Introduction to superconductor-ferromagnet hybrids and Andreev physics

Wolfgang Belzig

Quantum Transport Group

Universität Konstanz

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Universität Konstanz

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Message

- Superconductors in contact with ferromagnets lead to new pairing states
- Properties of superconductors can be efficiently manipulated by ferromagnets
- Supercurrent/Josephson effects can be tailored by ferromagnets
- Long-range spin-polarized supercurrents by non-collinear ferromagnets
- Andreev bound states are convenient to describe nanostructured superconductors

Superconductivity (generalized BCS state)

$$|BCS\rangle = \prod_k \left(u_k + \sum\nolimits_{\sigma\sigma'} v_{k\sigma\sigma'} c^+_{k\sigma}\, c^+_{-k\sigma'}\right)|0\rangle$$

Anomalous Greens function (pair amplitude, oder parameter:

$$F_{\sigma\sigma'}(r,t,r',t') = -i\langle\Psi_{\sigma'}(r',t')\Psi_\sigma(r,t)\rangle$$

Antisymmetry of F under spin, coordinate, time exchange (required by Pauli principle)

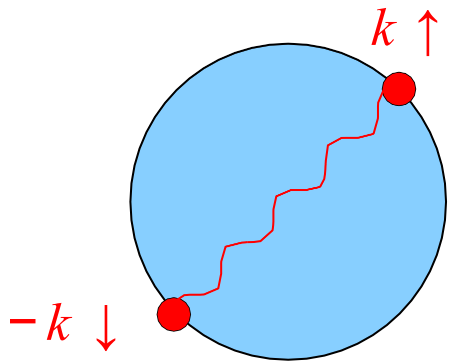
Spatial	Spin	Temporal	Robustness*	Abundance
s-wave (+) $F(k) \sim const.$	Singlet (-)	Even (+)	Yes	Yes (Ag, Nb, …)
	Triplet (+)	Odd (-)	Yes	Yes (Supra-Ferro)
p-wave (-) $F(k) \sim k_z$	Singlet (-)	Odd (-)	No	
	Triplet (+)	Even (+)	No	Yes (He_3, $SrRuO_4$?)
d-wave (+) $F(k) \sim k_x^2 - k_y^2$	Singlet (-)	Even (+)	No	High-T_c
	Triplet (+)	Odd (-)	No	

* = Robustness to structural disorder

General properties of Superconductor-Ferromagnet-Hybrids

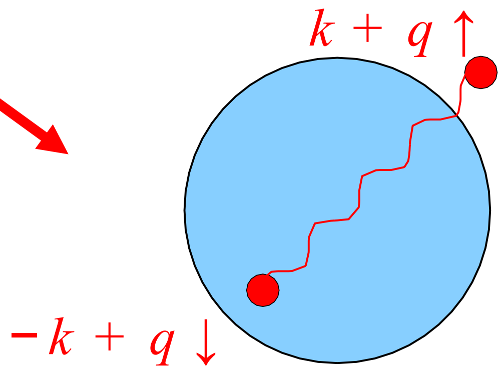
Superconductor: Cooper pairing (singlet)

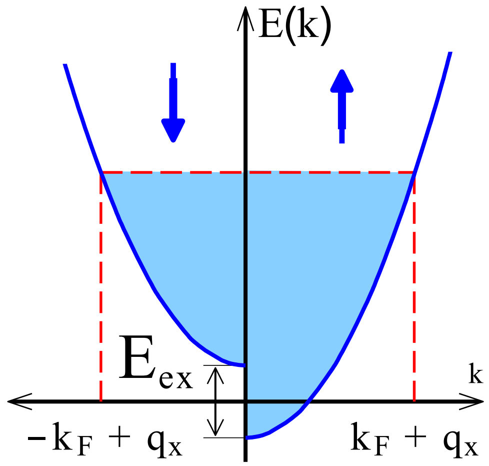
Magnetism: exchange splitting



total momentum

$k_\uparrow - k_\downarrow = 0$





Spin splitting

$k_\uparrow \neq k_\downarrow$

Cooper pairs with finite momentum $2q_x = \frac{2E_{ex}}{\hbar v_F} = \frac{1}{\xi_F}$

➔ Possible pairing state with spatial structure $\Delta(r) = \Delta_0 \cos(q_x r)$

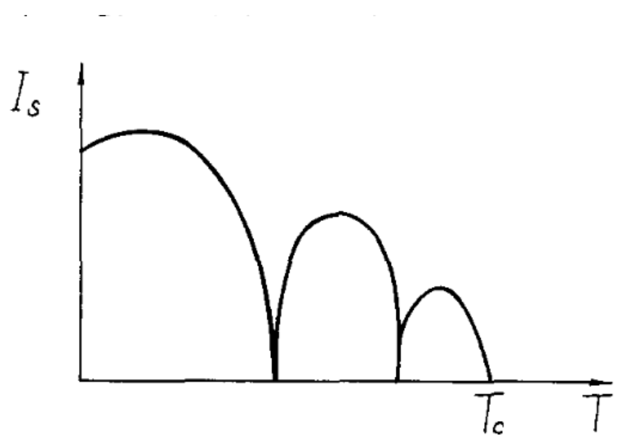
[Fulde+Ferrel (1964); Larkin+Ovchinnikov (1965)]

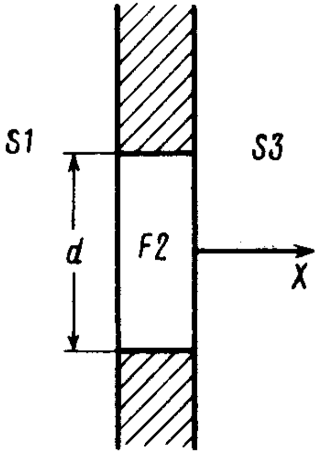
Josephson effect and Tc oscillations

- Buzdin, Bulaevskii, Panyukov (1978,1982)
  Critical current of an SFS Josephson
  junction oscillates!

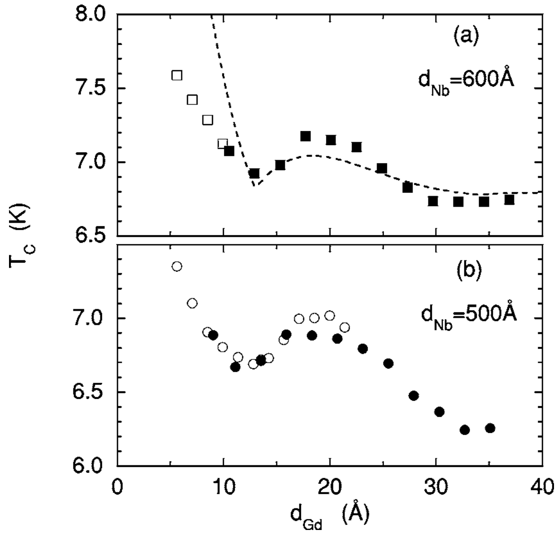
$$\boxed{I_c = I_c^0 e^{-q_{ex} d_F} \cos(q_{ex} d_F)}$$

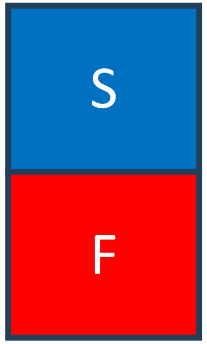
$$q_x = \frac{2E_{ex}}{\hbar v_F}$$





- Buzdin and Kuprianov (1990); Radovic *et al.* (1991)
  Critical temperature of SFS oscillates
  as function F-thickness
  (Exp. observed by Jiang et al.
  and Mercaldo et al 1995,…)

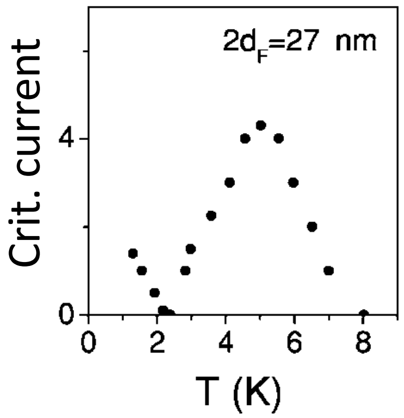




d_{Gd}

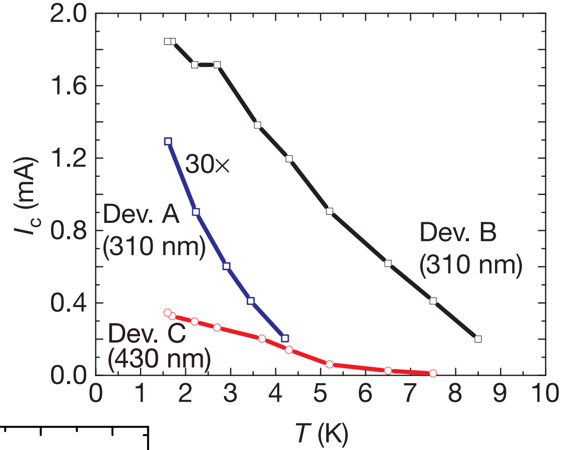
Experimental progress on Josephson effects

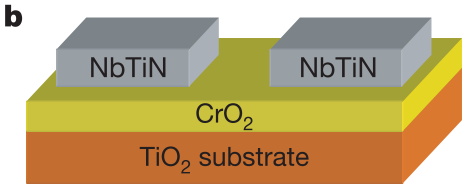
Critical current oscillations Ryazanov et al. (2001)



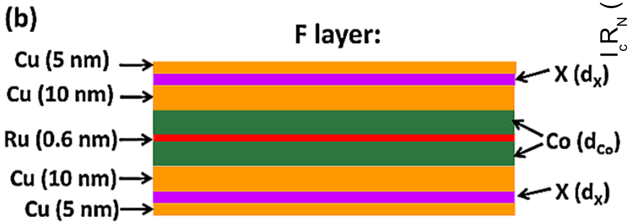
See also: Kontos, Aprili (2001)

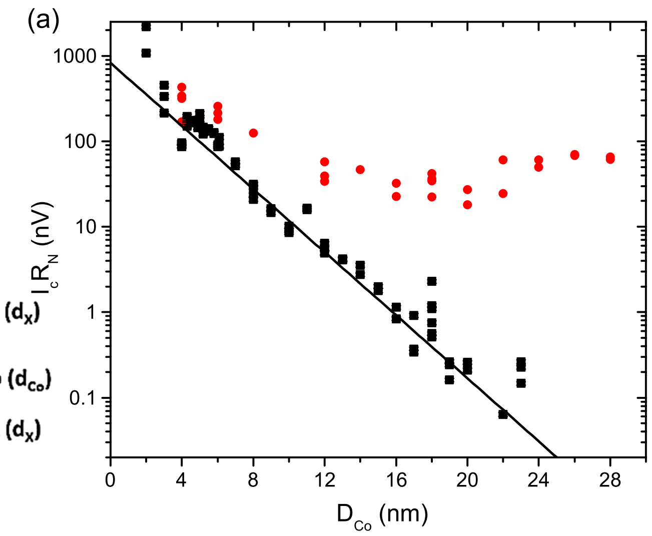
Long-range triplet supercurrent Keizer, Gönnenwein, et al. (2006)

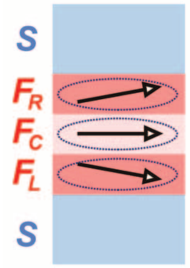


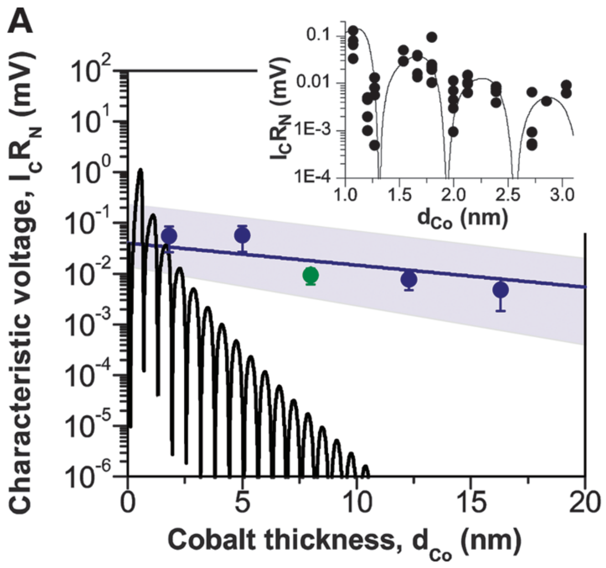


See also: Aarts (2012)







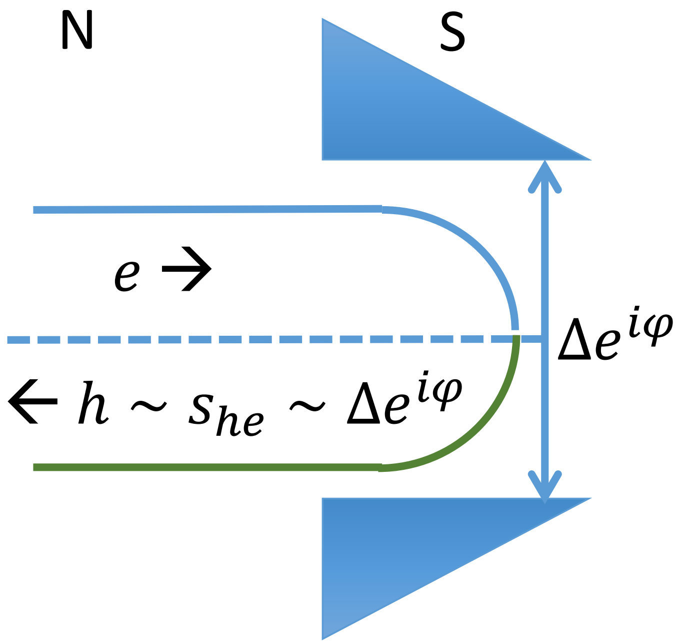


Long-range triplet supercurrent Birge group (2010)

Triplet supercurrent in conical ferromagnet Robinson et al. (2010)

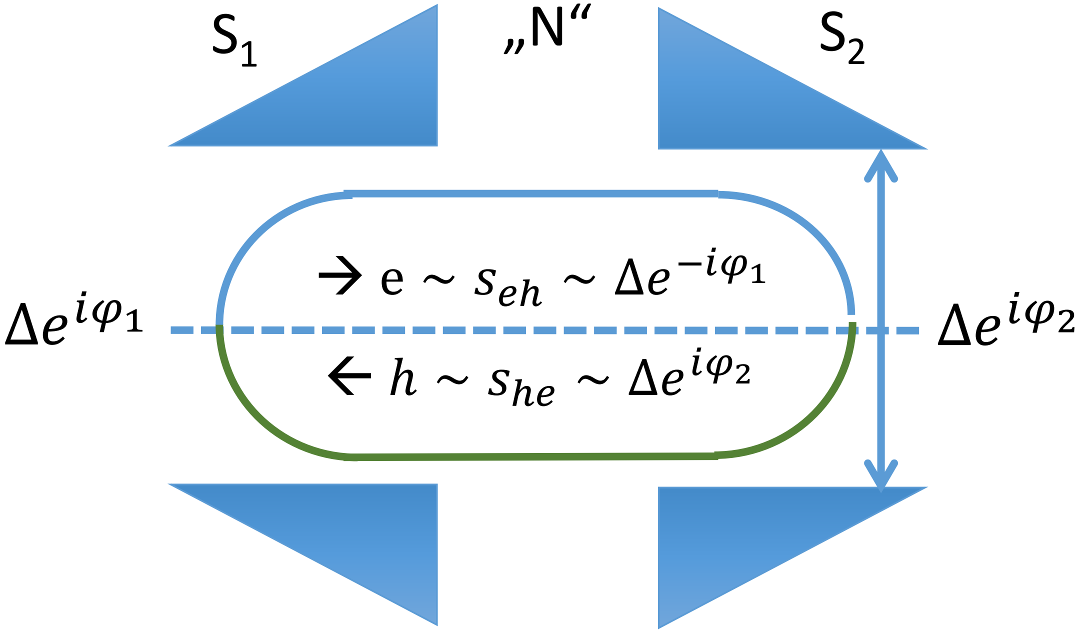
Andreev reflection and Andreev bound states (ABS)

Andreev reflection



Phase-conserving transition:
2 electrons ←→ Cooper pair

Andreev bound states



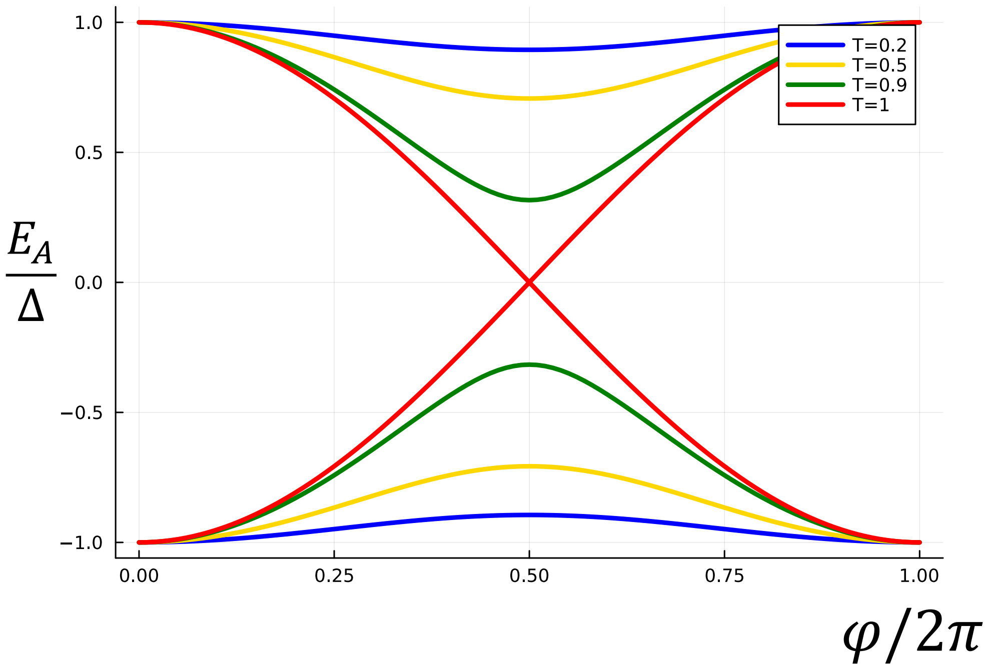
Constructive interference → bound state

Energy $E_A = \pm\Delta \cos\left[\frac{\varphi_1 - \varphi_2}{2}\right]$

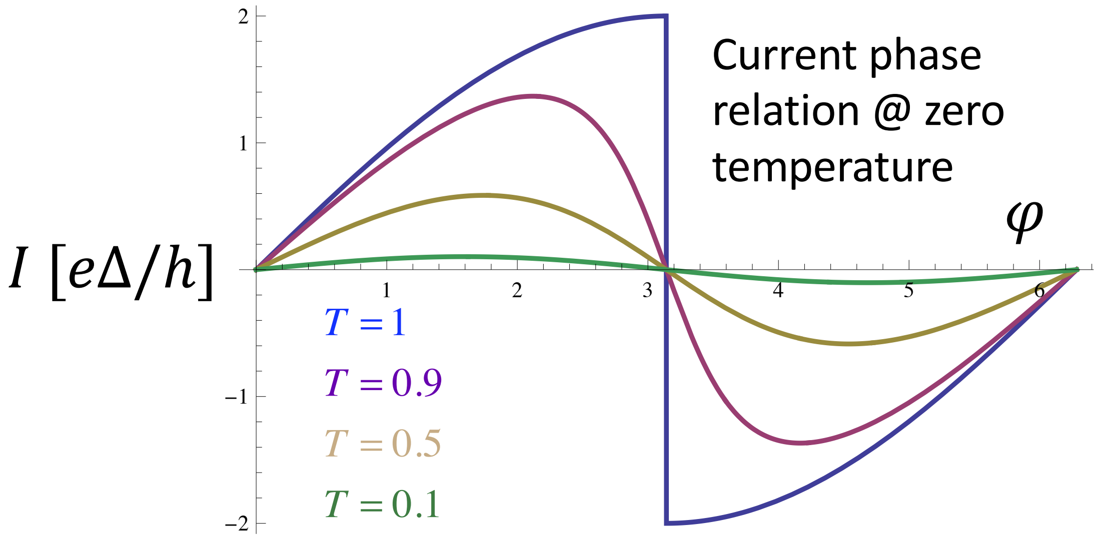
Josephson effect and Andreev bound states

Energy with transmission probability T:

$$E_A = \pm\Delta\sqrt{1 - T\sin^2\varphi/2}$$



Current



Current phase relation @ zero temperature

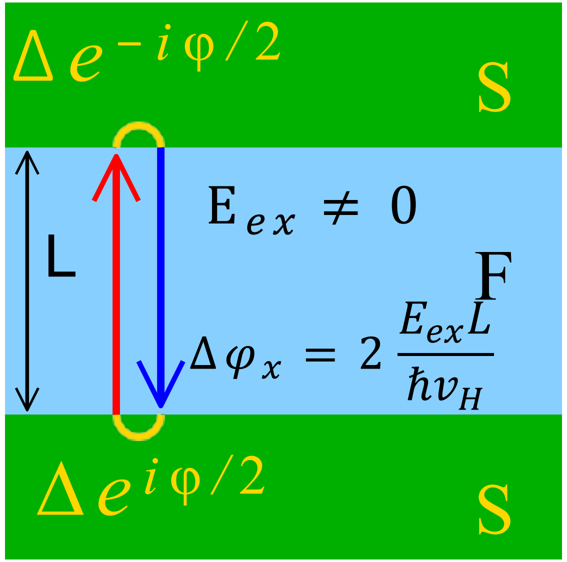
$$I = \frac{2e}{h}\frac{\partial E_A}{\partial\varphi} = \begin{cases} \frac{2e}{h}\Delta\sin\frac{\varphi}{2} & T = 1 \\ \frac{2eT}{h}\Delta\sin\varphi & T \ll 1 \end{cases}$$

Non-sinusoidal

"Josephson"

➔ Non-sinusoidal current-phase relation

Understanding the 0-Pi-Transition in SF

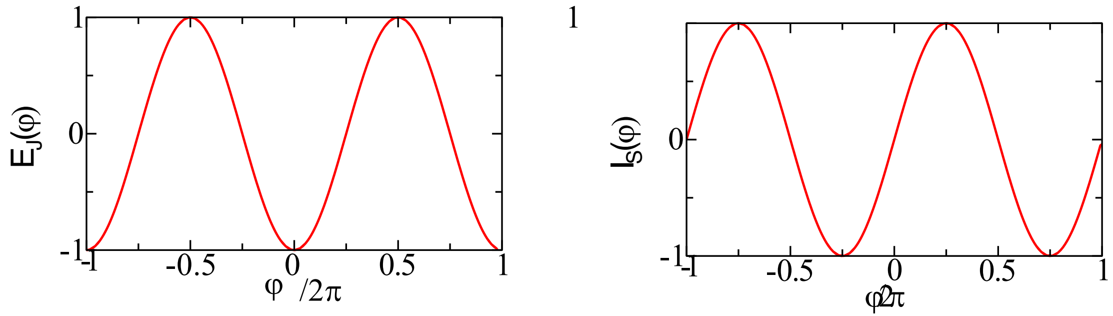


Josephson contact (without F):

$E_J(\varphi) = -\frac{h}{2e} I_c \cos\varphi$ depends on the phase difference φ

Supercurrent: $I_S = \frac{2e}{h}\frac{\partial E_J(\varphi)}{\partial\varphi} = I_c \sin\varphi$

Phase-dependent 'band structure': standard Josephson tunnel junction



Magnetic layer: Spin-dependent phase shift $\delta\varphi_x = 2q_x L = 2\frac{E_{ex}L}{\hbar v_H}$

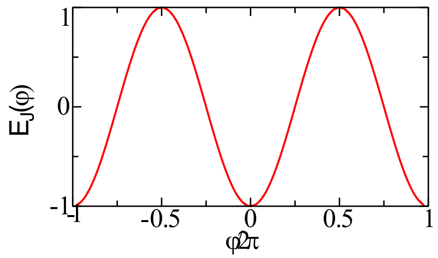
Spin-dependent band splitting: $E(\varphi, E_{ex}) = \frac{1}{2}\left[E_J(\varphi + \delta\varphi_x) + E_J(\varphi - \delta\varphi_x)\right]$

spin-↑ band spin-↓ band

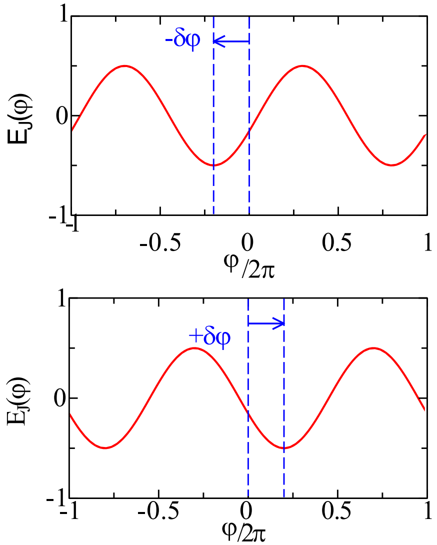
0-π transition for a tunnel contact

Band splitting in a magnetic tunnel junction (Transmission $T \ll 1$)

normal junction

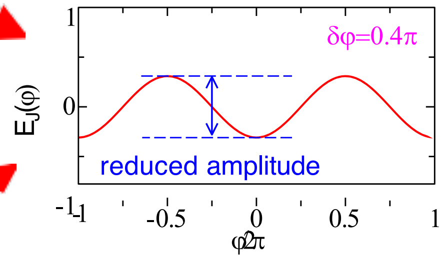


Spin-splitting



Σ

SFS junction

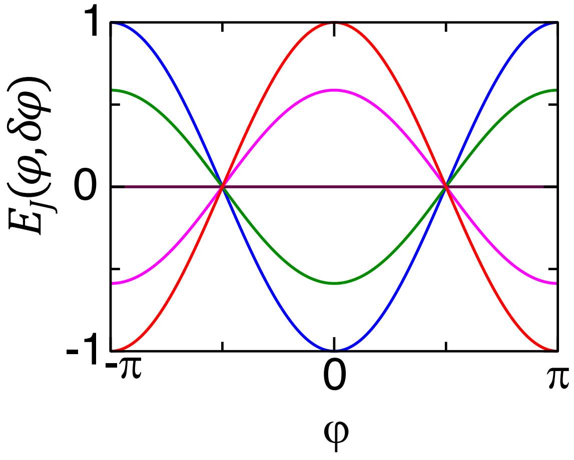


$E_J(\varphi) = -\frac{\hbar}{2e} I_c \cos(\varphi)$

Result: $E_J(\varphi,\delta\varphi) = -\frac{h}{2e} I_c(\delta\varphi)\cos(\varphi)$

$I_c(\delta\varphi) = I_c \cos(\delta\varphi)$

Sign change of $I_c(\delta\varphi)$ for $\delta\varphi > \pi/2$



$\delta\varphi = 0$

$\delta\varphi = \frac{\pi}{4}$

$\delta\varphi = \frac{\pi}{2}$

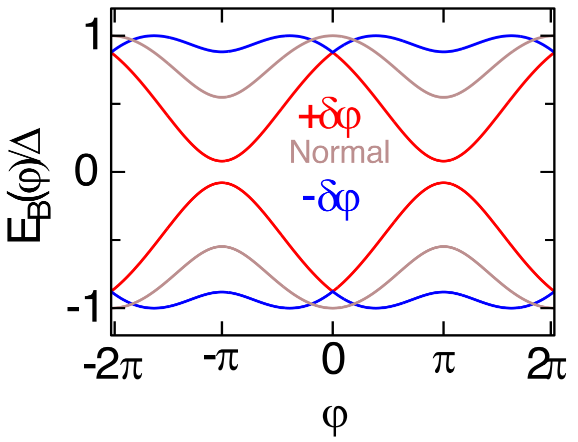
$\delta\varphi = \frac{3\pi}{4}$

$\delta\varphi = \pi$

0-π transition!

0-π transition for non-tunneling contacts

Open contacts: Andreev bound states with transmission T



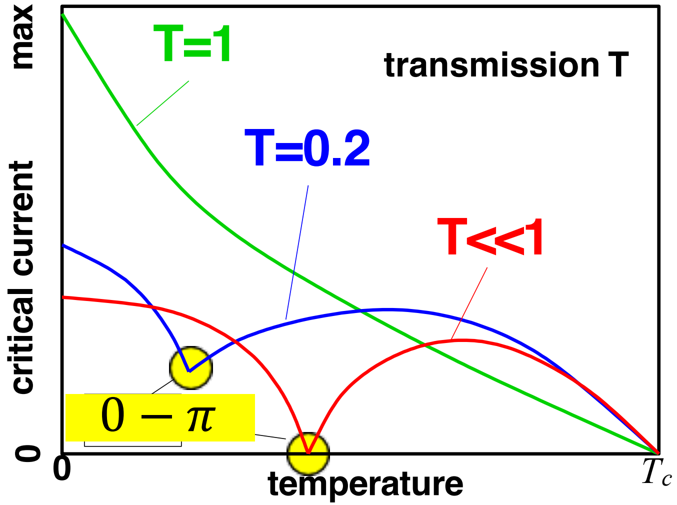
Energy of the bound state:

$E_{B\sigma}(\varphi) = \pm\Delta \cos((\gamma(\varphi) + \sigma\delta\varphi)/2)$

$\cos(\gamma(\varphi)) = 1 - T + T\cos(\varphi)$

➔ depends strongly on phase shift δφ

(Plot: $T = 0.7$, $\delta\varphi = \pi/2$)



Temperature-dependent 0-π transition

- π-phase at high temperature
- non-monotonic temperature dependence
- sharp 0-π-transition
- Ic =0 at transition only for T « 1

[Chtchelkatchev,Belzig,Nazarov,Bruder, JETPL (2001)]

Triplet Cooper pair generation due to spin-dependent scattering

$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle\right)$$

singlet

Fz $\uparrow$ $S_\sigma^z=\begin{pmatrix} r_\sigma^z & t'^z_\sigma \\ t_\sigma^z & r'^z_\sigma \end{pmatrix}$

ϑ_S = spin-dep. phase shift (SDIPS)

$$\frac{1}{\sqrt{2}}\left(t_\downarrow^z t_\uparrow^{z*}|\uparrow\downarrow\rangle - t_\uparrow^z t_\downarrow^{z*}|\downarrow\uparrow\rangle\right)=|t_\uparrow^z t_\downarrow^z|\left[\cos(\vartheta_s)\underbrace{\frac{|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle}{\sqrt{2}}}_{|S=0\rangle}-i\sin(\vartheta_s)\underbrace{\frac{|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle}{\sqrt{2}}}_{|S=1,M=0\rangle}\right]$$

$$|t_\uparrow^z t_\downarrow^z|\frac{-i\sin(\vartheta_s)}{\sqrt{2}}\left(|\uparrow_x\uparrow_x\rangle+|\downarrow_x\downarrow_x\rangle\right)$$

Fx $\rightarrow$ $S_\sigma^x=\begin{pmatrix} r_\sigma^x & t'^x_\sigma \\ t_\sigma^x & r'^x_\sigma \end{pmatrix}$

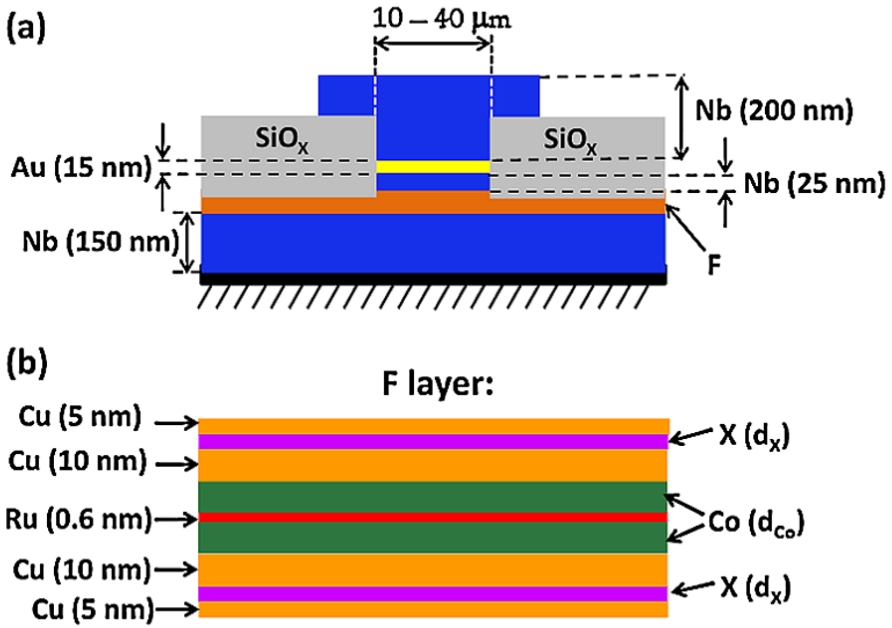
$t_\downarrow^x=0$

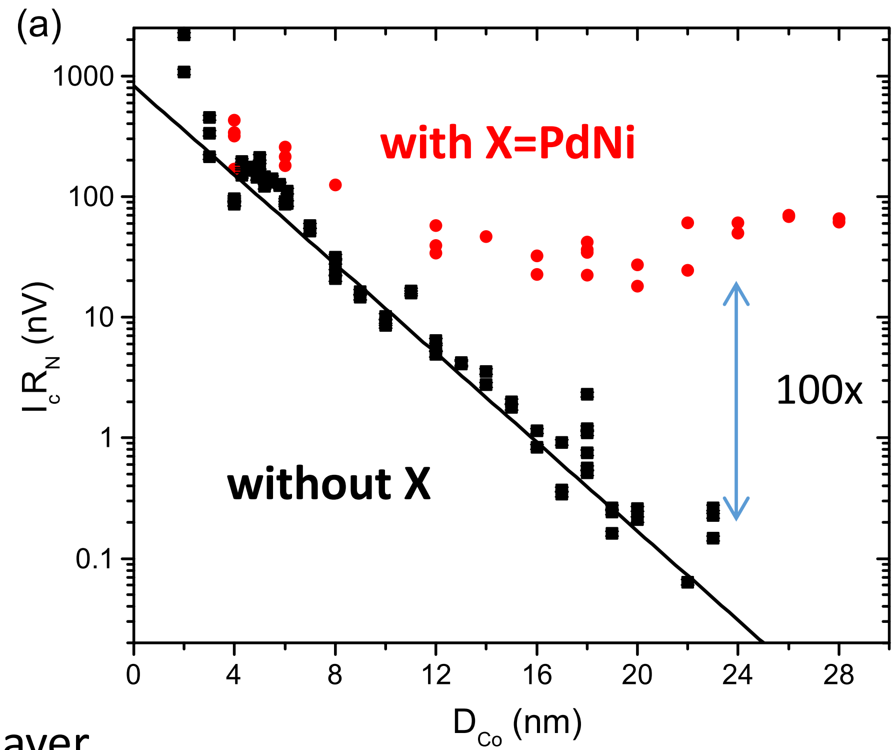
$$|t_\uparrow^x|^2|t_\uparrow^z t_\downarrow^z|\frac{-i\sin(\vartheta_s)}{\sqrt{2}}|\uparrow_x\uparrow_x\rangle$$

Equal spin triplet!

Theories: Bergeret, Volkov, Efetov (01); Eschrig, et al. (03); Nazarov, Heikkilä, Linder, Cottet….

Experiment: demonstration of long-range triplet proximity effect





- Josephson current through a ferromagnetic multilayer
- Adding non-collinear magnetic layers increases the critical current and leads to weaker decay

T. S. Khaire, M. A. Khasawneh, W. P. Pratt, and N. O. Birge, Phys. Rev. Lett. **104**, 137002 (2010)
Simultaneous: J.W.A. Robinson, J.D.S. Witt, and M.G. Blamire, Science **329**, 59 (2010)

Further experiments: Groups of Ryazanov, Kontos, Aprili, Klapwijk, Aarts, Strunk….

Message

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- Properties of superconductors can be efficiently manipulated by ferromagnets
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- Long-range spin-polarized supercurrents by non-collinear ferromagnets
- Andreev bound states are convenient to describe nanostructured superconductors

Topological states in multi-terminal superconducting structures

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Higher dimensional topology in multi-terminal Josephson Matter

- Topology in solid state physics and geometry of quantum states
- Synthetic dimensions in multi-terminal Josephson ‘matter’

I. Microwave spectroscopy of engineered Andreev bound states
   - Engineering Weyl singularities
   - Synthetic circularly polarized spectroscopy

II. Higher-dimensional topology: the 4D QHE and Josephson matter
   - Construction of degenerate states with nontrivial 2nd Chern number
   - Non-Abelian Berry phase

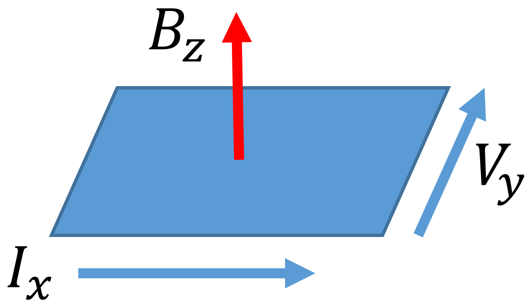
III. Fractional Cooper pair pump (analog fractional quantum Hall effect)

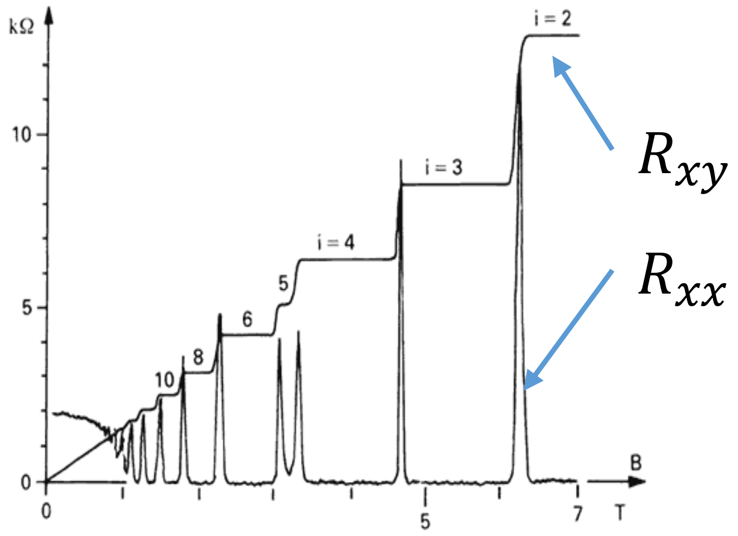
Topology in solid state physics: Quantum Hall Effect

- Two-dimensional electron gas in a strong magnetic field
- Quantized Hall conductance: [von Klitzing et al., PRL (1980)]

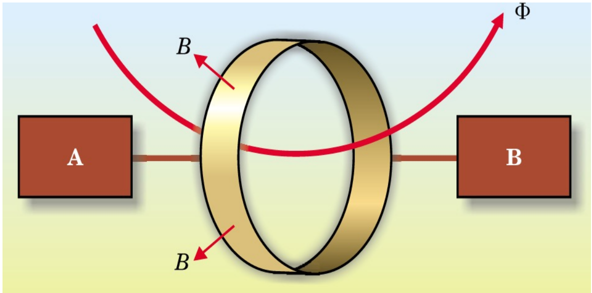
$$R_{xy} = \frac{V_y}{I_x} = \frac{1}{N}\frac{h}{e^2} \qquad N = 1,2,3,\ldots$$

- Quantization is exact: accuracy $\sim 10^{-6}$, despite disorder!





[nobelprize.org]



[Laughlin & Physics Today (2003)]

Adiabatic Pumping current:

(1e per flux quantum $\Phi_0 = \frac{h}{e}$)

$$I_{A\to B} = I_x = ef = e\frac{\dot{\Phi}}{\Phi_0} = \frac{e^2}{h}V_y$$

<u>Thouless (1982):</u>

Quantization is **topological** $G_{xy} = \frac{e^2}{h}C$

C = Chern number of the Landau level(s)

Berry curvature

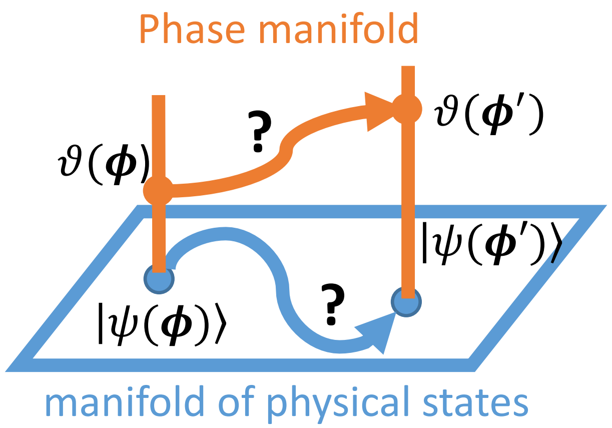
$$F_{xy} = \partial_{k_x}\langle\psi|\partial_{k_y}\psi\rangle - \partial_{k_y}\langle\psi|\partial_{k_x}\psi\rangle$$

Chern number

$$C = \frac{1}{2\pi}\int d^2k\, F_{xy} \in \mathbb{Z}$$

Quantum Geometry, Berry Phase and Topology

- Quantum states $|\psi(\boldsymbol{\phi})\rangle$ of a Hamiltonian $H(\boldsymbol{\phi})$ depend on parameters $\boldsymbol{\phi}$
- Quantum geometry: quantify the difference $|\psi(\boldsymbol{\phi})\rangle$ and $|\psi(\boldsymbol{\phi}+d\boldsymbol{\phi})\rangle$
- Naive answer: Berry connection $\boldsymbol{A}(\boldsymbol{\phi}) = i\langle\psi(\boldsymbol{\phi})|\partial_{\boldsymbol{\phi}}|\psi(\boldsymbol{\phi})\rangle$
- Gauge-invariant quantity: **Quantum geometric tensor (QGT)** χ

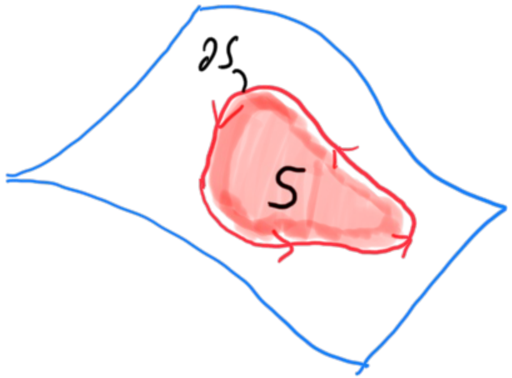


$$\boxed{\chi_{jk} = \langle\partial_j\psi|(1-|\psi\rangle\langle\psi|)|\partial_k\psi\rangle = g_{jk} - \frac{i}{2}F_{jk}}$$

$$|\partial_j\psi\rangle = \frac{\partial}{\partial\phi_j}|\psi\rangle$$

Symmetric part: Fubini-Study **quantum metric** (measures distance between physical states)

Antisymmetric part: **Berry curvature** (contains information about phase)

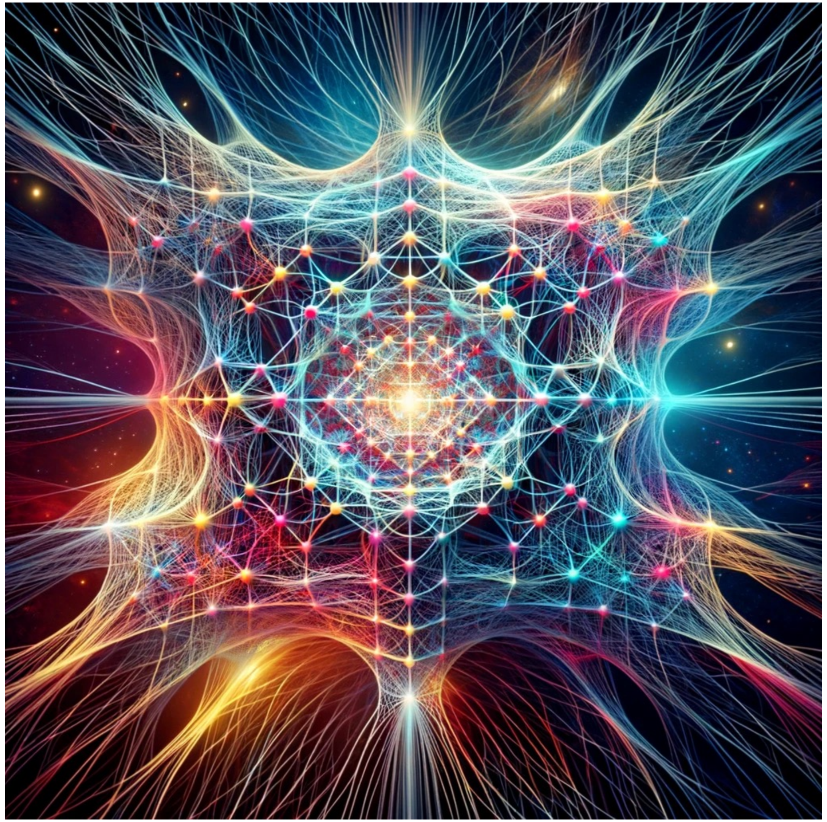


Berry phase: $\vartheta_{geo} = \oint_{\partial S} \boldsymbol{A}d\boldsymbol{\phi}$ $\rightarrow |\psi\rangle \xrightarrow{\partial S} e^{i\vartheta_{geo}}|\psi\rangle$ with $\vartheta_{geo} = \int_S F_{jk}d\phi_j d\phi_k$

Topology: First Chern number $C = \frac{1}{2\pi}\oiint_S F_{jk}\,\mathrm{d}\phi_j\,\mathrm{d}\phi_k \in \mathbb{Z}$ → global property

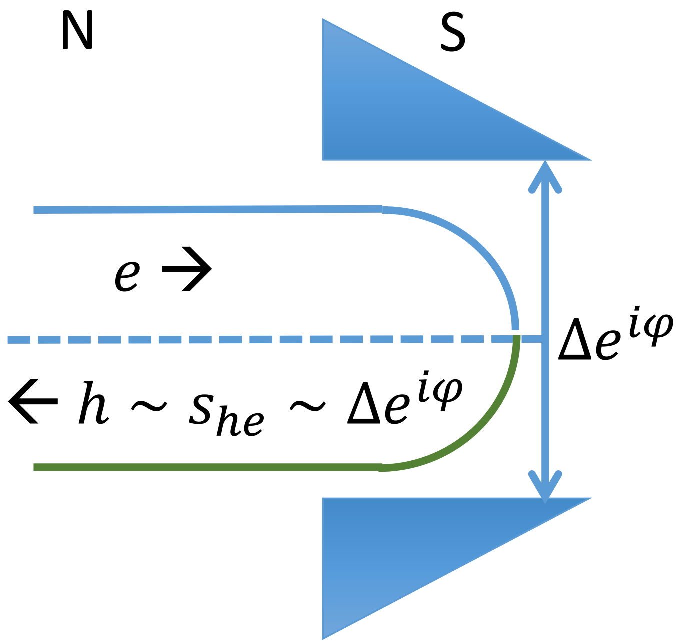
[M. Kolodrubetz et al., *Phys. Rep.* **697** (2017)]

Topological Josephson ‚Matter'



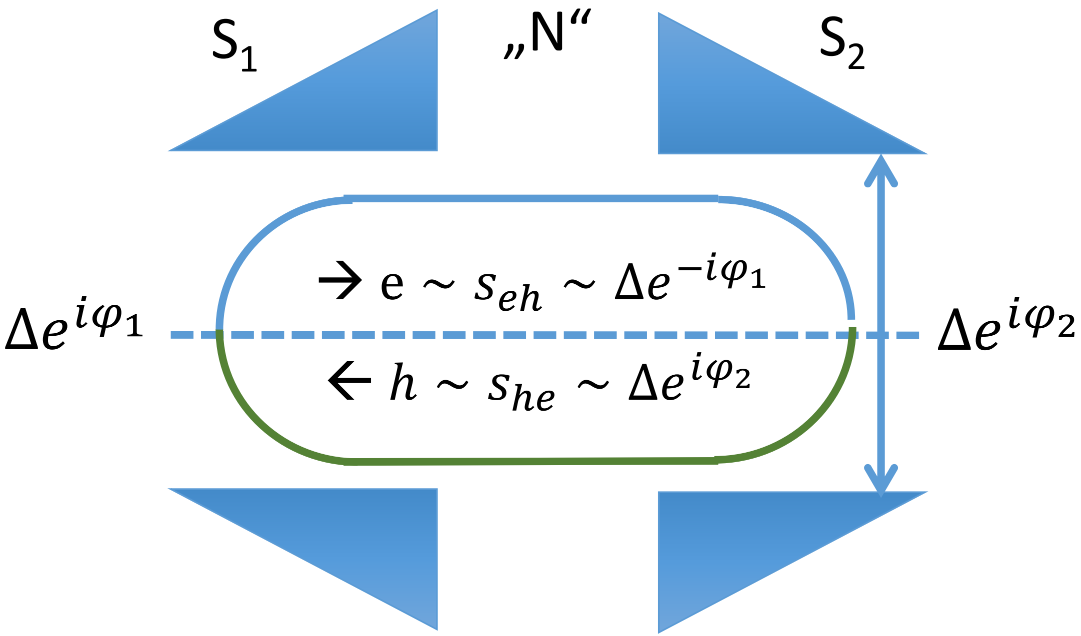
Andreev reflection and Andreev bound states (ABS)

Andreev reflection



Phase-conserving transition:
2 electrons ←→ Cooper pair

Andreev bound states



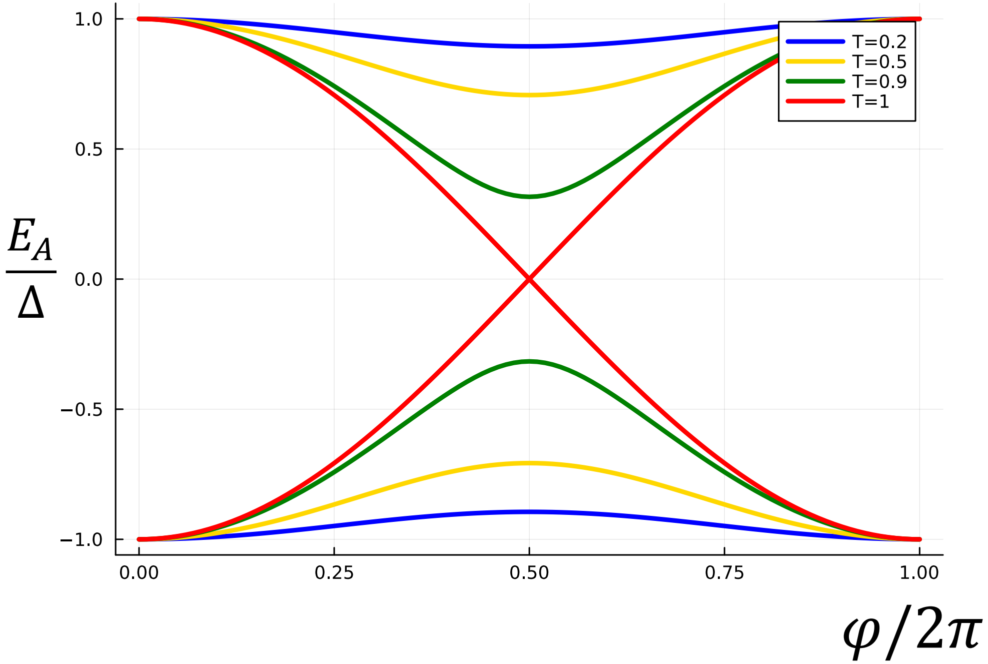
Constructive interference → bound state

Energy $E_A = \pm\Delta \cos\left[\frac{\varphi_1-\varphi_2}{2}\right]$

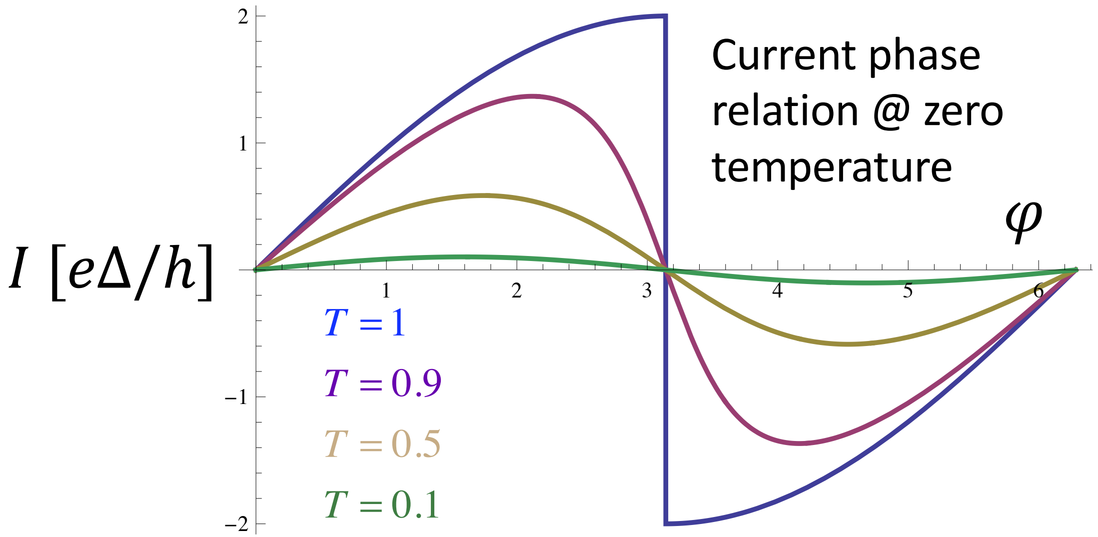
Josephson effect and Andreev bound states

Energy with transmission probability T:

$$E_A = \pm\Delta\sqrt{1 - T\sin^2\varphi/2}$$



Current



Current phase relation @ zero temperature

$$I = \frac{2e}{h}\frac{\partial E_A}{\partial\varphi} = \begin{cases} \frac{2e}{h}\Delta\sin\frac{\varphi}{2} & T = 1 \\ \frac{2eT}{h}\Delta\sin\varphi & T \ll 1 \end{cases}$$

Non-sinusoidal

"Josephson"

➔ Accidental zero-crossing

Weyl nodes (=3D Dirac cone)

Robustness of geometric singularities (1-D vs 3-D):

- 1D: **any** small perturbation

  → avoided crossing (unless symmetry protection)

- 3D: possible Weyl nodes are stable **without** any symmetry requirement

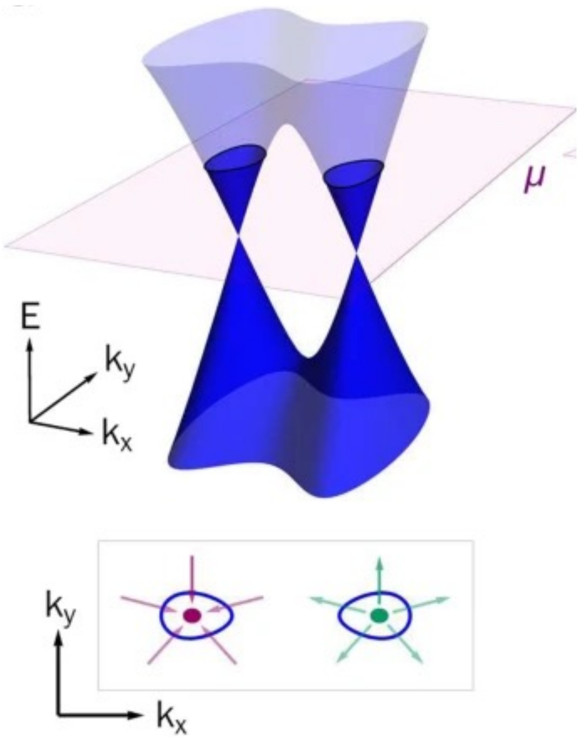
Weyl node: $\vec{k}$ = distance from singularity

$$H_W \approx v_F \vec{k}\hat{\vec{\sigma}} = v_F \begin{pmatrix} k_z & k_x - ik_y \\ k_x + ik_y & -k_z \end{pmatrix}$$

➔ Any perturbation → shift of Weyl node

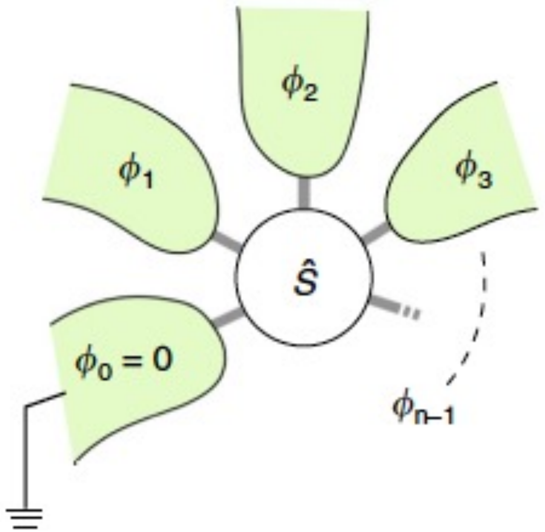
E.g. Weyl semimetal

$k_z = 0$

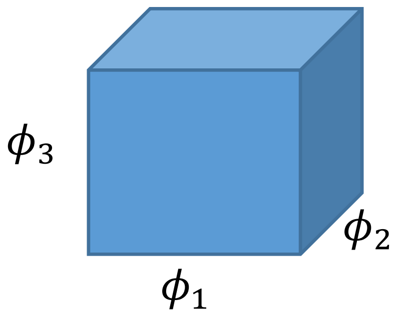


[Sci. Rep. 9, 2095 (2019)]

Topology in Multiterminal Josephson Junctions



- Scattering matrix $\hat{S}$ connecting **n superconductors**
- ABS determined by **n-1 superconducting phase differences:** $\boldsymbol{\phi} = (\phi_1 \quad \phi_2 \quad \phi_3 \ \dots)$
- **Synthetic (n-1)-dimensional** Brillouin zone
- Andreev bound states: $E_A = E_A(\boldsymbol{\phi})$



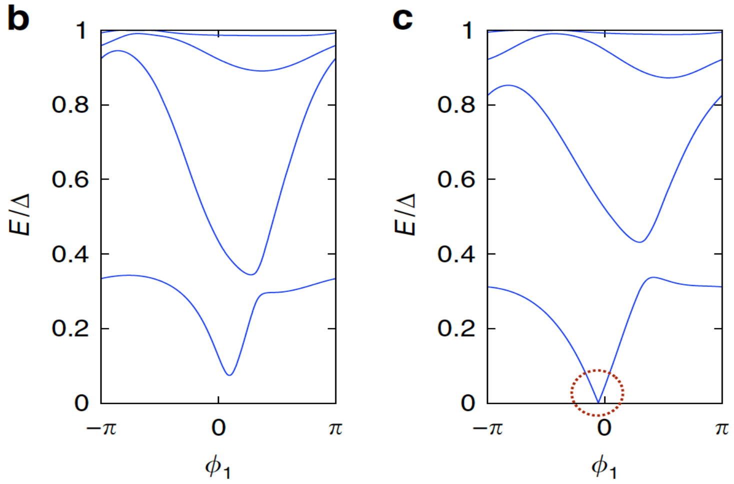
Search for Weyl nodes

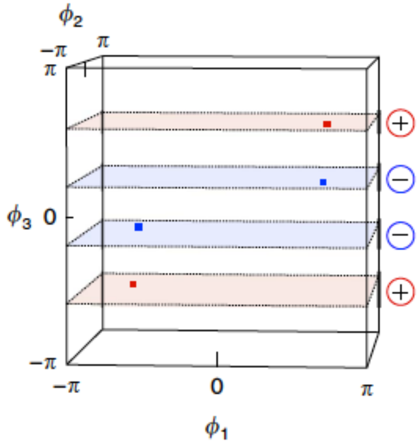
$$E_A(\boldsymbol{\phi_W}) = 0$$

Around each Weyl point:
Approximate 3D-Dirac cone

$$H_W \sim \delta\vec{\phi}\vec{\hat{\sigma}}$$

Randomly generated scattering matrices:





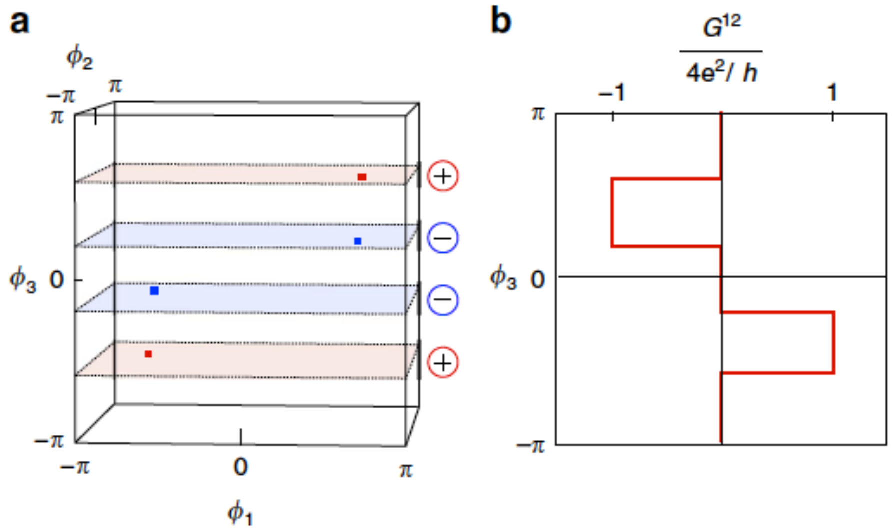
[Riwar, Meyer, Houzet, Nazarov, *Nat. Commun.* **7**, 11167 (2016)]

Topology in Josephson matter and synthetic QHE

Transconductance measurement: $\langle I_1 \rangle = G^{12} V_2$

- $\langle I_1 \rangle = \frac{1}{2\pi} \int I(\phi_1, \phi_2) d\phi_1 d\phi_2$
- $\dot{\phi}_2 = 2eV_2$
- $I_1(\phi_1, \phi_2) = \frac{2e}{h} \frac{\partial E_A}{\partial \phi_1} - \frac{2e}{h} F_{12} \dot{\phi}_2$

adiabatic non-adiabatic



[R.-P. Riwar et al., *Nat. Commun.* **7**, 11167 (2016).]

Further works by:
Meyer, Houzet, Riwar, Nazarov (PRL, PRB, 17-22)
Fatemi, Akhmerov, Brethau (PRR 21)
Peyruchat, Griesmar, Pillet, Girit (PRR 21)
…

Chern number $C = \sum_k C^k (n_k - \frac{1}{2})$ with

$$C^k = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\phi_1 \int_{-\pi}^{\pi} d\phi_2 \; F_{12}^k$$

Quantized transconductance

$$G^{12} = \frac{d\langle I_1 \rangle}{dV_2} = -\frac{4e^2}{h} C$$

I. Microwave spectroscopy of topological Andreev bound states

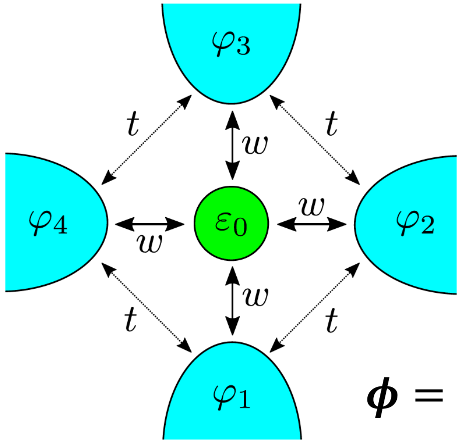
R. L. Klees, G. Rastelli, J. C. Cuevas, and W. Belzig

Microwave spectroscopy reveals the quantum geometric tensor of topological Josephson matter

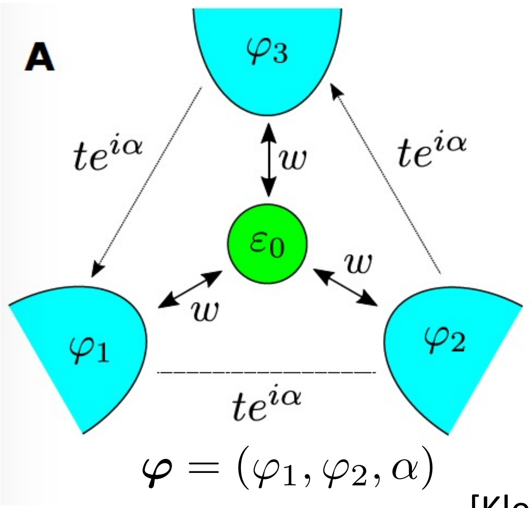
Phys. Rev. Lett. **124**, 197002 (2020)

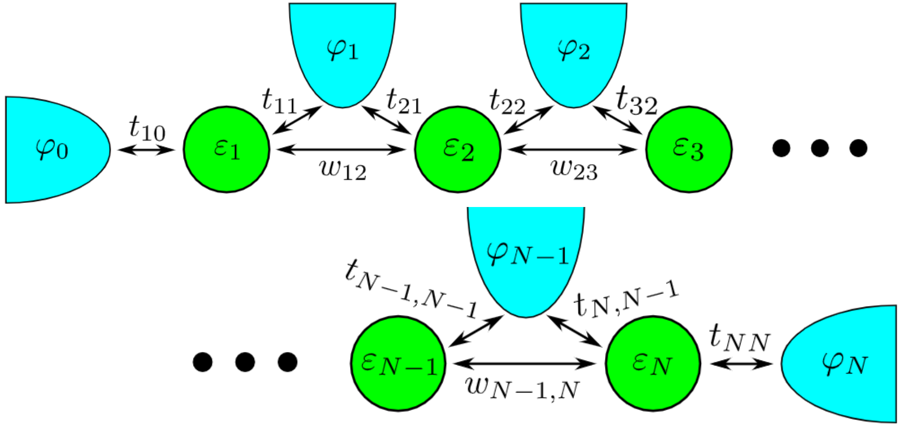
Engineering Weyl singularities

(Our) Proposed microscopic models of multiterminal Josephson junctions:



$$\boldsymbol{\phi} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}$$





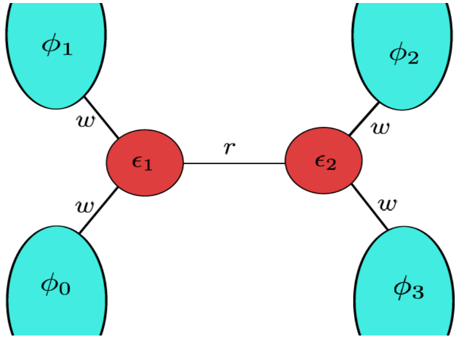
[Klees, Cuevas, WB, and Rastelli, Phys. Rev. B **103**, 014516 (2021)]

Hamiltonian of the dot →
pseudo-spin in an effective magnetic field

$$H_{\mathrm{D}} = \boldsymbol{d}(\boldsymbol{\phi}) \cdot \boldsymbol{\sigma}$$

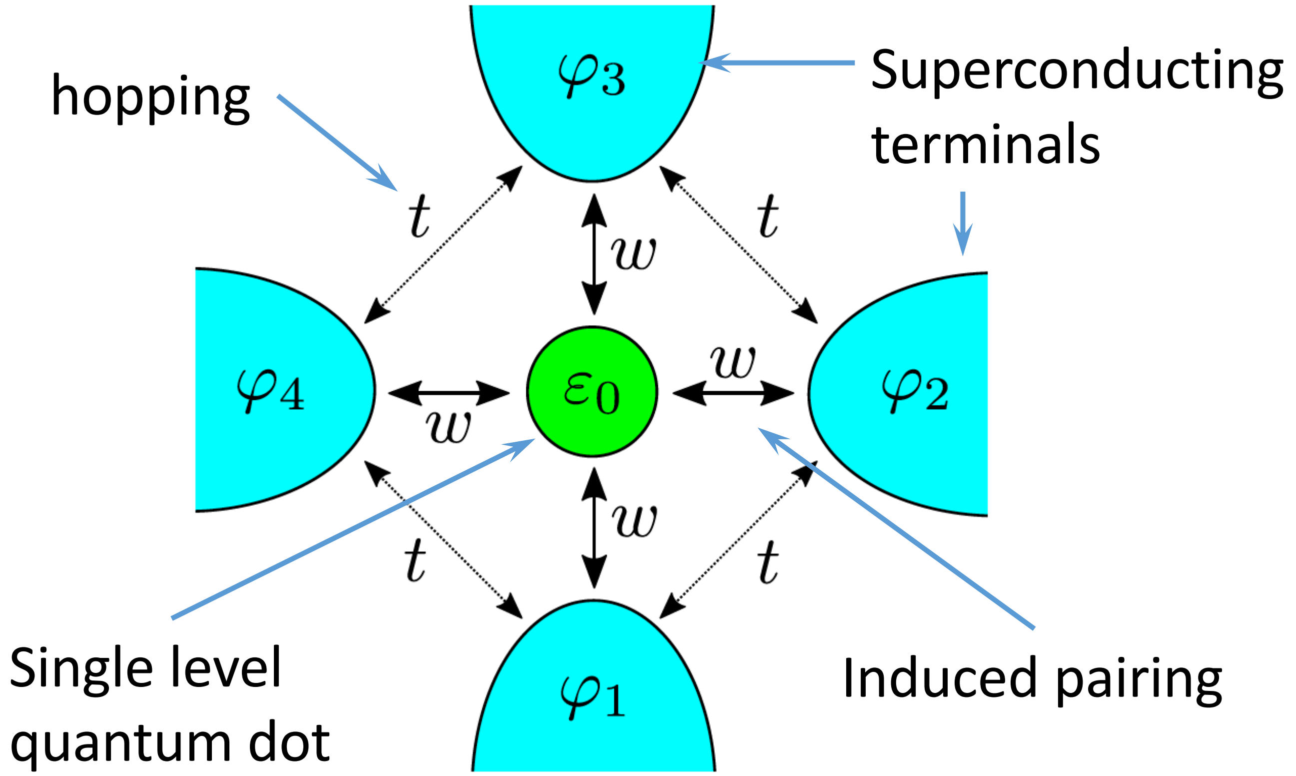
[Klees, Rastelli, Cuevas, WB,
Phys. Rev. Lett. **124**, 197002 (2020)]

Finally:
the simplest model ….



[Teshler, Weisbrich, Sturm, Klees, Rastelli,
WB, SciPost Phys. **15**, 214 (2023)]

4-Terminal Josephson Junction



ABS Hamiltonian ($E \ll \Delta$) =
Spin-½ in an effective magnetic field
controlled by superconducting phases

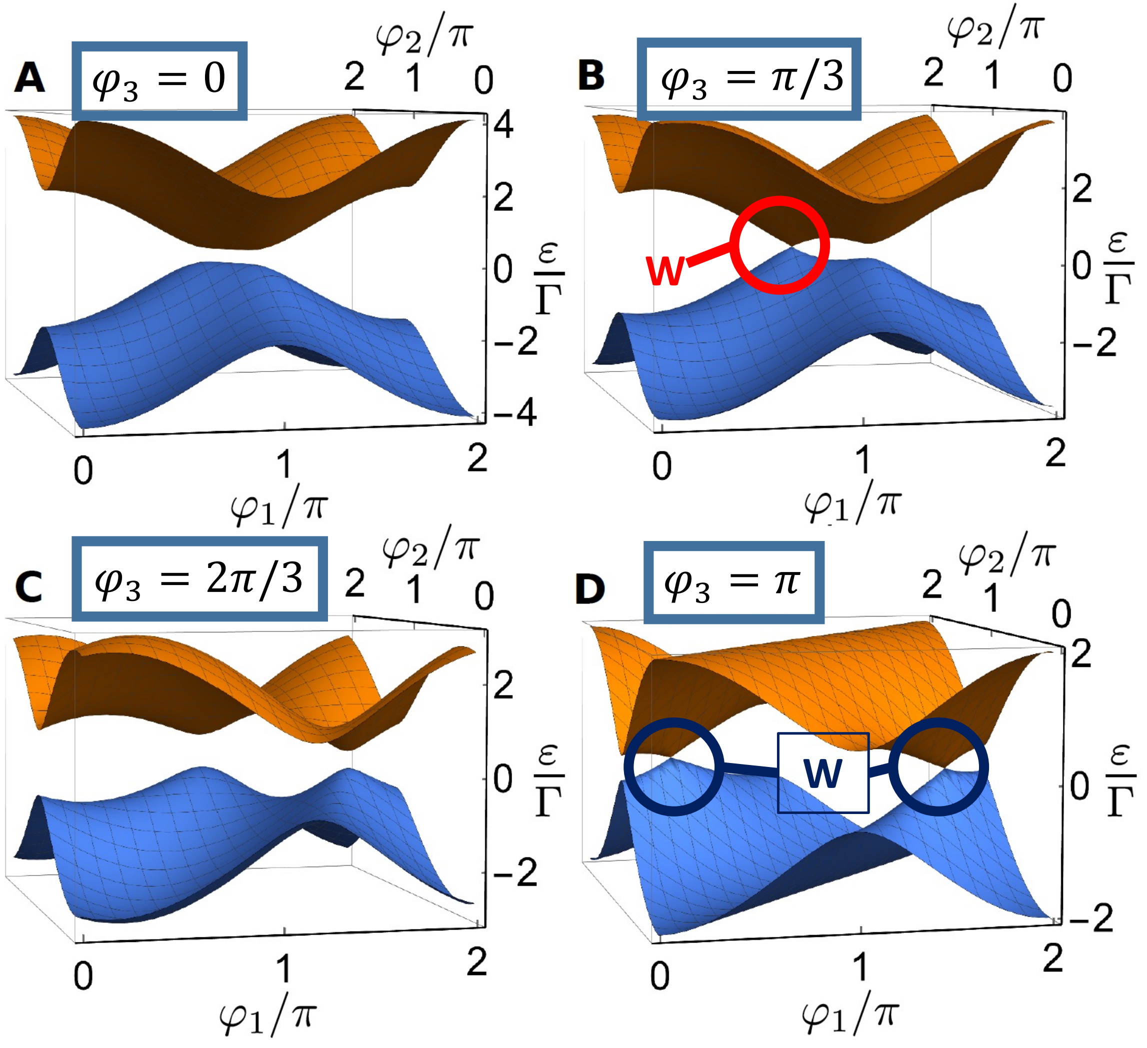
$$\begin{aligned} H &= \omega - G^{-1} \\ &= \epsilon_0 \hat{\sigma}_3 - \hat{\Sigma}(\phi) \\ &= \boldsymbol{d}(\boldsymbol{\phi})\hat{\boldsymbol{\sigma}} \end{aligned}$$

$$\Gamma = \pi\, N_0\, w^2 \qquad t_0 = \pi\, N_0\, t \ll 1$$

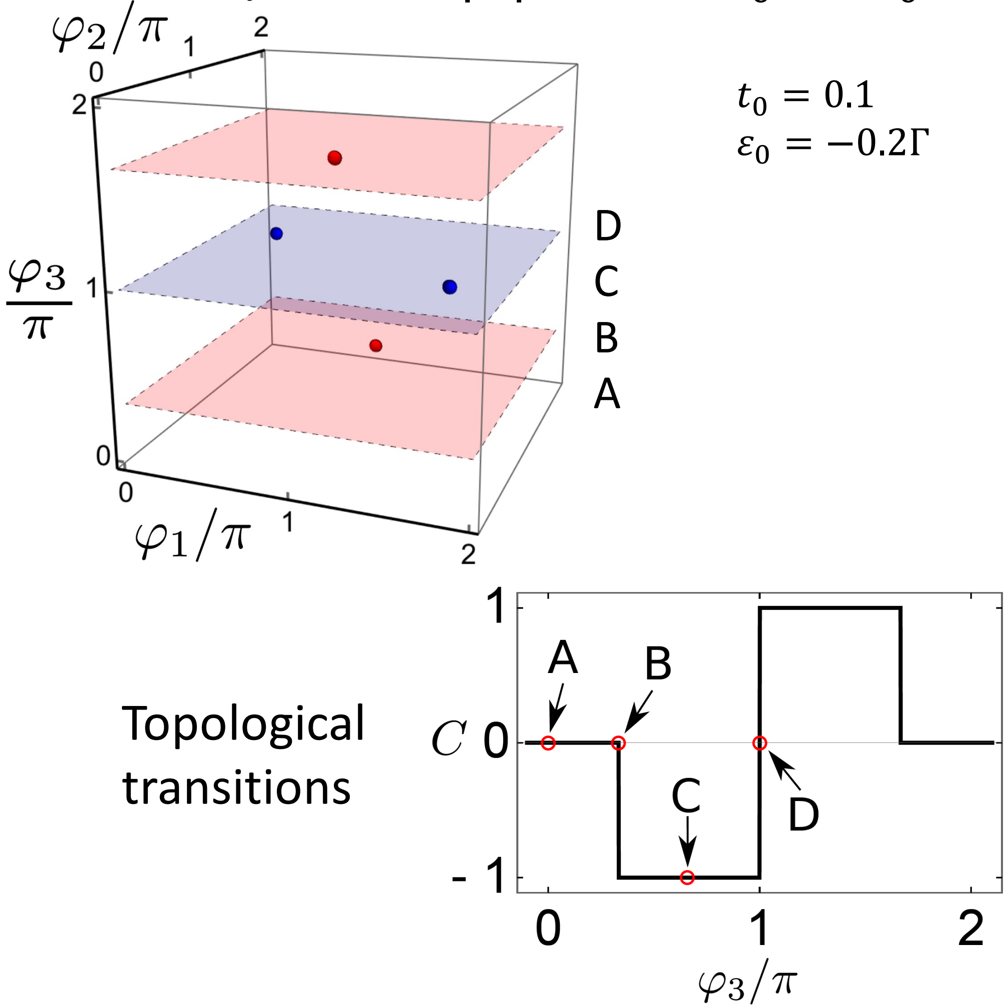
$$\boldsymbol{d}(\boldsymbol{\phi}) = \begin{pmatrix} \Gamma \sum_j \cos\varphi_j \\ -\Gamma \sum_j \sin\varphi_j \\ \varepsilon_0 - 2t_0\Gamma \sum_j \cos(\varphi_j - \varphi_{j+1}) \end{pmatrix}$$

[Klees, Rastelli, Cuevas, WB, Phys. Rev. Lett. **124**, 197002 (2020)]

4T Josephson junction: spectrum, Weyl nodes, topology



Existence of Weyl nodes? $|\boldsymbol{d}|$=0 → $-8t_0\Gamma < \varepsilon_0 < 0$



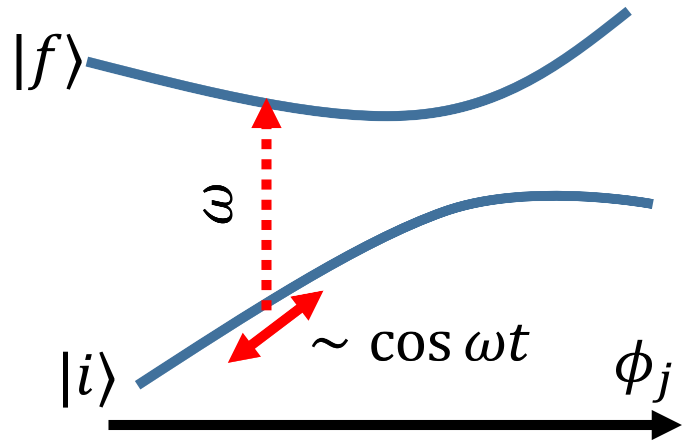
Topological transitions

Klees et al., arxiv

Spectroscopy and quantum geometry

- Hamiltonian $H(\boldsymbol{\phi})$ which depends on set of parameters $\boldsymbol{\phi} = \{\phi_j\}$
- Restriction to two levels (ground state $|i\rangle$ and excited state $|f\rangle$)
- Quantum geometric tensor $\chi_{jk} = g_{jk} - \frac{i}{2}F_{jk} = \langle \partial_j i | f \rangle \langle f | \partial_k i \rangle$
- Diagonal elements $g_{jj} = \left| \langle f | \partial_j i \rangle \right|^2$
- **Drive of one parameter:** $\phi_j \to \phi_j + (2E/\hbar\omega)\cos(\omega t)$, $E/\hbar\omega \ll 1$
- Fermi's Golden Rule:

$$\boxed{R_{jj}(\omega) = \frac{2\pi}{\hbar}\frac{E^2}{(\hbar\omega)^2}\left|\left\langle f \left| \frac{\partial H}{\partial \phi_j} \right| i \right\rangle\right|^2 \delta(E_f - E_i - \hbar\omega)}$$



Equivalence to quantum geometry by: $\left\langle f \left| \frac{\partial H}{\partial \phi_j} \right| i \right\rangle = (E_i - E_f)\,\langle f | \partial_j i \rangle$

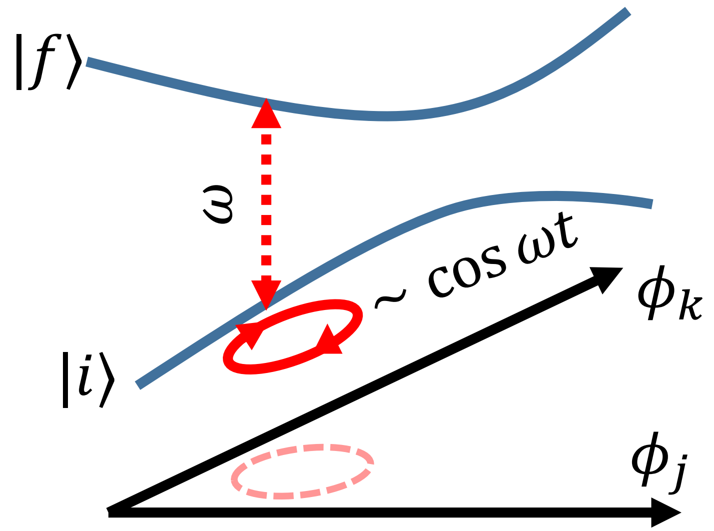
$$\boxed{\Rightarrow R_{jj}(\omega) = \frac{2\pi\, E^2}{\hbar^2} g_{jj}\delta\left(E_f - E_i - \hbar\omega\right) \equiv \bar{R}_{jj}\delta\left(E_f - E_i - \hbar\omega\right)}$$

Intensity $\bar{R}_{jj} \mathrel{\widehat{=}}$ Diagonal elements of the quantum metric!

[see also Ozawa et al., *Phys. Rev. B* (2018)]

Synthetically polarized (Berry) spectroscopy

- Hamiltonian $H(\boldsymbol{\phi})$ depending on set of parameters $\boldsymbol{\phi} = \{\phi_j\}$
- Quantum geometric tensor in two-level approximation $\chi_{jk} = \langle \partial_j i | f \rangle \langle f | \partial_k i \rangle$
- Drive of **two parameters:** $\phi_j \to \phi_j + (2E/\hbar\omega)\cos(\omega t)$,
  with a **phase shift** γ $\quad \phi_k \to \phi_k + (2E/\hbar\omega)\cos(\omega t - \gamma)$
- Fermi's Golden Rule (generalized)



$$R_{jk}^{(\gamma)}(\omega) = \frac{2\pi}{\hbar}\frac{E^2}{(\hbar\omega)^2}\left|\left\langle f\left|\frac{\partial H}{\partial \phi_j} + e^{i\gamma}\frac{\partial H}{\partial \phi_k}\right| i\right\rangle\right|^2 \delta(E_f - E_i - \hbar\omega)$$

$$= \frac{2\pi}{\hbar}E^2\left(g_{jj} + g_{kk} + 2g_{jk}\cos\gamma + F_{jk}\sin\gamma\right)\delta(E_f - E_i - \hbar\omega)$$

➔ Depends on **all** components of the **quantum geometry tensor**

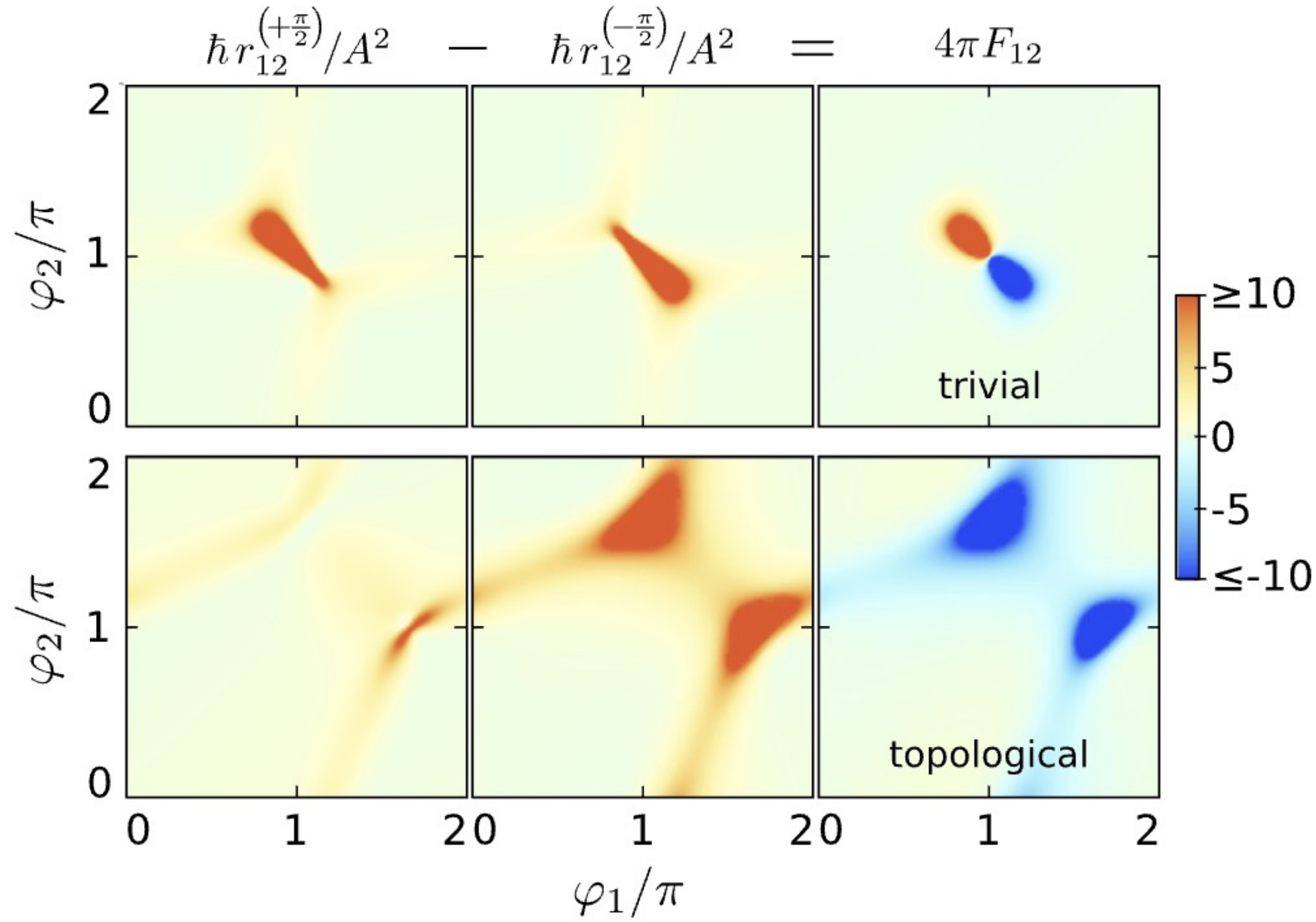
Combining different phase shifts: $\quad \bar{R}_{jk}^{(\pi/2)} - \bar{R}_{jk}^{(-\pi/2)} \sim F_{jk} \qquad \bar{R}_{jk}^{(0)} - \bar{R}_{jk}^{(\pi)} \sim g_{jk}$

E.g. Berry curvature from normalized intensity difference

$$\boxed{F_{jk} = \frac{1}{4\pi E^2}\left[\bar{R}_{jk}^{(\pi/2)} - \bar{R}_{jk}^{(-\pi/2)}\right] \to C(\varphi_3) = \frac{1}{2\pi}\int d\varphi_1 d\varphi_2 F_{12}(\boldsymbol{\varphi})}$$

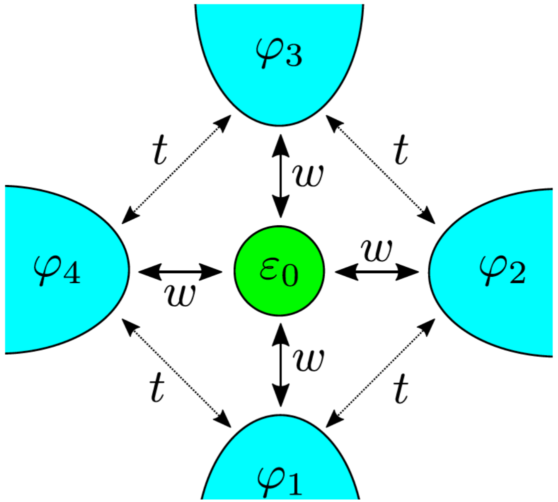
Measurement of the ground state **quantum geometry** in multiterminal Josephson junctions by **synthetically polarized microwave spectroscopy!**

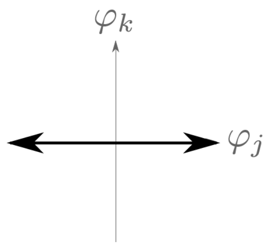
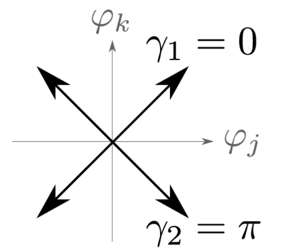
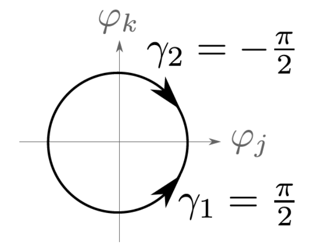
Distribution of Berry curvature:

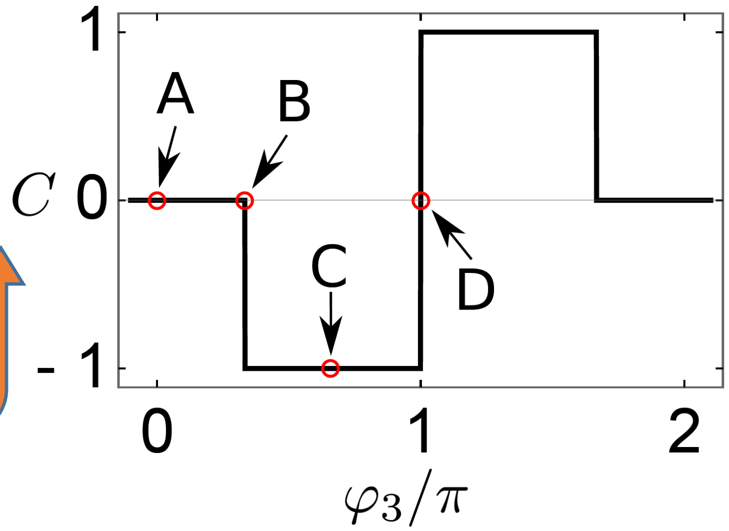


Summary: Berry spectroscopy of multi-terminal Andreev bound states

- Topology in multi-terminal Josephson matter
- 4-terminal junction with multiple paths shows Weyl-nodes and non-trivial topology
- Generalized microwave spectroscopy allows access to the full quantum geometric tensor (including quantum metric, Berry curvature and Chern number)



one phase	two phases	
	linear	circular
		
$R_{jj} \xrightarrow{\int d\omega} g_{jj}$	$R^{(0)}_{jk} - R^{(\pi)}_{jk}$ $\xrightarrow{\int d\omega} g_{jk}$	$R^{(+\frac{\pi}{2})}_{jk} - R^{(-\frac{\pi}{2})}_{jk}$ $\xrightarrow{\int d\omega} F_{jk}$



Experimental status:

nature communications

Article https://doi.org/10.1038/s41467-023-42356-6

Phase-engineering the Andreev band structure of a three-terminal Josephson junction

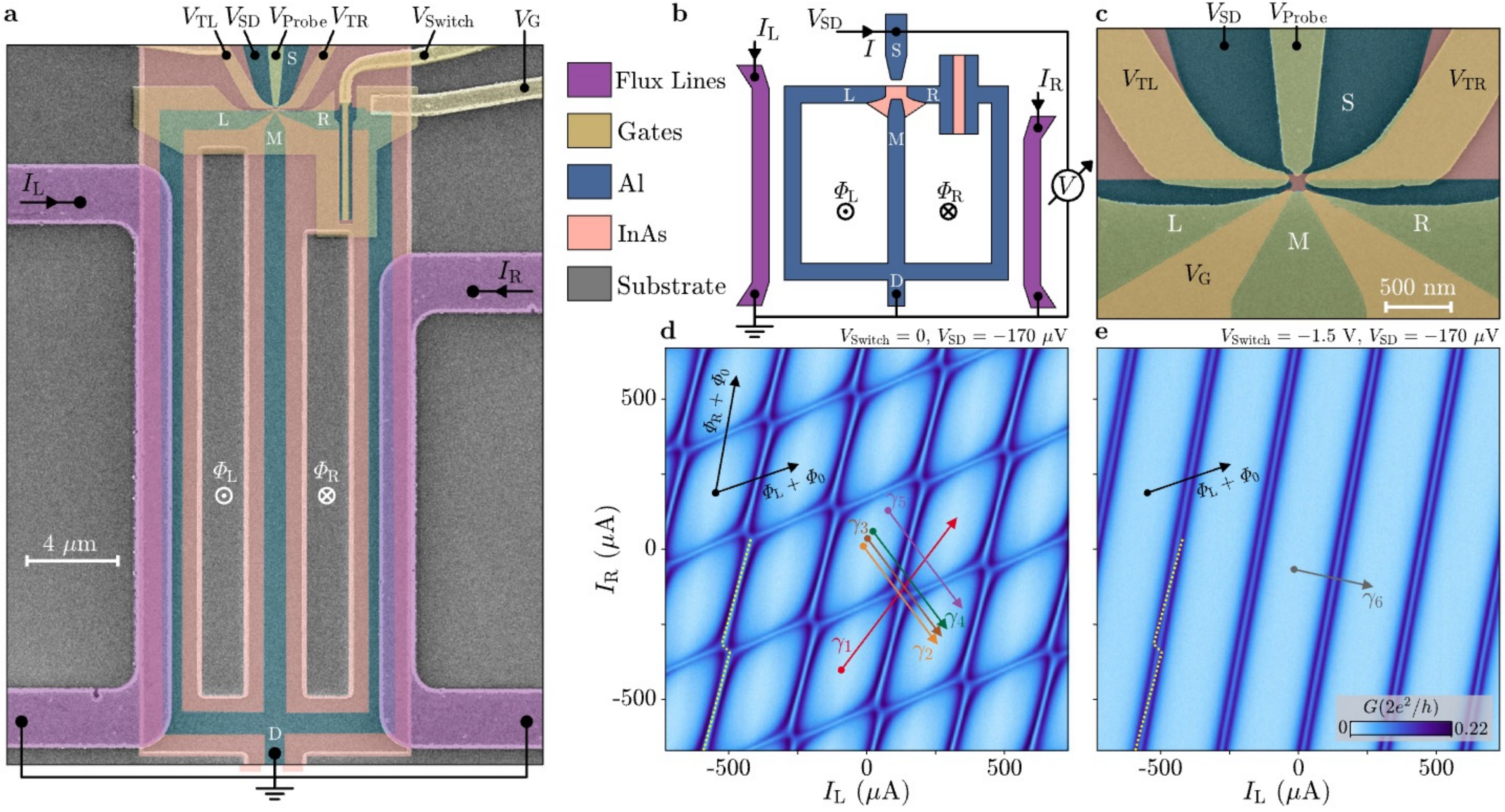
Received: 2 May 2023

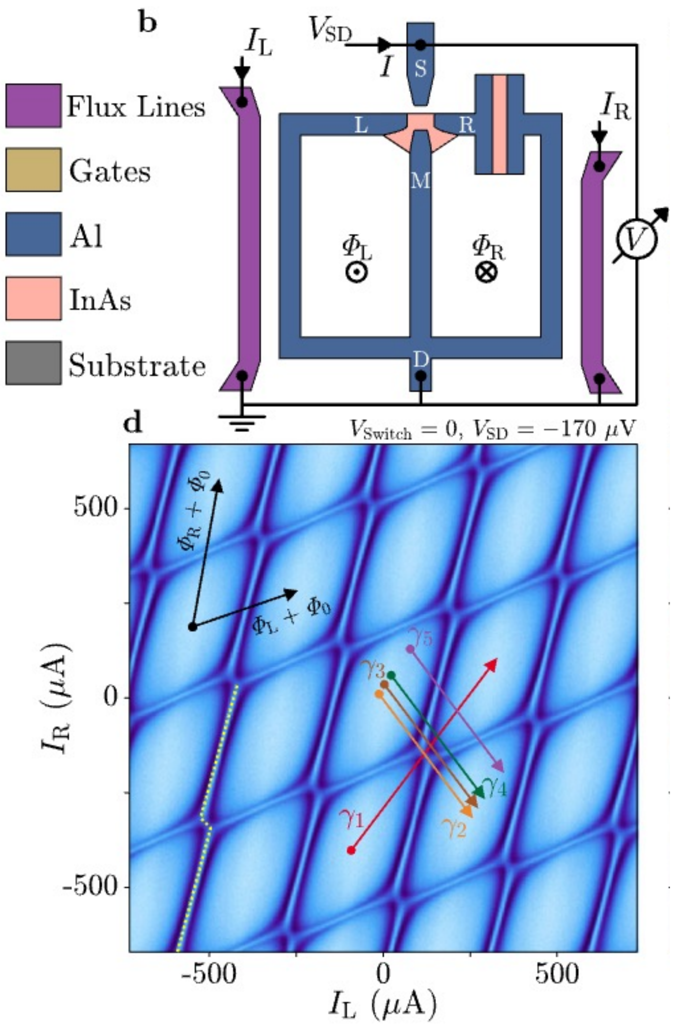
Accepted: 9 October 2023

Published online: 25 October 2023

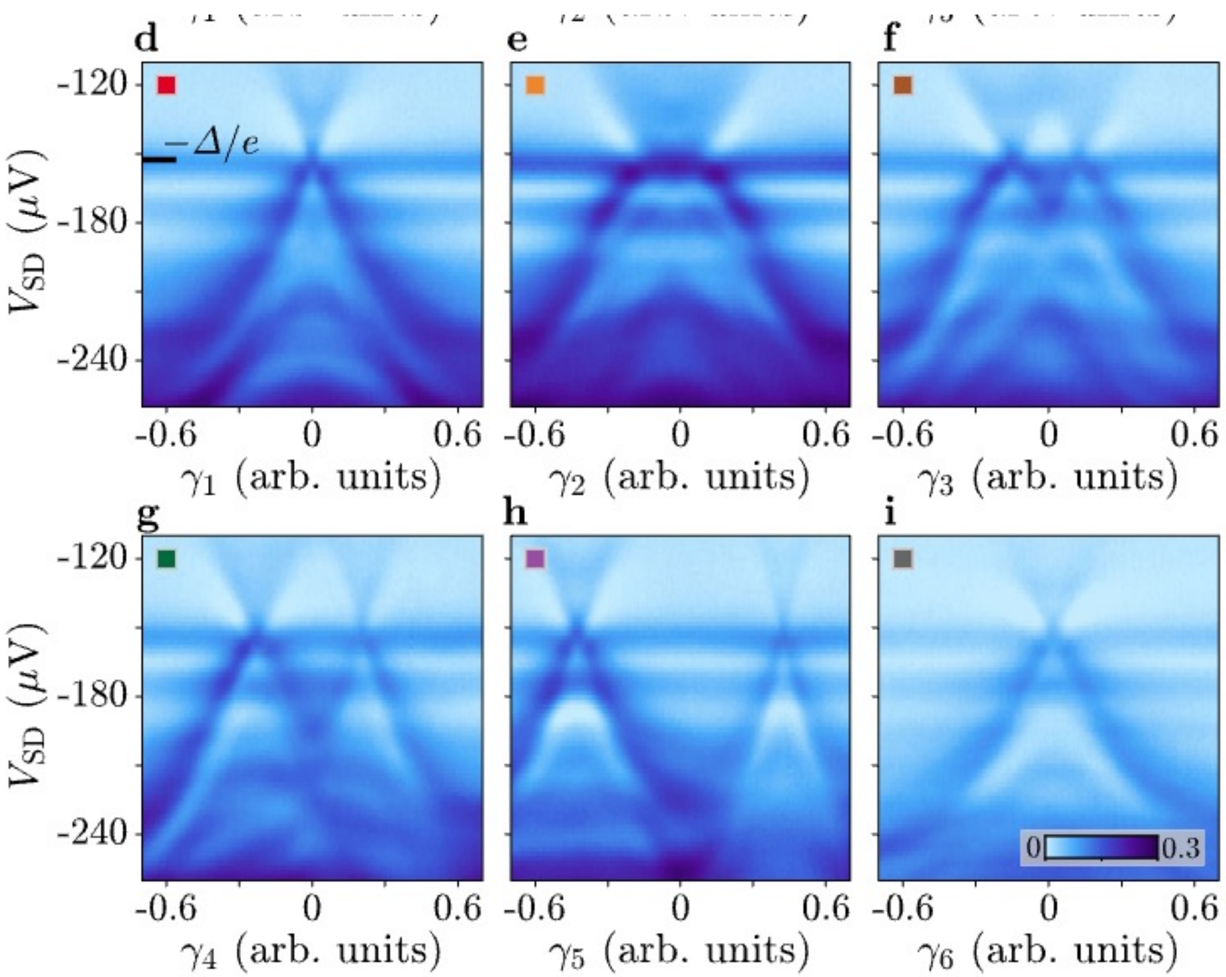
Marco Coraiola[1], Daniel Z. Haxell[1], Deividas Sabonis[1], Hannes Weisbrich[2], Aleksandr E. Svetogorov[2], Manuel Hinderling[1], Sofieke C. ten Kate[1], Erik Cheah[3], Filip Krizek[1,3,5], Rüdiger Schott[3], Werner Wegscheider[3], Juan Carlos Cuevas[4], Wolfgang Belzig[2] & Fabrizio Nichele[1]

Conventional tunneling spectroscopy

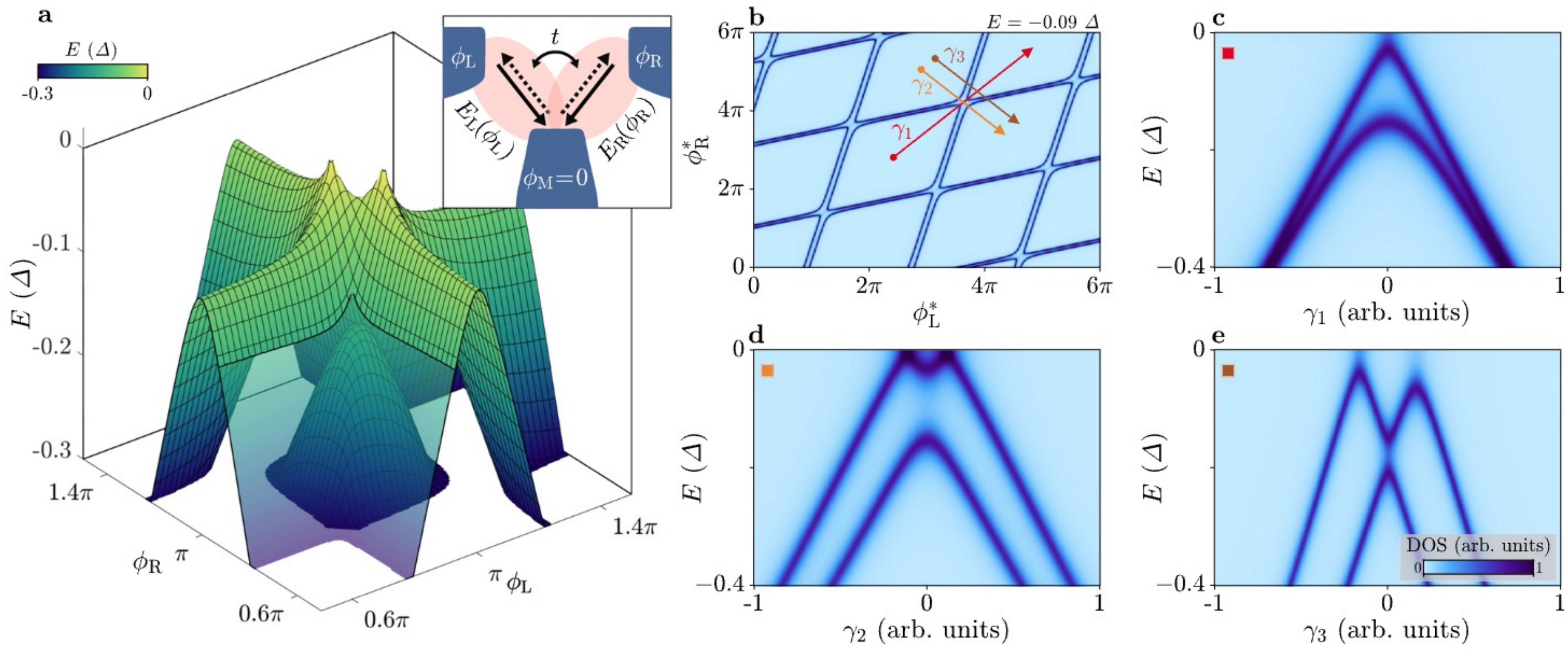




b
I_L
V_{SD}
I
S
I_R
Flux Lines
Gates
Al
InAs
Substrate
L
R
M
Φ_L
Φ_R
D
d
$V_{Switch} = 0$, $V_{SD} = -170\ \mu V$
500
0
-500
$I_R\ (\mu A)$
$\Phi_R + \Phi_0$
$\Phi_L + \Phi_0$
γ_1
γ_2
γ_3
γ_4
γ_5
-500
0
500
$I_L\ (\mu A)$

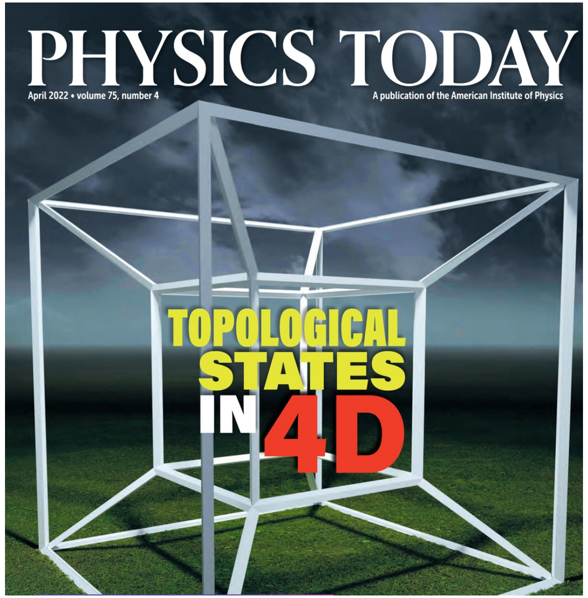


d
e
f
g
h
i
$-\Delta/e$
-120
-180
-240
$V_{SD}\ (\mu V)$
-0.6
0
0.6
γ_1 (arb. units)
γ_2 (arb. units)
γ_3 (arb. units)
γ_4 (arb. units)
γ_5 (arb. units)
γ_6 (arb. units)
0
0.3

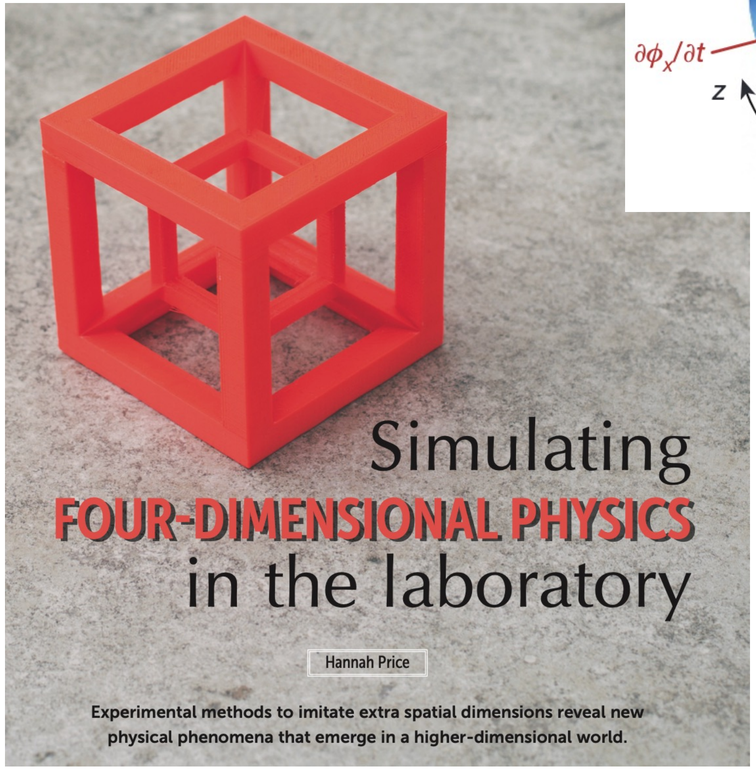


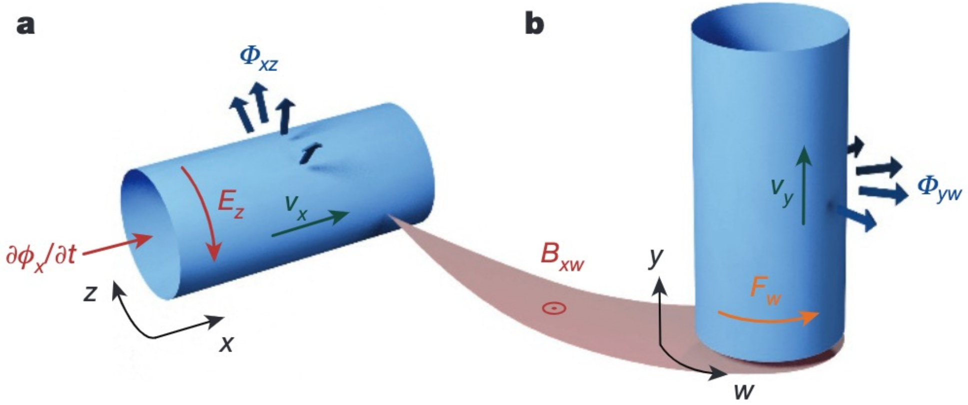
To do: four superconducting terminals…..

Higher-Dimensional Topology



April 2022





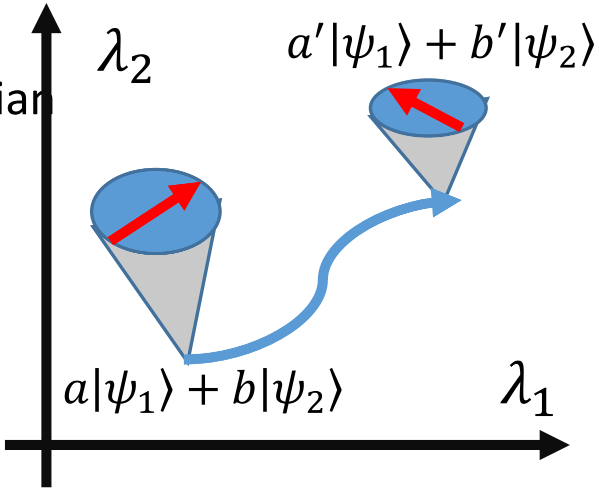
The 4D Quantum Hall Effect

- First idea: Zhang, Hu [Science (2001)]
- Idea for realization in ultracold atoms [Price et al., PRL (2015)].
- Realized experimentally [Lohse et al.; Zilberberg et al., Nature (2018)]

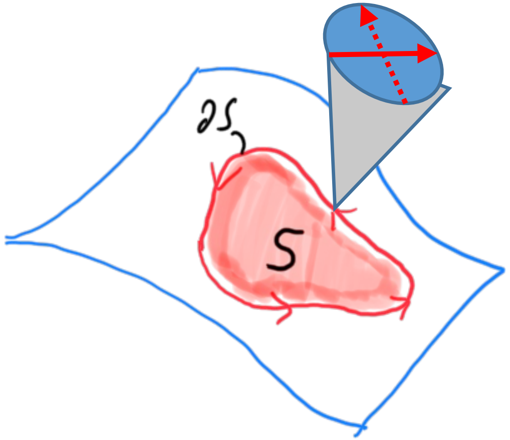
Higher-dimensional topology: 2nd Chern number

Two-fold **degenerate** state $|\psi_{\alpha=1,2}\rangle$, depending on parameters $\boldsymbol{\lambda} = (\lambda_1, \dots \lambda_d)$

- Moving an arbitrary superposition of the state requires **matrix-valued** non-Abelian Berry connection $\hat{A}_j$ with $\left(\hat{A}_j\right)_{\alpha\beta} = \langle\psi_\alpha|\partial_j\psi_\beta\rangle$ for each 'dimension' j.



- Matrix-valued Berry curvature $\hat{F}_{ij} = \partial_i\hat{A}_j - \partial_j\hat{A}_i - i\left[\hat{A}_i, \hat{A}_j\right]$
- Topological invariants (d=4)
  - 1st Chern number $C_{ij}^{(1)} = \frac{1}{2\pi}\int d\lambda_i d\lambda_j \,\mathrm{Tr}\left[\hat{F}_{ij}\right]$
    → characteristic for any 2D subspace of the 4D parameter space
    → depends only on diagonal elements of Berry curvature
  - 2nd Chern number $C^{(2)} = \frac{1}{4\pi^2}\int d^4\lambda\left(\mathrm{Tr}\left[\hat{F}_{12}\hat{F}_{34}\right] + \mathrm{Tr}\left[\hat{F}_{13}\hat{F}_{42}\right] + \mathrm{Tr}\left[\hat{F}_{14}\hat{F}_{23}\right]\right)$
    → characteristic of **4D space**
    → off-diagonal element of Berry curvature matrix enter



- Non-Abelian Berry phase: motion along closed curve ∂S: $|\psi\rangle \to \hat{R}(\partial S)|\psi\rangle$
  with rotation matrix $\hat{R}(\partial S) = \mathcal{P}\left[e^{i\oint_{\partial S}\hat{\boldsymbol{A}}\,d\boldsymbol{\phi}}\right] = e^{i\hat{\theta}_{geo}}$ and hence $\left[\hat{\theta}_{geo}^{\partial S_1}, \hat{\theta}_{geo}^{\partial S_2}\right] \neq 0$

II. Higher-Dimensional Topology in Josephson Matter

H. Weisbrich*, R. L. Klees*, G. Rastelli, and W. Belzig

Second Chern Number and Non-Abelian Berry Phase in Superconducting Systems,

PRX Quantum 2, 010310 (2021)

A) Construction of non-Abelian 4D-Josephson Matter

Two coupled quantum dots with at least D+1 superconductors

[Weisbrich, Klees, et al. PRX Quantum 2021]

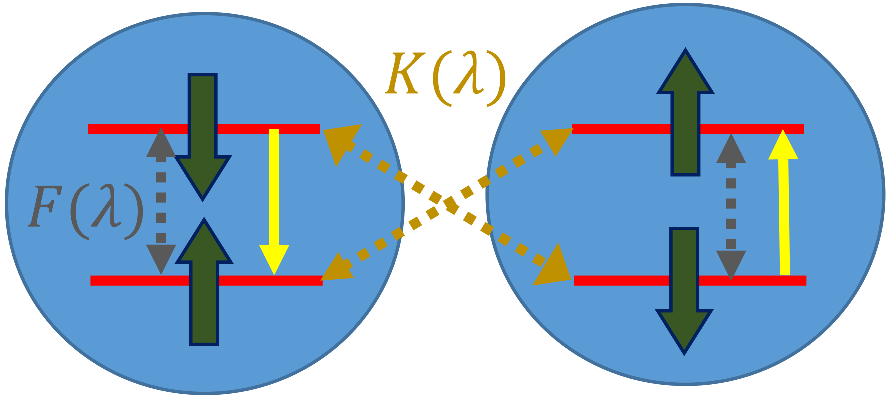
Effective Hamiltonian
in the 1-particle-subspace:

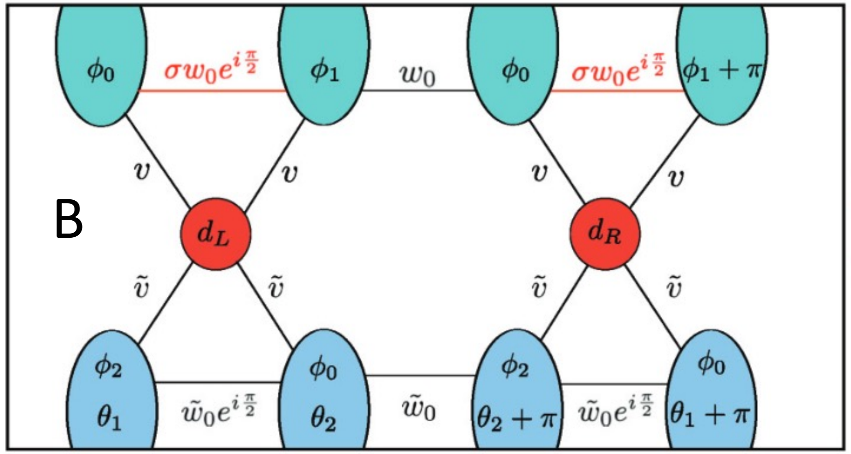
$$\boxed{H = \boldsymbol{c}^{+}[\vec{d}(\boldsymbol{\lambda})\vec{\hat{\gamma}}]\boldsymbol{c}}$$

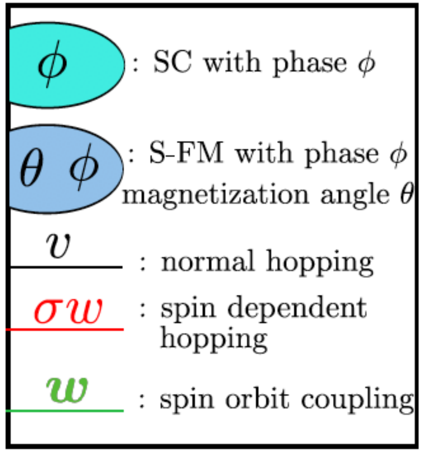
$$\{\hat{\gamma}_i, \hat{\gamma}_j\} = 2\delta_{ij} \quad i,j = 1, \dots, 5$$

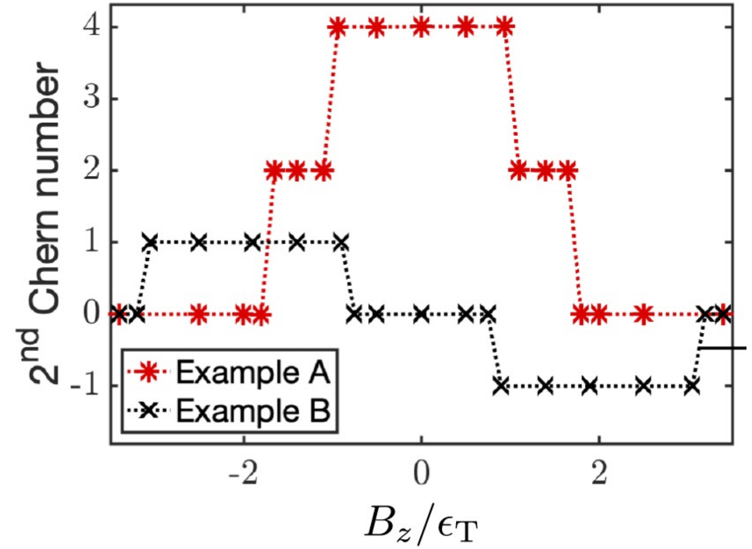
$$\boldsymbol{c} = (c_{L\uparrow}, c_{L\downarrow}, c_{R\uparrow}, c_{R\downarrow})^T$$

$$\vec{d}(\boldsymbol{\lambda}) = (B_z \quad \mathrm{Re}K \quad \mathrm{Im}K \quad \mathrm{Re}F \quad \mathrm{Im}F)$$



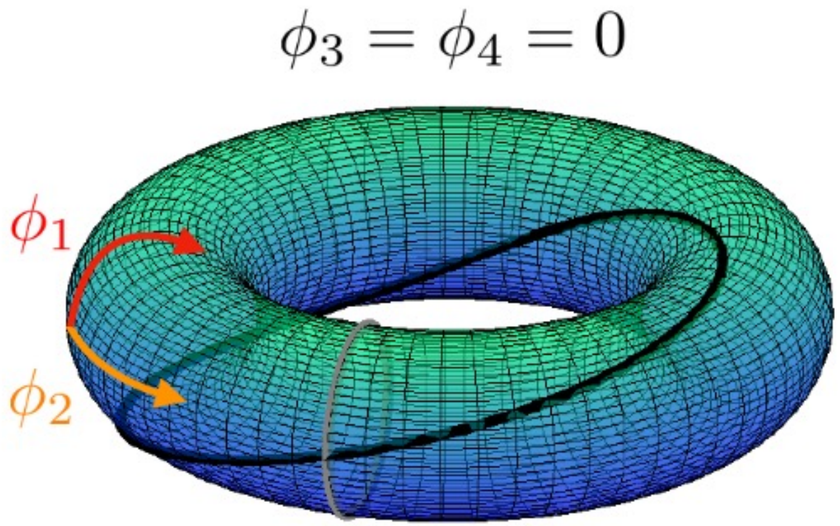




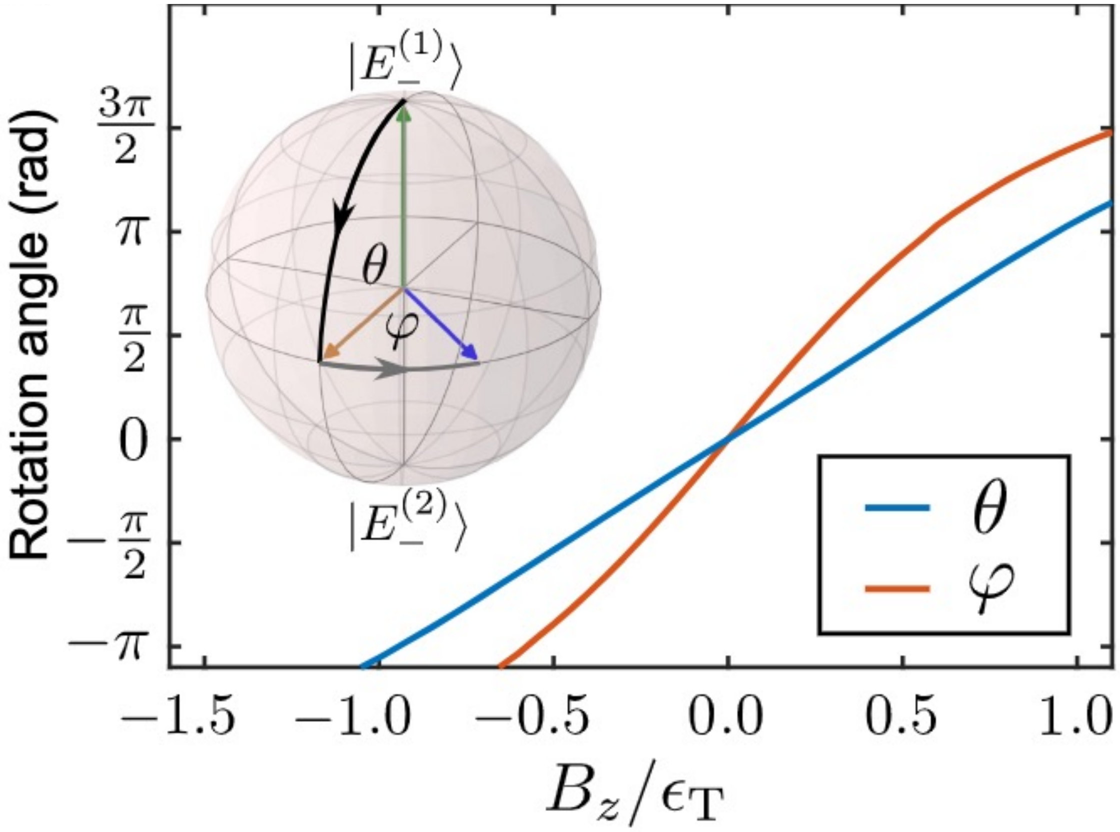


Non-Abelian Berry phase

Periodic parameter variation → rotation in degenerate subspace!



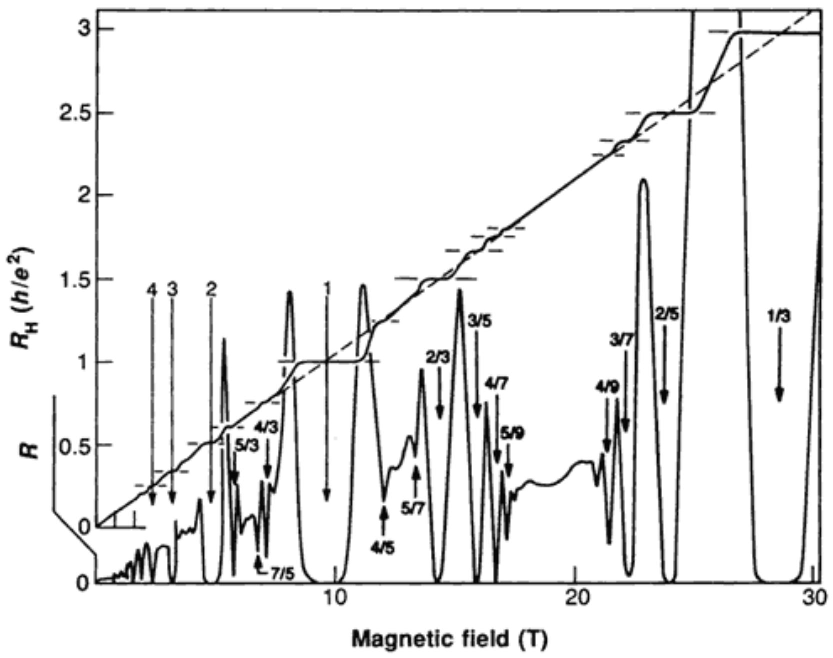
Black: rotation around x-axis
Grey: rotation around z-axis



Arbitrary single-qubit manipulation possible → platform for holonomic quantum computing?

III. Fractional transport in Josephson circuits

Intro 2a: Fractional quantum Hall effect



Fractional Hall conductance steps

$$G = \frac{1}{\nu}\frac{e^2}{h} \qquad \nu = 1,2,3,\ldots$$

(and more complicated fractions)

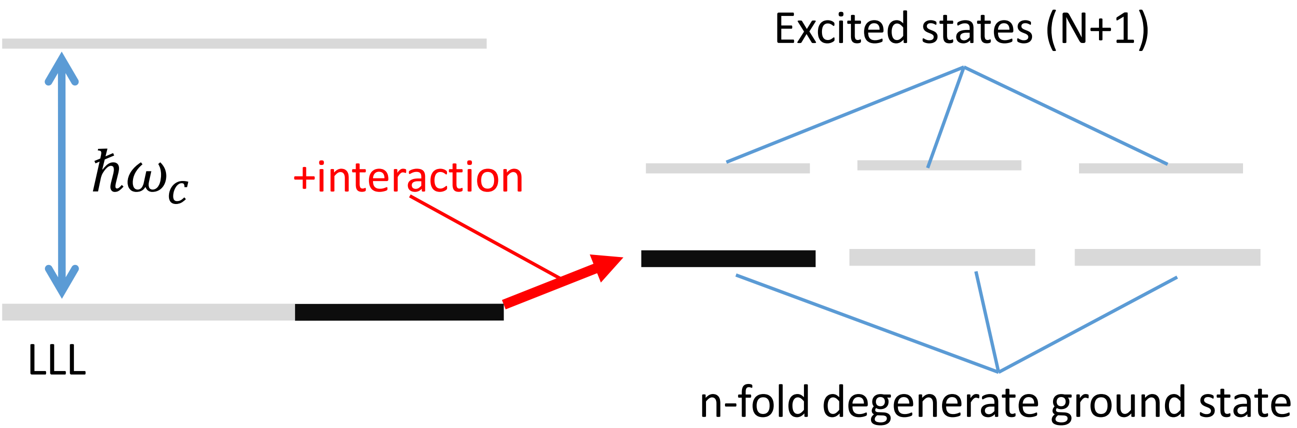
[Störmer, Tsui, 1982, Nobel Prize 1998]

Laughlin wave function ($n = \nu$) for $N = \Omega_{LLL}/n$

$$\psi_{n,N}(z_1, z_2, z_3, \ldots, z_N) =$$

$$D\left[\prod_{N\geqslant i>j\geqslant 1}(z_i - z_j)^n\right]\prod_{k=1}^{N}\exp(-\mid z_k\mid^2)$$

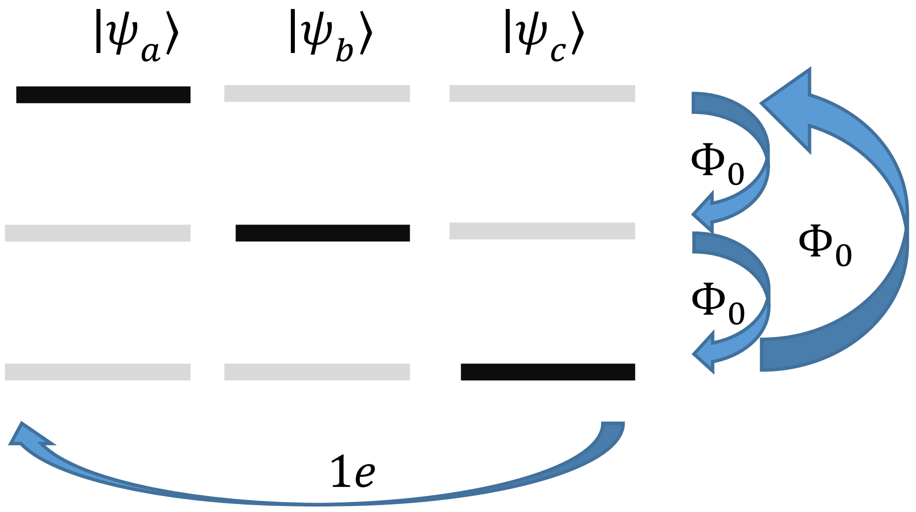
[Laughlin 1983, Nobel Prize 1998]



Fractional States in Transport

Fractionally quantized conductance:
Nontrivial topology of the ν-fold (quasi-) degenerate ground state manifold (Laughlin states)

[Thouless, e.g. TKNN 1982]



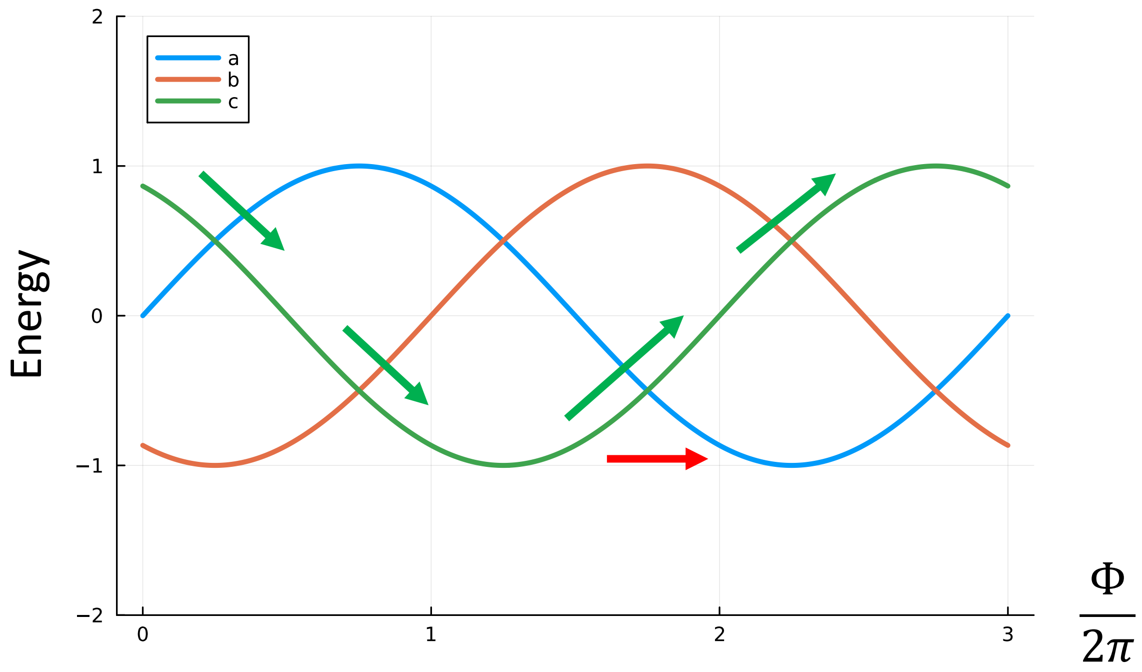
Topological invariant:

$$C = C_a + C_b + C_c \quad \in 0, \pm 1, \pm 2$$

Adiabatic Pumping current (1e per $\nu \times \Phi_0$)

$$I_{A\to B} = I_x = ef = e\,\dot{\Phi}/\nu\Phi_0 \quad = \left(\frac{1}{\nu}\right)\frac{e^2}{h}V_y$$

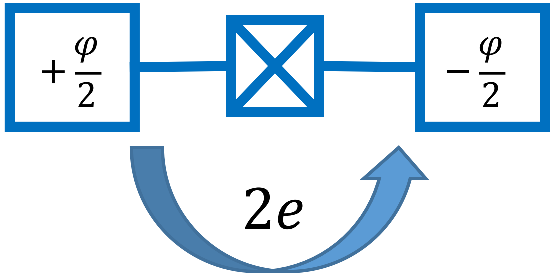
(Quasi-)Adiabatic pumping $\hbar\dot{\Phi} = eV_y$



Intro 2b: Josephson Circuits

Josephson effect in the Cooper pair picture with superconducting island (aka qubits):

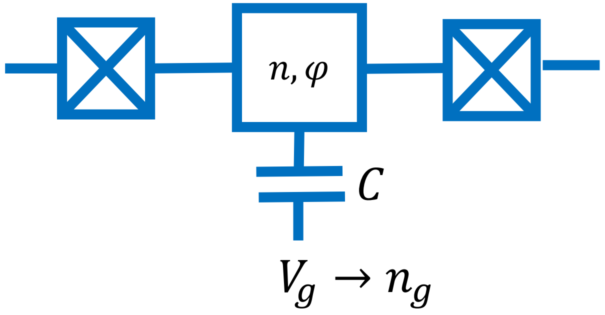
<u>Cooper pair tunneling</u>



$$E_J\cos(\varphi) \to \frac{E_J}{2}\sum_n \left[e^{i\varphi}|n\rangle\langle n-1| + h.c\right]$$

$$\boxed{\text{Charge-phase uncertainty } [\varphi, n] = i}$$

<u>Charging effect:</u>



$$\frac{2e^2(n-n_g)^2}{C} \to E_c\sum_n (n-n_g)^2|n\rangle\langle n|$$

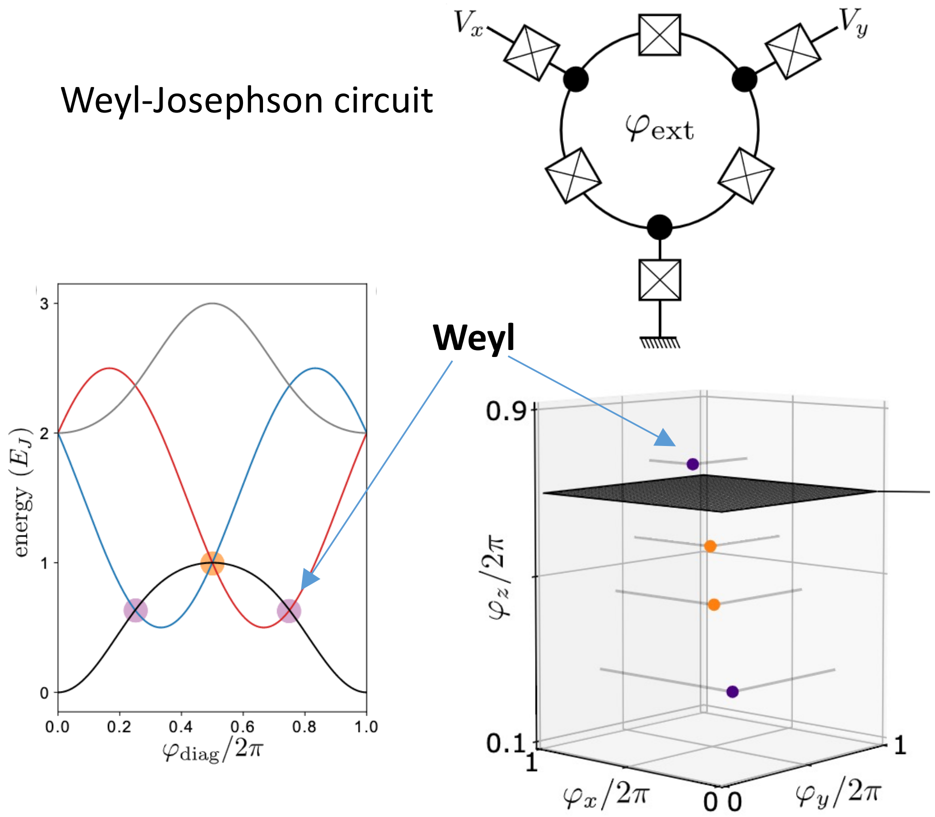
$$E_c = \frac{2e^2}{C}$$

[Devoret, Schön, Averin, Clarke, Tinkham, Mooji…. 90ties]

[Ingold, Nazarov, cond-mat/0508728 (1992)]

Topology in Multi-Island Josephson Circuits

Weyl-Josephson circuit



$$G^{xy} = \frac{dI_x}{dV_y} = \frac{4e^2}{h}C$$

Multi-island Josephson circuits

➔

Platform to simulate fractional physics?

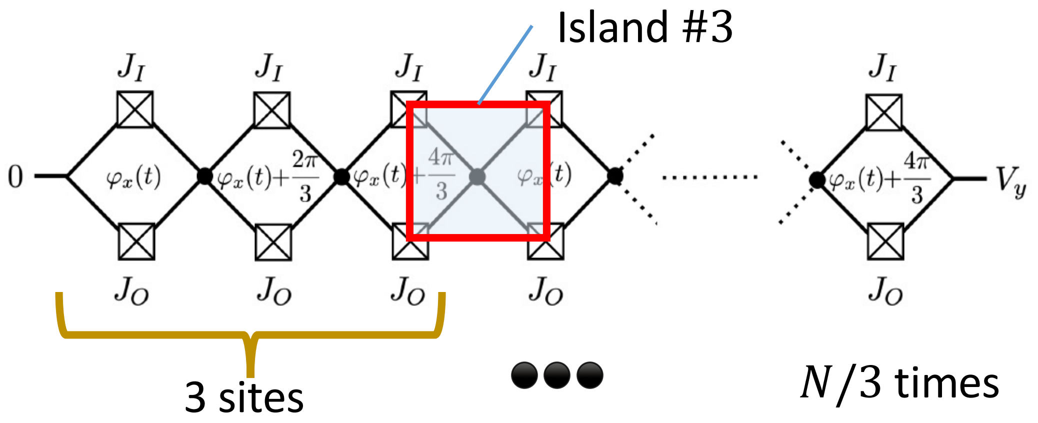
[Fatemi, Akhmerov, Bretheau, Phys. Rev. Res. (2021); Peyruchat, Griesmar, Pillet, Girit, Phys. Rev. Res. (2021)]

Josephson junction chain

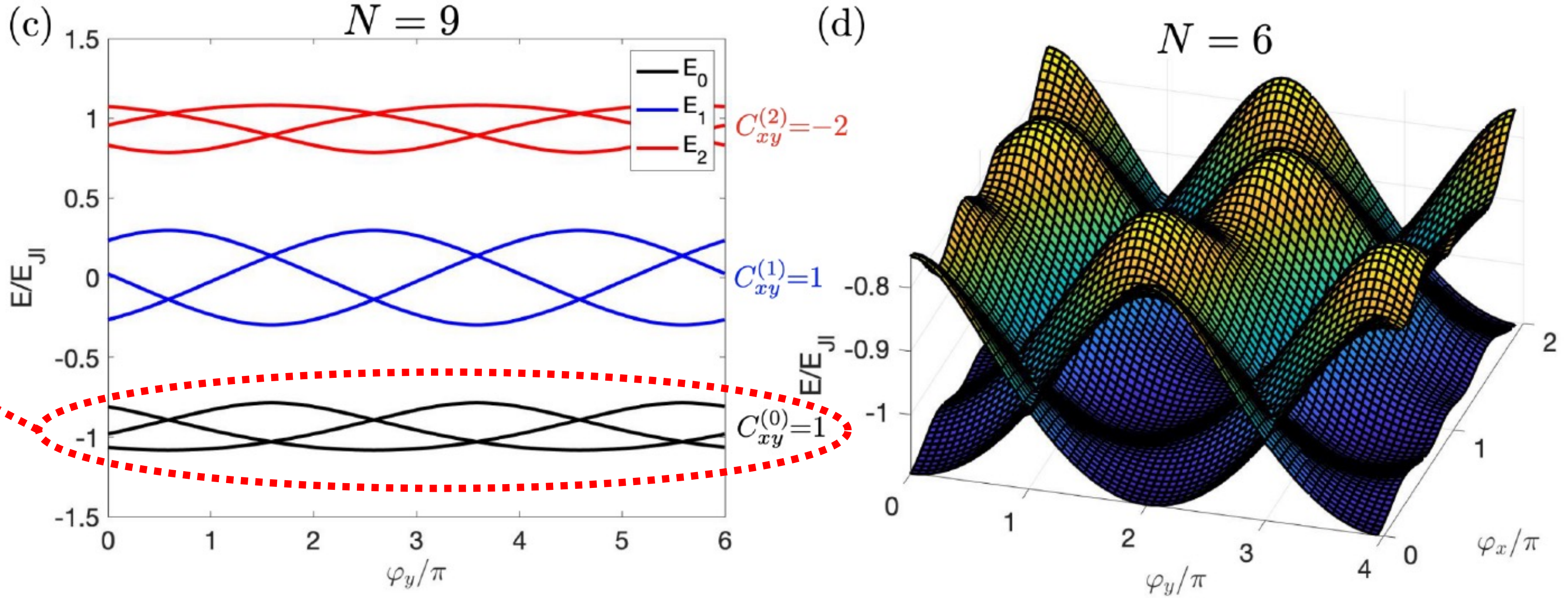
Josephson junction chain (3-cell unit, $E_C \gg E_J$)

→ $N-1$ islands with charging energy $E_C = \frac{2e^2}{C}$

$$|\psi\rangle = \begin{cases} |0\rangle & \text{0 Cooper pairs} \\ |j\rangle & \text{1 Cooper pair on island } \#j \end{cases}$$



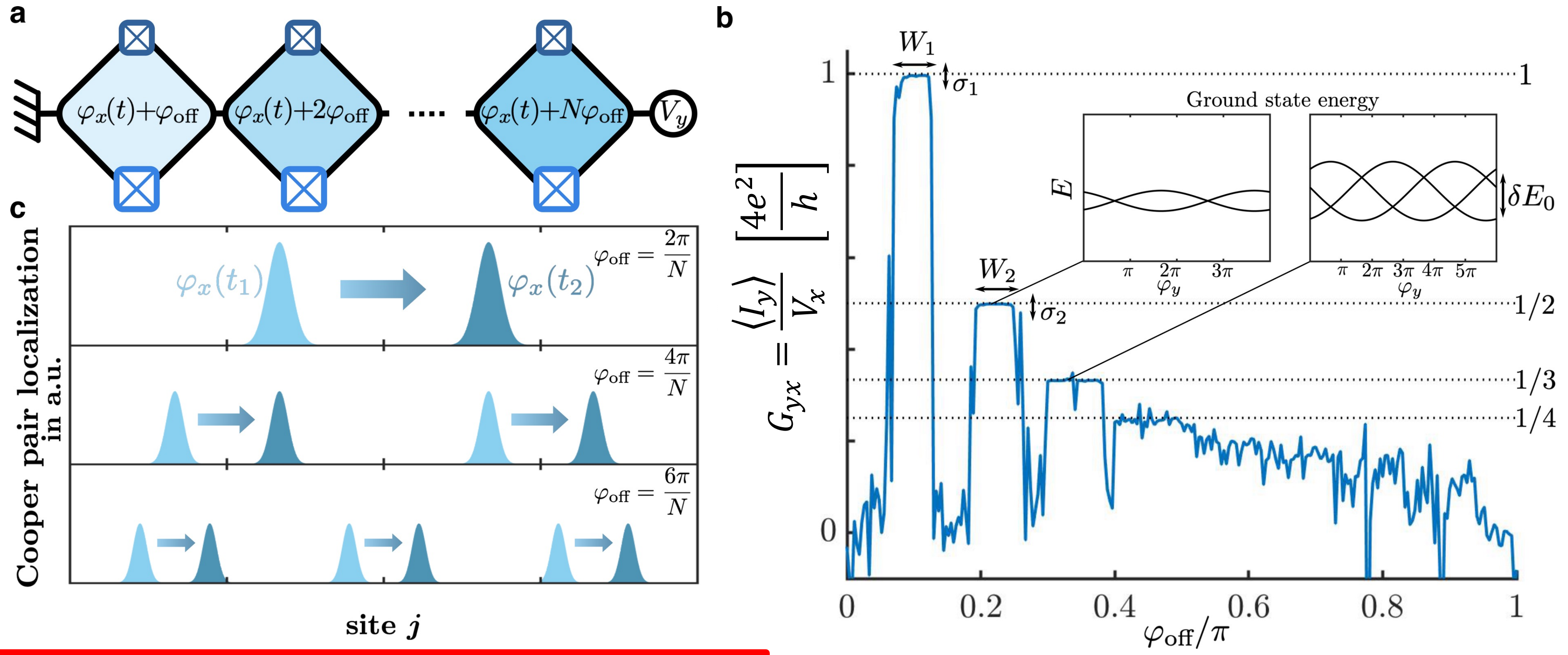
3 (quasi-)degenerate states



[Weisbrich, Klees, Zilberberg, Belzig, Phys. Rev. Res. **5**, 043045 (2023)]

Fractional Cooper pair pump (nonadiabatic)

Offset flux φ_{off} Pumping $2\mathrm{e}V_x = \dot{\varphi}_x$ Current I_y with offset flux (for N=18)

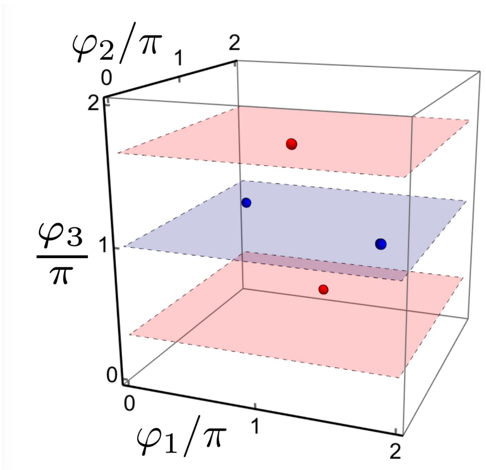


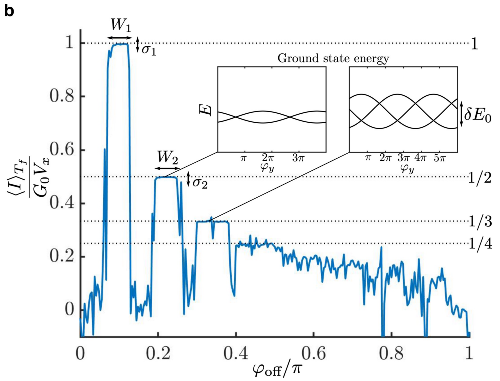
Fractional Transconductance $G_{xy} = -\frac{4e^2}{h}\frac{1}{\nu}$

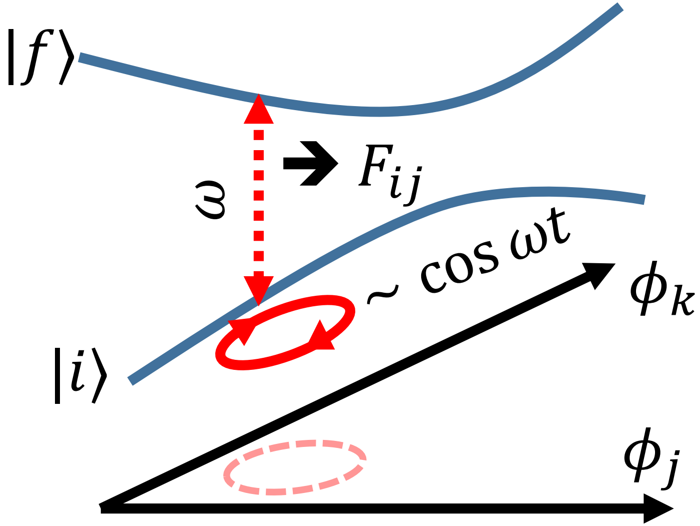
[Weisbrich, Klees, Zilberberg, Belzig, Phys. Rev. Res. **5**, 043045 (2023)]

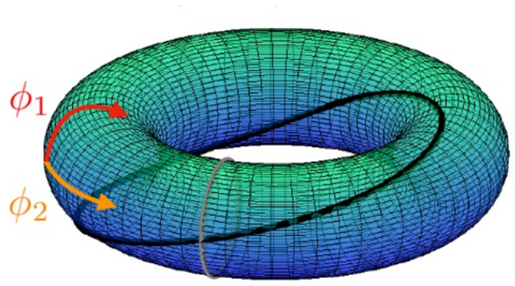
Summary

- **Multi-Terminal Josephson Matter** as platform for (high-dimensional) topology
- 4-terminal junctions or dot arrays show **Weyl Nodes** and finite **1st Chern Number**
- Synthetically polarized microwave spectroscopy: full **Quantum Geometric Tensor**
- **Higher Dimensional Topology** in a double dot coupled to superconducting terminals
- Nontrivial **2nd Chern Number** and **Non-Abelian Berry Phase**
- **Fractional** Cooper pair pump in a Josephson junction array









SFB 1432

Fluctuations and Nonlinearities in Classical and Quantum Matter beyond Equilibrium

<u>The Quantum Transport Group</u>
David Ohnmacht
Alexandr Svetogorov
Dennis Wuhrer
Joren Harms
Mahsa Heydari
Matthias Hübler
Sebastian Diaz
Michael Hein
Panch Ram
Daniel Boness
Sourabh Patil

Raffael Klees (Universität Augsburg)

Hannes Weisbrich (Vector Informatik)

Gianluca Rastelli (CNR-INO & BEC Group, Trento)

Juan Carlos Cuevas (Madrid)

Lev Teshler, Oded Zilberberg (Universität Konstanz)

Jonathan Sturm (Universität Würzburg)

M. Coriola, D. Haxell, D. Sabonis, F. Nichele (IBM Zürich)

References

- R. L. Klees, G. Rastelli, J. C. Cuevas, and W. Belzig, ***Microwave spectroscopy reveals the quantum geometric tensor of topological Josephson matter***, Phys. Rev. Lett. **124**, 197002 (2020)
- H. Weisbrich*, R. L. Klees*, G. Rastelli, and W. Belzig, ***Second Chern Number and Non-Abelian Berry Phase in Superconducting Systems***, PRX Quantum **2**, 010310 (2021)
- R. L. Klees, J. C. Cuevas, W. Belzig and G. Rastelli, ***Ground-state quantum geometry in superconductor–quantum dot chain***s, Phys. Rev. B **103**, 014516 (2021)
- H. Weisbrich, G. Rastelli and W. Belzig, ***Geometrical Rabi oscillations and Landau-Zener transitions in non-Abelian systems***, Phys. Rev. Research **3**, 033122 (2021)
- H. Weisbrich, M. Bestler, W. Belzig, ***Tensor Monopoles in superconducting systems***, Quantum **5**, 601 (2021)
- L. Teshler, H. Weisbrich, R. L. Klees, G. Rastelli, and W. Belzig, ***Ground state topology of a four-terminal superconducting double quantum dot,*** SciPost Phys. **15**, 214 (2023).
- D. Z. Haxell, M. Coraiola, M. Hinderling, S. C. ten Kate, D. Sabonis, A. E. Svetogorov, W. Belzig, E. Cheah, F. Krizek, R. Schott, W. Wegscheider, and F. Nichele, ***Demonstration of nonlocal Josephson effect in Andreev molecules***, Nano Lett. **23**, 7532 (2023)
- M. Coraiola, D. Z. Haxell, D. Sabonis, H. Weisbrich, A. E. Svetogorov, M. Hinderling, S. C. ten Kate, E. Cheah, F. Krizek, R. Schott, W. Wegscheider, J. C. Cuevas, W. Belzig, and F. Nichele, ***Phase-engineering the Andreev band structure of a three-terminal Josephson junction***, Nat. Commun. 14, 6784 (2023)
- H. Weisbrich, R. L. Klees, O. Zilberberg, and W. Belzig, **Fractional Transconductance via Nonadiabatic Topological Cooper Pair Pumping,** Phys. Rev. Res. **5**, 043045 (2023).
- M. Coraiola, A. E. Svetogorov, D. Z. Haxell, D. Sabonis, M. Hinderling, S. C. ten Kate, E. Cheah, F. Krizek, R. Schott, W. Wegscheider, J. C. Cuevas, W. Belzig and F. Nichele, ***Flux-Tunable Josephson Diode Effect in a Hybrid Four-Terminal Josephson Junction***, ACS Nano 18, 9221 (2024).
- D. C. Ohnmacht, M. Coraiola, J. J. García-Esteban, D. Sabonis, F. Nichele, W. Belzig and J. C. Cuevas, ***Quartet Tomography in Multiterminal Josephson Junctions***, *arXiv:2311.18544*