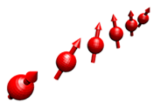
Tutorial on
**Interactions in magnonics**
Gerrit Bauer, バウアー ゲリット, 包格瑞



科研費
KAKENHI

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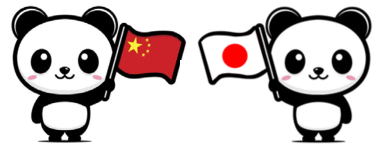
Institutes





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With thanks to:

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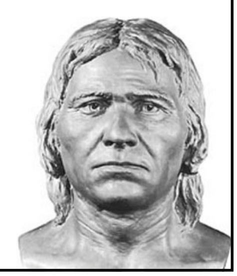
- **Magnon spin**
  - from micromagnetics
  - from spin models
- Magnon dipolar chirality
- Gated spin communities

$\hbar$?



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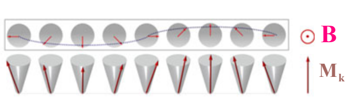
Phenomenology of spin waves

$$F[\mathbf{M}] = \int \left\{ -\mathbf{M}(\mathbf{r},t)\cdot\mathbf{B}(\mathbf{r},t) + \frac{D}{M_0}\left[\nabla\mathbf{M}(\mathbf{r},t)\right]^2 \right\} d\mathbf{r} \text{ and } \left|\mathbf{M}(\mathbf{r},t)\right| = M_0$$

Landau-Lifshitz (LL) equation of motion: $\dot{\mathbf{M}} = -\gamma \mathbf{M}\times\mathbf{B}; \quad \mathbf{B} = -\frac{\delta E[\mathbf{M}]}{\delta \mathbf{M}}$

Plane wave ansatz: $\mathbf{M}_\mathbf{k} = M_0\left(\hat{\mathbf{z}} + \mathbf{m}_\mathbf{k} e^{i\omega_k t} e^{-i\mathbf{k}\cdot\mathbf{r}}\right)$ $\quad \mathbf{B} \parallel \hat{\mathbf{z}}$, $\mathbf{m}_\mathbf{k} \perp \hat{\mathbf{z}}$

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Exchange spin waves in thin films

Linearized LL equation for $|\mathbf{m}_k| \ll M_0$ :

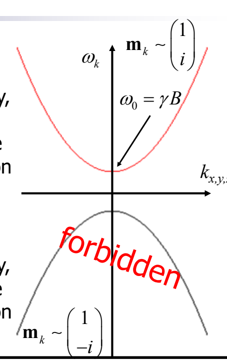
$$i\omega_k \mathbf{m}_\mathbf{k} = -\gamma\left(B + Dk^2\right)\mathbf{m}_\mathbf{k}\times\hat{\mathbf{z}}$$

$$\begin{pmatrix} -\frac{i\omega_k}{\gamma} & B + Dk^2 \\ B + Dk^2 & -\frac{i\omega_k}{\gamma} \end{pmatrix}\begin{pmatrix} m_{x,\mathbf{k}} \\ m_{y,\mathbf{k}} \end{pmatrix} = 0$$

$$\Rightarrow \omega_k = \pm\left(\gamma B + Dk^2\right)$$

positive frequency, counter-clockwise precession

negative frequency, clockwise precession

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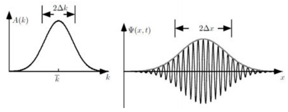
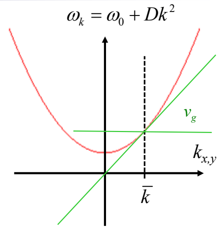
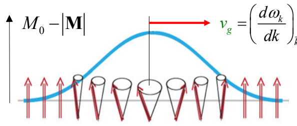
## Spin wave packet

$$\omega_k = \omega_0 + Dk^2$$




$$v_g = \left(\frac{d\omega_k}{dk}\right)_{\bar{k}} = 2D\bar{k}$$

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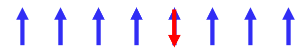
## Magnon quantization and spin



$$\Delta M = -g\mu_B \qquad S_m = g \times \frac{1}{2}$$

$$\mathbf{M_k} = M_0\left(\hat{\mathbf{z}} + \mathbf{m_k} e^{i\omega_k t} e^{-i\mathbf{k}\cdot\mathbf{r}}\right) \qquad \omega_k = \omega_0 + 2Dk^2$$

$$f[\mathbf{M_k}] = -BM_{k,z} + \frac{Dk^2}{M_0}\left|M_{k,\perp}\right|^2 \equiv \hbar\omega_k$$

$$\mathbf{m}_k = \sqrt{\frac{g\mu_B}{M_0 V}}\begin{pmatrix}1\\ i\end{pmatrix}$$ single magnon transverse amplitude

$$m_z = -g\mu_B \rightarrow S_m = 1$$ **magnon spin**

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## Magnetic film (Kittel formula, $k$=0)

$f(\mathbf{M}) = -M_z B + \frac{K}{M_0} M_y^2$ and $|\mathbf{M}(\mathbf{r},t)| = M_0$

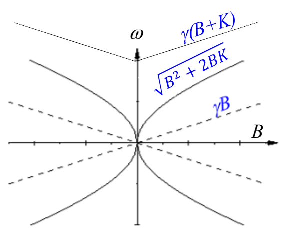
$K > 0$ easy $(x,y)$ plane

$$\omega = \pm\sqrt{B(B+2K)}$$

$$\dot{\mathbf{M}} = -\gamma \mathbf{B}_{\text{eff}} \times \mathbf{M}$$

$$\mathbf{B}_{\text{eff}} = -\frac{\partial f(\mathbf{M})}{\partial \mathbf{M}} = \left(0, -\frac{2K}{M_0} M_z, B\right)$$

$$\begin{pmatrix} \omega + \gamma(B+K) & K \\ -K & \frac{\omega}{\gamma} - B + K \end{pmatrix}\begin{pmatrix} m_+ \\ m_- \end{pmatrix} = 0$$

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## Magnon quantization and spin (Kittel mode)

$$\begin{pmatrix} m_x \\ m_z \end{pmatrix} \sim \begin{pmatrix} \sqrt{\frac{B}{B+2K}} \\ i \end{pmatrix} = \begin{cases} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{ when } B \to 0 \\ \begin{pmatrix} 1 \\ i \end{pmatrix} \text{ when } K \to 0 \end{cases}$$

linearly polarized

circularly polarized

$$-M_z B + \frac{K}{M_0} M_y^2 = \hbar\omega \qquad \omega = \pm\gamma\sqrt{B(B+2K)}$$

$$m_z = -\gamma\hbar\frac{B+K}{\sqrt{B(B+2K)}} \rightarrow S_m = \frac{B+K}{\sqrt{B(B+2K)}} = \begin{cases} \infty \\ 1 \end{cases} \text{when } \begin{matrix} B \to 0 \\ K \to 0 \end{matrix}$$

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## Holstein-Primakoff transformation (single spin)

$$\hat{H} = -\hat{\mathbf{M}}\cdot\mathbf{B} = \hbar\gamma\hat{\mathbf{S}}\cdot\mathbf{B} \overset{\mathbf{B}=(0,0,B)}{=} \hbar\gamma B\hat{S}_z$$

$$\left[\hat{S}_x, \hat{S}_y\right] = i\hat{S}_z; \quad \hat{\mathbf{S}}\cdot\hat{\mathbf{S}} = S(S+1)$$

Charles Kittel,
Quantum Theory of Solids,
1963

Mapping the spin problem on a Boson problem:

$$|\Psi\rangle = |S, S_z\rangle = |S, n\rangle = \frac{1}{\sqrt{n!}}\left(a^+\right)^n |S, 0\rangle$$

With $\left[\hat{a}, \hat{a}^+\right] = 1$ and $\left[\hat{a}, \hat{a}\right] = \left[\hat{a}^+, \hat{a}^+\right] = 0$:

$$E = \hbar\omega_0(n - S)$$

$$\hat{S}^+ = \hat{S}_x + i\hat{S}_y \equiv \frac{1}{\sqrt{2S}}\sqrt{1 - \frac{\hat{a}^+\hat{a}}{2S}}\hat{a} \approx \frac{1}{\sqrt{2S}}\left(1 - \frac{\hat{a}^+\hat{a}}{4S}\right)\hat{a}$$

$$\hat{S}^- = \hat{S}_x - i\hat{S}_y \equiv \frac{1}{\sqrt{2S}}\hat{a}^+\sqrt{1 - \frac{\hat{a}^+\hat{a}}{2S}} \approx \frac{1}{\sqrt{2S}}\hat{a}^+\left(1 - \frac{\hat{a}^+\hat{a}}{4S}\right)$$

$$\hat{S}_z = S - \hat{a}^+\hat{a}$$

$\hat{H} = \hbar\omega_0\left(\hat{a}^+\hat{a} - S\right)$ with $\omega_0 = \gamma B$

higher-order terms in expansion introduce interactions between magnons.

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## Heisenberg model with anisotropy

Single spin $\hat{H} = \hbar\gamma B\hat{S}_y + K\hat{S}_z^2 \approx (H+K)\hat{a}^\dagger\hat{a} + (K/2)(\hat{a}\hat{a} + \hat{a}^\dagger\hat{a}^\dagger) + \text{const.}$

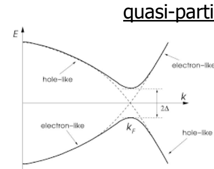
quasi-particles are not conserved!

Bogoliubov-de Gennes transformation: $\hat{b} = u\hat{a} - v^*\hat{a}^\dagger \Rightarrow$

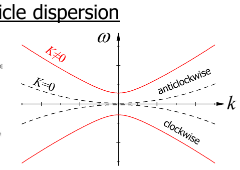
$$\hat{H} = \hbar\omega b^\dagger b + \text{const.}$$

$$\omega = \gamma\sqrt{B(B+2K)}$$

quasi-particle dispersion




superconductor




anisotropic magnet

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## Magnon magnetic moment

For $\mathcal{H}\psi_i = \varepsilon_i \psi_i$ and $\hat{\mathcal{H}} = \hat{\mathcal{H}}(\lambda)$ then $\frac{\partial \varepsilon_i}{\partial \lambda} = \langle \psi_i | \frac{\partial \hat{\mathcal{H}}}{\partial \lambda} | \psi_i \rangle$

Zeeman interaction:

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 - \hat{m} B_{\parallel} \quad \Rightarrow \quad m_i = \langle \psi_i | \frac{\partial \hat{\mathcal{H}}}{\partial B_{\parallel}} | \psi_i \rangle = -\frac{\partial \varepsilon_i}{\partial B_{\parallel}}$$

Example: exchange magnons

$$\varepsilon_{\mathbf{k}} = \hbar \gamma B + A k^2 \Rightarrow m_{\mathbf{k}} = -\frac{\partial \varepsilon_{\mathbf{k}}}{\partial B} = -g \mu_B$$

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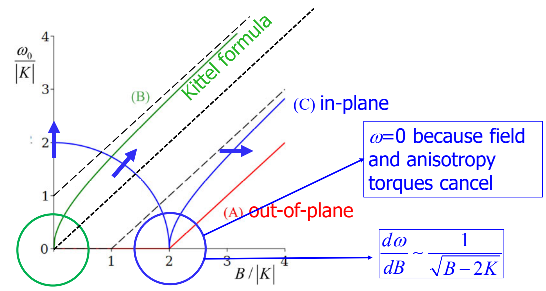
## Band edges of anisotropic ferromagnets

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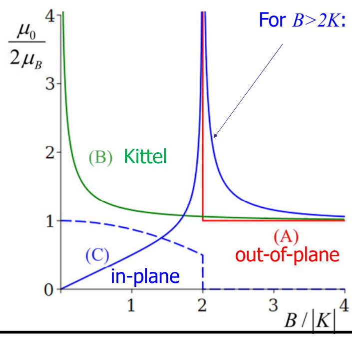
## Band-edge magnon spin (GB *et al.*, 2023)




For $B>2K$:

$$|\mathbf{m}_0| = -g\gamma\hbar \frac{B-K}{\sqrt{B(B-2K)}}$$

- Diverging magnon spins cause enhanced magnon conductivities.
- However, magnon approximations breaks down

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  - ➢ from spin models
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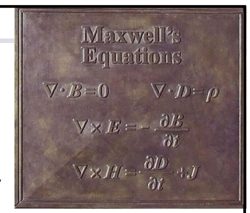
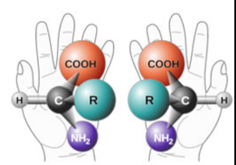
- Gated spin communities

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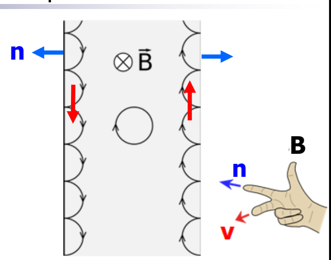
## Chirality in chemistry and transport

DOI: 10.1002/evl3.31




quantum Hall effect

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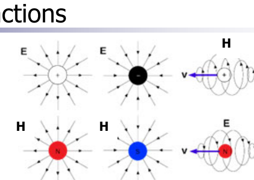
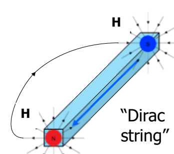
## Magnetodipolar interactions

magnetic vs. electric monopoles (Wikipedia)




magnetic wire, spin ice

$$\rho_m = \nabla \cdot \mathbf{M}$$ magnetic charges

$$B_m(\mathbf{r}) = \nabla \int \frac{\rho_m(\mathbf{r}')}{4\pi\mu_0 |\mathbf{r}-\mathbf{r}'|} d\mathbf{r}'$$ Coulomb's Law

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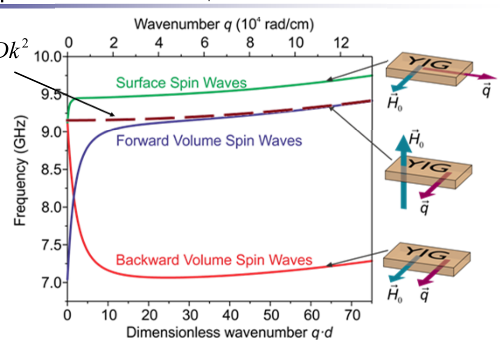
## Magnon dispersion of a $d = 5\ \mu$m YIG film

$\hbar\omega_k = \hbar\omega_0 + 2Dk^2$

$\omega_0 = \gamma B$

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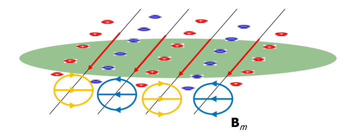
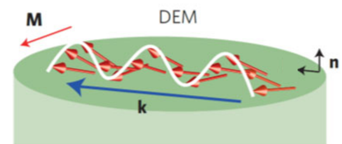
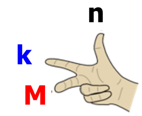
## Magnetostatic surface waves (Damon & Eshbach, 1959)




$\mathbf{B}_\mathbf{k} = \nabla \int \frac{d\mathbf{r}'}{4\pi\mu_0} \frac{i\mathbf{k}\cdot\mathbf{M}_\mathbf{k}}{|\mathbf{r}-\mathbf{r}'|}$

$E_k = -\mathbf{B}_\mathbf{k}\cdot\mathbf{M}_\mathbf{k}$




chiral propagation

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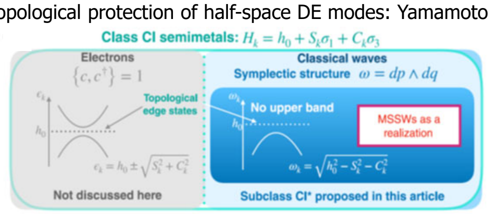
## Magnetostatic surface waves

\+ Topological protection of half-space DE modes: Yamamoto et al. (2019)




- Small group velocity — Yu et al. (2019)
- Dephasing by weak disorder
- Efficient backscattering to opposite surface in films

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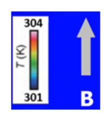
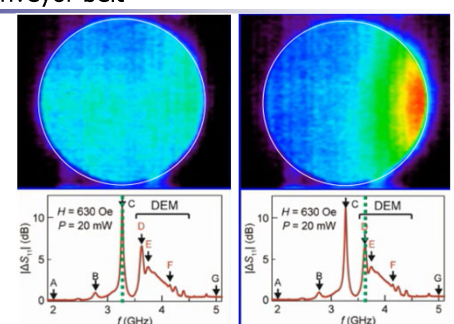
## Magnetic conveyor belt

Thermography:
by An et al. (2013)

Theory:
T. Yu et al. (2019)




Kittel mode — Damon-Eshbach mode

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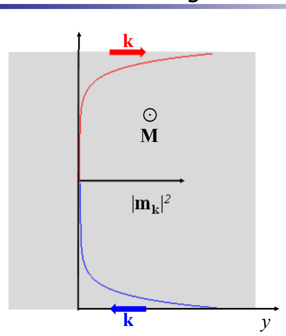
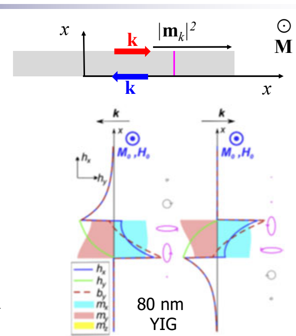
## Thick vs. thin magnetic films




Mohseni et al. (2021)

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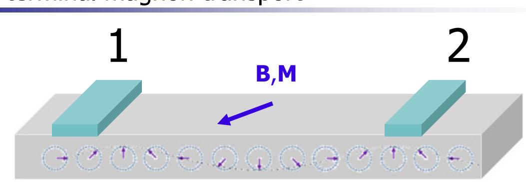
## 2-terminal magnon transport




$$\mathbf{S}(\omega_1,\omega_2) = \begin{pmatrix} S_{11} & S_{21} \\ S_{12} & S_{22} \end{pmatrix} (\omega_1,\omega_2)$$

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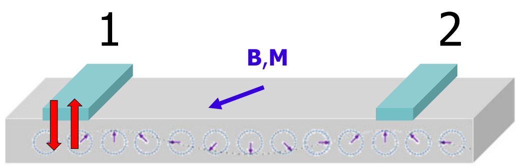
## 2-terminal magnon transport




$$\mathbf{S}(\omega_1,\omega_2) = \begin{pmatrix} \mathbf{S}_{11} & S_{21} \\ S_{12} & S_{22} \end{pmatrix} (\omega_1,\omega_2)$$

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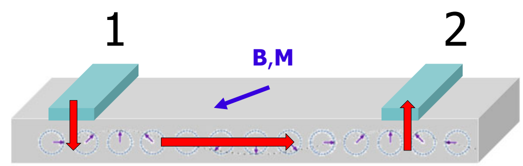
## 2-terminal magnon transport




$$\mathbf{S}(\omega_1,\omega_2) = \begin{pmatrix} S_{11} & \mathbf{S}_{21} \\ S_{12} & S_{22} \end{pmatrix} (\omega_1,\omega_2)$$

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## Non-local magnon Transport

**Contact 1: microwave stripline at frequency $\omega$**
**Contact 2: inductive detection at frequency $\omega$** $S_{12}(\omega,\omega)$

V. Vlaminck and M. Bailleul (2008)
H. Wang *et al.* (2020), De Wal et al. 2023)

Contact 1: electric injection by SHE
Contact 2: electric detection by ISHE $\int \left|S_{12}(\omega_1,\omega_2)\right|^2 d\omega_1 d\omega_2$

L. Cornelissen *et al.* (2015)
X. Wei *et al.* (2022)

Contact 1: microwave stripline at frequency $\omega$
Contact 2: electric detection by ISHE $\int \left|S_{12}(\omega,\omega_2)\right|^2 d\omega_2$

L. Sheng *et al.* (2023)

**Three terminal devices ("magnon transistors")**

A.V. Chumak *et al.* (2014)
Baumgaertl & Grundler (2022)
T. Yu *et al.* (2023)

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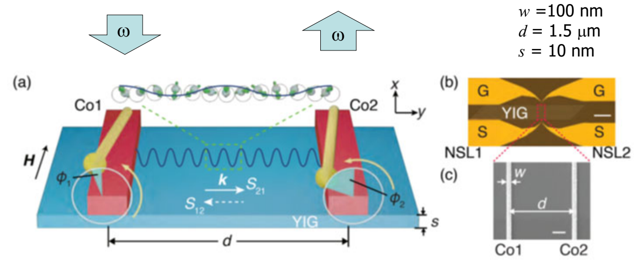
## Spin wave spectroscopy in thin films

$w$ =100 nm
$d$ = 1.5 μm
$s$ = 10 nm




H. Wang *et al.* (2020)

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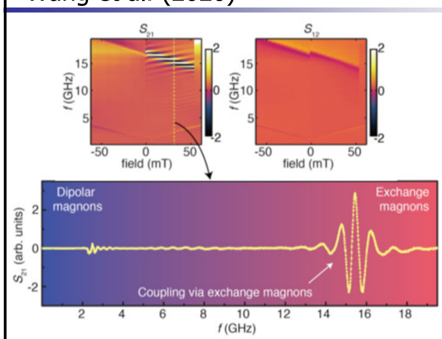
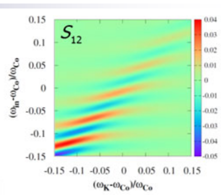
## Wang et al. (2020)




T. Yu *et al.* (2021)

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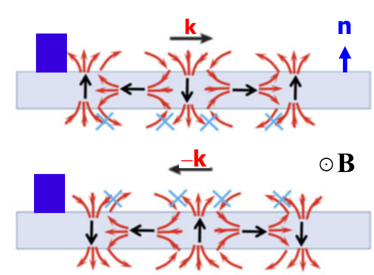
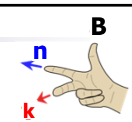
## Chirality by classical electrodynamics




Stray magnetic fields:

$$\mathbf{h}_{\mathbf{k}} = \nabla \int \frac{d\mathbf{r}'}{4\pi} \frac{\nabla \cdot \mathbf{M}_{\mathbf{k}}}{|\mathbf{r}-\mathbf{r}'|}$$

$$\mathbf{h}_{|k|} = \mathbf{h}^{\mathrm{surface}}_{|k|} + \mathbf{h}^{\mathrm{bulk}}_{|k|} = 2\mathbf{h}^{\mathrm{surface}}_{|k|}$$

$$\mathbf{h}_{-|k|} = \mathbf{h}^{\mathrm{surface}}_{|k|} - \mathbf{h}^{\mathrm{bulk}}_{|k|} = 0$$

Tao Yu *et al.*, *Chirality as generalized spin–orbit interaction in spintronics*, Physics Reports, 2023.

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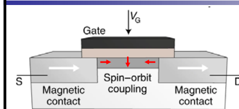
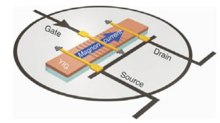
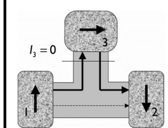
## Spin transistors



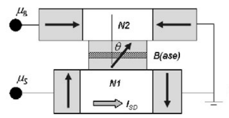

Datta & Das (1990)



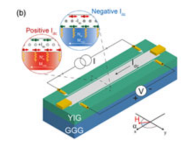

Chumak et al., 2014




Brataas et al. (2000)




Bauer et al. (2003)
Chiba et al. (2013)




Cornelissen et al., 2018

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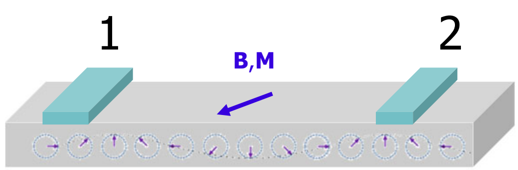
## 2-terminal magnon transport




$$\mathbf{S}(\omega_1,\omega_2)=\begin{pmatrix} S_{11} & S_{21} \\ S_{12} & S_{22} \end{pmatrix}(\omega_1,\omega_2)$$

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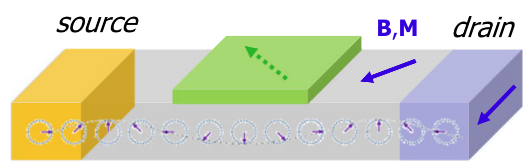
## Chiral magnon transistor




$S_{21} = S_{12}$ without gate

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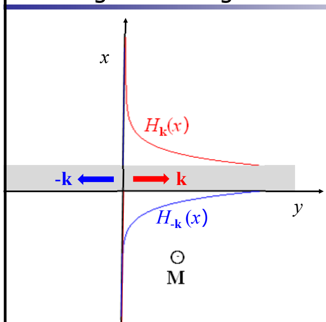
## Metal gate on magnetic film

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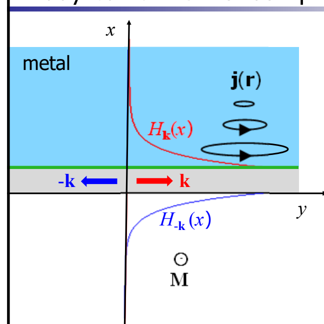
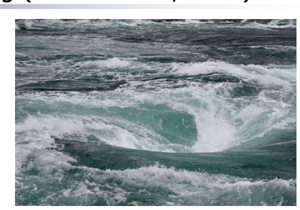
## Eddy-current Chiral damping (Bertelli et al., 2021)



$p = \frac{j^2}{\sigma}$ Joule dissipation

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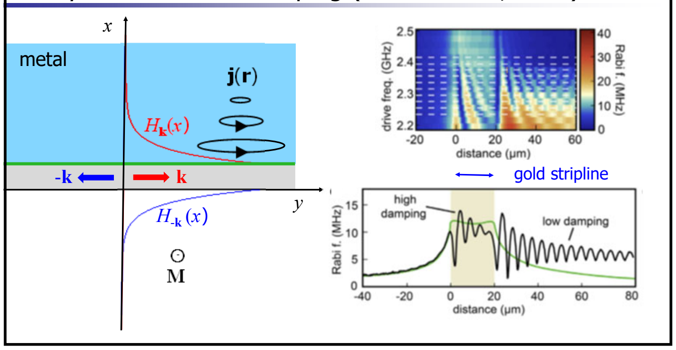
## Eddy-current Chiral damping (Bertelli et al., 2021)

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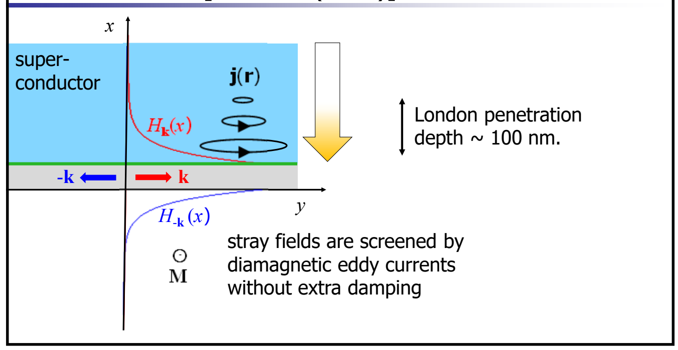
## Meissner effect [Yu *et al.* (2022)]

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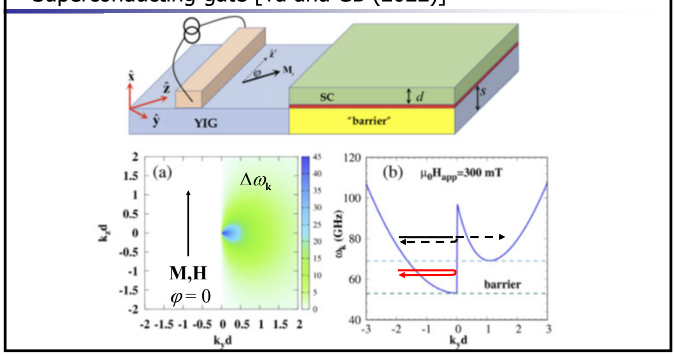
## Superconducting gate [Yu and GB (2022)]

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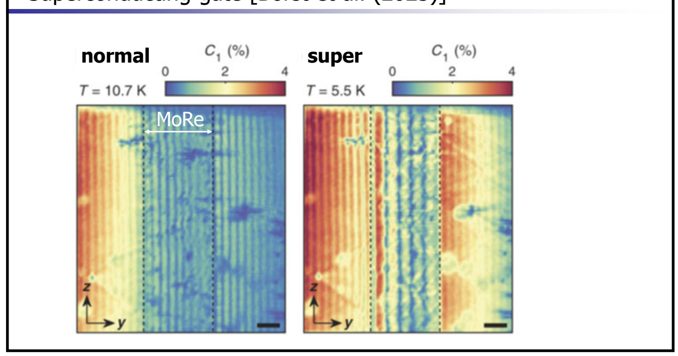
## Superconducting gate [Borst et al. (2023)]

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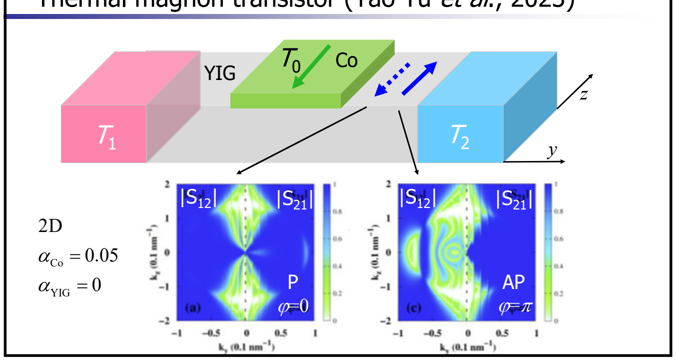
## Thermal magnon transistor (Tao Yu *et al.*, 2023)

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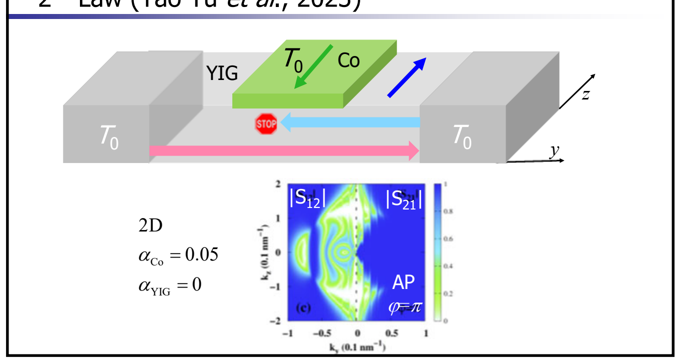
## 2nd Law (Tao Yu *et al.*, 2023)

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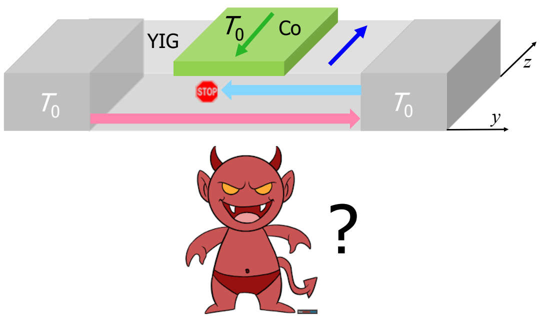
## 2nd Law (Tao Yu *et al.*, 2023)

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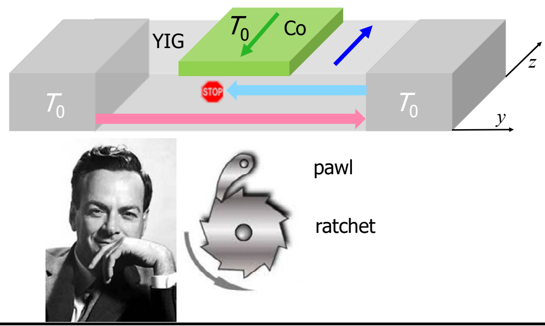
## 2nd Law (Tao Yu *et al.*, 2023)

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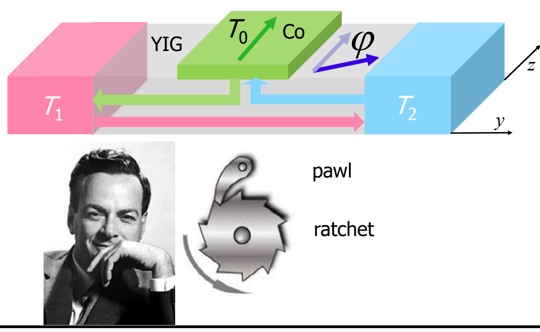
## Non-local Peltier effect (Tao Yu *et al.*, 2023)

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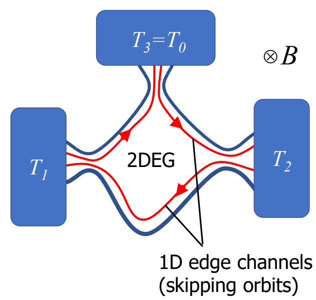
## 3-terminal quantum Hall effect at high fields




$T_{1/2} = T_0 \pm \Delta T/2$

In absence of gate 3 (or chiral damping):

$J_1^{(Q)} = -K_0\left(T_2 - T_1\right)$

$K_0 = \dfrac{\pi^2 k_B^2 T^2}{3h}$

In presence of gate:

$J_1^{(Q)} = -K_0 \Delta T$

$J_{2/3}^{(Q)} = K_0 \Delta T / 2$

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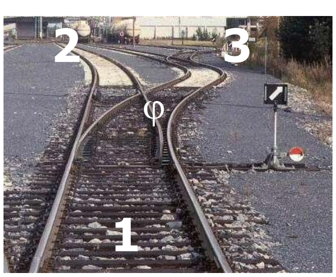
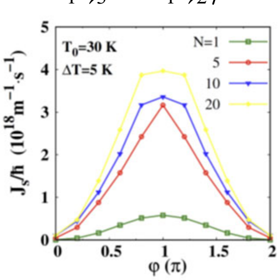
## Heat current rerouting

$$J_{1\to3} \leq J_{1\to2}/2$$

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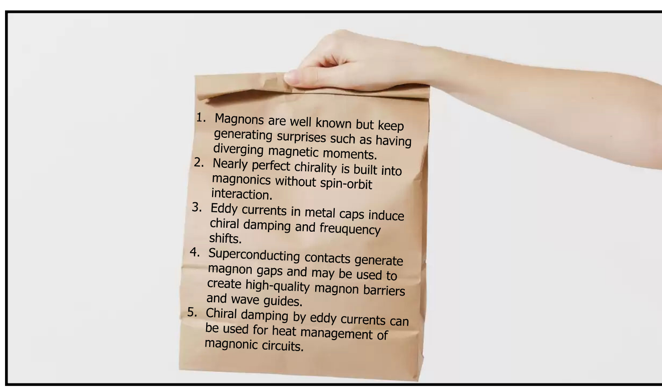


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