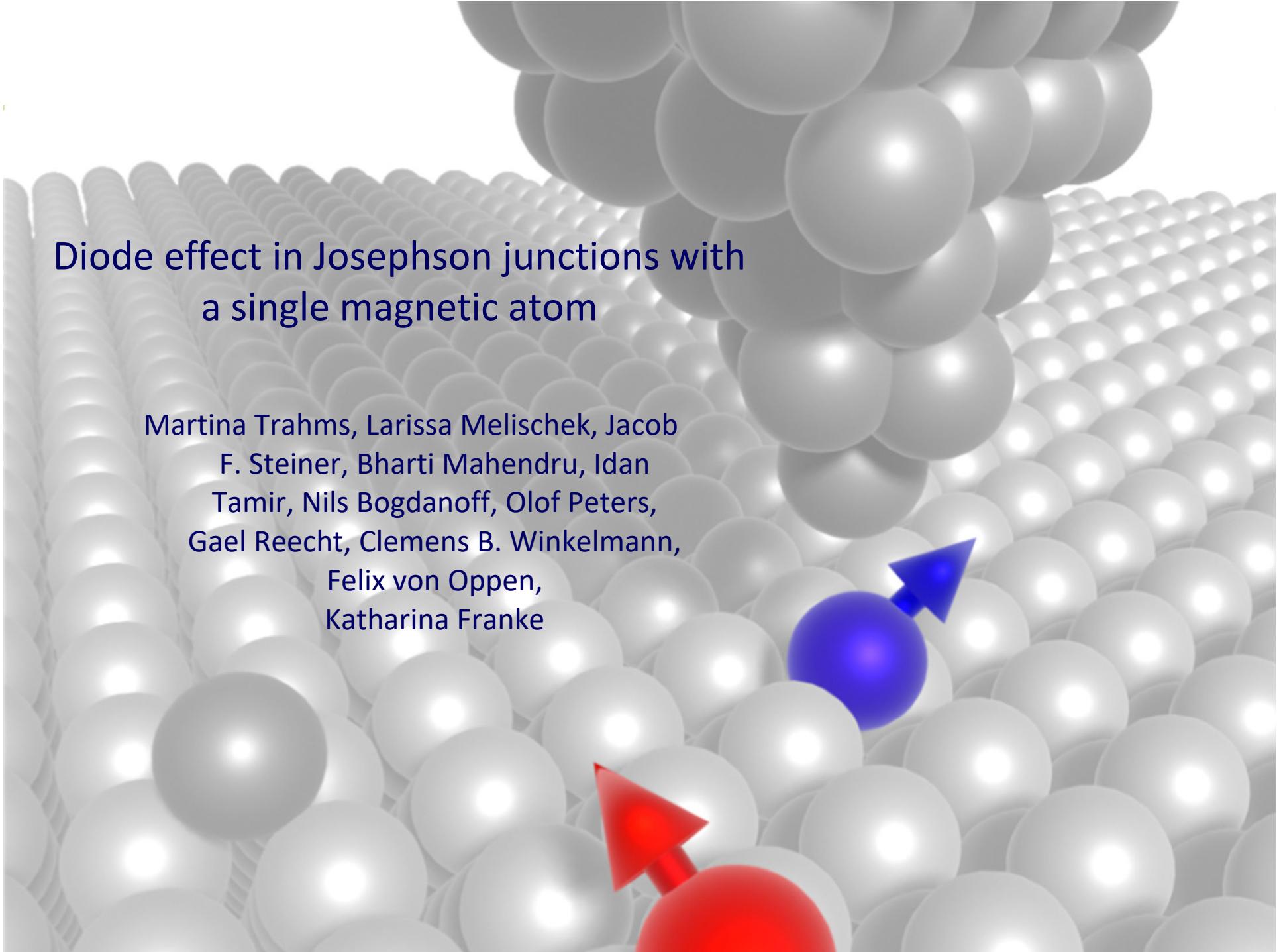


# Diode effect in Josephson junctions with a single magnetic atom

Martina Trahms, Larissa Melischek, Jacob  
F. Steiner, Bharti Mahendru, Idan  
Tamir, Nils Bogdanoff, Olof Peters,  
Gael Reecht, Clemens B. Winkelmann,  
Felix von Oppen,  
Katharina Franke

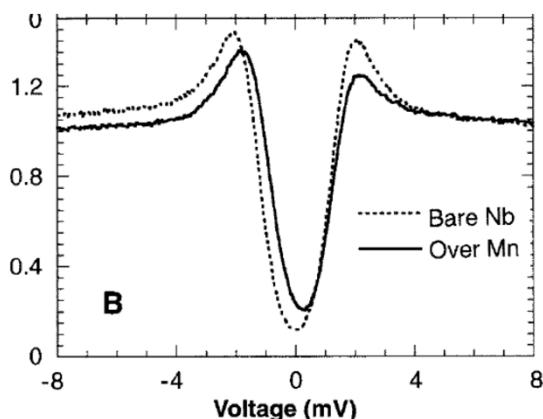
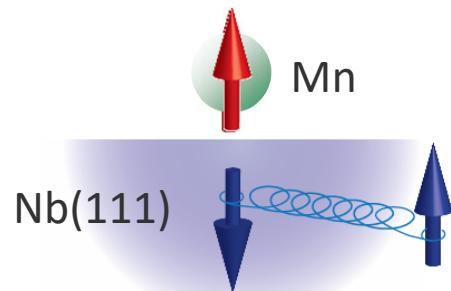


# Yu-Shiba-Rusinov states

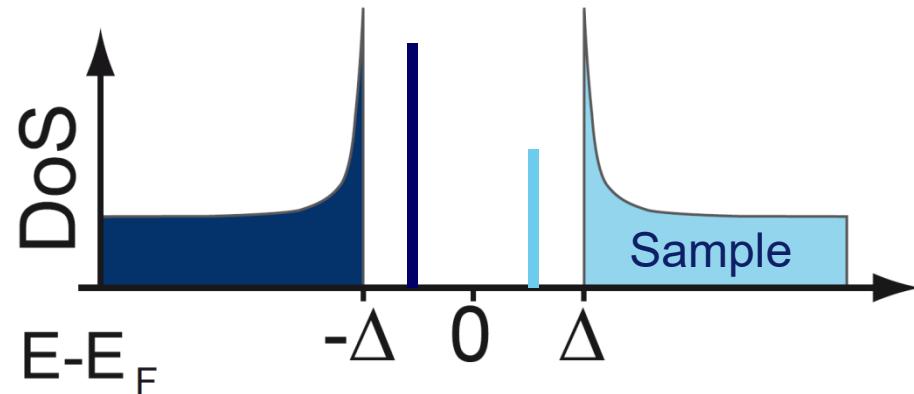


On superconductors:

- Yu-Shiba-Rusinov states



Yazdani et al., Science (1997)



- Spins exchange scatter at a magnetic center:

$$\epsilon_s = \Delta \frac{1 - A^2 + B^2}{\sqrt{4A^2 + (1 - A^2 + B^2)}}$$

$$A = \frac{\pi}{2} J S \nu_0 \quad B = \pi K \nu_0$$

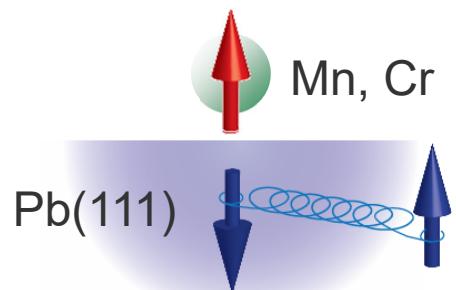
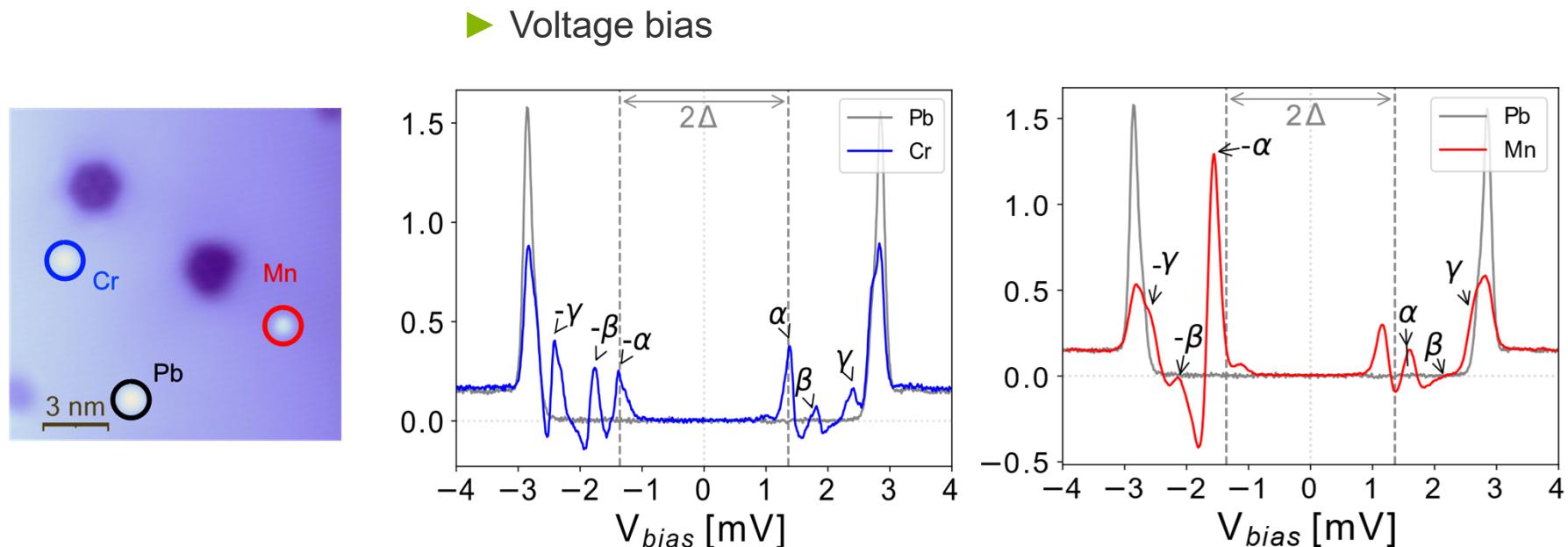
- Yu-Shiba-Rusinov states

L. Yu, Acta Phys. Sin. 21, 75 (1965)

H. Shiba, Prog. Theor. Phys. 40, 435 (1968)

A.I. Rusinov, JETP Lett. 9, 85 (1969)

# YSR states of Mn and Cr on Pb(111)



► Yu-Shiba-Rusinov states:

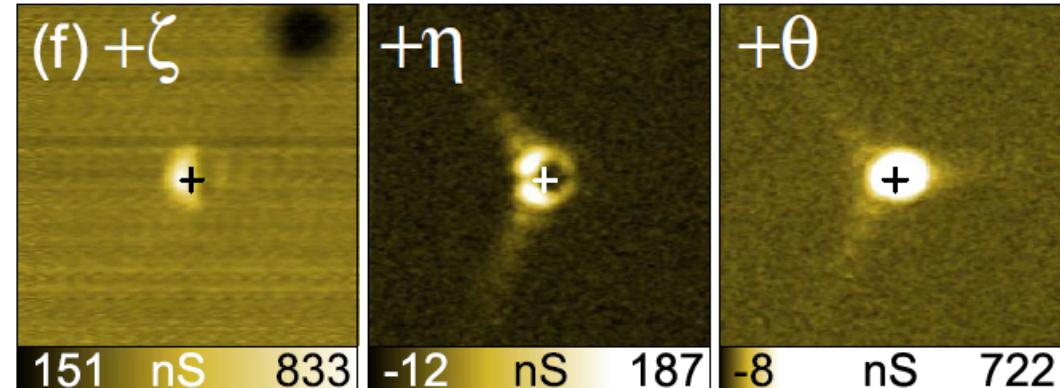
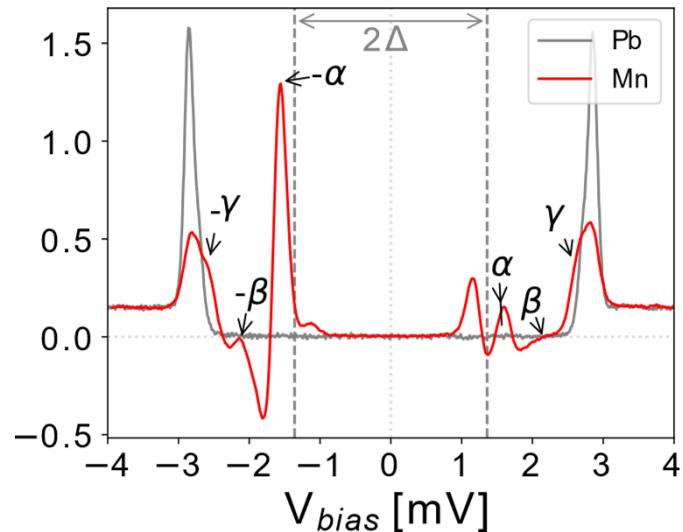
$$\epsilon_S = \Delta \frac{1 - A^2 + B^2}{\sqrt{4A^2 + (1 - A^2 + B^2)}}$$

$$A = \frac{\pi}{2} J S \nu_0 \quad B = \pi K \nu_0$$

► Different electron and hole weight

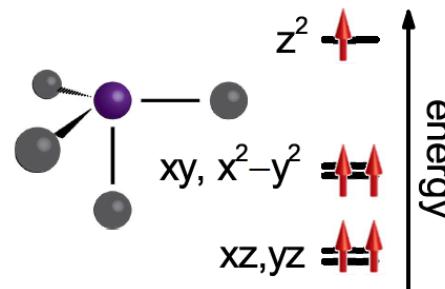
- L. Yu, Acta Phys. Sin. 21, 75 (1965)  
 H. Shiba, Prog. Theor. Phys. 40, 435 (1968)  
 A.I. Rusinov, JETP Lett. 9, 85 (1969)

# Crystal-field split YSR states



$dI/dV$  maps,  $8.8 \times 8.8 \text{ nm}^2$

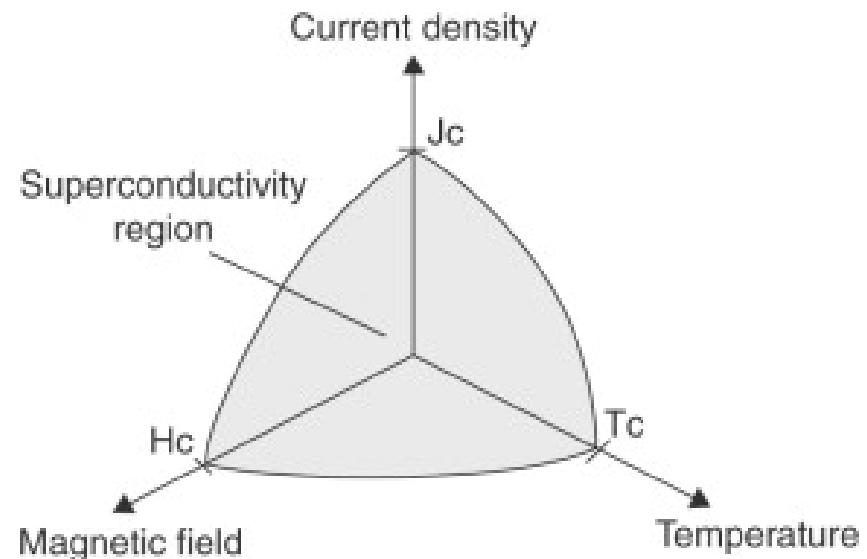
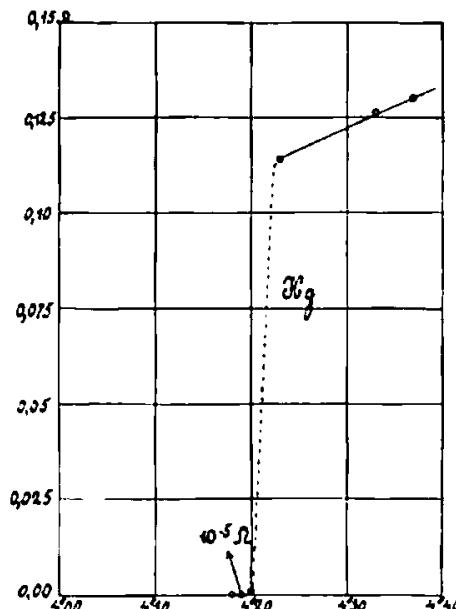
- ▶ different adsorption sites yield different YSR splittings
- ▶ crystal field splitting



# Superconductivity

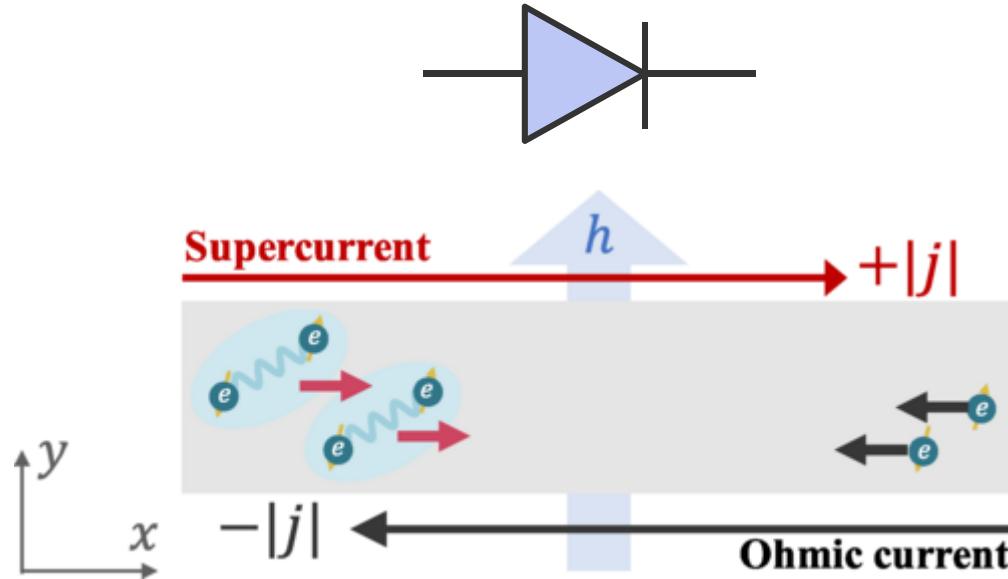


- Current flow with zero resistance  
(1911 Kamerlingh Onnes)



Control of dissipationless vs.  
resistive regime?

# Superconducting diode



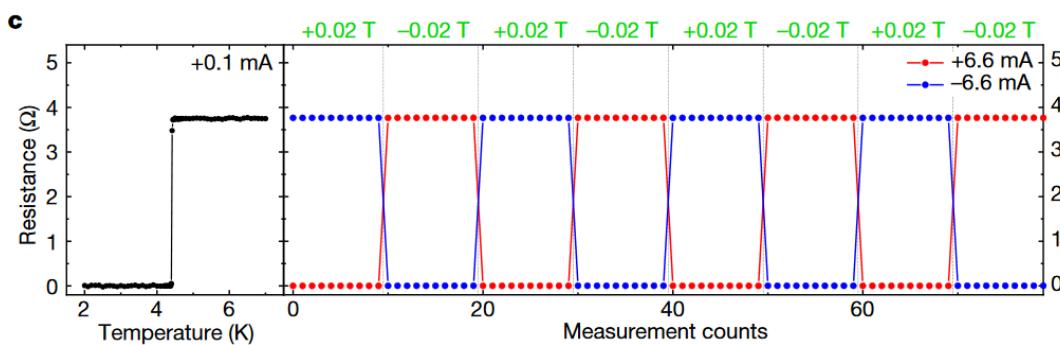
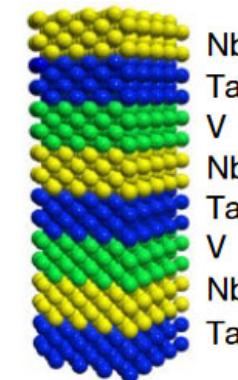
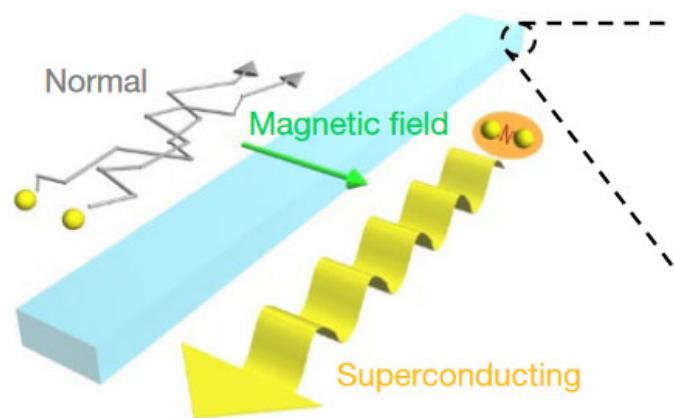
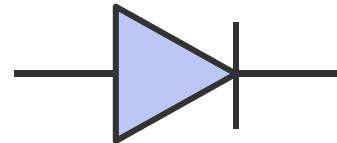
Daido, et al., Phys. Rev. Lett. 128, 037001 (2022)

- ▶ Supercurrent in one direction
- ▶ Resistive current in reverse direction

Superconducting diode

# Superconducting diode

Freie Universität Berlin



Ando, ...Ono, Nature 584, 373 (2020)

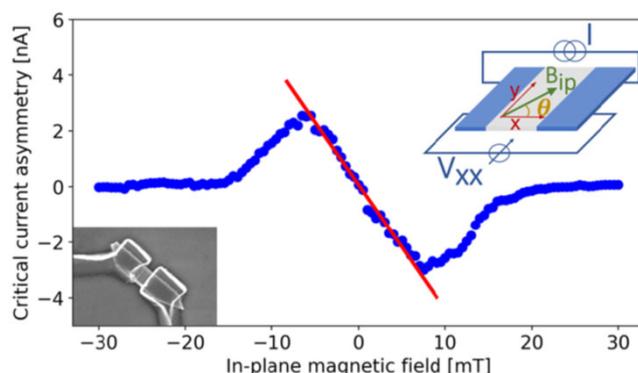
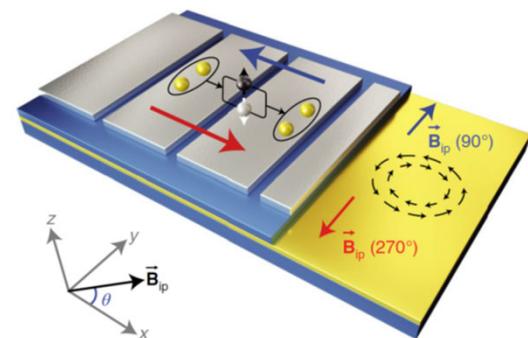
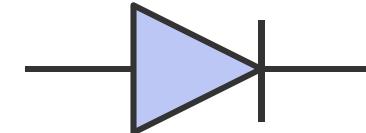
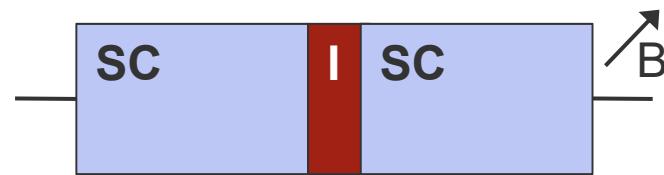
- ▶ Supercurrent in one direction
- ▶ Resistive current in reverse direction
- ▶ Magnetochiral anisotropy

$$R = R_0 [1 + \gamma \hat{\mathbf{e}}_z (\mathbf{B} \times \mathbf{I})]$$

Superconducting diode

# Josephson diode: realization

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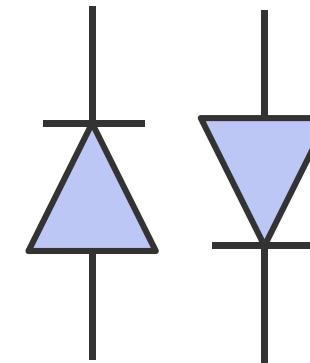
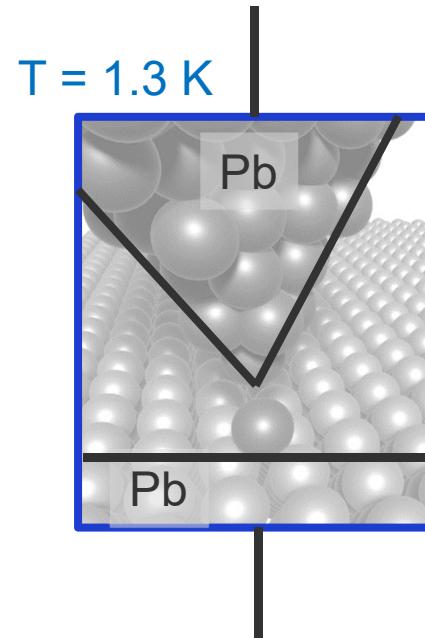
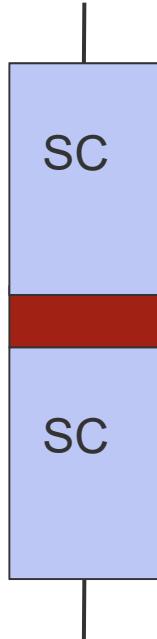
- ▶ Proximity-coupled two-dimensional electron gas/material with strong spin-orbit interaction

- ▶ Spatial-inversion symmetry broken
- ▶ Time-reversal symmetry broken in B field
- ▶ Anomalous current-phase relationship  $I(\phi) \neq -I(-\phi)$

- ▶ Typical diode behavior in critical current

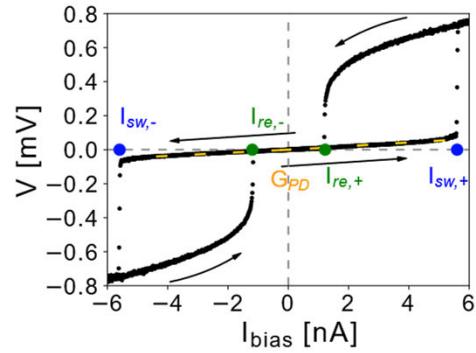
# Atomic Josephson junctions as diodes?

Freie Universität Berlin

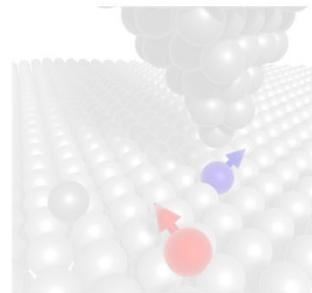


- ▶ So far:  
layered structures
- ▶ Atomic-scale  
Josephson junction in an  
STM
- ▶ Diode behavior?

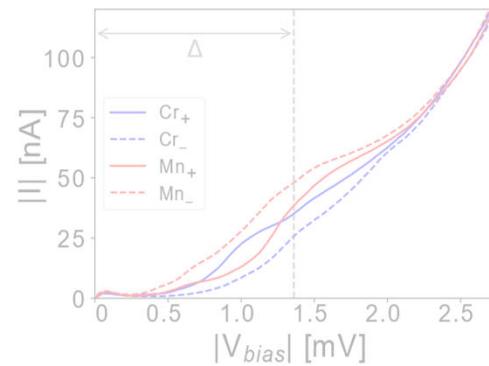
# Outline



- ▶ Current-biased spectroscopy in a STM
- ▶ Switching and retrapping currents



- ▶ Magnetic adatoms on superconductors
- ▶ Asymmetry of retrapping currents
- ▶ Diode behavior



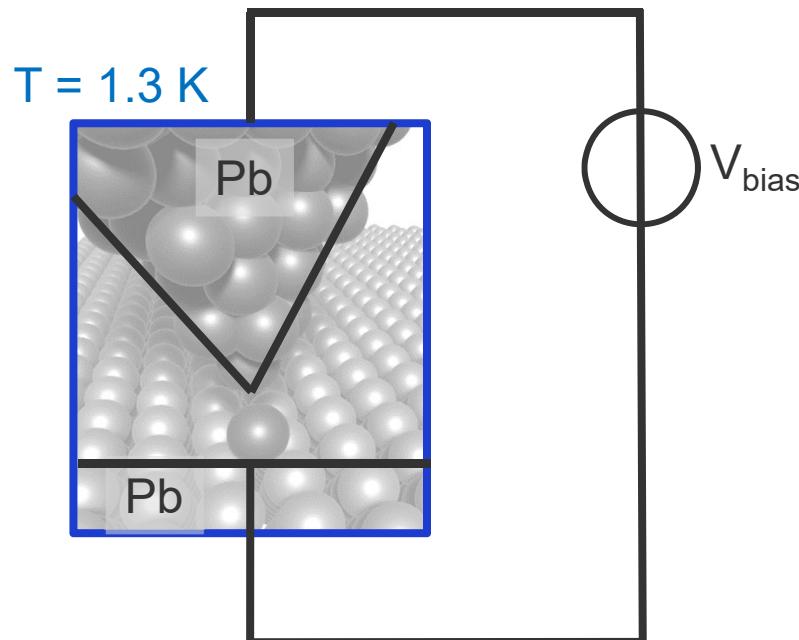
- ▶ Origin of diode behavior

# Voltage-biased STM

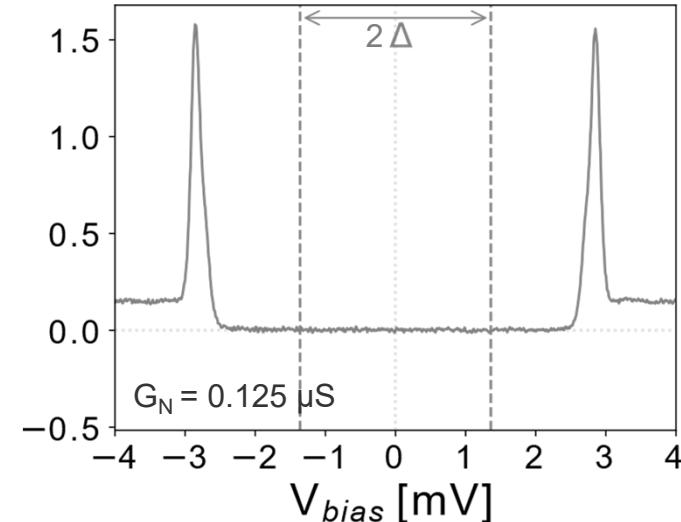
Freie Universität Berlin



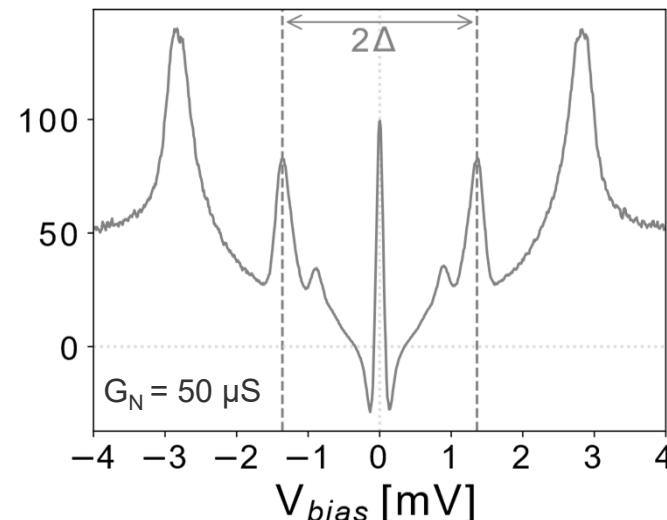
- ▶ Superconducting tip and superconducting substrate



- ▶ Tunnel junction: tip far away

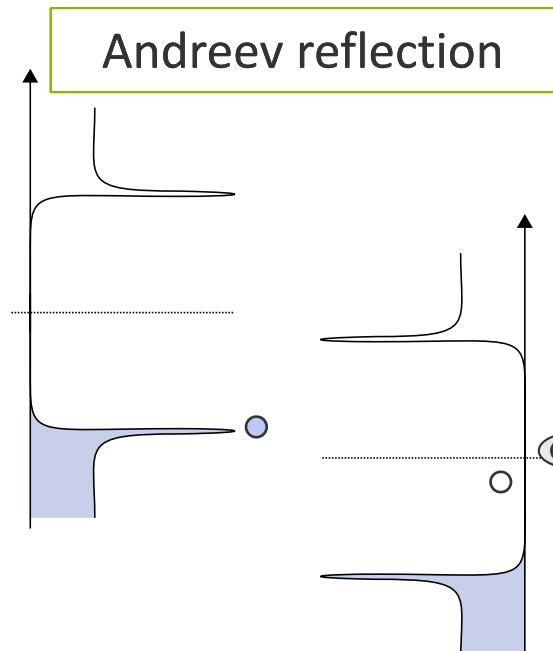
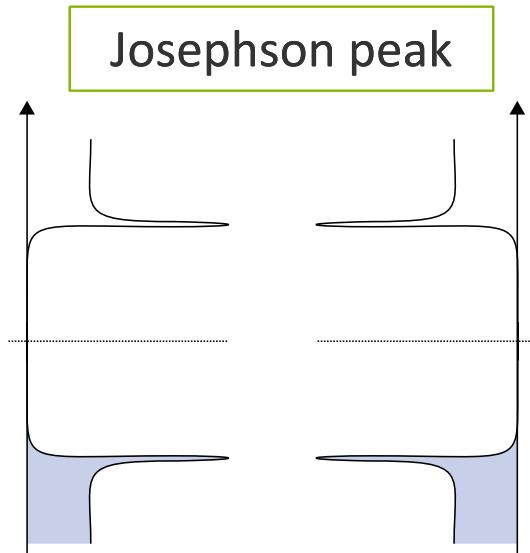


- ▶ Josephson junction: tip close to substrate

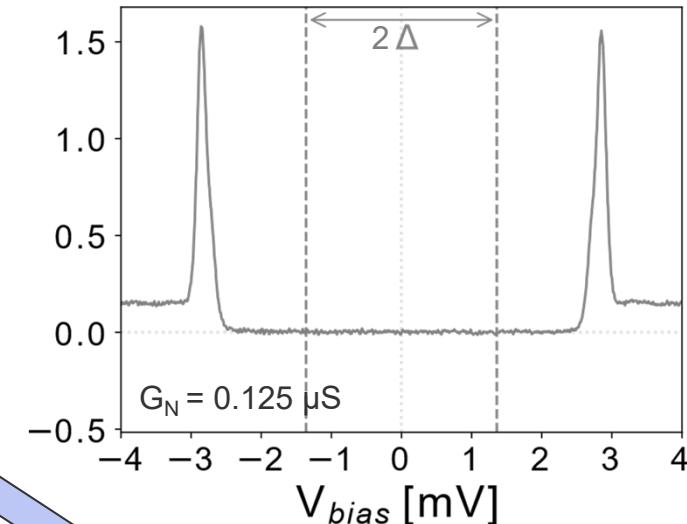


# Voltage-biased STM

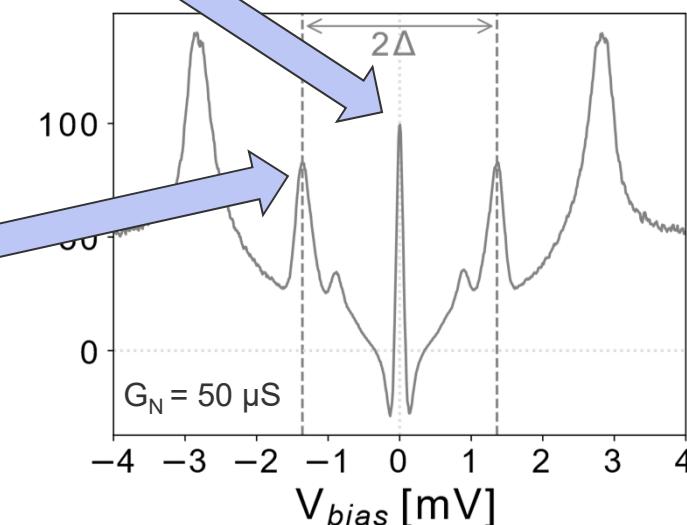
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► Tunnel junction: tip far away



► Josephson junction: tip close to substrate

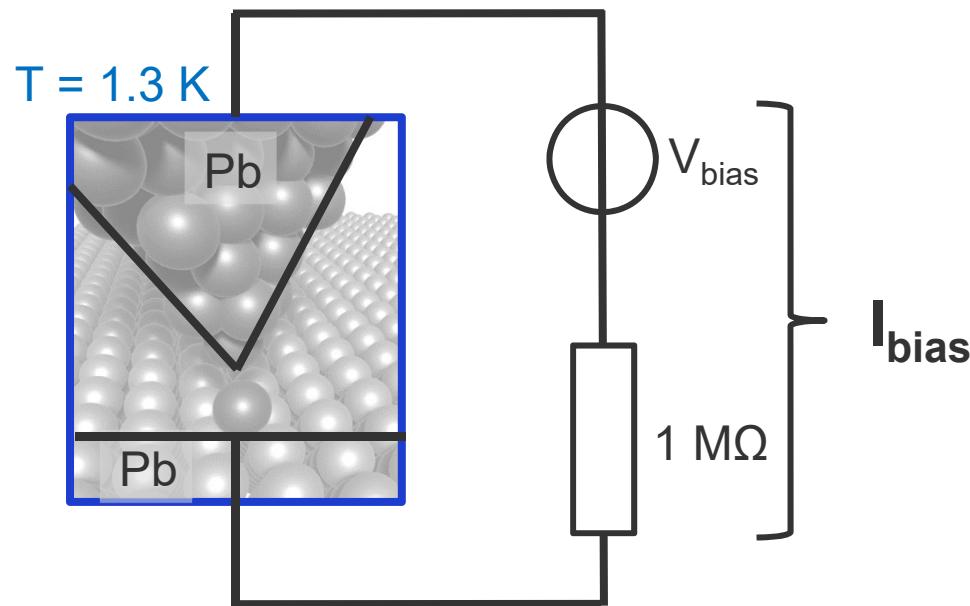


# Current-biased Josephson spectroscopy in STM

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Berlin



- Effective current control by large resistor

- Josephson equations

$$I_S(t) = I_{c0} \sin(\phi(t))$$

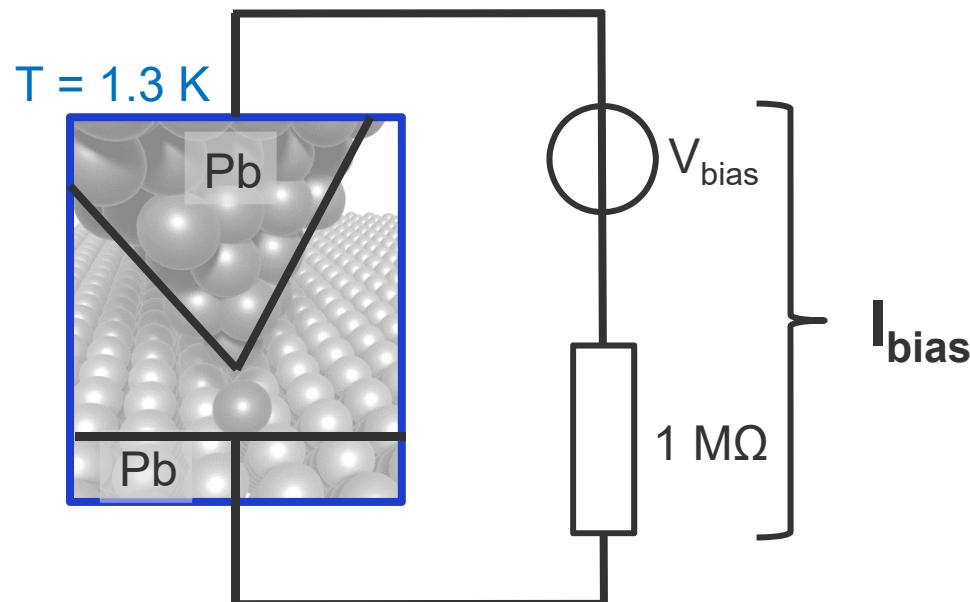
$$V_J = \frac{\hbar}{2e} \frac{d\phi}{dt}$$

# Current-biased Josephson spectroscopy in STM

Freie Universität

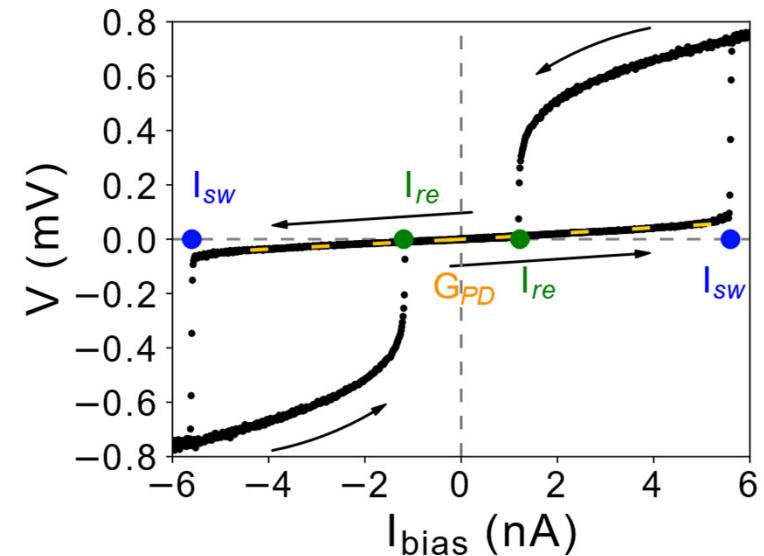


Berlin



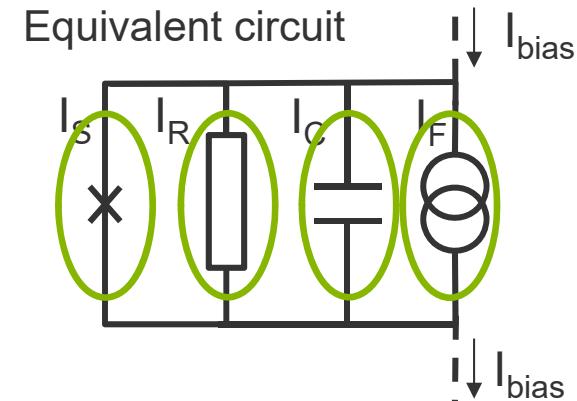
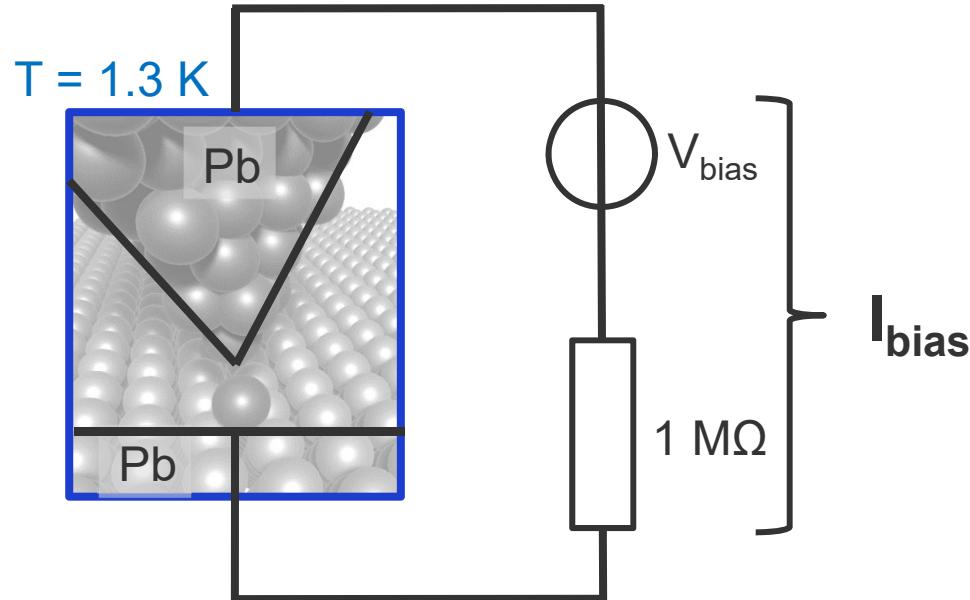
- ▶ Effective current control by large resistor

Current biased Pb-Pb junction:



- ▶ Switching current ( $\sim 5.8 \text{ nA}$ )
- ▶ Retrapping current ( $\sim 1.2 \text{ nA}$ )
- ▶ Hysteretic behavior

# RCSJ model: current contributions



► Resistively and capacitively shunted Josephson junction (RCSJ)

$$I(t) = I_{c0} \sin(\phi(t)) + \frac{V}{R_N} + C \frac{dV}{dt} + \delta I$$

$$I(t) = I_{c0} \sin(\phi(t)) + \frac{\hbar}{2e} \frac{1}{R_N} \dot{\phi} + \frac{\hbar}{2e} C \ddot{\phi} + \delta I$$

► Josephson equations

$$I_S(t) = I_{c0} \sin(\phi(t))$$

$$V_J = \frac{\hbar}{2e} \frac{d\phi}{dt}$$

# RCSJ model: trapped state

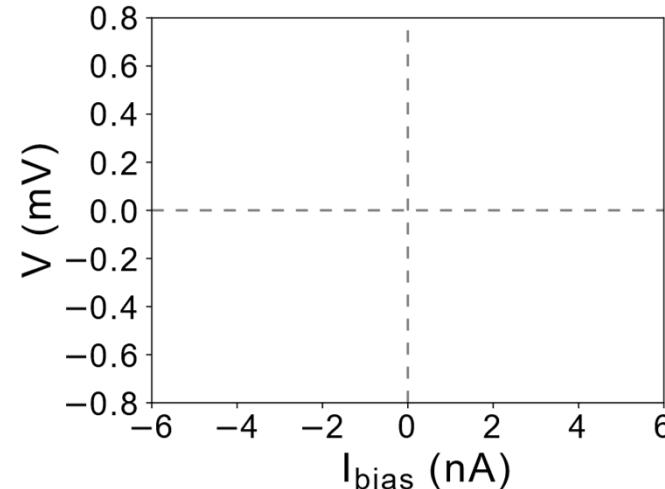
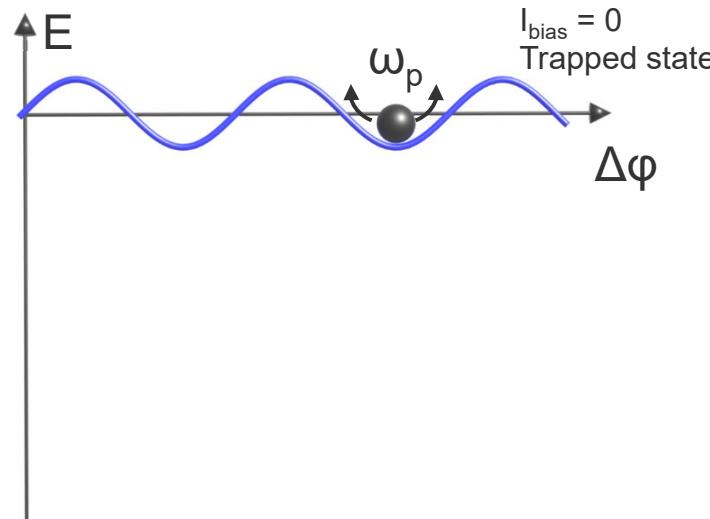
Freie Universität



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- Resistively and capacitively shunted Josephson junction (RCSJ)

$$I(t) = I_{c0} \sin(\phi(t)) + \frac{\hbar}{2e} \frac{1}{R_N} \dot{\phi} + \frac{\hbar}{2e} C \ddot{\phi} + \delta I$$



# RCSJ model: trapped state

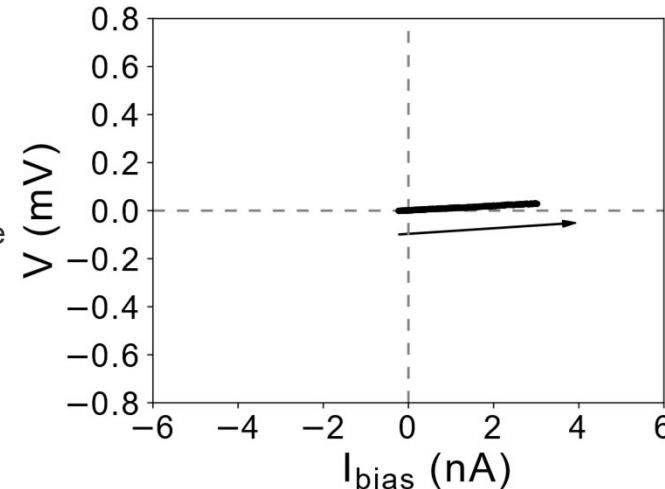
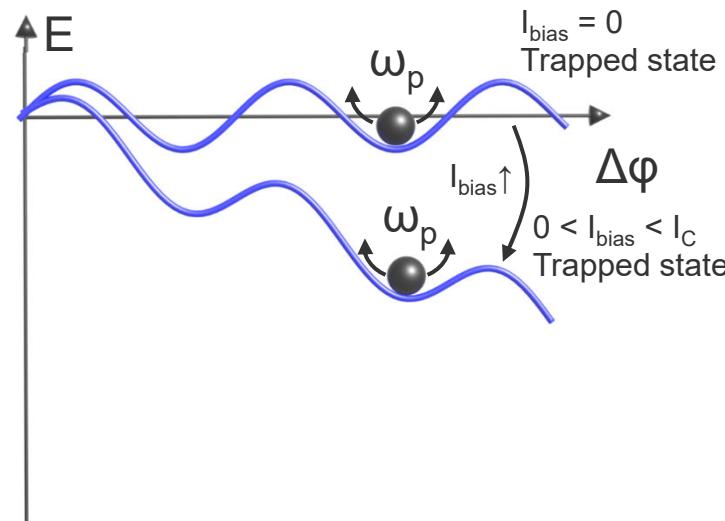
Freie Universität



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- Resistively and capacitively shunted Josephson junction (RCSJ)

$$I(t) = I_{c0} \sin(\phi(t)) + \frac{\hbar}{2e} \frac{1}{R_N} \dot{\phi} + \frac{\hbar}{2e} C \ddot{\phi} + \delta I$$



# RCSJ model: switching into running state

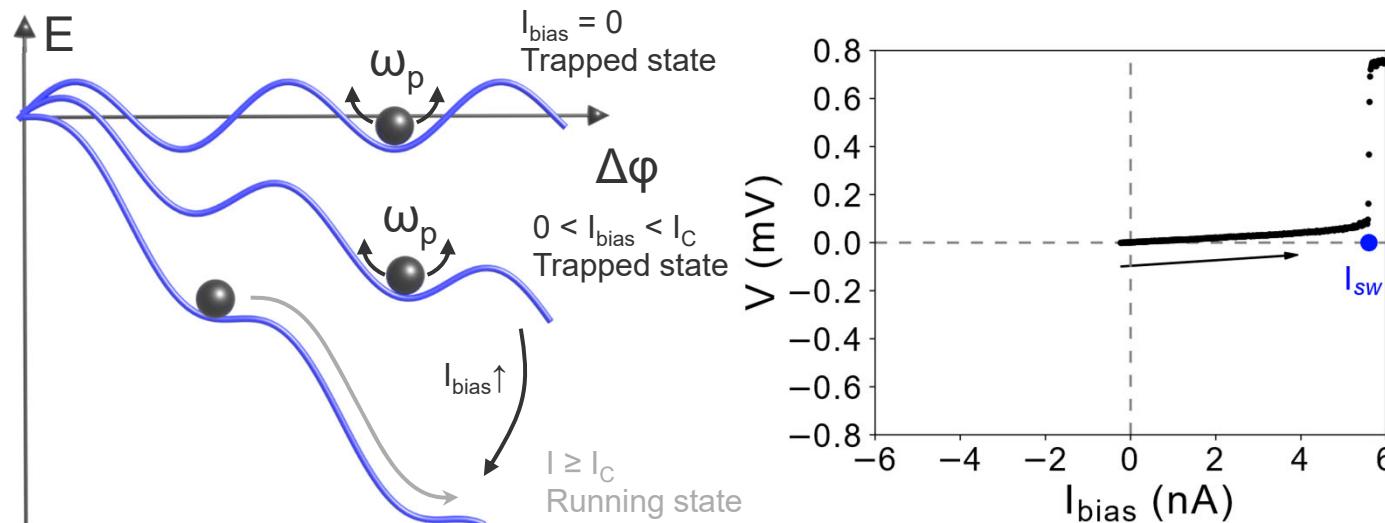
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- Resistively and capacitively shunted Josephson junction (RCSJ)

$$I(t) = I_{c0} \sin(\phi(t)) + \frac{\hbar}{2e} \frac{1}{R_N} \dot{\phi} + \frac{\hbar}{2e} C \ddot{\phi} + \delta I$$

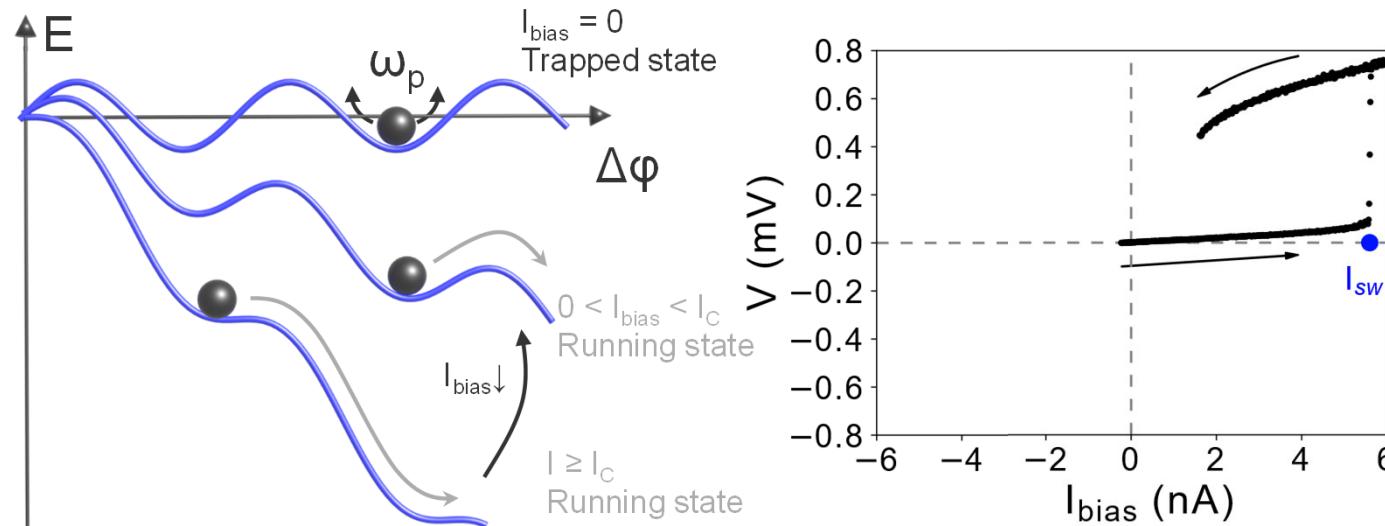


# RCSJ model: running state



- Resistively and capacitively shunted Josephson junction (RCSJ)

$$I(t) = I_{c0} \sin(\phi(t)) + \frac{\hbar}{2e} \frac{1}{R_N} \dot{\phi} + \frac{\hbar}{2e} C \ddot{\phi} + \delta I$$



# RCSJ model: retrapping of phase particle

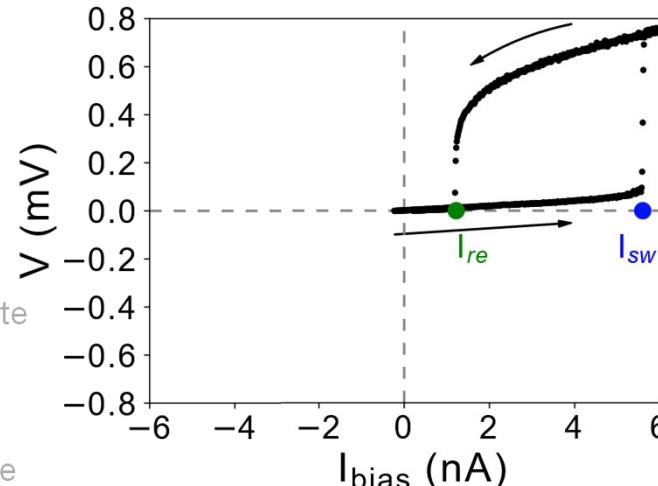
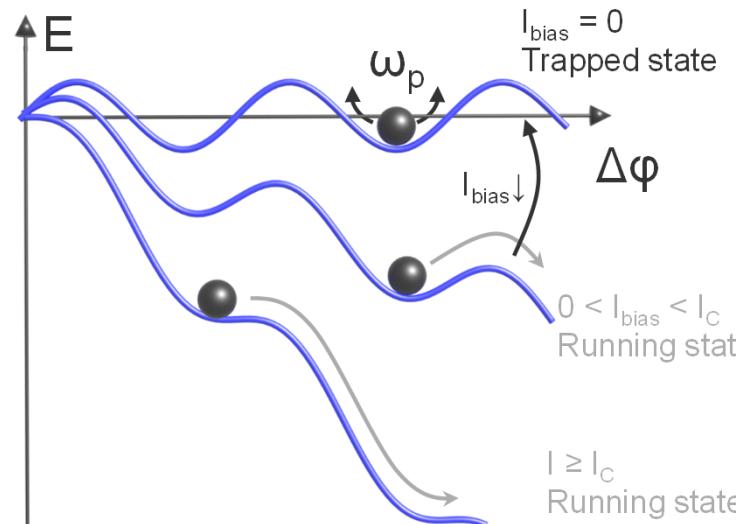
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Berlin

- Resistively and capacitively shunted Josephson junction (RCSJ)

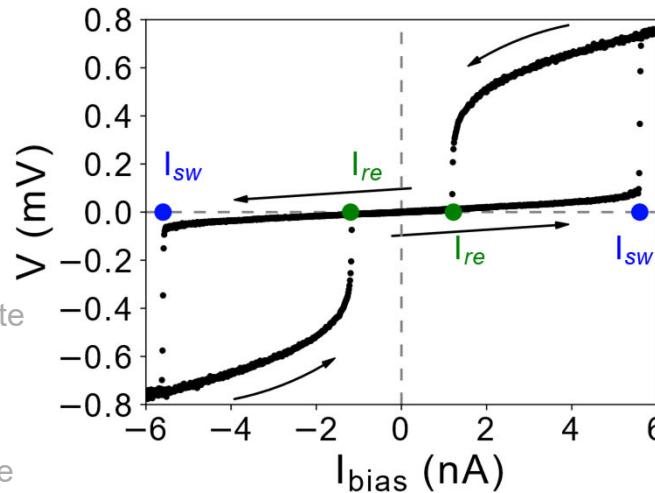
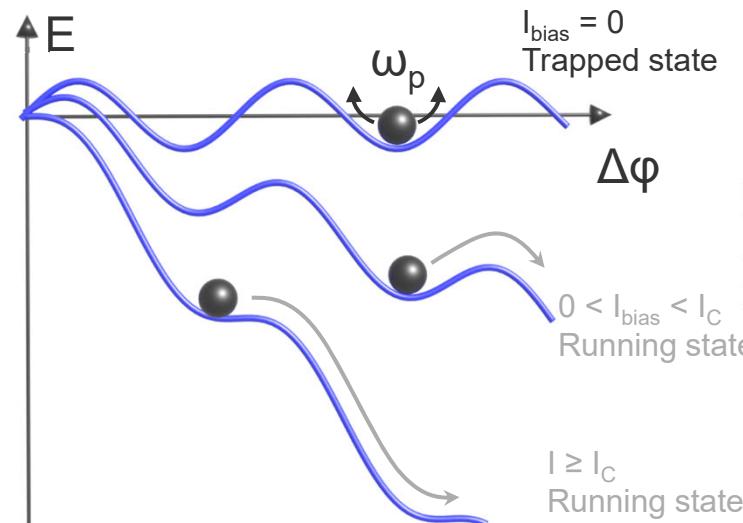
$$I(t) = I_{c0} \sin(\phi(t)) + \frac{\hbar}{2e} \frac{1}{R_N} \dot{\phi} + \frac{\hbar}{2e} C \ddot{\phi} + \delta I$$



# RCSJ model

- Resistively and capacitively shunted Josephson junction (RCSJ)

$$I(t) = I_{c0} \sin(\phi(t)) + \frac{\hbar}{2e} \frac{1}{R_N} \dot{\phi} + \frac{\hbar}{2e} C \ddot{\phi} + \delta I$$

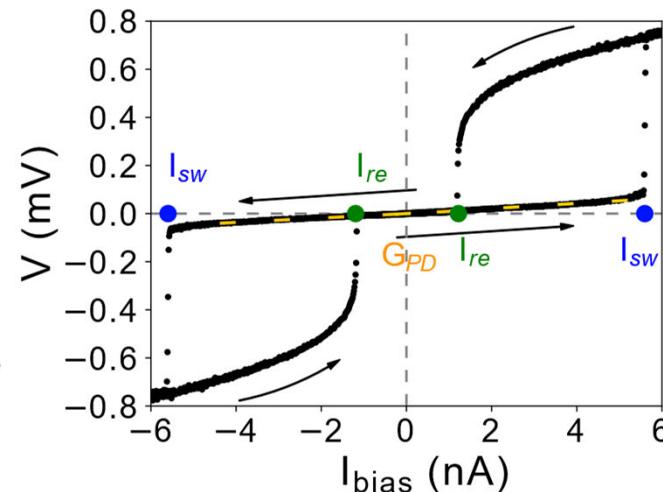
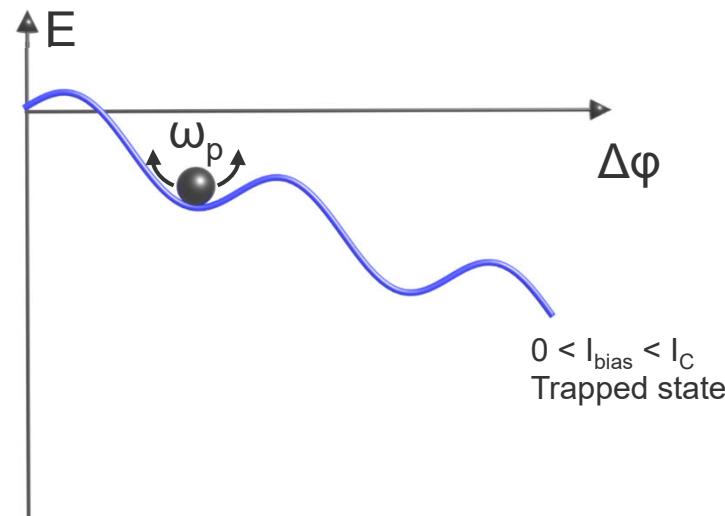


# RCSJ model



- Resistively and capacitively shunted Josephson junction (RCSJ)

$$I(t) = I_{c0} \sin(\phi(t)) + \frac{\hbar}{2e} \frac{1}{R_N} \dot{\phi} + \frac{\hbar}{2e} C \ddot{\phi} + \delta I$$

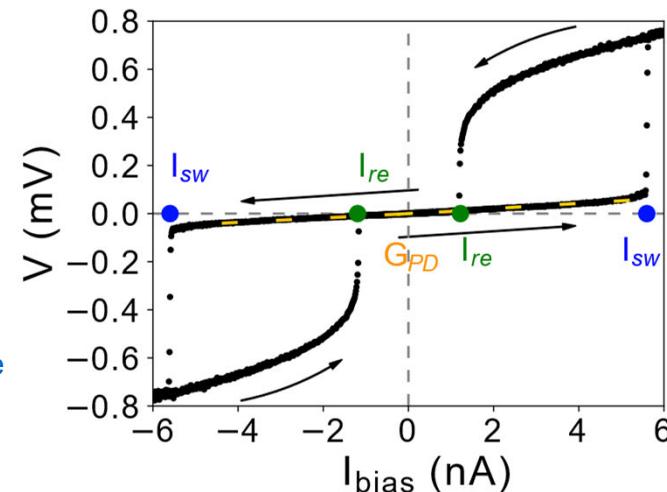
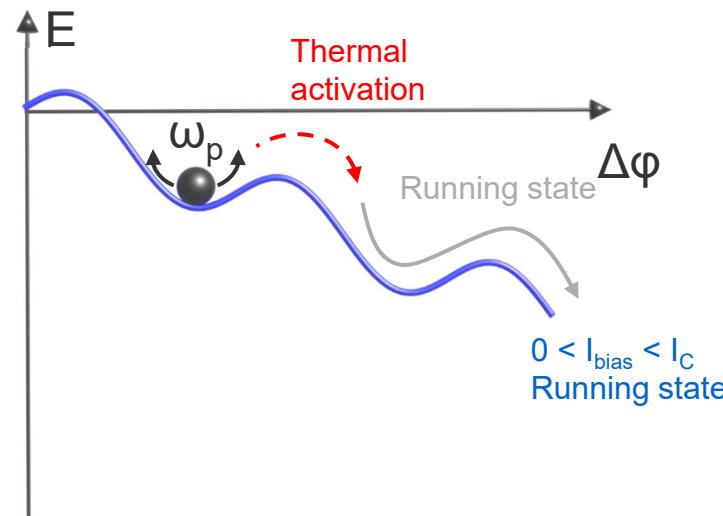


# RCSJ model: phase diffusion



- Resistively and capacitively shunted Josephson junction (RCSJ)

$$I(t) = I_{c0} \sin(\phi(t)) + \frac{\hbar}{2e} \frac{1}{R_N} \dot{\phi} + \frac{\hbar}{2e} C \ddot{\phi} + \delta I$$

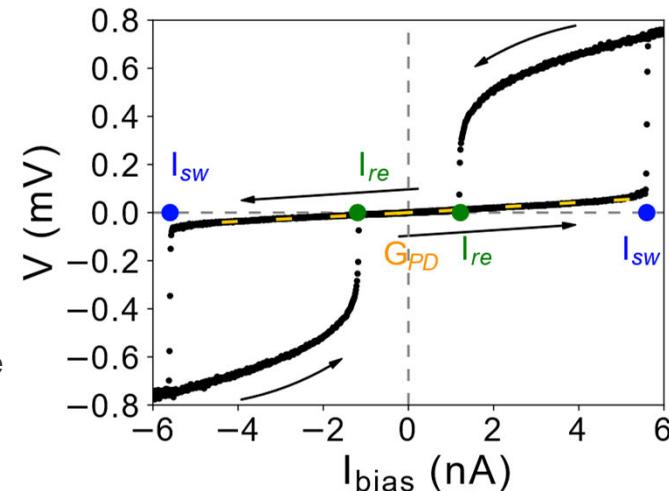
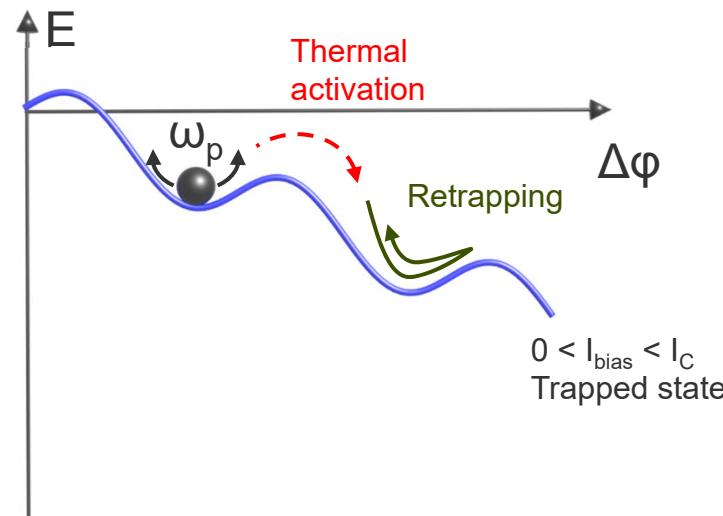


# RCSJ model: phase diffusion



- Resistively and capacitively shunted Josephson junction (RCSJ)

$$I(t) = I_{c0} \sin(\phi(t)) + \frac{\hbar}{2e} \frac{1}{R_N} \dot{\phi} + \frac{\hbar}{2e} C \ddot{\phi} + \delta I$$



# RCSJ model with frequency-dependent damping

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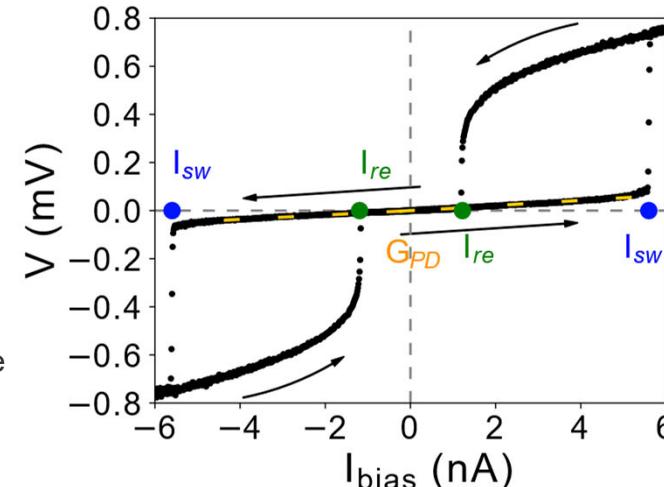
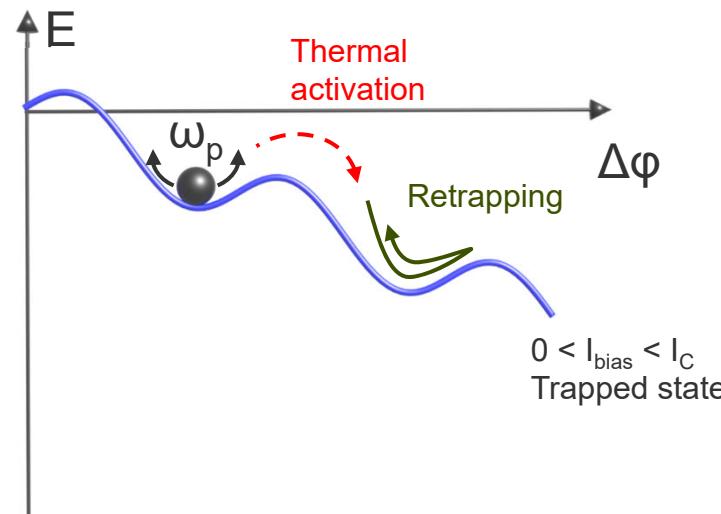
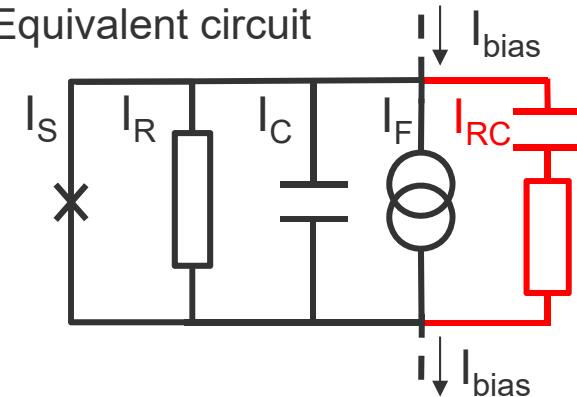


Berlin

- Resistively and capacitively shunted Josephson junction (RCSJ)

$$I(t) = I_{c0} \sin(\phi(t)) + \frac{\hbar}{2e} \frac{1}{R_N} \dot{\phi} + \frac{\hbar}{2e} C \ddot{\phi} + \delta I$$

Equivalent circuit



- Frequency-dependent damping (Kautz, Martinis, Phys. Rev B 42, 16 (1990))

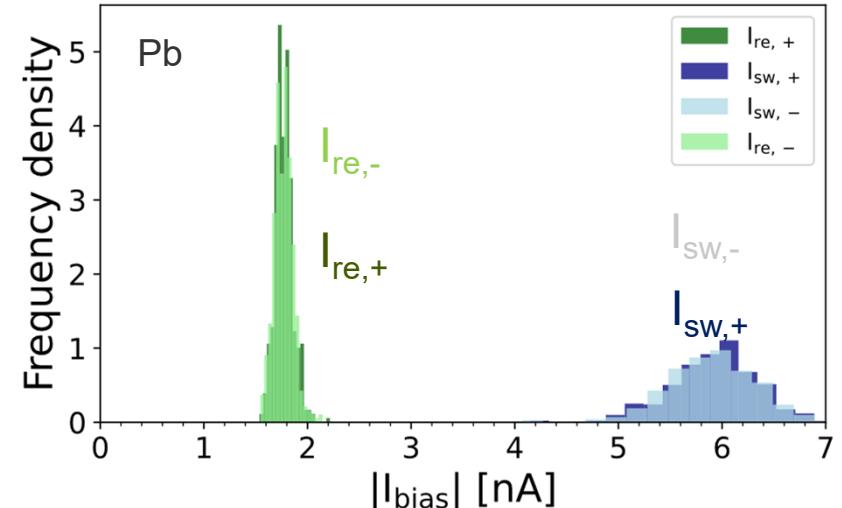
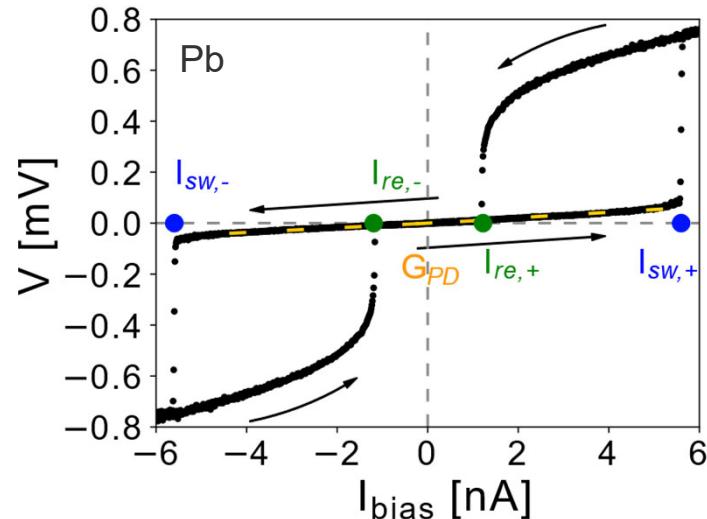
# RCSJ model of Pb-Pb Josephson junction

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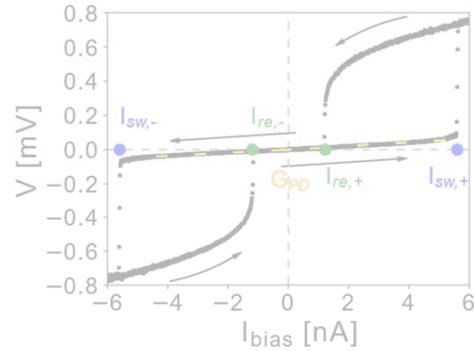
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- Resistively and capacitively shunted Josephson junction (RCSJ)

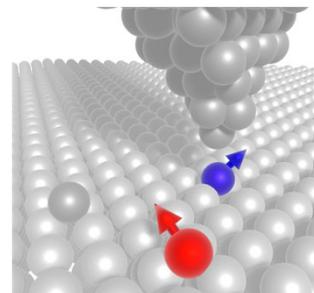


- Statistical distribution of switching and retrapping currents due to fluctuations

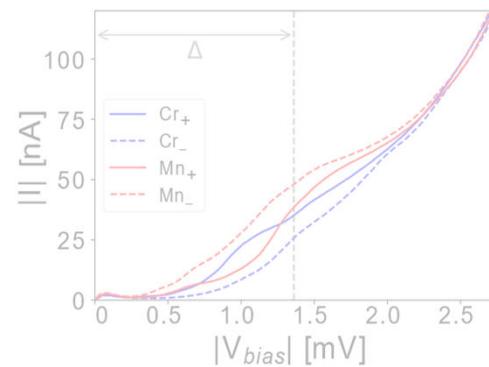
# Outline



- ▶ Current-biased spectroscopy in a STM
- ▶ Switching and retrapping currents



- ▶ Magnetic adatoms on superconductors
- ▶ Asymmetry of retrapping currents
- ▶ Diode behavior



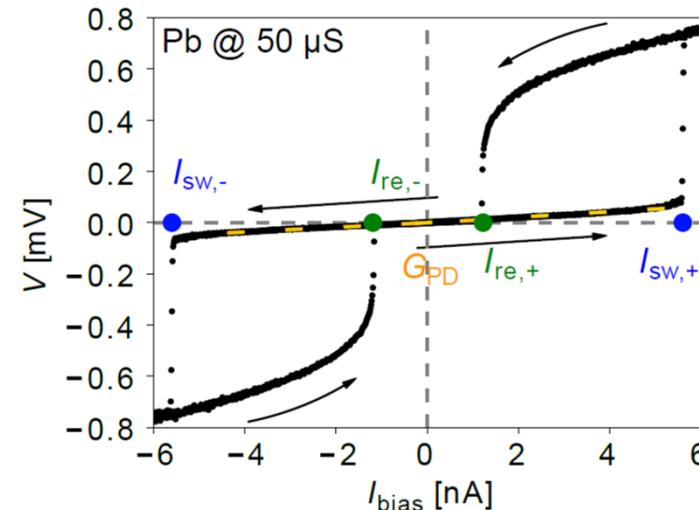
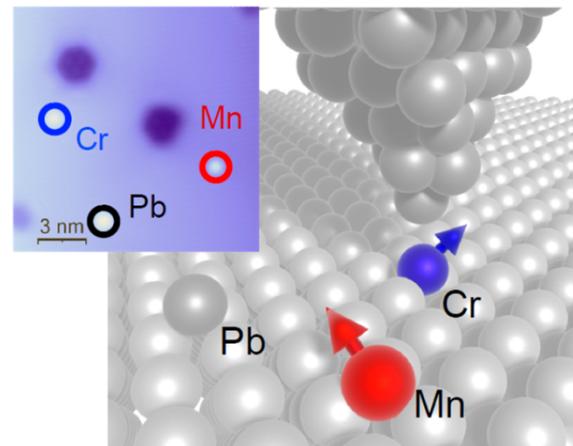
- ▶ Origin of diode behavior

# Josephson junctions with magnetic adatoms

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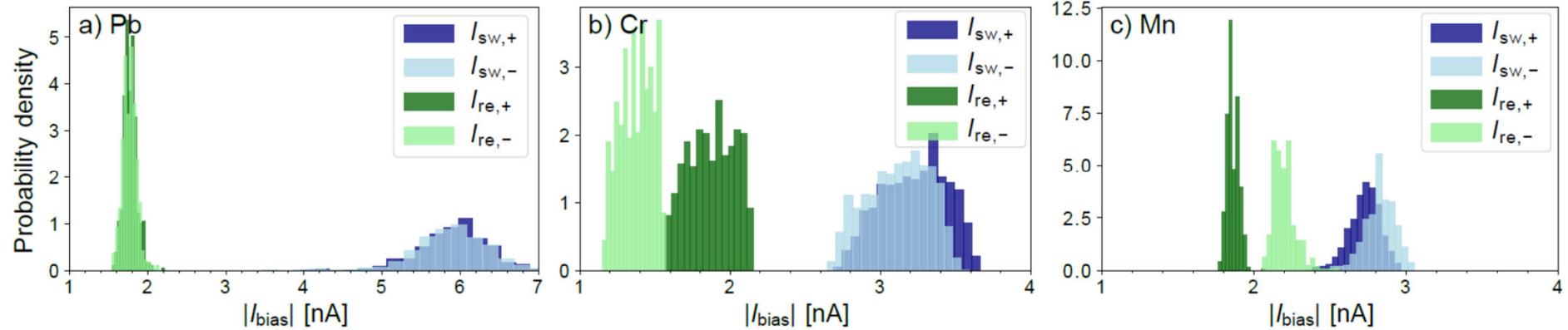
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- ▶ Asymmetric retrapping currents
- ▶ Asymmetry in opposite direction for Mn and Cr atoms

# Statistics of asymmetry

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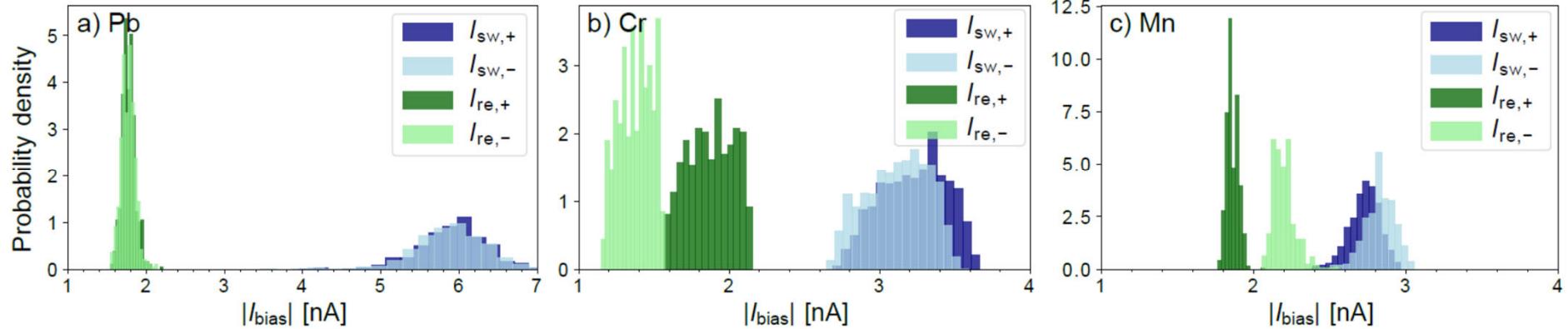


- ▶ Symmetric Pb-Pb junctions
- ▶ Strong asymmetry in retrapping currents
- ▶ Small asymmetry in switching currents

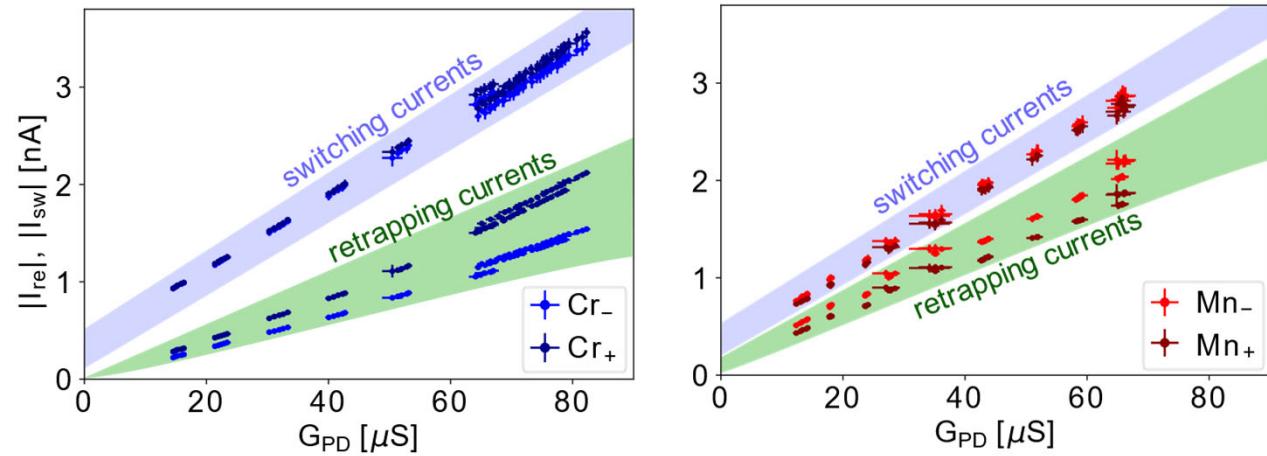
Diode behavior

# Statistics of asymmetry

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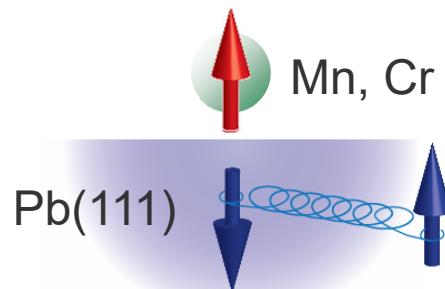
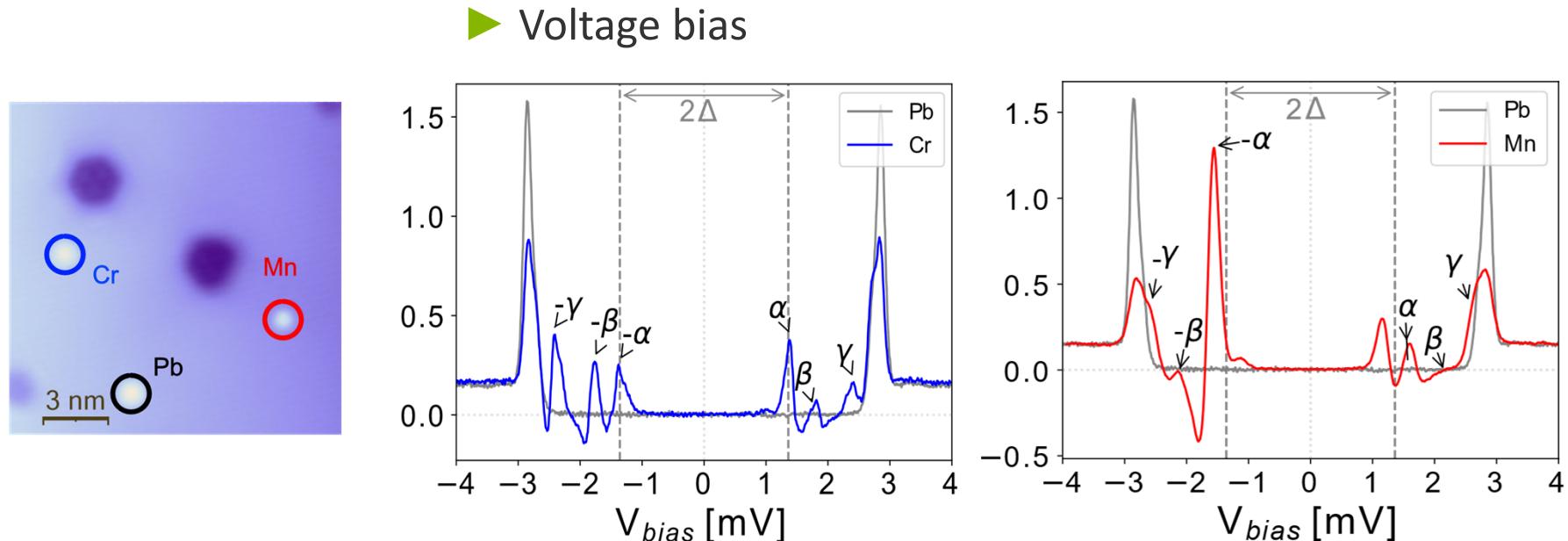


- ▶ Increase in asymmetry with increasing junction conductance
- ▶ Diode behavior



# YSR states of Mn and Cr on Pb(111)

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► Different electron and hole weight due to potential scattering

Influence of broken electron-hole symmetry on Josephson junction?

L. Yu, Acta Phys. Sin. 21, 75 (1965)  
H. Shiba, Prog. Theor. Phys. 40, 435 (1968)  
A.I. Rusinov, JETP Lett. 9, 85 (1969)

# YSR states of Mn and Cr on Pb(111)

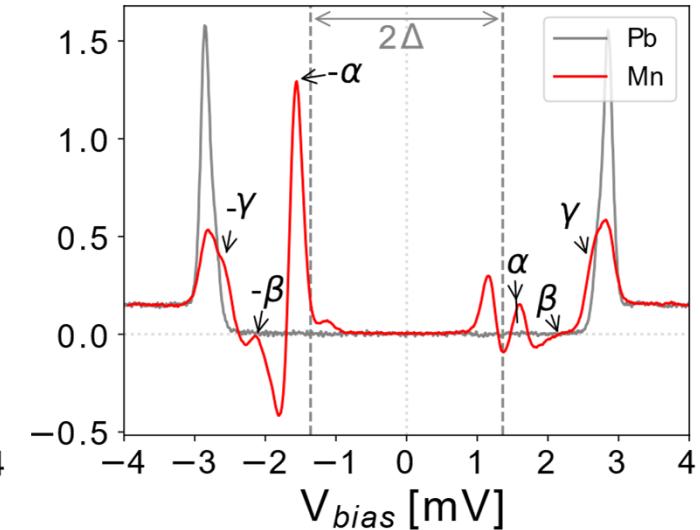
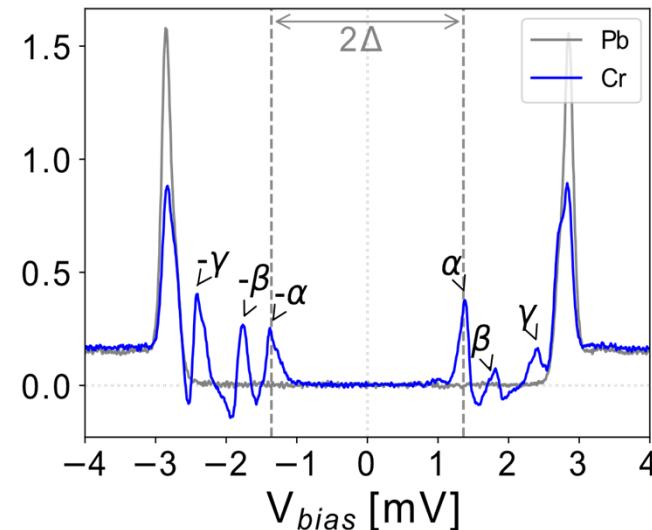
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Far distance

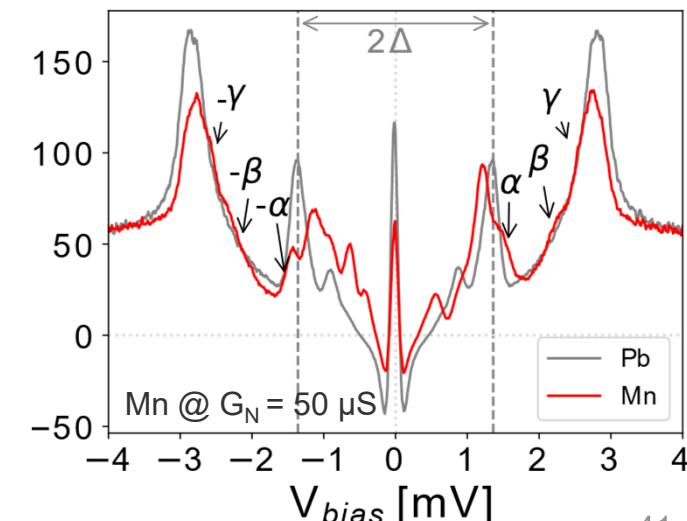
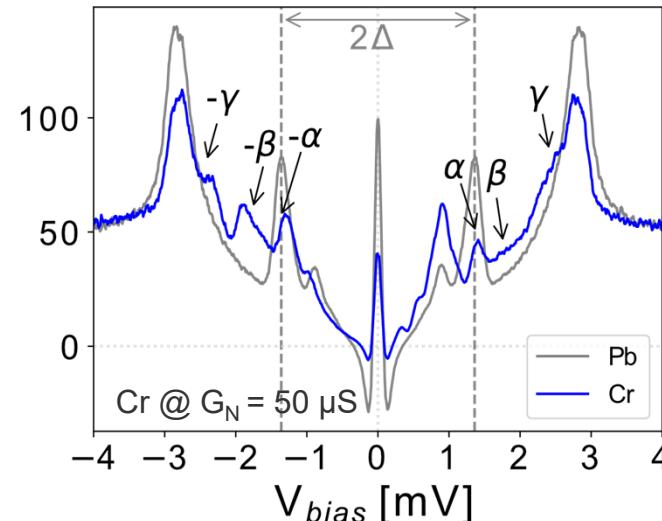
► Particle-hole  
asymmetric YSR  
states

► Voltage bias



Josephson regime

► Particle-hole  
asymmetric dI/dV



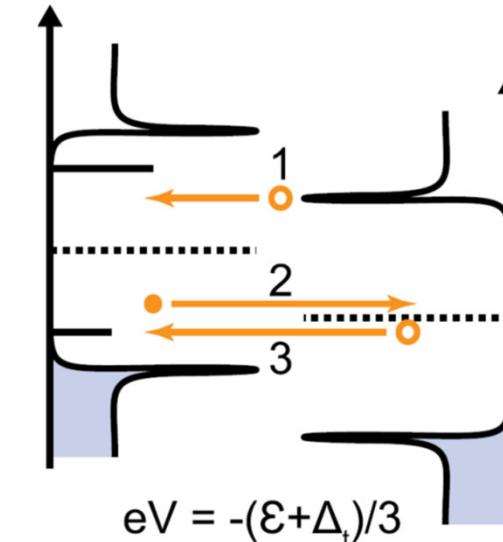
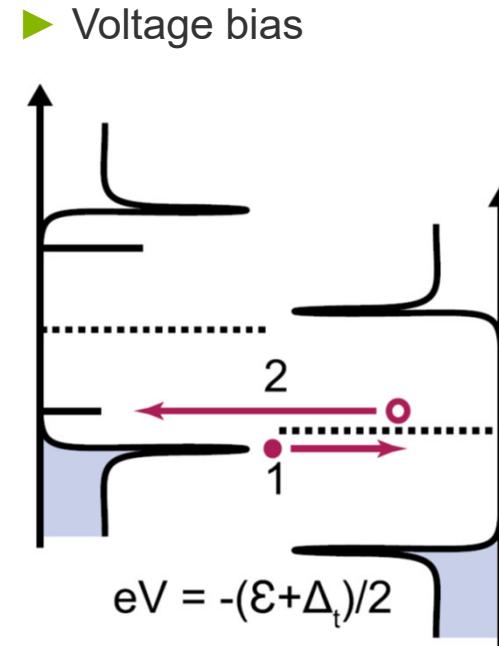
# Excitation of YSR states by Andreev reflections

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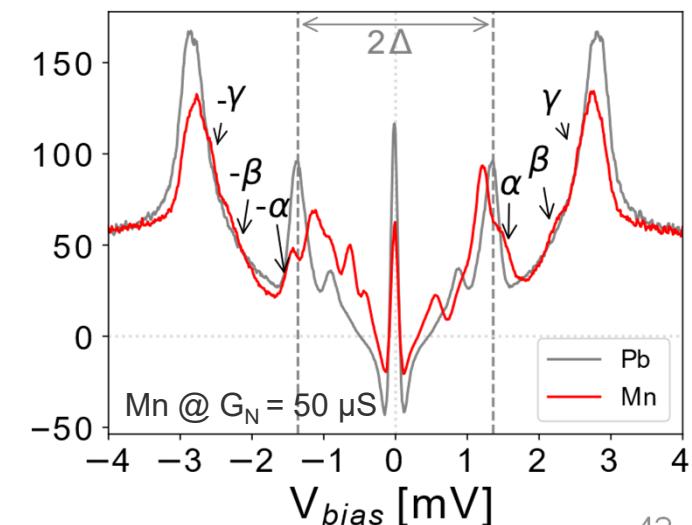
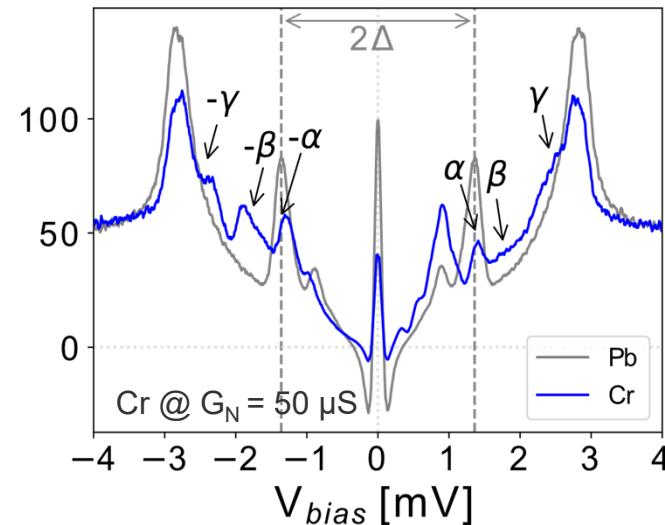


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- ▶ Voltage bias
- Josephson regime
- ▶ Particle-hole asymmetric YSR excitations by Andreev reflections



- ▶ Voltage bias
- Josephson regime
- ▶ Particle-hole asymmetric  $dI/dV$



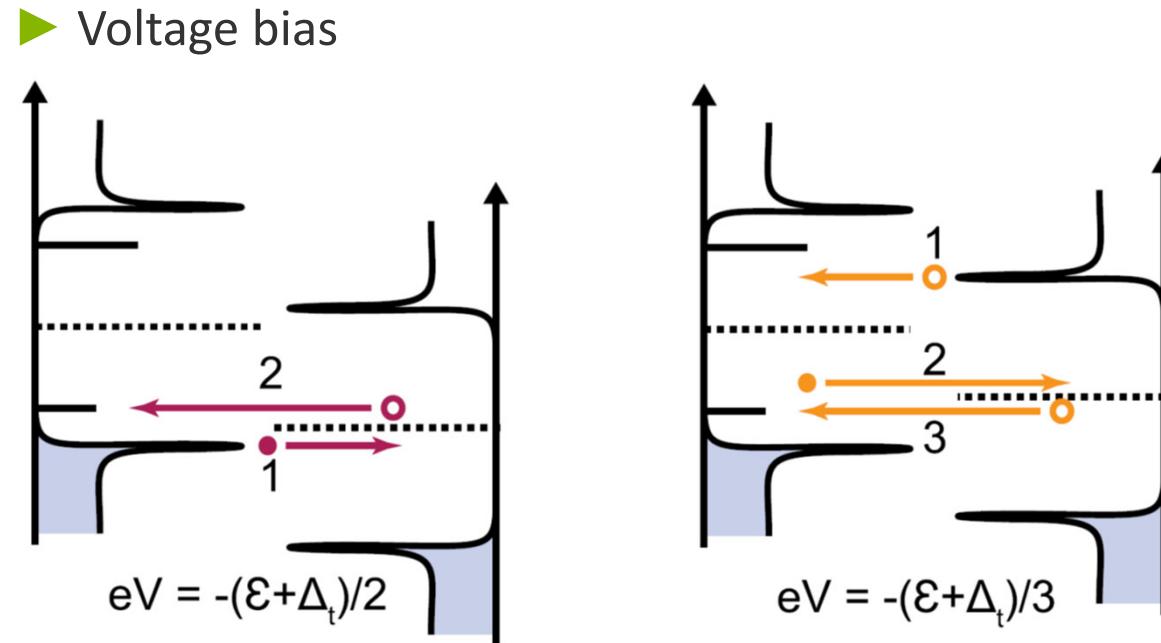
# Excitation of YSR states by Andreev reflections

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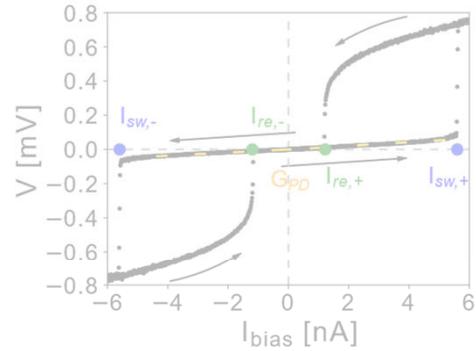
Berlin

- Josephson regime
- Particle-hole asymmetric YSR excitations by Andreev reflections

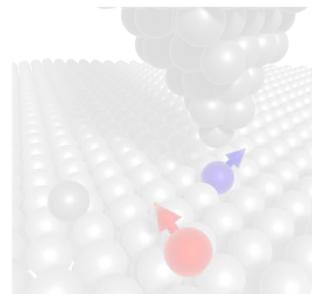


Influence of dissipative excitation of YSR states on diode effect?

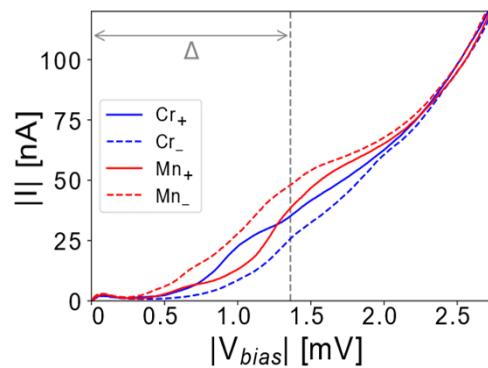
# Outline



- ▶ Current-biased spectroscopy in a STM
- ▶ Switching and retrapping currents



- ▶ Magnetic adatoms on superconductors
- ▶ Asymmetry of retrapping currents
- ▶ Diode behavior



- ▶ Origin of diode behavior

# Particle-hole asymmetry of YSR states of Mn and Cr

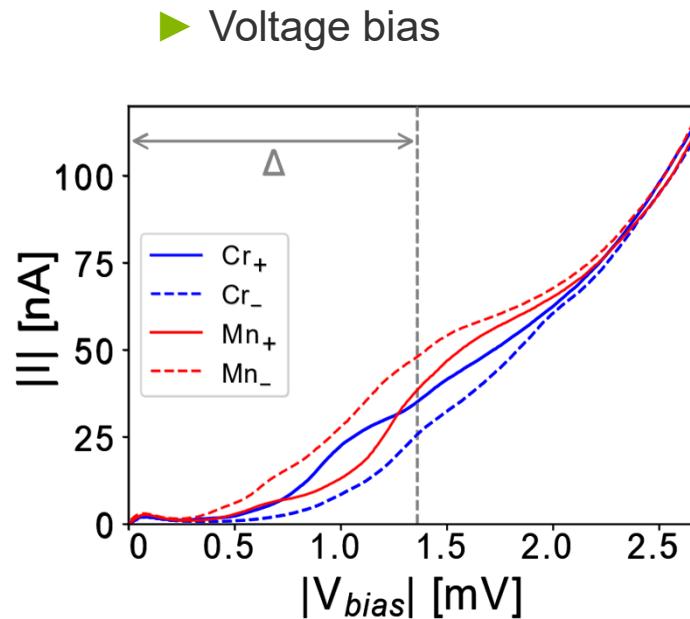
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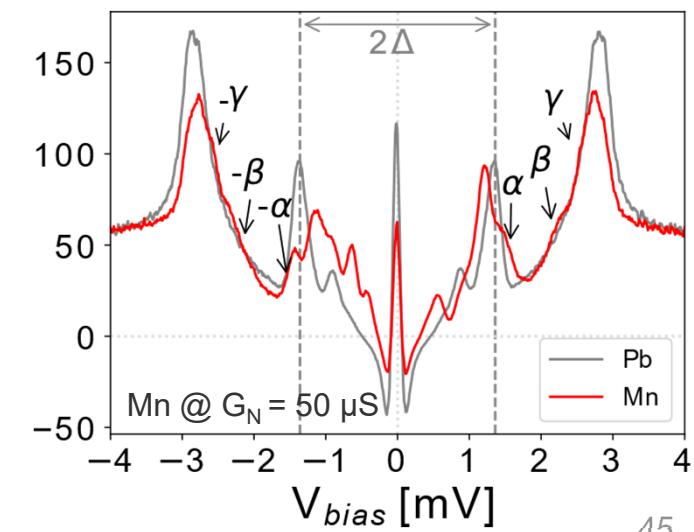
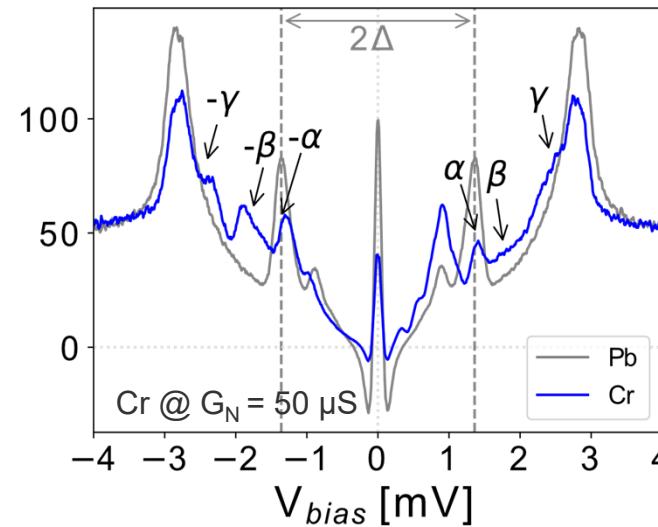
Josephson regime

- ▶ Particle-hole asymmetric YSR excitations by Andreev reflections



Josephson regime

- ▶ Particle-hole asymmetric dI/dV

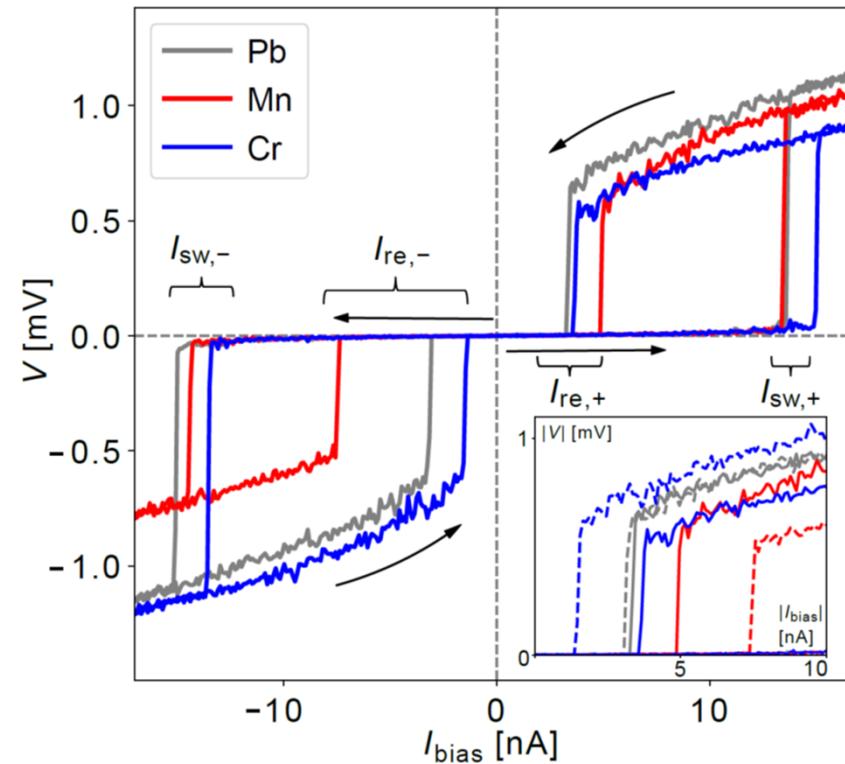


# Simulations of V-I curves

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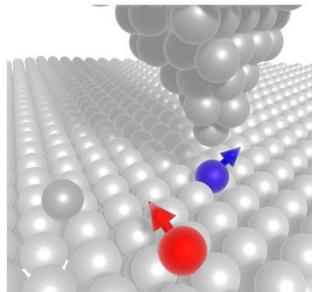


- ▶ Numerical simulations
- ▶ frequency-dependent damping
- ▶ Johnson-Nyquist noise
- ▶ Extended RCSJ model with asymmetric quasi-particle current (in contrast to ohmic resistance)

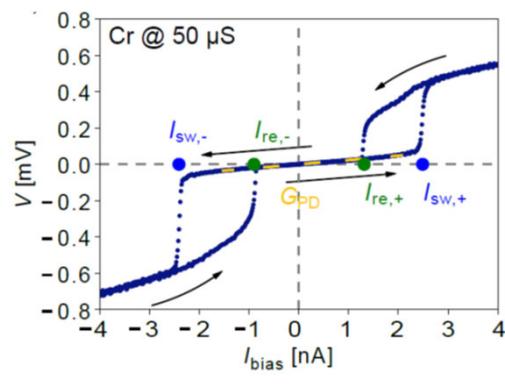


- ▶ Asymmetric retrapping currents reproduced
- ▶ Diode behavior due to broken e-h symmetry by YSR states

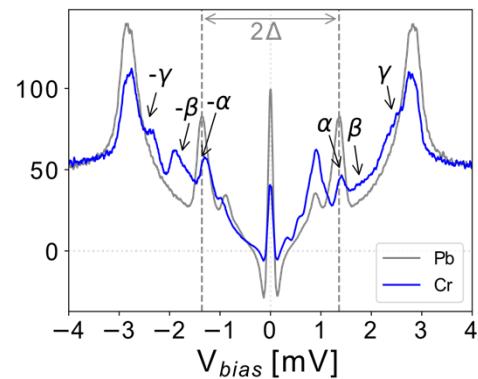
# Conclusions



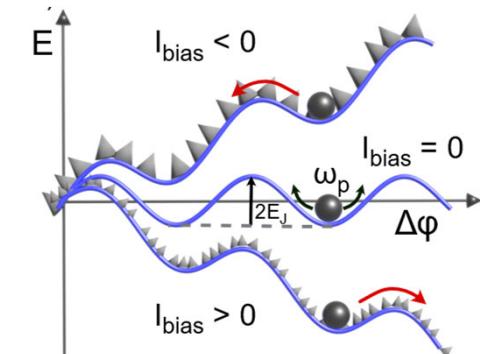
- ▶ Atomic-scale Josephson junctions



- ▶ Diode behavior when magnetic atoms are included
- ▶ Diode behavior in retrapping current (in contrast to earlier experiments)



- ▶ Diode behavior due to asymmetric damping
- ▶ Extended RCSJ model captures diode behavior



M. Trahms, L. Melischek, J.F. Steiner, B. Mahendru, I. Tamir, N. Bogdanoff, O. Peters, G. Reecht, C.B. Winkelmann, F. von Oppen, K.J. Franke, *Nature* 615, 628 (2023)

Related theory: J.F. Steiner, L. Melischek, M. Trahms, K.J. Franke, F. von Oppen, *Phys. Rev. Lett.* 130, 177002 (2023)

# Thank you!

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Katharina Biel, Verena Caspari, Florian Faaber, Melvin Grumser, Margarete Huiszinga, Christian Lotze, **Bharti Mahendru**, Vibhuti Rai, Lisa Rütten, Junyoung Sim, Rika Simon, **Martina Trahms**, Werner van Weerdenburg, Paul Wiechers



**Larissa Melischek, Jacob Steiner, Felix von Oppen**