrow$ Large skyrmion "1"

[Kravchuk, Sheka, Kákay, Volkov, Röbler, van den Brink, Makarov, Gaididei, PRL (2018)]

Geometrical magnetochiral effects: Nonlocal chiral symmetry breaking

Micromagnetism of flat thin films

	Local interaction	Non-local interaction
Anisotropy	a 	c
Chiral	b 	d does not exist

$$-\nabla \cdot \vec{m} = \rho(\vec{r}) + g(\vec{r})$$

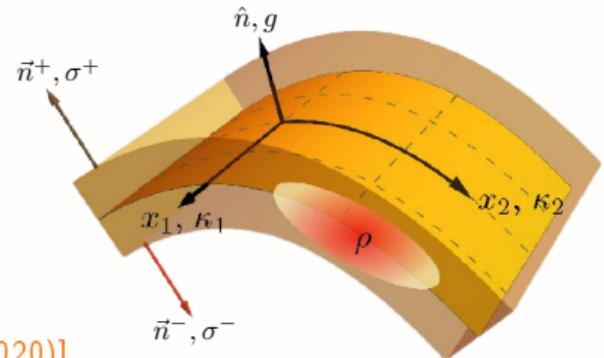
$$\rho(\vec{r}) = -\partial_\alpha m_\alpha : \quad \text{tangential charge}$$

$$g(\vec{r}) = (\kappa_1 + \kappa_2) \vec{m} \cdot \hat{n} : \quad \text{geometrical charge}$$

$$\sigma^\pm(\vec{r}) = \vec{m} \cdot \vec{n}^\pm : \quad \text{surface charge}$$

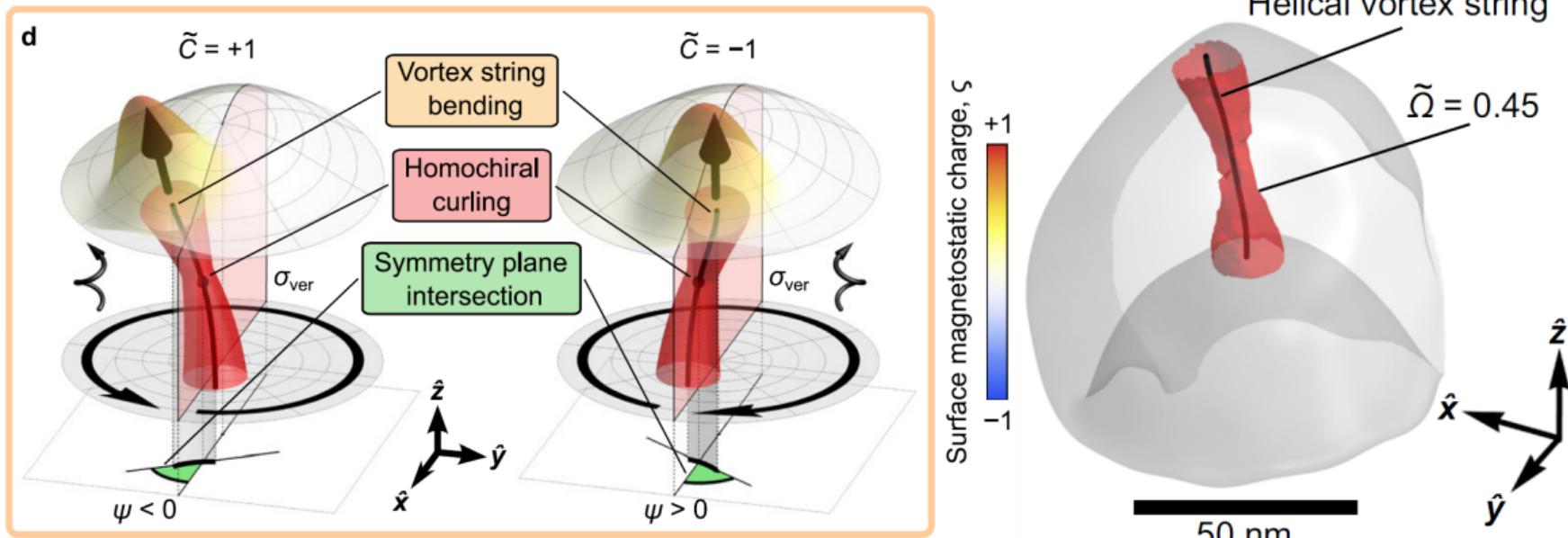
Micromagnetism in curved geometries

	Local interaction	Non-local interaction
Anisotropy	e 	g
Chiral	f 	h



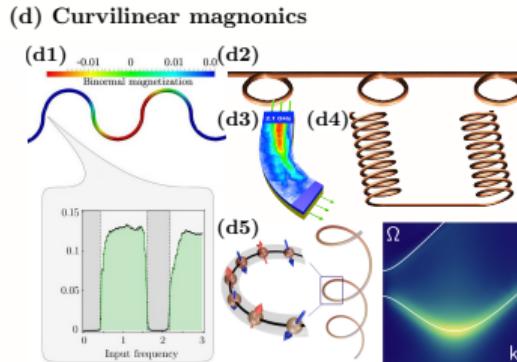
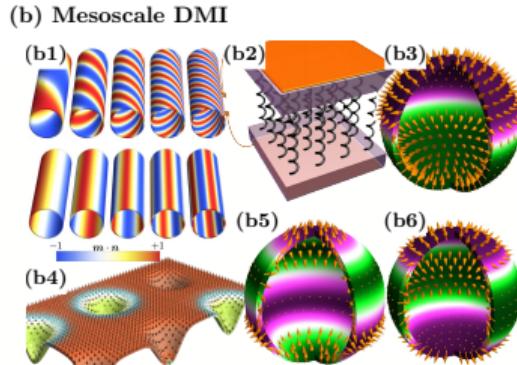
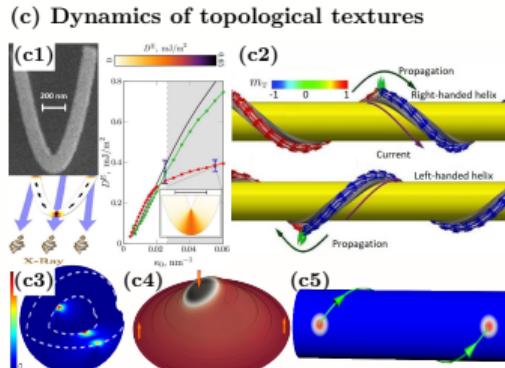
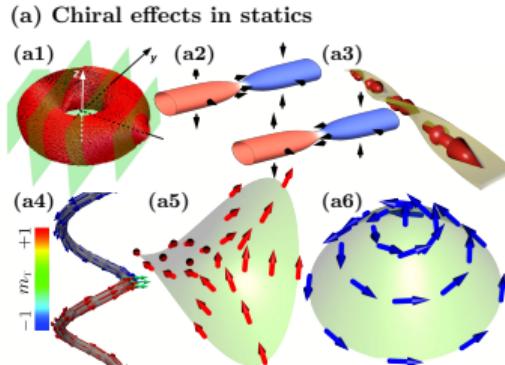
[Sheka, Pylypovskiy, Landeros, Gaididei, Kákay, Makarov, Comms. Phys (2020)]

experimental validation



[Volkov, Wolf, Pylypowskyi, Kákay, Sheka, Büchner, Fassbender, Lubk, Makarov, Nature Comms. (2023)]

Current and future challenges



Fundamental researches:

- Curvilinear antiferromagnetism
- Tayloring cross section in curvilinear magnetism

Prospect for applications:

- Shapeable magnetoelectronics
- Curvilinear spintronics
- Curvilinear magnonics
- Curvilinear skyrmionics
- Magnetic soft robotics

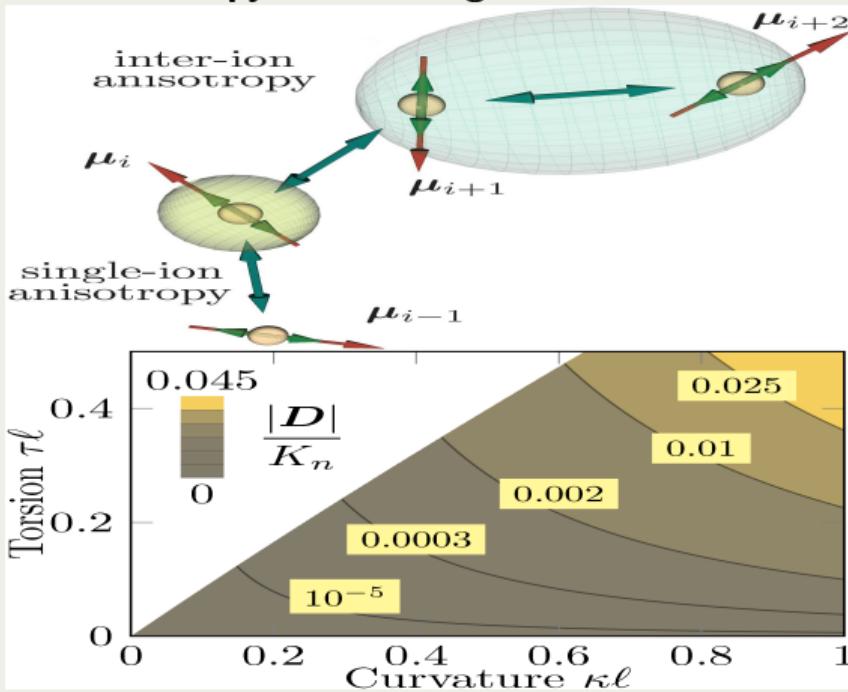
[Sheka, 'A perspective on curvilinear magnetism', APL (2021)]

Curvilinear antiferromagnetism

Curvilinear antiferromagnetism: curvature effects

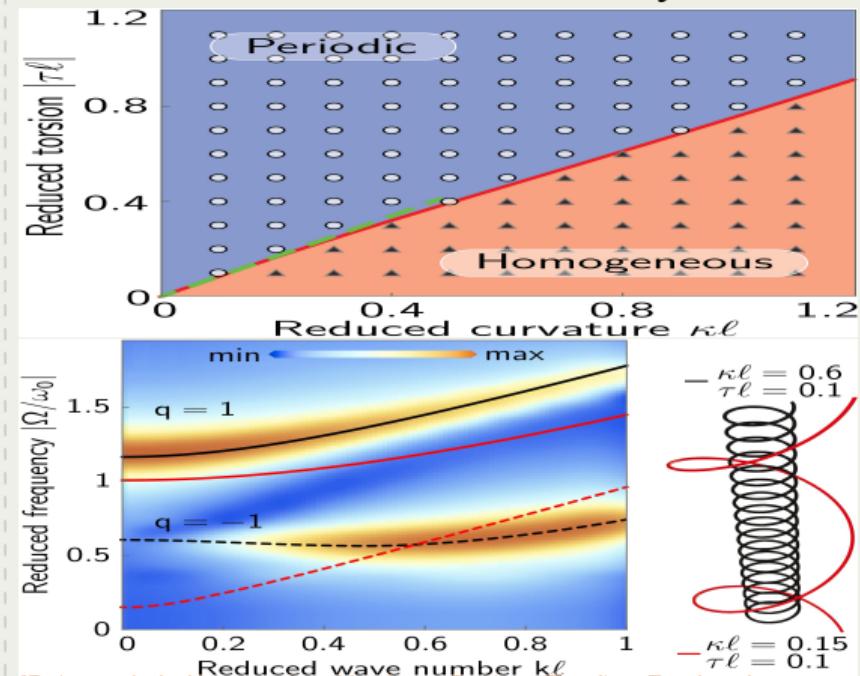
Theory

Anisotropy and homogeneous DMI



[Pylypovskiy, Borysenko, Fassbender, Sheka, Makarov, APL (2021)]

Helix as a case study

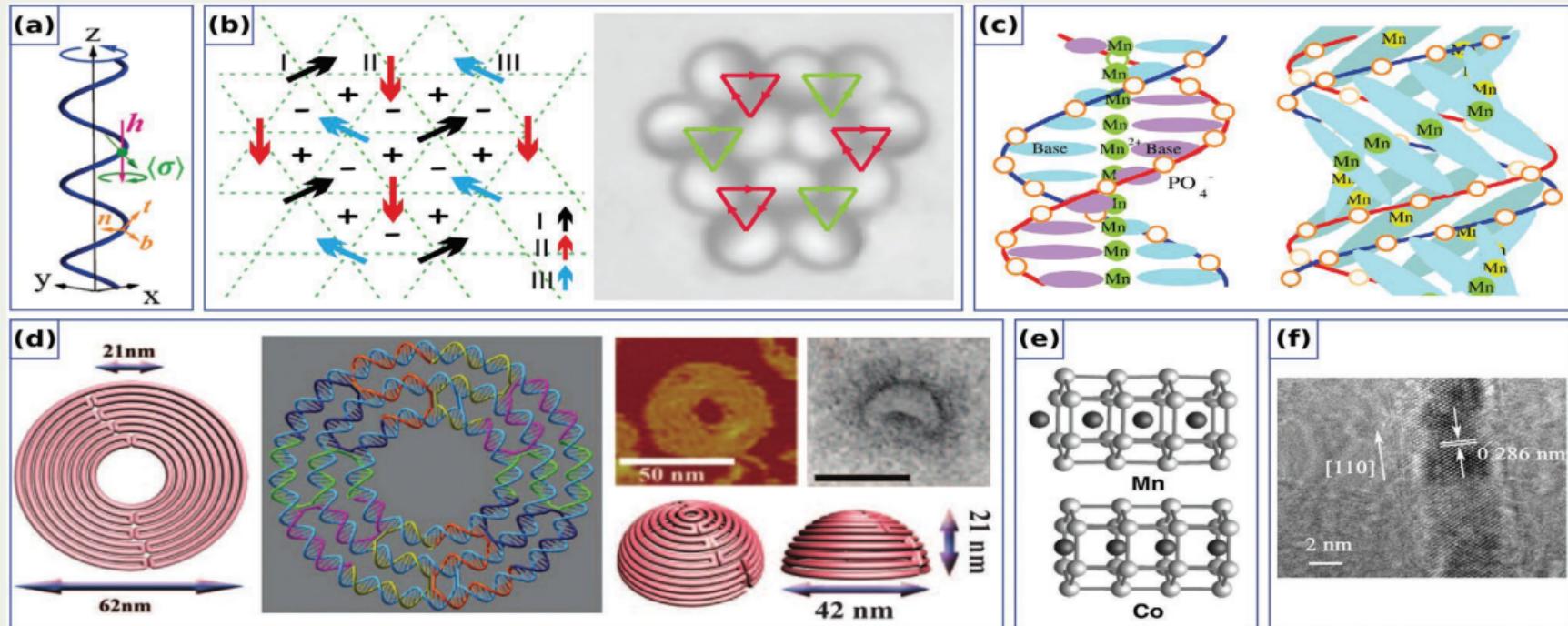


[Pylypovskiy, Kononenko, Yershov, Röbler, Tomilo, Fassbender, van den Brink, Makarov, Sheka, Nano Lett (2020)]

Curvilinear antiferromagnetism

Curvilinear antiferromagnetism: curvature effects

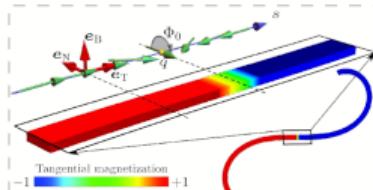
Perspectives of curvilinear antiferromagnetism



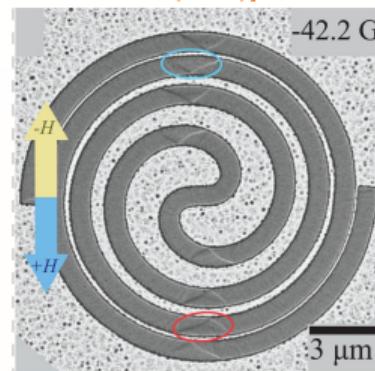
[Makarov, Volkov, Kákay, Pylypovskiy, Budinská, Dobrovolskiy (review), Adv. Mat. (2021)]

Curvilinear magnetism of nanowires with varying cross section

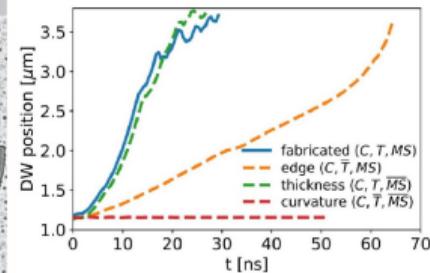
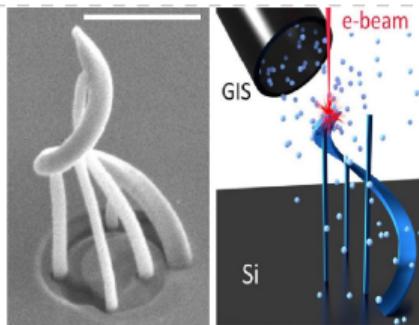
Geometry-induced automotion



[Yershov, Kravchuk, Sheka, Pylypovskiy, Makarov, Gaididei, PRB (2018)]



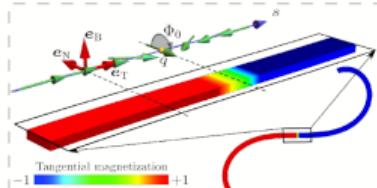
[Brajuskovic, Phatak, APL (2021)]



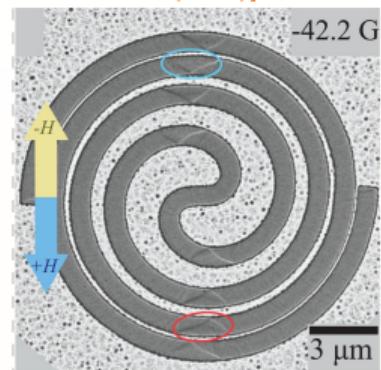
[Skoric, Donnelly, Hierro-Rodriguez, Cascales Sandoval, Ruiz-Gómez, Foerster, Niño Ortí, Belkhou, Abert, Suess, Fernández-Pacheco, ACS Nano (2022)]

Curvilinear magnetism of nanowires with varying cross section

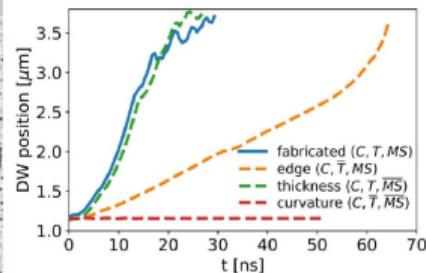
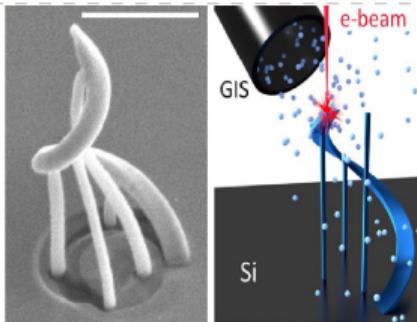
Geometry-induced automotion



[Yershov, Kravchuk, Sheka, Pylypovskiy, Makarov, Gaididei, PRB (2018)]

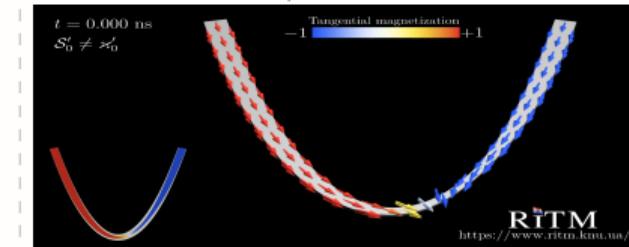
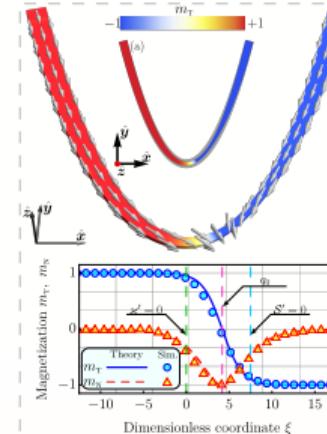


[Brajuskovic, Phatak, APL (2021)]



[Skoric, Donnelly, Hierro-Rodriguez, Cascales Sandoval, Ruiz-Gómez, Foerster, Niño Ortí, Belkhou, Abert, Suess, Fernández-Pacheco, ACS Nano (2022)]

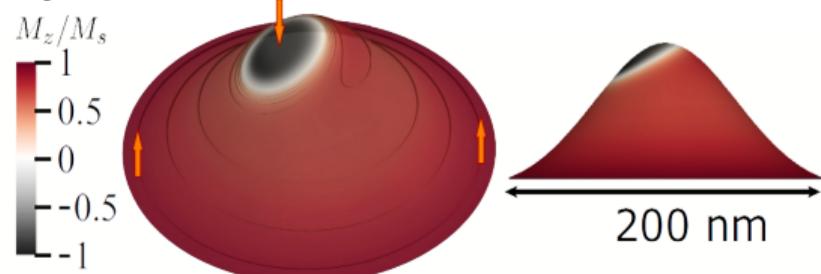
Control of magnetic response by tailoring the cross section



[Yershov, Sheka, PRB (2023)]

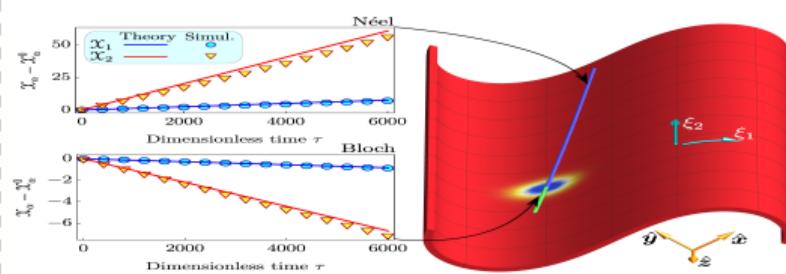
Curvature-induced automotion

Gyromotion



[Korniienko, Kákay, Sheka, Kravchuk, PRB (2020)]

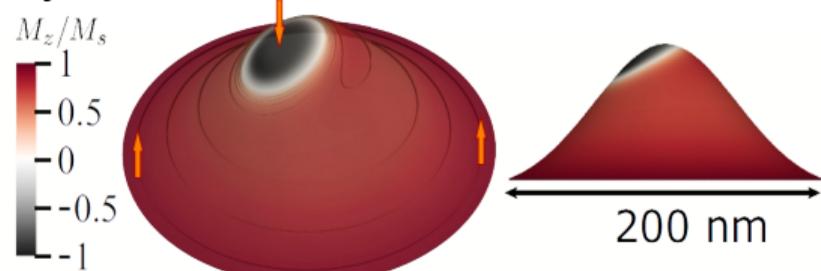
Drift



[Yershov, Kákay, Kravchuk, PRB (2022)]

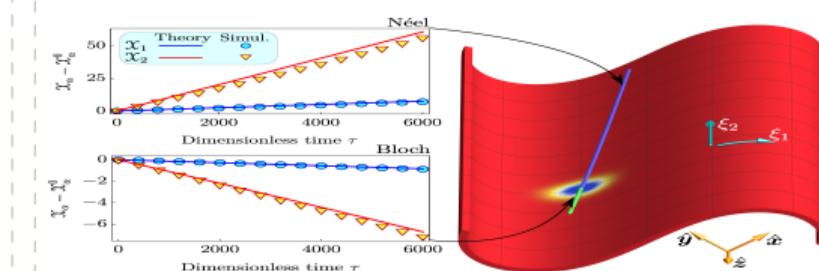
Curvature-induced automotion

Gyromotion



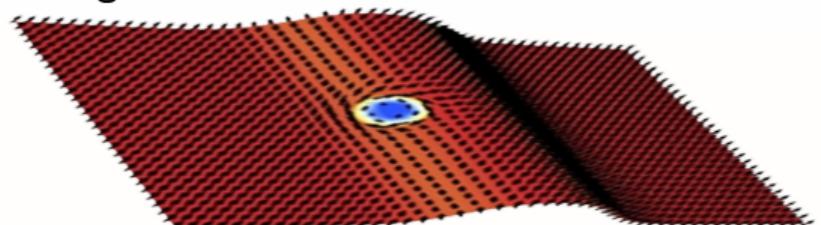
[Korniienko, Kákay, Sheka, Kravchuk, PRB (2020)]

Drift



[Yershov, Kákay, Kravchuk, PRB (2022)]

Along a curved racetrack



[Carvalho-Santos, Castro, Salazar-Aravena, Laroze, Corona, Allende, Altibr, APL (2021)]

Current-induced propagation

Along a nanotube



[Wang, Wang, Wang, Yang, Cao, Yan, APL (2019)]

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Our team (RiTМ):: <http://ritm.knu.ua>

Yelyzaveta Borysenko

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